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ESTIMATION PROCEDURES FOR DETERMINING ANNUAL TREE  
MORTALITY CAUSED BY THE MOUNTAIN PINE BEETLE

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## ABSTRACT

Multistage sampling techniques were used in 1978 to estimate ponderosa pine mortality caused by the mountain pine beetle in the Black Hills of South Dakota and Wyoming. Using aerial sketch mapping information, the survey area of 651,000 acres (263,000 ha) was stratified into 3 intensity classes based on the estimated number of dead trees per acre. Each of the 2 highest intensity classes or strata were sampled from large scale (1:6,000) color aerial photographs and on the ground. Samples were selected using systematic random procedures in the first stage and probabilities proportional to size in the second stage.

Correlations between photo and ground counts were very good for each stratum sampled. On the 303,000 acres sampled, an estimated 318,000 ponderosa pine representing a volume of 5,629,000 cubic feet were killed in 1978. These number and volume estimates had standard errors of 4.2 and 7.2 percent, respectively.

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ESTIMATION PROCEDURES FOR DETERMINING ANNUAL TREE  
MORTALITY CAUSED BY THE MOUNTAIN PINE BEETLE

(Dendroctonus ponderosae)

by

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## INTRODUCTION

In the western United States the mountain pine beetle (*Dendroctonus ponderosae* Hopkins) causes more coniferous tree mortality than any other insect. The location and extent of beetle-caused tree mortality is imperative information for proper short- and long-term management of pine forests. Thus, a fast, accurate method of obtaining this information is needed.

A survey was conducted in 1978 to establish efficient sampling methods for estimating annual tree mortality and volume loss attributed to the mountain pine beetle (MPB). The survey was conducted using multistage sampling techniques in the Black Hills area of South Dakota and Wyoming on the ponderosa pine (*Pinus ponderosa* Laws.) resource. Reported here are the techniques and results of this 1978 ponderosa pine mortality survey.

The primary objectives of the 1978 Black Hills survey were: (1) to demonstrate a multistage survey technique designed to measure annual ponderosa pine mortality caused by MPB, and to estimate (2) total number of 1977-infested ponderosa pine, (3) total volume of 1977-infested ponderosa pine, and (4) the ratio of ponderosa pine infested in 1978 to those infested in 1977.

## DESCRIPTION OF SURVEY SITE

The 1978 survey involved about 1.5 million acres (0.61 million ha) of ponderosa pine in the Black Hills of southwestern South Dakota and northeastern Wyoming. Most of the timber is located on two specific geological types: (1) a central area composed of crystalline rocks and characterized by rough to rounded hills and divides with elevations ranging from 4,300 to 7,000 feet (1,310 to 2,130 m) and (2) an extensive limestone plateau on the western side consisting of rather level divides separated by narrow, steep valleys with elevations ranging from 4,500 feet (1,370 m) to slightly over 7,000 feet (2,130 m).

Ponderosa pine is the dominant tree species in the survey area with interspersed aspen (*Populus tremuloides* Michx.), white spruce (*Picea glauca* (Moench)), paper birch (*Betula papyrifera* Marsh.) and bur oak (*Quercus macrocarpa* Michx.).



The area surveyed is principally within the Black Hills National Forest with intermingled and adjoining lands owned and/or administered by the Bureau of Land Management, South Dakota State Division of Forestry, Wyoming State Forestry Division, the National Park Service, or private concerns.

## METHODS AND MATERIALS

### Survey Design

A stratified two-stage sampling design was employed (Fig. 1). Stratification was based on the results of aerial sketch mapping. First stage sampling was based on 90-acre (36.4-ha) photo plots selected by a systematic random process for each of two strata. Second stage samples were selected with probabilities proportional to the number of 1977-infested trees (i.e., faders) counted on the photo plots (Cochran 1977, p. 251). The final 2.5-acre (1.0-ha) subplots to be ground-checked were selected, again by probabilities proportional to size (pps), from second stage samples. The number of faders and volume estimates were determined on the ground. Specific details for each step follow.

### Aerial Sketch Mapping

Aerial sketch mapping was initiated when an estimated 95 percent of the 1977-attacked trees exhibited obvious discoloration. The survey areas were sketch mapped from a light aircraft at a maximum height of 1,500 feet (460 m) above the terrain. Polygons containing discolored ponderosa pines were segregated, an estimate of the number of faders in each area was made, and the boundaries of each area were outlined on 1:126,720 scale planimetric maps (Fig. 2). Approximately 26 hours of flying were required to map about 1.9 million acres (0.8 million ha) during the period of July 31 through August 3.

### Stratification

Following sketch mapping, polygon boundaries were traced from the maps onto a plastic (mylar) surface. The area of each sketch mapped polygon was then measured using a digitizing planimeter and the number of faders per acre for each polygon was calculated. Polygons were segregated into three strata based on the number of faders per acre (Fig. 3 and Table 1). The stratum boundaries were



established for stratification purposes and have no biological significance. Stratum 1 ( $< 0.1$  faders/acre) was excluded from the further sampling because of its low level of mortality. Sampling was confined to strata 2 and 3 with 302,983 acres (122,616 ha).

### First Stage Sampling

The primary sampling unit (PSU) was a 90-acre (36.4-ha) photo plot. Samples were selected independently from each of strata 2 and 3. A systematic random procedure was used to select 109 and 77 plots in strata 2 and 3 respectively. The sample allocation for the first stage follows:

<u>Stratum</u>	<u>Size of stratum (acres)</u>	<u>Size of photo plot (acres)</u>	<u>No. of PSU's</u>	<u>No. of Samples</u>	<u>Sampling fraction</u>
2	222,554	90	2473	109	.044
3	80,429	90	894	77	.086

To facilitate operational constraints, photo plots were selected along north-south flight lines spaced one mile (1.6 km) apart (Fig. 4). These plots were photographed on August 20-26 using a Fairchild KA-2 (12 inch (30.5 cm) focal length) camera mounted in a Cessna T-337 aircraft. Color aerial (MS-2448) film 9x9 inch (3.54 x 3.54 cm) stereo triplets were taken of each of the 186 plots at a scale of approximately 1:6,000. The altitude of the aircraft was recorded several times at specific points along each flight line so that the true scale could be determined for each photograph.

### Photo Interpretation

A series of acetate templates were prepared for a range of scales (1:5,400 - 1:6,600). Etched on each acetate template were boundaries of a square 90-acre (36.4-ha) plot and a grid within this plot delineating 36 square 2.5-acre (1.0-ha) subplots. A template of the proper scale was placed onto the center of the effective area of each photo prior to interpretation. Photo plots were interpreted using Old Delft scanning stereoscopes. Interpreters recorded the number of visible faders within each 2.5-acre (1.0-ha) subplot.



## Second Stage Sampling

Twenty-five photo plots were chosen with replacement from the 109 and 77 plots of strata 2 and 3 respectively using pps. Secondly, the same procedure was used to select 2 of the 36 2.5-acre subplots from each of the 25 photo plots of each stratum. These 100 subplots were then surveyed on the ground.

Selected subplot boundaries were etched on the emulsion sides of the appropriate stereo pairs. These stereo pairs along with 1:24,000 U. S. -eological Survey topographic maps were used in the field to locate subplot boundaries. Transparencies were placed into light-weight plastic film holders and examined with a pocket stereoscope using transmitted sunlight as the primary light source. If available sunlight was below suitable illumination levels, it was supplemented with portable fluorescent lights.

Ground truth data from the 100 subplots were obtained during the period of September 5 to October 20, 1978. Each crew of 2-3 people located subplots and marked the boundaries with string. One-chain-wide (20.1-m-wide) strips within each subplot were also bounded with string to expedite collection and insure accuracy of the data. Numbers of faders and 1978-attacked ponderosa pines 5 inches (12.7 cm) DBH and larger were recorded by 1-inch (2.5-cm) diameter class. Heights of all ponderosa pines 20 inches (50.8 cm) DBH or larger were measured to augment existing volume tables of Myers (1964).

Five variable radius plots were established within each subplot. One variable plot was located at the center of the subplot and one at each cardinal direction 2 chains (40.2 m) from plot center. A Spiegel relaskop (BAF 10) on a Jacob staff was used to determine trees to be tallied. Recorded by species within each variable plot were all live trees 5 inches (12.7 cm) DBH and greater. Also recorded were all ponderosa pines 5 inches (12.7 cm) DBH and greater that had been infested by MPB in 1977 (faded needles; no MPB present) or in 1978 (green needles; MPB present). The distance from plot center to tree center was measured for borderline trees. These distances were compared to those of limiting distance tables to determine which trees were to be tallied. For each subplot the elevation of plot center and length of one boundary were determined as a check on scale accuracy. Most of the subplots were actually smaller than 2.5-acres (1.0-ha), thus, the number and volume estimates were adjusted accordingly.





## Data Analysis

Standard multistage estimators were used to derive the specific formulae used for the sample design (Appendices A and B). The analysis consisted of (1) generating the number and volume of ponderosa pine killed annually and (2) determining the relationship between photo interpreter counts of faders and counts of faders made on the ground.

## RESULTS

The total number of faders (i.e., 1977-attacked trees) was 318,417 from 302,983 acres (122,616 ha) surveyed having an associated volume of 5.6 million cubic feet (0.16 million m<sup>3</sup>) (Table 2). Relative sampling errors were 4.5 and 7.2 percent for number of faders and volume of faders respectively. These relative standard errors are at levels which are acceptable for management purposes. The 95% confidence intervals indicate that 19 out of 20 times the number of faders would fall between 289,633 and 347,301, and fader volume between 4.8 and 6.4 million cubic feet (0.14 and 0.18 million m<sup>3</sup>).

Numbers of faders determined from ground counts and from photo counts were well correlated for both strata 2 and 3 with coefficient of determination ( $R^2$ ) values of 0.96 and 0.90 respectively, (Figs. 5 and 6). Combined data from these two strata showed an  $R^2$  value of 0.93 (Fig. 7). Linear regression slope (b) estimates using strata 2 and 3 data indicated that the ratios of photo counts to ground counts were very close to 1:1. The ratio of the number of trees attacked by MPB in 1978 to those attacked in 1977 (i.e., attack ratio) was estimated for each stratum and for the two strata combined. Attack ratios were 0.79, 1.62, and 1.14 for strata 2, 3, and combined, respectively (Table 3). Because of the large standard errors of the estimates of newly attacked pines (Table 3) these attack ratios have little precision and should be used with caution.

The basal areas of trees by species were estimated from data gathered in the variable radius plots (Table 4). The basal area ratios of total dead <sup>1/</sup> to total living ponderosa pine were 0.23 and 0.38 for strata 2 and 3 respectively.

---

<sup>1/</sup>

This includes all trees infested by MPB in 1977 and 1978 as well as those that had been killed in previous years.



From ground plot data the greatest percentage (16.6) of faders was in the 8-inch (20.3-cm) diameter class (Fig. 8) as was the greatest percentage (15.3) of 1978-attacked trees (Fig. 9). The highest percentage (10.9) of uninfested ponderosa pines fell into the 9-inch (22.9-cm) diameter class (Fig. 10).

## DISCUSSION

Mortality estimates derived from aerial sketch mapping comprised an effective base for stratification of the survey area even though these estimates were only 0.2 to 0.4 times as great as those derived from multistage sampling.

The multistage sampling design using pps for second stage sample selection seems to be a practical and efficient procedure for estimating tree mortality attributable to MPB. A similar survey conducted in the Black Hills in 1977 (Hostetler and Young 1979) using techniques involving double sampling and regression did not provide adequate results due to the patchy distribution of MPB-killed ponderosa pine. However, second stage samples of the 1977 survey were selected randomly. Low sampling errors of the 1978 survey were the result of selecting second stage samples using pps. This procedure greatly reduced the probability of selecting sample plots which contained no or very few 1977-attacked trees.

A lodgepole pine (*Pinus contorta* var. *latifolia* Engelm.) mortality survey in Idaho in 1977 (Klein et al. 1979) did find the techniques using double sampling and regression to be successful. However, the distribution of MPB-killed lodgepole pine is much more homogeneous than that of ponderosa pine.

Fader counts made on ground plots and from corresponding photographs were highly correlated (Figs. 5-7). Thus, the photo interpreters were consistent in their counting on all photos. In addition the slopes of regression lines determined from these counts were approximately equal to 1.0 indicated that all faders counted on the ground were also counted on the photographs.



The multistage survey methods used were successful in estimating ponderosa pine mortality caused by MPB in the Black Hills. Even though the area covered in this survey contained predominately pure ponderosa pine forests, we suggest that these survey methods will, with some modification, be successful in forested areas containing ponderosa pine mixed with other tree species.



MULTISTAGE SAMPLING

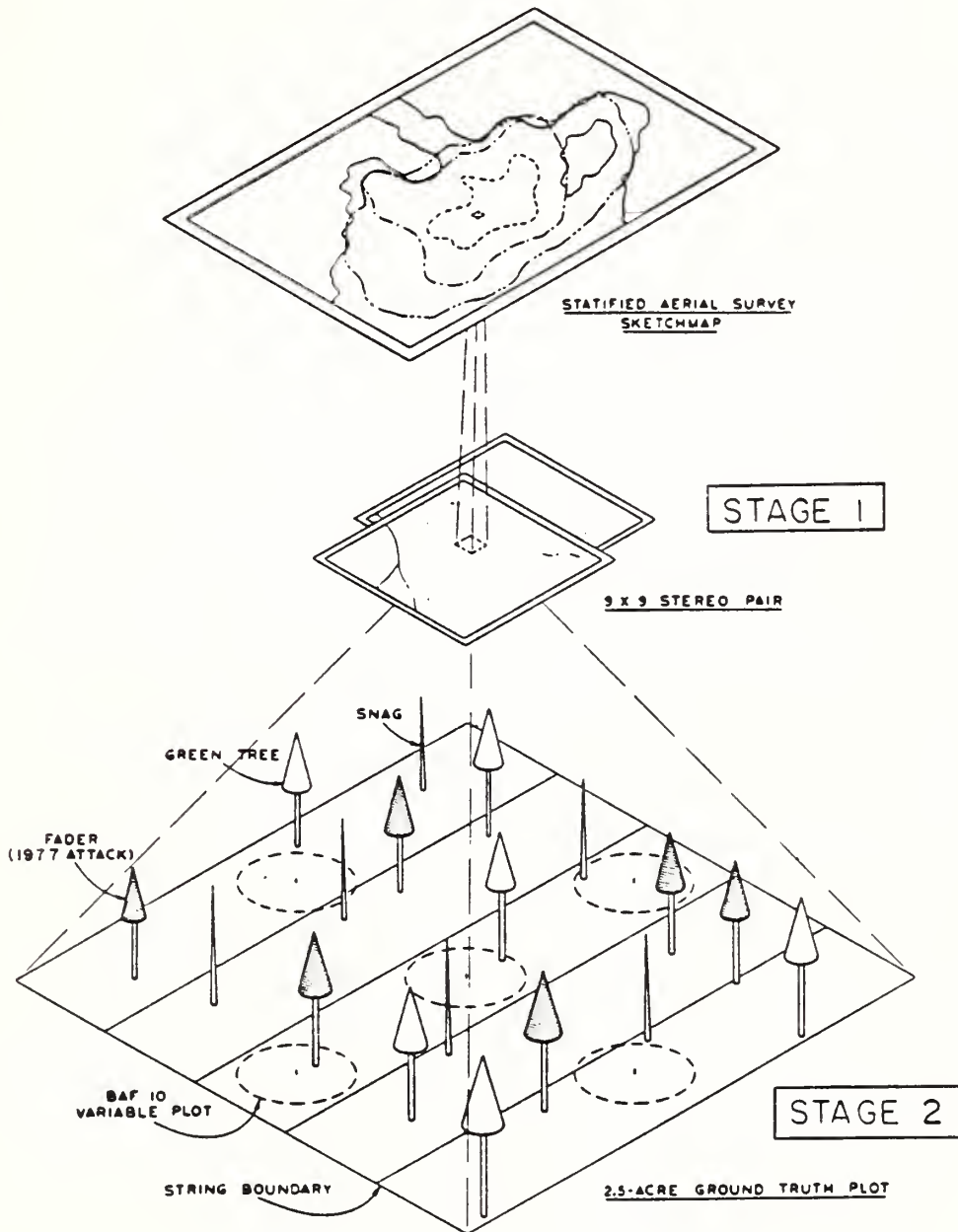


Figure 1. Diagrammatic representation of the stages and procedures used for the 1978 ponderosa pine mortality survey, Black Hills of South Dakota and Wyoming. (Modified and taken from Klein *et al.* (1979, p. 5))





FIGURE 2. Delineation of polygon boundaries as determined by aerial sketch mapping

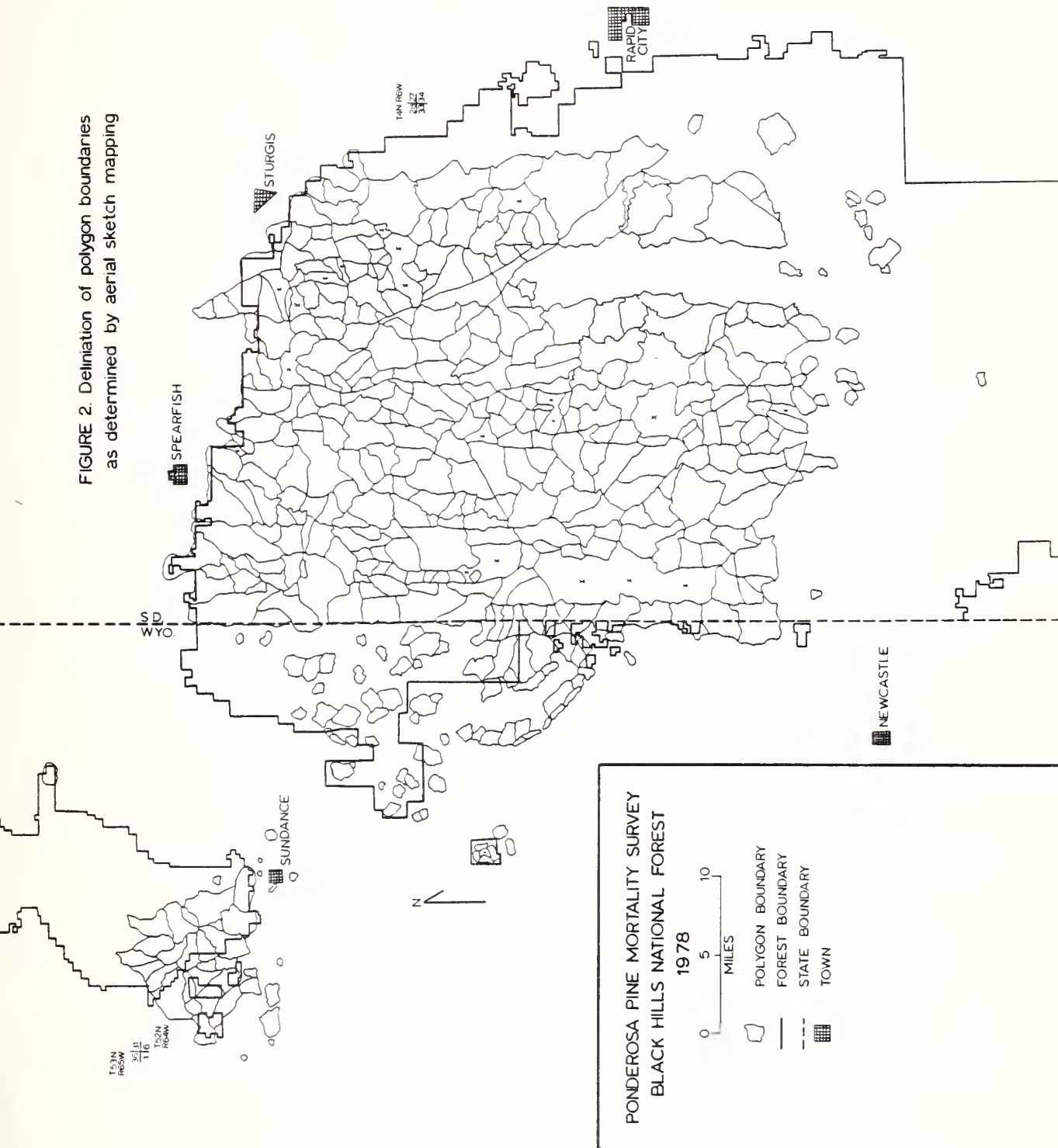




FIGURE 3 Distribution of MPB-killed ponderosa pine stratified by infestation intensity

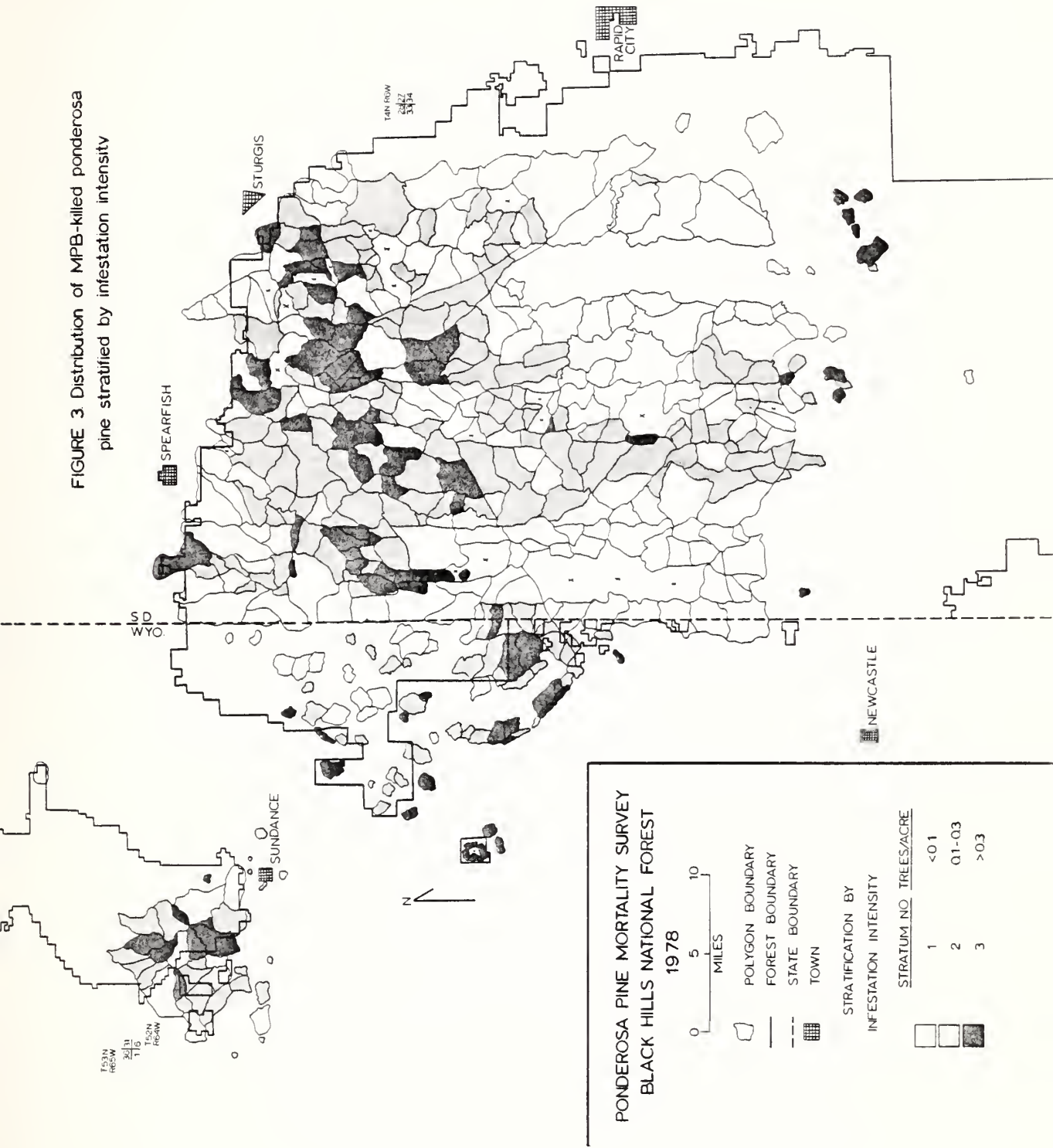
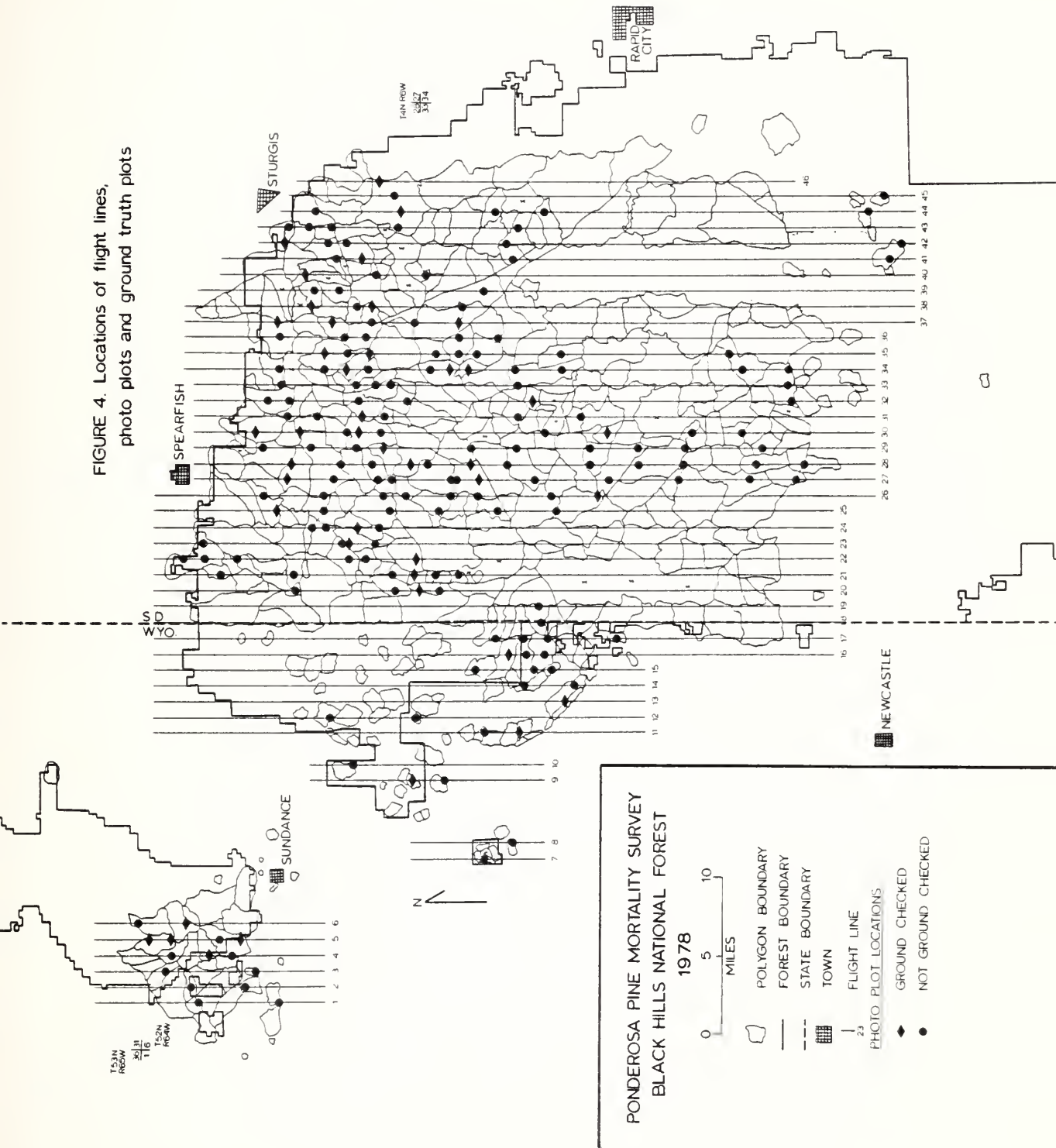




FIGURE 4. Locations of flight lines, photo plots and ground truth plots





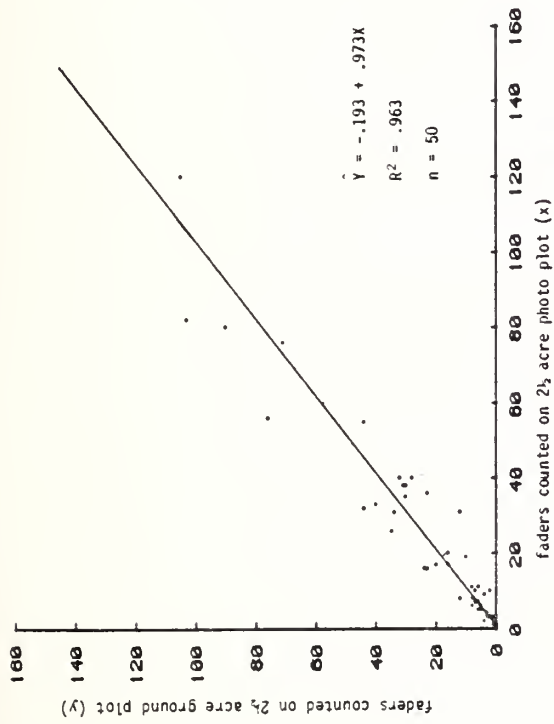


Figure 5. Photo and ground counts regression, Stratum 2, Black Hills pine mortality survey, 1978.

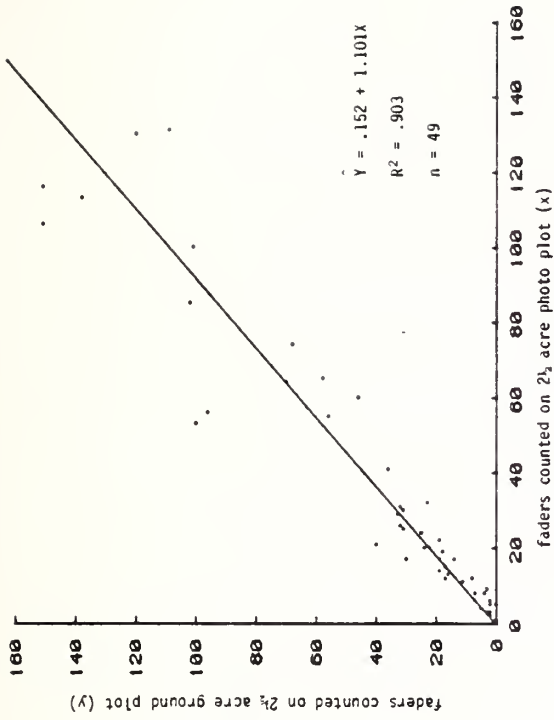


Figure 6. Photo and ground counts regression, Stratum 3, Black Hills pine mortality survey, 1978.

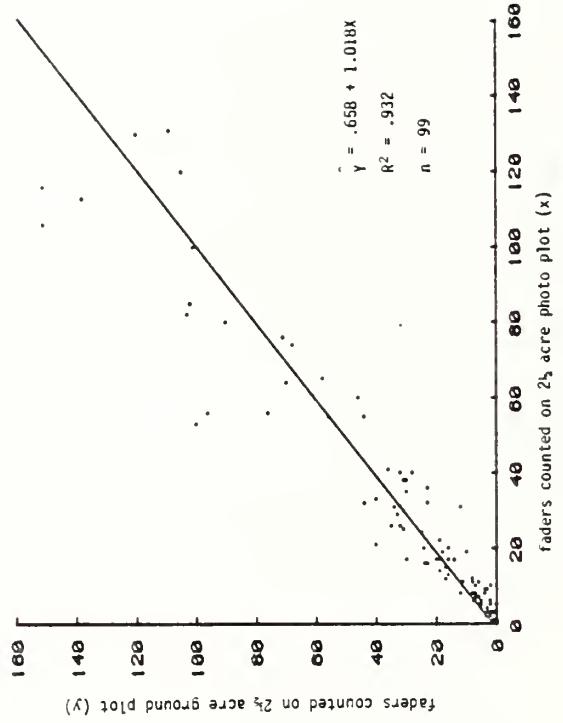


Figure 7. Photo and ground counts regression, combined data, Black Hills pine mortality survey, 1978.





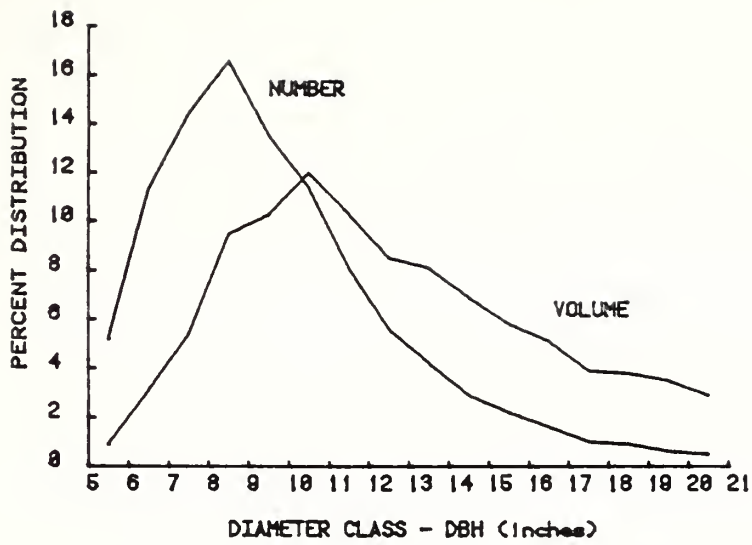


Figure 8. Distribution of the number and volume of faders by diameter class, Black Hills of South Dakota and Wyoming, 1978.

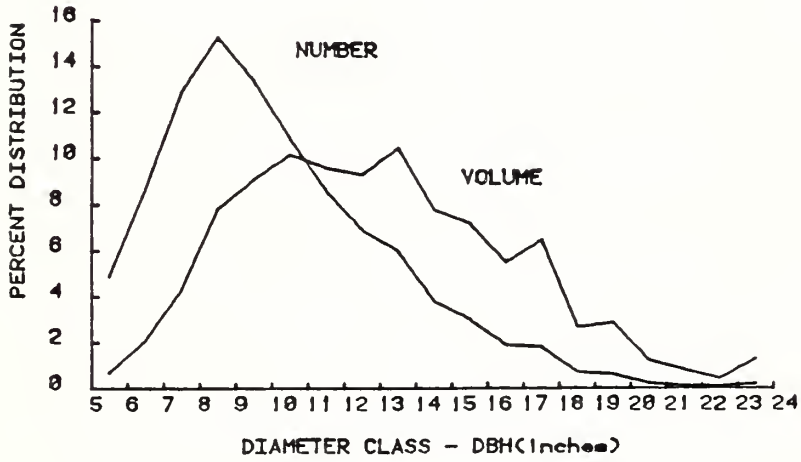


Figure 9. Distribution of the number and volume of newly infested ponderosa pine by diameter class, Black Hills of South Dakota and Wyoming, 1978.

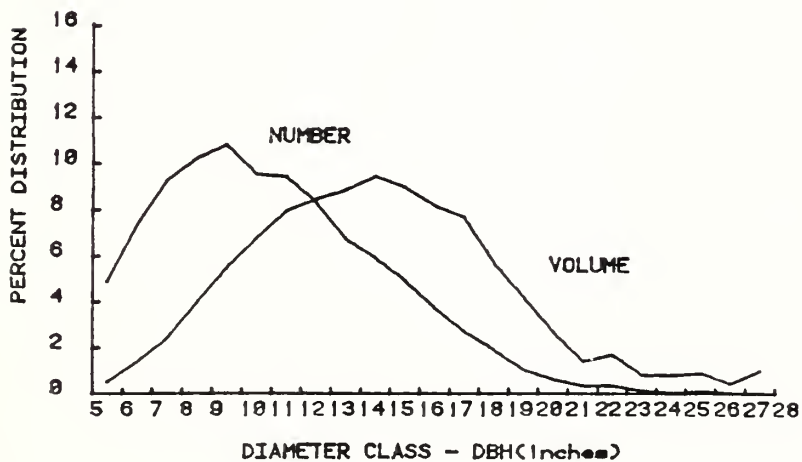


Figure 10. Distribution of the number and volume of uninfested ponderosa pine by diameter class, Black Hills of South Dakota and Wyoming, 1978.



Table 1. Stratification by intensity class of discolored *P. ponderosa* (i.e., faders) as determined from 1978 aerial sketch mapping in the Black Hills of South Dakota and Wyoming.

<u>Stratum</u>	<u>Class boundaries (faders/acre)</u>	<u>Estimated No. of faders</u>	<u>Size of area (acres)</u>	<u>No. of polygons</u>
1	< 0.1	12,510	347,805	177
2	0.1 - 0.3	38,690	222,554	186
3	> 0.3	54,460	80,429	94
Total		105,660	650,788	457



Table 2. 1978 multistage estimates of numbers and volume of *P. ponderosa* infested in 1977 by *D. ponderosae* in the survey area of the Black Hills of South Dakota and Wyoming.

Stratum	Area (acres)	No. of faders <sup>1/</sup>	Standard error	Percent error	Volume <sup>2/</sup> per tree (cu. ft.)	Total volume (M cu. ft.)	Standard error (M cu. ft.)	Percent error
2	222,554	182,748	10,985	6.0	18.7	3,422	331	9.7
3	80,429	135,669	9,299	6.9	16.3	2,207	238	10.8
Combined	302,983	318,417	14,392	4.5	17.7	5,629	408	7.2

<sup>1/</sup> See Appendix A for formulae used to compute these estimates.

<sup>2/</sup> Volumes were calculated using information from Table 1, page 5 of: Myers, C. A. 1964. Volume tables and point-sampling factors for ponderosa pine in the Black Hills. USDA For. Serv., Rocky Mt. For. and Range Exp. Sta. Res. Paper RM-8. 16 pp.



Table 3. Estimates of numbers of newly (1978) attacked ponderosa pines and the ratio of those attacked by MPB in 1978 to those attacked in 1977, Black Hills of South Dakota and Wyoming.

<u>Stratum</u>	<u>Size of area (acres)</u>	<u>No. of faders</u>	<u>No. of new (1978) attacks</u>	<u>Attack ratio</u>
2	222,554	182,748 (6.0) <u>1/</u>	143,572 (31.5)	0.79
3	80,429	135,669 (6.9)	219,788 (27.5)	1.62
2 + 3	302,983	318,417 (4.5)	363,360 (20.8)	1.14

1/

All numbers in parentheses are relative standard errors expressed as percentages.





Table 4. Estimates of percent total basal area by tree species in 1978 survey area of the Black Hills of South Dakota and Wyoming.

<u>Stratum</u>	<i>P. ponderosa</i>		<i>Picea glauca</i>	<u>Other species</u>	<u>Total</u>
	<u>Living</u>	<u>Dead</u>			
2	77.5	17.6	2.2	2.7	100.0
3	68.5	25.9	4.0	1.6	100.0

1/ This includes all trees infested by MPB in 1977 and 1978 as well as those that had been killed in previous years.



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Appendix A. Formulae and procedures used for determining multistage sampling estimates

I. Notation

Subscripts:  $h = 1, 2$  denotes the number of strata that are sampled  
 $i = 1, \dots, m_h$  denotes the sampled 90-acre photo plots  
 $j = 1, \dots, n_h$  denotes the 90-acre photo plots selected for ground sampling  
 $k = 1, 2$  denotes the 2.5-acre subplots selected from each 90-acre photo plot

$A_h$  : Number of acres in  $h^{\text{th}}$  stratum

$M_h$  : Number of primary sample units in the  $h^{\text{th}}$  stratum,  

$$M_h = \frac{A_h}{90 \text{ acres}}$$

$m_h$  : Number of photo plots sampled in the  $h^{\text{th}}$  stratum

$n_h$  : Number of photo plots selected for ground sampling in the  $h^{\text{th}}$  stratum

$x_{hi}$  : Number of faders counted in the  $i^{\text{th}}$  photo plot in the  $h^{\text{th}}$  stratum

$x_h$  : Total number of faders counted in the  $h^{\text{th}}$  stratum,  

$$x_h = \sum_{i=1}^{m_h} x_{hi}$$

$x_{hjk}$  : Number of faders counted from the photo in the  $k^{\text{th}}$  subplot of the  $j^{\text{th}}$  photo plot in the  $h^{\text{th}}$  stratum





Appendix A. (continued)

$y_{hjk}$  : Number of faders counted on the ground, in the  $k^{\text{th}}$  subplot of the  $j^{\text{th}}$  photo plot in the  $h^{\text{th}}$  stratum

$\hat{y}_{hj}$  : Estimate of the number of faders in the  $j^{\text{th}}$  photo plot of the  $h^{\text{th}}$  stratum

$\hat{Y}_h$  : Estimate of the number of faders in the  $h^{\text{th}}$  stratum

S.E. $\hat{Y}$  : Standard error of the estimate

R.S.E. $\hat{Y}$  : Relative standard error of the estimate,

$$\text{R.S.E.}\hat{Y} = \frac{\text{S.E.}\hat{Y}}{\hat{Y}} \cdot 100, \text{ expressed as a percent}$$

II. Estimators

$$\hat{Y}_h = \left[ \frac{A_h}{(90)(m_h)(n_h)} \right] \sum_{j=1}^{n_h} \hat{y}_{hj}$$

where

$$\hat{y}_{hj} = \frac{x_h \left[ \sum_{k=1}^2 \frac{(x_{hj})(y_{hjk})}{x_{hjk}} \right]}{(x_{hj})(2)}$$

$$\text{S.E.}\hat{Y}_h = \sqrt{\left[ \frac{A_h}{(90)(m_h)} \right]^2 \left[ \frac{\sum_{j=1}^{n_h} \left( \hat{y}_{hj} - \hat{Y}_h \right)^2}{n_h (n_h - 1)} \right]}$$



Appendix A. (continued)

Since  $\hat{Y}_h$  is the mean of the  $\hat{y}_{hj}$  values, the above formula can be reduced to the following more easily calculated form:

$$S.E. \hat{Y}_h = \sqrt{\left[ \frac{A_h}{(90)(m_h)} \right]^2 \left[ \frac{\sum_{j=1}^{m_h} y_{hj}^2 - \frac{\left( \sum_{j=1}^{m_h} \hat{y}_{hj} \right)^2}{m_h}}{n_h(n_h-1)} \right]}$$

$$R.S.E. \hat{Y}_h = \frac{S.E. \hat{Y}_h}{\hat{Y}_h} \cdot 100, \text{ expressed as a percent}$$

$$\hat{Y} = \sum_{h=1}^2 \hat{Y}_h$$

$$S.E. \hat{Y} = \sqrt{\sum_{h=1}^2 S.E. \hat{Y}_h^2}$$

$$R.S.E. \hat{Y} = \frac{S.E. \hat{Y}}{\hat{Y}} \cdot 100, \text{ expressed as a percent}$$



Appendix A. (continued)

III. Comments

The sample design consists of a systematic random selection of the  $m_h$  photo plots from the  $M_h$  possible photo plots in the  $h$ <sup>th</sup> stratum. Photo plots from each stratum were selected independently. Plots for ground sampling were selected in two stages both using probabilities proportional to an estimate of size. Since there is high correlation between the photo counts and what is expected to be on the ground, the photo counts provide a good estimate of size. The  $n_h$  samples were selected from the  $m_h$  photos using the total photo count ( $X_h$ ) and the individual photo count ( $x_{hi}$ ) following the procedure outlined in Cochran (1977, p. 251) an using sampling with replacement. Once the  $n_h$  samples were selected, the same procedure was used to sample 2 of the 36 2.5-acre subplots.

The estimators used for this survey are modifications of the basic probability proportional to size:

$$\hat{Y}_{pps} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{z_i}$$

where  $y_i$  represents the ground data and  $z_i$  represents the probability of a particular photo plot being selected. The variance for that function is as follows:

$$V(\hat{Y}) = \left[ \frac{\sum_{i=1}^n \left( \frac{y_i}{z_i} - \hat{Y} \right)^2}{n(n-1)} \right]$$



Appendix B. Formulae used for calculating linear regressions for each stratum.

Note: Linear regressions were based on matched data pairs of faders per acre on the ground and on photos of the 2.5-acre subplots.

Regression estimates of slope (b) and y-intercept (a) <sup>1/</sup>

$$b = \frac{\sum x_i y_i - \frac{(\sum x_i \sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$a = \bar{y} - b\bar{x}$$

where, n = number of data pairs

$x_i$  = number of faders counted on photo in  $i^{\text{th}}$  subplot

$y_i$  = number of faders counted on ground in  $i^{\text{th}}$  subplot

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\bar{y} = \frac{\sum y_i}{n}$$

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<sup>1/</sup> Calculations were made for each stratum.





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