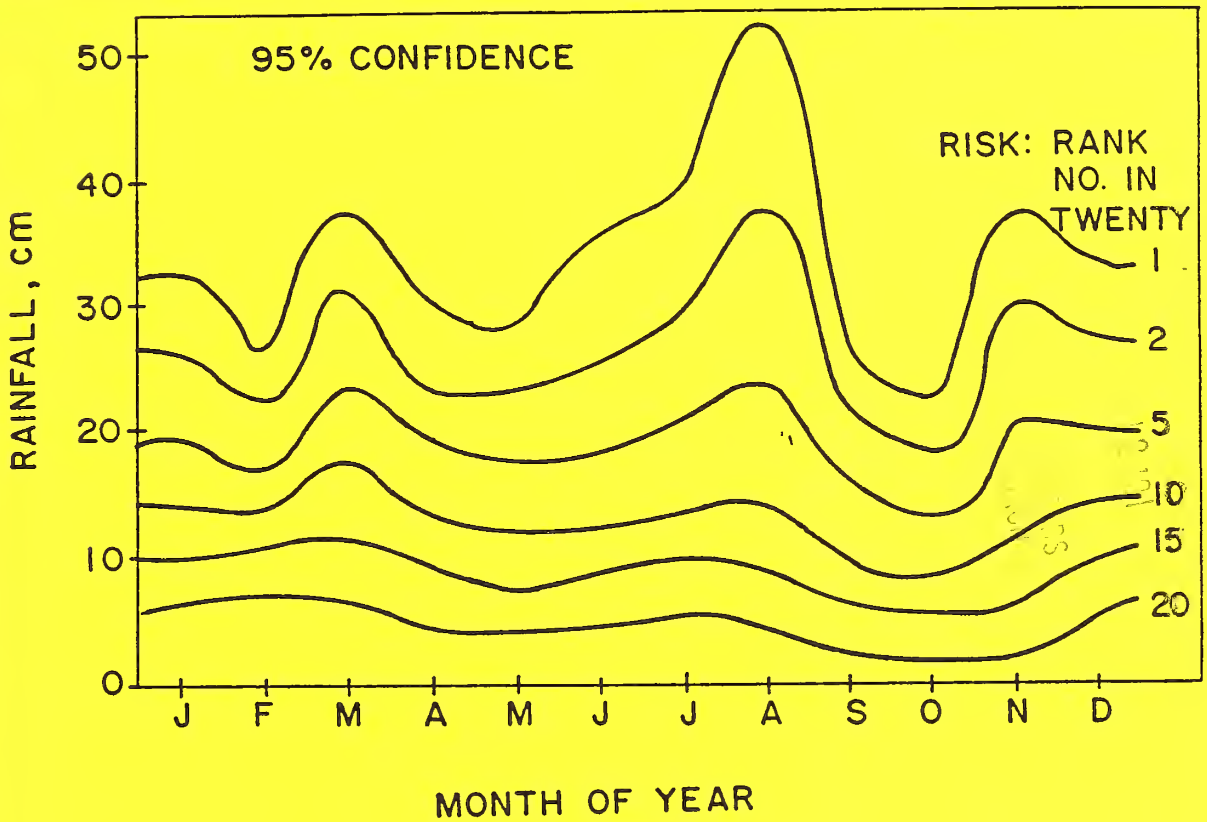
COMPUTER PROGRAMS FOR ANALYSIS AND SIMULATION OF SEASONALLY CONTINUOUS PROBABILITY DISTRIBUTIONS



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RESEARCH REPORT
No. IRC 070187
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Research Report[1/]
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Computer Programs for Analysis and Simulation of Seasonally Continuous Probability Distributions

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[1/] This report describes computer programs and their operational details that calculate continuous seasonal cycles of probabilities for values of climatic variables and then synthesize distributions that will describe probabilistic levels of risk.

PREFACE

This report is one of a series of "Research Reports" published by the Integrated Row Crop Management Systems Research Unit at the Southern Piedmont Conservation Research Center, Watkinsville, GA. The purpose of these reports is to provide a mechanism for technology transfer to potential users of information developed by the scientists in this Research Unit. Below is a list of the reports developed in this series. Any report may be obtained by requesting it at the Watkinsville address (phone 404/769-5631 or FTS 250-2425).

Research Report No.	Authors and Title
IRC 060183	White, A. W., Jr., R. R. Bruce, A. W. Thomas, G. W. Langdale, and H. F. Perkins. The effects of soil erosion on soybean production in the Southern Piedmont of Georgia in 1982.
IRC 093083	Welch, R., T. R. Jordan, A. W. Thomas, and J. W. Ellis. Photogrammetric techniques for monitoring soil erosion.
IRC 060184	White, A. W., Jr., R. R. Bruce, A. W. Thomas, G. W. Langdale, and H. F. Perkins. Effect of soil erosion on soybean yields and characteristics of Cecil-Pacolet soils.
IRC 070184	Thomas, A. W. and W. M. Snyder. Computer programs for analysis and simulation of probability distribution using sliding polynomials.
IRC 010186	Thomas, A. W., W. M. Snyder, and A. L. Dillard. A computer program for transforming stochastic data and evaluating probability distributions.
IRC 060686	Harper, L. A. and W. M. Snyder. A laboratory guide to smoothing and use of stochastic integrals for time-distribution data.
IRC 070686	Harper, L. A. and W. M. Snyder. A laboratory guide to sliding polynomial smoothing and testing for significant difference of functions in non-standard data sets.
IRC 070187	Thomas, A. W., W. M. Snyder, and A. L. Dillard. Computer programs for analysis and simulation of seasonally continuous probability distributions

INTRODUCTION

We in agriculture readily recognize how dependent we are on seasonal patterns of rainfall, temperature, streamflow, and other climatological variables. Because of this dependency, we are motivated to develop methodology useful in the study and treatment of climatological variates that impact agriculture. We wish to isolate and define quantitatively the seasonal variation of natural risk and uncertainty in order to provide improved inputs into the planning and implementation of seasonally dependent activities.

We have developed a series of papers under the general theme of "Stochastic Impacts on Farming" that treat the determination of risk and uncertainty in relation to resource management and agricultural production. In the first paper, Thomas and Snyder (1986a) presented a data transformation method that was developed and tested against selected climatological records. Then, Thomas et al. (1986) published a companion research report that describes a computer program which transforms stochastic data. In the second paper, Snyder and Thomas (1986) presented a method for computing seasonally continuous probability distributions. The third paper (Thomas and Snyder, 1986b) used simulation techniques to develop seasonal variation of risk. This research report gives the computer programs and their operational details used in the studies of paper two (analysis) and paper three (simulation).

In the analysis phase, we adapt two-dimensional sliding polynomials to form-free frequency distributions having mathematically continuous cyclic variability through the annual march of season. The season is defined by monthly samples of data consisting of one observation per

month for the period of record. Such data may be extremes of maxima or minima or may be monthly averages or totals. Bi-modal distributions or statistical outliers require no special or subjective treatment; however, this method is presently limited to samples containing only positive skew. The monthly data are standardized by a separate transformation for each of the twelve calendar months. The standardized data are smoothed by fitting the two-dimensional sliding polynomial surface by least squares. Smoothing is constrained by imposition of boundary conditions for extreme magnitude and for maintenance of full seasonal cyclic continuity. Thus, we develop variate probabilities that are defined by the mathematical surface based on magnitude of event and month of the year.

During the analysis phase the sliding polynomial surface establishes the relationship between magnitudes of an event and the probability of occurrence of the event. In the simulation phase, the numerical relationship is reversed. The surface is now used to calculate event magnitudes from computer-generated and linearly distributed probabilities. The process simply reduces to reverse interpolation. In the analysis, the derivation of form-free seasonally continuous probability distributions establishes the stochastic characteristics of the climatological variate that includes seasonal cycles. Simulation converts these characteristics into utilitarian expressions of risk and uncertainty.

ANALYSIS

The method of analysis consists of two parts. First, transformation of the variate is necessary to bring into alignment the samples from different months of the year, factoring out the different

means and standard deviations to produce nearly homogeneous data that can be smoothed by a mathematical surface. Resulting from this first step are 12 graphs of sample probabilities versus an abstract variate v. The reader is referred to Thomas and Snyder (1986a) and Thomas et al. (1986) for a full discussion of the variate transform and a computer program that performs the transformation.

The second part of the methodology is to smooth simultaneously the 12 monthly sample probabilities and the month-to-month transition in order to produce continuous probability functions that follow the natural continuity of seasonal transitions. A surface of smoothing is required which may be visualized as a set of contours on a cylinder of the seasons. The smoothing space is defined by the magnitude of events in v-scale versus the time by months. The height of the surface across this space is probability. Two-dimensional sliding polynomials produce the continuous but piece-wise form-free surface. Boundaries of the space in v-scale are given by Snyder and Thomas (1986), while in time scale they have a particular requirement. Seasonal continuity requires that the smoothing surface pass through December at the end of the year and join with January at the beginning of the year with mathematical continuity. We used the cyclically continuous form of sliding polynomials as presented by Snyder (1980).

The smoothing surface in the time vs v space is found by least squares determination of the values of a grid of nodes. Our nodal arrangement is shown as Fig. 1. We place lines of nodes at January, April, June, October, and again at January. By imposing boundary conditions, the initial 5x7 grid is reduced to a 4x5 grid. First, all nodes at v=0 are set to zero, and the nodes at v=-1 are set equal to

the nodes at v=1 (Thomas and Snyder, 1986a). Five nodes in v-scale remain. The right boundary at v=4 is a free boundary. Next, the nodes of the upper January are set equal to the nodes of the lower January to produce the cyclindrical surface. This leaves four nodes in the seasonal time scale. This boundary-constrained 4x5 grid of nodes is then evaluated by two-dimensional least-squares smoothing of the 12 monthly samples.

SIMULATION

The method of simulation consists of two numerical steps: the computer-generation of linearly distributed pseudo-random numbers and a transformation of these numbers to other, different distributions. The pseudo-random numbers are generated in a scale from zero to 100 and each may be considered a probability. Each value of probability has an equal chance of occurrence. In the second step, the numerical relationship as developed in the stochastic analysis is reversed as described earlier. The computed mathematical surface is now used to calculate event magnitudes from the computer-generated and linearly distributed probabilities. Through the simulation process, we synthesize distributions that will describe probablistic levels of risk, and that will detail the variation of this risk in a continuous fashion by months throughout the year.

The complete process of seasonal simulation of risk requires consideration of four elements. The first two, probability-event magnitude relation and the seasonal variation of this relationship, are quantified by the least-square determination of the sliding polynomial smoothing surface. These first two elements are thus internal to the stochastic analysis. The other two elements are external and require

subjective operational decision. One of these elements is the length of the synthetic samples that are to be generated. This number could be described as the length of the planning period. Thus, simulation establishes the levels of risk of occurrence of events during the planning period. The last element is the number of synthetic samples to be generated. This number sets a level of precision, or a level of confidence, that can be placed on the statements of risk.

DISCUSSION

A rainfall sample was selected to demonstrate the results of the analysis and simulation. This sample was a 99-year record (1885-1983) of monthly total rainfall measured at Athens, GA. The monthly values of mean, standard deviation, skew, range, and transformation parameters for this sample are given by Snyder and Thomas (1986).

The smoothing of this monthly historical data set produces a surface of probability. As shown in Fig. 2, this surface can be displayed by contours of equal probability drawn in the season-event magnitude space of the problem. The figure displays the strong natural seasonality of rainfall that is typical of the region. These contours are calculated on the full set of 24-nodes defining the two-dimensional surface as in Fig. 1 of Snyder et al. (1984). This includes the four boundary nodes set to zero at v=0. The values of these nodes in probability scale and their standard deviations are given in Table 3 of Snyder and Thomas (1986). The standard deviations are expressions of localized variance (Snyder et al., 1984) and are alternatives to the cumbersome errors of prediction of conventional regression analysis. They provide measures of the instability of the surface at these nodes and are one measure of the goodness of fit. Since all deviations are

less than 1.5%, we conclude that the contours of probability are closely defined.

To demonstrate the simulation process, we chose the length of the planning period to be 20 years. We generated 100 samples for this planning period. The results of simulation are presented in Fig. 3 in two parts. Part "a" shows simulated values of rainfall which is number 5 in ranking in the 100 samples simulated. Part "b" shows values which is number 25 in ranking in the 100 samples. These figure parts are thus simulated approximations to confidence levels of 95% and 75% since risk values would be exceeded on the average in only 5% or 25%, respectively, of occurrences at selected risk levels.

Fig. 3 shows event magnitude versus month for coordinate scale. The items plotted to this scale are selected rank values within the planning period and therefore express risk. For example, we see that in January, there is a 1-in-20 risk that monthly rainfall will equal or exceed about 33 cm. This statement is made with 95% confidence. These contours of equal risk show patterns closely resembling the seasonal probabilities of Fig. 2. However, this does not mean that we should perform the stochastic analysis and omit the simulation. The simulation is necessary to quantify risk and uncertainty.

In this simulation, two elements of variation are incorporated. First, the values of the nodes defining the mathematical surface are varied in conformance with their standard deviations. After varying the nodes, random numbers are generated and converted to event magnitude using the mathematical surface through the pulsed nodes. Therefore, both natural stochasticity and derived system variance determine the variances of the simulations.

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5. Thomas, A. W. and W. M. Snyder. 1986b. Stochastic Impacts on Farming: III. Simulation of Seasonal Variation of Climatic Risk. Trans. ASAE. 29(4):1026-1031.
6. Thomas, A. W., W. M. Snyder, and A. L. Dillard. 1986. A Computer Program for Transforming Stochastic Data and Evaluating Probability Distributions. Technical Research Report No. IRC 010186. Watkinsville, GA.

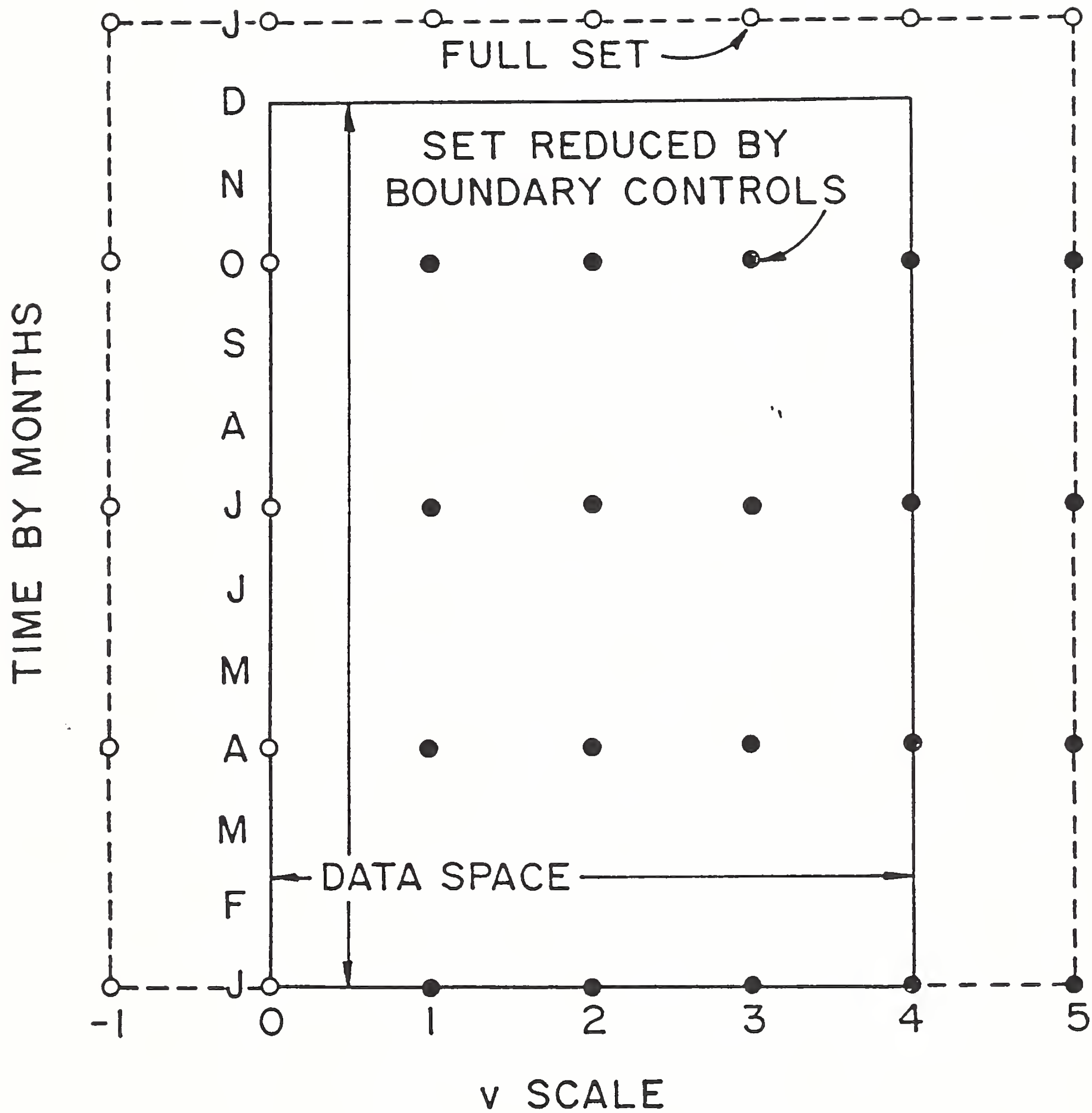

Fig. 1. Arrangement of nodes for smoothing probabilities in two dimensions.

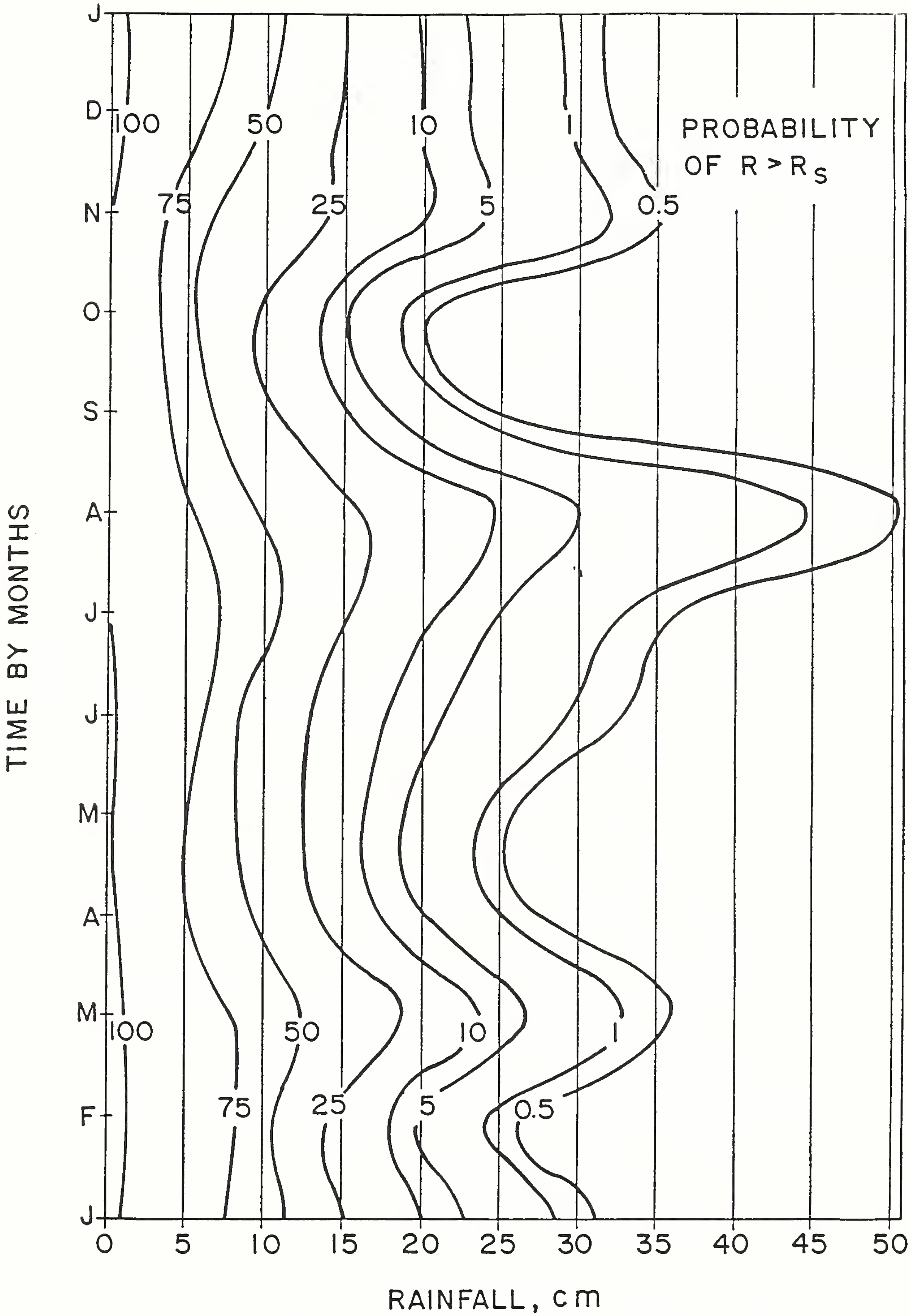

Fig. 2. Seasonal contours of probability for Athens monthly total rainfall.

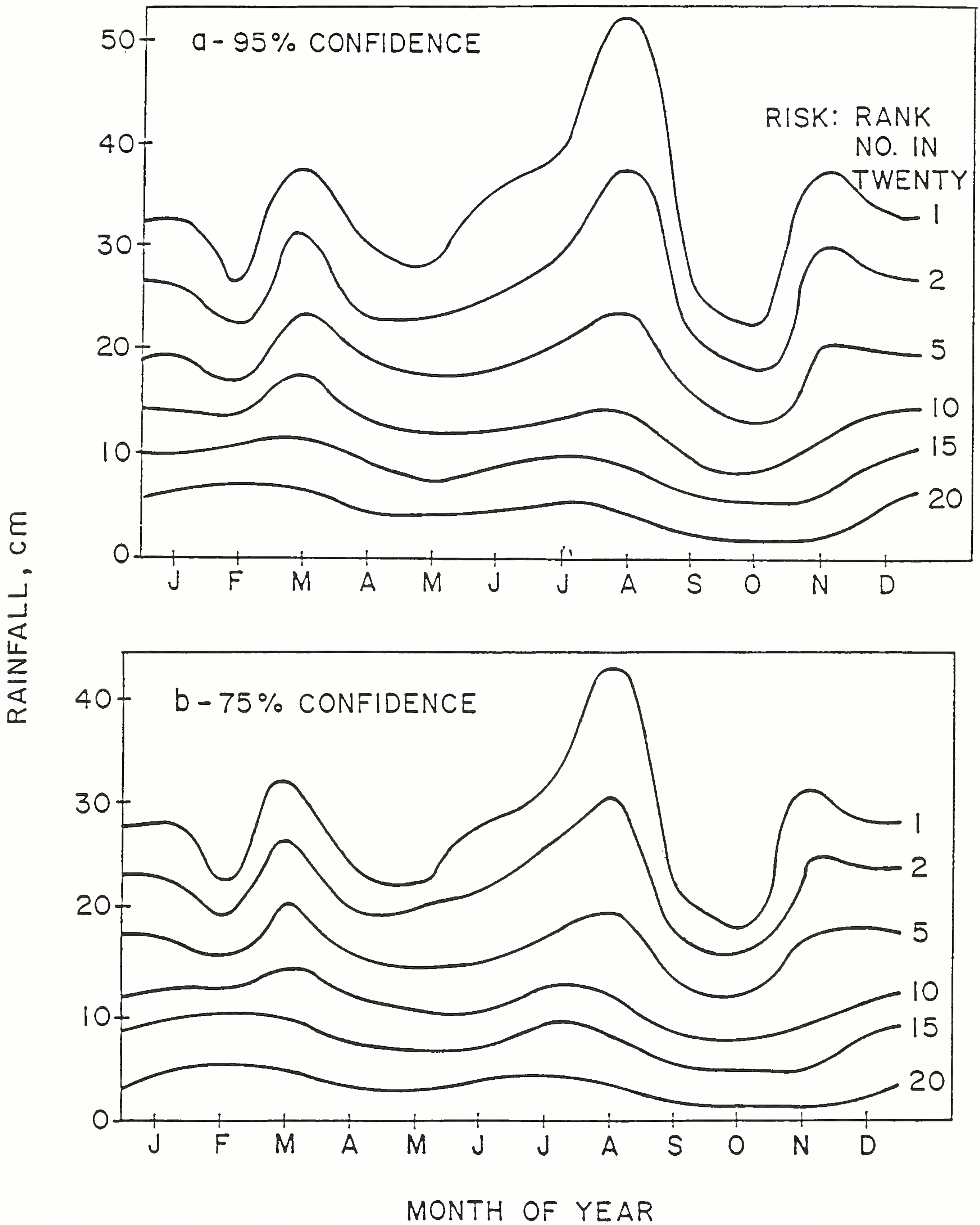

Fig. 3. Seasonal contours of risk for Athens monthly total rainfall.

APPENDIX

The Appendix includes a description of input and output variables, program listing, and sample output in the determination of risk and uncertainty (including analysis and simulation) of a 99-year monthly-total rainfall record.

The programs are presented for the convenience of potential users. While the programs have been run and tested on various data sets, the originators of the programs assume no responsibility for their accuracy or adequacy. Such responsibility must rest solely on the user. We stand ready to assist and advise within the limitations imposed by our operating resources. The analysis program is listed in Hewlett-Packard BASIC with Ext. 2.1. The simulation program is listed in FORTRAN V. (Trade name is included for the benefit of the reader and does not imply an endorsement or preferential treatment of the named product.)

ANALYSIS PROGRAM

This seasonally continuous probability program fits a two-dimensional sliding polynomial surface to a sample made up of one event per month. Record for each month is converted to a sample accumulated probability curve across 40 classes. Height of the surface is probability. Data are input from disk by calendar month and classified in v-scale by month.

The program 'WRITEWTINV' provides data files needed by the seasonally continuous probability program. The data files contain sliding polynomial weights and the inverse sums of square matrix. No inputs are required since it is programmed for the fixed field of nodes as shown in Fig. 1.

Analysis Program Input Variables

Variable name	Comment
Prtr	Printer output device control. (1=CRT, 706=Printer)
Dsname$	Sample input disk file name
Stnam$	Output disk file name
T$	Problem title
Ny	Number of years in record
Ck$	Program control variable.
H(I)	Monthly sample variate For I=1 to 12
Wt(*)	Nodal weights for least square smoothing For *=1 to 41
B(*,**)	Inverse sums-of-squares matrix for least squares For *=1 to 20:**=1 to 20

Analysis Program Output Variables

Variable name	Comment
T$	Problem title
Hb(I)	Monthly sample mean For I=1 to 12
Hsd(I)	Monthly sample std. deviation For I=1 to 12
Hsk(I)	Monthly sample coeff. of skew For I=1 to 12
Hp	Minimum value of intermediate variate, h', at minimum class limit
Cp	Common point of two exponential limbs (in v-scale)
Vr	Right asymptotic boundary for limb v2 (in v-scale)
F	Shape parameter
D	Shape parameter
Hmin(I)	Monthly minimum class limit For I=1 to 12
Cf(J,I)	Sample class probabilities For I=1 to 41:J=1 to 12
Lim(J,I)	Sample class limits For I=1 to 41:J=1 to 12
Yn(M)	Sliding polynomial solution nodes For M=1 to 20
Py(I)	Smoothed probability values For I=1 to 492
E(I)	Residual errors For I=1 to 492
Nvar(J)	Nodal variances For J=1 to 20
Lu3(J)	Nodal sum of weights For J=1 to 20
Ssy	Total adjusted sum of squares

Ses	Total residual error square
Rs	Coefficient of determination
Ab(*,**)	Smoothed probabilities For *=1 to 12:**=1 to 41

Analysis Program Listing

```
10    REM  ##################################################################
20    REM  #########                    "SEACONPRB"                #############
30    REM  ######### THIS PROGRAM FITS A 2-D SLIDING POLYNOMIAL    #############
40    REM  ######### SURFACE TO A SAMPLE MADE UP OF ONE EVENT      #############
50    REM  ######### PER MONTH. RECORD FOR EACH MONTH IS CONVERTED ###########
60    REM  ######### TO A SAMPLE ACCUMULATED PROBABILITY CURVE     #############
70    REM  ######### ACROSS 40 CLASSES. HEIGHT OF THE SURFACE IS #############
80    REM  ######### PROBABILITY. DATA ARE INPUT FROM DISK BY      #############
90    REM  ######### CALENDAR MONTH AND CLASSIFIED IN V-SCALE BY #############
100   REM  ######### MONTH.                                        #############
110   REM  ##################################################################
120   REM  ######### CHECK LEFT DISK DRIVE FOR NECESSARY FILES     #############
130   REM  ##################################################################
140   CAT ":INTERNAL,4,1"
150   LINPUT " ARE  'PROBWTSP' AND 'PRBINVMATP' ON LEFT DRIVE? Y OR N",Ck$
160   REM  ######### IF YES, CONTINUE WITH PROGRAM                 #############
170   IF Ck$="Y" THEN 200
180   REM  ######### IF NO, RUN PROGRAM TO GENERATE FILES          #############
190   LOAD "WRITEWTINV:INTERNAL,4,1",10
200   REM  ##################################################################
210   OPTION BASE 1
220   DIM H(100),Hb(12),Hsd(12),Hsk(12),Lim(12,41),Cf(12,41),T$[50],U(12)
230   DIM B(20,20),Wt(20),Py(492),E(492),Sy(20),Yn(20),Ab(12,41),Hmin(12)
240   DIM Lu1(20),Lu2(20),Lu3(20),Nvar(20),Hku(16),Dsname$[30],Stnam$[30]
250   DIM Ct(40),Ch(40)
260   INTEGER Ny,I,J,K,N,L,M,Knt
270   REAL Ny1
280   INPUT "TO PRINT TO CRT ENTER<1>, HARDCOPY ENTER<706>",Prtr
290   LINPUT "INPUT SAMPLE DATA SET NAME",Dsname$
300   LINPUT "STORAGE DATA SET NAME",Stnam$
310   CREATE BDAT Stnam$,50
320   INPUT "PROBLEM TITLE",T$
330   OUTPUT Prtr;T$
340   ASSIGN @Four TO Dsname$
350   INPUT "NUMBER OF YEARS IN RECORD",Ny
360   Ny1=Ny
370   LINPUT " CAN HMIN BE NEGATIVE VALUE? Y OR N",Ck$
380                    Ks=.2
390   OUTPUT Prtr;"                                        COEFFS     "
400   OUTPUT Prtr;"  MONTH    MEAN         STD DEV     SKEW       ";
410   OUTPUT Prtr;"HP        CP        VR        F         D";
420   OUTPUT Prtr;"         HMIN "
430   REM  #########      BEGIN MONTH LOOP                          #############
440   FOR I=1 TO 12
450   MAT Ch= (0)
460   MAT Ct= (0)
470   REM  #########    FIND FIRST THREE MOMENTS FOR MONTH I      #############
480   PRINT I;
490   Sh=0.
500   Shh=0.
510   Shhh=0.
520   Hmin(I)=99999.
540   FOR J=1 TO Ny
```

```
550   PRINT J;
560   ENTER @Four;H(J)
570   Check=H(J)
580   IF Check<Hmin(I) THEN
590      Hmin(I)=Check
600      END IF
610   Sh=Sh+H(J)
620   Hh=H(J)*H(J)
630   Thh=Thh+Hh
640   Th=Th+H(J)
650   Shh=Shh+Hh
660   Shhh=Shhh+Hh*H(J)
670   NEXT J
680   PRINT
690   Hb(I)=Sh/Ny1
700   M2=(Shh-Sh*Sh/Ny)/Ny1
710   Hsd(I)=SQR(M2)
720   Hb3=Hb(I)*Hb(I)*Hb(I)
730   Hs3=Hsd(I)*Hsd(I)*Hsd(I)
740   Hsk(I)=((Shhh/Ny1)-(3*Hb(I)*Shh)/Ny1+(2*Hb3))/Hs3
750   REM  ######### KS=0.2 SETS Hmin 20% OF 1 STD DEV BELOW      #############
760   REM  ######### MINIMUM DATA VALUE.                           #############
770   Hmin(I)=Hmin(I)-Ks*Hsd(I)
780   REM  #########      MAY LIMIT Hmin TO  ZERO                  #############
790   IF Ck$="N" THEN
800     IF Hmin(I)<0. THEN
810     Hmin(I)=0.
820     END IF
830   END IF
840   OUTPUT Prtr USING 850;I,Hb(I),Hsd(I),Hsk(I)
850   IMAGE #,5D,7D.4D,2X,6D.4D,2X,3D.4D
860   IMAGE 5(5D.4D),6D.2D,2X
870   REM  #########            SET TRANSFORM PARAMETERS          #############
880   Hp=(Hmin(I)-Hb(I))/Hsd(I)+Hsk(I)/8
890   Cp=2.+.375*Hsk(I)
900   Vr=4.05+(Cp-2.)
910   F=LOG((4.-Vr)/(Cp-Vr))/Hp
920   D=F*(Vr-Cp)/Cp
930   OUTPUT Prtr USING 860;Hp,Cp,Vr,F,D,Hmin(I)
940   V1=0.
950   REM  #########     CALCULATE CLASS LIMITS                   #############
960   FOR K=1 TO 39
970   V1=V1+.1
980   IF V1>Cp THEN 1020
990   REM  #########         LEFT SIDE OF TRANSFORM               #############
1000  Ch(K)=-Hsd(I)*(LOG(V1/Cp)/D+Hsk(I)/8)+Hb(I)
1010  GOTO 1040
1020  REM  #########        RIGHT SIDE OF TRANSFORM               #############
1030  Ch(K)=Hsd(I)*(LOG((V1-Vr)/(Cp-Vr))/F-Hsk(I)/8)+Hb(I)
1040  NEXT K
1050  Ch(40)=Hmin(I)
1060  Itt=1
1070  Cf(I,1)=0.
1080  Lim(I,1)=99999.99
```

```
1090  REM  #########    CALCULATE SAMPLE CLASS PROBABILITIES     #############
1100  FOR J=1 TO Ny
1110  FOR K=Itt TO 40
1120  IF H(J)>Ch(K) THEN 1140
1130  GOTO 1160
1140  Ct(K)=Ct(K)+1
1150  GOTO 1170
1160  NEXT K
1170  NEXT J
1180  REM
1190  FOR J=2 TO 40
1200  Ct(J)=Ct(J)+Ct(J-1)
1210  NEXT J
1220  FOR K=Itt TO 40
1230  Ct(K)=Ct(K)/Ny
1240  NEXT K
1250  REM  ######### REALIGN LIMITS AND SAMPLE PROBS INTO 41 CLASSES   #######.
1260  FOR N=1 TO 40
1270  Cf(I,N-Itt+2)=Ct(N)
1280  Lim(I,N-Itt+2)=Ch(N)
1290  NEXT N
1300  FOR N=1 TO 41
1310  Tp=Tp+Cf(I,N)
1320  Tp2=Tp2+Cf(I,N)*Cf(I,N)
1330  NEXT N
1340  NEXT I
1350  REM  #########         END OF MONTH LOOP                        #############
1360  ASSIGN @Four TO *
1370  OUTPUT Prtr;"     SAMPLE  CLASS PROBABILITIES"
1380  FOR I=1 TO 12
1390  OUTPUT Prtr USING 1400;I
1400  IMAGE #,5X,5D
1410  NEXT I
1420  OUTPUT Prtr
1430  IMAGE #,4D
1440  FOR I=1 TO 41
1450  OUTPUT Prtr USING 1430;I-1
1460  FOR J=1 TO 12
1470  OUTPUT Prtr USING 1480;Cf(J,I)
1480  IMAGE #,3D.3D,3X
1490  NEXT J
1500  OUTPUT Prtr
1510  NEXT I
1520  OUTPUT Prtr
1530  OUTPUT Prtr;"       CLASS LIMITS"
1540  FOR I=1 TO 12
1550  OUTPUT Prtr USING 1400;I
1560  NEXT I
1570  OUTPUT Prtr
1580  FOR I=1 TO 41
1590  OUTPUT Prtr USING 1430;I-1
1600  FOR J=1 TO 12
1610  IF Lim(J,I)=99999.99 THEN
1620     OUTPUT Prtr;"  ++++++  ";
```

```
1630     GOTO 1670
1640     END IF
1650  OUTPUT Prtr USING 1660;Lim(J,I)
1660  IMAGE #,6D.D,2X
1670  NEXT J
1680  OUTPUT Prtr
1690  NEXT I
1700  OUTPUT Prtr
1710  REM  #########  READ IN WEIGHTS A DATA POINT AT A TIME     #############
1720  REM  #########  ACCUMULATE XY VECTOR.                     #############
1730  ASSIGN @Wrts TO "PROBWTSP:INTERNAL,4,1"
1740  FOR I=1 TO 12
1750  PRINT I;
1760  FOR J=1 TO 41
1770  ENTER @Wrts;Wt(*)
1780  FOR L=1 TO 20
1790  Sy(L)=Sy(L)+Wt(L)*Cf(I,J)
1800  NEXT L
1810  NEXT J
1820  NEXT I
1830  ASSIGN @Wrts TO *
1840  REM  #########      READ IN INVERSE MATRIX                 #############
1850  ASSIGN @Ivn TO "PRBINVMATP:INTERNAL,4,1"
1860  ENTER @Ivn;B(*)
1870  FOR L=1 TO 20
1880  FOR I=1 TO 20
1890  Yn(L)=Yn(L)+Sy(I)*B(L,I)
1900  NEXT I
1910  NEXT L
1920  REM  ################################################################
1930  OUTPUT Prtr;"SOLUTION NODES"
1940  FOR M=1 TO 8
1950  OUTPUT Prtr USING 1960;M,Yn(M)
1960  IMAGE #,4D,2X,3D.5D
1970  NEXT M
1980  OUTPUT Prtr
1990  FOR M=9 TO 16
2000  OUTPUT Prtr USING 1960;M,Yn(M)
2010  NEXT M
2020  OUTPUT Prtr
2030  FOR M=17 TO 20
2040  OUTPUT Prtr USING 1960;M,Yn(M)
2050  NEXT M
2060  OUTPUT Prtr
2070  ASSIGN @Ivn TO *
2080  REM  ######### LAY IN SLIDING POLYNOMIAL SURFACE THROUGH   #############
2090  REM  ######### SOLUTION NODES.                             #############
2100  ASSIGN @Wts TO "PROBWTSP:INTERNAL,4,1"
2110  Ip=0
2120  FOR I=1 TO 12
2130  FOR J=1 TO 41
2140  Ip=Ip+1
2150  PRINT Ip;
```

```
2160  ENTER @Wts;Wt(*)
2170  FOR L=1 TO 20
2180  Py(Ip)=Py(Ip)+Wt(L)*Yn(L)
2190  NEXT L
2200  REM  #########              CALCULATE RESIDUALS             #############
2210  E(Ip)=Cf(I,J)-Py(Ip)
2220  T1=E(Ip)*E(Ip)
2230  Ses=Ses+T1
2240  FOR L=1 TO 20
2250  T2=Wt(L)*Wt(L)
2260  Lu2(L)=Lu2(L)+T1*T2
2270  Lu1(L)=Lu1(L)+T2
2280  Lu3(L)=Lu3(L)+Wt(L
2290  NEXT L
2300  NEXT J
2310  NEXT I
2320  ASSIGN @Wts TO *
2330  OUTPUT Prtr2340  OUTPUT Prtr;"   SMOOTHED PROBABILITY VALUES"
2350  FOR I=1 TO 12
2360  OUTPUT Prtr USING 1400;I
2370  NEXT I
2380  OUTPUT Prtr
2390  FOR J=1 TO 41
2400  OUTPUT Prtr USING 1430;J-1
2410  FOR I=J TO (451+J) STEP 41
2420  OUTPUT Prtr USING 1480;Py(I)
2430  NEXT I
2440  OUTPUT Prtr
2450  NEXT J
2460  OUTPUT Prtr;"    RESIDUAL ERRORS"
2470  FOR J=1 TO 12
2480  OUTPUT Prtr USING 1400;J
2490  NEXT J
2500  OUTPUT Prtr
2510  FOR I=1 TO 41
2520  OUTPUT Prtr USING 1430;I-1
2530  FOR J=I TO (451+I) STEP 41
2540  OUTPUT Prtr USING 2550;E(J)
2550  IMAGE #,SDD.5D,1X
2560  NEXT J
2570  OUTPUT Prtr
2580  NEXT I
2590  FOR L=1 TO 20
2600  IF Lu3(L)<=0. THEN 2630
2610  Nvar(L)=Lu2(L)/Lu1(L)*Ny*12/(Ny*12-16)/Lu3(L)
2620  GOTO 2640
2630  Nvar(L)=9999.999
2640  NEXT L
2650  OUTPUT Prtr;"  NODAL VARIANCE";"   NODAL STANDARD DEV.";" SUM OF WTS."
2660  FOR J=1 TO 20
2670  OUTPUT Prtr USING "4D,5D.6D,5D.6D,6D.4D";J,Nvar(J),SQR(Nvar(J)),Lu3(J)
2680  NEXT J
2690  Ssy=Tp2-Tp*Tp/492
2700  OUTPUT Prtr;"TOTAL ADJUSTED SUM OF SQUARES ";Ssy
```

```
2710 OUTPUT Prtr;"TOTAL RESIDUAL ERROR SQUARE ";Ses
2720 OUTPUT Prtr;"  MEAN RESIDUAL ERROR";SQR(Ses/492)
2730 Rs=(Ssy-Ses)/Ssy
2740 OUTPUT Prtr;"CORRELATION           ";Rs;"     ";SQR(Rs)
2750 REM  ######### REORDER SMOOTHED VALUES AND OUTPUT TO DISK  #############
2760 Knt=0
2770 FOR I=1 TO 12
2780 PRINT I
2790 FOR J=1 TO 41
2800 Knt=Knt+1
2810 PRINT Knt;
2820 Ab(I,J)=Py(Knt)
2830 NEXT J
2840 PRINT
2850 NEXT I
2860 !
2870 ASSIGN @Out TO Stnam$
2880 OUTPUT @Out;Cf(*),Lim(*),Ab(*)
2890 ASSIGN @Out TO *
2900 STOP
2910 END
```

Analysis Program Output (Abbreviated)

99 YR RAIN TEST

MONTH	MEAN	STD DEV	COEFF SKEW	HP	CP	VR	F	D	HMIN
1	11.9317	5.9421	.8437	-1.7776	2.3164	4.3664	.9687	.8573	.74
2	12.1458	5.8473	.2992	-1.9618	2.1122	4.1622	1.2931	1.2550	.45
3	13.6520	7.0386	.8246	-1.6793	2.3092	4.3592	1.0371	.9207	1.12
4	10.0254	5.7386	.7283	-1.6559	2.2731	4.3231	1.1158	1.0062	0.00
5	9.7955	5.5481	.7439	-1.6725	2.2790	4.3290	1.0939	.9840	0.00
6	9.9746	5.7531	1.3860	-1.4559	2.5197	4.5697	.8795	.7155	.61
7	12.6152	6.8534	1.1808	-1.6931	2.4428	4.4928	.8419	.7066	0.00
8	10.7795	7.6685	2.1525	-1.1366	2.8072	4.8572	.7671	.5602	0.00
9	8.6164	6.0005	.9615	-1.3158	2.3606	4.4106	1.2222	1.0614	0.00
10	7.5573	5.6241	.7956	-1.2442	2.2984	4.3484	1.4244	1.2705	0.00
11	7.7813	5.9126	2.0567	-1.0590	2.7712	4.8212	.8638	.6390	0.00
12	11.4282	5.9205	.8581	-1.7527	2.3218	4.3718	.9741	.8601	.41

SAMPLE CLASS PROBABILITIES

	1	2	3	4	5	6	7	8	9	10	11	12
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.000	.020	0.000	.010	.010	0.000	0.000	0.000	.020	.010	.010	0.000
2	.010	.030	.010	.030	.020	.010	.020	.020	.051	.071	.010	.010
3	.010	.081	.030	.040	.051	.020	.020	.020	.071	.081	.010	.040
4	.051	.141	.071	.061	.061	.020	.030	.020	.071	.111	.010	.061
5	.081	.152	.111	.071	.091	.030	.030	.020	.101	.152	.010	.081

(Abbreviated)

36	.919	.808	.899	.859	.879	.889	.889	.879	.778	.747	.869	.879
37	.939	.859	.960	.919	.899	.960	.949	.909	.869	.788	.879	.919
38	.980	.909	.980	.960	.919	.990	.970	.960	.939	.869	.919	.960
39	.990	.960	.980	.980	.970	1.000	.990	.980	.970	.879	.970	.990
40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

CLASS LIMITS

	1	2	3	4	5	6	7	8	9	10	11	12
0	++++++	++++++	++++++	++++++	++++++	++++++	++++++	++++++	++++++	++++++	++++++	++++++
1	33.0	26.2	36.8	27.4	26.9	34.8	42.7	54.4	25.6	20.8	36.8	32.5
2	28.2	22.9	31.8	23.4	23.1	29.5	35.8	45.0	21.8	17.8	30.5	27.7
3	25.4	21.1	28.4	21.1	20.8	26.2	32.0	39.4	19.6	16.0	26.9	24.9
4	23.4	19.6	26.4	19.3	19.0	23.9	29.2	35.3	18.0	14.7	24.1	22.9
5	21.8	18.5	24.6	18.0	17.8	22.1	26.9	32.3	16.8	13.7	22.1	21.3

(Abbreviated)

36	5.3	6.1	6.1	4.1	4.1	4.1	4.8	3.8	3.3	3.0	2.8	4.8
37	4.3	5.1	5.3	3.3	3.3	3.3	3.8	3.0	2.8	2.5	2.0	4.1
38	3.3	4.1	4.1	2.5	2.3	2.5	2.8	2.0	2.0	1.8	1.5	3.0
39	2.3	2.5	2.8	1.3	1.3	1.8	1.5	1.0	1.0	1.0	.8	1.8
40	.8	.5	1.0	0.0	0.0	.5	0.0	0.0	0.0	0.0	0.0	.8

SOLUTION NODES

1	.18232	2	.40672	3	.67558	4	1.00311	5	.94817	6	.24166	7	.46774	8	.60273
9	1.00112	10	1.12607	11	.10099	12	.33605	13	.63674	14	1.00892	15	.73348	16	.21467
17	.36658	18	.55385	19	.99293	20	1.62713								

SMOOTHED PROBABILITY VALUES

	1	2	3	4	5	6	7	8	9	10	11	12
0	0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	-0.000
1	.003	.005	.004	.005	.002	.004	.001	.003	.002	.004	.003	.005
2	.012	.015	.016	.018	.012	.010	.005	.010	.011	.016	.015	.015
3	.027	.031	.035	.037	.028	.019	.011	.018	.026	.035	.033	.030
4	.045	.051	.059	.063	.049	.031	.019	.029	.045	.058	.056	.049
5	.066	.074	.088	.092	.073	.045	.029	.042	.067	.084	.082	.071
					(Abbreviated)							
36	.897	.889	.863	.849	.865	.894	.903	.871	.817	.791	.815	.866
37	.931	.925	.904	.893	.908	.935	.943	.914	.862	.839	.858	.904
38	.961	.958	.942	.935	.947	.970	.975	.952	.908	.889	.903	.940
39	.986	.984	.975	.971	.979	.994	.998	.983	.953	.940	.948	.972
40	1.003	1.003	1.002	1.001	1.004	1.008	1.009	1.004	.997	.993	.995	1.000

RESIDUAL ERRORS

	1	2	3	4	5	6	7	8	9	10	11	12
0	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	+0.00000	-0.00000	+0.00000
1	-.00327	+.01552	-.00351	+.00544	+.00790	-.00360	-.00132	-.00346	+.01824	+.00574	+.00683	-.00455
2	-.00235	+.01487	-.00552	+.01265	+.00825	-.00004	+.01508	+.01057	+.03950	+.05425	-.00457	-.00483
3	-.01647	+.04954	-.00446	+.00294	+.02259	+.00084	+.00897	+.00193	+.04493	+.04599	-.02252	+.01023
4	+.00585	+.09019	+.01134	-.00201	+.01186	-.01080	+.01089	-.00890	+.02574	+.05314	-.04548	+.01128
5	+.01507	+.07720	+.02327	-.02089	+.01783	-.01448	+.00081	-.02166	+.03378	+.06709	-.07190	+.00941
					(Abbreviated)							
36	+.02237	-.08057	+.03633	+.00978	+.01388	-.00537	-.01435	+.00737	-.03952	-.04369	+.05363	+.01298
37	+.00828	-.06680	+.05569	+.02579	-.00895	+.02420	+.00688	-.00524	+.00625	-.05106	+.02038	+.01488
38	+.01865	-.04862	+.03803	+.02487	-.02740	+.02034	-.00509	+.00754	+.03167	-.01991	+.01628	+.01940
39	+.00431	-.02449	+.00500	+.00863	-.00944	+.00556	-.00771	-.00290	+.01692	-.06120	+.02127	+.01755
40	-.00311	-.00299	-.00151	-.00112	-.00387	-.00772	-.00892	-.00438	+.00279	+.00707	+.00516	+.00035

NODAL	VARIANCE.	NODAL STANDARD DEV.	SUM OF WTS.
1	.000054	.007335	30.0000
2	.000081	.009013	30.0000
3	.000034	.005815	31.2375
4	.000052	.007184	16.5000
5	9999.999000*	99.999995*	-1.2375
6	.000030	.005505	30.0000
7	.000033	.005754	30.0000
8	.000021	.004542	31.2375
9	.000032	.005624	16.5000
10	9999.999000	99.999995	-1.2375
11	.000066	.008139	30.0000
12	.000060	.007755	30.0000
13	.000042	.006446	31.2375
14	.000010	.003157	16.5000
15	9999.999000	99.999995	-1.2375
16	.000195	.013970	30.0000
17	.000105	.010270	30.0000
18	.000026	.005055	31.2375
19	.000062	.007884	16.5000
20	9999.999000	99.999995	-1.2375

TOTAL ADJUSTED SUM OF SQUARES 42.6867316094
TOTAL RESIDUAL ERROR SQUARE .780820470719
MEAN RESIDUAL ERROR .0398375887457
CORRELATION .981708122377 .99081185014

*Defaults for undefined values

"WRITEWTINV" Program Listing

```
10    REM  ############################################################
20    REM  ##########        "WRITEWTINV"                ###############
30    REM  ######### PROGRAM TO WRITE WEIGHTS AND INVERSE #############
40    REM  ######### SUMS OF SQUARES TO DISK. FILES NEEDED ############
50    REM  ######### FOR SEASONALLY CONTINUOUS ANALYSIS ###############
60    REM  ############################################################
70    OPTION BASE 1
80    DIM Gc(7,8),C(16),Wt(20),Sx(20,20),Sxinv(20,20)
90    INTEGER Id,Js,Is,L,M,Ic,Iw,Ix,Im,N,K
100   REM  ############################################################
110   REM  ######### OPEN DOUBLE DO LOOP TO SCAN ENTIRE ###############
120   REM  ######### 12 X 41 SURFACE MATRIX. FIND U AND ###############
130   REM  ######### Z VALUES AND GET WEIGHTS FOR EACH  ###############
140   REM  ######### POINT. VERTICAL SCALE (W) IS MONTH ###############
150   REM  ######### HORIZONTAL SCALE (X) IS V TRANSFORM ##############
160   REM  ############################################################
170   PRINT "WEIGHTS FOR SEASONALLY CONTINUOUS PROB    "
180   PRINT
190   K1=3
200   K2=1
210   K3=3
220   K4=7
230   CREATE BDAT "PROBWTSP:INTERNAL,4,1",310
240   ASSIGN @Owt TO "PROBWTSP:INTERNAL,4,1"
250   REM  #########      MONTH LOOP BEGINS              ###############
260   REM  #########     LOCATE MONTH ON SURFACE         ###############
270   Id=0
280   FOR Wm=1 TO 12
290   W=1+(Wm-1)/3
300   Js=INT(W+.000001)
310   Tw=(W+.000001)-FRACT(W+.000001)
320   U=-.5+W-Tw
330   PRINT
340   REM  #########      V TRANSFORM LOOP BEGINS        ###############
350   REM  #########        LOCATE V ON SURFACE          ###############
360   FOR Xc=1 TO 41
370   PRINT "JS=";Js;"     IS=";
380   Id=Id+1
390   X=1+(Xc-1)/10
400   Is=INT(X)
410   Tx=X-FRACT(X)
420   Z=-.5+X-Tx
430   PRINT Is;
440   PRINT " ID= ";Id
450   REM  ######### GET 16 WEIGHTS FROM SUBROUTINE      ###############
460   REM  ######### Weight_gen. PUT IN PROPER 4 X 4     ###############
470   REM  ######### NODAL SUB MATRIX.                   ###############
480   GOSUB Weight_gen
490   FOR L=1 TO 7
500   FOR M=1 TO 7
510   Gc(L,M)=0.
520   NEXT M
530   NEXT L
```

```
540  Ic=0
550  FOR L=1 TO 4
560  FOR M=1 TO 4
570  Ic=Ic+1
580  Gc(L+Js-1,M+Is-1)=C(Ic)
590  NEXT M
600  NEXT L
610  REM  ######### SET BOUNDARIES. LEFT BOUNDARY HAS  ###############
620  REM  ######### ZERO NODE AND ZERO SLOPE            ###############
630  FOR Iw=1 TO 7
640  Gc(Iw,K1)=Gc(Iw,K1)+Gc(Iw,K2)
650  NEXT Iw
660  REM  ######### TOP AND BOTTOM BOUNDARIES WRAP      ###############
670  REM  ######### AROUND FOR CYLNDER FUNCTION.        ###############
680  FOR Ix=K3 TO K4
690  Gc(2,Ix)=Gc(2,Ix)+Gc(6,Ix)
700  Gc(5,Ix)=Gc(5,Ix)+Gc(1,Ix)
710  Gc(3,Ix)=Gc(3,Ix)+Gc(7,Ix)
720  NEXT Ix
730  REM  ######### LINEARIZE THE Gc( , ) MATRIX, SAVE ###############
740  REM  ######### AND TAKE THE SUM OF SQUARES.        ###############
750  Im=0
760  FOR Iw=2 TO 5
770  FOR Ix=K3 TO K4
780  Im=Im+1
790  Wt(Im)=Gc(Iw,Ix)
800  PRINT Wt(Im);
810  NEXT Ix
820  NEXT Iw
830  PRINT
840  OUTPUT @Owt;Wt(*)
850  PRINT
860  FOR N=1 TO Im
870  FOR K=N TO Im
880  Sx(N,K)=Sx(N,K)+Wt(N)*Wt(K)
890  NEXT K
900  NEXT N
910  NEXT Xc
920  NEXT Wm
930  ASSIGN @Owt TO *
940  REM  ######### SQUARE THE TRIANGULAR SUM-OF-      ###############
950  REM  ######### SQUARES MATRIX AND PRINT           ###############
960  FOR N=1 TO Im
970  FOR K=N TO Im
980  Sx(K,N)=Sx(N,K)
990  NEXT K
1000 NEXT N
1010 PRINT "    SUMS OF SQUARES MATRIX"
1020 FOR N=1 TO 20
1030 FOR K=1 TO 10
1040 PRINT Sx(N,K);
1050 NEXT K
1060 PRINT
1070 FOR K=11 TO 20
```

```
1080 PRINT Sx(N,K);
1090 NEXT K
1100 PRINT
1110 PRINT
1120 NEXT N
1130 PRINT
1140 FOR K=1 TO 8
1150 PRINT Sx(K,17);
1160 NEXT K
1170 PRINT
1180 FOR K=9 TO 16
1190 PRINT Sx(K,17);
1200 NEXT K
1210 PRINT
1220 REM  ######### USE SUBROUTINE Matt_inv TO INVERT  ###############
1230 REM  ######### MATRIX.                             ###############
1240 GOSUB Matt_inv
1250 REM  ######### LOAD AND EXECUTE SEASONALLY         ###############
1260 REM  ######### CONTINUOUS ANALYSIS PROGRAM         ###############
1270 PRINT "  DISK FILES HAVE BEEN GENERATED "
1280 LOAD "SEACONPST",200
1290 STOP
1300 Weight_gen:   !
1310  IF U=Us THEN 1350
1320  Us=U
1330  Tr=1
1340  GOTO 1360
1350  Tr=2
1360  ON Tr GOTO 1370,1450
1370  U1=(((8*U-4)*U-2)*U+1)/256
1380  U2=(((-32*U+4)*U+8)*U-1)/128
1390  U3=(((-8*U+4)*U+2)*U-1)/64
1400  U4=(((32*U-4)*U-8)*U+1)/32
1410  U5=(((-72*U+36)*U+18)*U-9)/256
1420  U6=(((160*U-44)*U-40)*U+11)/128
1430  U7=(((8*U-4)*U-2)*U+1)/64
1440  U8=(((-96*U+12)*U+24)*U-3)/32
1450  C(1)=((U4*Z+U3)*Z+U2)*Z+U1
1460  C(2)=((U8*Z+U7)*Z+U6)*Z+U5
1470  C(3)=((-U8*Z+U7)*Z-U6)*Z+U5
1480  C(4)=((-U4*Z+U3)*Z-U2)*Z+U1
1490  ON Tr GOTO 1500,1580
1500  V1=(((-24*U+4)*U+22)*U-9)/256
1510  V2=(((96*U-4)*U-40)*U+9)/128
1520  V3=(((24*U-4)*U-22)*U+9)/64
1530  V4=(((-96*U+4)*U+40)*U-9)/32
1540  V5=(((216*U-36)*U-198)*U+81)/256
1550  V6=(((-480*U+44)*U+296)*U-99)/128
1560  V7=(((-24*U+4)*U+22)*U-9)/64
1570  V8=(((288*U-12)*U-120)*U+27)/32
1580  C(5)=((V4*Z+V3)*Z+V2)*Z+V1
1590  C(6)=((V8*Z+V7)*Z+V6)*Z+V5
1600  C(7)=((-V8*Z+V7)*Z-V6)*Z+V5
1610  C(8)=((-V4*Z+V3)*Z-V2)*Z+V1
```

```
1620  ON Tr GOTO 1630,1710
1630  G1=(((24*U+4)*U-22)*U-9)/256
1640  G2=(((-96*U-4)*U+40)*U+9)/128
1650  G3=(((-24*U-4)*U+22)*U+9)/64
1660  G4=(((96*U+4)*U-40)*U-9)/32
1670  G5=(((-216*U-36)*U+198)*U+81)/256
1680  G6=(((480*U+44)*U-296)*U-99)/128
1690  G7=(((24*U+4)*U-22)*U-9)/64
1700  G8=(((-288*U-12)*U+120)*U+27)/32
1710  C(9)=((G4*Z+G3)*Z+G2)*Z+G1
1720  C(10)=((G8*Z+G7)*Z+G6)*Z+G5
1730  C(11)=((-G8*Z+G7)*Z-G6)*Z+G5
1740  C(12)=((-G4*Z+G3)*Z-G2)*Z+G1
1750  ON Tr GOTO 1760,1840
1760  H1=(((-8*U-4)*U+2)*U+1)/256
1770  H2=(((32*U+4)*U-8)*U-1)/128
1780  H3=(((8*U+4)*U-2)*U-1)/64
1790  H4=(((-32*U-4)*U+8)*U+1)/32
1800  H5=(((72*U+36)*U-18)*U-9)/256
1810  H6=(((-160*U-44)*U+40)*U+11)/128
1820  H7=(((-8*U-4)*U+2)*U+1)/64
1830  H8=(((96*U+12)*U-24)*U-3)/32
1840  C(13)=((H4*Z+H3)*Z+H2)*Z+H1
1850  C(14)=((H8*Z+H7)*Z+H6)*Z+H5
1860  C(15)=((-H8*Z+H7)*Z-H6)*Z+H5
1870  C(16)=((-H4*Z+H3)*Z-H2)*Z+H1
1880  RETURN
1890 Matt_inv:        !
1900 REM  ######### THIS SUBROUTINE USES THE ADVANCED  ###############
1910 REM  ######### PROGRAM FEATURE TO INVERT A MATRIX ###############
1920    CREATE BDAT "PRBINVMATP:INTERNAL,4,1",13
1930    ASSIGN @Siv TO "PRBINVMATP:INTERNAL,4,1"
1940 REM  ######### CLEAR THE TARGET MATRIX             ###############
1950   MAT Sxinv= (0)
1960 REM  ######### AND PUT THE INVERTED MATRIX IN      ###############
1970   MAT Sxinv= INV(Sx)
1980   Check_det=DET
1990   PRINT
2000   PRINT "INVERSE MATRIX"
2010   PRINT
2020   FOR I=1 TO 20
2030   FOR N=1 TO 10
2040   PRINT Sxinv(I,N);
2050   NEXT N
2060   PRINT
2070   FOR N=11 TO 20
2080   PRINT Sxinv(I,N);
2090   NEXT N
2100   PRINT
2110   PRINT
2120   NEXT I
2130 REM  ######### CHECK THE DETERMINANT AND RETURN    ###############
2140   PRINT "  DETERMINANT = ";Check_det
2150   BEEP 500,2
```

```
2160   WAIT 4
2170   OUTPUT @Siv;Sxinv(*)
2180   ASSIGN @Siv TO *
2190   RETURN
2200   END
```

SIMULATION PROGRAM

This program simulates the seasonal variation of climatic risk by randomly drawing and ordering events from a 2-D sliding polynomial surface. This surface or smoothing space is defined by the magnitude of events in v-scale versus time by month. The height of the surface across this space is probability.

Simulation Program Input Variables

Variable name	Comment
COEF(I)	Sliding polynomial coefficients For I=1 to 4
AT(I)	Problem title For I=1 to 80
NR, N1, N2	Random number seeds
NS	Number of samples
NI	Planning period
HB(I)	Monthly mean For I=1 to 12
HSD(I)	Monthly standard deviation For I=1 to 12
HSK(I)	Monthly coeff. of skew For I=1 to 12
HMIN(I)	Monthly minimum class limit For I=1 to 12
YN(I,J)	Solution nodes For I=2 to 5: J=3 to 7
SN(I,J)	Nodal standard deviation For I=2 to 5: J=3 to 7

Simulation Program Output Variables

Variable name	Comment
AT(I)	Problem title For I=1 to 80
NR, N1, N2	Random number seeds
NS	Number of samples
NI	Planning period
HB(I)	Monthly mean For I=1 to 12
HSD(I)	Monthly sample std. deviation For I=1 to 12
HSK(I)	Monthly sample coeff. of skew For I=1 to 12
YN(I,J)	Solution nodes For I=2 to 5: J=3 to 7
SN(I,J)	Nodal standard deviation For I=2 to 5: J=3 to 7
HP	Minimum value of intermediate variate, h', at minimum class limit
CP	Common point of two exponential limbs (in v-scale)
VR	Right asymptotic boundary for limb v2 (in v-scale)
F	Shape parameter
D	Shape parameter
H(M,K,N)	Simulated variate For N=1 to 12: M=1 to NS: K=1 to NI

Simulation Program Listing

```
C ####################################################################
C ############  SIMULATION OF SEASONAL CONTINUOUS VARIATION ##########
C ############  OF CLIMATIC RISK. MONTHLY EVENTS ARE DRAWN  ##########
C ############  FROM A 2-D SLIDING POLYNOMIAL SURFACE AS    ##########
C ############  DEFINED BY SEASONALLY CONTINUOUS PROB.      ##########
C ############  ANALYSIS. HEIGHT OF SURFACE IS PROBABILITY. ##########
C ############  MONTHLY EVENTS ARE ORDERED BY PLANNING      ##########
C ############  PERIODS AND SAMPLES.                        ##########
C ####################################################################
      DIMENSION H(100,50,12),HSK(12),C(4),RNOD(12,7)
      DIMENSION AT(80),HB(12),HSD(12),HMIN(12)
      DIMENSION CP(12),HP(12),VR(12),F1(12),D1(12)
      COMMON YN(7,7),SN(7,7),COEF(4),SNOD(12,7),YNOD(12,7)
      READ(5,8110) (COEF(I),I=1,4)
 8110 FORMAT(4F12.0)
      READ(5,8080) (AT(I),I=1,80)
 8080 FORMAT(80A1)
      WRITE(6,8081) (AT(I),I=1,80)
 8081 FORMAT(' ',80A1)
      READ(5,1000) NR,N1,N2,NS,NI
      WRITE(6,8010)
 8010 FORMAT(' '/,20X,'INPUT DATA'/,9X,'SEEDS',9X,'NS',9X,'NI')
 1000 FORMAT(3I5,2I10)
      WRITE(6,50) NR,N1,N2,NS,NI
   50 FORMAT(' ',6X/,5I8)
      WRITE(6,8020)
 8020 FORMAT(' '/,' MONTH   MEAN  STD DEV    SKEW   HMIN')
 8022 FORMAT(' ',I4,2F12.3,2F12.3)
      READ(5,1001) (HB(I),HSD(I),HSK(I),HMIN(I),I=1,12)
      WRITE(6,8022) (I,HB(I),HSD(I),HSK(I),HMIN(I),I=1,12)
      KNT=1
      DO 41 I=2,5
      DO 40 J=3,7
      READ(5,8082) YN(I,J),SN(I,J)
 8082 FORMAT(2F10.0)
      YNOD(KNT,J)=YN(I,J)
      SNOD(KNT,J)=SN(I,J)
   40 CONTINUE
      KNT=KNT+3
      WRITE(6,8035) (YN(I,J),SN(I,J),J=3,7)
 8035 FORMAT(' ',2F12.3)
   41 CONTINUE
 1001 FORMAT(4F10.0)
C ###########  EXPAND 20 SOLUTION NODES TO 12 X 7 ###################
C ###########  FIELD OF NODES                     ###################
      CALL INTERP
      WRITE(6,332)
  332 FORMAT(' ',8X,'HP',8X,'CP',8X,'VR',8X,'F',9X,'D'/)
C ###########    SET TRANSFORM   ####################################
      DO 1060 I=1,12
      HP(I)=(HMIN(I)-HB(I))/HSD(I)+HSK(I)/8
      CP(I)=2.+0.375*HSK(I)
      VR(I)=4.05+(CP(I)-2.)
```

```
      F1(I)=ALOG((4.-VR(I))/(CP(I)-VR(I)))/HP(I)
      D1(I)=F1(I)*(VR(I)-CP(I))/CP(I)
      WRITE(6,333) HP(I),CP(I),VR(I),F1(I),D1(I)
  333 FORMAT(' ',5F10.4)
      DO 1060 L=1,7
      RNOD(I,L)=YNOD(I,L)
 1060 CONTINUE
C ##########  INITIALIZE THE RANDOM NUMBER MATRIX  ###################
      CALL RAND(NR,N1,N2,R,1)
C ###########  BEGIN LOOPS:                          ###################
C ###########               M = NUMBER OF SAMPLE    ###################
C ###########               I = PLANNING PERIOD     ###################
C ###########               K = MONTH               ###################
      DO 1003 M=1,NS
      DO 1004 I=1,NI
      DO 1080 K=1,12
      VS=1.
C ###########  DRAW RANDOM NUMBER AND LOCATE ON MONTHLY ##############
C ###########  ARC USING INTERVAL-HALVING METHOD        ##############
      CALL RAND(NR,N1,N2,R,2)
      IF (R.LT.RNOD(K,3)) GO TO 1005
      IF (R.LT.RNOD(K,4)) GO TO 1006
      IF (R.LT.RNOD(K,5)) GO TO 1007
      IF (R.LT.RNOD(K,6)) GO TO 1715
      H(M,I,K)=HMIN(K)
      GO TO 1080
 1005 BL=0.0
      BR=VS
      IS=1
      GO TO 1008
 1006 BL=VS
      BR=2.0*VS
      IS=2
      GO TO 1008
 1007 BL=2.0*VS
      BR=3.0*VS
      IS=3
      GO TO 1008
 1715 BL=3.0*VS
      BR=4.0*VS
      IS=4
 1008 A=(-BL)/(BR-BL)-0.5
      C(1)=(9.*(RNOD(K,IS+1)+RNOD(K,IS+2))-RNOD(K,IS)-RNOD(K,IS+3))/16.
      C(2)=(11.*(RNOD(K,IS+2)-RNOD(K,IS+1))+RNOD(K,IS)-RNOD(K,IS+3))/8.
      C(3)=(RNOD(K,IS)-RNOD(K,IS+1)-RNOD(K,IS+2)+RNOD(K,IS+3))/4.
      C(4)=(3.*(RNOD(K,IS+1)-RNOD(K,IS+2))-RNOD(K,IS)+RNOD(K,IS+3))/2.
      B=1./(BR-BL)
 1100 Z=A+B*(BL+BR)/2.0
      PC=((C(4)*Z+C(3))*Z+C(2))*Z+C(1)
      IF(R.LT.PC) GO TO 1009
      BL=(Z-A)/B
      GO TO 1010
 1009 BR=(Z-A)/B
 1010 IF(ABS(BR-BL).LT.0.001) GO TO 1011
```

```
      GO TO 1100
 1011 V=(BL+BR)/2.
C  #############    COMPUTE A VALUE H(M,I,K)       ####################
      IF(V.LE.CP(K)) GO TO 1012
      HS=HSD(K)*(ALOG((V-VR(K))/(CP(K)-VR(K)))/F1(K)-HSK(K)/8)+HB(K)
      GO TO 1111
 1012 CONTINUE
      HS=-HSD(K)*(ALOG(V/CP(K))/D1(K)+HSK(K)/8)+HB(K)
 1111 H(M,I,K)=HS
C  ###########  PULSE NODES WITH RANDOM NORMAL DEVIATE ################
      DO 1112 L=3,7
      CALL RANDN(NR,N1,N2,RANUM)
      RNOD(K,L)=YNOD(K,L)+RANUM*SNOD(K,L)
 1112 CONTINUE
      RNOD(K,1)=RNOD(K,3)
 1080 CONTINUE
C  ################      END OF MONTH LOOP         ####################
 1004 CONTINUE
C  ################      END OF NI LOOP            ####################
      DO 1035 K=1,12
      DO 10 J=1,NI
      X6=H(M,J,K)
      DO 20 J1=1,NI
      IF(H(M,J1,K).GE.X6) GO TO 20
      H(M,J,K)=H(M,J1,K)
      H(M,J1,K)=X6
      X6=H(M,J,K)
   20 CONTINUE
   10 CONTINUE
 1035 CONTINUE
C        REMOVE COMMENTS ON CARDS TO PRINT EACH SAMPLE
C     WRITE(6,1017) M
 1017 FORMAT(' '/,10X,'SAMPLE NO.',I4)
C     DO 1045 K1=1,12
C     WRITE(6,8083) K1
 8083 FORMAT(' '/,10X,'MONTH',I4)
C     WRITE(6,1018) (K,H(M,K,K1),K=1,NI)
 1018 FORMAT(' ',8(I4,F8.3)/)
 1045 CONTINUE
 1003 CONTINUE
C  ###########          END OF NS LOOP             ####################
C  ###########  SORT SAMPLES IN DESCENDING ORDER    ####################
C  ###########  ACROSS PLANNING PERIOD AND PRINT    ####################
      DO 2000 J1=1,12
      DO 2010 K=1,NI
      DO 2005 I=1,NS
      CHECK=H(I,K,J1)
      DO 2004 J=1,NS
      IF(H(J,K,J1).GE.CHECK) GO TO 2004
      H(I,K,J1)=H(J,K,J1)
      H(J,K,J1)=CHECK
      CHECK=H(I,K,J1)
 2004 CONTINUE
 2005 CONTINUE
```

```
 2010 CONTINUE
 2000 CONTINUE
      DO 2233 N=1,12
      WRITE(6,2235) N
 2235 FORMAT(' '/,10X,'MONTH',I4)
      WRITE(6,2223) (L,L=1,NI)
 2223 FORMAT(' ',//11X,'  SORTED DATA '/,8X,20(I2,4X))
      DO 2001 M=1,NS
      WRITE(6,2222) M,(H(M,K,N),K=1,NI)
 2222 FORMAT(' ',I3,2X,20F6.2)
 2001 CONTINUE
 2233 CONTINUE
   70 STOP
      END
C   ####################################################################
C   ###########  SUBROUTINE RAND                            ###############
C   ###########  RANDOM DRAW FROM A 10 X 10 MATRIX          ###############
      SUBROUTINE RAND(NR,N1,N2,DRAW,IENT)
      DIMENSION TAB(10,10)
      IF(IENT.NE.1) GO TO 1003
      CALL RANDO(N1,N12,XRN)
      N1=N12
      II=INT(10.0*XRN)+1
      CALL RANDO(N2,N22,XRN)
      N2=N22
      JJ=INT(10.0*XRN)+1
      DO 1000 I=1,10
      DO 1001 J=1,10
      CALL RANDO(NR,NR2,XRN)
      NR=NR2
 1001 TAB(I,J)=XRN
 1000 CONTINUE
      IC=1
 1003 DRAW=TAB(II,JJ)
      CALL RANDO(NR,NR2,XRN)
      NR=NR2
      TAB(II,JJ)=XRN
      IF(MOD(IC,2).EQ.0) GO TO 1005
      CALL RANDO(N1,N12,XRN)
      N1=N12
      II=INT(10.0*XRN)+1
      GO TO 1006
 1005 CALL RANDO(N2,N22,XRN)
      N2=N22
      JJ=INT(10.0*XRN)+1
 1006 IC=IC+1
      RETURN
      END
      SUBROUTINE RANDO(IX,IY,YFL)
      IY=IX*65539
      IF(IY) 5,6,6
    5 IY=IY+2147483647+1
    6 YFL=FLOAT(IY)*0.4656613E-9
      RETURN
```

```
      END
C ######################################################################
C ###########  SUBROUTINE INTERP                          #################
C ###########  EXPAND 20 SOLUTION NODES TO FILL THE       #################
C ###########  12 X 7 FIELD OF NODES                      #################
      SUBROUTINE INTERP
      COMMON YNN(7,7),SNN(7,7),COEFN(4),SNODN(12,7),YNODN(12,7)
C ####### FILL IN BOUNDARY NODES FOR CYLINDER FUNCTION  #############
      DO 11 J=3,7
      YNN(1,J)=YNN(5,J)
      YNN(6,J)=YNN(2,J)
      SNN(1,J)=SNN(5,J)
      SNN(6,J)=SNN(2,J)
      YNN(7,J)=YNN(3,J)
      SNN(7,J)=SNN(3,J)
   11 CONTINUE
C ###########  INTERPOLATE TO FILL IN GRID         ####################
      KNT=0
      DO 17 I=1,10,3
      DO 16 J=3,7
      YNODN(I+1,J)=0.0
      YNODN(I+2,J)=0.0
      SNODN(I+1,J)=0.0
      SNODN(I+2,J)=0.0
      VAR1=0.0
      VAR11=0.0
      VAR2=0.0
      DO 15 K=1,4
      YNODN(I+1,J)=YNODN(I+1,J)+COEFN(K)*YNN(K+KNT,J)
      YNODN(I+2,J)=YNODN(I+2,J)+COEFN(K)*YNN(5-K+KNT,J)
      VAR1=VAR1+COEFN(K)*COEFN(K)*SNN(K+KNT,J)*SNN(K+KNT,J)
      VAR2=VAR2+COEFN(K)*COEFN(K)
      COEF2=COEFN(K)*COEFN(K)
      SNOD2=SNN(5-K+KNT,J)*SNN(5-K+KNT,J)
      VAR11=VAR11+COEF2*SNOD2
   15 CONTINUE
      SNODN(I+1,J)=SQRT(VAR1/VAR2)
      SNODN(I+2,J)=SQRT(VAR11/VAR2)
   16 CONTINUE
      KNT=KNT+1
   17 CONTINUE
      DO 19 I=1,12
      YNODN(I,1)=YNODN(I,3)
      SNODN(I,1)=SNODN(I,3)
      YNODN(I,2)=0.0
      SNODN(I,2)=0.0
   19 CONTINUE
      RETURN
      END
C ######################################################################
C #########          SUBROUTINE RANDN                     ###############
C #########  DRAW RANDOM NORMAL NUMBER                     ###############
      SUBROUTINE RANDN(NR,N1,N2,RANUM)
      RANUM=0.
```

```
      DO 10 I=1,12
      CALL RAND(NR,N1,N2,X,2)
      RANUM=RANUM+X
   10 CONTINUE
      RANUM=RANUM-6
      RETURN
      END
```

Simulation Program Output (Abbreviated)

SEACONPRB 99 YR MONTHLY TOTAL PRECIP

INPUT DATA

	SEEDS		NS	NI
29899	55376	61832	100	20

MONTH	MEAN	STD DEV	SKEW	HMIN
1	11.932	5.942	0.844	0.740
2	12.146	5.847	0.299	0.450
3	13.652	7.039	0.825	1.120
4	10.025	5.739	0.728	0.000
5	9.795	5.548	0.744	0.000
6	9.975	5.753	1.386	0.610
7	12.615	6.853	1.181	0.000
8	10.780	7.668	2.153	0.000
9	8.616	6.000	0.961	0.000
10	7.557	5.624	0.796	0.000
11	7.781	5.913	2.057	0.000
12	11.428	5.920	0.858	0.410

SOLUTION NODES	STD DEV
0.182	0.007
0.407	0.009
0.676	0.006
1.003	0.007
0.948	0.007
0.242	0.006
0.468	0.006
0.603	0.005
1.001	0.006
1.126	0.006
0.101	0.008
0.336	0.008
0.637	0.006
1.009	0.003
0.733	0.003

0.215	0.014
0.367	0.010
0.554	0.005
0.993	0.008
1.627	0.008

HP	CP	VR	F	D
-1.7780	2.3164	4.3664	0.9685	0.8571
-1.9628	2.1122	4.1622	1.2924	1.2544
-1.6774	2.3092	4.3592	1.0383	0.9218
-1.6560	2.2731	4.3231	1.1157	1.0062
-1.6726	2.2790	4.3290	1.0939	0.9840
-1.4545	2.5197	4.5697	0.8803	0.7162
-1.6931	2.4428	4.4928	0.8419	0.7065
-1.1366	2.8072	4.8572	0.7671	0.5602
-1.3158	2.3606	4.4106	1.2222	1.0614
-1.2443	2.2983	4.3483	1.4244	1.2705
-1.0590	2.7713	4.8213	0.8638	0.6390
-1.7538	2.3218	4.3718	0.9735	0.8595

MONTH 1

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	44.59	30.61	28.26	23.87	21.32	20.14	18.66	17.61	16.39	15.70	15.32	14.39	13.87	12.56	12.23	12.03	11.36	11.18	9.74	7.64
2	36.97	29.16	25.58	23.28	21.20	19.97	18.38	16.76	16.35	15.41	14.71	12.96	12.71	12.51	11.95	11.52	11.34	9.65	8.51	6.78
3	36.41	28.75	24.78	22.52	21.08	19.29	18.37	16.71	15.99	15.36	14.48	12.89	12.58	11.67	10.98	10.82	10.43	9.40	8.50	6.75
4	35.98	28.61	24.44	21.69	20.18	18.95	18.21	16.70	15.98	14.98	13.91	12.81	12.35	11.60	10.93	10.66	10.22	9.07	8.28	6.66
5	35.78	28.47	24.27	21.48	19.98	18.91	17.97	16.65	15.86	14.23	13.90	12.80	12.18	11.55	10.88	10.19	9.47	8.78	7.78	6.34
							(Abbreviated)													
96	19.69	16.41	15.26	13.50	12.85	11.71	11.02	10.64	10.17	9.53	8.70	8.13	7.39	7.05	6.49	4.75	4.47	3.62	2.16	1.23
97	19.36	15.93	14.84	13.36	12.57	11.62	10.92	10.62	10.07	9.49	8.59	7.90	7.08	7.01	6.38	4.70	4.25	3.47	1.73	1.10
98	18.96	15.76	14.65	13.29	12.07	11.46	10.86	10.60	9.69	9.44	5.54	7.79	6.90	6.82	6.23	4.56	3.98	3.47	1.47	0.74
99	17.88	15.07	14.63	12.71	11.81	11.27	10.78	10.60	9.66	9.21	8.33	7.63	6.50	5.76	4.75	4.48	3.81	3.24	1.23	0.74
100	17.66	14.44	12.30	12.15	11.52	11.06	10.67	10.35	9.64	8.36	7.63	7.10	5.67	5.53	4.26	4.15	2.62	1.25	0.74	0.74

MONTH 2

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	30.14	26.57	24.11	19.82	19.62	17.70	17.18	15.42	14.63	14.11	14.02	12.95	12.67	12.35	12.08	11.12	10.62	9.58	8.87	8.57
2	28.92	25.47	21.58	19.25	18.17	17.60	15.95	15.04	14.20	13.98	13.31	12.76	12.50	12.27	11.75	10.79	10.52	9.43	8.69	6.92
3	28.16	23.33	21.16	18.91	18.14	17.28	15.60	14.76	14.11	13.96	13.26	12.68	12.41	11.94	11.47	10.68	10.35	9.16	8.65	6.63
4	27.63	22.55	21.01	18.84	17.53	16.76	15.47	14.71	14.06	13.81	13.08	12.61	12.26	11.59	11.13	10.64	10.34	9.13	8.18	6.59

5	27.15	22.01	10.70	18.82	17.53	16.65	15.36	14.61	14.01	13.45	12.80	12.59	12.09	11.43	11.03	10.42	9.77	8.84	8.13	6.41
							(Abbreviated)													
96	16.80	15.02	13.52	13.11	12.71	11.93	11.25	10.77	9.95	9.38	8.85	8.40	7.90	7.20	6.71	6.05	5.55	4.28	2.61	0.45
97	16.78	14.97	13.27	13.05	12.35	11.92	11.24	10.49	9.80	9.33	8.80	8.39	7.82	7.09	6.70	6.04	5.31	4.01	2.40	0.45
98	15.80	14.55	13.22	12.93	12.35	11.76	11.08	10.22	9.62	9.21	8.78	8.35	7.31	7.05	6.59	5.94	5.05	3.99	2.33	0.45
99	15.55	14.33	13.20	12.92	12.22	11.76	10.94	9.80	9.22	8.95	8.29	8.18	7.21	6.67	6.52	5.58	4.84	3.92	2.26	0.45
100	14.69	13.64	13.04	12.87	12.14	11.34	9.47	9.35	9.08	8.54	7.75	7.49	6.70	6.59	6.51	5.40	3.58	3.48	1.26	0.45

MONTH 3

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	45.66	35.11	33.86	30.44	26.20	24.68	23.50	22.23	20.41	18.84	16.93	15.90	15.16	14.97	14.33	13.18	11.26	10.07	9.33	8.11
2	43.09	34.93	29.65	28.94	25.92	24.31	22.74	22.08	19.85	17.95	16.64	15.67	14.73	14.44	13.97	12.96	11.13	9.72	8.91	6.88
3	42.93	34.71	29.57	26.37	25.72	23.70	22.34	21.67	19.29	17.74	16.48	15.30	14.65	14.23	13.97	12.50	10.84	9.52	8.26	6.53
4	42.61	31.58	28.96	26.35	25.47	23.24	22.24	21.60	19.28	17.74	16.41	15.24	14.62	14.05	13.49	12.26	9.74	9.35	8.13	6.21
5	39.86	31.29	28.36	26.04	24.69	22.92	21.90	21.17	18.24	17.69	16.38	15.19	14.46	14.01	13.34	11.61	9.70	9.29	7.62	5.93
							(Abbreviated)													
96	23.03	19.76	18.09	16.92	15.27	13.83	13.05	11.84	9.95	9.60	8.53	7.67	7.31	6.26	5.90	4.88	4.13	3.12	2.35	1.12
97	22.24	19.06	18.02	16.74	15.23	13.64	12.97	11.24	9.78	9.45	8.14	7.61	7.25	6.22	5.49	4.81	4.09	3.10	2.07	1.12
98	21.84	18.80	17.99	15.33	14.98	13.63	12.64	10.14	9.68	8.97	8.10	7.49	6.61	6.14	5.47	4.72	4.03	2.71	2.01	1.12
99	21.46	18.55	16.30	15.33	14.58	13.03	12.53	9.81	9.52	8.78	7.83	7.42	6.61	5.83	5.28	4.60	3.41	2.68	1.84	1.12
100	19.54	16.48	14.87	14.78	14.44	10.93	10.60	9.68	8.54	8.28	7.80	7.37	5.90	5.72	4.92	4.32	2.93	2.57	1.74	1.12

MONTH 4

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	35.96	26.02	23.21	22.74	20.99	19.36	18.29	17.57	16.04	14.12	12.75	12.43	12.30	10.99	10.27	8.74	6.98	6.32	6.11	4.92
2	30.71	24.72	22.74	20.65	19.92	18.21	17.40	16.38	16.03	13.52	12.73	12.42	11.22	10.51	10.03	8.70	6.94	6.24	6.05	4.54
3	29.70	24.58	21.86	20.45	19.88	17.57	16.86	16.00	15.51	13.41	12.62	11.95	11.05	10.18	9.14	7.99	6.91	6.00	5.63	4.43
4	29.61	23.87	21.82	20.37	19.04	18.50	16.70	15.63	13.80	13.14	12.54	11.62	10.72	9.97	9.08	7.84	6.77	5.89	5.63	3.91
5	27.96	23.40	21.76	20.16	17.90	17.21	16.51	15.25	13.49	13.14	12.40	11.33	10.65	9.94	8.77	7.62	6.69	5.86	5.29	3.88
							(Abbreviated)													
96	16.29	14.91	13.21	12.18	11.51	10.70	9.39	8.53	7.36	6.96	5.72	5.3	4.81	4.43	4.19	3.68	2.46	1.62	0.80	0.00
97	16.05	14.86	13.19	12.18	11.48	10.69	9.35	8.23	7.05	6.84	5.44	5.2	4.81	4.40	4.01	3.44	2.30	1.45	0.69	0.00
98	16.00	14.83	13.16	11.82	11.33	10.56	8.97	8.12	6.80	5.83	5.41	5.1	4.69	4.27	3.73	3.01	2.07	1.29	0.69	0.00
99	15.77	14.59	13.13	11.67	11.27	9.57	8.88	7.52	6.75	5.78	5.35	5.1	4.61	4.16	3.63	2.93	1.84	1.18	0.64	0.00
100	15.47	14.12	12.83	11.31	10.93	7.76	7.56	7.28	6.55	5.65	5.35	4.9	4.55	3.95	3.33	2.84	1.68	0.99	0.40	0.00

MONTH 5

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	31.48	25.85	25.41	22.28	19.39	18.84	15.94	14.43	14.36	12.99	12.18	12.05	11.56	9.64	9.50	8.71	7.40	7.29	6.36	5.88

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
							15.40	14.03	13.01	12.72	12.05	11.83	11.54	9.58	9.06	8.28	7.23	6.45	5.83	5.35
2	29.97	24.41	23.44	19.16	17.30	16.32	14.32	13.65	12.69	12.46	11.62	11.26	10.84	9.47	8.84	8.20	7.21	6.33	5.63	5.23
3	29.78	23.71	21.01	18.59	16.65	15.44	13.78	13.32	12.64	12.31	11.33	11.16	10.35	9.38	8.83	8.02	6.94	6.11	5.47	4.55
4	29.69	23.56	20.08	18.36	16.56	15.32	13.64	13.19	12.50	11.91	10.99	10.87	9.89	9.33	8.58	7.84	6.86	6.05	5.46	4.35
5	29.51	23.50	19.87	18.01	16.51	15.29	(Abbreviated)													
							8.81	7.81	6.85	6.64	6.13	5.74	4.92	4.79	4.01	3.60	2.95	2.18	1.30	0.34
96	15.39	13.27	12.23	11.10	10.52	9.76	8.73	7.70	6.82	6.31	5.95	5.42	4.85	4.58	3.92	3.50	2.80	2.14	1.03	0.29
97	14.96	13.26	11.90	11.07	10.37	9.68	8.65	7.66	6.78	6.22	5.86	5.21	4.72	4.51	3.78	3.33	2.56	2.03	1.00	0.23
98	14.81	13.11	11.21	10.88	10.12	9.51	8.60	6.55	5.97	5.87	5.40	4.96	4.59	4.41	3.58	3.25	2.54	1.95	1.00	0.00
99	14.03	12.83	11.21	10.85	9.82	9.40	6.84	5.71	5.47	4.84	4.80	3.96	3.93	3.36	3.30	2.85	2.25	1.82	0.81	0.00
100	13.50	12.71	11.12	9.86	8.90	8.43														

MONTH 6

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
							17.59	14.71	14.45	13.68	12.99	11.21	10.70	10.38	10.11	9.59	8.58	8.48	8.05	6.55
1	51.79	32.89	24.61	21.14	20.16	18.09	16.58	14.62	14.24	13.51	11.88	11.07	10.65	10.35	9.36	9.06	8.34	8.03	7.05	6.37
2	42.76	32.77	24.29	20.91	19.85	17.95	15.90	14.49	14.03	13.08	11.64	10.87	10.63	9.82	9.36	8.84	7.84	7.06	6.87	5.56
3	42.55	31.44	23.14	20.18	19.37	17.91	15.59	14.39	13.81	12.61	11.30	10.80	10.39	9.65	9.23	8.81	7.82	6.90	6.57	5.45
4	40.11	29.54	22.93	20.10	18.56	17.72	15.54	14.27	13.53	12.45	11.30	10.68	10.35	9.48	9.15	8.55	7.67	6.59	6.25	5.06
5	38.01	29.26	22.45	20.10	18.55	17.60	(Abbreviated)													
							9.30	8.15	7.14	6.71	6.32	5.83	5.66	5.04	4.59	4.09	3.41	2.77	2.28	1.55
96	15.47	13.89	12.93	11.32	10.29	9.83	9.28	8.03	7.08	6.67	6.19	5.78	5.52	5.00	4.59	4.01	3.26	2.71	2.14	1.44
97	14.98	13.25	12.63	10.84	10.21	9.57	8.81	7.47	7.01	6.59	6.00	5.70	5.38	4.77	4.50	3.70	3.12	2.71	2.04	1.42
98	13.90	13.15	11.70	10.69	10.17	9.28	8.46	7.46	6.72	6.31	5.82	5.39	5.31	4.69	4.27	3.47	2.85	2.49	2.03	1.28
99	13.57	12.98	10.44	10.43	9.57	9.16	8.29	6.66	6.61	6.03	5.73	5.38	5.14	4.59	4.23	3.37	2.80	2.42	2.01	0.88
100	12.66	11.93	9.90	9.84	9.36	9.14														

MONTH 7

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
							19.06	18.11	16.73	14.86	14.55	14.27	14.19	12.37	12.02	11.54	10.35	9.91	7.50	6.97
1	45.06	31.90	31.87	25.87	23.72	21.97	18.80	18.00	16.48	14.80	13.87	13.29	12.72	12.10	11.26	10.44	10.30	9.04	7.43	6.52
2	44.70	31.63	28.76	25.57	23.70	21.76	18.58	16.68	15.65	14.78	13.87	12.84	12.52	11.94	11.09	9.65	9.61	8.86	7.15	6.09
3	42.78	31.56	28.56	24.91	22.53	20.74	17.60	16.12	15.33	14.47	13.63	12.82	12.08	11.53	11.06	9.53	9.23	8.82	7.13	6.07
4	42.40	30.67	26.15	24.61	21.97	20.38	17.57	15.75	15.16	14.41	13.44	12.69	11.79	11.34	10.76	9.48	9.07	8.30	7.03	5.82
5	41.78	28.83	25.66	23.79	21.68	19.95	(Abbreviated)													
							11.30	10.50	10.01	9.04	8.41	7.63	7.09	6.52	5.70	5.05	4.41	3.45	2.61	1.59
96	19.15	15.80	14.46	13.60	12.66	11.99	11.17	10.16	9.56	8.77	8.34	7.62	6.98	6.45	5.65	5.04	4.26	3.44	2.61	1.57
97	18.62	15.71	14.14	13.55	12.59	11.62	10.97	10.06	9.28	8.53	8.25	7.46	6.98	6.25	5.58	5.00	4.10	3.42	2.44	1.57
98	17.32	15.58	13.68	13.43	12.32	11.42	9.86	9.47	9.28	8.47	8.24	7.17	6.94	6.24	5.25	4.65	3.85	3.24	2.28	1.51
99	16.84	14.43	13.02	11.94	11.23	10.33	9.48	9.40	8.89	8.29	8.17	7.12	6.71	6.15	4.43	4.42	3.65	3.08	2.00	1.27
100	14.32	13.83	12.74	11.36	10.55	10.09														

MONTH 8

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	67.06	56.52	37.96	31.57	29.44	25.05	22.61	21.35	20.12	16.61	14.53	14.47	13.29	11.48	8.84	8.64	7.38	7.17	6.99	4.84
2	60.91	46.99	34.36	31.02	29.01	23.76	22.11	21.05	18.10	14.96	14.22	13.95	11.62	10.75	8.79	8.24	7.21	6.87	5.32	4.74
3	58.72	40.77	34.18	30.61	28.00	23.64	21.30	18.69	16.60	14.84	13.82	12.37	10.46	9.59	8.71	7.92	7.16	6.51	5.07	4.38
4	58.18	37.84	33.58	29.53	24.54	23.38	20.23	18.52	16.15	14.52	12.66	11.14	10.17	9.56	8.41	7.41	6.58	6.39	5.05	4.24
5	57.83	37.64	33.46	29.46	24.42	23.26	18.69	17.92	15.51	14.05	12.62	10.92	10.05	9.25	8.37	7.36	6.57	6.09	5.01	4.21
(Abbreviated)																				
96	21.39	16.05	15.08	13.12	11.98	10.99	9.04	8.17	7.76	6.89	6.35	5.98	5.37	4.55	3.99	3.33	2.49	1.99	1.17	0.24
97	21.31	15.81	13.62	12.72	11.75	10.15	9.02	8.11	7.44	6.81	6.35	5.72	5.34	4.48	3.87	3.33	2.41	1.97	1.13	0.24
98	20.32	15.43	13.37	12.58	11.47	9.69	8.44	7.82	7.42	6.64	6.20	5.69	5.26	4.15	3.65	3.14	2.35	1.74	1.06	0.07
99	19.89	14.17	13.25	12.38	11.05	9.35	8.10	7.68	6.89	6.45	6.02	5.61	4.75	3.72	2.87	2.65	2.07	1.70	0.74	0.06
100	16.03	13.41	12.63	12.21	9.67	9.17	8.02	7.06	6.82	6.21	5.92	4.92	4.68	3.12	2.42	2.42	1.69	1.69	0.16	0.06

MONTH 9

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	31.01	23.05	20.15	19.52	18.91	17.88	15.22	13.83	12.50	10.38	9.38	8.69	8.05	8.05	7.23	6.40	6.36	5.28	4.91	3.55
2	29.33	22.44	19.80	18.30	17.11	15.44	15.12	13.14	11.25	9.69	9.09	8.68	7.86	7.80	6.63	6.31	5.73	5.10	4.01	3.37
3	29.03	22.29	19.34	17.76	16.48	15.44	14.58	12.98	10.57	9.65	9.04	8.51	7.85	7.02	6.54	6.20	5.64	4.74	3.92	3.16
4	28.56	21.80	19.28	17.12	15.99	15.33	14.00	12.05	10.45	9.63	8.93	8.39	7.66	6.75	6.42	6.03	5.35	4.52	3.84	3.02
5	26.86	21.75	18.33	17.10	15.98	14.75	13.87	11.89	10.40	9.40	8.78	8.12	7.44	6.73	6.23	6.01	5.04	4.41	3.76	2.70
(Abbreviated)																				
96	13.87	11.39	9.59	8.91	7.80	7.03	6.68	5.95	5.42	4.76	4.56	4.07	3.40	3.19	2.53	2.27	1.56	1.01	0.43	0.00
97	13.01	10.72	9.51	8.71	7.61	7.01	6.39	5.91	4.89	4.66	4.35	3.80	3.32	3.16	2.45	2.14	1.20	0.99	0.37	0.00
98	12.96	9.90	9.49	8.60	7.05	7.00	5.90	5.87	4.82	4.65	4.31	3.67	3.27	2.87	2.44	2.04	1.08	0.89	0.34	0.00
99	12.44	9.55	8.10	7.04	6.67	6.51	5.62	5.60	4.73	4.41	4.15	3.59	3.19	2.82	2.34	1.81	1.02	0.67	0.21	0.00
100	10.52	9.39	6.74	6.56	6.32	5.35	4.97	4.95	4.65	4.06	3.97	3.42	3.00	2.70	2.10	1.16	1.01	0.45	0.13	0.00

MONTH 10

SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	30.96	19.29	15.98	15.23	15.03	12.50	12.29	11.75	10.49	9.74	8.39	8.19	7.92	6.57	6.08	5.46	4.58	4.42	3.19	2.42
2	29.23	17.54	15.96	14.52	13.63	12.47	11.78	10.91	9.96	9.38	8.32	7.50	7.04	5.96	5.72	4.97	4.52	3.98	2.97	2.24
3	26.69	17.48	15.70	13.90	13.17	12.42	11.55	10.91	9.85	9.14	8.22	7.24	6.94	5.91	5.21	4.67	4.33	3.72	2.96	2.17
4	23.75	17.42	15.36	13.89	13.10	12.23	11.24	10.51	9.72	8.97	8.08	7.08	6.55	5.71	5.15	4.63	4.27	3.69	2.75	2.11
5	23.35	17.28	15.04	13.88	12.99	11.59	11.18	10.27	9.43	8.53	8.05	6.98	6.45	5.58	5.13	4.61	4.14	3.62	2.71	1.99
(Abbreviated)																				
96	12.12	10.17	8.82	7.52	6.52	6.11	5.74	4.92	4.65	4.46	3.98	3.64	3.15	2.74	2.30	1.62	1.06	0.63	0.15	0.00
97	12.08	10.14	8.58	7.00	6.30	5.98	5.46	4.88	4.51	4.42	3.57	3.45	3.14	2.57	2.21	1.60	1.04	0.59	0.07	0.00
98	11.45	9.56	8.49	6.92	6.27	5.96	5.07	4.77	4.38	4.31	3.50	3.27	2.87	2.44	1.97	1.49	0.96	0.49	0.00	0.00

99	11.12	8.89	7.74	6.68	6.26	5.55	5.03	4.73	4.37	4.29	3.34	2.99	2.84	2.38	1.83	1.43	0.79	0.45	0.00	0.00
100	10.06	7.83	7.01	6.23	5.76	5.52	4.65	4.58	4.35	4.21	3.27	2.96	2.83	2.11	1.80	1.26	0.55	0.02	0.00	0.00

MONTH 11
SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	41.42	33.31	26.56	24.71	23.56	20.74	18.38	16.47	14.58	12.08	11.62	10.08	10.07	9.72	9.31	6.83	6.01	4.90	4.09	3.21
2	39.11	32.90	26.27	24.04	21.71	20.03	18.26	16.38	13.81	12.05	10.72	9.88	9.27	7.96	7.73	6.29	5.92	4.63	4.09	2.64
3	37.55	32.17	25.48	23.84	21.59	19.53	18.22	16.24	13.74	11.82	10.49	9.63	8.94	7.50	6.58	5.27	5.12	4.60	3.94	2.53
4	37.26	31.80	24.47	23.73	21.54	18.92	17.79	14.86	13.50	11.72	10.35	9.27	8.41	7.16	6.19	5.20	5.00	4.57	3.69	2.28
5	37.08	30.83	24.15	22.58	20.91	18.47	16.51	14.47	12.98	11.59	10.16	8.81	8.37	6.67	6.08	5.14	4.97	4.57	3.40	2.16
						(Abbreviated)														
96	21.34	15.89	13.47	10.99	9.26	8.64	7.05	6.13	5.38	4.73	4.28	3.87	3.47	3.06	2.76	2.08	1.20	0.52	0.20	0.00
97	20.51	15.59	12.93	10.44	8.12	7.27	6.83	6.07	5.31	4.57	4.20	3.79	3.37	2.97	2.48	1.93	1.20	0.48	0.16	0.00
98	19.99	15.50	12.55	9.60	7.83	7.09	6.74	6.03	5.23	4.28	4.15	3.56	3.31	2.88	2.38	1.79	0.84	0.48	0.12	0.00
99	19.85	13.42	12.33	8.85	6.98	6.61	6.28	5.30	5.15	4.14	4.13	3.46	3.30	2.64	2.28	1.50	0.64	0.40	0.09	0.00
100	15.31	13.07	8.63	7.74	6.61	6.31	6.19	5.03	4.43	4.09	4.05	3.33	3.23	2.53	2.16	1.06	0.54	0.39	0.00	0.00

MONTH 12
SORTED DATA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	37.19	30.79	25.92	24.20	23.32	22.96	18.90	18.47	18.32	15.63	15.04	13.75	12.07	11.55	11.31	10.52	9.31	8.95	8.61	6.47
2	34.20	28.74	24.41	23.63	21.79	20.21	18.60	17.27	16.12	14.64	13.26	12.72	11.67	11.52	11.08	10.49	9.29	7.95	7.44	5.77
3	34.11	28.55	24.08	23.22	20.77	19.51	17.82	17.07	15.80	13.90	13.05	12.18	11.35	11.24	10.64	9.93	9.28	7.58	7.39	5.76
4	33.86	27.39	24.02	23.18	20.56	19.45	17.53	16.85	15.63	13.83	13.02	11.80	11.35	10.54	10.38	9.83	9.02	7.54	6.96	5.66
5	33.62	26.96	23.56	22.44	20.17	19.20	17.47	16.54	15.21	13.83	12.80	11.80	11.33	10.49	10.19	9.61	8.55	7.47	6.92	5.51
						(Abbreviated)														
96	19.01	15.94	13.58	12.24	11.58	10.84	9.82	9.21	8.69	7.96	7.33	6.79	6.59	6.00	5.03	4.31	3.19	2.26	1.29	0.41
97	18.79	14.90	13.45	12.17	11.56	10.76	9.73	9.21	8.51	7.60	7.29	6.74	6.45	5.60	4.97	4.00	3.11	2.14	1.28	0.41
98	17.02	14.82	13.35	11.82	11.21	10.56	9.66	9.15	8.30	7.51	7.03	6.70	6.38	5.56	4.71	3.56	2.97	1.94	1.11	0.41
99	15.87	14.21	11.97	11.37	10.99	10.48	9.47	9.00	8.08	7.50	6.84	6.53	5.99	5.52	4.65	3.35	2.26	1.68	0.74	0.41
100	13.83	11.61	11.43	11.04	10.90	10.34	9.30	8.33	7.71	7.13	6.66	6.31	5.97	4.87	4.64	3.20	2.01	1.46	0.71	0.41