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THESIS

ANALYSIS OF FIRST PRICE SEALED BIDDING (FPSB)
USING GAME THEORY

by

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December 1997

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Thesis
T76738

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.

1. AGENCY USE ONLY

2. REPORT DATE
December 1997

3. REPORT TYPE AND DATES COVERED
Master's Thesis

4. TITLE AND SUBTITLE
ANALYSIS OF FIRST PRICE SEALED BIDDING (FPSB) USING GAME THEORY

5. FUNDING NUMBERS

6. AUTHOR(S)
Tozendemir, Suat

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT

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14. SUBJECT TERMS

First Price Sealed Bidding, Game Theory, Static Games with Incomplete Information, Game Simulations

15. NUMBER OF PAGES
304

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT
Unclassified

20. LIMITATION OF ABSTRACT
UL

Approved for public release; distribution is unlimited

**ANALYSIS OF FIRST PRICE SEALED BIDDING (FPSB)
USING GAME THEORY**

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

from the

**NAVAL POSTGRADUATE SCHOOL
December 1997**

NPS Archive

1997.12

Tozendeniv, S.

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199738
1-76~~

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ACKNOWLEDGEMENT

I would like to first acknowledge my lovely wife, Eda, for her patience and support throughout the drafting of this Thesis in our very first year of marriage. Her support and understanding created a comfortable environment for me during my research.

Furthermore, I thank and appreciate my thesis advisors, Prof. Katsuaki L. Terasawa and Prof. Mark W. Stone, for their guidance and comments for my research.

I. INTRODUCTION

The Turkish Armed Forces (TAF) have been continually trying to enhance their war-fighting capabilities through either direct procurement of new systems or modernization of existing equipment. In either case, these efforts involve acquisition of systems and several support equipment as well as the acquisition of new equipment for existing systems' modernization. The Turkish Government is preparing to launch massive defense procurement programs to bolster its Army, Navy and Air Force and intends to spend \$31 billion for the contracts over the next 10 years. According to the Turkish Government and Industry sources this Figure may reach as much as \$150 billion through 2025. [Ref. 1]

The scarcity of resources mandates practicing more effective and more efficient ways of purchasing. The international military market differs day by day because of economic reasons and the change in global military needs. The recent example of this is the mergers of giant companies like Boeing and McDonnell Douglas.

To better equip the war-fighters with scarce resources, TAF plans to integrate smart acquisition programs. In order to achieve efficiency and effectiveness, TAF should be able to understand the strategic behavior of the prospective bidders.

This thesis will analyze the strategic behavior of the bidders in the First Price Sealed Bid (FPSB) procurement method. To understand the behavior of the bidders and to find opportunities to influence the future procurement practice of the Turkish Naval forces (TNF), this Thesis proposes a model of the FPSB. The model describes the behavior of the bidders and their actions in similar conditions to actual bidding conditions using the findings of the Game Theory.

A. AN OVERVIEW OF TURKISH NAVY PROCUREMENT ACTIVITY

The State Procurement Act of 1983 (the Act) has established general rules guiding the Government agencies' procurement procedures. Turkish Navy mainly uses four of the procurement methods outlined in this act and they are: [Ref. 2]

1. Sealed Bidding.
2. Sealed Bidding among Selected Contractors.
3. Open Bidding.
4. Negotiation.

1. Sealed Bidding

The Act bases the Sealed Bidding method as the main procurement method. This method is generally used to:

- a. provide offerors to submit their bids confidentially.
- b. provide full and open competition.

The Act established the rules of the FPSB and these rules apply to all the government agencies. According to the act, FPSB is the main method for buying goods and services.

The nature of the goods and services make them vital to national security. Therefore, when acquiring them, we must take into account some special considerations. First Price Sealed Bid method is one of the methods TNF has been using.

As the international market continually changes, FPSB method seems to remain fair and efficient whereas applicable. It provides a competitive environment for public procurement as well.

There are mainly two major bidding methods, first and second price bidding. In the first-price bidding, the bidders submit their bids, bids are opened simultaneously and the lowest bidder wins the award. In the second-price bidding, the lowest bidder wins the award but gets the second lowest bid-price. While the second-price sealed bidding has some useful properties, there is no regulation governing it in the public procurement practices of Turkey.

2. Sealed Bidding among Selected Contractors (SBASC)

The Act allows the use of SBASC for the acquisition of aircraft, Navy vessels, ammunition, electronic devices, other weapon systems and its spares and related defense industry services if there are at least three capable offerors.

If it is necessary to receive bids from less than three offerors, the soliciting agency needs to get approval from the Cabinet.

Publication of IFB is not mandatory for this type of procurement.

The goods and services that can be procured through the use of SBASC may be grouped in five categories:

- *Special Military Needs:* Aircraft, fighting ships, ammunition, electronic devices, and other weapon systems and their spares.
- *Technical Equipment for Civilian Purposes:* Aircraft, electronic devices and systems, equipment related to railroads and their spares.
- *Large and Special Buildings:* Construction of dams, power stations, irrigation plants, ports, airports and roads.
- *Special Art Works*

- *City Planning Works*

3. Open Bidding

This type of procurement is done by orally in front of the agency's procurement commission. This type of procurement is the same as auctioning either positive or negative.

4. Negotiation

Acquisition by negotiation is the method where buyer and seller try to reach an agreement through negotiation. An acquisition commission receives the proposals and negotiates with the offerors. Negotiations may be made with one or more offerors. However, the Act suggests taking at least three proposals to be able to provide competition in the acquisition. A government estimate is made and documented. The commission determines the winner and documents the rationale for the selection.

B. AREA OF RESEARCH

This thesis research will develop a computer simulation model to apply the "Nash Equilibrium Theory" into the bidding process.

C. RESEARCH QUESTIONS AND METHODOLOGY

1. Research Questions

The primary research question for this thesis is as follows:

“How do profit maximizing bidders choose their bids in a competitive environment?”

This thesis research examines the following subsidiary research questions:

1. What are the equilibrium strategies of the bidders?
2. Do the bidders have a dominant strategy in FPSB?
3. How does the number of bidders affect the outcome of the FPSB?
4. How does the cost distribution affect the outcome of the FPSB?
5. How can the procuring agencies use the findings of “Game Theory” in their acquisition practice?

2. Methodology

The thesis is conducted first by reviewing the literature for first price sealed bid activity, game theory, use of order statistics and some basic statistical procedures.

The second step in conducting the thesis was to develop a model of FPSB method and simulate the model with Visual Basic programming language.

The third step was to conduct experimentation by the simulation program under uniform and triangular statistical cost distributions. The experiments were conducted for different number of bidders.

The final step was to analyze the results of the experiments and conclude upon the analysis.

D. SCOPE OF THE RESEARCH AND ORGANIZATION OF THESIS

This thesis will:

1. Review the bidding methods used by TNF.
2. Review the game theory providing the background for constructing a model of the FPSB.
3. Develop a computer simulation to analyze the FPSB and experiment with the model.
4. Provide recommendations to use computer simulations of different bidding methods used by the TNF in its contracting practice.

This thesis has been organized in four chapters. Next chapters are as follows. Chapter II outlines the background information for developing the simulation model and mentions about the development of auctioning theory as well.

Chapter III first develops a model for simulation and then describes the experiments conducted as well as developing the bidding functions under uniform and triangular cost distributions. Then, the chapter analyzes the results of the experiments conducted with two, three, five, ten and fifteen bidders under different scenarios.

Conclusions and recommendations are provided in Chapter IV. This chapter addresses each of the primary and secondary research questions written in this chapter. In addition to this, it gives recommendations for areas for further research.

E. DEFINITIONS

Strategy -- refers to the definition, given by von Neuman and Morgenstern, founders of the Game theory, “a complete plan: a plan that specifies what choices [the player] will make in every possible situation.” [Ref. 3]

Bidder -- refers to a responsive and responsible prospective supplier, where:

Responsiveness of the supplier implies the offeror has the ability to comply with the specifications, quantities to be delivered, and terms and conditions of the invitation for bids (IFB) and

Responsibility of the supplier means offeror,

1. has adequate financial resources to perform the contract or the ability to obtain such resources,
2. is able to comply with the contracted delivery schedule,
3. has the necessary organization , experience, and technical skills,
4. has the necessary production, construction or technical equipment and facility to perform the contract obligations. [Ref. 4]

Buyer -- as the name implies, refers to a government agency soliciting an invitation for bids for a specific contract.

Bidding for contract game -- refers to the definition of a single state static game with incomplete information played by bidders.

II. BACKGROUND

This chapter of the thesis will explore the basic background information used to develop the simulation model for FPSB. The first section mentions about the elements of and conditions for sealed bidding. The second section will analyze the games to which auctioning theory applies. The final section of the chapter will provide information about the order statistics, which will be used to develop bidding functions for contract games.

A. SEALED BIDDING

Sealed bidding is a method of contracting that employs competitive bids, public opening of bids, and awards. The following steps are involved:

1. Preparation of Invitations for Bids

Invitations must describe the requirements of the Government clearly, accurately, and completely. Unnecessarily restrictive specifications or requirements that might unduly limit the number of bidders are prohibited. The invitation includes all documents (whether attached or incorporated by reference) furnished prospective bidders for the purpose of bidding.

2. Publicizing the Invitation for Bids

Invitations must be publicized through distribution to prospective bidders, posting in public places and such other means as may be appropriate. Publicizing must occur a sufficient time before public opening of bids to enable prospective bidders to prepare and submit bids.

3. Submission of Bids

Bidders must submit sealed bids to be opened at the time and place stated in the solicitation for the public opening of bids.

4. Evaluation of Bids

Bids are evaluated without discussions. Price and price related factors are the only evaluation factors.

5. Contract Award

After bids are publicly opened, an award will be made with reasonable promptness to that responsible bidder whose bid, conforming to the invitation for bids, will be most advantageous to the Government, considering only price and the price-related factors included in the invitation. [Ref. 5]

Sealed bidding can operate efficiently only when the following conditions are present:

1. There is a complete, detailed and realistic specification or purchase description.
2. There are two or more suppliers available, willing and able to compete effectively for the government's business.
3. Selection of the successful bidder can be made, without discussions of the bid, on the basis of price or price-related factors alone.
4. Enough time is available to prepare a complete statement of the government's needs and the terms under which it will do business and to carry out its administrative procedures. [Ref. 6]

B. THEORY OF FIRST PRICE SEALED BIDDING

This section will define the representation of the bidding for contract game, after studying non-cooperative games with incomplete information. Defining the equilibrium in non-cooperative games with incomplete information will provide the theoretical basis to determine the bidders' equilibrium strategies. The appearance of the equilibrium in the game will facilitate surveying the bidders' strategies in FPSB.

1. Normal-form Representation of Non-cooperative Games with Incomplete Information

This thesis surveys non-cooperative games of incomplete information based on the study of R. Gibson. [Ref. 7] To develop a normal form representation of the static game with incomplete information, also called Bayesian games; we have to consider non-cooperative games of complete information. We can represent the normal form of an n player game with complete information, as follows:

$$G = \{S_1, S_2, \dots, S_n; u_1, u_2, \dots, u_n\}$$

Where: S_i -- player i 's strategic space

u_i -- player i 's payoff function when the player selects strategy s_i

In the static non-cooperative game of complete information, the timing of moves is as follows: the players simultaneously choose an action from the feasible set of actions, A_i (i.e., player i chooses action a_i), and the payoff $u_i(u_1, \dots, u_n)$ is received.

The first step in developing the normal-form representation of the non-cooperative game with incomplete information is denoting the idea that each player has private information about his/her payoff. The players, however, are uncertain about other

players' payoff functions. Let player i 's possible payoff function be represented as $u_i(u_1, \dots, u_n, t_i)$, where t_i is called the player i 's type. t_i belongs to a set of possible types (i.e., type space). Each t_i corresponds to a different payoff function. J. Harsanyi first applied this notion for representing the payoff functions in games of incomplete information. [Ref. 8]

Given the definition of players' types, if players know their payoff functions it means they know their types. Likewise, saying the player i may be uncertain about the other players' payoff function means that player i is uncertain about the other players' types.

Players have beliefs about the other players' types. We denote the probability distribution of player i 's belief about the probability distribution of other players' types, given that player i 's type is t_i , as $p_i(t_{-i} | t_i)$, where t_{-i} represents the set of all possible types. In our analysis, the players' types (production costs as well) are identical and independent. In this case, $p_i(t_{-i} | t_i)$ does not depend on t_i , thus we can write player i 's belief as $p_i(t_{-i})$.

We can derive the normal form representation of the non-cooperative game with incomplete information by joining the normal form representation of the non-cooperative game with complete information and the concepts of type and distribution of beliefs.

The normal form representation of an n -player non-cooperative game with incomplete information specifies the players' action space, A_1, \dots, A_n , their type space, T_1, \dots, T_n , their beliefs, p_1, \dots, p_n , and their payoff functions, u_1, \dots, u_n . Player i 's type, privately known by player i , determines player i 's payoff function $u_i(a_1, \dots, a_n; t_i)$, and is a member of the set of possible types. We denote this game:

$$G = \{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

As stated by Harsanyi [Ref. 8], assume that the timing of the static Bayesian game is as follows:

- a. nature draws a type vector t_i where it is drawn from the set of all possible types of T_i ,
- b. nature reveals t_i to player i but not to any other player,
- c. the players simultaneously choose actions; and player i chooses action a_i .
- d. payoff u_i is received

Introducing the fictional move by nature, in steps a and b, produces a game with incomplete information that also satisfies the requirement for the games with imperfect information. Because nature only reveals player i 's type to player i , but not to others, other players do not know the complete history of the game when taking their actions. This is a condition of the game with imperfect information.

Two technical assumptions complete the discussion about the normal form representation of an n -player non-cooperative game with incomplete information. First, player i has private information about the type of some of other player(s), as well as his/her own. We can not exclude this condition explicitly, however, we can assume that the signal received about the other players' types may be false, and bidders do not consider this information when taking their actions.

The second point involves beliefs about the other players. It is known that the timing of the game is common knowledge, so is the $p(t)$ from which the nature draws

type vector t_i . When nature reveals t_i to player i , he/she can compute the belief $p_i(t_{-i} | t_i)$ using Bayes' rule of conditional probability. [Ref. 9]

$$p_i(t_{-i} | t_i) = p(t_{-i}, t_i) / p(t_{-i}) = p(t_{-i}, t_i) / \sum p(t_{-i}, t_i)$$

Furthermore, a player can compute the beliefs that the other players might hold. We assume that the type distribution is common knowledge and takes the form of either uniform or triangular distribution.

2. Definition of Bayesian Nash Equilibrium

To define the equilibrium in the static Bayesian game, we have to first define the strategic space of players. The players' strategy is a complete plan of action, specifying a worthwhile action in every circumstance in which the player might be engaged. In a static game with incomplete information, nature begins the game by drawing the players' type. Thus, a strategy for a player must specify a feasible action for each of players' possible type....

In static Bayesian games, unlike games with complete information, the strategic space is not given in the normal form representation of the game. In the games of incomplete information, the strategic space is constructed from the type and action space. Player i 's set of possible strategies is the set of all possible functions with range A_i and domain T_i .

It is seemingly unnecessary for the player i to specify actions for each of player i 's possible type. Once nature has revealed a specific type to the player, that player should not be concerned about the other possible types. However, in choosing a strategy, player i has to consider what other players will do. What other players will do largely depend on what they think player i will do...

[Ref. 10:pp. 12-13]

3. Normal-form Representation of Bidding for Contract Game

The first price sealed bidding is a non-cooperative game with incomplete information. These games are often called Bayesian games. In these games, each bidder knows their cost, but does not know any other bidders' cost. Bids are submitted in sealed

envelopes and the envelopes are opened publicly, so we can assume that the bidders act simultaneously.

The normal form representation of the bidding for contract game with two bidders competing for the contract is represented as:

$$u_i(b_i, b_j, c_i, c_j) = \begin{cases} b_i - c_i & \text{if } b_i < b_j \\ (1/2) * (b_i - c_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i > b_j \end{cases}$$

When more than two bidders are competing for a contract, the normal-form representation of the bidding for contract game is represented as:

$$u_i(b_1, \dots, b_n; c_1, \dots, c_n) = \begin{cases} b_i - c_i & \text{if } b_i = \min(b_1, \dots, b_n) \\ (1/z) * (b_i - c_i) & \text{if } b_i = b_j = \dots = b_n \\ 0 & \text{if } b_i > \min(b_1, \dots, b_n) \end{cases}$$

$$i=1, 2, \dots, n$$

Assuming no more than z bidders submit the same bid.

C. ORDER STATISTICS: AN OVERVIEW

This section of the chapter will outline the basics of order statistics theory using the guidelines provided in R.Hogg and A. Craig book of mathematical statistics. [Ref. 11]

Theory of order statistics deals with the ranked values of a sample of random variables having drawn from a probability distribution. The ranking of the order statistics goes from the smallest to largest. Order statistics have some remarkable characteristics. For example: properties of the order statistics do not depend upon the distribution from which the random sample has been drawn.

This thesis uses order statistics to analyze the outcome of the bidding simulations. Bids and the bidders' production costs are considered to be random variables drawn from a probability distribution. Ranking the submitted bids and the expected costs in ascending order we generate order statistics.

Let X_1, X_2, \dots, X_n denote a random sample from a continuous distribution, having probability density function of $f(x)$ that is positive over the interval of $a < x < b$. Let Y_1 be the smallest of these X_i , Y_2 be the second smallest, ..., and Y_n be the largest. That is, $Y_1 < Y_2 < \dots < Y_n$ represents X_1, \dots, X_n when they are arranged in ascending order. It can be proven that the joint probability distribution of $Y_1 < Y_2 < \dots < Y_n$ is given by:

$$g(y_1, y_2, \dots, y_n) = n! \cdot [f(x_1) \cdot f(x_2) \cdot \dots \cdot f(x_n)] \quad \text{if } a < y_1 < y_2 < \dots < y_n < b$$

$$= 0 \quad \text{otherwise}$$

The proof of this theorem is found in R. Hogg and A. Craig. [Ref. 11] Marginal probability density function represents the probability density function of the order statistics. This is given by:

$$g_k(y_k) = [n! / (n-k)!] \cdot [F(y_k)]^{n-1} \cdot [1-F(y_k)]^{n-k} \cdot f(y_k) \quad \text{for } a < y_k < b$$

$$= 0 \quad \text{otherwise}$$

D. SUMMARY

This chapter has explored elements of sealed bidding process and theory of FPSB. At this point, the chapter analyzed and defined the non-cooperative games with incomplete information as well as providing information about the Bayesian Nash Equilibrium. The third section summarized the basics of order statistics.

III. SIMULATION AND EXPERIMENTATION

This chapter of the thesis will explain the model used to develop the simulation of the FPSB and then provide the results obtained from the experimentation. While developing the simulation, two different cost distributions were used, uniform and triangular distributions. First section of this chapter explains the model used to simulate the FPSB. The next two sections summarize the results of the simulations.

A. SIMULATION OF FPSB

This section of the thesis will develop a FPSB model for simulating the bidding for contract game. The mathematical model for simulation will be transformed into computer code using Microsoft Excel's macro editor.

1. Simulation of FPSB

A model is defined as representation of a system for the purposes of studying the system. For most studies, it is not necessary to consider all the details of a system; thus a model is not only a substitute for the system, it is also a simplification of the system. On the other hand, the model should be sufficiently detailed to permit valid conclusions to be drawn about the real system. [Ref. 12]

By developing a simulation model we can study the processes and behavior of the system, and its changes over time.

Models can be classified as being mathematical or physical. A mathematical model uses symbolic notation and mathematical equations to simulate a system. Mathematical models can be classified as deterministic or stochastic simulation models.

A stochastic model uses one or more random variables as inputs. These random inputs generate random outputs. Since the outputs of the model are random, stochastic models can imitate the real system. However, the simulation results must be treated as a statistical estimate of the real-world system's characteristics.

FPSB model represents the bidders. It includes the rules and regulations of the bids and the bidders' behavior. The actual market conditions, the availability of resources and other factors also affect the FPSB system. Using more than one cost distribution incorporated some of these variables. However, to keep the model manageable, a number of simplifications and assumptions have been made.

2. Information Space of the Game and Strategic Behavior of the Bidder

This subsection will analyze one of the most important aspects of the model, the bidders' and the buyer's characteristics. The bidding environment will also be analyzed as well. Because of the wide variety of bidding regulations, this thesis assumes both bidders and buyers meet regulations. Both parties comply with the applicable laws and regulations at the time of bidding.

a. Information Space of the FPSB

We assume that the FPSB is a non-cooperative game in which the players have limited information. However, the players know the following:

1. The buyer is fully committed not to deviate from the FPSB rules

during the bidding process, even if the deviation is in the buyer's best interest. All participants know the rules of bidding.

2. The bidders' utility function is defined by a Von Neuman Morgenstein utility function, $U(\cdot)$, and it is common for all bidders.

3. The bidders know their production cost c_i , with certainty when they bid and the cost is private information known only to the bidder. However, the bidders have subjective assumptions about the range and distribution of production costs for other bidders. This thesis assumes that the probability distribution for each bidder is the same; it follows either a uniform or a triangular probability distribution over the production cost range.

4. The bidders know the number of the bidders with certainty.

5. While preparing the bid proposals, the bidders send signals, sometimes misleading, about their cost to other bidders. However, cooperation among bidders is restricted. [Ref. 10]

6. Bidders' proposal preparation costs are not incorporated into the model. Therefore, unsuccessful bidders' losses because of bid preparation activities were neglected.

b. The Players' Strategic Behavior

One of the assumptions about the bidders is that the buyer and the bidders are expected to act rationally. The rationality of the bidders means that:

1. The bidders pursue their own interest; they attempt to maximize their

profit by bidding for the contract. Bidders maximize profits by submitting the highest possible bid. However, the bidders recognize that they are constrained by the other bidders' bids. The higher the bid the lower the probability of winning the award. This self-regulating mechanism provides an efficient solution for the game.

2. Bidders consider their production costs and the production costs of the other bidders. The resulting bids form a Bayesian Nash equilibrium. Bidders using the equilibrium strategy simultaneously maximize the expected profit regarding both their and other participants' expected bids. [Ref. 10]

3. Model Description

Consider a competitive bidding model in which the buyer solicits an IFB to procure a specified commodity or service. This thesis assumes that there are n bidders for a particular procurement and they are responsive and responsible. It is assumed that bids are submitted and the contract awarded to the lowest responsive and responsible bidder. The contract specifies the winning firm's total receipts from the buyer. The winner's expected profit depends both on the bid submitted and the cost incurred. In turn, the bids are influenced by the firm's expectation about the competing bids.

The constructed model attempts to capture the major and decisive characteristics of the real FPSB process. However, this model is only a first approximation of many procurement procedures. The model provides an opportunity to experiment with the bidders' possible actions and decisions during FPSB. The proposed FPSB model assumes symmetry of information and preferences, which makes it possible to concentrate exclusively on a symmetric Bayesian Nash equilibrium.

4. Computer Simulation Methodology

This thesis uses a personal computer and Excel 7.0 spreadsheet software to simulate the FPSB process. Two different computer programs were written to conduct the necessary operations to simulate the process. The simulation results were collected in separate worksheets. The composed computer programs allow some of the data to be input interactively. Other necessary experiment data may be input using the Microsoft Excel 7.0 macro editor.

The bidders' initial production costs, which were random variables during the entire simulation process, were generated by Excel's built-in random number generator. The built-in random number generator provides a random number distributed uniformly over the interval $[0, 1]$.

B. EXPERIMENTATION UNDER UNIFORM COST DISTRIBUTION

This section of the chapter describes the FPSB simulation when the production costs are distributed according to the uniform distribution.

1. Mathematical Model for Simulation

For products and services for which fairly mature, simple and common technologies are used, the costs of production are almost identical. Assume that the bidders' potential production costs are distributed uniformly over an interval $[h, k]$.

For the uniform distribution:

The probability density function of cost: $f(c)=1/(h-k)$

The cumulative distribution function of cost: $F(c)=(c-a)/(h-k)$

Where h is the upper limit and k is the lower limit of the cost range.

A bidding strategy defines the relationship between the bidder's proposal, b_i , and cost, c_i . The following section is the derivation of the bidding function under uniform cost distribution.

2. General-Form Bidding Function for the Uniform Cost Distribution

Assume that n players are competing for a contract and the bidders' production/service cost c_i has a uniform cost distribution over the range of $[k, h]$.

Assume players 1, 2, 3,..., n-1, n adopt the strategy $b(\cdot)$, and $b(\cdot)$ is strictly increasing and differentiable. For a given value of the player i's cost, player i's optimal bidding strategy solves:

$\text{Max}\{(b_i-c_i) * \text{Prob}[b_i < b(c_1), \dots, b_i < b(c_n)]\}$ where n: number of bidders.

Let $b^{-1}(b_i), b^{-1}(b_{i+1}), \dots, b^{-1}(b_n)$ denote the costs that the bidders must have in order to bid b_i, b_{i+1}, \dots, b_n . That is, $b^{-1}(b_i) = c_i, \dots, b^{-1}(b_n) = c_n$ if $b_i = b(c_i), \dots, b_n = b(c_n)$.

We can define the probability of a given player i's bid to be the lowest of all as:

$\text{Prob}(b_i < b_{i+1}, \dots, b_i < b_n) = [1 - b^{-1}(b_i)]^{n-1}$

The first order condition for player i's optimization problem is then:

$d\{(b_i-c_i) * [1 - b^{-1}(b_i)]^{n-1}\} / db_i = 0$

$[1 - b^{-1}(b_i)]^{n-1} + (b_i-c_i) * (n-1) * [1 - b^{-1}(b_i)]^{n-2} * d[1 - b^{-1}(b_i)] / db_i = 0$

The first order condition is an implicit equation for i^{th} bidder's best response to the strategy $b(\cdot)$ played by other bidders, given that the i^{th} bidder's cost is c_i . If the strategy $b(\cdot)$ is to be a symmetric Nash equilibrium, we require that the solution of the first order condition be $b(c_i)$. That is, for each of i^{th} bidder's costs, the bidder does not want to deviate from the strategy $b(\cdot)$, given that the other players play the same strategy. To impose this, we substitute b_i with $b(c_i)$ into the first order condition:

$$\{1-b^{-1}[b(c_i)]\}^{n-1} + [b(c_i)-c_i] * (n-1) * \{1-b^{-1}[b(c_i)]\}^{n-2} * d\{1-b^{-1}[b(c_i)]\}/db_i=0$$

$$\text{where } b^{-1}[b(c_i)]=c_i \text{ and } d\{1-b^{-1}[b(c_i)]\}/db_i=-1/b'(c_i)$$

By substituting these two into the first order differential equation we get:

$$(1-c_i)^{n-1} - \{[b(c_i)-c_i] * (n-1) * (1-c_i)^{n-2}\} / b'(c_i)=0$$

$$(1-c_i)^{n-1} = \{[b(c_i)-c_i] * (n-1) * (1-c_i)^{n-2}\} / b'(c_i)$$

$$b'(c_i) * (1-c_i)^{n-1} - (n-1) * b(c_i) = - (n-1) * c_i$$

The left-hand side of the equation can be written as:

$$b'(c_i) * (1-c_i)^{n-1} - (n-1) * b(c_i) = [1/(1-c_i)^{n-2}] * \{d[b(c_i) * (1-c_i)^{n-1}]/dc_i$$

Then, the original equation can be written as:

$$[1/(1-c_i)^{n-2}] * \{d[b(c_i) * (1-c_i)^{n-1}]\}/dc_i = - (n-1) * c_i$$

$$\{d[b(c_i) * (1-c_i)^{n-1}]\}/dc_i = - c_i * (n-1) * (1-c_i)^{n-2}$$

Integrating both sides of this equation using by parts method results in:

$$\int \{d[b(c_i) * (1-c_i)^{n-1}]\}/dc_i = - \int c_i * (n-1) * (1-c_i)^{n-2}$$

$$b(c_i) * (1-c_i)^{n-1} = c_i * (1-c_i)^{n-1} - \int (1-c_i)^{n-1} dc_i$$

$$b(c_i) * (1-c_i)^{n-1} = c_i * (1-c_i)^{n-1} + [(1-c_i)^n]/n + k$$

Finally, by using the boundary conditions we determine the value of k . That is, $b(c_i) \geq c_i$. If $c_i=1$ then $b(1)$ is finite, which is true; thus $k=0$.

Therefore, the bidding function, $b(c_i)$, takes the form:

$$b(c_i) = c_i + [(1-c_i)]/n$$

$$b(c_i) = [1 + (n-1) * c_i]/n$$

Under the assumption that the players' strategies are strictly increasing and differentiable, we have a linear and symmetric Nash equilibrium in the n -person bidding game.

We can determine the bidding function for the cost interval, $[k, h]$, then, modify the function as:

$$b(c_i) = [(h-k) + (n-1) * c_i]/n$$

Where $k < h$ and,

k : the lower limit of the uniform cost distribution.

h : the upper limit of the uniform cost distribution.

3. Experimentation

During the experimentation, five different scenarios were used. In these scenarios, the number of bidders was changed to analyze the FPSB process and also verify the accuracy of the simulation. The number of bidders used are two, three, five, 10 and 15. Therefore, the bidding functions for these scenarios over the cost interval $[0, 1]$ are:

$$b_i = (1+c_i)/2, \quad b_i = (1+2*c_i)/3, \quad b_i = (1+4*c_i)/5, \quad b_i = (1+9*c_i)/10, \quad b_i = (1+14*c_i)/15$$

To allow different cost intervals to be used in the simulation, the required random number was generated as:

$$R=(h-k)*\text{Rand}()+k$$

Where R is the required random number and Rand() is the Excel generated random number.

The simulation program for FPSB contract game is in Appendix A.

a. Experimentation With Two Bidders

During the simulation, three different scenarios were experimented with and analyzed. The first scenario was conducted as if both bidders used equilibrium strategies. In the second scenario, one of the bidders underbid while the other used the equilibrium strategy. Finally, in the third scenario, both of the bidders did not use the equilibrium strategy and underbid.

The different scenarios were designed to show that using the equilibrium bidding strategy maximizes the bidders' profit and any deviation from the equilibrium strategy would reduce the expected profit for the deviating bidder.

Figure 1 shows the graphical results of the simulation when both bidders used equilibrium strategies. The results show that there exists an equilibrium in the contract game and the winners' expected profits approach 0.333 as the number of simulated games approaches 2000. Another result was that the bidders won almost equally; bidder1 won 982 of the games while bidder2 won the remaining 1018 games.

Average profit per game considers all games in which the bidders bid. As shown in Table 1 in Appendix C, bidder1 won 44 of the first 100 games and its total profit is 15.2916. Therefore its average profit is $15.2916/100$, which is 0.1529. The average profit is another indicator of bidders' expectations, motivations and behavior.

The average profit graph for this particular simulation is shown in Figure 2. This graph also reinforces the result of existing equilibrium in the bidding game. Both bidders' average profit approaches to 0.167 by bidding in 2000 games.

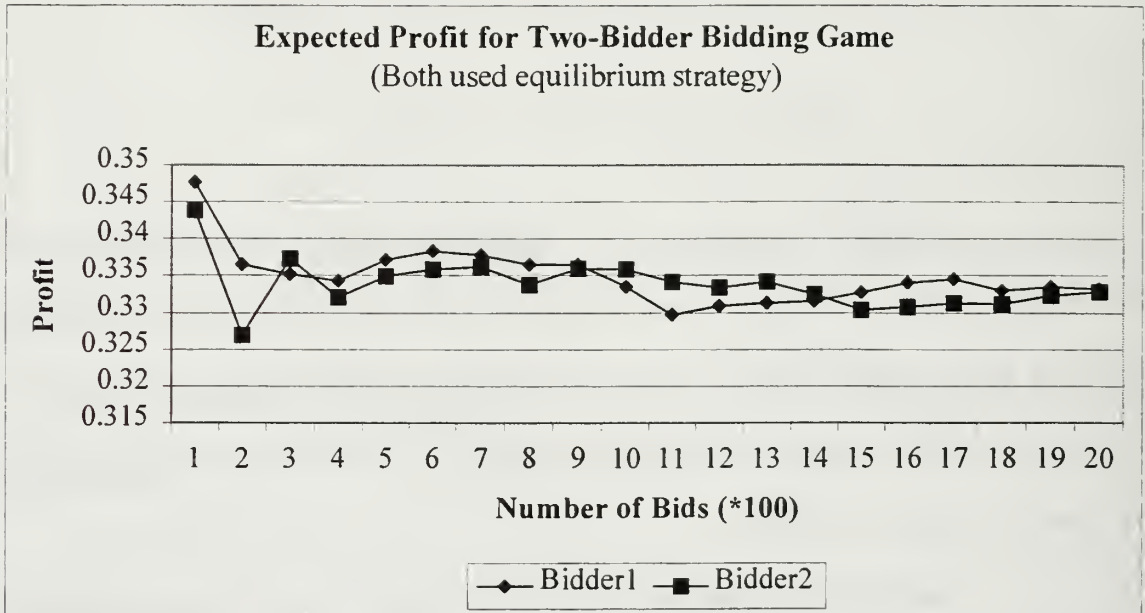


Figure 1

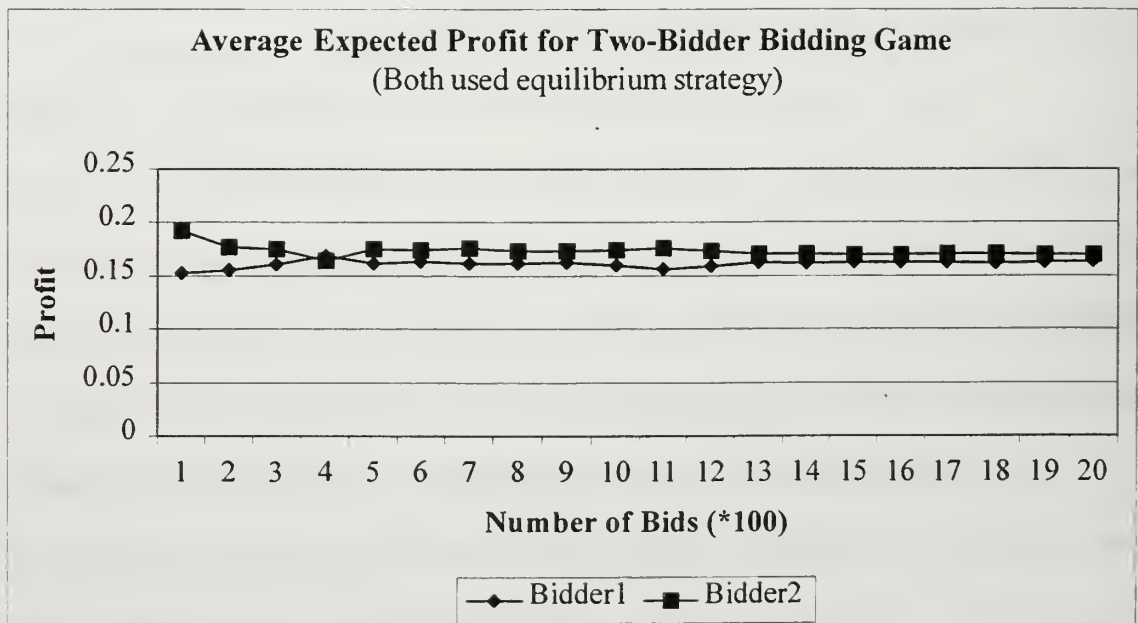


Figure 2

Total profits for both bidders resulted almost the same; 327.263 for bidder1 and 338.872 for bidder2 in 2000 games and are shown graphically in Figure 3.

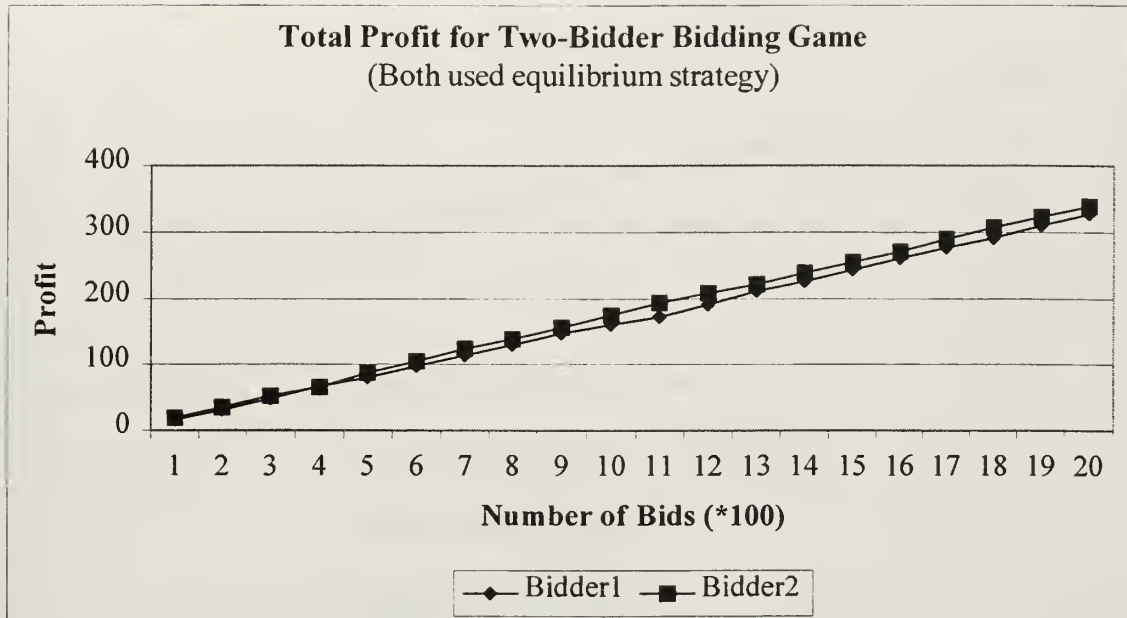


Figure 3

The next experimentation was conducted as bidder1 underbid by 0.2 while bidder2 used equilibrium strategy. Bidder1's bidding function therefore, was:

$$b_1 = [(1+c_1)/2] - 0.2$$

The results of the experimentation is shown in Table 2 in Appendix C. The experimentation resulted in lower profit for bidder1, 0.085, while bidder2 made 0.40 which is more than the equilibrium amount of 0.333. The expected profit graph is shown in Figure 4.

The average expected profit for bidder1 decreased to 0.069 and 0.074 for bidder2. The graph for average profit is shown in Figure 5.

Total profit for bidder1 also decreased to 104.514 and 111.956 for bidder2

as shown in Figure 6. Bidder1 won 1221 of 1500 games while bidder2 won remaining 279.

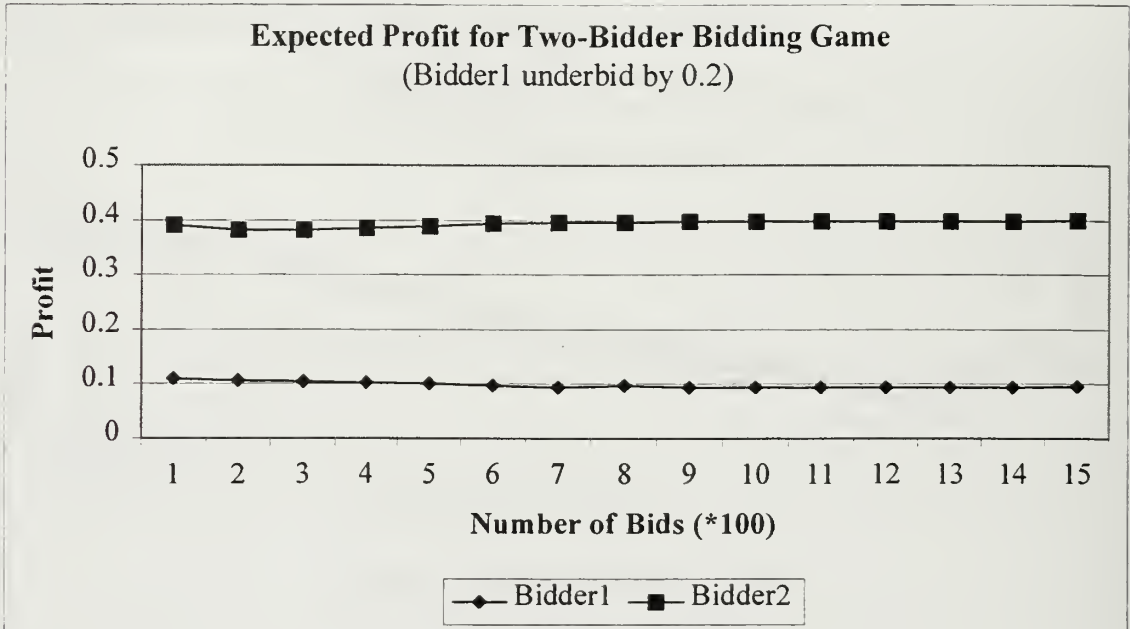


Figure 4

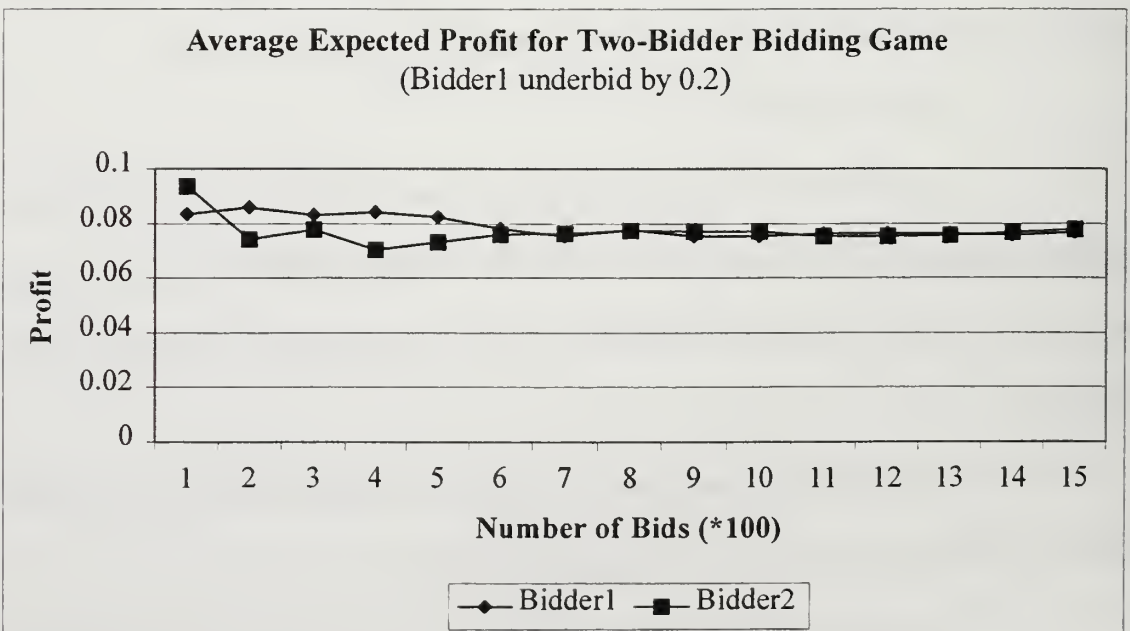


Figure 5

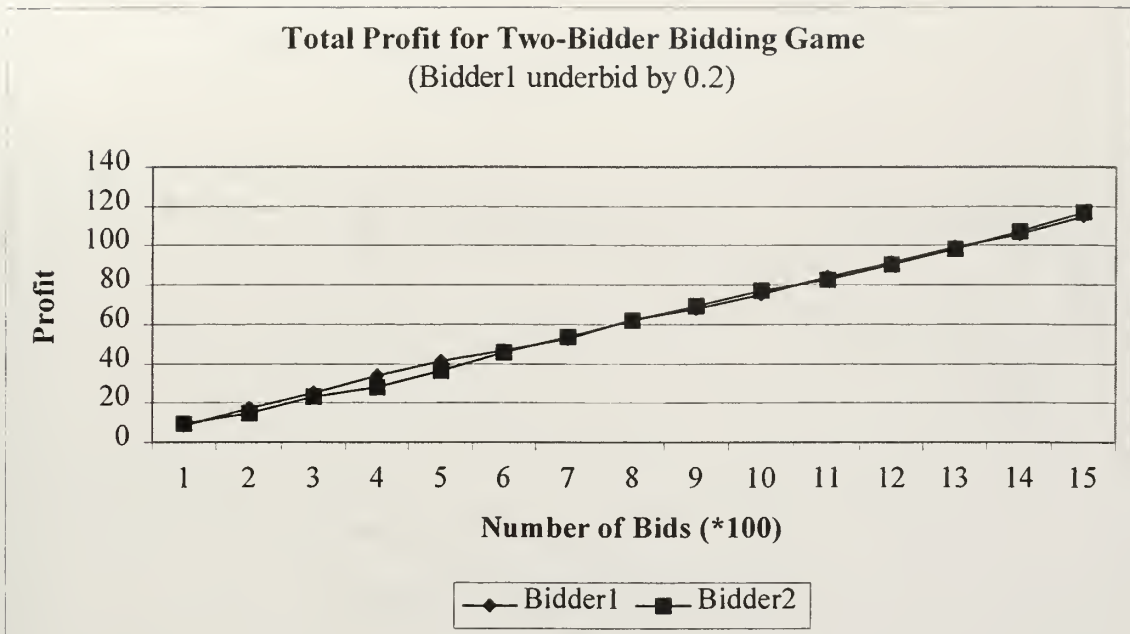


Figure 6

The last experiment with two bidders was conducted as they both did not use equilibrium strategies. Both bidders underbid by 0.1. Therefore, bidding function for both bidders was:

$$b = [(1+c)/2] - 0.1$$

In this scenario, both bidders' expected profits approached 0.23 in 1500 games, which is lower than the equilibrium amount of 0.33. The graphical representation of expected profit is shown in Figure 7.

The average profit for both bidders also decreased to 0.11 and is shown in Figure 8 graphically. Total profits in 1500 games as shown in Table 3, 171.08 for bidder1 and 175.54 for the other, were also lower than the equilibrium total profit of 243.236 and 253.983 respectively. The graphical representation of total profit curves is in Figure 9. The bidders won almost equal number of games. Bidder1 won 742 games while bidder2 won 758.

Expected Profit for Two-Bidder Bidding Game
(Both underbid by 0.1)

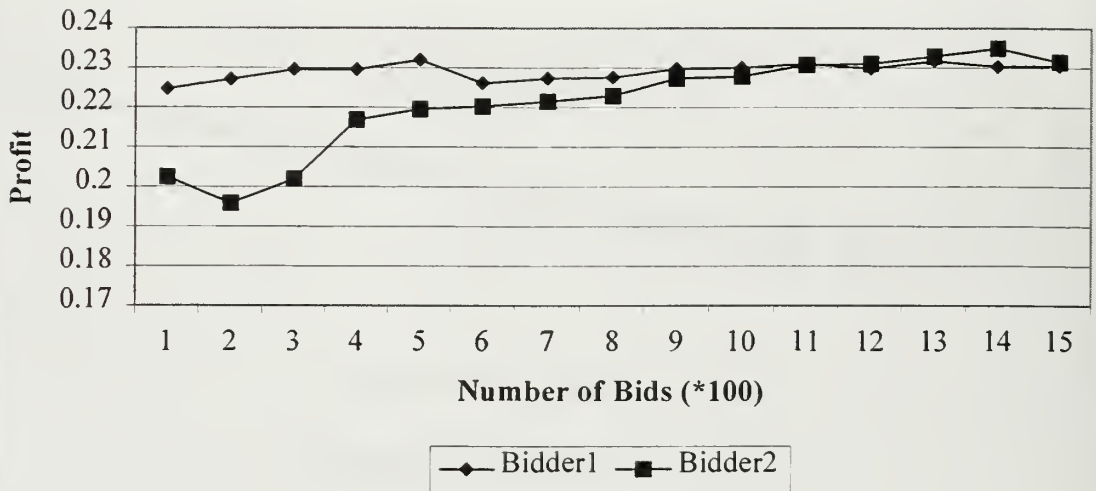


Figure 7

Average Expected Profit for Two-Bidder Bidding Game
(Both underbid by 0.1)

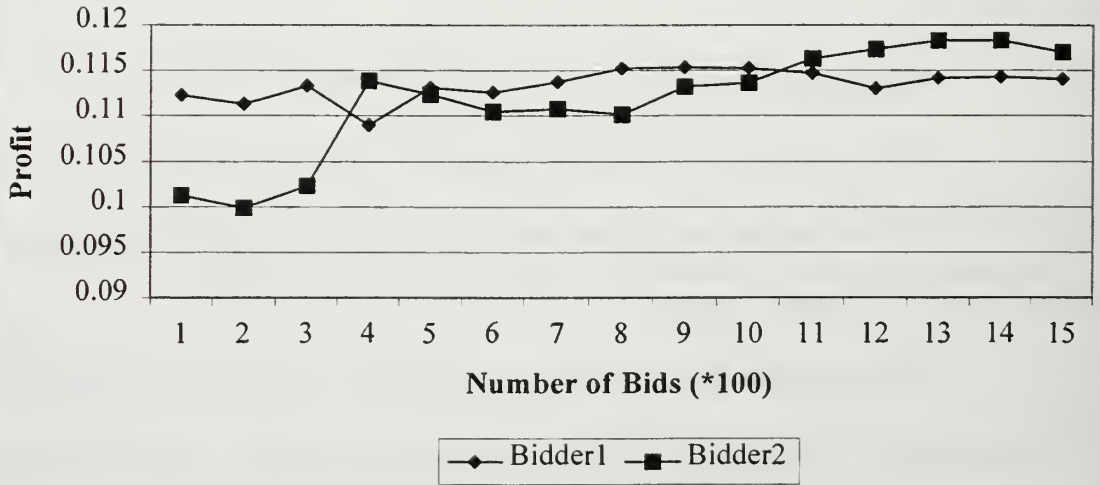


Figure 8

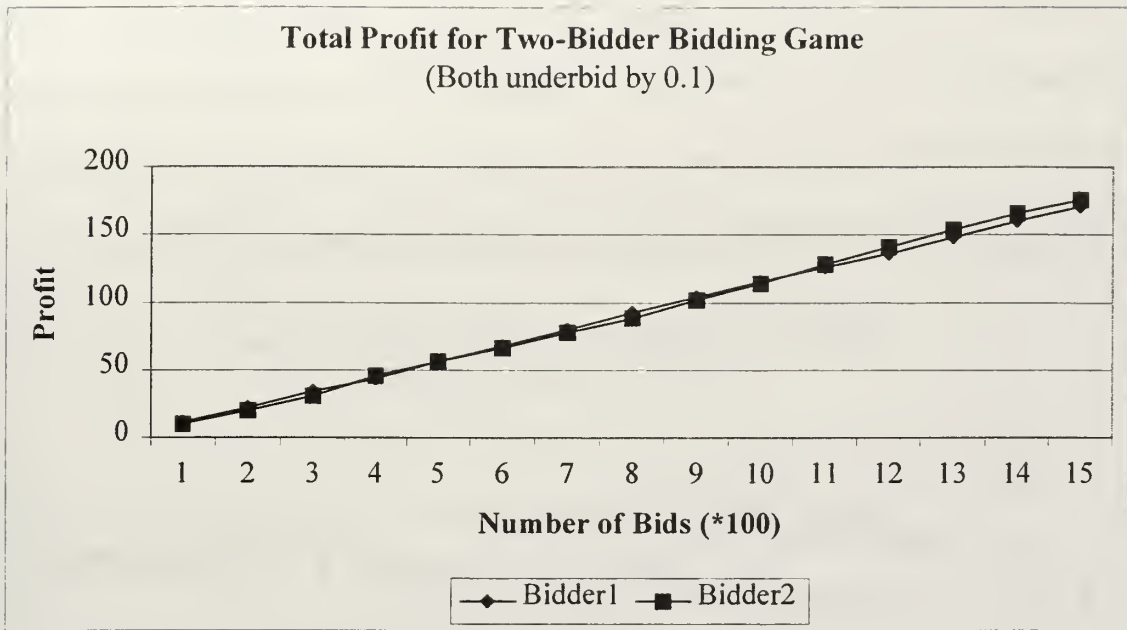


Figure 9

b. Experimentation With Three Bidders

This simulation along with the other multiple bidder simulations were conducted to find out the effect of number of bidders on the game as well as analyzing the behavior of bidders and the accuracy of the simulation. The simulation was conducted with three bidders and the cost was within the interval [0, 1] under uniform distribution.

During the simulation, three different scenarios were analyzed. In the first scenario, all off the bidders used equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. Finally in the third scenario, bidders “1” and “2” underbid by different amounts while bidder3 used equilibrium strategy.

Figure 10 shows the results of the first experimentation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the three-bidder bidding game as well as in the two-bidder bidding game.

According to the simulation results, the winners' expected profit from bidding approaches 0.25 as the number of games approaches 2000. Introducing another bidder to the game reduced the expected profit from bidding. This is true, because as the number of bidders increase, the probability of winning the award decreases, thus bidders bid lower.

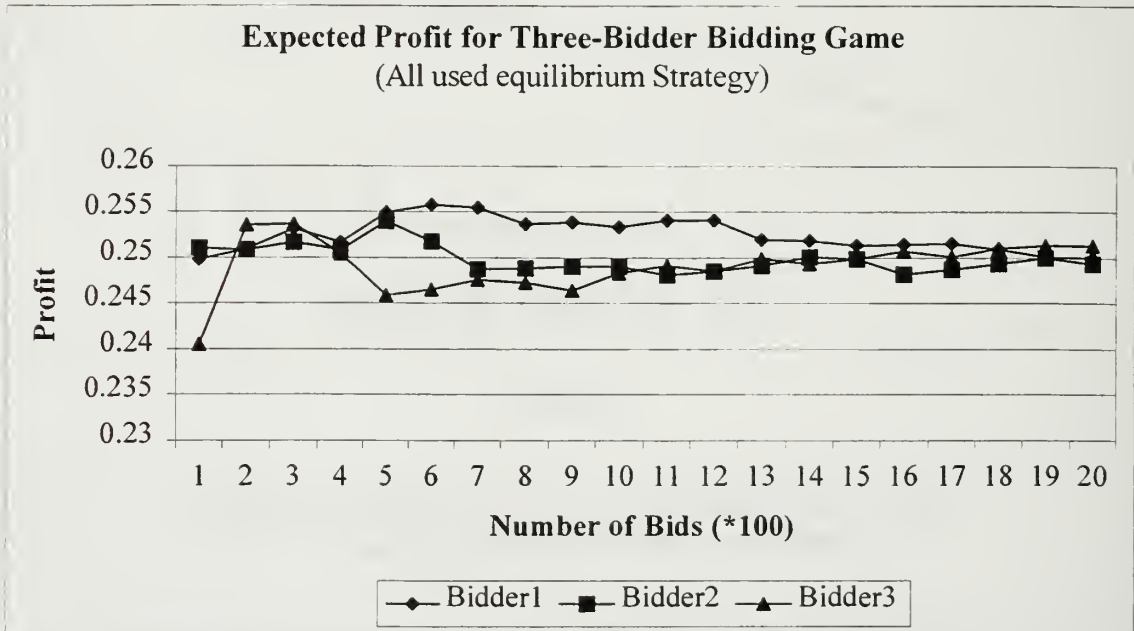


Figure 10

The average expected profit graph is shown in Figure 11 and the overall results of the experiment are shown in Table 4 in Appendix C. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached 0.83 while the number of games approached 2000.

As seen in Table 4, each bidder won the games almost equally. Overall, bidders won 683, 652 and 665 games respectively. Total profit for bidders as seen in Figure 12, were almost the same; 170.72, 162.6 and 167.13 respectively.

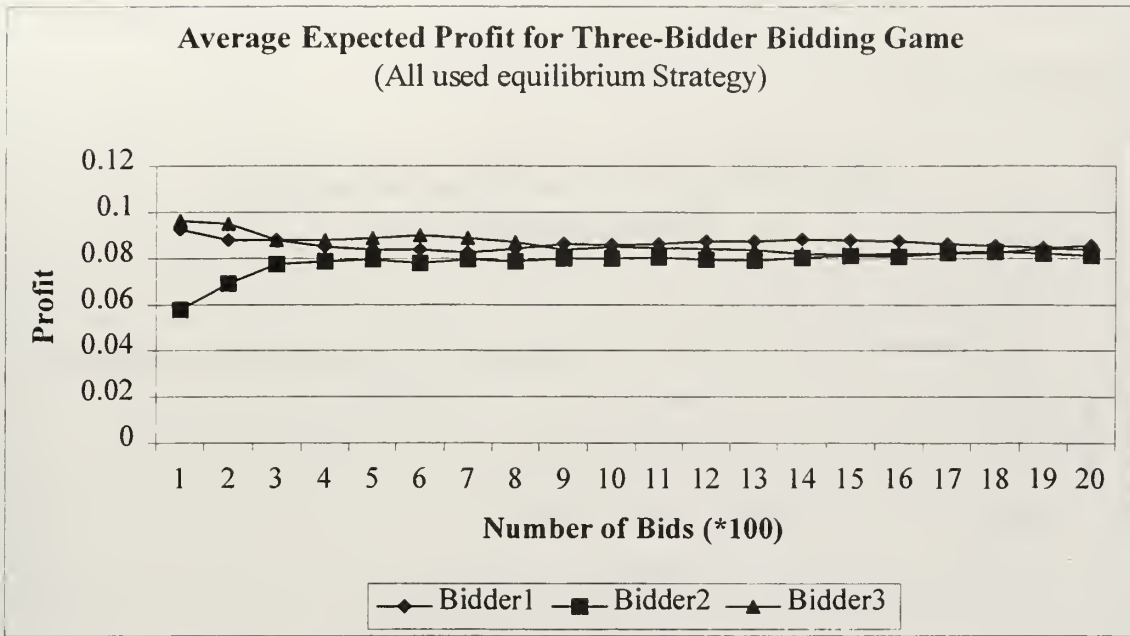


Figure 11

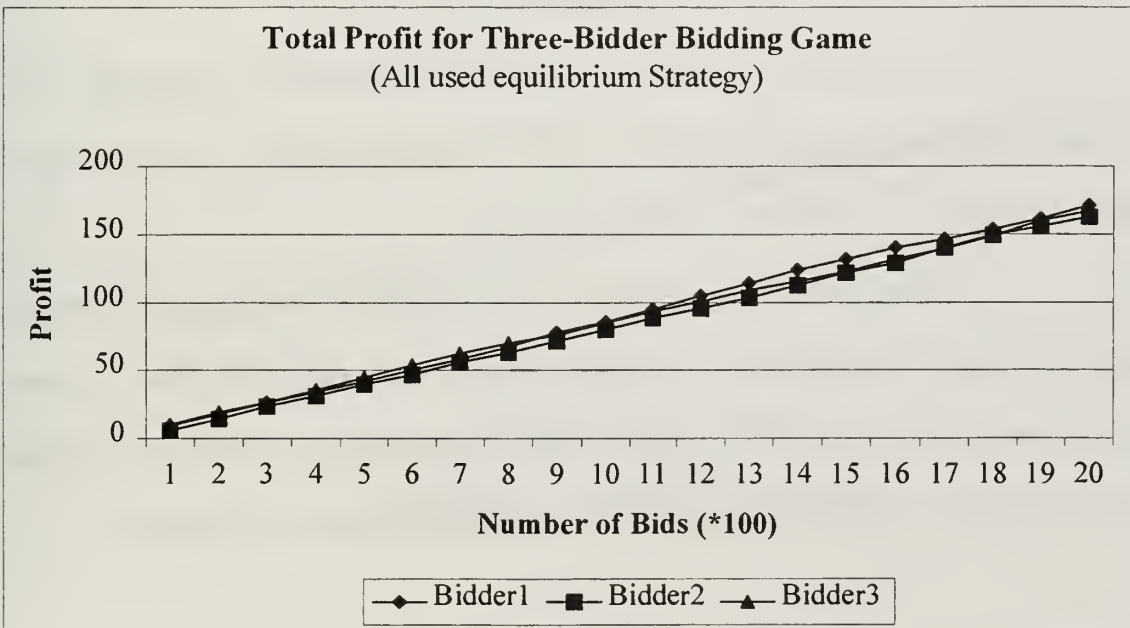


Figure 12

In the next experiment, bidder1 underbid by 0.1 while other two used equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to

0.13 while the other two bidders' slightly increased to 0.258 and .259 respectively. The expected profit graph is shown in Figure 13.

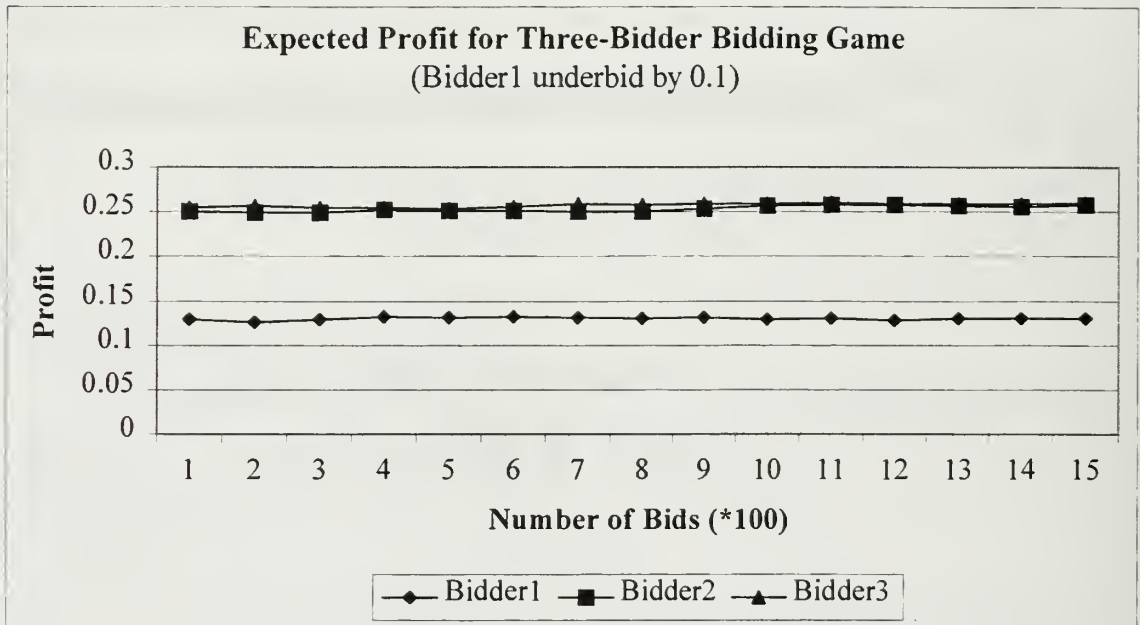


Figure 13

Average profit scheme was different than the equilibrium game experiment. Bidder1's average profit was 0.059 while others higher, 0.708 and 0.706 as graphically shown in Figure 14.

Bidder1 won 680 of 1500 games while others won 412 and 408 games as shown in Table 5 in Appendix C. Although bidder1 won more games than the others won, its total profit was 88.5022 which is lower than the others'; which were 106.2 and 105.87 respectively. Total profit graph for this experiment is shown in Figure 15.

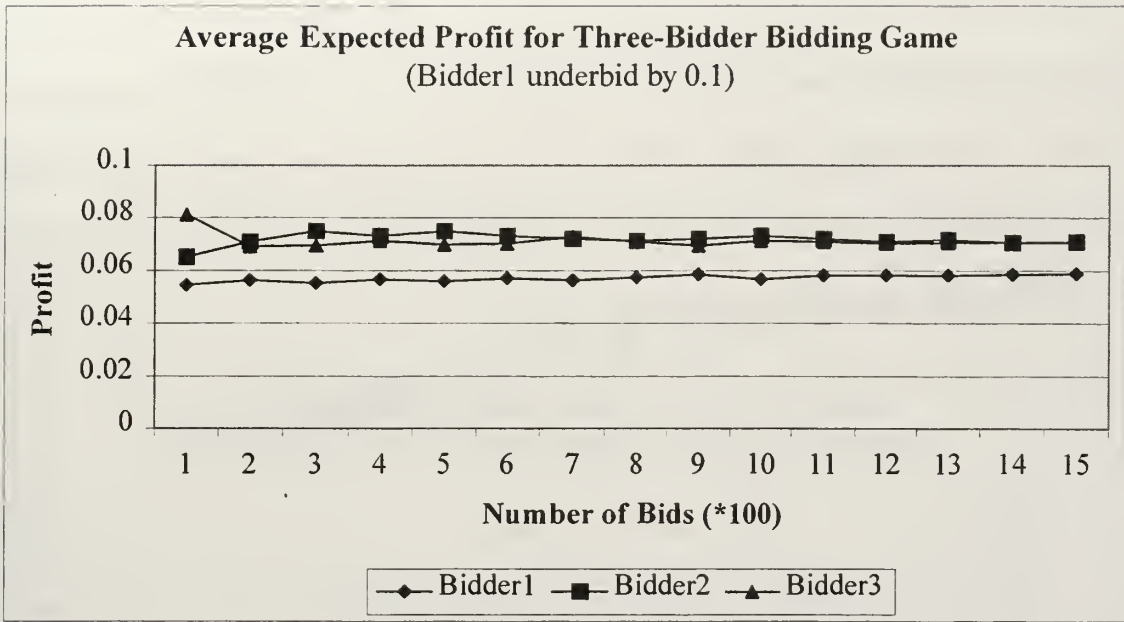


Figure 14

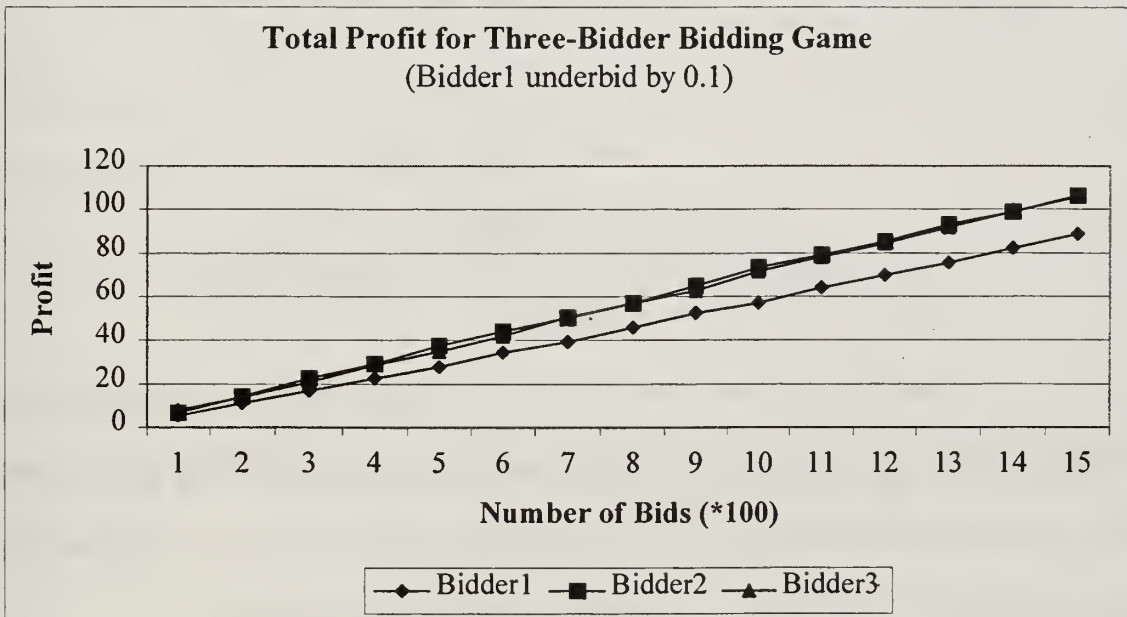


Figure 15

The last scenario for three-bidder simulation was bidder3 used equilibrium strategy while bidders “1” and “2” did not. Bidders “1” and “2” underbid by 0.1 and 0.15 respectively. The experiment resulted in lower expected profits for the first two bidders

while it was even higher for the third. Bidder1's profit approached 0.1441, which is slightly higher than in the previous game, which was 0.13 and bidder2's was even lower, 0.0857 in 1500 games. Bidder3, which had used equilibrium strategy, made slightly higher profit than it did in the first two experiments, 0.2645 as opposed to 0.25 and 0.259. The expected profit scheme is shown in Figure 16:

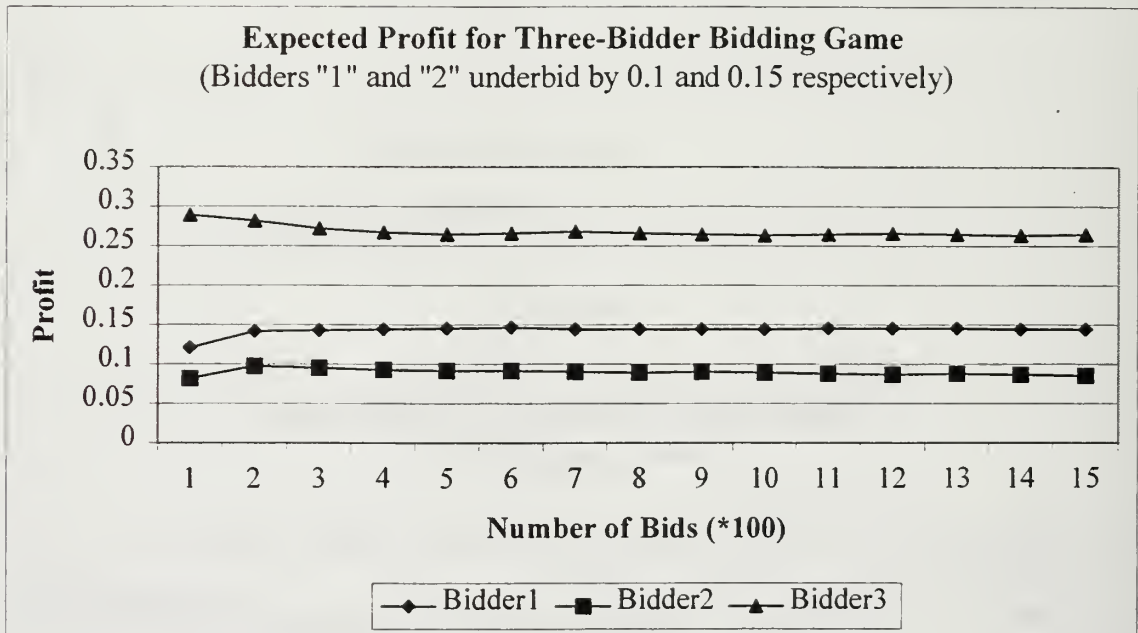


Figure 16

Total profit scheme was also very different than previous experiments. Bidder1's total profit turned out to be higher than bidders "2" and "3". It made 77.961 while others made 60.2550 and 67.722 in 1500 games. Bidders won 541, 703 and 256 games respectively. Due to the high number of games won by bidder1 made him gain more profit. In spite of winning more games, bidder2 made the lowest profit as seen in Table 6 in Appendix C and shown in Figure 17.

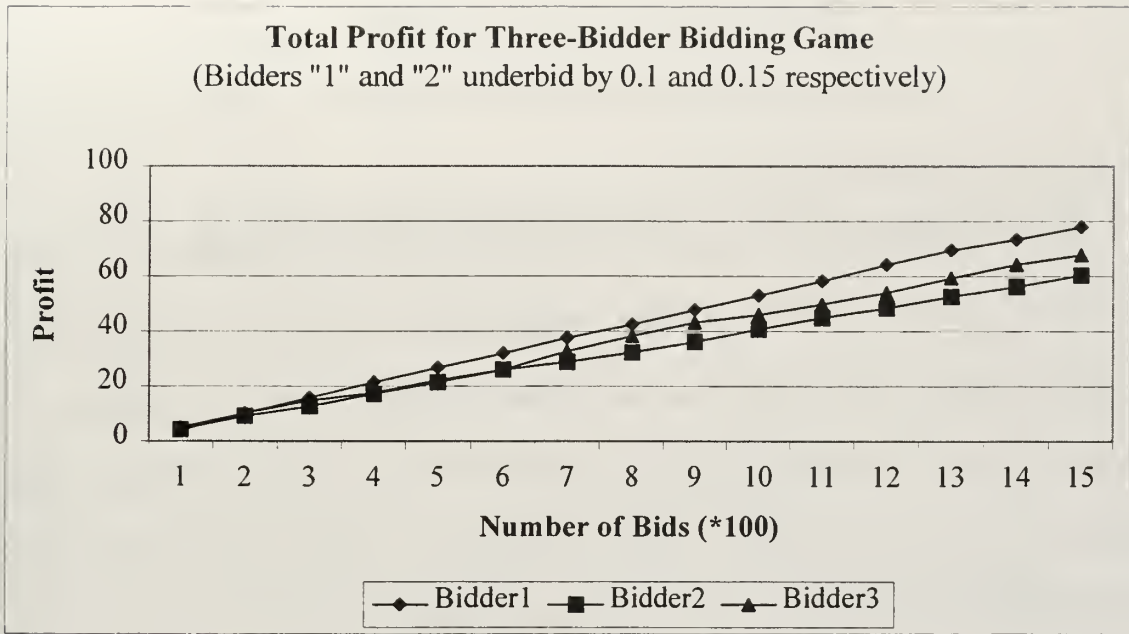


Figure 17

Though, bidder1 had higher average profit than the others did. It made 0.052 as opposed to 0.0402 and 0.0451 made by bidders "2" and "3" respectively. The average profit graph is shown in Figure 18:

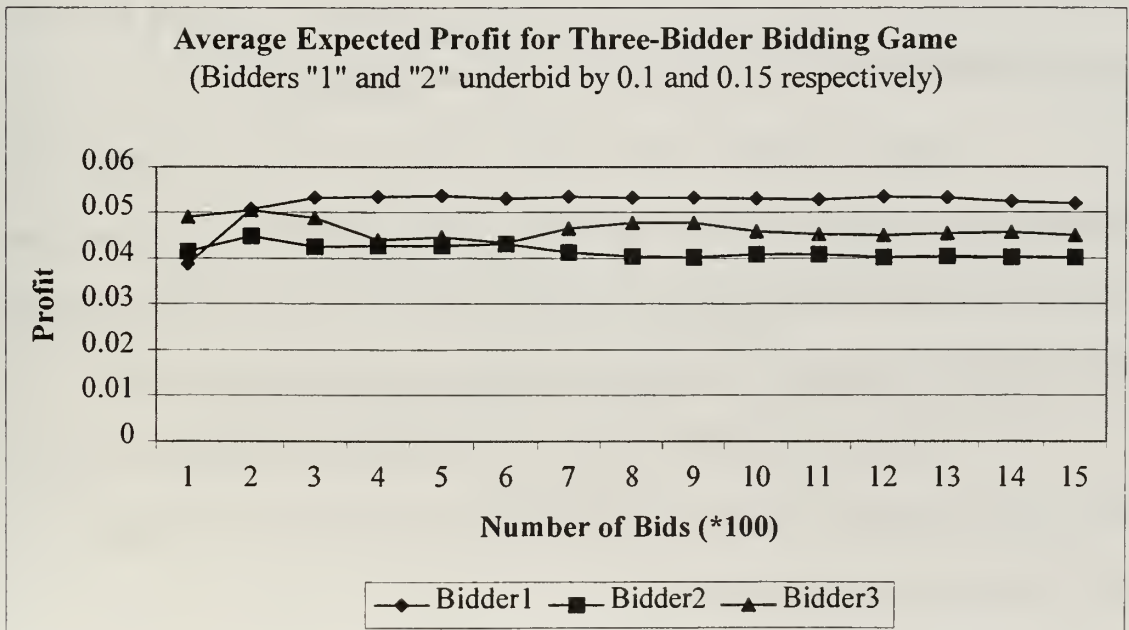


Figure 18

c. Experimentation With Five Bidders

The simulation was conducted with five bidders according to uniform cost distribution within interval $[0, 1]$.

During the simulation four different scenarios were analyzed. In the first scenario, all bidders used the equilibrium strategy. The second scenario was conducted as bidder1 underbid while others used equilibrium strategy. In the third scenario, both bidders “1” and “2” underbid by the same amount while others used equilibrium strategy and in the last scenario, bidders “1” and “2” underbid by different amounts while others used equilibrium strategy.

Figure 19 shows the results obtained from the simulation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the game as in the two and three-bidder bidding games. According to the simulation, each winner’s expected profit approached approximately 0.166 as the number of games approached 1500. As the number of bidders increased, the expected profit from bidding decreased as it was the case in the three-bidder bidding game.

Average expected profit graph is shown in Figure 20 and overall simulation results obtained are shown in Tables 7 and 8 in Appendix C. One more time, the average expected profits reinforced the game equilibrium. The average expected profits for the bidders approached 0.033 as the number of games approached 1500.

The experiment resulted in very close total profits, ranging from 45.383 to 51.889, as seen in Table 8. The total profit graph is shown in Figure 21. The bidders won 277 to 313 games during the experiment.

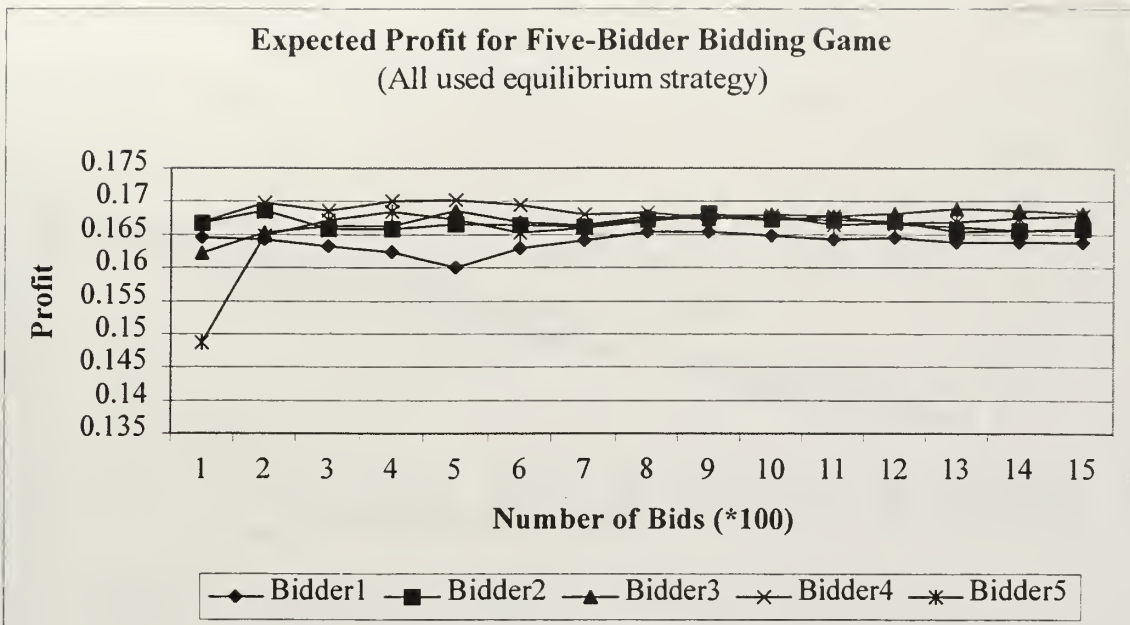


Figure 19

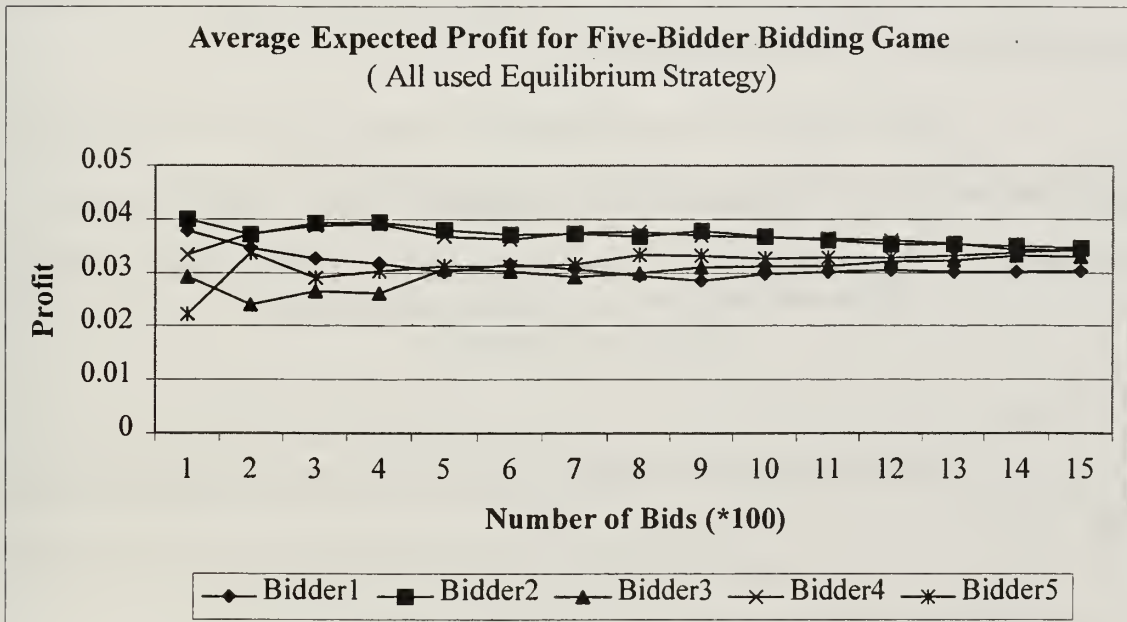


Figure 20

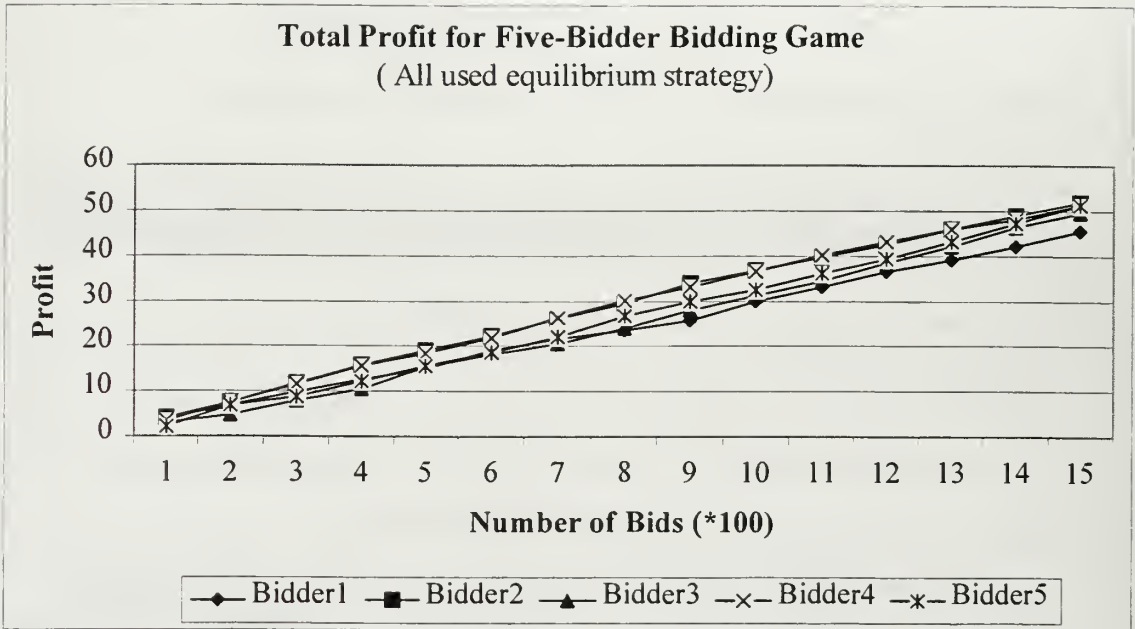


Figure 21

The second experiment, where bidder1 underbid by 0.09 and others used the equilibrium strategy, resulted in following:

Bidder1 made a lower expected profit, 0.07, than the others did, which is approximately 0.168, as the number of games approached 1500 as shown in Figure 22:

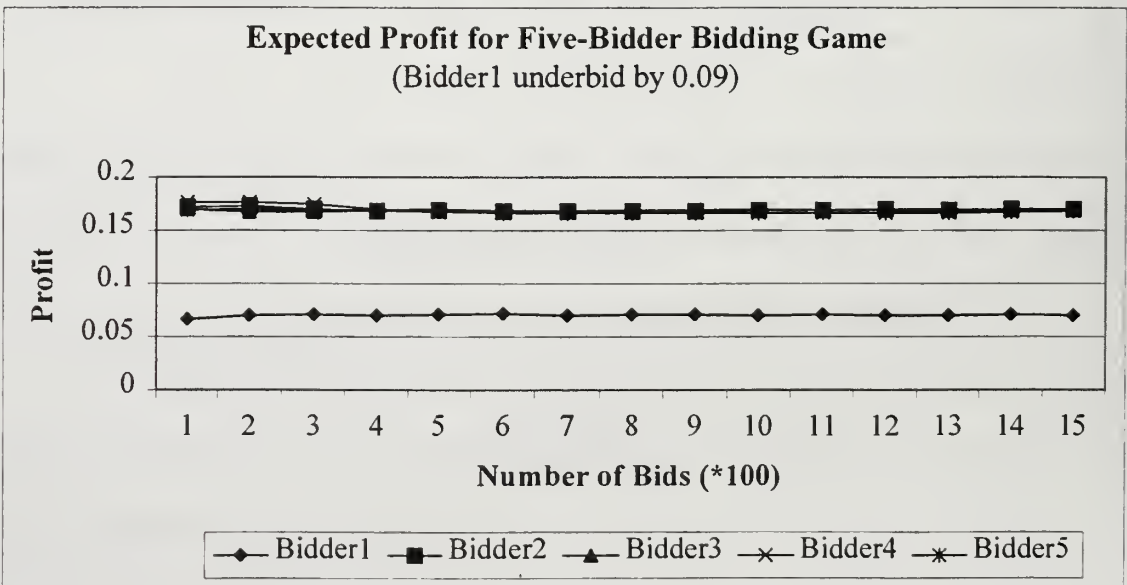


Figure 22

The average expected profit for bidder1 approached 0.0209 while others approached to a range from 0.0269 to 0.0328. The average profit curves are shown in Figure 23:

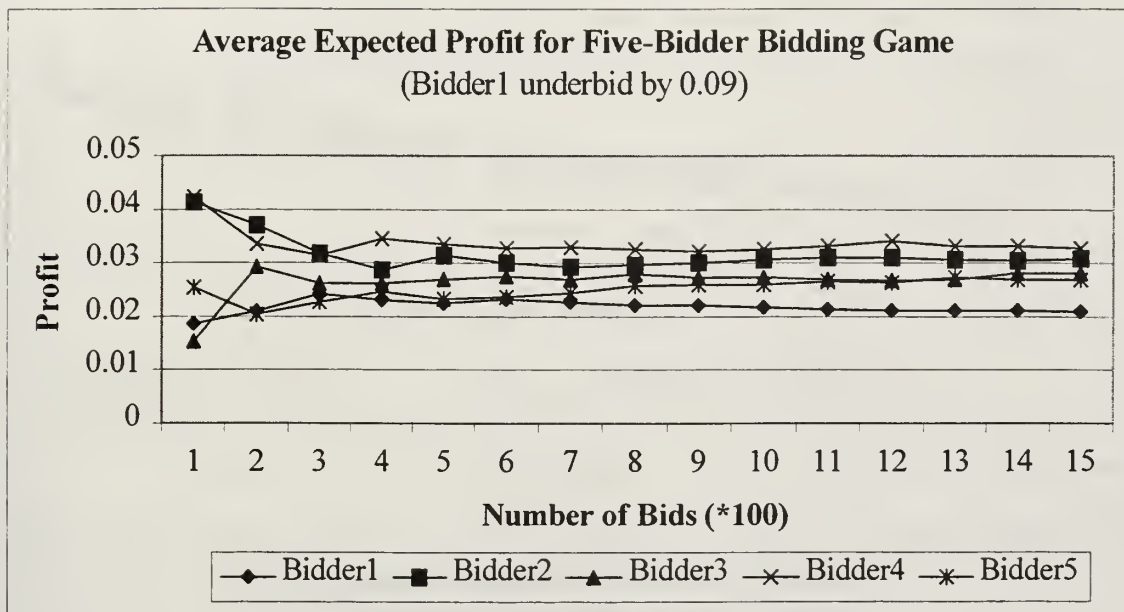


Figure 23

Bidder1 won 446 games while others won the games in a range of 241 to 292 times. Overall results of the experiment is outlined in Tables 9 and 10 in Appendix C.

Total profit for bidder1 was also lower than the others', 31.3722 as opposed to 40.4257-49.2515, even though bidder1 won more games than the others did. The total profit curves are graphically shown in Figure 24.

In the third scenario, both bidders "1" and "2" underbid by 0.12 while the rest of the bidders used equilibrium strategy. Bidders "1" and "2" made lower profits, 0.0378 and 0.0407 respectively, than the rest did, approximately 0.169, as shown in Figure 25.

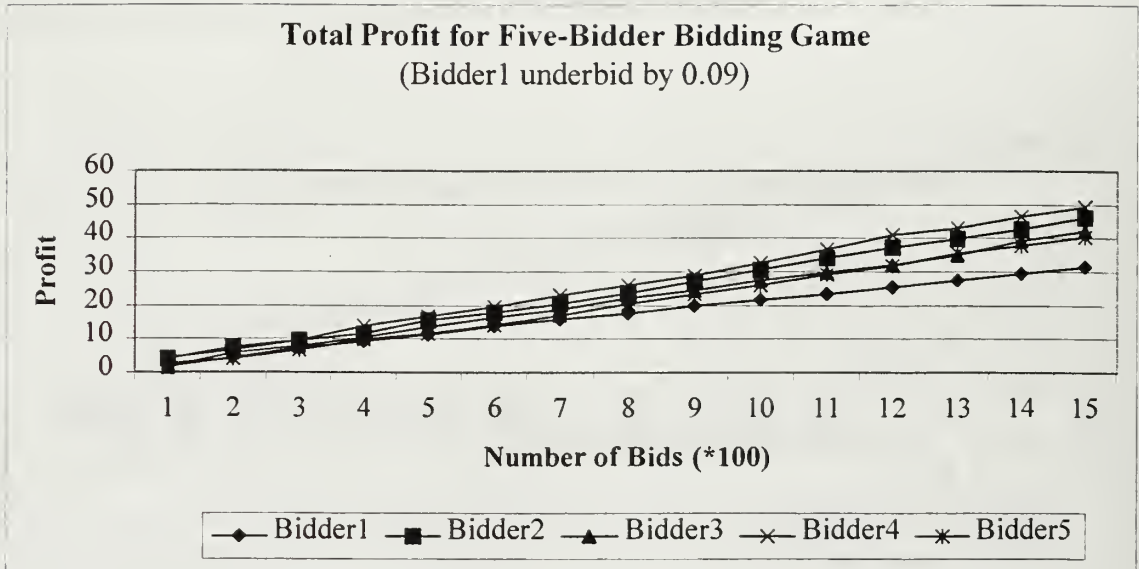


Figure 24

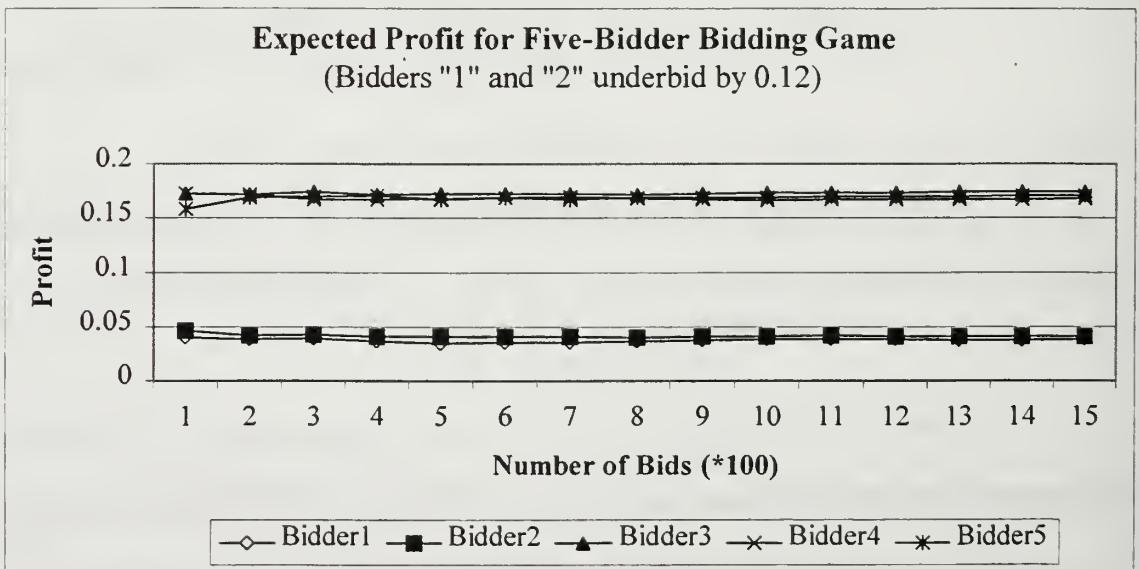


Figure 25

The average profit scheme resulted the same; bidders "1" and "2" made 0.011 while others did 0.024, 0.0245 and 0.0215 respectively, as seen in Figure 26.

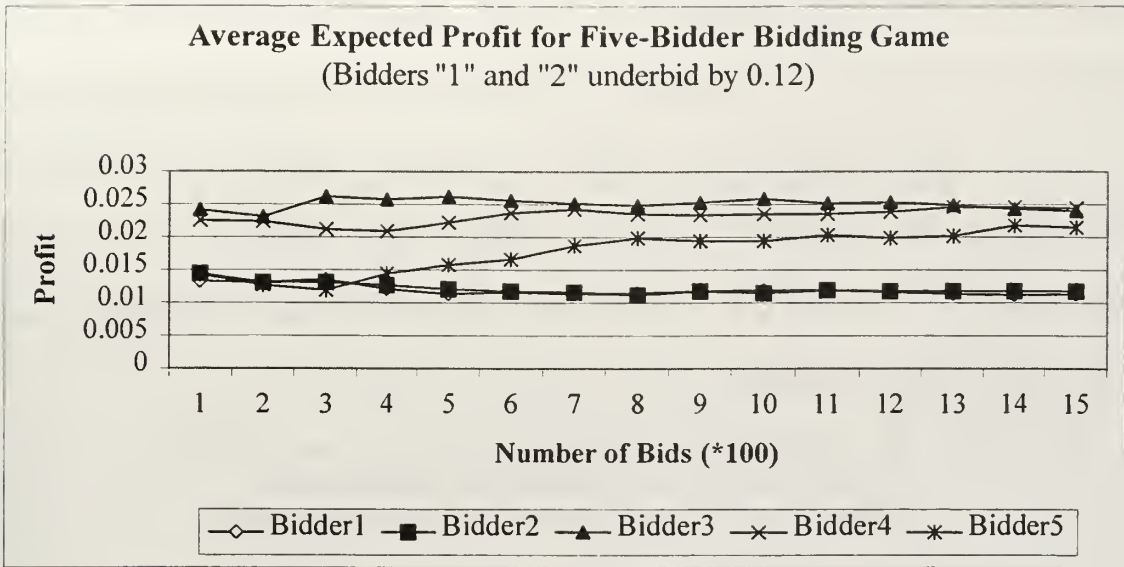


Figure 26

Total profit for bidders “1” and “2” also were lower than the others’ profits. They made 16.992 and 17.737 respectively, while others profit ranged from 32.21 to 36.711. Total profit curves for bidders are shown in Figure 27.

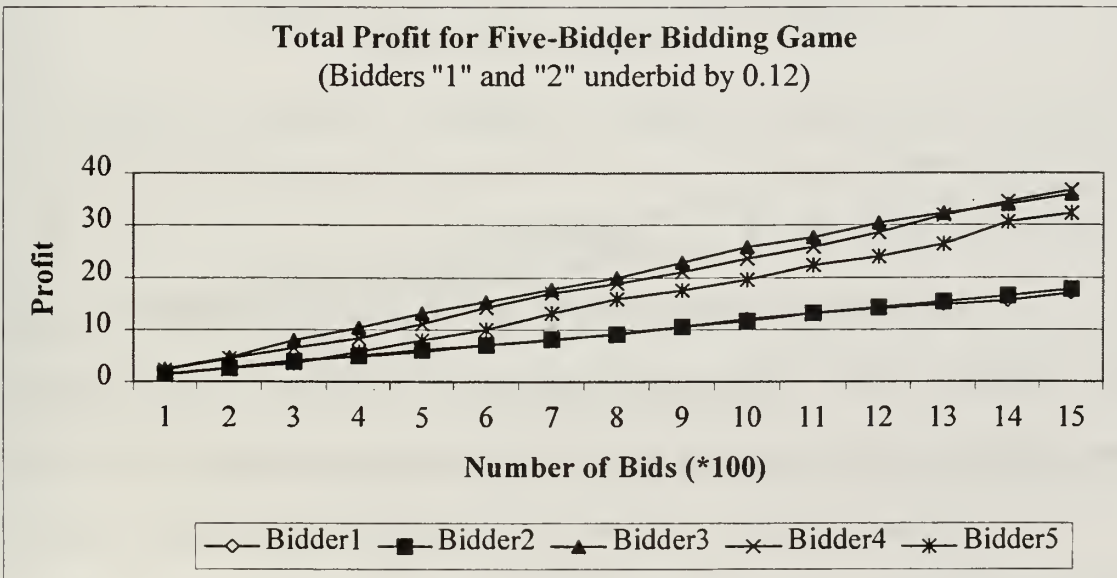


Figure 27

Bidders “1” and “2” won 449 and 436 games out of 1500 games. The rest won the games 189-219 times. As it was the case in previous experiments, underbidding

bidders made lower profits, although they won more games than the equilibrium strategy bidders. Overall results of the experiment are shown in Tables 11 and 12 in Appendix C.

In the last scenario, bidders “1” and “2” underbid by 0.12 and 0.1 respectively while others used equilibrium strategy. The underbidding bidders made lower profits of 0.03781 and 0.06149 respectively while others made 0.169. The expected profit graph is shown in Figure 28.

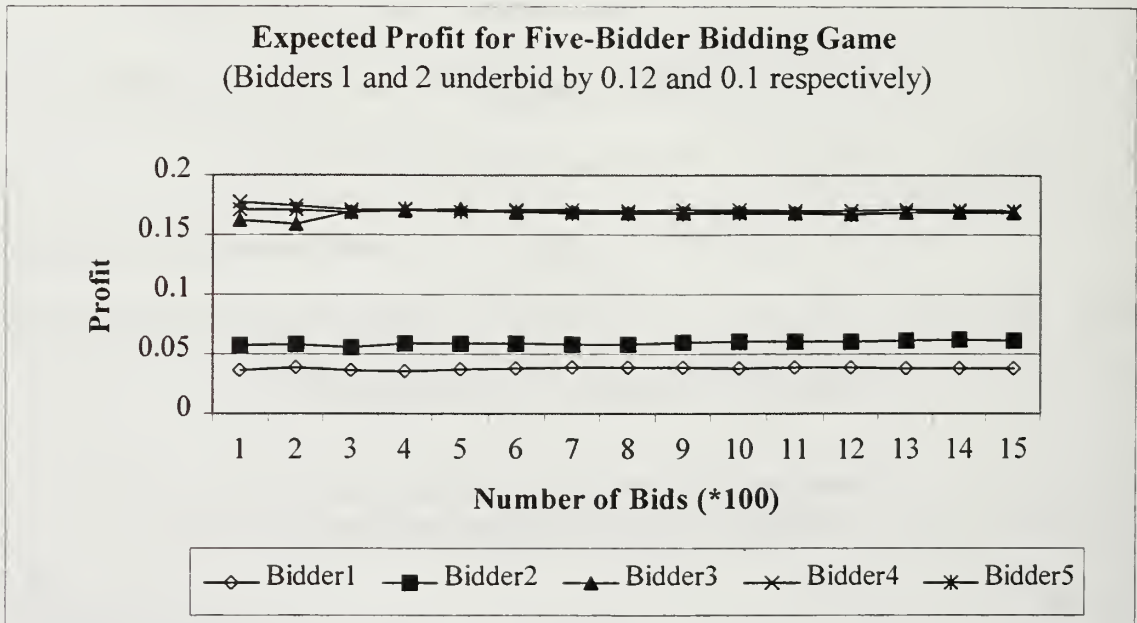


Figure 28

The average profit for bidders “1” and “2” were 0.01129 and 0.0169 while others’ approached almost 0.024 in 1500 games as seen in Figure 29. The equilibrium strategy bidders made higher expected and average profits than underbidding bidders did.

Bidders “1” and “2” won 448 and 414 games while the others won very close number of games ranging from 209 to 218 as seen in Table 13 in Appendix C. Despite bidders “1” and “2” won much more games than others, they made lower total

profits in 1500 games. The lowest bidder, bidder1, made the lowest total profit, 16.9386. The second lowest total profit was bidder2's, 25.4564.

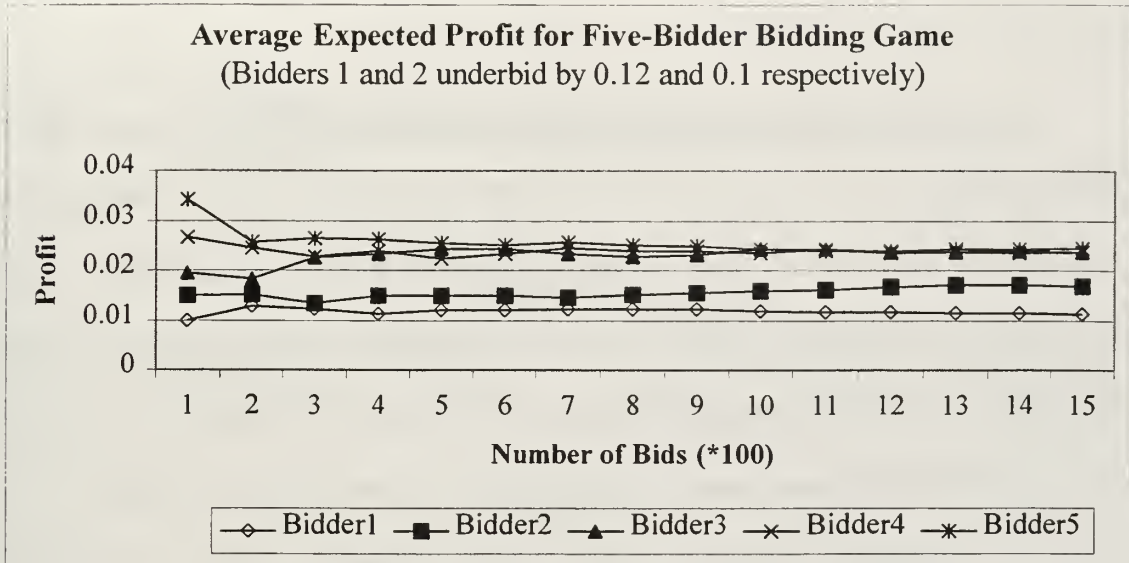


Figure 29

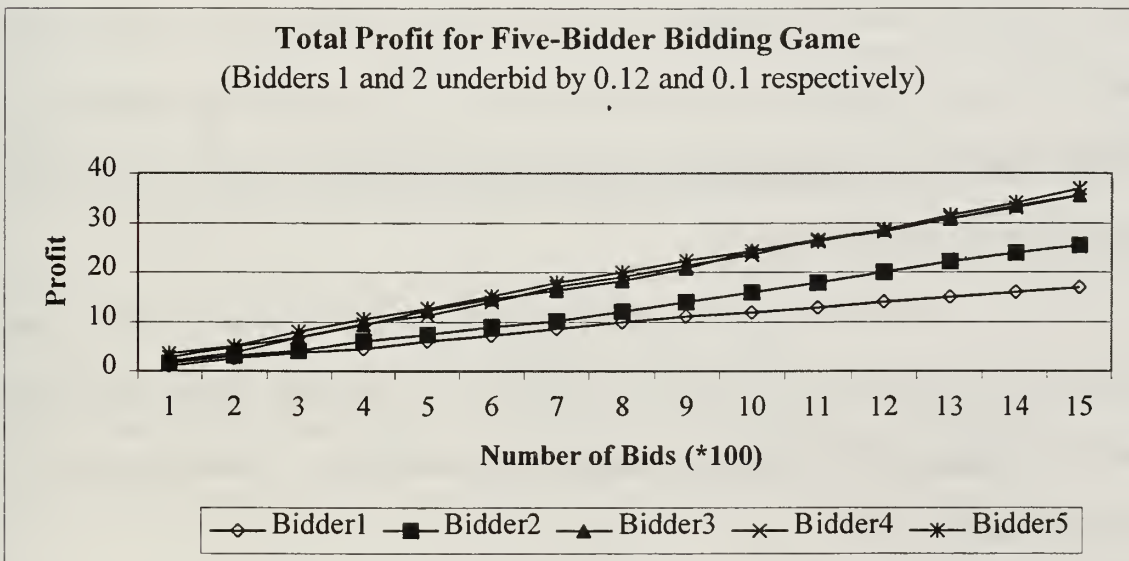


Figure 30

Finally, equilibrium strategy bidders made very close total profits ranging from 35.57 to 36.84. Total profit curves are shown in Figure 30. Results of the experiment are shown in Tables 13 and 14 in Appendix C.

d. Experimentation With Ten Bidders

The simulation was conducted with ten bidders according to uniform cost distribution within interval $[0, 1]$.

In these series of experiments, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. The second one was conducted as bidder1 underbid while others used the equilibrium strategy again. In the third one, both bidders “1” and “2” underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders “1”, “2”, “3” and “4” underbid by different amounts while the rest used the equilibrium strategy.

The first experiment, which all used equilibrium strategy, resulted in a well-defined equilibrium. All bidders’ expected profit approached approximately 0.091 as the number of games approached 1500. The graphical representation of the expected profit is shown in Figure 31. As can be seen in the overall results of the experiment in Tables 15 through 18 in Appendix C, introducing more bidders to the game reduced the expected profit from bidding more.

The average expected profit graph is shown in Figure 32. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders’ average expected profit approaches approximately 0.009 while the number of games approaches 1500.

As seen in Table 16 in Appendix C, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 139 to 162 games. Total profits as seen in Figure 33, were almost the same, ranging between 12.6196 and 14.7461.

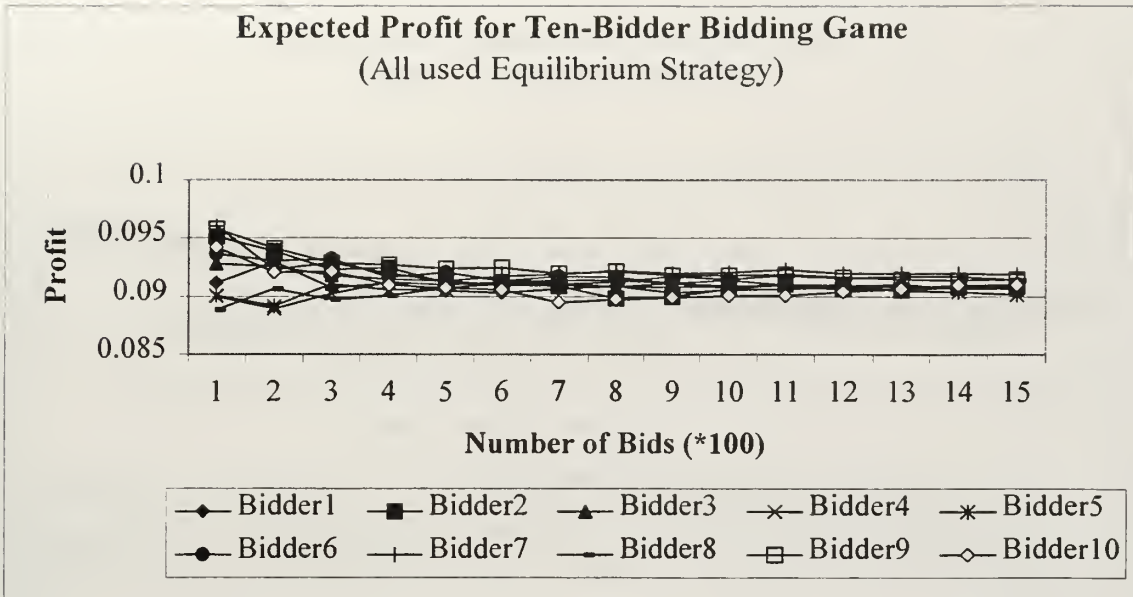


Figure 31

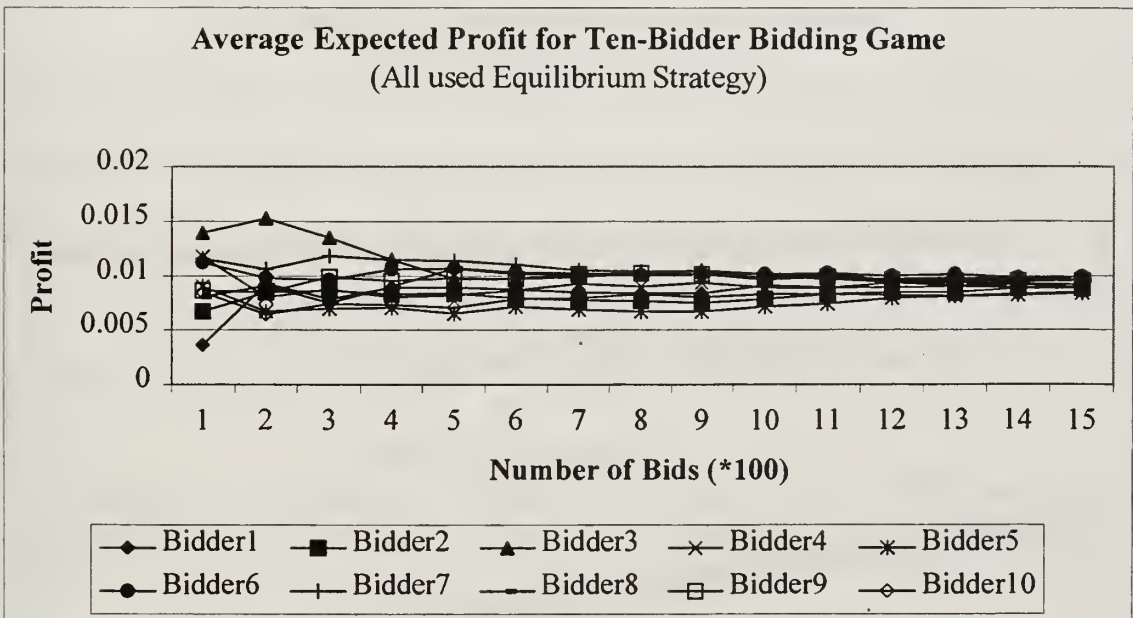


Figure 32

In the next experiment, bidder1 underbid by 0.0825, which was 15 percent of the average bid price, while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.057 while other bidders' remained at 0.091. The expected profit graph is shown in Figure 34.

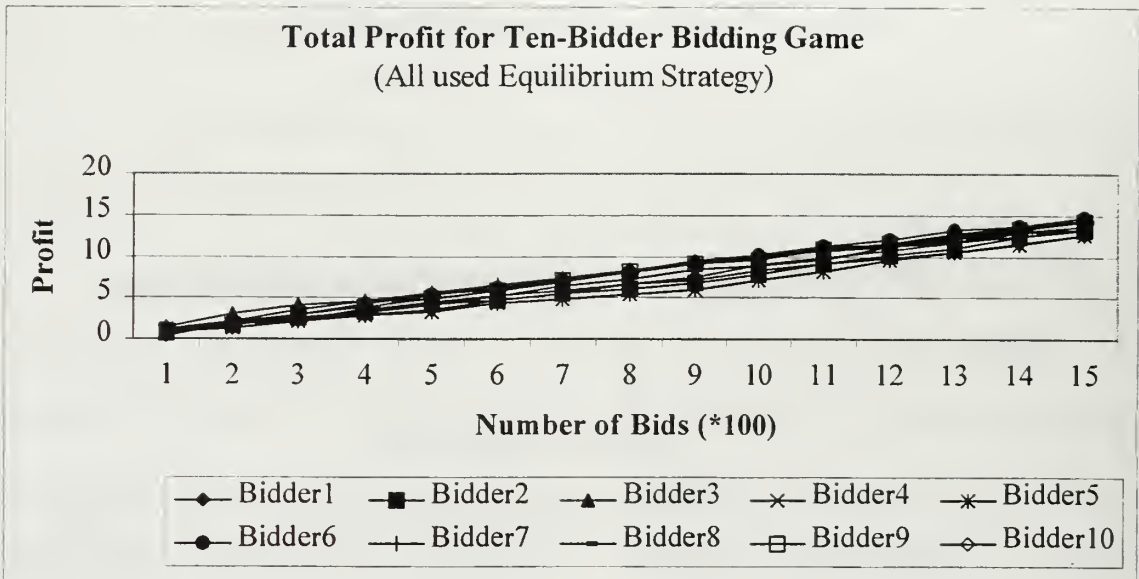


Figure 33

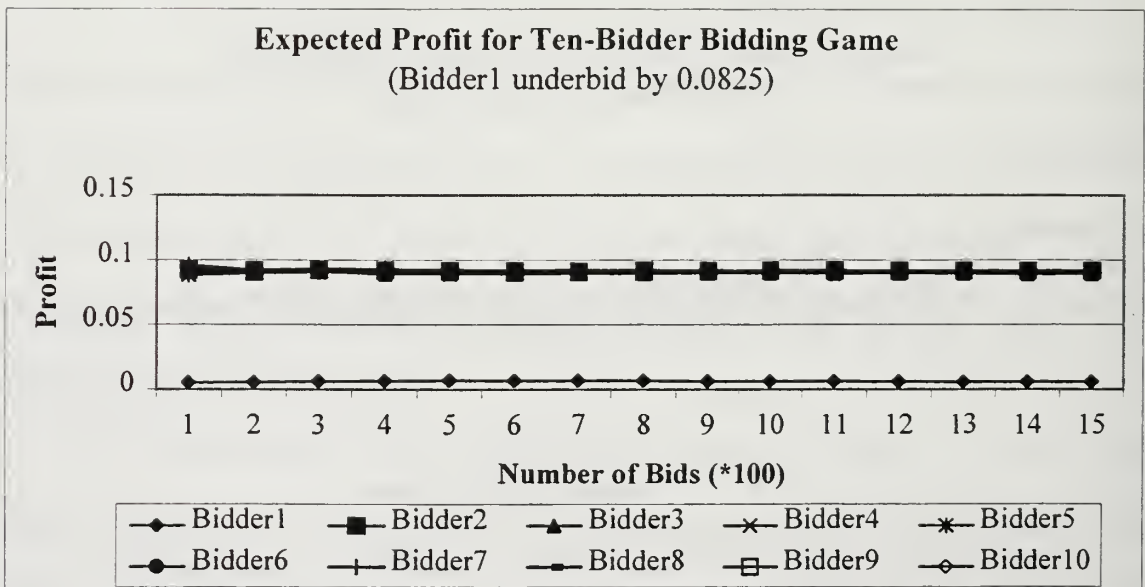


Figure 34

Average profit for the bidders also reinforced the same scheme as the equilibrium game had shown. Bidder1's average profit was 0.0011 while others were very close, approximately 0.008, as graphically shown in Figure 35. Overall results of the experiment are outlined in Tables 19 through 21 in Appendix C.

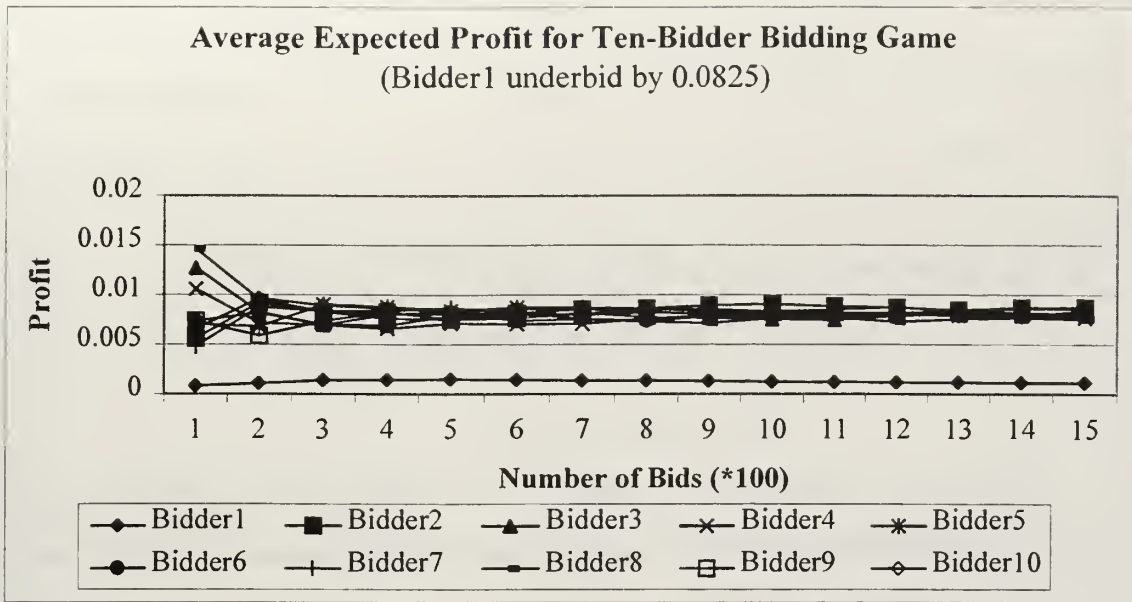


Figure 35

Bidder1 won 296 of 1500 games while others won in a range of 124 to 143 games as shown in Table 20. Although bidder1 won more games than the others won, its total profit was 1.6872 which is much lower than the others' profits ranging from 11.3013 to 13.1131. Total profit graph for this experiment is shown in Figure 36:

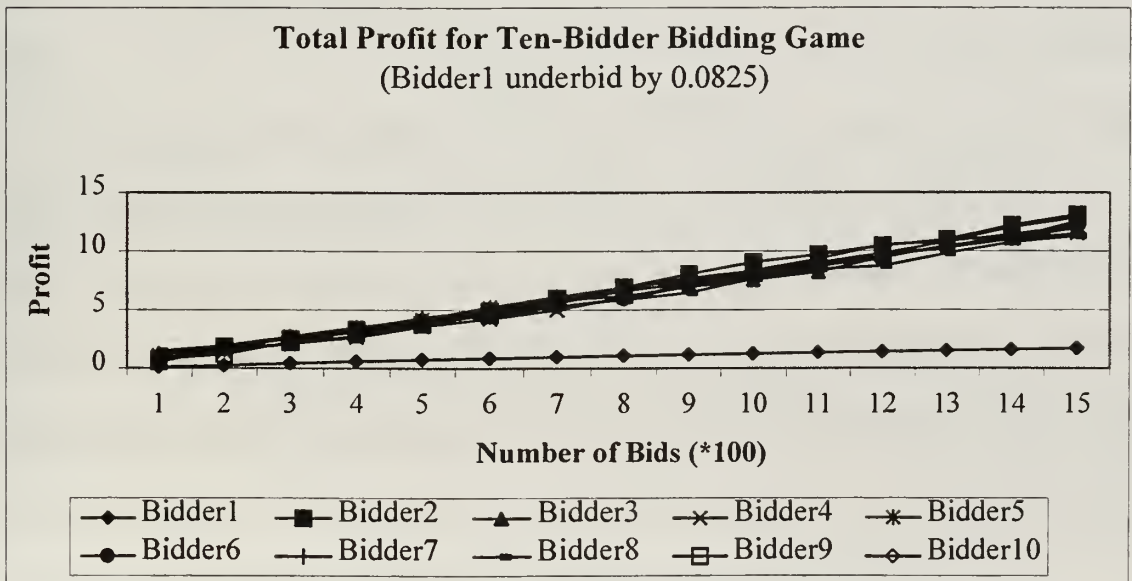


Figure 36

In the third experiment, bidders “1” and “2” underbid by 0.0825 while others bid according to the equilibrium strategy. As a result of 1500 games, bidders “1” and “2” made expected profits, 0.055 and 0.059, while the other bidders’ profits remained at 0.091. The expected profit graph is shown in Figure 37.

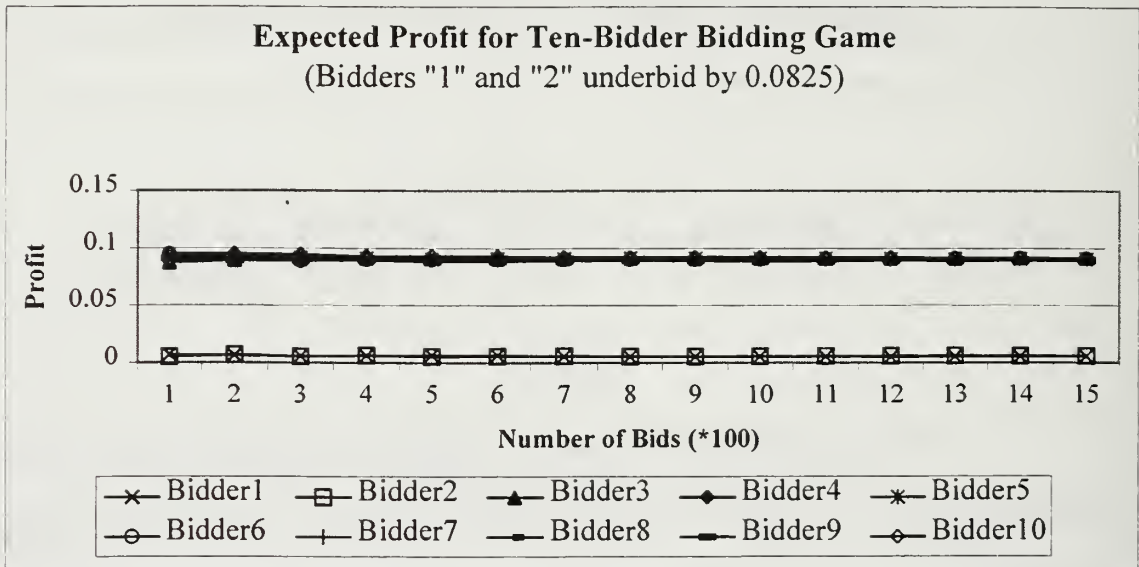


Figure 37

Average profit for the bidders also reinforced the same scheme as the expected profit results had shown. Underbidding bidders’ average profits were 0.001 while others were ranging from 0.0064 to 0.0082, as graphically shown in Figure 38.

Bidders “1” and “2” won 264 and 259 games respectively while others won the games in a range of 109 to 135 times as shown in Table 24 in Appendix C. Total profits for underbidding bidders were 1.4591 and 1.5343 which are much lower than the others’ profits ranging from 9.7221 to 12.3395. Total profit graph for this experiment is shown in Figure 39.

The overall results of the experiment are shown in Tables 23 through 26 in Appendix C.

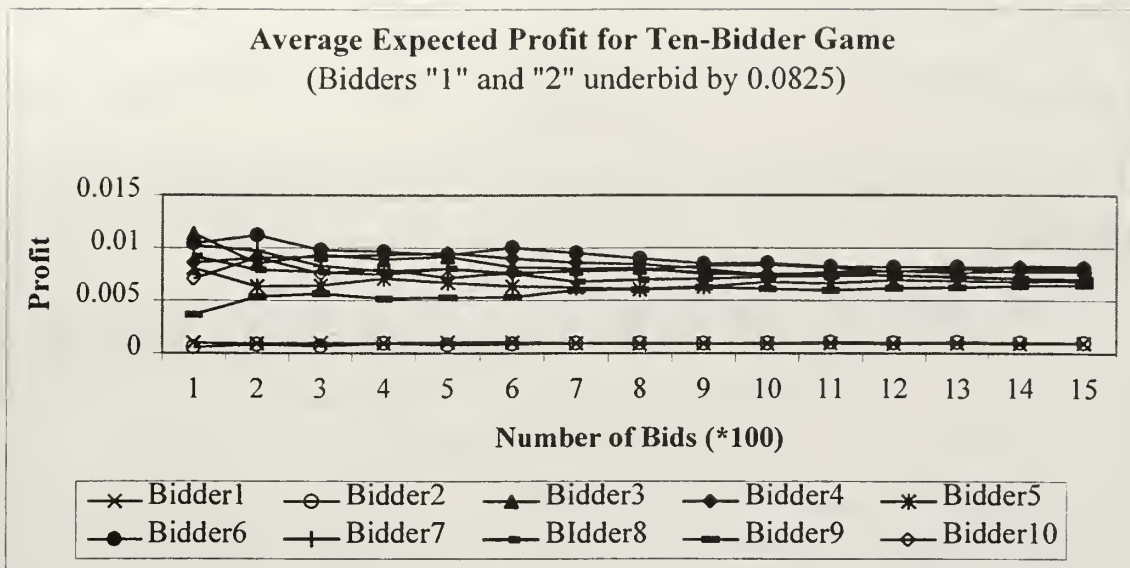


Figure 38

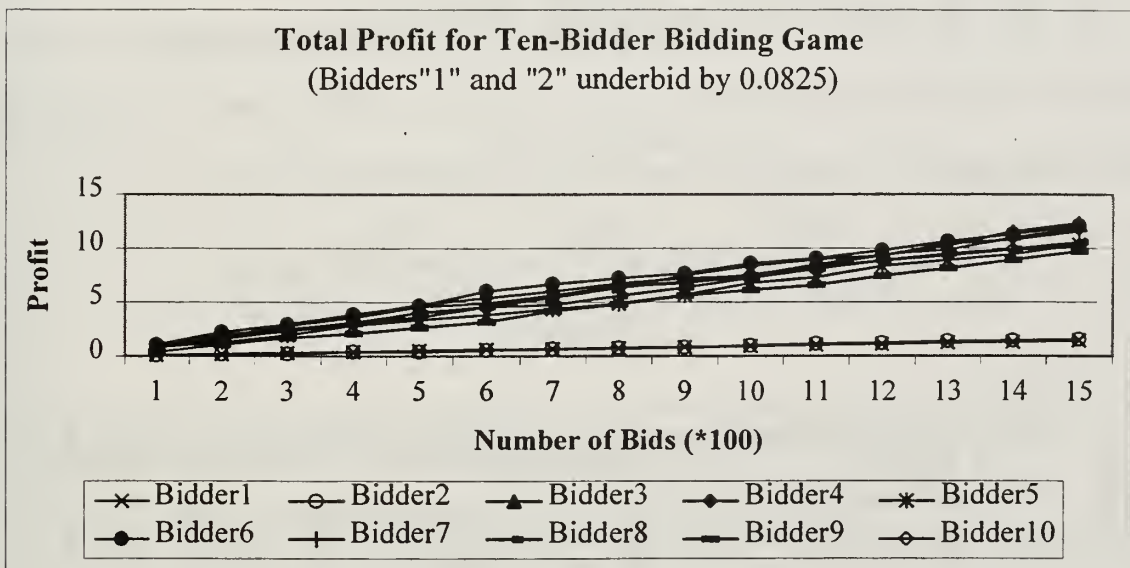


Figure 39

In the last experiment, bidders "1", "2", "3" and "4" underbid by 0.0825, 0.075, 0.0675 and 0.06 respectively in 1500 games. The remaining six bidders used equilibrium strategy. The underbidding bidders made lower profits, 0.0063, 0.0148, 0.0229 and 0.03, than the equilibrium strategy bidders did, which was 0.091. The expected profit curves are shown in Figure 40.

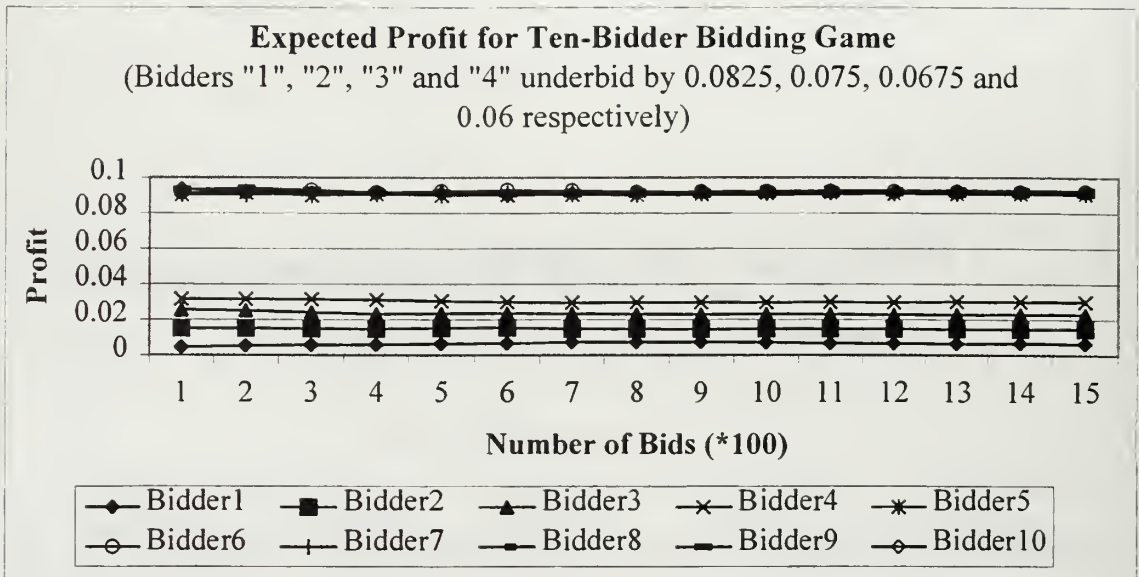


Figure 40

The equilibrium strategy bidders made higher average profits as seen in Figure 41. The underbidding bidders average profits were 0.00098, 0.00234, 0.00328 and 0.004 while equilibrium strategy bidders' ranged from 0.0053 to 0.0073.

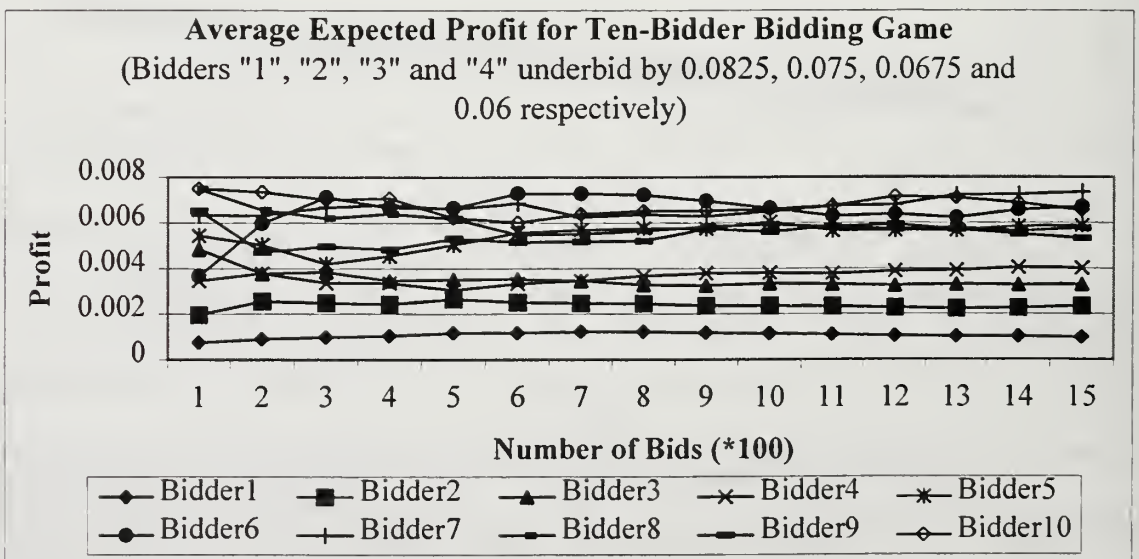


Figure 41

The underbidding bidders won more games than the others did. They won the games in a range of 200 to 233 times while equilibrium strategy bidders won in a

range of 88 to 121 times. Despite this result, the equilibrium bidders made higher total profits than underbidding bidders. The underbidding bidders' total profits ranged from 1.4674 to 6.0083 while others' ranged from 7.9767 to 11.0308. Total profit curves are shown in Figure 42. Overall results of the experiment are written in Tables 27 through 30 in Appendix C.

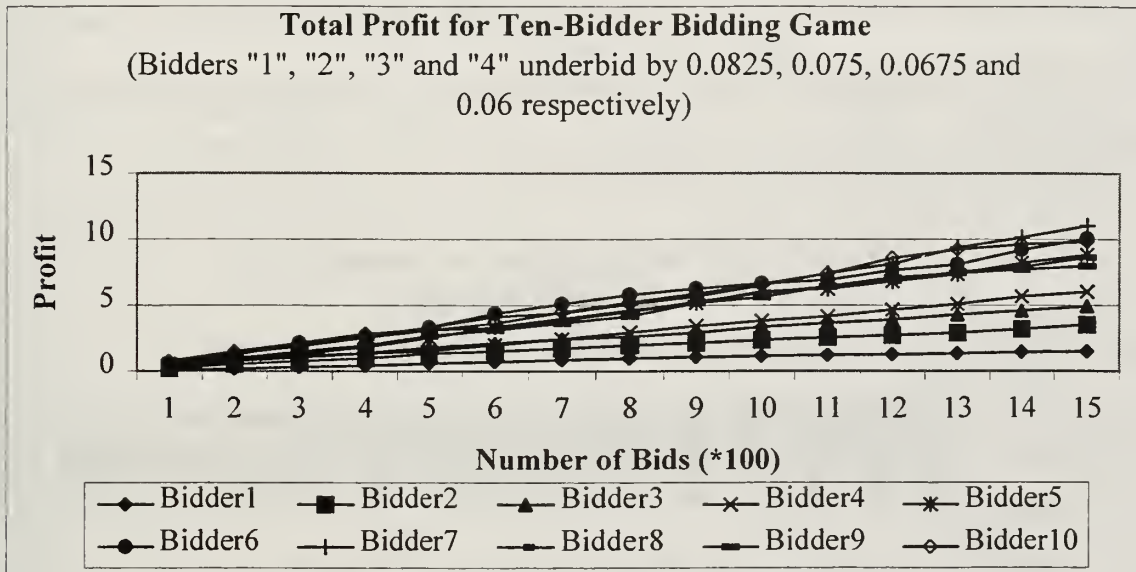


Figure 42

e. Experimentation With Fifteen Bidders

The simulation was conducted with fifteen bidders according to uniform cost distribution within interval [0, 1].

In these last series of experiments under uniform cost distribution, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. In the second one, bidder1 underbid while others used the equilibrium strategy again. In the third one, bidders "1", "2" and "3" underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario,

bidders “1”, “2”, “3” and “4” underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategy resulted in a well-defined equilibrium. All bidders’ expected profit approached approximately 0.062 as the number of games approached 1500. The graphical representation of the expected profit is shown in Figure 43. As it was the result of the previous experiments, introducing more bidders to the game reduced the expected profit from bidding more. The results are outlined in Tables 31 through 34 in Appendix C.

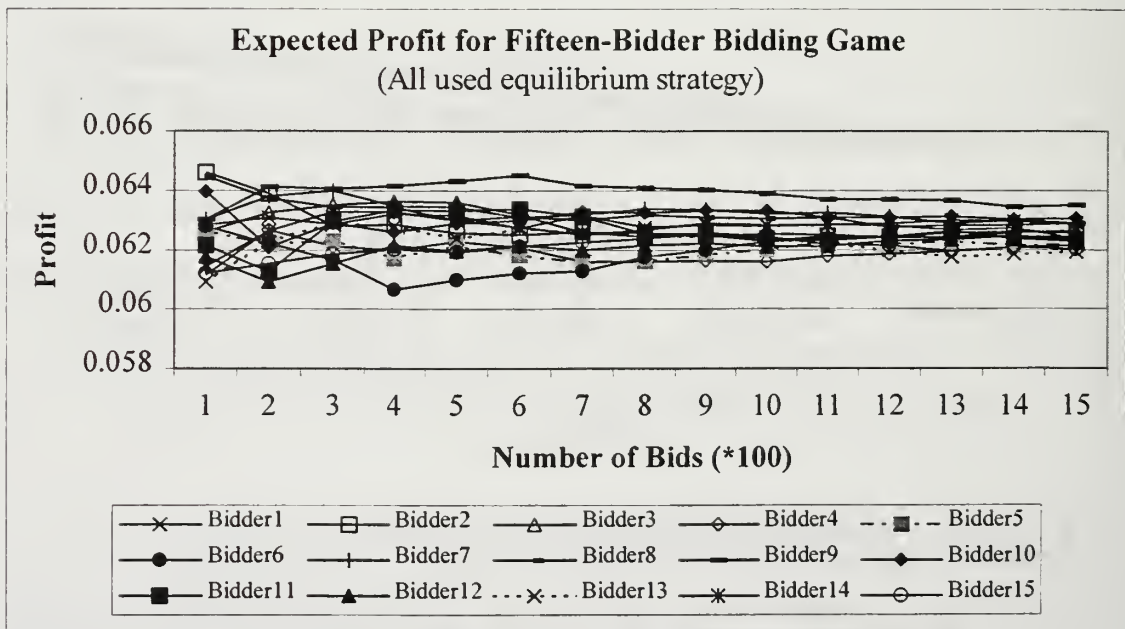


Figure 43

The average expected profit graph is shown in Figure 44. The average expected profit results also reinforce the equilibrium of the game. In this simulation, bidders’ average expected profit approaches 0.004 while the number of games approaches 1500.

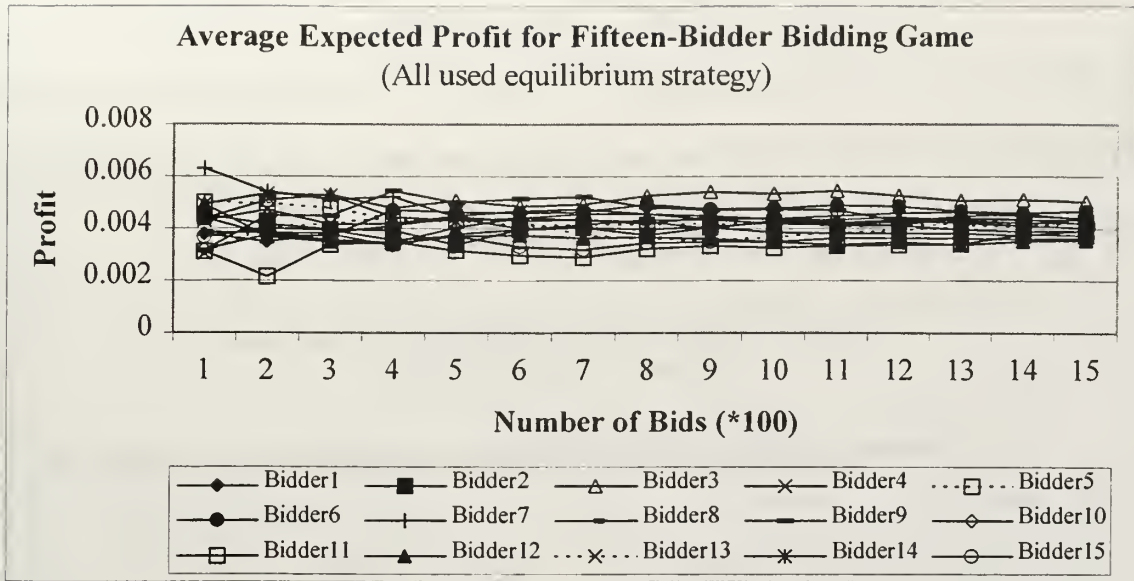


Figure 44

As seen in Table 32, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 86 to 121 games each. Total profit for bidders as seen in Figure 45, ranged between 5.361 to 7.578.

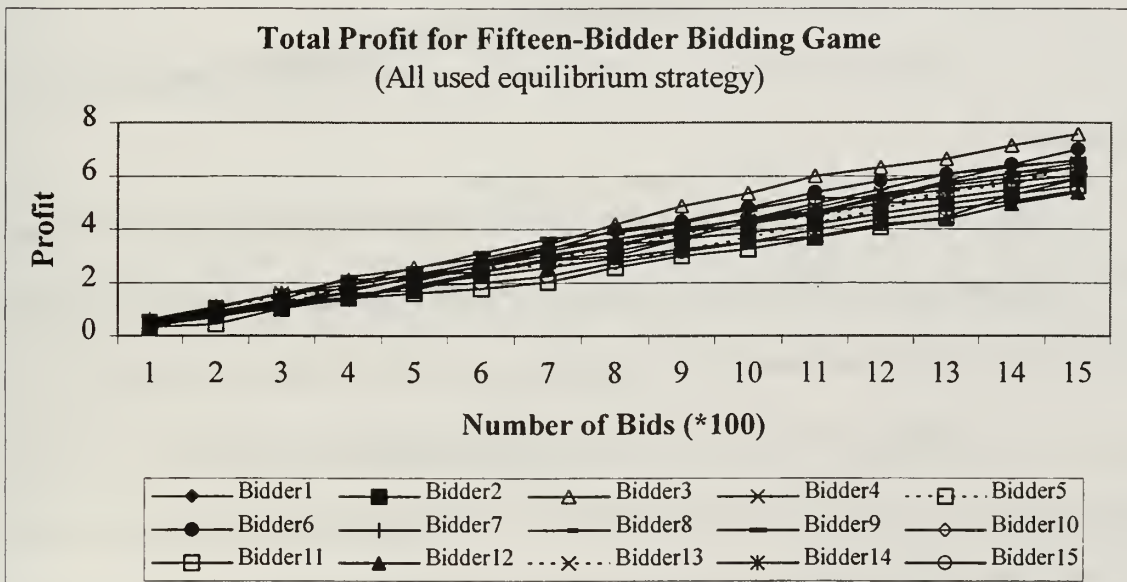


Figure 45

In the next experiment, bidder1 underbid by 0.075 while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.0116 while the other bidders' remained at 0.062. The expected profit graph is shown in Figure 46.

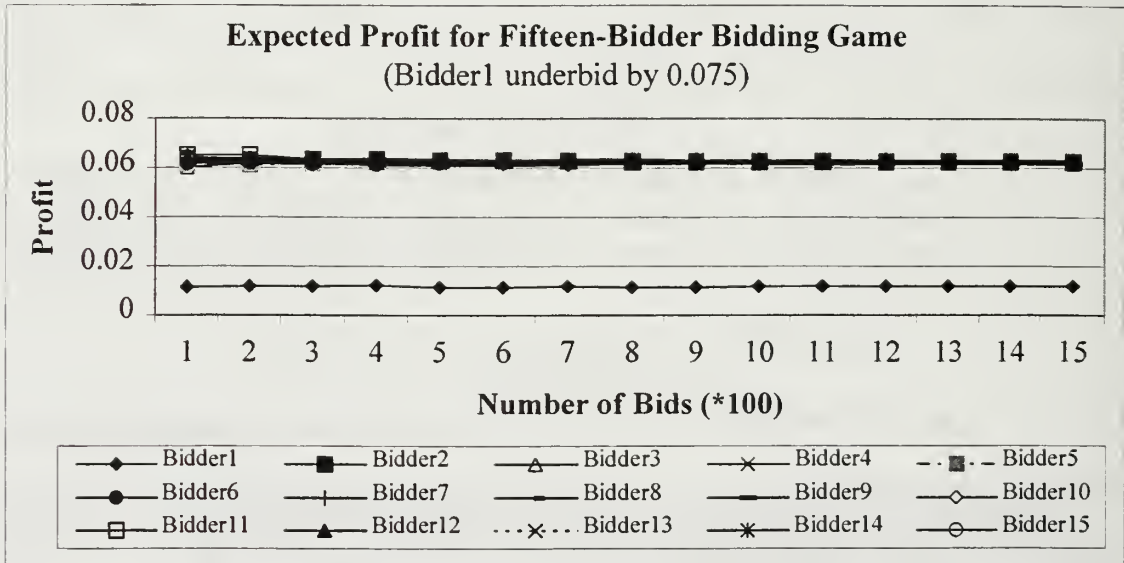


Figure 46

The overall results of the experiment are outlined in Tables 35 through 38 in Appendix C.

Average expected profit for the bidders also reinforced the same scheme as the equilibrium game had shown, except for bidder1. Bidder1's average profit was 0.00139 while others were almost the same, 0.004, as graphically shown in Figure 47.

Bidder1 won 179 of 1500 games while others won in a range of 69 to 109 games as shown in Table 36. Although bidder1 won more games than the others won, its total profit was 2.07798 which is quite lower than the others' profits ranging from 4.2818 to 6.8147. Total profit graph for this experiment is shown in Figure 48.

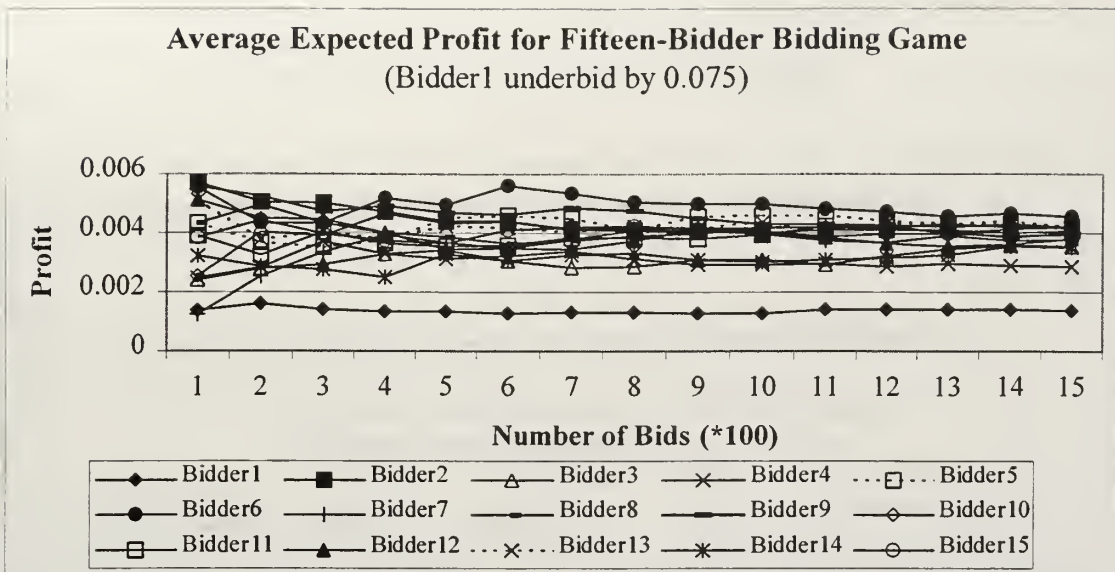


Figure 47

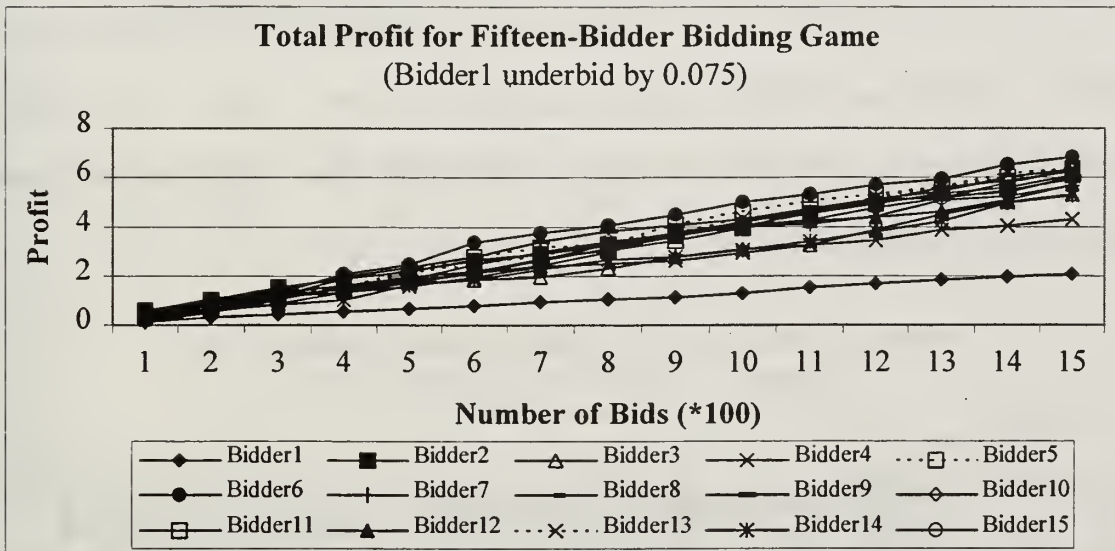


Figure 48

In the third experiment, bidders “1”, “2” and “3” underbid by 0.055 while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders’ expected profit approached 0.0057 while others’ approached approximately 0.062. The expected profit graph is shown in Figure 49.

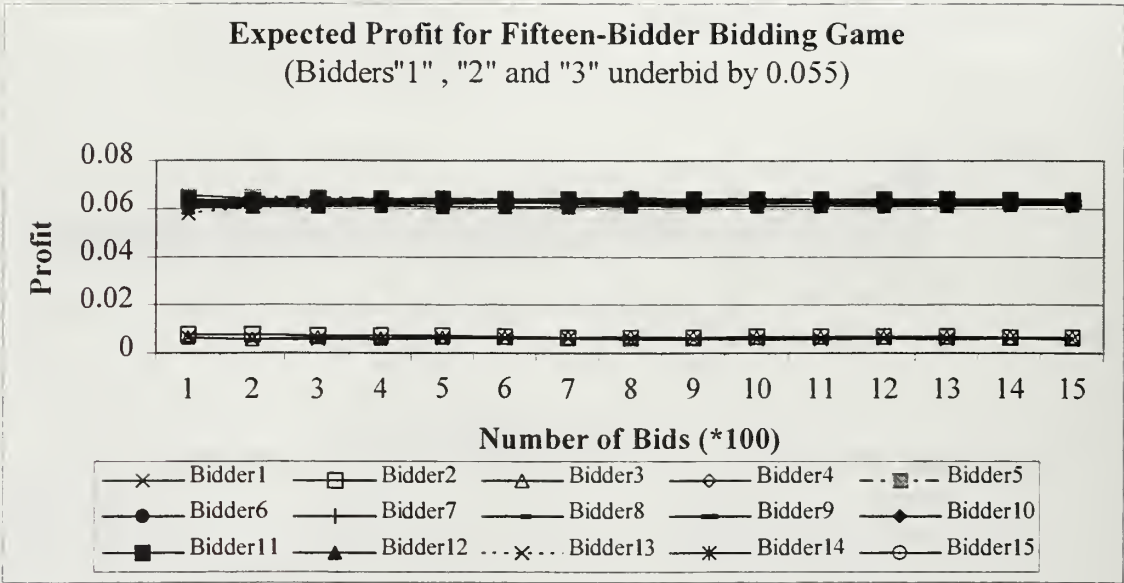


Figure 49

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0007 while others' approached to a range of 0.0028 to 0.0041 in 1500 games. The average expected profit graph is shown in Figure 50.

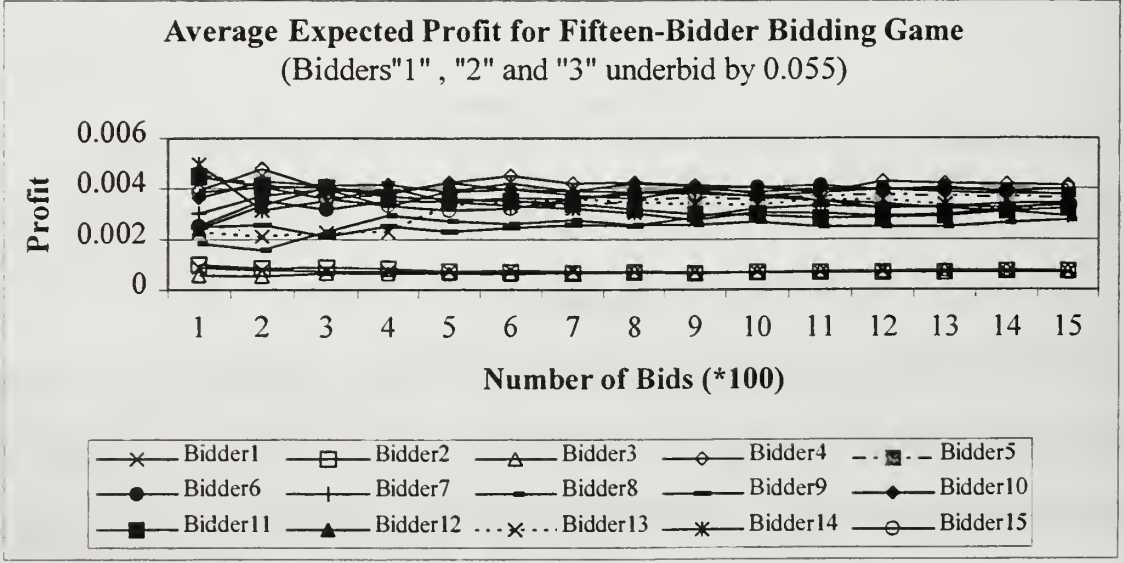


Figure 50

Total profits for underbidding bidders, ranging from 1.0074 to 1.0914, were also lower than the equilibrium strategy bidders' total profits, 4.1325 to 6.1872. The underbidding bidders won 163 to 176 games while others won 67 to 99 games as seen in Table 40 in Appendix C. The total profit curves are shown in Figure 51 and overall results of the experiment are shown in Tables 39 through 42 in Appendix C.

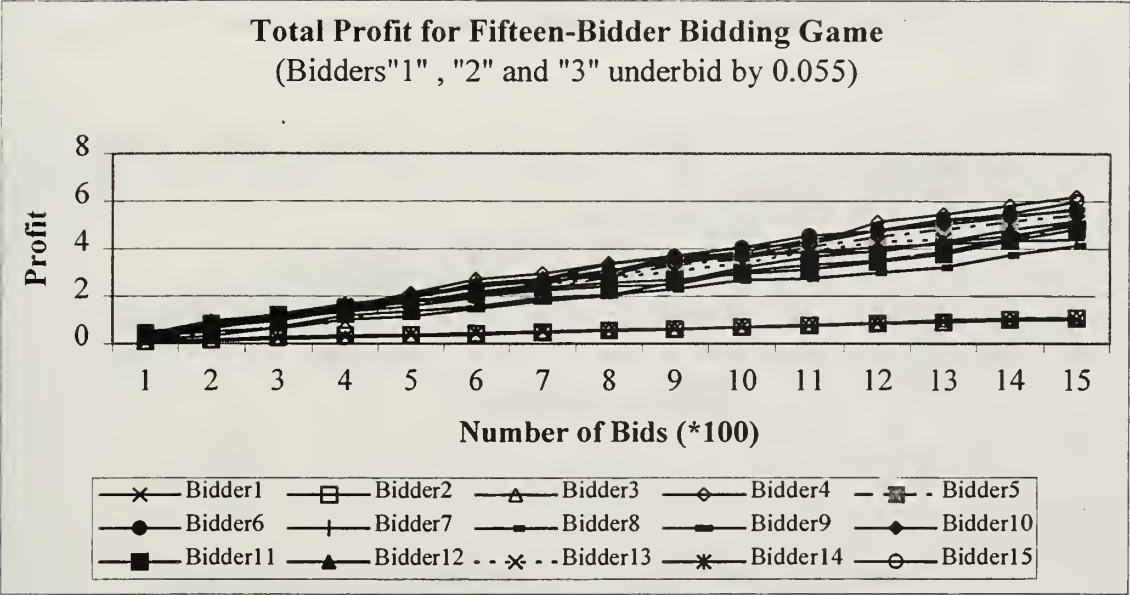


Figure 51

In the last experiment with fifteen bidders, bidders “1”, “2”, “3” and “4” underbid by 0.055, 0.05, 0.045 and 0.04 respectively, while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders' expected profit approached 0.006, 0.0115, 0.0168 and 0.022 respectively, while others' approached approximately 0.063. The expected profit graph is shown in Figure 52.

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0063, 0.00119, 0.00182 and 0.00229 while others' approached a range of 0.0028 to 0.004 in 1500 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in

contract games. The deviant bidders lose profit even if they win more games than the equilibrium strategy bidders. The average profit graph for this experiment is shown in Figure 53.

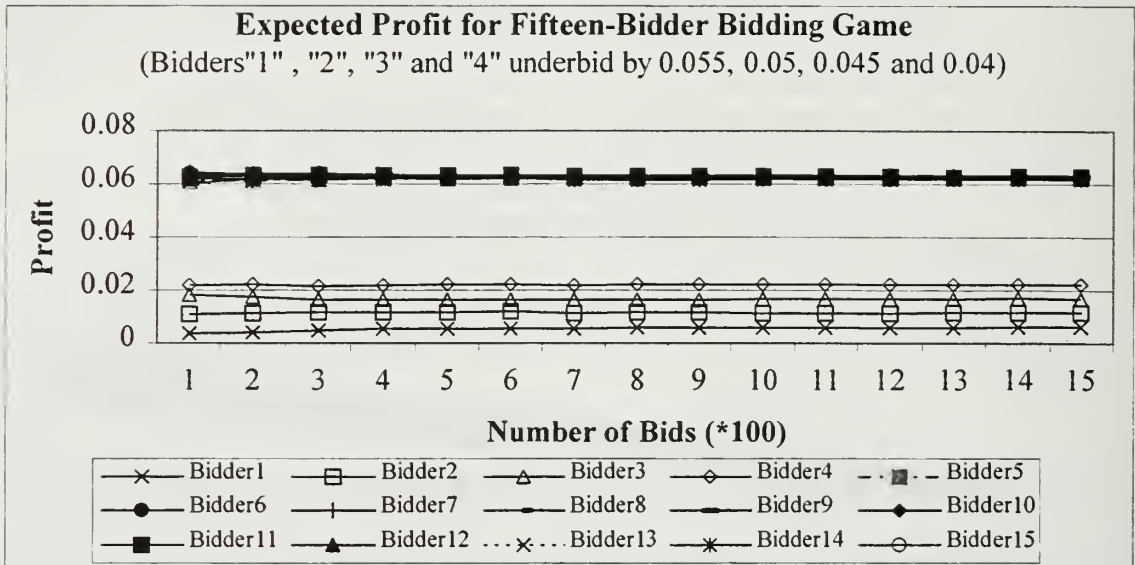


Figure 52

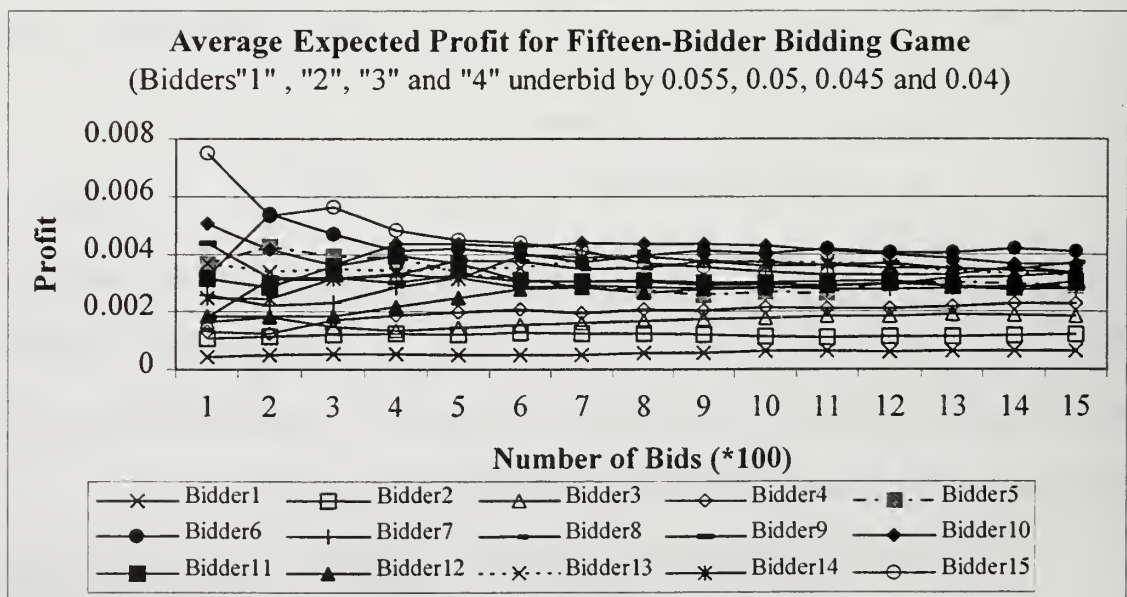


Figure 53

Total profits for underbidding bidders, ranging from 0.9441 to 3.4353, were also lower than the equilibrium strategy bidders' total profits, 4.2041 to 6.1235. The underbidding bidders won 155 to 162 games as opposed to others, which won 67 to 97 games as seen in Table 44. Total profit curves are shown in Figure 54.

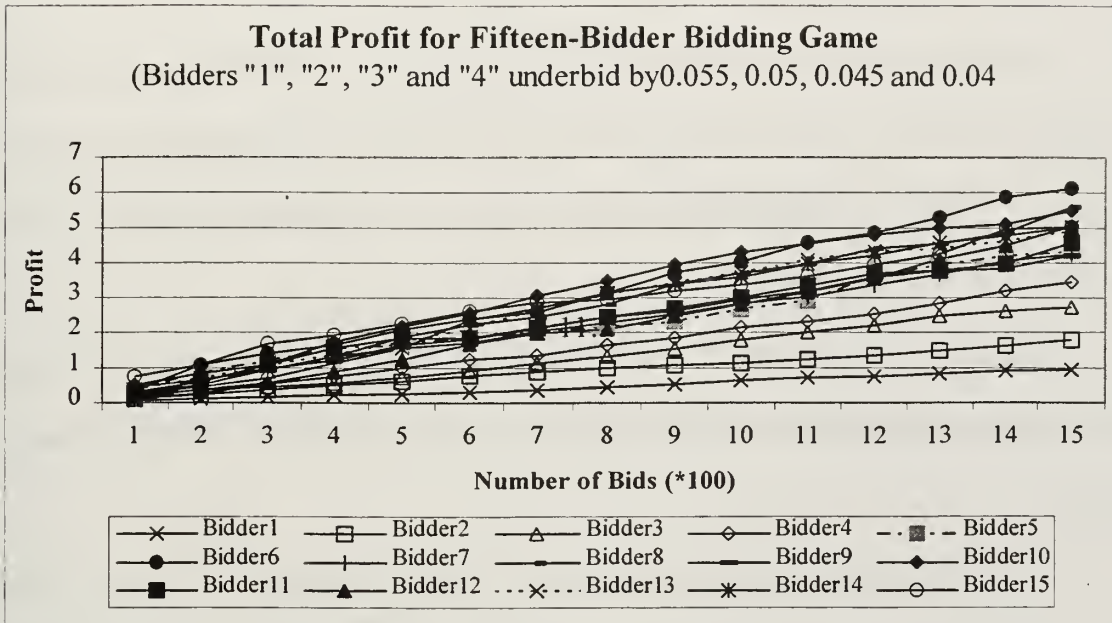


Figure 54

The overall results of the experiment are shown in Tables 43 through 46 in Appendix C.

C. EXPERIMENTATION UNDER TRIANGULAR COST DISTRIBUTION

This section of the chapter describes the FPSB simulation when the production costs are distributed according to triangular distribution. The FPSB simulation using a triangular production cost distribution defines the bidders' behavior under a different cost condition. The uniform cost distribution was primarily used for simplicity, though it was reasonable when bidders' production costs were identical due to the similar production

methods used. Triangular distribution may be a better reflection of reality when some bidders have competitive advantage over the others because of better production techniques, management controls or different technologies used for the product.

1. Mathematical Model for Simulation

For products for which relatively new, more complex and different manufacturing methods are used, the costs of production vary according to the manufacturers' business practices. Triangular distribution, then, becomes a closer representation of costs associated with that product.

Assume that the bidders' potential production costs are distributed according to the triangular distribution over an interval $[h, k]$ with a mode value, m , with probability $2/(h-k)$.

A bidding strategy defines the relationship between the bidder's proposal, b_i , and cost, c_i . The following section is the derivation of the bidding function under uniform cost distribution.

2. General-Form Bidding Function for the Triangular Cost Distribution

The probability density function, $f_X(x)$, of the triangular distribution is shown as:

$$f_X(x) = \begin{cases} [2*(x-k)]/[(h-k)*(m-k)] & \text{for } k < x < m \\ [2*(h-x)]/[(h-k)*(h-m)] & \text{for } m < x < h \\ 0 & \text{otherwise} \end{cases}$$

Where $k < m < h$ and,

k : the lower limit of the triangular distribution.

m: the mode of the distribution.

h: the upper limit of the triangular distribution.

The graphic representation of the triangular distribution is shown in Figure 55:

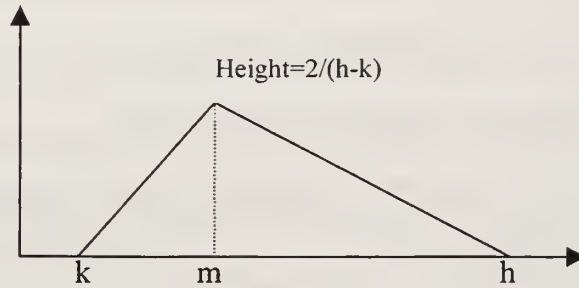


Figure 55 PDF of the triangular distribution

The cumulative distribution function, $F_X(x)$, of the triangular distribution is:

$$F_X(x) = \begin{cases} 0 & \text{for } x < k \\ \frac{[(x-k)^2]}{[(h-k)*(m-k)]} & \text{for } k < x < m \\ \frac{[1-(h-x)^2]}{[(h-k)*(h-m)]} & \text{for } m < x < h \\ 1 & \text{for } x > h \end{cases}$$

Assume players 1, 2, 3, ..., n-1, n adopt the strategy $b(\cdot)$, and $b(\cdot)$ is strictly increasing and differentiable. For a given value of the i^{th} player's cost, i^{th} player's optimal bidding strategy solves:

$$\text{Max}\{(b_i - c_i) * \text{Prob}[b_i < b(c_1), \dots, b_i < b(c_n)]\} \text{ where } n: \text{ number of bidders.}$$

The probability of a given player i 's bid to be the lowest of all is:

$$\text{Prob}(b_i < b_{i+1}, \dots, b_i < b_n) = [1 - b^{-1}(b_i)]^{n-1}$$

The first order condition for player i 's optimization problem is then:

$$d\{(b_i - c_i) * [1 - b^{-1}(b_i)]^{n-1}\} / db_i = 0$$

The triangular distribution has two different distributions over the interval [k, h], that is, the first one is from the lower limit, k, through the mode, m, and the second one is from m through the upper limit, h, of the distribution. Because of this unique characteristic, the triangular distribution of the bidders' costs may be a better representative of the costs of items when the manufacturing methods are complex and the bidders use different techniques to manufacture those items.

This section of the thesis will define the bidding functions according to the interval costs fall into over [k, h].

a. The Bidding Function for the cost interval [k, m]:

The bidders' expected profit, $E[\pi]$, has a maximum if the derivative of the expected profit with respect to b equals to zero. That is:

$$d(E[\pi])/db=0$$

The expected profit is:

$$E[\pi]=(b-c)*[1-F_X(x)]^{n-1}$$

By taking the derivative of expected profit with respect to b and then equal it to zero, we are going to find the bidding function that maximizes bidders' profits.

$$d(E[\pi])/db=0$$

$$d\{(b-c)*[1-F_X(x)]^{n-1}\}=0$$

$$d\{(b-c)*\{[1-(b-k)^2]/[(h-k)*(m-k)]\}^{n-1}\}/db=0$$

Let's use K instead of $[(h-k)*(m-k)]$ to make the calculations easier.

$$d\{(b-c)*\{[1-(b-k)^2]/K\}^{n-1}\}/db=0$$

$$\{[1-(b-k)^2]/K\}^{n-1}+\{(b-c)*(n-1)*\{[1-(b-k)^2]/K\}^{(n-2)}*[-2*(b-k)/K]\}=0$$

$$[K-(b-k)^2]^{n-1} + \{(b-c) \cdot (n-1) \cdot [K-(b-k)^2]^{(n-2)} \cdot [-2 \cdot (b-k)]\} = 0$$

by factoring out $[K-(b-k)^2]^{n-2}$:

$$K-(b-k)^2 - 2 \cdot (n-1) \cdot (b-c) \cdot (b-k) = 0$$

$$K - (b^2 - 2 \cdot b \cdot k + k^2) - 2(n-1) \cdot (b^2 - c \cdot b - b \cdot k + c \cdot k) = 0$$

$$- [2 \cdot (n-1) + 1] \cdot b^2 + 2 \cdot [n \cdot k + (n-1) \cdot c] \cdot b + [K - k^2 - 2 \cdot (n-1) \cdot c \cdot k] = 0$$

$$- [(n-1) + 0.5] \cdot b^2 + [n \cdot k + (n-1) \cdot c] \cdot b + \{ [K - k^2 - 2 \cdot (n-1) \cdot c \cdot k] / 2 \} = 0$$

By solving the equation for b using the quadratic formula, we get the general formula for the bidding function under the triangular cost distribution when actual costs are in the interval $[k, m]$:

$$b = \frac{n \cdot k + (n-1) \cdot c + \{ [n \cdot k + (n-1) \cdot c]^2 + 2 \cdot \{ [(n-1) + 0.5] \cdot [K - k^2 - 2 \cdot (n-1) \cdot c \cdot k] \} \}^{0.5}}{2 \cdot [(n-1) + 0.5]}$$

The bidding function is a function of number of bidders, cost and cost distribution functions. Using this general formula, we can derive all the necessary bidding functions where the cost is distributed triangularly and cost is in the interval $[k, m]$.

b. The Bidding Function For The Interval $[m, h]$

The bidders' expected profit, $E[\pi]$, has a maximum if the $dE[\pi]/db = 0$.

$$E[\pi] = (b-c) \cdot [1 - F(x)]^{n-1}$$

$$dE[\pi]/db = d \{ (b-c) \cdot [1 - \{ [1 - (h-b)^2] / [(h-k) \cdot (h-m)] \}]^{n-1} \} / db = 0$$

Let $T = (h-k) \cdot (h-m)$ in order to simplify the derivation. The equation then becomes:

$$d \{ (b-c) \cdot [1 - \{ 1 - (h-b)^2 \} / T]^{n-1} \} / db = 0$$

$$\{ 1 - [1 - (h-b)^2] / T \}^{n-1} + (b-c) \cdot (n-1) \cdot \{ 1 - [1 - (h-b)^2] / T \}^{n-2} \cdot [-2 \cdot (h-b) / T] = 0$$

$$[-(h-b)^2/T]^{n-1} + (b-c) * (n-1) * \{-(h-b)^2/T\}^{n-2} * [-2*(h-b)/T] = 0$$

$$[-(h-b)^2]^{n-1} + (b-c) * (n-1) * [-(h-b)^2]^{n-2} * [-2*(h-b)] = 0$$

Factoring out $[-(h-b)^2]^{n-2}$ yields:

$$(h-b)^2 - [2*(n-1)*(h-b)*(b-c)] = 0$$

Factoring out $(h-b)$ yields:

$$h-b-2*(n-1)*(b-c) = 0$$

Solving for b gives the general formula for the bidding function when the cost distribution is triangular and the costs are within the interval $[m, h]$:

$$b = \frac{h + 2*(n-1)*c}{2*n-1}$$

3. Experimentation

During the experimentation, five different scenarios were used. In this scenarios, the number of bidders were changed to analyze the FPSB process and verify the accuracy of the simulation as well. The number of bidders used are two, three, five, ten and 15. Since the bidding function depends on the interval where the cost is distributed (i.e., either in $[k, m]$ or $[m, h]$), they take one of the either form of the below formulas using following parameters:

Lower limit of the triangular distribution, $k=0$.

Mode of the distribution, $m=1$.

Upper limit of the triangular distribution, $h=2$.

Two-bidder equilibrium strategy game:

$$bi = \frac{c_i + (c_i^2 + 6)^{0.5}}{3} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 2 * c_i}{3} \quad \text{if } m < c_i < h$$

Three-bidder equilibrium strategy game:

$$bi = \frac{2 * c_i + (4 * c_i^2 + 10)^{0.5}}{5} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 4 * c_i}{5} \quad \text{if } m < c_i < h$$

Five-bidder equilibrium strategy game:

$$bi = \frac{4 * c_i + (16 * c_i^2 + 18)^{0.5}}{9} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 8 * c_i}{9} \quad \text{if } m < c_i < h$$

Ten-bidder equilibrium strategy game:

$$bi = \frac{9 * c_i + (81 * c_i^2 + 38)^{0.5}}{19} \quad \text{if } k < c_i < m$$

$$b_i = \frac{2+18*c_i}{19} \quad \text{if } m < c_i < h$$

Fifteen-bidder equilibrium strategy game:

$$b_i = \frac{14*c_i + (196*c_i^2 + 58)^{0.5}}{29} \quad \text{if } k < c_i < m$$

$$b_i = \frac{2+28*c_i}{29} \quad \text{if } m < c_i < h$$

a. Random Number Generation

For simulation, the Excel 7.0 built-in random number generator, which provides random numbers uniformly distributed over the interval [0, 1], was used. To convert these numbers to be distributed according to triangular distribution, inverse transformation technique was used. [Ref 12:p. 300]

According to this technique:

$$c = k + [\text{Rand()} * (m-k) * (h-k)]^{0.5} \quad \text{for } k < c < m \text{ and}$$

$$c = h - [\text{Rand()} * (h-m) * (h-k)]^{0.5} \quad \text{for } m < c < h$$

where c is triangularly distributed random number and $\text{Rand}()$ is uniformly distributed random number generated by Excel 7.0.

The simulation program for FPSB contract game is in Appendix B.

b. Experimentation With Two Bidders

During the simulation, three different scenarios were experimented and analyzed. In the first scenario, both bidders used equilibrium strategies. In the second scenario, one of the bidders underbid while the other used the equilibrium strategy. Finally in the third scenario, both of the bidders did not use the equilibrium strategy.

The different scenarios were designed to show that using the equilibrium bidding strategy maximizes the bidders' profit and any deviation from the equilibrium strategy would reduce the expected profit for the deviating bidder.

Figure 56 shows the graphical result of the simulation when both bidders used equilibrium strategies. The results show that there exists an equilibrium in the contract game and the winners' expected profit approaches 0.22 as the number of simulated games approaches 2500.

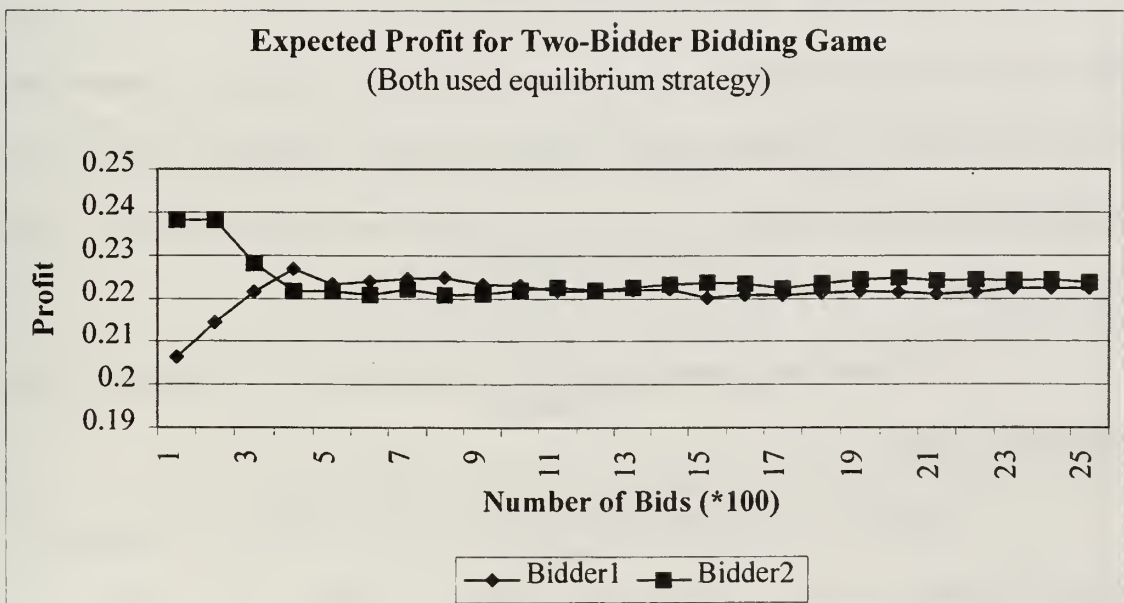


Figure 56

The average profit graph for this particular simulation is shown in Figure 57. This result also reinforces the result of existing equilibrium in the bidding game. Both bidders' average profit approaches approximately 0.11 by bidding in 2500 games.

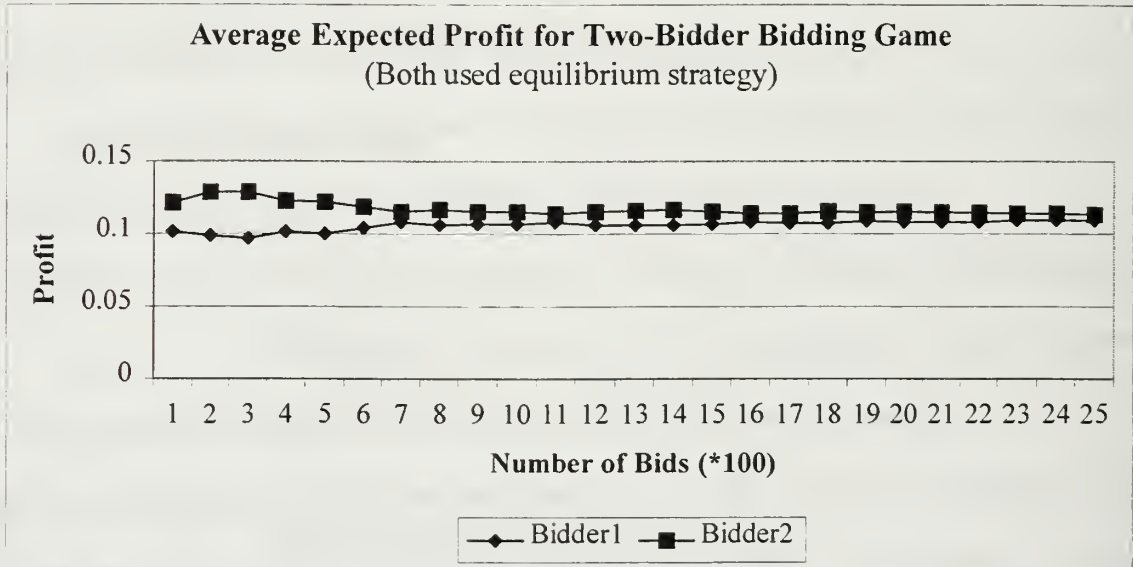


Figure 57

Total profits for both bidders resulted almost the same; 273.6609 for bidder1 and 283.7462 for bidder2 and are shown graphically in Figure 58. Another result was that the bidders won almost equally; bidder1 won 1231 of the games while bidder2 won the remaining 1269 games. Overall results of the experiment are outlined in Table 47 in Appendix C.

In the next experimentation, bidder1 underbid by 0.15 while bidder2 used equilibrium strategy.

The results of the experimentation are shown in Table 48 in Appendix C. The experimentation resulted in lower expected profit for bidder1, 0.048, while bidder2 made 0.243, which is more than the equilibrium amount of 0.22. The expected profit graph is shown in Figure 59.

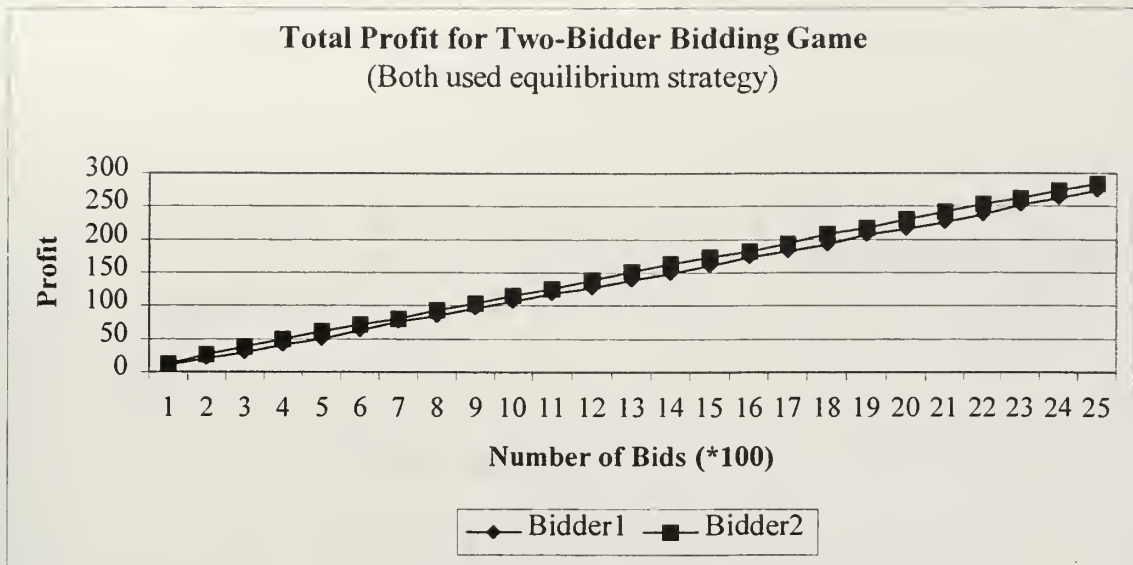


Figure 58

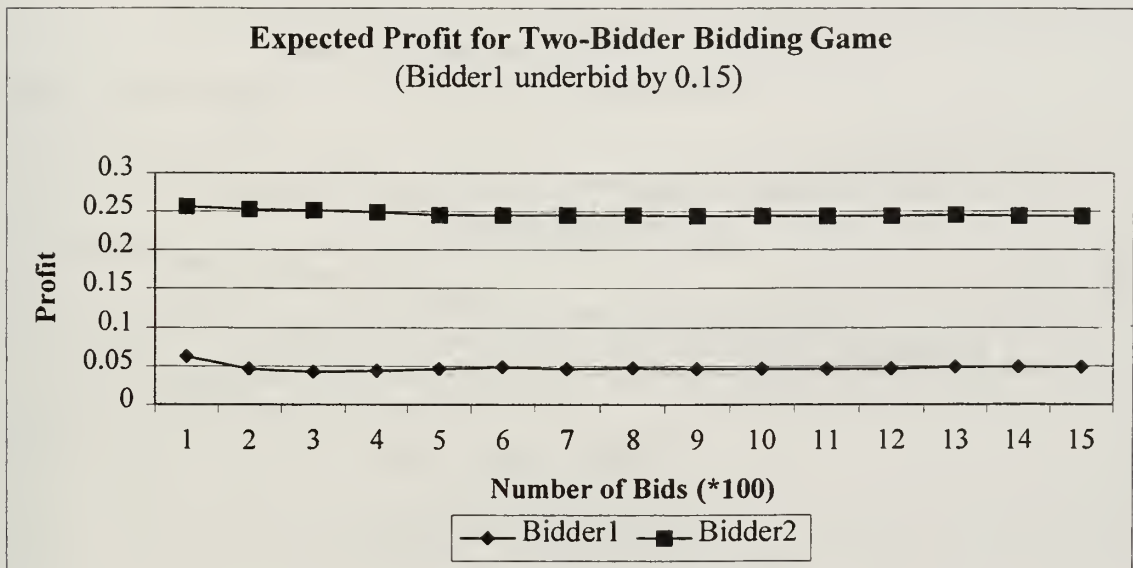


Figure 59

The average profit for bidder1 decreased to 0.034 and 0.07 for bidder2.

The graph for average profit is shown in Figure 60.

Total profit for bidder1 also dramatically decreased to 51.4114 and 105.1904 for bidder2 in 1500 games as shown in Figure 61. Bidder1 won 1068 of 1500 games while bidder2 won only 432.

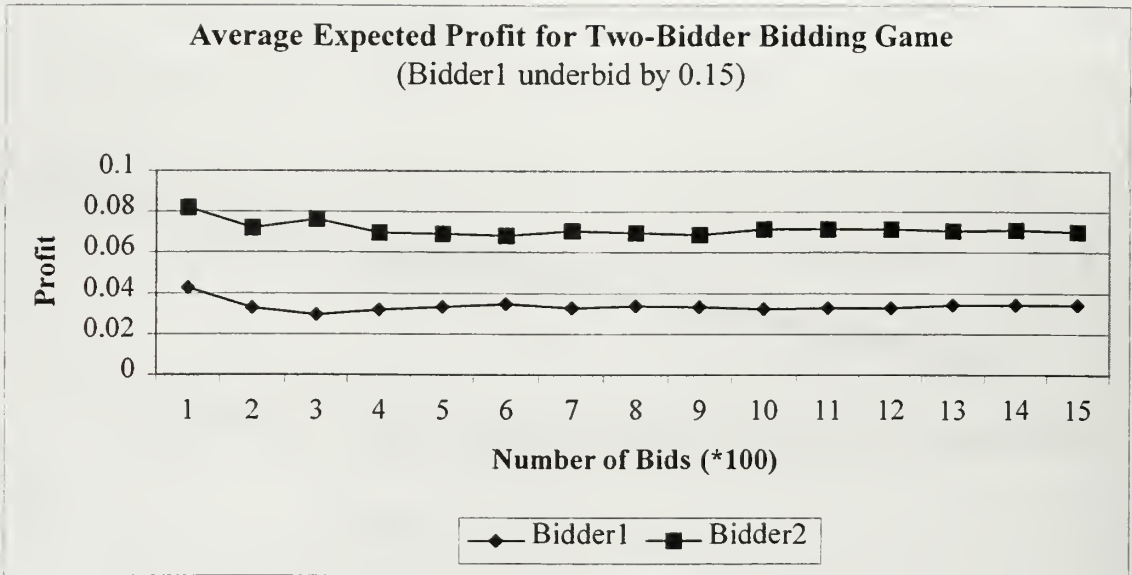


Figure 60

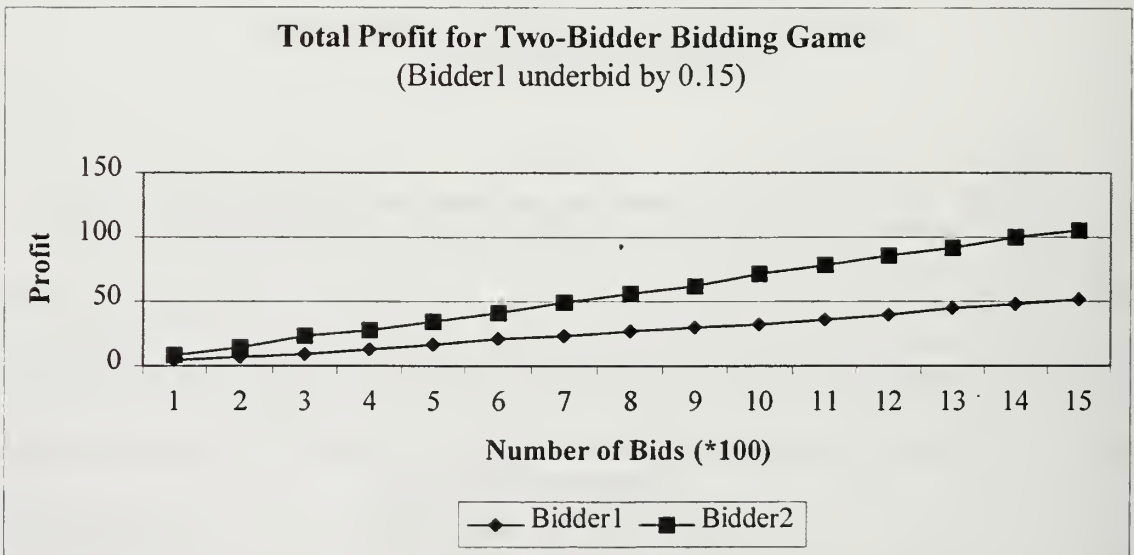


Figure 61

The last experiment with two bidders was conducted as they both did not use the equilibrium strategy. Both bidders underbid by 0.1, which was approximately five percent of their average bids. In this scenario, both bidders' expected profits approached 0.122, which is lower than the equilibrium amount of 0.22 as the number of games approached 2500. The graphical representation of expected profit is shown in Figure 62.

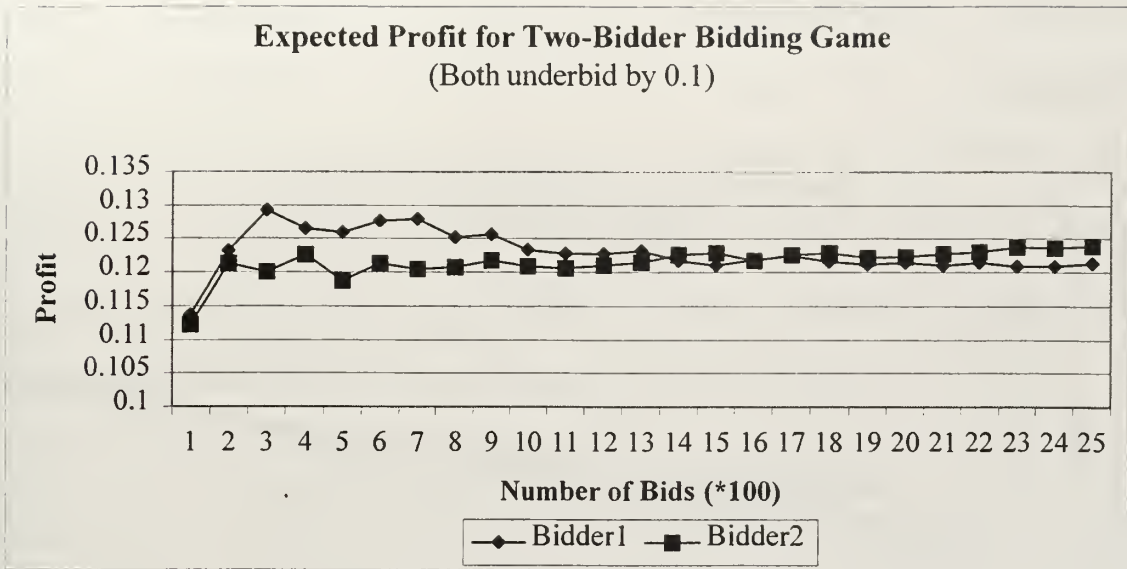


Figure 62

The average profit for both bidders also decreased to 0.06 and is shown in Figure 63 graphically.

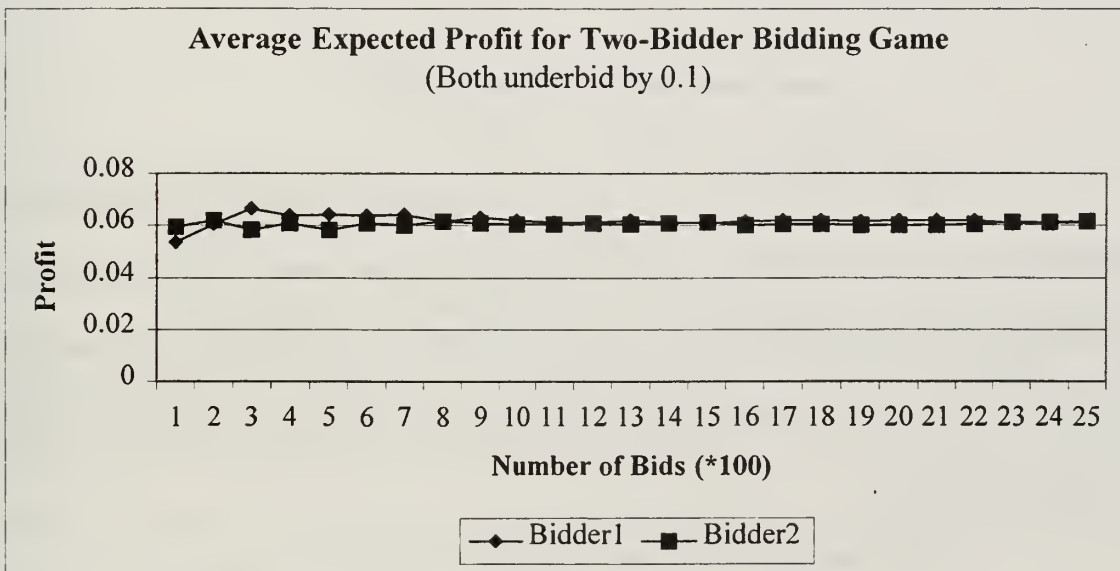


Figure 63

Total profits in 2500 games shown in Table 49 in Appendix C, 152.6738 for bidder1 and 153.8211 for the other, were also lower than the equilibrium total profit

of 273.6609 and 283.7462 respectively. The graphical representation of total profit curves is in Figure 64.

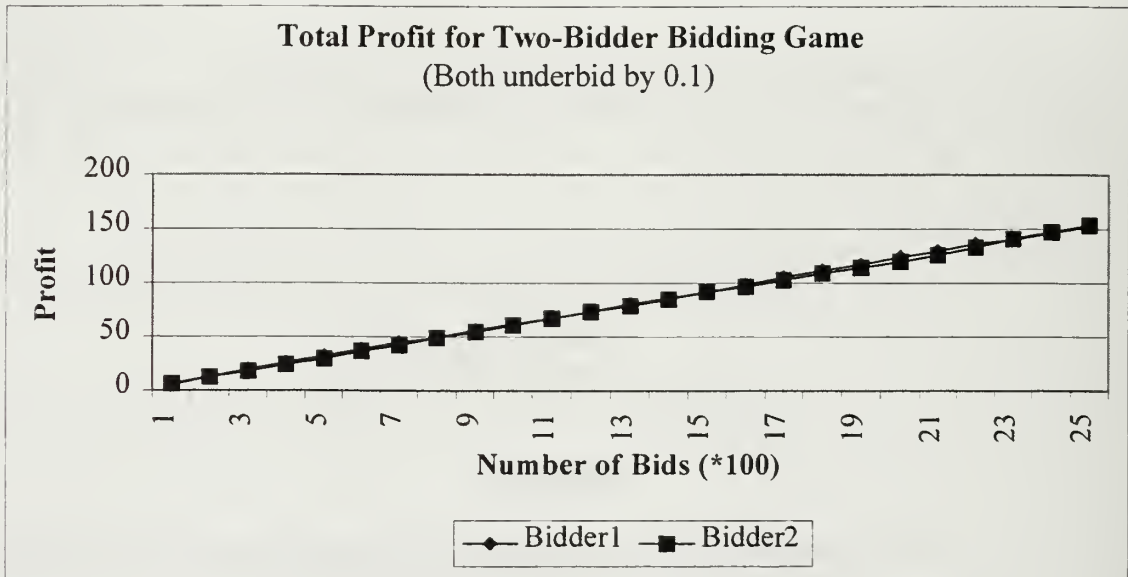


Figure 64

c. Experimentation With Three Bidders

This simulation along with the other multiple bidder simulations was conducted to find out the effect of number of bidders on the game as well as analyzing the behavior of bidders and the accuracy of the simulation. The simulation was conducted with three bidders and the cost is within the interval $[0, 2]$ with a mode value of 2 under triangular distribution.

During the simulation, three different scenarios were analyzed. In the first scenario, all off the bidders used equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. Finally in the third scenario, bidders “1” and “2” underbid by different amounts while bidder3 used equilibrium strategy.

Figure 65 shows the results of the first experimentation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the three-bidder bidding game as well as in the two-bidder bidding game. According to the simulation results, the winners' expected profit from bidding approached approximately 0.149 as the number of games approached 2000. Introducing another bidder to the game reduced the expected profit from bidding.

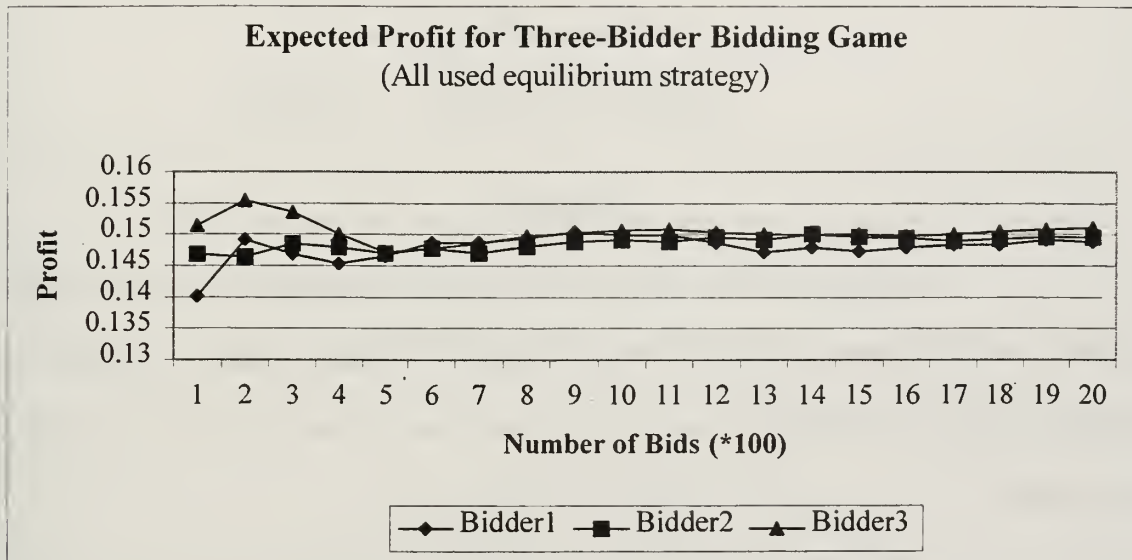


Figure 65

The average expected profit graph is shown in Figure 66 and the overall results of the experiment are shown in Table 50 in Appendix C. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached approximately 0.05 while the number of games approached 2000.

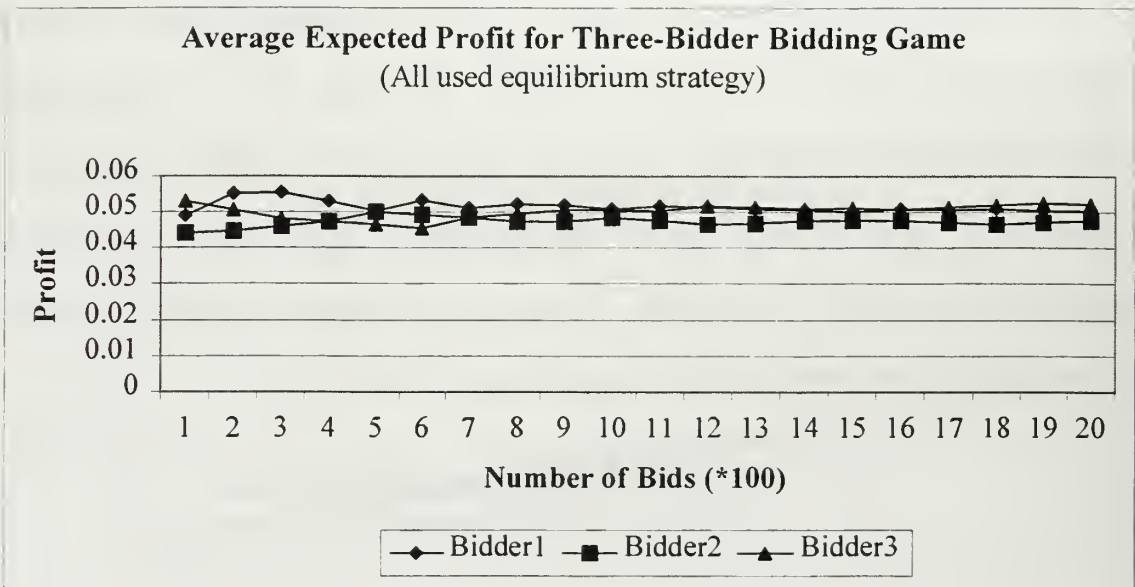


Figure 66

As seen in Table 50, each bidder won the games almost equally. Overall, bidder1 won 675 games, bidder2 635 and bidder3 690 games each. Total profit for bidders as seen in Figure 67, were almost the same; 100.41, 94.983 and 104.24 respectively.

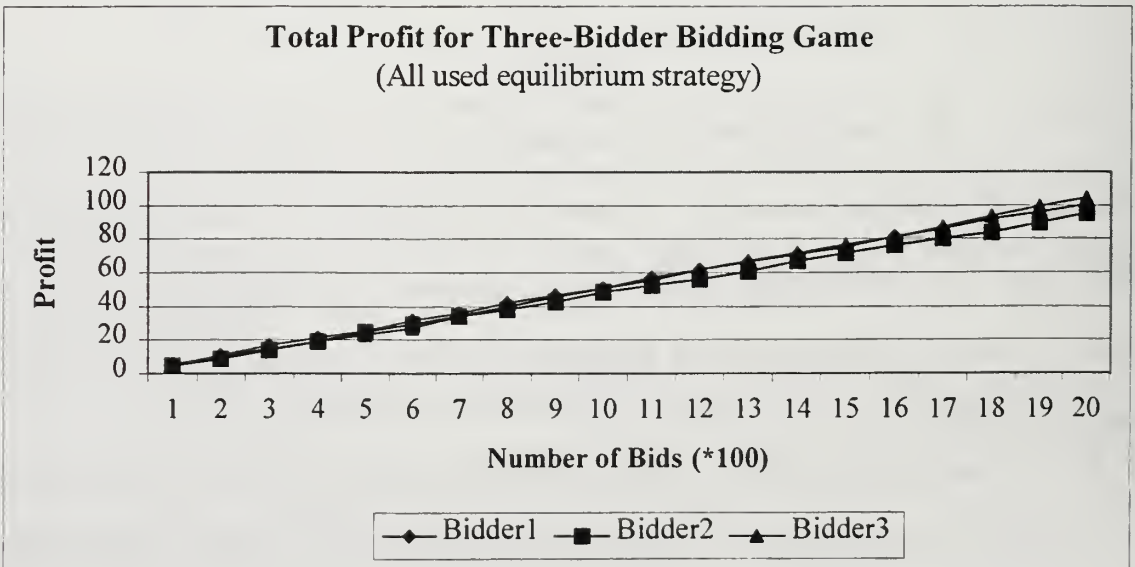


Figure 67

In the next experiment, bidder1 underbid by 0.1, which was approximately six percent of the average bid amount, while other two used equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.041 while the other two bidders' slightly increased to 0.1529 and .1541 respectively. The expected profit graph is shown in Figure 68

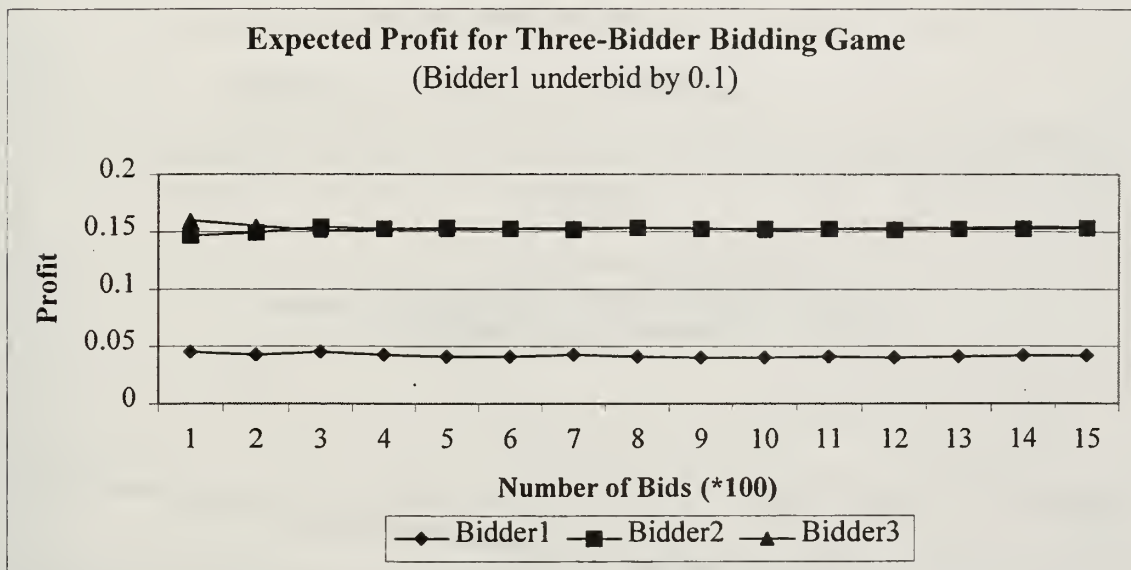


Figure 68

Average profit scheme was different than the equilibrium game experiment for bidder1. Bidder1's average profit was 0.019 while others higher, 0.041 as graphically shown in Figure 69.

Bidder1 won 687 of 1500 games while others won 405 and 408 games as shown in Table 51 in Appendix C. Although bidder1 won more games than the others won, its total profit was 28.554 which is lower than the others'; which were 61.925 and 62.879 respectively. Total profit graph for this experiment is shown in Figure 70.

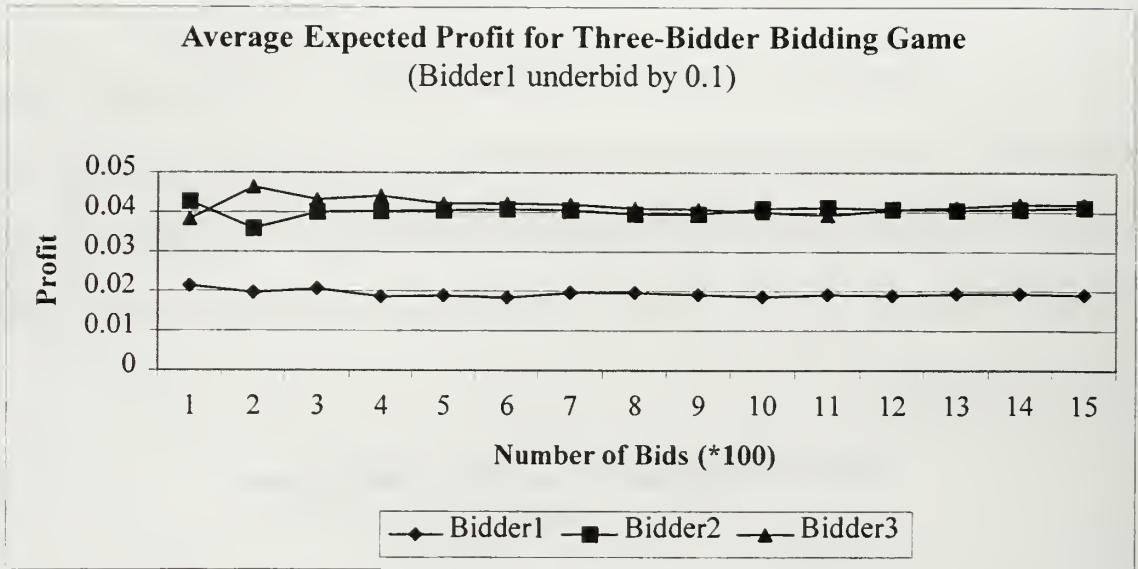


Figure 69

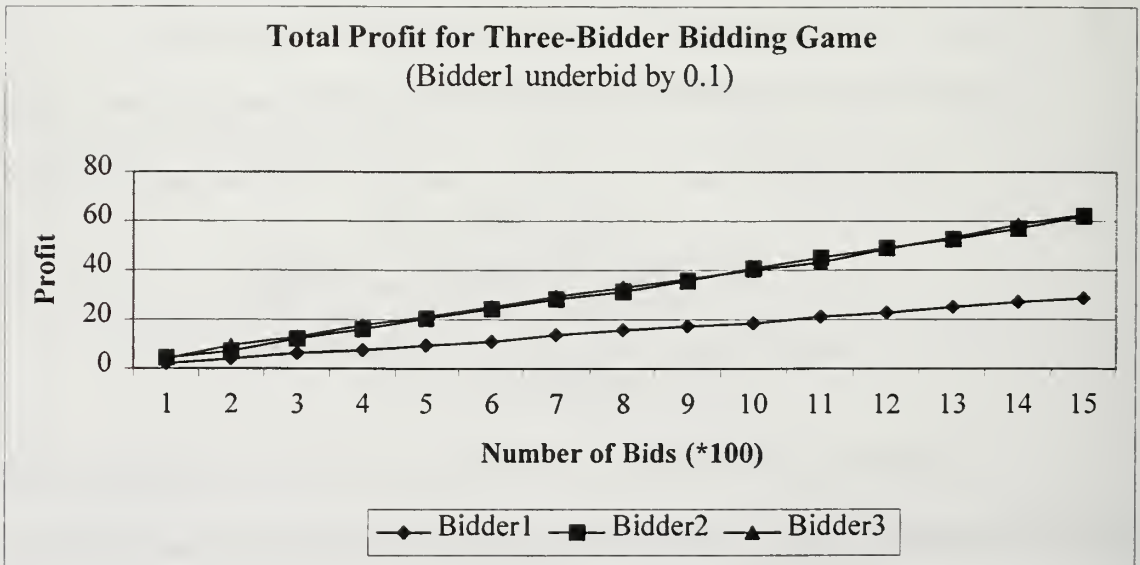


Figure 70

The last scenario for three-bidder simulation was while bidder3 used equilibrium strategy bidders “1” and “2” did not. Bidders “1” and “2” underbid by 0.1 and 0.12 respectively. The experiment resulted in lower expected profits for the first two bidders while it was even higher for the third. Bidder1’s profit approached 0.0469, which is slightly higher than in the previous game which was 0.041 and bidder2’s was even

lower, 0.0239 in 2000 games. Bidder3, which had used equilibrium strategy, made higher profit than it did in the first two experiments, 0.157 as opposed to 0.149 and 0.1541. The expected profit graph is shown in Figure 71:

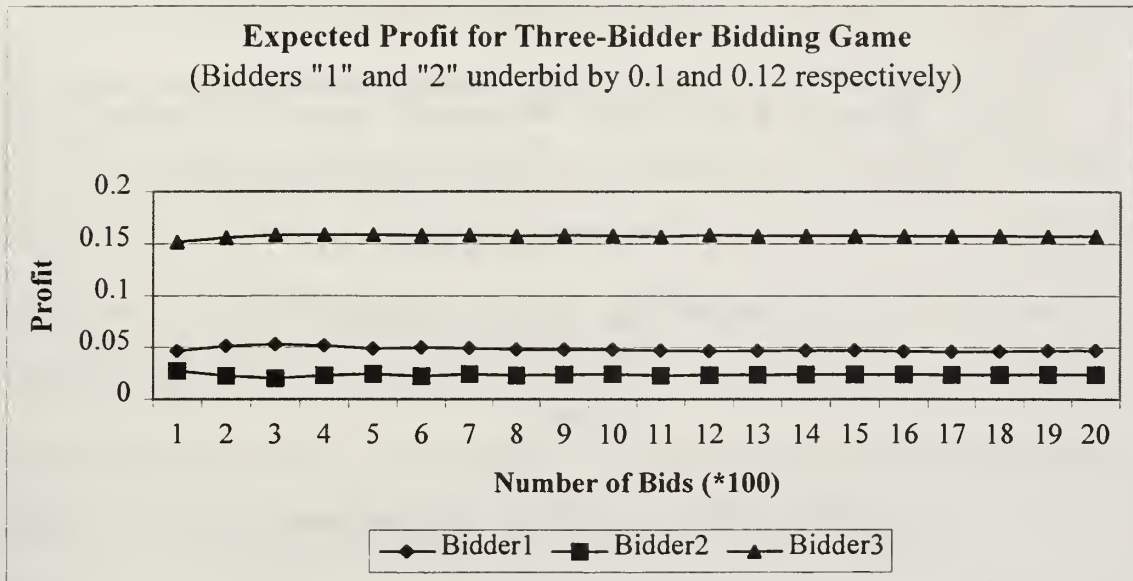


Figure 71

Bidder2 had lowest average profit than the others. It made 0.01 as opposed to 0.0168 and 0.033 made by bidders “1” and “3” respectively. The average profit graph is shown in Figure 72.

Total profit scheme was also different than previous experiments, but the general trend, which is the more a bidder underbids the less profit it makes, was the same. Bidder2 made the lowest profit, 20.3517 while bidder1 and bidder3 made 33.7713 and 67.483 respectively in 2000 games. Bidders won 720, 851 and 429 games respectively. Total profit curves are shown in Figure 73.

Overall results of the game are outlined in Table 52 in Appendix C.

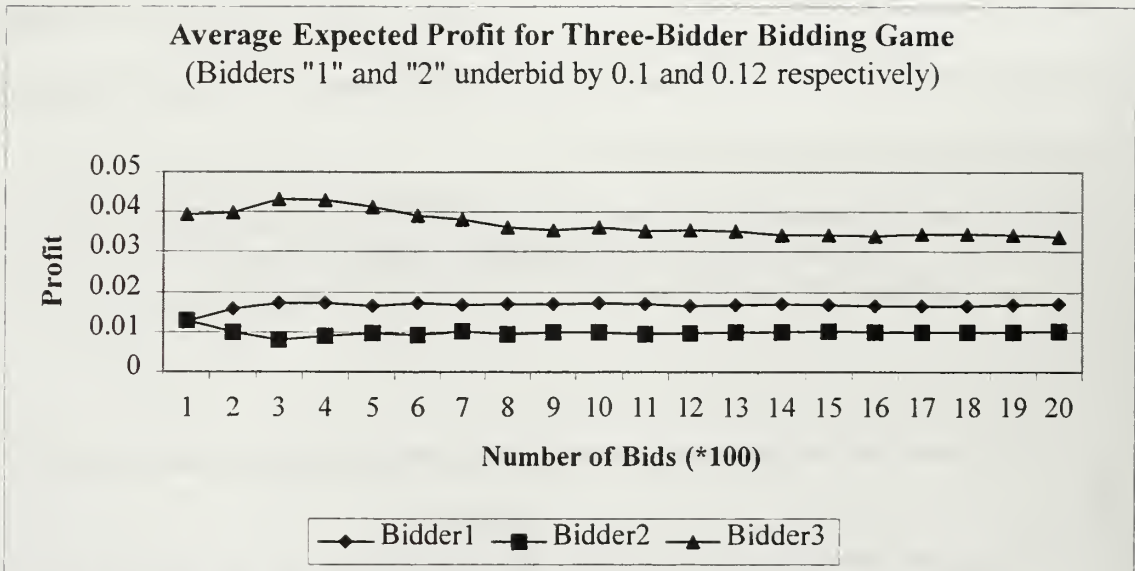


Figure 72

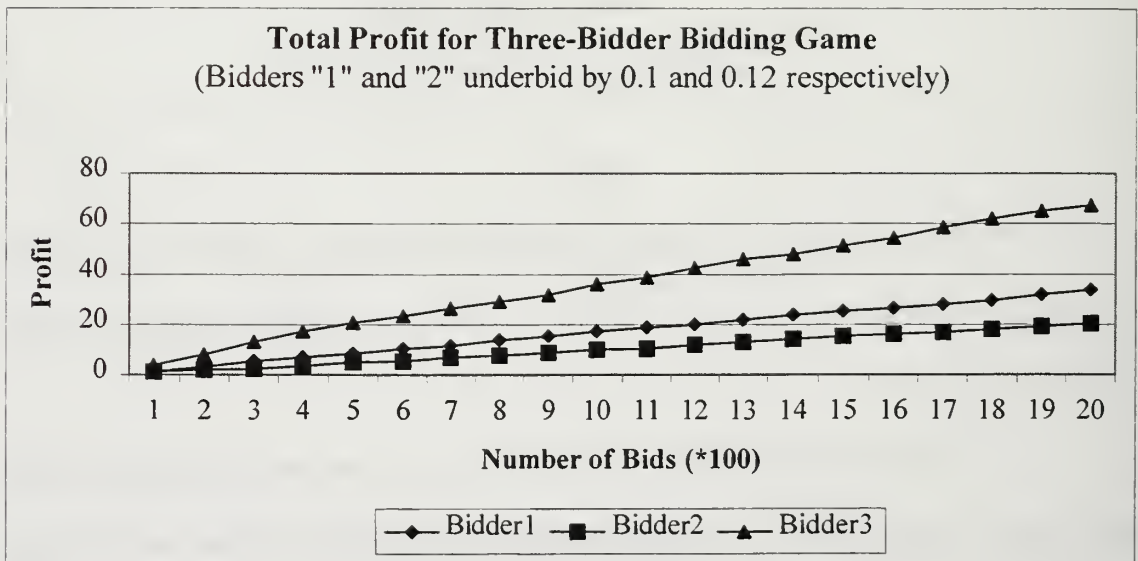


Figure 73

d. Experimentation With Five Bidders

The simulation was conducted with five bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

During the simulation four different scenarios were analyzed. In the first scenario, all bidders used the equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. In the third scenario, both bidders “1” and “2” underbid while others used equilibrium strategy and in the last scenario, bidders “1” and “2” underbid by different amounts while others used equilibrium strategy.

Figure 74 shows the results obtained from the simulation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the game as in the two and three-bidder bidding games. According to the simulation, each winner’s expected profit approached approximately 0.092 as the number of games approached 1500. As the number of bidders increased, the expected profit from bidding decreased, as it was the case in the three-bidder bidding game.

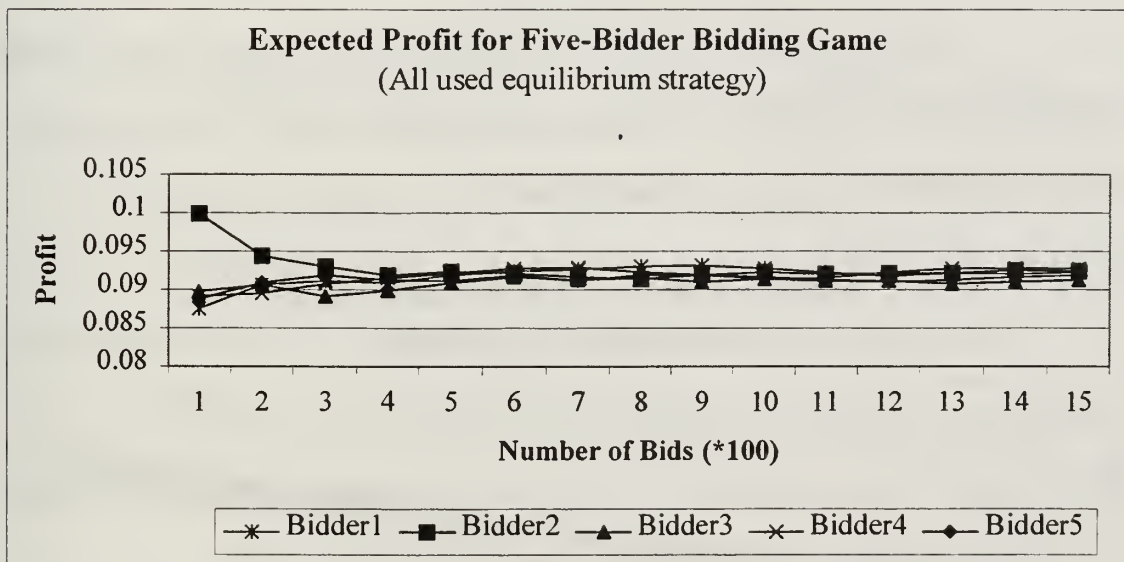


Figure 74

Average expected profit graph is shown in Figure 75 and overall simulation results obtained are shown in Tables 53 and 54 in Appendix C. One more time, the average expected profits reinforced the game equilibrium. The average expected

profit for the bidders approached approximately 0.018 as the number of games approached 1500.

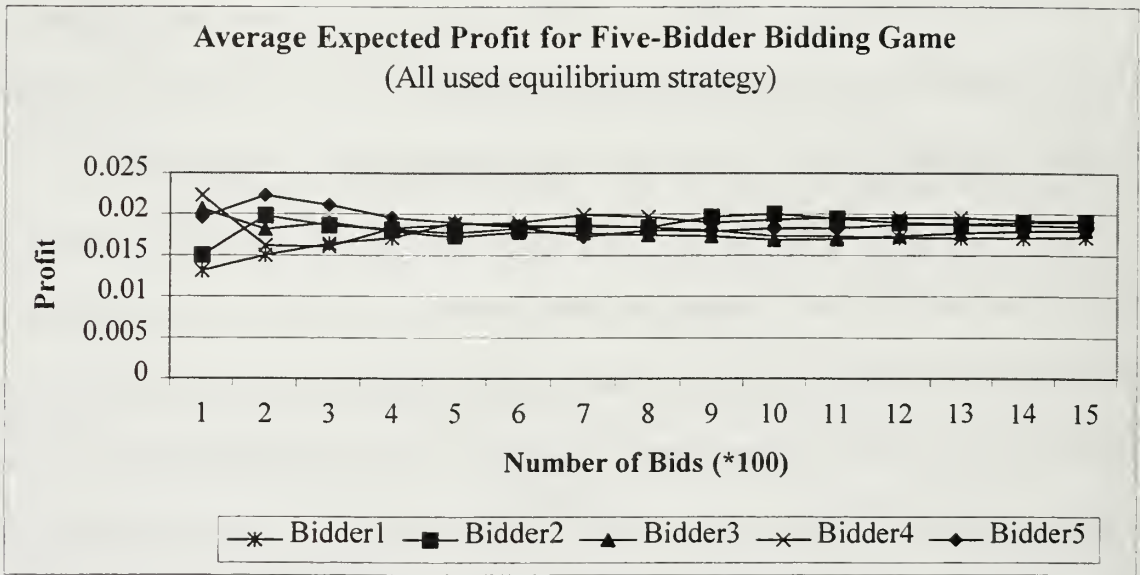


Figure 75

The experiment resulted in very close total profits, ranging from 25.7489 to 28.8431, as seen in Table 54. Total profit graph is shown in Figure 76. The number of games won by bidders was also very close to each other, ranging from 279 to 311.

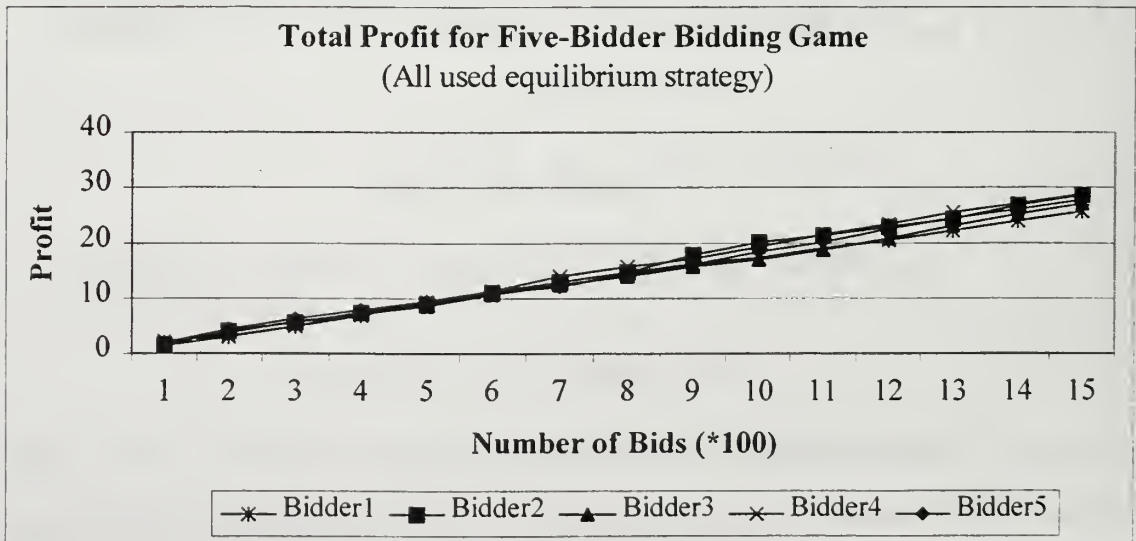


Figure 76

The second experiment, where bidder1 underbid by 0.05 and others used the equilibrium strategy, resulted in following:

Bidder1 made a lower expected profit, 0.04, than others did, which was 0.093, as the number of games approached 1500. The expected profit graph is shown in Figure 77.

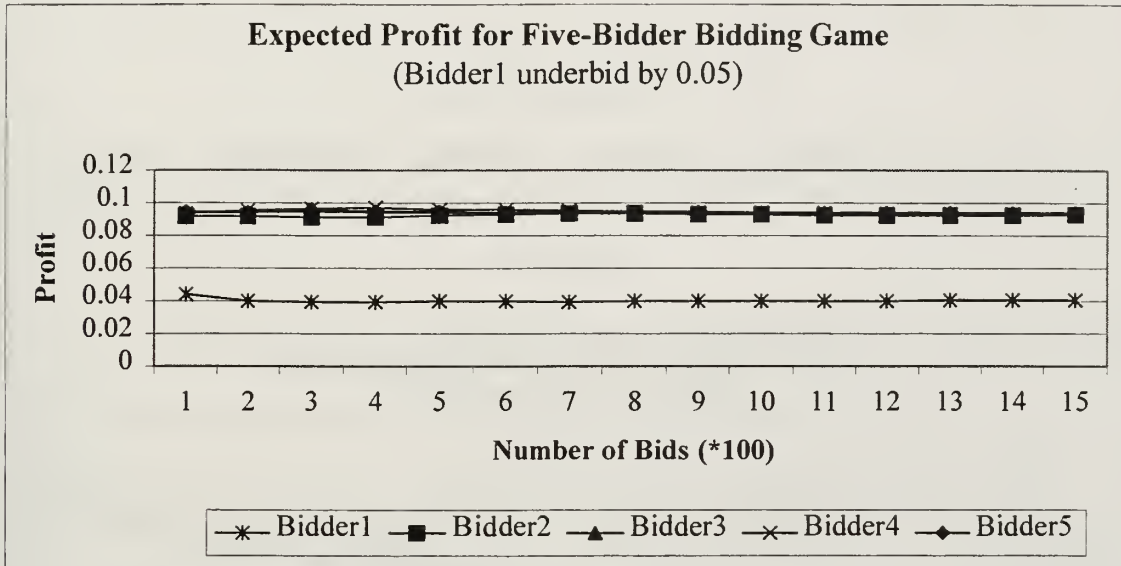


Figure 77

The average profit for bidder1 approached 0.0109 while others did between 0.0167 and 0.0178. The average profit curves are shown in Figure 78.

Bidder1 won 402 games while others won the games almost equally, ranging from 269 to 286. Total profit for bidder1 was also lower than the others', 16.3539 as opposed to 25.1509-26.7629, even though bidder1 won more games than the others did. The total profit curves are graphically shown in Figure 79.

Overall results of the experiment are outlined in Tables 55 and 56 in Appendix C.

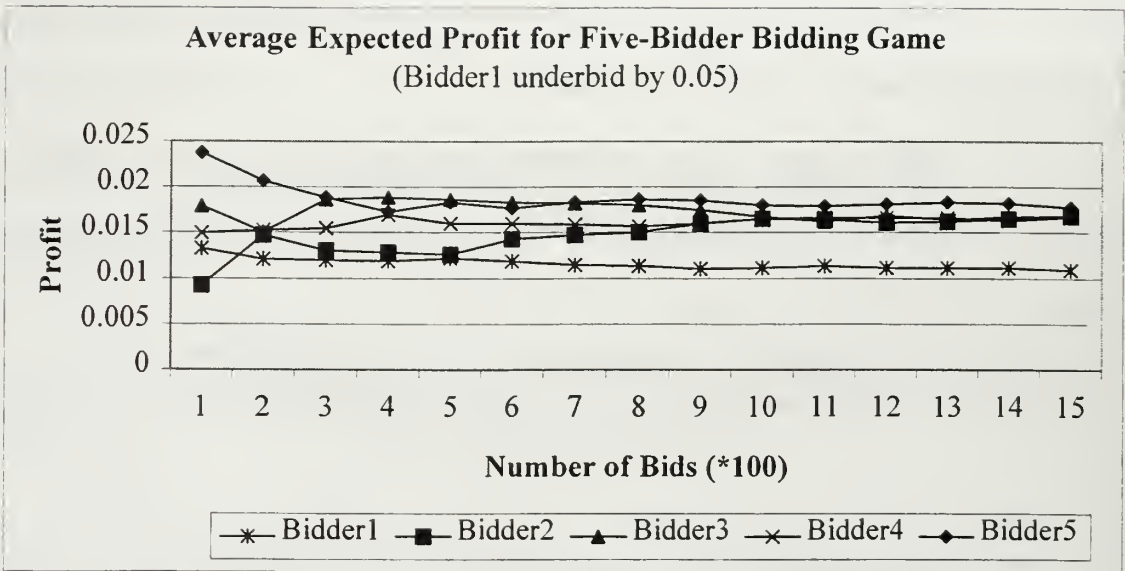


Figure 78

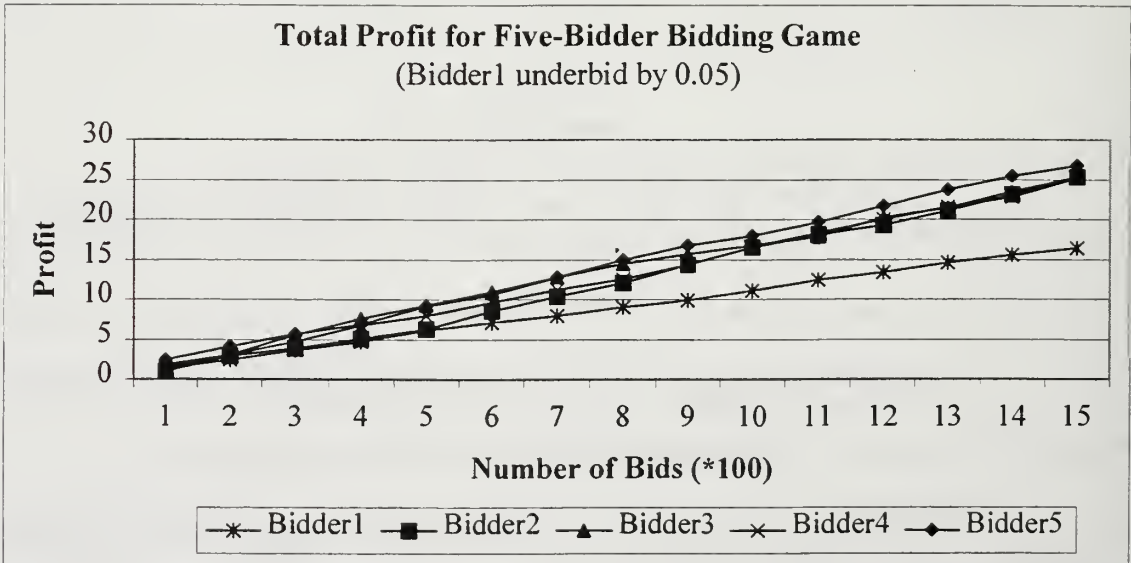


Figure 79

In the third scenario, both bidders “1” and “2” underbid by 0.05 while the rest of the bidders used equilibrium strategy. Overall results of the experiment are shown in Tables 57 and 58 in Appendix C. As for the expected profits, bidders “1” and “2” made lower profits, approximately 0.041, than the rest did, 0.093, as shown in Figure 80:

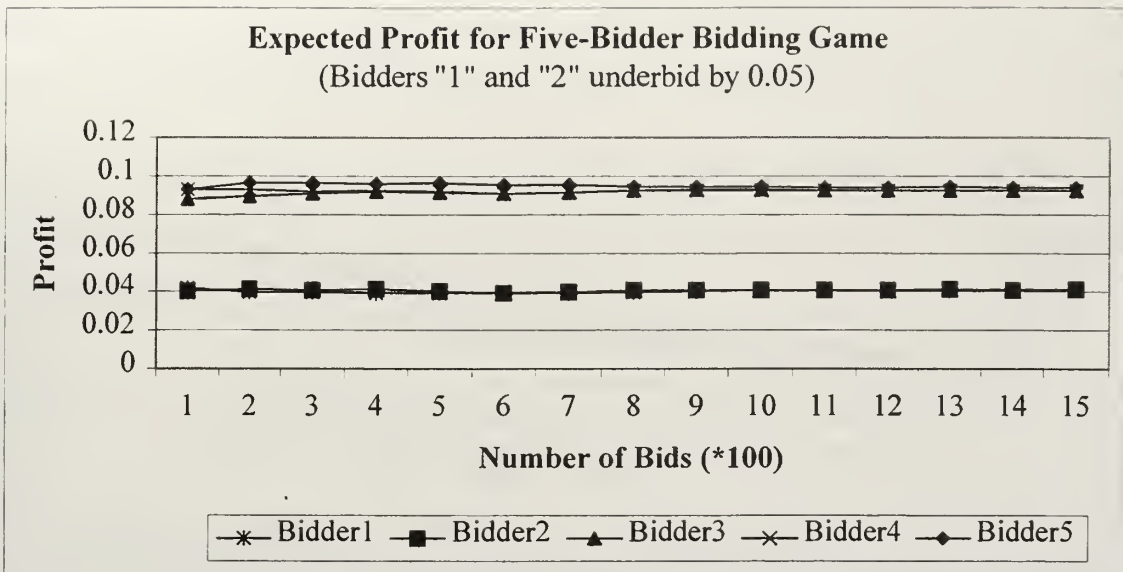


Figure 80

The average profit scheme resulted the same; bidders “1” and “2” made 0.009 while others did between 0.01487 and 0.01724 as seen in Figure 81:

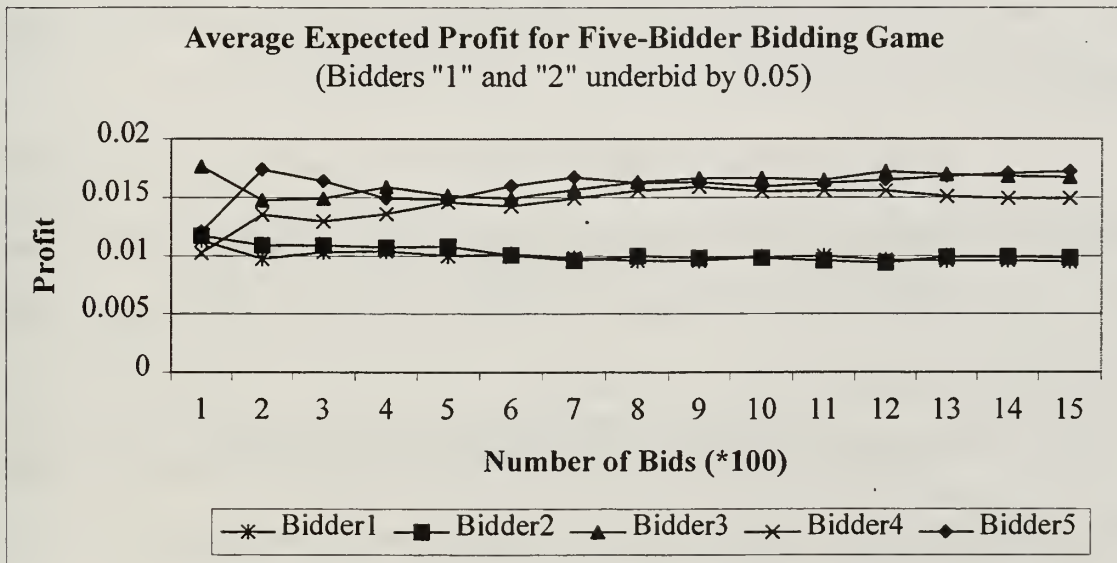


Figure81

Total profit for bidders “1” and “2” also were lower than the others’ profits. They made 14.2425 and 14.7406 respectively, while others profit ranged from 22.3112 to 25.8637. Total profit curves for bidders are shown in Figure 82:

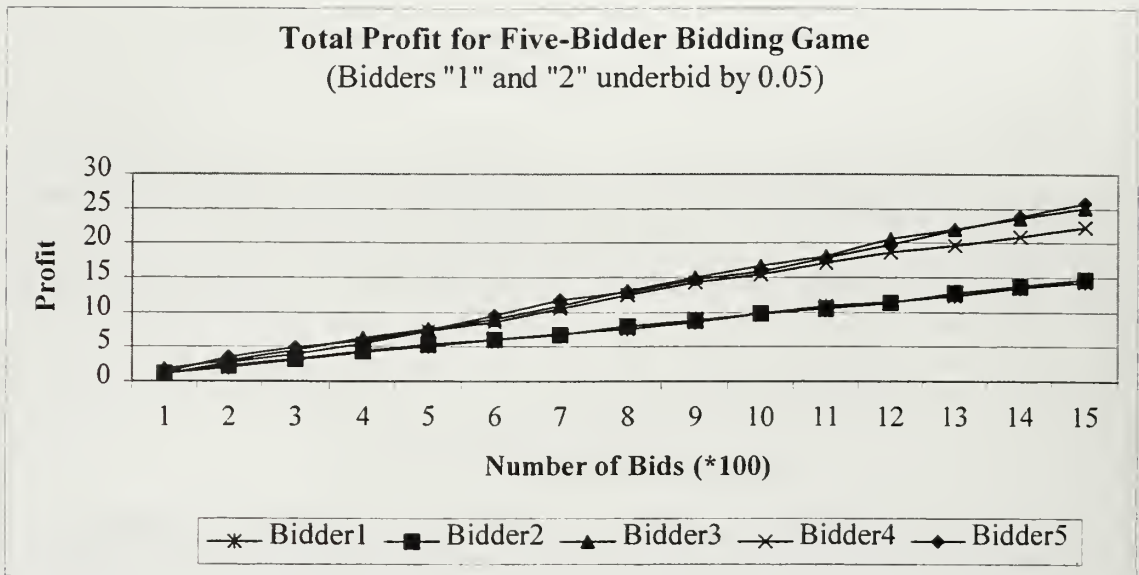


Figure 82

Bidders "1" and "2" won 313 and 329 games out of 1500 games. The rest won the games 282-293 times. As it was the case in previous experiments, underbidding bidders made lower profits, although they won more games than the equilibrium strategy bidders.

In the last scenario, bidders "1" and "2" underbid by 0.05 and 0.075 respectively while others used equilibrium strategy. The underbidding bidders made lower profits of 0.041 and 0.015 respectively while others made approximately 0.093. The expected profit graph is shown in Figure 83.

The average profit for bidders "1" and "2" were 0.00997 and 0.0041 while others' approached 0.015 in 2500 games as seen in Figure 84. The equilibrium strategy bidders made higher expected and average profit than underbidding bidders did.

Bidders "1" and "2" won 582 and 674 games while the others won very close number of games ranging from 402 to 424 as seen in Table 59 in Appendix C. Despite bidders "1" and "2" won much more games than others, they made lower total

profits in 2500 games. The lowest bidder, bidder2, made the lowest total profit, 10.194. The second lowest total profit was bidder2's, 24.156. Finally, equilibrium strategy bidders made very close total profits ranging from 36.97 to 39.499. Total profit curves are shown in Figure 85 and overall results of the experiment are in Tables 59 and 60 in Appendix C.

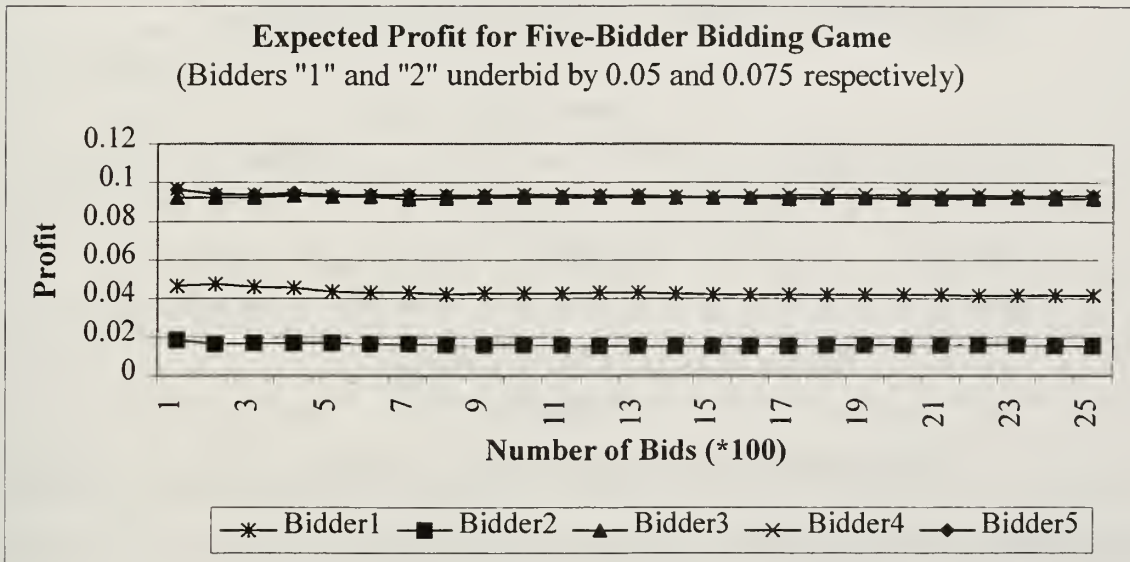


Figure 83

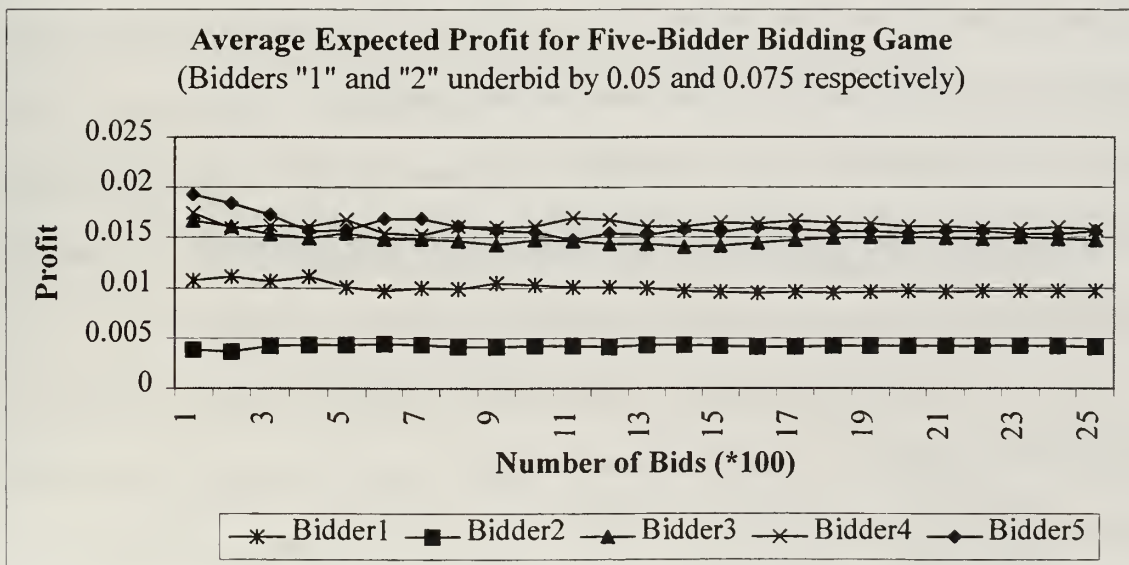


Figure 84

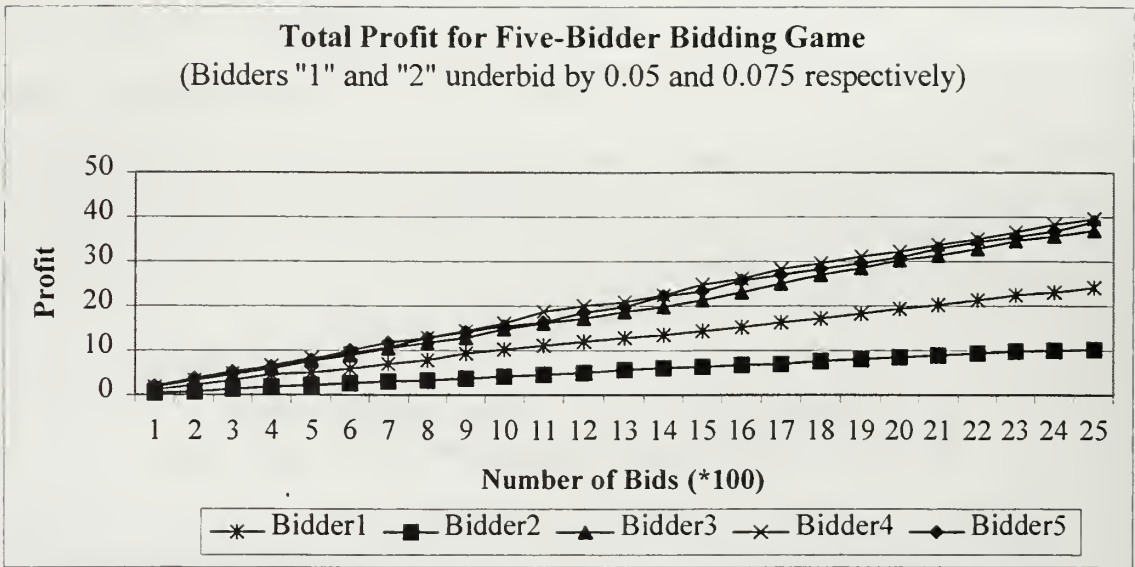


Figure 85

e. Experimentation With Ten Bidders

The simulation was conducted with ten bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

In these series of experiments, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. In the second one, bidder1 underbid while others used the equilibrium strategy again. In the third one, both bidders “1” and “2” underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders “1”, “2”, “3” and “4” underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategy resulted in a well-defined equilibrium. All bidders’ expected profit approached approximately 0.048 as the number of games approached 1500. The graphical representation of the expected profit curves is shown in Figure 86. As can be seen in the overall results of the

experiment in Tables 61 through 64 in Appendix C, introducing more bidders to the game reduced the expected profit from bidding more.

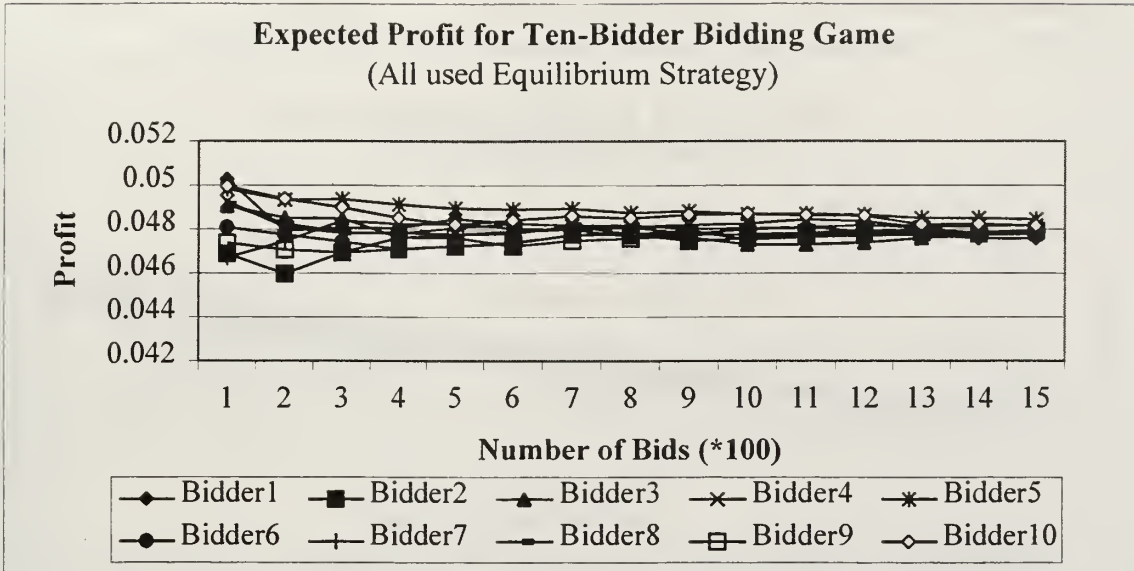


Figure 86

The average expected profit graph is shown in Figure 87. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached 0.005 in 1500 games.

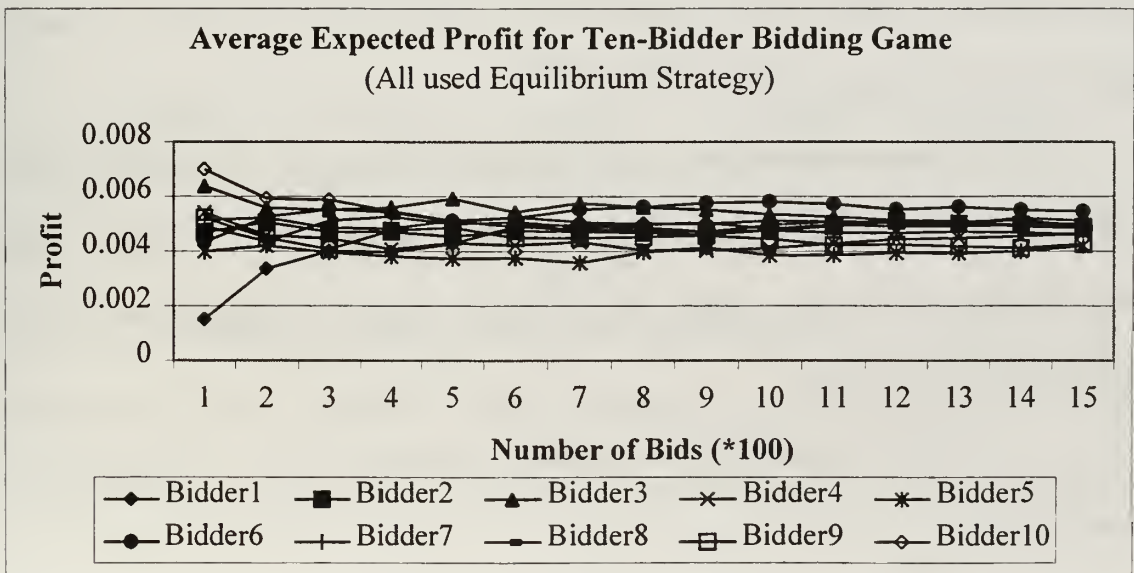


Figure 87

As seen in Table 62, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 130 to 161 games each. Total profit for bidders as seen in Figure 88, were almost the same, ranging between 6.2965 and 7.66.

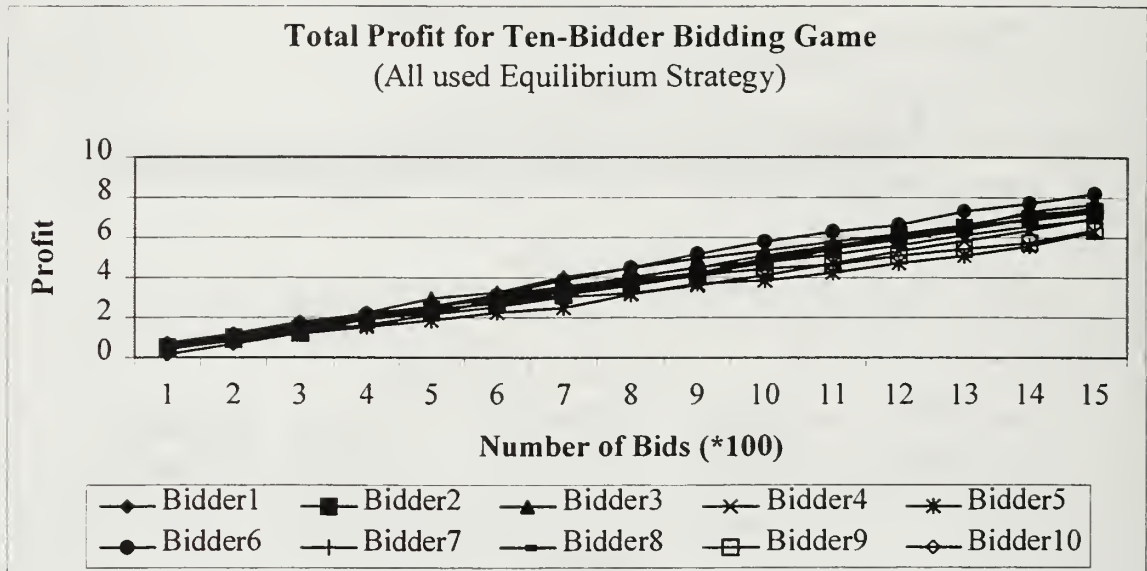


Figure 88

In the next experiment, bidder1 underbid by 0.025 while others bid according to the equilibrium strategy. As a result of 2500 games, bidder1's expected profit decreased to 0.0225 while other bidders' remained at 0.048. The expected profit graph is shown in Figure 89.

Average expected profits for the bidders also reinforced the same scheme as the expected profit results had shown. Bidder1's average profit was 0.0026 while others were almost the same, ranging from 0.0043 to 0.0052. Average expected profit graph is shown in Figure 90.

Overall results of the experiment are shown in Tables 65 through 68 in Appendix C.

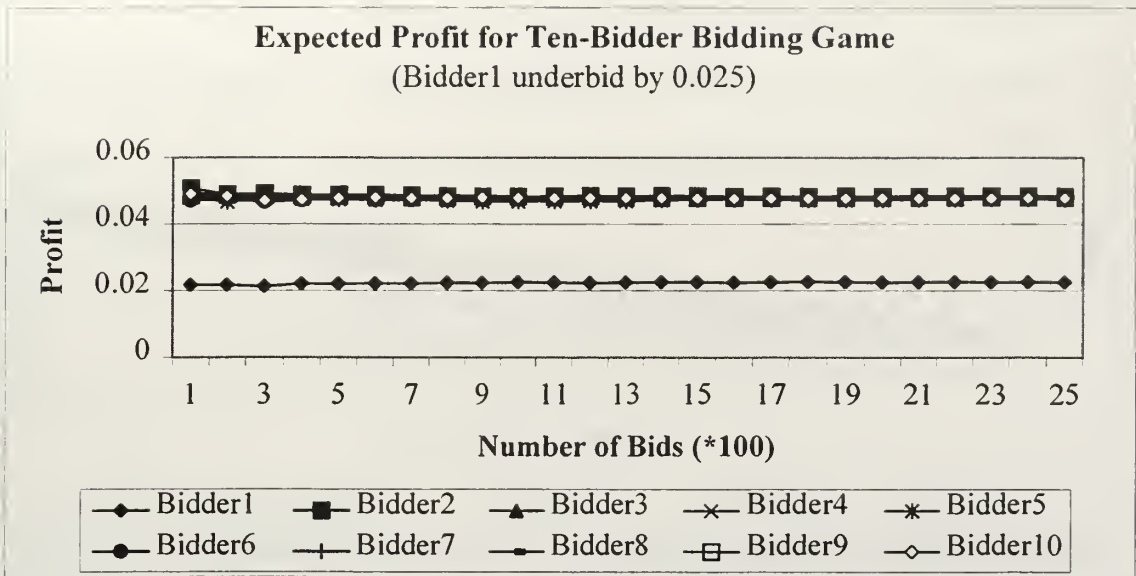


Figure 89

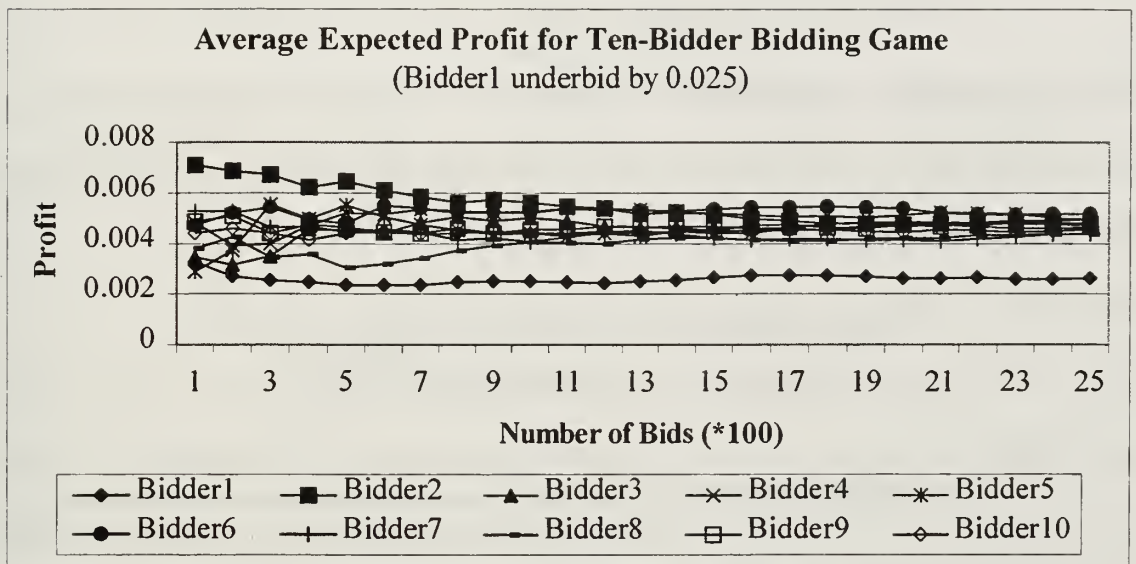


Figure 90

Bidder1 won 289 of 2500 games while others won in a range of 226 to 270 games as shown in Table 66. Although bidder1 won more games than the others won, its total profit was 6.5026 which is much lower than the others' profits ranging from 10.859 to 12.876. Total profit graph for this experiment is shown in Figure 91.

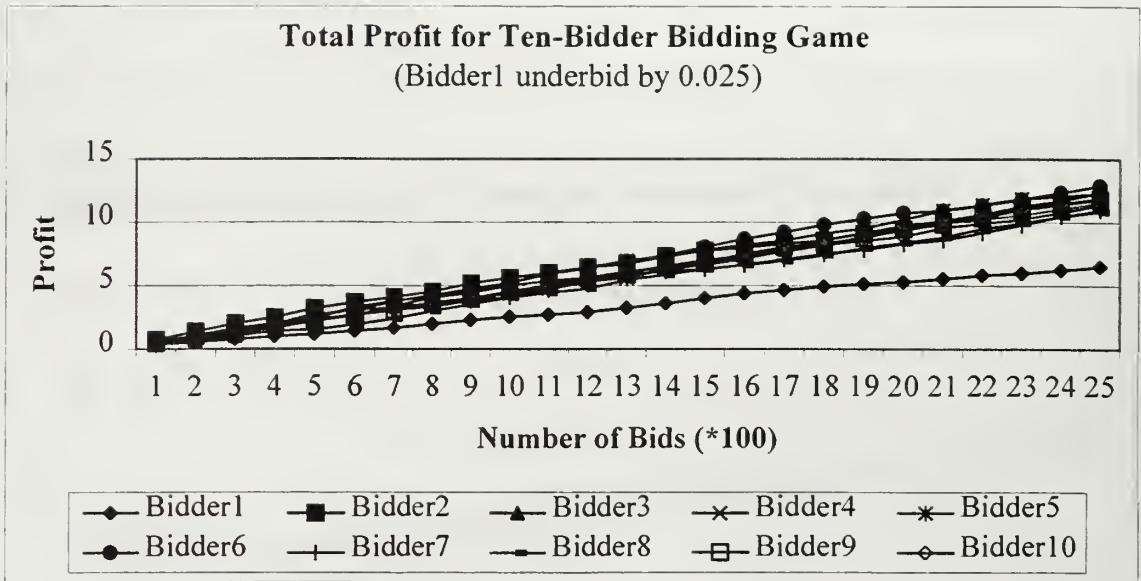


Figure 91

In the third experiment, bidders “1” and “2” underbid by 0.025 while others bid according to the equilibrium strategy. As a result of 2500 games, bidders “1” and “2” made expected profits, 0.0225 and 0.023, while other bidders’ remained at 0.048 again. The expected profit graph is shown in Figure 92:

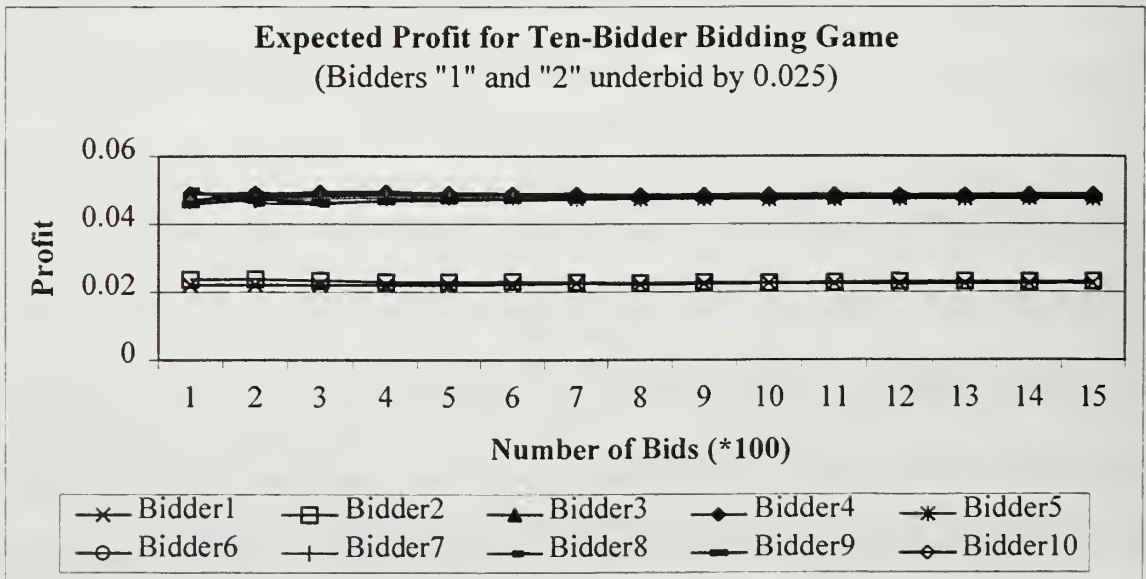


Figure 92

Average expected profits for the bidders also resulted in the same trend as the expected profits did. Underbidding bidders' average profits were 0.00277 and 0.00248, while others were almost the same, ranging from 0.0043 to 0.0054, as graphically shown in Figure 93.

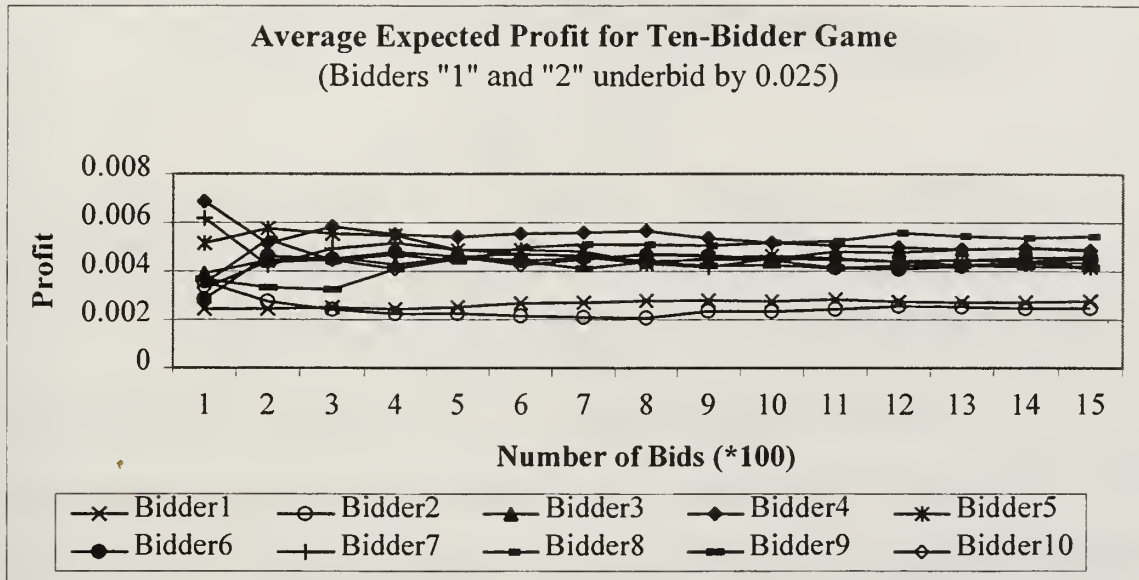


Figure 93

Bidders "1" and "2" won 184 and 161 games respectively while others won in a range of 131 to 170 games. Total profits for underbidding bidders were 4.1538 and 3.7136, which are much lower than the others' profits ranging from 6.234 to 8.1608. Total profit graph for this experiment is shown in Figure 94.

Overall results of the experiment are shown in Tables 69 through 72 in Appendix C.

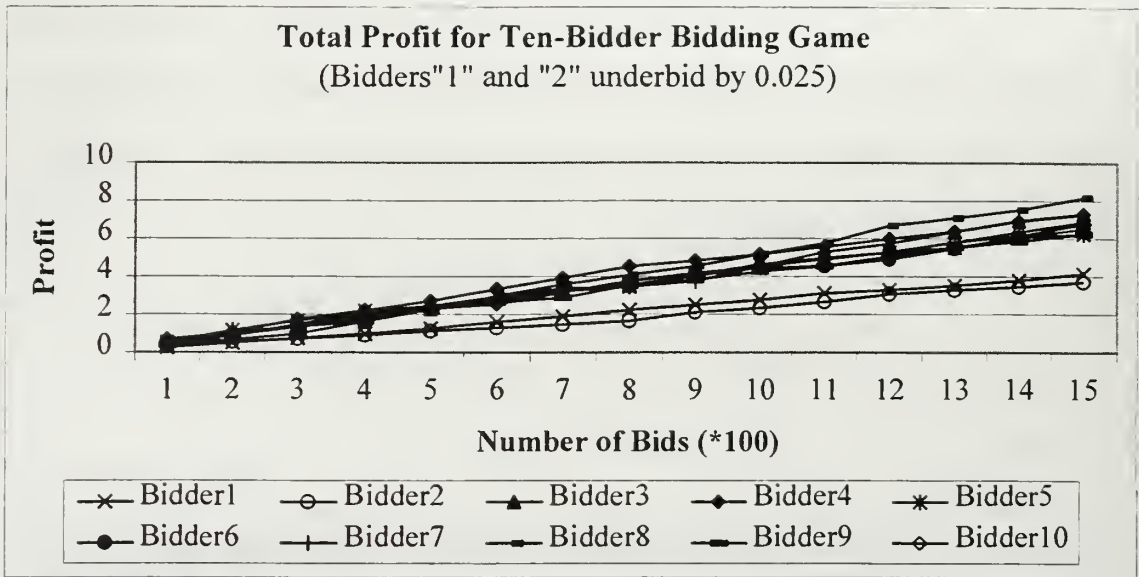


Figure 94

In the last experiment, bidders "1", "2", "3" and "4" underbid by 0.03, 0.025, 0.02 and 0.015 respectively in 2000 games. The remaining six bidders used equilibrium strategy. The underbidding bidders made lower profits, 0.0074, 0.0129, 0.0174 and 0.0225, than the equilibrium strategy bidders did, which was 0.048 again. The expected profit curves are shown in Figure 95.

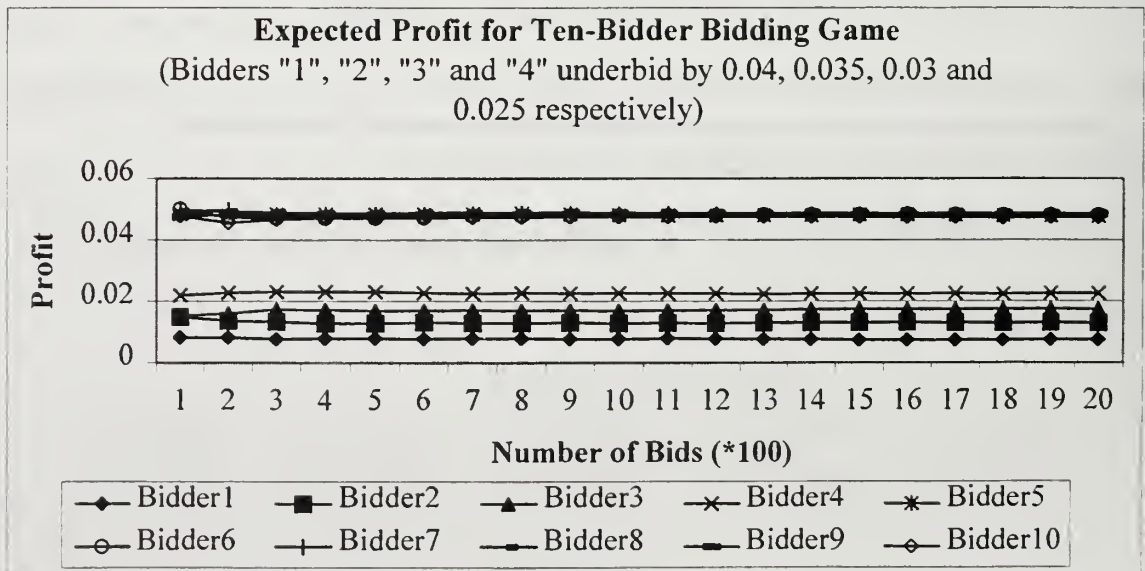


Figure 95

Average profit results were almost identical to similar previous experiments. The equilibrium strategy bidders made higher average profits as seen in Figure 96. The underbidding bidders average profits were 0.001, 0.0018, 0.0021 and 0.0023 while equilibrium strategy bidders' ranged from 0.0036 to 0.0043.

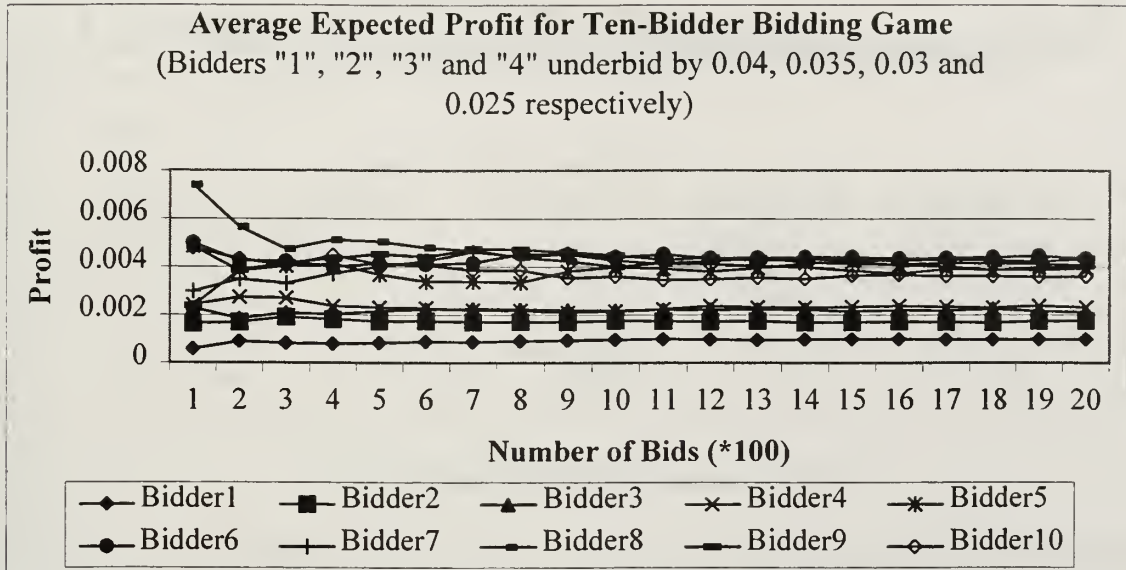


Figure 96

The underbidding bidders won more games than the others did. They won the games in a range of 206 to 273 times while equilibrium strategy bidders won in a range of 152 to 180 times. Despite this result, the equilibrium bidders made higher total profits than underbidding bidders as seen in Table 75 in Appendix C. According to this, underbidding bidders made profits; 1.9694, 3.5223, 4.2502 and 4.6414 respectively, while others did in a range of 7.1935 to 8.7005. The total profit curves are graphically shown in Figure 97.

Overall detailed results of the experiment are shown in Tables 73 through 76 in Appendix C.

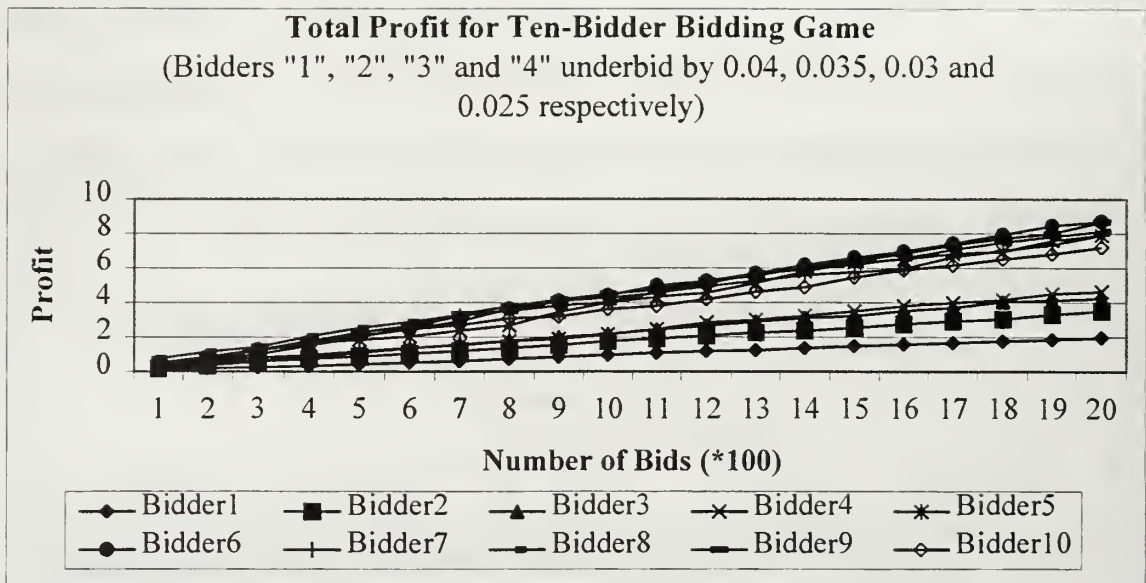


Figure 97

f. Experimentation With Fifteen Bidders

The simulation was conducted with fifteen bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

In these last series of experiments under triangular cost distribution, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used equilibrium strategies. In the second one, bidder1 underbid while others used the equilibrium strategy again. Third experiment was conducted as bidders "1", "2" and "3" underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders "1", "2", "3", "4" and "5" underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategies resulted in a well-defined equilibrium. All bidders' expected profit approached 0.032 as the number of

games approached 1500. The graphical representation of the expected profit results is shown in Figure 98.

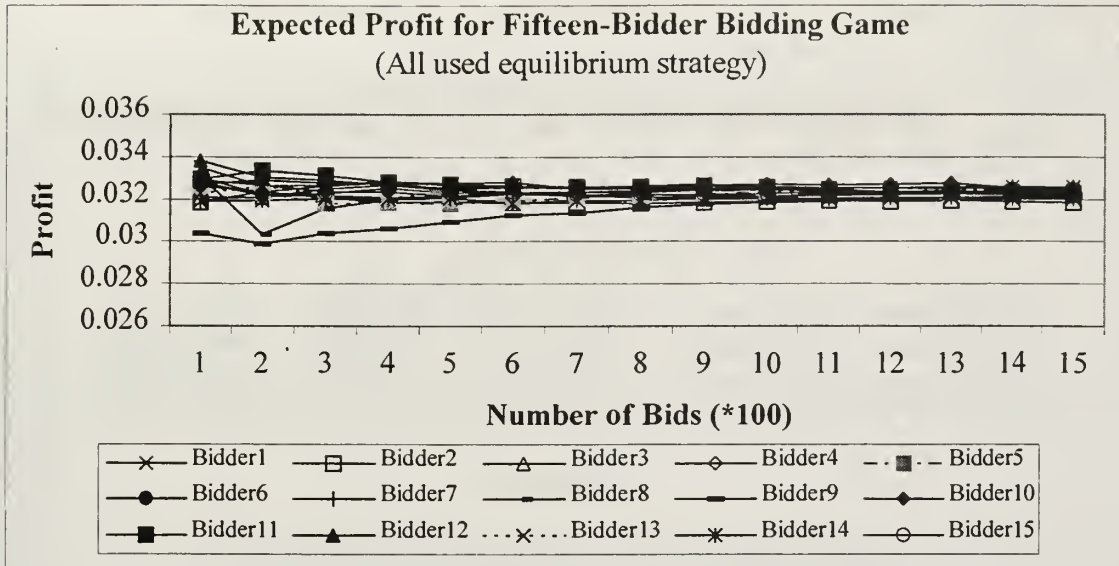


Figure 98

As it was the result of the previous experiments, introducing more bidders to the game reduced the expected profit from bidding more. The results of the experiment are outlined in Tables 77 through 80 in Appendix C.

The average expected profit graph is shown in Figure 99. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached approximately 0.002 while the number of games approached 1500.

As seen in Table 78, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 82 to 139 games each. Total profit for bidders as seen in Figure 100, ranged between 2.6708 and 4.1422.

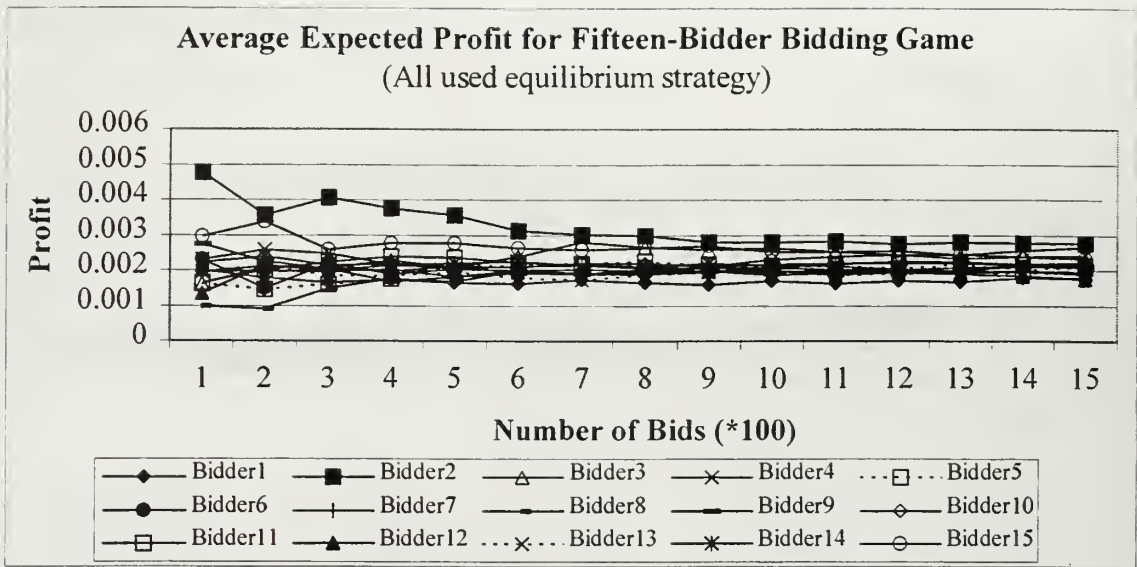


Figure 99

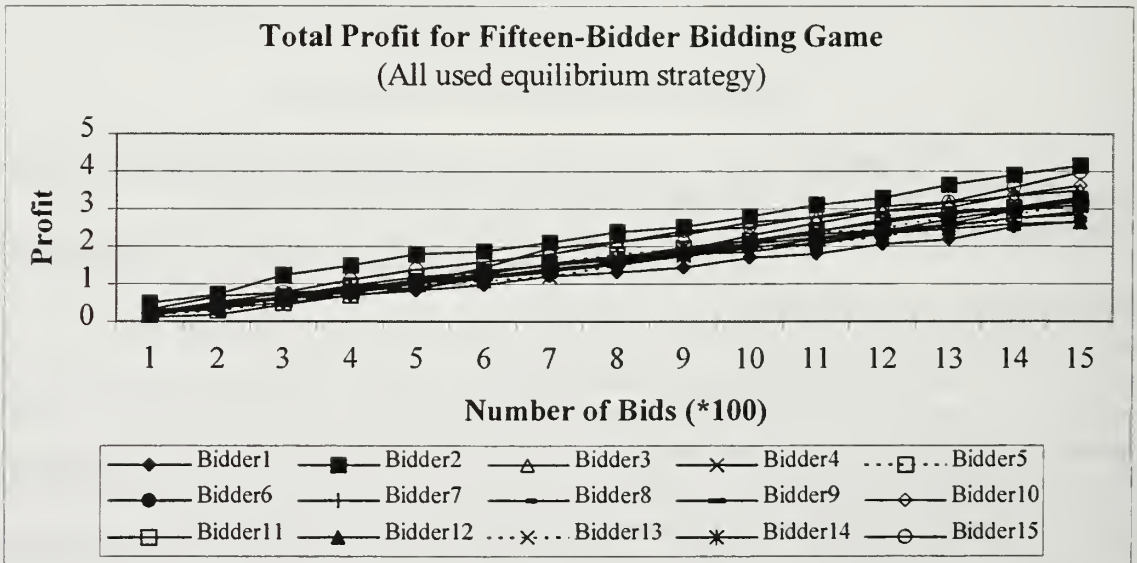


Figure 100

In the next experiment, bidder1 underbid by 0.015 while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.01735 while the other bidders' remained at 0.032. The expected profit graph is shown in Figure 101.

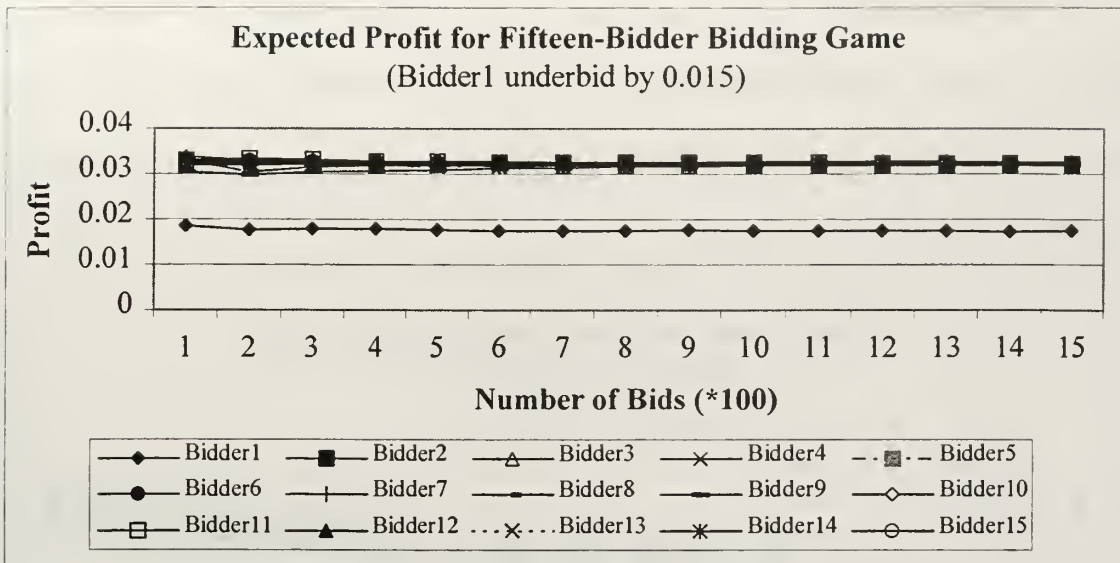


Figure 101

Average profit for the bidders also reinforced the same scheme as the equilibrium game had shown. Bidder1's average profit was 0.0011 while others ranged from 0.00174 to 0.00272, as graphically shown in Figure 102.

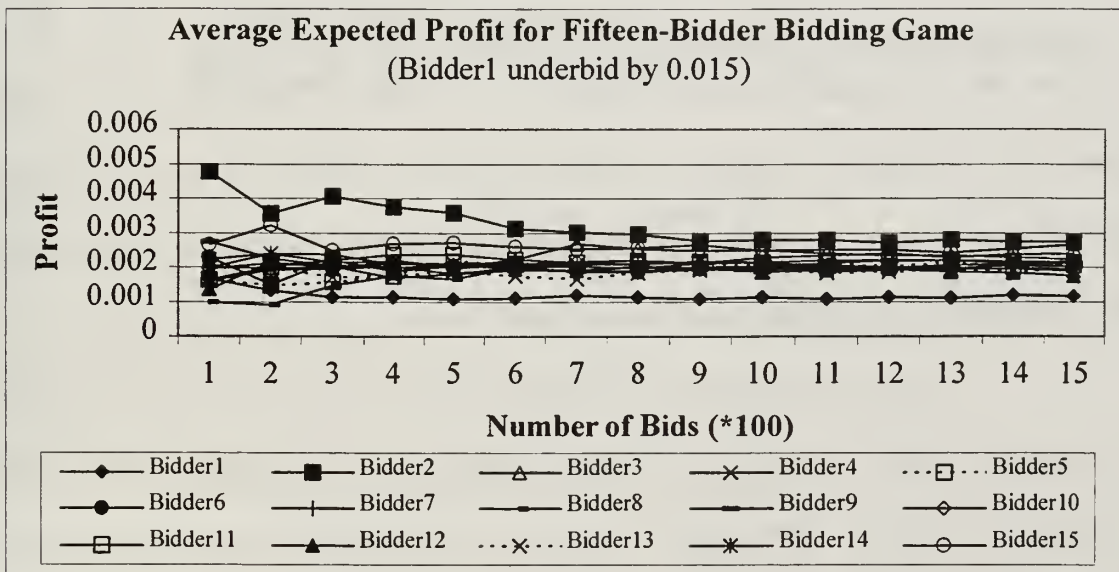


Figure 102

Bidder1 won 100 of 1500 games while others won in a range of 81 to 128 games as shown in Table 82 in Appendix C. Although bidder1 underbid, it won less

games than some of the others won. Total profit for bidder1 was 1.735, which is lower than the others' profits ranging from 2.6137 to 4.0855. Total profit graph for this experiment is shown in Figure 103 and detailed results of the experiment are shown in Tables 81 through 84 in Appendix C.

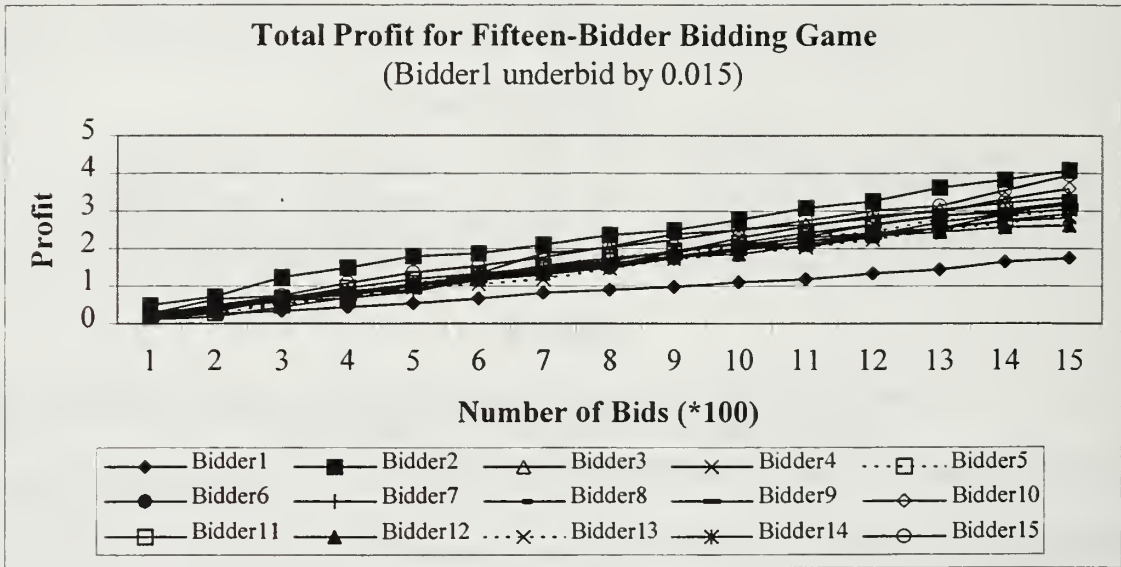


Figure 103

In the third experiment, bidders “1”, “2” and “3” underbid by 0.02 while others used equilibrium strategy in 2000 games. As a result of the experiment, underbidding bidders' expected profit approached 0.0125 while others' approached 0.032. The expected profit graph is shown in Figure 104.

Average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.001 while others' approached approximately 0.002 in 2000 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in contract games and any underbidding bidder would get less profit than equilibrium strategy bidders. The average expected profit graph is shown in Figure 105.

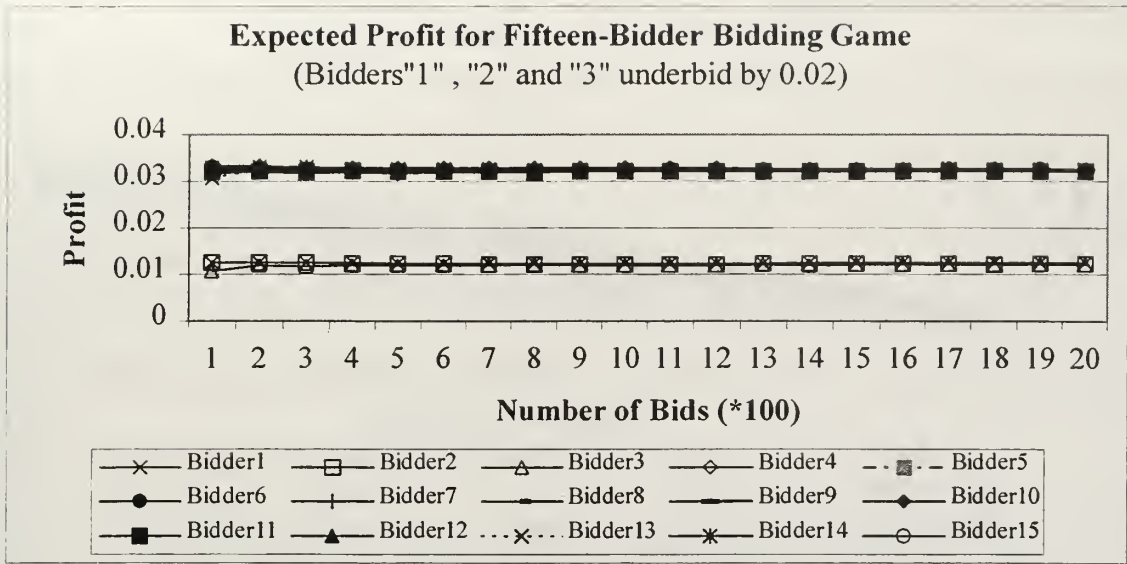


Figure 104

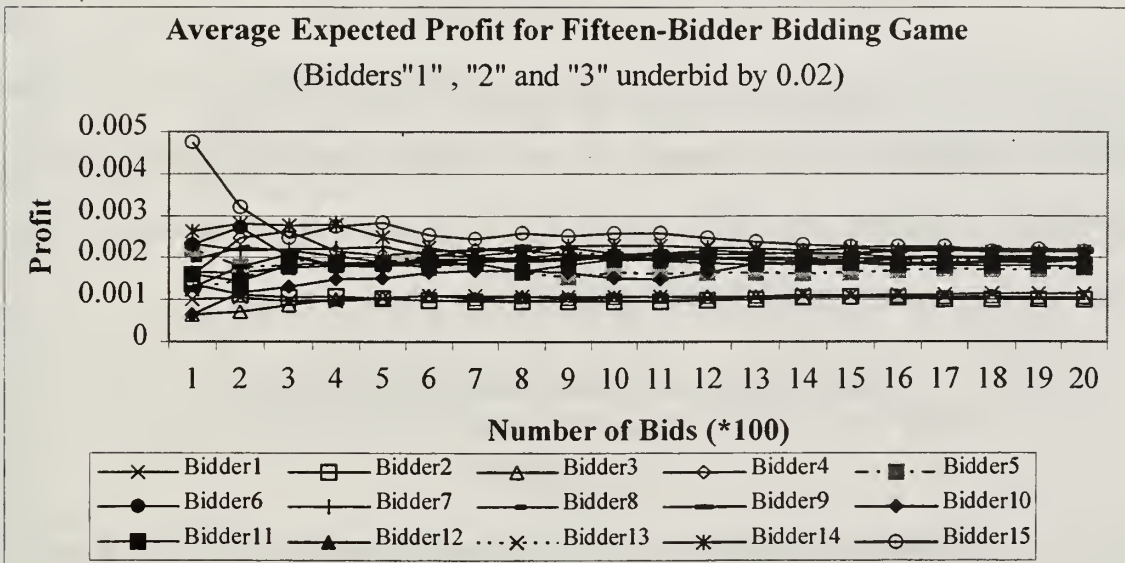


Figure 105

Total profits for underbidding bidders, ranging from 2.0129 to 2.277, were also lower than the equilibrium strategy bidders' total profits, ranging from 3.5074 to 4.3862. The underbidding bidders won 165 to 182 games as opposed to others won games of 108 to 135. Total profit curves are shown in Figure 106 and the results of the experiment are outlined in Tables 85 through 88 in Appendix C.

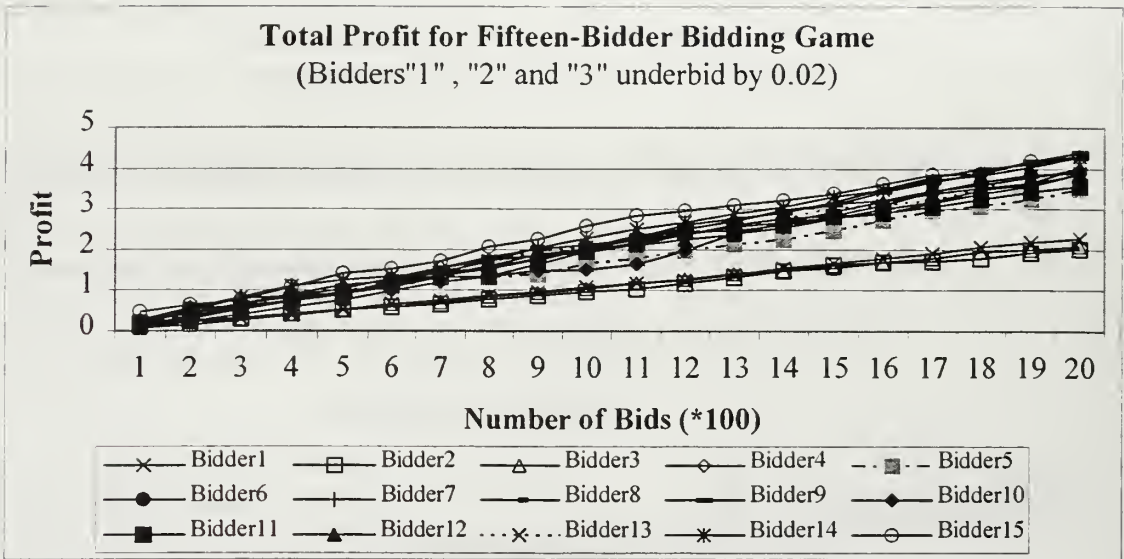


Figure 106

In the last experiment with fifteen bidders, bidders "1", "2", "3", "4" and "5" underbid by 0.025, 0.0225, 0.02, 0.0175 and 0.015 respectively, while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders' expected profit approached 0.0074, 0.0095, 0.012, 0.014 and 0.017 respectively, while others' approached 0.032. The expected profit graph is shown in Figure 107.

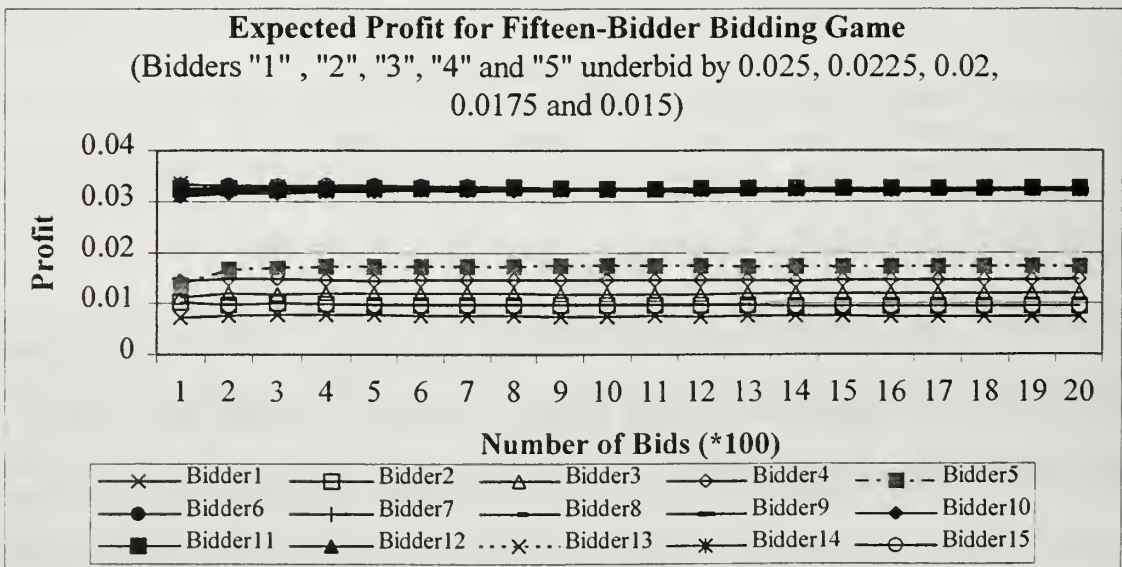


Figure 107

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0006, 0.0009, 0.00091, 0.0011 and 0.0011 respectively while others' approached a range between 0.0017 and 0.0023 in 2000 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in contract games. The deviant bidders lose profit even if they win more games than the equilibrium strategy bidders. The average expected profit graph for this experiment is shown in Figure 108.

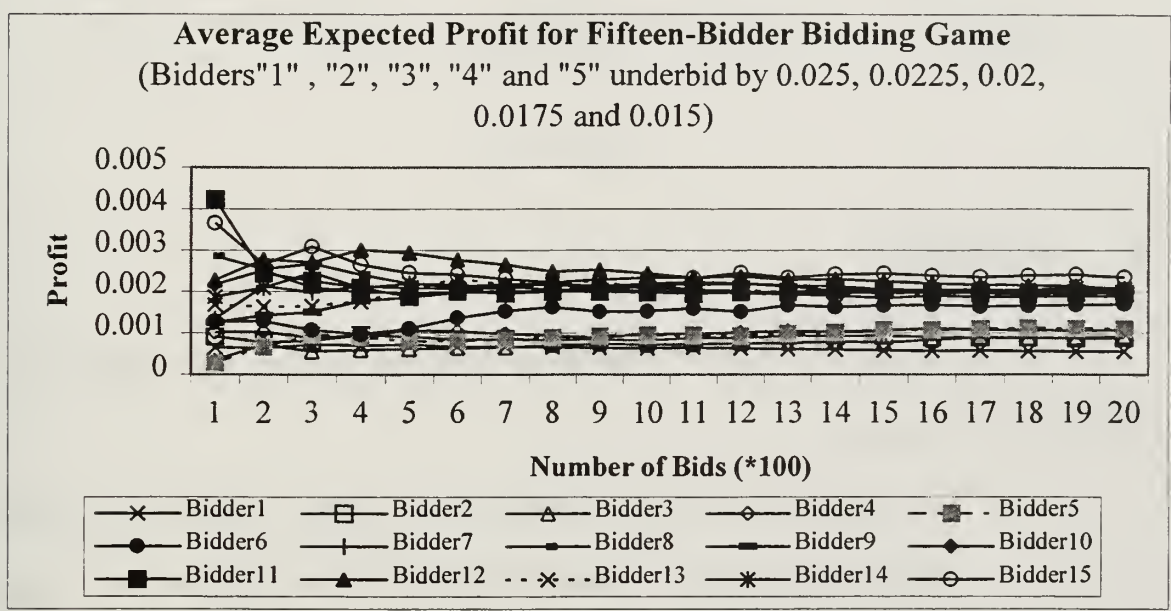


Figure 108

Total profits for underbidding bidders, ranging from 1.1038 to 2.2021, were also lower than the equilibrium strategy bidders' total profits, ranging from 3.4011 to 4.6884. The underbidding bidders won 127 to 186 games as opposed to others, who won the games 108 to 135 times. Total profit curves are shown in Figure 109 and results of the game are outlined in Tables 89 through 92 in Appendix C.

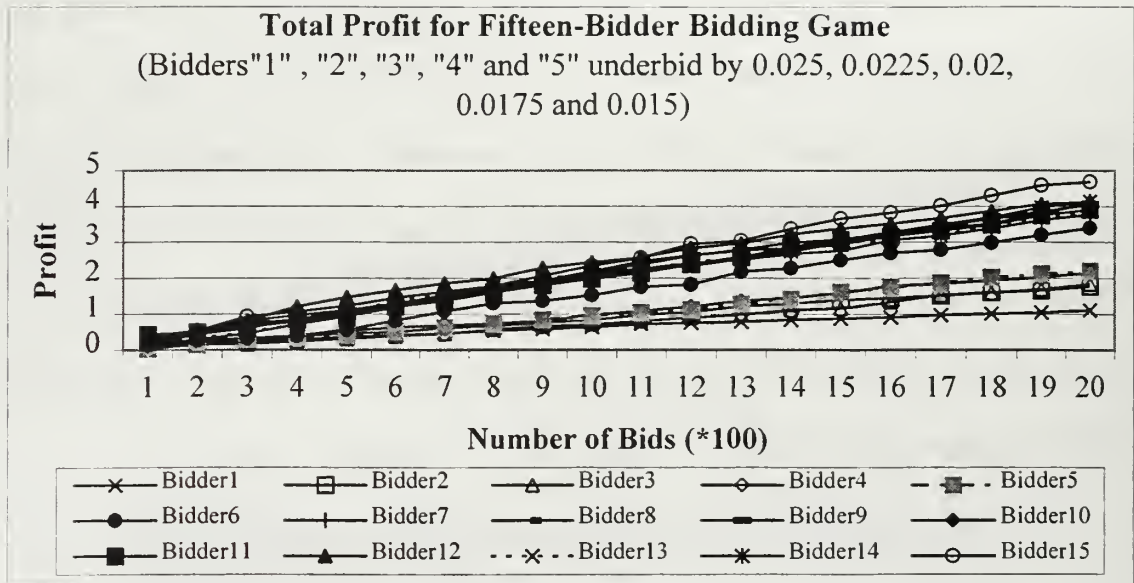


Figure 109

D. SUMMARY

After analyzing the results of experiments with both uniform and triangular cost distributions, we can say:

1. The equilibrium strategy maximizes the bidders expected profit and average expected profit. Underbidding bidders' both expected and average expected profits are lower than the equilibrium strategy bidders' profits.
2. If one of the bidders underbid, it reduces not only its total profit but also other bidders' total profits.
3. Underbidding bidders always won more games assuming they were responsive and responsible bidders.
4. Changing the cost distribution did not effect the general trend in simulations, but substantially altered the expected quantitative outcomes of the simulated FPSB in all simulations. The difference between the

expected profit outcomes resulted from the probability distributions' characteristics.

5. Adding additional competitors to the game does not change the general trend found in the games, but reduces both the expected and average expected profits. In fact, the number of competing bidders is one of the most influential factors in FPSB simulations.
6. Different scenarios used throughout the simulations showed that the more a bidder underbids the less profit it makes.

IV. CONCLUSION

This chapter of the thesis will review and discuss the research questions written in Chapter I. In answering these questions, the findings of the experiments will be the foundation for discussion and conclusions. Following this section, recommendations will be provided and areas for further research will also be identified.

A. ANSWERS TO RESEARCH QUESTIONS

The primary question for this research was: “*How do profit maximizing bidders choose their bids in a competitive environment?*” In answering this question, discussing and answering the subsidiary questions will be helpful, that is why, first the subsidiary questions will be answered.

1. Answers to Subsidiary Questions

a. Subsidiary Question 1

“*What are the equilibrium strategies of the bidders?*” As it was analyzed and discussed in Chapter II of the thesis, the bidding for contract game had been identified as a static non-cooperative game with incomplete information. The theory of contract games predicts that these games have an equilibrium. This thesis found a strong and well-defined equilibrium in the games during the simulation and experimentation as discussed in Chapter III.

This thesis used uniform and triangular production cost distributions for bidders. Then, equilibrium strategies, also called “bidding functions”, were developed in

Chapter III, under these two distributions. These strategies were used to develop the simulation programs and conduct the experimentation.

The first experiments were conducted under a uniform cost distribution with several different numbers of bidders and scenarios. As the bidders used the developed theoretical equilibrium strategy, the experiments resulted in very close outcomes that showed the existence of a well-defined equilibrium in these games. This is called symmetric Bayesian Nash equilibrium.

The resulting game equilibrium and the proven existence of equilibrium strategy provide the opportunity of predicting the behavior of the prospective suppliers. However, the reader should be careful about the probabilistic characteristics of the simulated games. The simulation cannot predict the actual behavior of the bidders in a particular bid, but it can approximate the general trend and expected average winning bid with a reasonable accuracy.

b. Subsidiary Question 2

“Do the bidders have a dominant strategy in FPSB?” An equilibrium strategy should maximize the bidders’ profits simultaneously. If a strategy maximizes the bidders’ profits, no bidder is willing to deviate from this strategy. This is the equilibrium of the game. This equilibrium condition was experimented with during the simulations and found that whatever the number of underbidding bidders in a game, deviations from the equilibrium strategy resulted in an expected loss for the underbidding bidders. In some of the experiments, scenarios were set up as if all the bidders did not use the equilibrium strategy and the result was they all made lower profits. The bidders choose

their bids (i.e., their strategies) based on their assumptions of the decisions of other bidders. That is, there is not a dominant strategy for the bidders in FPSB. The assumption for the information symmetry made it possible to analyze only the symmetric Nash equilibrium.

c. Subsidiary Question 3

“How does the number of bidders affect the outcome of the FPSB?”

Number of bidders was found to be one of the most influential factors affecting the game outcomes in FPSB.

Experiments were conducted with up to 15 bidders under both uniform and triangular cost distributions. The simulation programs in Appendices A and B were designed to experiment with up to 25 and 16 bidders respectively. As discussed throughout Chapter III of the Thesis, as the number of bidders increased, bidders' profits decreased. To be able to analyze this situation, a series of experiments were conducted with different numbers of bidders. Figure 110 shows the results of these experiments. As the Figure indicates, the bidders' expected profits decrease as the number of bidders increases from two to fifteen.

In conclusion, the number of participants in FPSB significantly affects the outcome by influencing the bidders' decisions in FPSB. The bidders consider it a potential factor affecting the probability of winning the bidding game. The higher the number of bids, the higher the probability of that any given bid will not be the lowest bid. Therefore, expected bid price decreases as the number of bidders increases, though, expected profit for the winners decreases.

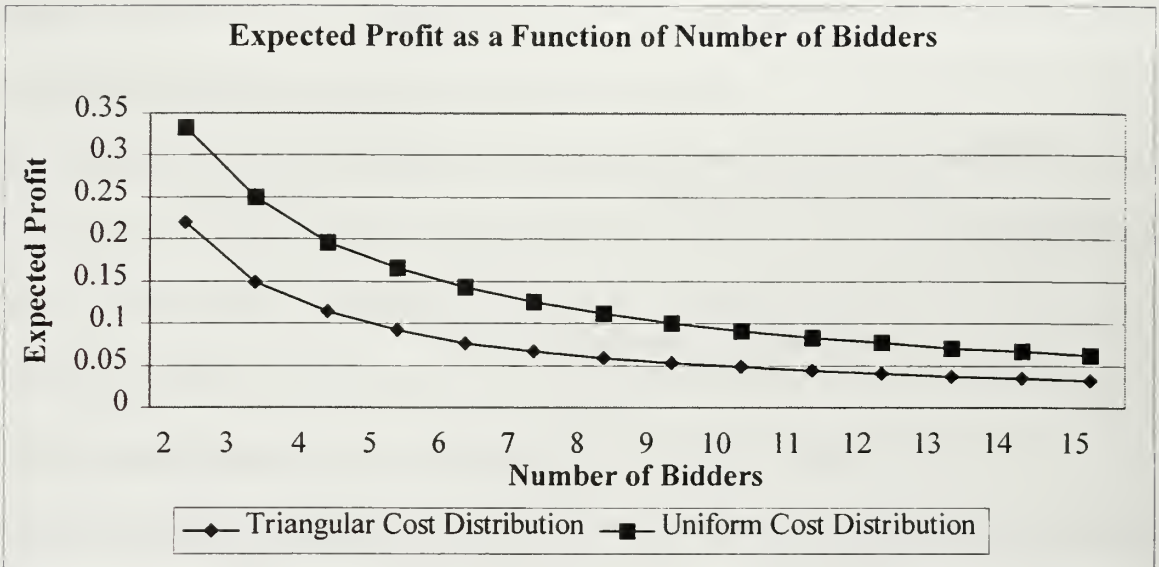


Figure 110

d. Subsidiary Question 4

“How does the cost distribution affect the outcome of the FPSB?”

Changing the production cost distribution did not affect the general pattern of the FPSB. However, it had a significant effect on the value of bids and the bidders’ expected profits. To be able to compare the outcomes of the two simulations under these two distributions, another experiment was conducted under uniform distribution over the cost interval [0, 2] with two bidders. The bidders’ expected profit was 0.667 under uniform cost distribution when using equilibrium strategy. This is almost three times higher than the equilibrium strategy profit of 0.22 under triangular distribution over the same cost interval with mode [1].

The change in the cost distribution has a significant effect on the FPSB outcomes. However, it is not just the cost distribution that affects the bidding outcomes; bidders’ perceptions about the other bidders’ production costs, influences their bids as

well. Because, as discussed in the first secondary question, bidders choose their bidding strategies according to their guess on the other participants probable behavior and production costs. This thesis assumed that the bidders' cost distributions are common knowledge among the bidders and identical to each other. It did not analyze the potential influence of bidders' cost assumptions on the other participants.

In reality, bidders do not have specific information about the other participants' cost distributions. They have some perceptions about it. Sometimes they develop their perceptions based on the signals received from other bidders. It is in the senders interest to distort this information, mainly in an increasing direction. The bidders have to be aware of this strategic misrepresentation.

d. Subsidiary Question 5

“How can the procuring agencies use the findings of “Game Theory” in their acquisition practice?” The simulations revealed that the equilibrium strategy is the strategy that maximizes the bidders' expected profits simultaneously. The suppliers will most likely use this strategy when formulating their bids in an actual bidding. Procuring agencies can use this finding to project the expected winning bid. The agencies can use these projections to forecast the expected spending on procurement. If the procuring agencies choose to negotiate with the offerors, they can develop negotiation positions and prepare the negotiators. The essence of these projections is forecasting the accurate expected cost range. In other words, this finding can help procuring agencies to develop competitive ranges for the bids.

Findings regarding the number of bidders and their effect on the expected bid can be used to establish the number of bidders to be invited. Although, the benefits of competition are known throughout the countries with free economy, there aren't many defense industry firms developing unique defense systems, such as aircraft, ships, missiles and so on. Therefore, the simulation of the FPSB can provide some insight through analysis of bidders behavior under changing situations.

2. Primary Research Question and Discussion

Bidders in a FPSB have some information available to them on which they base their bids. This thesis assumed symmetry of information among bidders. That is, all bidders' production cost distributions are common knowledge and are identical. However, the bidders' own production cost is private information. Another important factor in building the FPSB model was the number of bidders.

Bidders consider their own production costs, other bidders' production cost distribution and the number of competing bidders while preparing their bids. Since they have their own production cost information, they estimate their production cost for a given work while projecting their competitors' production costs based on experience, signals from them etc. Based on these assumptions, it can also be said, in a competitive world, bidders should know their potential competitors' bidding habits and production potentials.

This thesis applied two probability distributions on the production costs, uniform and triangular distributions. It is assumed that uniform cost distribution is applicable to the production of standard, non-complex items while triangular cost distribution is

applicable to more complex productions and where some firms have production advantages over the competitors.

It is furthermore assumed that, bidders are risk neutral and act to maximize their profits. It is this assumption that made it possible for us to construct the FPSB model and derive the equilibrium bidding functions. Bidders want to maximize their profits but their bid should be low enough to have a reasonable chance to win the award. This thesis found that if the bidders act rationally (i.e., bid to maximize their profits), they should use the equilibrium strategy. Simulation of FPSB also provided an opportunity for bidders to analyze tradeoffs between profit maximizing and probability of winning the award.

One of the results of the experiments with the FPSB was underbidding bidders won significantly more games than the equilibrium strategy bidders. This was the result of the artificially low bids they chose. However, these bidders made lower profits than the equilibrium strategy bidders. That is, equilibrium strategy bidders made higher profits in less number of contracts. Therefore, they can devote their time to other areas, which means they have more time to gain more profits.

First Price Sealed Bidding is an ancient market institution, which has been used for thousands of years. Bidders' behavior in FPSB is determined by several factors and perceptions. This thesis used the game theory approach in microeconomics to describe and analyze this behavior. However, there are several different approaches in game theory to construct different models or a model including many different approaches in one simulation. The game simulation developed in this thesis is one of the methods to explore the FPSB. However, simulation gives some insight to the reality and one should not confuse the simulation with reality. Data resulted from the simulation give an average

information about the real life. The model does not prescribe the bidders' behavior; it forecasts and describes the average behavior expected from the players over time.

B. AREAS FOR FURTHER RESEARCH

There are several areas for further research related to this thesis. Some are as follows:

1. The second price sealed bidding is one of the top areas for further research. Simulation of the second price sealed bidding technique, and experimentation would probably provide more insights to the auctioning theory. It can also be said that, since second price sealed bidding is being experimented to be an alternative for FPSB because of inefficiencies described as “winner’s curse” and “buyer’s angst”[Ref. 13], analysis of this technique would surely be a valuable research.

2. The competitive range is a measure established by the buyer to influence the bidders' behavior. Further research would analyze the effect of this measure on the FPSB.

APPENDIX A

SIMULATION PROGRAM WITH UNIFORM COST DISTRIBUTION

Sub Uniform-Simulation ()

' Uniform-Simulation Macro

' Macro recorded 8/28/97 by Suat Tozendemir

'Input for experiment

Sheets("sheet1").Select

Range("a1").Select

ActiveCell.FormulaR1C1 = "Write # of Bidders (n) [2,25]"

Range("A2").Select

ActiveCell.FormulaR1C1 = "Write Lower Limit of Uniform Cost Dist. (k)"

Range("A3").Select

ActiveCell.FormulaR1C1 = "Write Upper Limit of Uniform Cost Dist. (h)"

Columns("B:B").Select

'Page design

Range("A1:B3").Select

Selection.Borders(xlDiagonalDown).LineStyle = xlNone

Selection.Borders(xlDiagonalUp).LineStyle = xlNone

With Selection.Borders(xlEdgeLeft)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeTop)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeBottom)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeRight)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlInsideVertical)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlInsideHorizontal)

.LineStyle = xlContinuous

```

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

Selection.Borders(xlDiagonalDown).LineStyle = xlNone

Selection.Borders(xlDiagonalUp).LineStyle = xlNone

With Selection.Borders(xlEdgeLeft)

.LineStyle = xlContinuous

.Weight = xlMedium

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeTop)

.LineStyle = xlContinuous

.Weight = xlMedium

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeBottom)

.LineStyle = xlContinuous

.Weight = xlMedium

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeRight)

.LineStyle = xlContinuous

.Weight = xlMedium

```

```
.ColorIndex = xlAutomatic
```

```
End With
```

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With Selection.Borders(xlInsideVertical)
```

```
.LineStyle = xlContinuous
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.Weight = xlThin
```

```
.ColorIndex = xlAutomatic
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End With
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With Selection.Borders(xlInsideHorizontal)
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.Weight = xlThin
```

```
.ColorIndex = xlAutomatic
```

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End With
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Selection.Font.Bold = True
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Columns("A:A").EntireColumn.AutoFit
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'Assigning cost variables

```
Range("B5").Select
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```
ActiveCell.FormulaR1C1 = "=IF(R[-4]C>=2, ""Cost1"", "" "")"
```

```
Range("C5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-1]>=2, ""Cost2"", "" "")"
```

```
Range("D5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-2]>=3, ""Cost3"", "" "")"
```

```
Range("E5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-3]>=4, ""Cost4"", "" "")"
```

Range("F5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-4]>=5, ""Cost5"", "" "")"

Range("G5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-5]>=6, ""Cost6"", "" "")"

Range("H5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-6]>=7, ""Cost7"", "" "")"

Range("I5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-7]>=8, ""Cost8"", "" "")"

Range("J5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-8]>=9, ""Cost9"", "" "")"

Range("K5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-9]>=10, ""Cost10"", "" "")"

Range("L5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-10]>=11, ""Cost11"", "" "")"

Range("M5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-11]>=12, ""Cost12"", "" "")"

Range("N5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-12]>=13, ""Cost13"", "" "")"

Range("O5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-13]>=14, ""Cost14"", "" "")"

Range("P5").Select

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Range("Q5").Select

```
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```

```
Range("R5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=17, ""Cost17"", "" "")"
```

```
Range("S5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=18, ""Cost18"", "" "")"
```

```
Range("T5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=19, ""Cost19"", "" "")"
```

```
Range("U5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=20, ""Cost20"", "" "")"
```

```
Range("V5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=21, ""Cost21"", "" "")"
```

```
Range("W5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=22, ""Cost22"", "" "")"
```

```
Range("X5").Select
```

```
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```

```
Range("Y5").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=24, ""Cost24"", "" "")"
```

```
Range("Z5").Select
```

```
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```

'Assigning bid variables

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Sheets("Sheet2").Select
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```
Range("A1").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=2, ""Bid1"", "" "")"
```

Range("B1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=2, ""Bid2"", "" "")"

Range("C1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=3, ""Bid3"", "" "")"

Range("D1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=4, ""Bid4"", "" "")"

Range("E1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=5, ""Bid5"", "" "")"

Range("F1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=6, ""Bid6"", "" "")"

Range("G1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=7, ""Bid7"", "" "")"

Range("H1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=8, ""Bid8"", "" "")"

Range("I1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=9, ""Bid9"", "" "")"

Range("J1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=10, ""Bid10"", "" "")"

Range("K1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=11, ""Bid11"", "" "")"

Range("L1").Select

ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=12, ""Bid12"", "" "")"

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ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=20, ""Payoff20"", "" "")"  
Range("U1").Select  
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=21, ""Payoff21"", "" "")"
```

```
Range("V1").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=22, ""Payoff22"", "" "")"
```

```
Range("W1").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=23, ""Payoff23"", "" "")"
```

```
Range("X1").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=24, ""Payoff24"", "" "")"
```

```
Range("Y1").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(Sheet1!R1C2>=25, ""Payoff25"", "" "")"
```

```
For I = 1 To 1500 ' NUMBER OF EXPERIEMENTS TO BE DONE
```

'Calculation of costs

```
Sheets("sheet1").Select
```

```
Range("B6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=2,((Sheet1!R3C2-  
Sheet1!R2C2)*RAND()+Sheet1!R2C2), "" "")"
```

```
Range("C6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=2,((Sheet1!R3C2-  
Sheet1!R2C2)*RAND()+Sheet1!R2C2), "" "")"
```

```
Range("D6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=3,((Sheet1!R3C2-  
Sheet1!R2C2)*RAND()+Sheet1!R2C2), "" "")"
```

```
Range("E6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=4,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("F6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=5,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("G6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=6,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("H6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=7,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("I6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=8,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("J6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=9,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("K6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=10,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("L6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=11,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

Range("M6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=12,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("N6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=13,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("O6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=14,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("P6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=15,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("Q6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=16,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("R6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=17,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("S6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=18,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("T6").Select

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=19,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("U6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=20,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("V6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=21,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("W6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=22,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("X6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=23,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("Y6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=24,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("Z6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=25,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

'Copying costs to another row

```
Sheets("sheet1").Select
```

```
Range("b7").Select
```

```
Selection.EntireRow.Insert
Range("b6:z6").Select
Selection.Copy
Range("b7").Select
Selection.PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Range("b6:z6").Select
Selection.Clear
```

'Calculation of bids

```
Sheets("sheet2").Select
    Range("A2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=2,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("B2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=2,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("C2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=3,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("D2").Select
```

```

ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=4,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("E2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=5,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("F2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=6,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("G2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=7,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("H2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=8,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"
Range("I2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=9,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

```


Range("J2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=10,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("K2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=11,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("L2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=12,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("M2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=13,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("N2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=14,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("O2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=15,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("P2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=16,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("Q2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=17,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("R2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=18,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("S2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=19,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("T2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=20,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("U2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=21,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

```
Range("V2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=22,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("W2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=23,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("X2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=24,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("Y2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=25,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

'Copying bids to another row

```
Sheets("Sheet2").Select
```

```
Range("a3").Select
```

```
Selection.EntireRow.Insert
```

```
Sheets("sheet2").Select
```

```
Range("a2:y2").Select
```

```
Selection.Copy
```

```
Sheets("Sheet2").Select
Range("a3").Select
Selection.PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Sheets("Sheet2").Select
Range("a2:y2").Select
Selection.Clear
```

'Selection of winner and its payoff

```
Sheets("sheet3").Select
Range("A2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet2!R3C1=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C1-
Sheet1!R[5]C[1],""""")"
Range("b2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet2!R3C2=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C2-
Sheet1!R[5]C[1],""""")"
Range("c2").Select
ActiveCell.FormulaR1C1 = _
    "=IF(Sheet2!R3C3=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C3-
Sheet1!R[5]C[1],""""")"
Range("d2").Select
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C4=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C4-  
Sheet1!R[5]C[1],""""")"
```

```
Range("e2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C5=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C5-  
Sheet1!R[5]C[1],""""")"
```

```
Range("f2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C6=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C6-  
Sheet1!R[5]C[1],""""")"
```

```
Range("g2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C7=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C7-  
Sheet1!R[5]C[1],""""")"
```

```
Range("h2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C8=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C8-  
Sheet1!R[5]C[1],""""")"
```

```
Range("i2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C9=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C9-  
Sheet1!R[5]C[1],""""")"
```

```
Range("j2").Select
```

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C10=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C10-Sheet1!R[5]C[1],""""")"

Range("k2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C11=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C11-Sheet1!R[5]C[1],""""")"

Range("l2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C12=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C12-Sheet1!R[5]C[1],""""")"

Range("m2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C13=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C13-Sheet1!R[5]C[1],""""")"

Range("n2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C14=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C14-Sheet1!R[5]C[1],""""")"

Range("o2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C15=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C15-Sheet1!R[5]C[1],""""")"

Range("p2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C16=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C16-

Sheet1!R[5]C[1],""""")"

Range("q2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C17=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C17-

Sheet1!R[5]C[1],""""")"

Range("r2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C18=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C18-

Sheet1!R[5]C[1],""""")"

Range("s2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C19=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C19-

Sheet1!R[5]C[1],""""")"

Range("t2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C20=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C20-

Sheet1!R[5]C[1],""""")"

Range("u2").Select

ActiveCell.FormulaR1C1 = _

```
"=IF(Sheet2!R3C21=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C21-  
Sheet1!R[5]C[1],""""")"
```

```
Range("v2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C22=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C22-  
Sheet1!R[5]C[1],""""")"
```

```
Range("w2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C23=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C23-  
Sheet1!R[5]C[1],""""")"
```

```
Range("x2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C24=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C24-  
Sheet1!R[5]C[1],""""")"
```

```
Range("y2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C25=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C25-  
Sheet1!R[5]C[1],""""")"
```

'Copying winner's payoff to another row

```
Sheets("Sheet3").Select
```

```
Range("a3").Select
```

```
Selection.EntireRow.Insert
```

```
Sheets("sheet3").Select
```



```
Range("a2:y2").Select
Selection.Copy
Sheets("Sheet3").Select
Range("a3").Select
Selection.PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Sheets("Sheet3").Select
Range("a2:y2").Select
Selection.Clear

Next
    Beep
End Sub
```


APPENDIX B

SIMULATION PROGRAM WITH TRIANGULAR COST DISTRIBUTION

Sub Triangular-Simulation()

' Triangular-Simulation Macro

' Macro recorded 9/1/97 by Suat Tozendemir

Application.ScreenUpdating = False

'Assigning variables

Dim h, m, k, n, num, rand1, rand2, rand3, rand4, rand5, rand6, rand7, rand8, rand9,
cost1, cost2, cost3, cost4, cost5, cost6, cost7, cost8, cost9, cost10, cost11, cost12, cost13,
cost14, cost15, cost16 As Variant

Dim bid1, bid1a, bid1b, bid1c, bid2, bid2a, bid2b, bid3, bid3a, bid3b, bid4, bid4a,
bid4b, bid5, bid5a, bid5b, bid6, bid6a, bid6b, bid7, bid7a, bid7b, bid8, bid8a, bid8b, bid9,
bid9a, bid9b, bid10, bid10a, bid10b, bid11, bid11a, bid11b, bid12, bid12a, bid12b, bid13,
bid13a, bid13b, bid14, bid14a, bid14b, bid15, bid15a, bid15b, bid16, bid16a, bid16b As
Variant

Dim payoff1, payoff2, payoff3, payoff4, payoff5, payoff6, payoff7, payoff8, payoff9,
payoff10, payoff11, payoff12, payoff13, payoff14, payoff15, payoff16 As Variant

'giving initial values to variables

Sheets("sheet2").Select

Range("a1").Select

ActiveCell.FormulaR1C1 = "k"

k = 0 'lower limit of cost range

Range("b1").Select

ActiveCell.Value = k

Range("c1").Select

ActiveCell.FormulaR1C1 = "m"

m = 1 'mode of the cost distribution

Range("d1").Select

ActiveCell.Value = m

Range("e1").Select

ActiveCell.FormulaR1C1 = "h"

h = 2 'the higher limit of the cost range

Range("f1").Select

ActiveCell.Value = h

bid1a = 0

bid1b = 0

bid1c = 0

Range("g1").Select

ActiveCell.FormulaR1C1 = "n"

n = 2 'number of bidders

Range("h1").Select

ActiveCell.Value = n

'start loop enter the number of required after "To"

num = 0

```

For num = 1 To 3
If n >= 2 Then
Sheets("sheet2").Select
Range("a4").Select
ActiveCell.FormulaR1C1 = " cost1"
Range("b4").Select
ActiveCell.FormulaR1C1 = " cost2"
Sheets("sheet3").Select
Range("a4").Select
ActiveCell.FormulaR1C1 = " bid1"
Range("b4").Select
ActiveCell.FormulaR1C1 = " bid2"
Sheets("sheet4").Select
Range("a4").Select
ActiveCell.FormulaR1C1 = " payoff1"
Range("b4").Select
ActiveCell.FormulaR1C1 = " payoff2"

```

'computation of cost1

```

rand1 = Rnd()
If rand1 < ((m - k) / (h - k)) Then
    cost1 = ((rand1 * (h - k) * (m - k)) ^ 0.5) + 1
Else
    cost1 = h - ((1 - rand1) * (h - k) * (h - m)) ^ 0.5

```

End If

'cost1 to sheet2

Sheets("sheet2").Select

Range("a3").Select

ActiveCell.Value = cost1

'computation of cost2

rand2 = Rnd()

If rand2 < ((m - k) / (h - k)) Then

cost2 = ((rand2 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost2 = h - ((1 - rand2) * (h - k) * (h - m)) ^ 0.5

End If

'cost2 to sheet2

Sheets("sheet2").Select

Range("b3").Select

ActiveCell.Value = cost2

'computation of bid1

bid1a = (n - 1) + 0.5

If cost1 < m Then

bid1b = n * k + (n - 1) * cost1

bid1c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost1 * (n - 1) * k)

bid1 = (bid1b + (bid1b ^ 2 + 2 * bid1c) ^ 0.5) / (2 * bid1a)

Else

$bid1 = (h + (2 * (n - 1) * cost1)) / (2 * bid1a)$

End If

'bid1 to sheet2

Sheets("sheet3").Select

Range("a3").Select

ActiveCell.Value = bid1

'computation of bid2

If cost2 < m Then

$bid2b = n * k + (n - 1) * cost2$

$bid2c = bid1a * (h - k) * (m - k) - (k^2) - (2 * cost2 * (n - 1) * k)$

$bid2 = (bid2b + (bid2b^2 + 2 * bid2c)^{0.5}) / (2 * bid1a)$

Else

$bid2 = (h + (2 * (n - 1) * cost2)) / (2 * bid1a)$

End If

'bid2 to sheet2

Sheets("sheet3").Select

Range("b3").Select

ActiveCell.Value = bid2

End If

If n >= 3 Then GoTo row10:

'defining the winner and its payoff

If bid1 < bid2 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

Else

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

row10:

Sheets("sheet2").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " cost3"

Sheets("sheet3").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " bid3"

Sheets("sheet4").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " payoff3"

'computation of cost3

rand3 = Rnd()

If rand3 < ((m - k) / (h - k)) Then

cost3 = ((rand3 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost3 = h - ((1 - rand3) * (h - k) * (h - m)) ^ 0.5

End If

'cost3 to sheet2

Sheets("sheet2").Select

Range("c3").Select

ActiveCell.Value = cost3

'computation of bid3

If cost3 < m Then

bid3b = n * k + (n - 1) * cost3

bid3c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost3 * (n - 1) * k)

bid3 = (bid3b + (bid3b ^ 2 + 2 * bid3c) ^ 0.5) / (2 * bid1a)

Else

bid3 = (h + (2 * (n - 1) * cost3)) / (2 * bid1a)

End If

'bid3 to sheet2

Sheets("sheet3").Select

Range("c3").Select

ActiveCell.Value = bid3

If n >= 4 Then GoTo row11:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 Then

 payoff1 = bid1 - cost1

 Sheets("sheet4").Select

 Range("a3").Select

 ActiveCell.Value = payoff1

 GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 Then

 payoff2 = bid2 - cost2

 Sheets("sheet4").Select

 Range("b3").Select

 ActiveCell.Value = payoff2

 GoTo row100:

Else

 payoff3 = bid3 - cost3

 Sheets("sheet4").Select

 Range("c3").Select

 ActiveCell.Value = payoff3

 GoTo row100:

End If

row11:

```
Sheets("sheet2").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " cost4"
```

```
Sheets("sheet3").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " bid4"
```

```
Sheets("sheet4").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff4"
```

```
'computation of cost4
```

```
rand4 = Rnd()
```

```
If rand4 < ((m - k) / (h - k)) Then
```

```
    cost4 = ((rand4 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
    cost4 = h - ((1 - rand4) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost4 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("d3").Select
```

```
ActiveCell.Value = cost4
```

```
'computation of bid4
```

```
If cost4 < m Then
```

$$\text{bid4b} = n * k + (n - 1) * \text{cost4}$$

$$\text{bid4c} = \text{bid1a} * (h - k) * (m - k) - (k ^ 2) - (2 * \text{cost4} * (n - 1) * k)$$

$$\text{bid4} = (\text{bid4b} + (\text{bid4b} ^ 2 + 2 * \text{bid4c}) ^ 0.5) / (2 * \text{bid1a})$$

Else

$$\text{bid4} = (h + (2 * (n - 1) * \text{cost4})) / (2 * \text{bid1a})$$

End If

'bid4 to sheet3

Sheets("sheet3").Select

Range("d3").Select

ActiveCell.Value = bid4

If n >= 5 Then GoTo row12:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 Then

$$\text{payoff1} = \text{bid1} - \text{cost1}$$

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 Then

$$\text{payoff2} = \text{bid2} - \text{cost2}$$

Sheets("sheet4").Select

Range("b3").Select

```

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 Then

    payoff3 = bid3 - cost3

    Sheets("sheet4").Select

    Range("c3").Select

    ActiveCell.Value = payoff3

    GoTo row100:

Else

    payoff4 = bid4 - cost4

    Sheets("sheet4").Select

    Range("d3").Select

    ActiveCell.Value = payoff4

    GoTo row100:

End If

row12:

    Sheets("sheet2").Select

    Range("e4").Select

    ActiveCell.FormulaR1C1 = " cost5"

    Sheets("sheet3").Select

    Range("e4").Select

    ActiveCell.FormulaR1C1 = " bid5"

```

```
Sheets("sheet4").Select
```

```
Range("e4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff5"
```

```
'computation of cost5
```

```
rand5 = Rnd()
```

```
If rand5 < ((m - k) / (h - k)) Then
```

```
cost5 = ((rand5 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost5 = h - ((1 - rand5) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost5 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("e3").Select
```

```
ActiveCell.Value = cost5
```

```
'computation of bid5
```

```
If cost5 < m Then
```

```
bid5b = n * k + (n - 1) * cost5
```

```
bid5c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost5 * (n - 1) * k)
```

```
bid5 = (bid5b + (bid5b ^ 2 + 2 * bid5c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
bid5 = (h + (2 * (n - 1) * cost5)) / (2 * bid1a)
```

```
End If
```

'bid5 to sheet3

Sheets("sheet3").Select

Range("e3").Select

ActiveCell.Value = bid5

If n >= 6 Then GoTo row13:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

Else

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

row13:

Sheets("sheet2").Select

Range("f4").Select

ActiveCell.FormulaR1C1 = " cost6"

Sheets("sheet3").Select

Range("f4").Select


```
ActiveCell.FormulaR1C1 = " bid6"
```

```
Sheets("sheet4").Select
```

```
Range("f4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff6"
```

```
'computation of cost6
```

```
rand6 = Rnd()
```

```
If rand6 < ((m - k) / (h - k)) Then
```

```
    cost6 = ((rand6 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
    cost6 = h - ((1 - rand6) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost6 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("f3").Select
```

```
ActiveCell.Value = cost6
```

```
'computation of bid6
```

```
If cost6 < m Then
```

```
    bid6b = n * k + (n - 1) * cost6
```

```
    bid6c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost6 * (n - 1) * k)
```

```
    bid6 = (bid6b + (bid6b ^ 2 + 2 * bid6c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
    bid6 = (h + (2 * (n - 1) * cost6)) / (2 * bid1a)
```

End If

'bid6 to sheet3

Sheets("sheet3").Select

Range("f3").Select

ActiveCell.Value = bid6

If n >= 7 Then GoTo row14:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

Else

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

row14:

Sheets("sheet2").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " cost7"

Sheets("sheet3").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " bid7"

Sheets("sheet4").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " payoff7"

'computation of cost7

rand7 = Rnd()

If rand7 < ((m - k) / (h - k)) Then

cost7 = ((rand7 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost7 = h - ((1 - rand7) * (h - k) * (h - m)) ^ 0.5

End If

'cost7 to sheet2

Sheets("sheet2").Select

Range("g3").Select

ActiveCell.Value = cost7

'computation of bid7

If cost7 < m Then

bid7b = n * k + (n - 1) * cost7

bid7c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost7 * (n - 1) * k)

bid7 = (bid7b + (bid7b ^ 2 + 2 * bid7c) ^ 0.5) / (2 * bid1a)

Else

bid7 = (h + (2 * (n - 1) * cost7)) / (2 * bid1a)

End If

'bid7 to sheet3

Sheets("sheet3").Select

Range("g3").Select

ActiveCell.Value = bid7

If n >= 8 Then GoTo row15:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 Then

 payoff2 = bid2 - cost2

 Sheets("sheet4").Select

 Range("b3").Select

 ActiveCell.Value = payoff2

 GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 Then

 payoff3 = bid3 - cost3

 Sheets("sheet4").Select

 Range("c3").Select

 ActiveCell.Value = payoff3

 GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 Then

 payoff4 = bid4 - cost4

 Sheets("sheet4").Select

 Range("d3").Select

 ActiveCell.Value = payoff4

 GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And

bid5 < bid7 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And

bid6 < bid7 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

Else

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

row15:

Sheets("sheet2").Select

```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " cost8"
```

```
Sheets("sheet3").Select
```

```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " bid8"
```

```
Sheets("sheet4").Select
```

```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff8"
```

```
'computation of cost8
```

```
rand8 = Rnd()
```

```
If rand8 < ((m - k) / (h - k)) Then
```

```
cost8 = ((rand8 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost8 = h - ((1 - rand8) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost8 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("h3").Select
```

```
ActiveCell.Value = cost8
```

```
'computation of bid8
```

```
If cost8 < m Then
```

```
bid8b = n * k + (n - 1) * cost8
```


bid8c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost8 * (n - 1) * k)

bid8 = (bid8b + (bid8b ^ 2 + 2 * bid8c) ^ 0.5) / (2 * bid1a)

Else

bid8 = (h + (2 * (n - 1) * cost8)) / (2 * bid1a)

End If

'bid8 to sheet3

Sheets("sheet3").Select

Range("h3").Select

ActiveCell.Value = bid8

If n >= 9 Then GoTo row16:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 Then

payoff5 = bid5 - cost5

```

Sheets("sheet4").Select
Range("e3").Select
ActiveCell.Value = payoff5
GoTo row100:
End If
If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 Then
    payoff6 = bid6 - cost6
    Sheets("sheet4").Select
    Range("f3").Select
    ActiveCell.Value = payoff6
    GoTo row100:
End If
If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 Then
    payoff7 = bid7 - cost7
    Sheets("sheet4").Select
    Range("g3").Select
    ActiveCell.Value = payoff7
    GoTo row100:
Else
    payoff8 = bid8 - cost8
    Sheets("sheet4").Select

```

```
Range("h3").Select
```

```
ActiveCell.Value = payoff8
```

```
GoTo row100:
```

```
End If
```

```
row16:
```

```
Sheets("sheet2").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " cost9"
```

```
Sheets("sheet3").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " bid9"
```

```
Sheets("sheet4").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff9"
```

```
'computation of cost9
```

```
rand9 = Rnd()
```

```
If rand9 < ((m - k) / (h - k)) Then
```

```
cost9 = ((rand9 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost9 = h - ((1 - rand9) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost9 to sheet2
```

```
Sheets("sheet2").Select
```

Range("i3").Select

ActiveCell.Value = cost9

'computation of bid9

If cost9 < m Then

bid9b = n * k + (n - 1) * cost9

bid9c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost9 * (n - 1) * k)

bid9 = (bid9b + (bid9b ^ 2 + 2 * bid9c) ^ 0.5) / (2 * bid1a)

Else

bid9 = (h + (2 * (n - 1) * cost9)) / (2 * bid1a)

End If

'bid9 to sheet3

Sheets("sheet3").Select

Range("i3").Select

ActiveCell.Value = bid9

If n >= 10 Then GoTo row17:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

Else

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

row17:

Sheets("sheet2").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " cost10"

Sheets("sheet3").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " bid10"

Sheets("sheet4").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " payoff10"

'computation of cost10

rand10 = Rnd() .

If rand10 < ((m - k) / (h - k)) Then

cost10 = ((rand10 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost10 = h - ((1 - rand10) * (h - k) * (h - m)) ^ 0.5

End If

'cost10 to sheet2

Sheets("sheet2").Select

Range("j3").Select

ActiveCell.Value = cost10

'computation of bid10

If cost10 < m Then

bid10b = n * k + (n - 1) * cost10

bid10c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost10 * (n - 1) * k)

bid10 = (bid10b + (bid10b ^ 2 + 2 * bid10c) ^ 0.5) / (2 * bid1a)

Else

$bid10 = (h + (2 * (n - 1) * cost10)) / (2 * bid1a)$

End If

'bid10 to sheet3

Sheets("sheet3").Select

Range("j3").Select

ActiveCell.Value = bid10

If n >= 11 Then GoTo row18:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

Else

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

End If

row18:

Sheets("sheet2").Select

Range("k4").Select

ActiveCell.FormulaR1C1 = " cost11"

Sheets("sheet3").Select

Range("k4").Select

ActiveCell.FormulaR1C1 = " bid11"

```
Sheets("sheet4").Select
```

```
Range("k4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff11"
```

```
'computation of cost11
```

```
rand11 = Rnd()
```

```
If rand11 < ((m - k) / (h - k)) Then
```

```
cost11 = ((rand11 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost11 = h - ((1 - rand11) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost11 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("k3").Select
```

```
ActiveCell.Value = cost11
```

```
'computation of bid11
```

```
If cost11 < m Then
```

```
bid11b = n * k + (n - 1) * cost11
```

```
bid11c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost11 * (n - 1) * k)
```

```
bid11 = (bid11b + (bid11b ^ 2 + 2 * bid11c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
bid11 = (h + (2 * (n - 1) * cost11)) / (2 * bid1a)
```

```
End If
```

'bid11 to sheet3

Sheets("sheet3").Select

Range("k3").Select

ActiveCell.Value = bid11

If n >= 12 Then GoTo row19:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

Else

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

```

GoTo row100:

End If

row19:

    Sheets("sheet2").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " cost12"

    Sheets("sheet3").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " bid12"

    Sheets("sheet4").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " payoff12"

'computation of cost12

rand12 = Rnd()

If rand12 < ((m - k) / (h - k)) Then

    cost12 = ((rand12 * (h - k) * (m - k)) ^ 0.5) + 1

Else

    cost12 = h - ((1 - rand12) * (h - k) * (h - m)) ^ 0.5

End If

'cost12 to sheet2

Sheets("sheet2").Select

Range("l3").Select

ActiveCell.Value = cost12

```

'computation of bid12

If cost12 < m Then

bid12b = n * k + (n - 1) * cost12

bid12c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost12 * (n - 1) * k)

bid12 = (bid12b + (bid12b ^ 2 + 2 * bid12c) ^ 0.5) / (2 * bid1a)

Else

bid12 = (h + (2 * (n - 1) * cost12)) / (2 * bid1a)

End If

'bid12 to sheet3

Sheets("sheet3").Select

Range("l3").Select

ActiveCell.Value = bid12

If n >= 13 Then GoTo row20:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5

And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <

bid11 And bid10 < bid12 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5

And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <

bid10 And bid11 < bid12 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

Else

payoff12 = bid12 - cost12


```
Sheets("sheet4").Select
Range("l3").Select
ActiveCell.Value = payoff12
GoTo row100:
```

```
End If
```

```
row20:
```

```
Sheets("sheet2").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = " cost13"
Sheets("sheet3").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = ". bid13"
Sheets("sheet4").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = " payoff13"
```

```
'computation of cost13
```

```
rand13 = Rnd()
```

```
If rand13 < ((m - k) / (h - k)) Then
```

```
cost13 = ((rand13 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost13 = h - ((1 - rand13) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost13 to sheet2
```

Sheets("sheet2").Select

Range("m3").Select

ActiveCell.Value = cost13

'computation of bid13

If cost13 < m Then

bid13b = n * k + (n - 1) * cost13

bid13c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost13 * (n - 1) * k)

bid13 = (bid13b + (bid13b ^ 2 + 2 * bid13c) ^ 0.5) / (2 * bid1a)

Else

bid13 = (h + (2 * (n - 1) * cost13)) / (2 * bid1a)

End If

'bid13 to sheet3

Sheets("sheet3").Select

Range("m3").Select

ActiveCell.Value = bid13

If n >= 14 Then GoTo row21:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And

bid1 < bid12 And bid1 < bid13 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

```

Range("a3").Select
ActiveCell.Value = payoff1
GoTo row100:
End If
If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 Then
    payoff2 = bid2 - cost2
    Sheets("sheet4").Select
    Range("b3").Select
    ActiveCell.Value = payoff2
    GoTo row100:
End If
If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 Then
    payoff3 = bid3 - cost3
    Sheets("sheet4").Select
    Range("c3").Select
    ActiveCell.Value = payoff3
    GoTo row100:
End If

```

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And

bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And

bid7 < bid12 And bid7 < bid13 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And

bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And

bid8 < bid12 And bid8 < bid13 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <
bid10 And bid11 < bid12 And bid11 < bid13 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5

And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <

bid10 And bid12 < bid11 And bid12 < bid13 Then

payoff12 = bid12 - cost12

Sheets("sheet4").Select

Range("l3").Select

ActiveCell.Value = payoff12

GoTo row100:

Else

payoff13 = bid13 - cost13

Sheets("sheet4").Select

Range("m3").Select

ActiveCell.Value = payoff13

End If

row21:

Sheets("sheet2").Select

Range("n4").Select

ActiveCell.FormulaR1C1 = " cost14"

Sheets("sheet3").Select

```
Range("n4").Select
```

```
ActiveCell.FormulaR1C1 = " bid14"
```

```
Sheets("sheet4").Select
```

```
Range("n4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff14"
```

```
'computation of cost14
```

```
rand14 = Rnd()
```

```
If rand14 < ((m - k) / (h - k)) Then
```

```
cost14 = ((rand14 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost14 = h - ((1 - rand14) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost14 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("n3").Select
```

```
ActiveCell.Value = cost14
```

```
'computation of bid14
```

```
If cost14 < m Then
```

```
bid14b = n * k + (n - 1) * cost14
```

```
bid14c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost14 * (n - 1) * k)
```

```
bid14 = (bid14b + (bid14b ^ 2 + 2 * bid14c) ^ 0.5) / (2 * bid1a)
```

```
Else
```


$bid14 = (h + (2 * (n - 1) * cost14)) / (2 * bid1a)$

End If

'bid14 to sheet3

Sheets("sheet3").Select

Range("n3").Select

ActiveCell.Value = bid14

If n >= 15 Then GoTo row22:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 And bid1 < bid13 And bid1 < bid14 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 Then

 payoff8 = bid8 - cost8

 Sheets("sheet4").Select

 Range("h3").Select

 ActiveCell.Value = payoff8

 GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 Then

 payoff9 = bid9 - cost9

 Sheets("sheet4").Select

 Range("i3").Select

 ActiveCell.Value = payoff9

 GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 Then

 payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5

And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <

bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 Then

 payoff11 = bid11 - cost11

 Sheets("sheet4").Select

 Range("k3").Select

 ActiveCell.Value = payoff11

 GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5

And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <

bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 Then

 payoff12 = bid12 - cost12

 Sheets("sheet4").Select

 Range("l3").Select

 ActiveCell.Value = payoff12

 GoTo row100:

End If

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 Then

payoff13 = bid13 - cost13

Sheets("sheet4").Select

Range("m3").Select

ActiveCell.Value = payoff13

GoTo row100:

Else

payoff14 = bid14 - cost14

Sheets("sheet4").Select

Range("n3").Select

ActiveCell.Value = payoff14

GoTo row100:

End If

row22:

Sheets("sheet2").Select

Range("o4").Select

ActiveCell.FormulaR1C1 = " cost15"

Sheets("sheet3").Select

Range("o4").Select

ActiveCell.FormulaR1C1 = " bid15"

Sheets("sheet4").Select

```
Range("o4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff15"
```

```
'computation of cost15
```

```
rand15 = Rnd()
```

```
If rand15 < ((m - k) / (h - k)) Then
```

```
cost15 = ((rand15 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost15 = h - ((1 - rand15) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost15 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("o3").Select
```

```
ActiveCell.Value = cost15
```

```
'computation of bid15
```

```
If cost15 < m Then
```

```
bid15b = n * k + (n - 1) * cost15
```

```
bid15c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost15 * (n - 1) * k)
```

```
bid15 = (bid15b + (bid15b ^ 2 + 2 * bid15c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
bid15 = (h + (2 * (n - 1) * cost15)) / (2 * bid1a)
```

```
End If
```

```
'bid15 to sheet3
```

Sheets("sheet3").Select

Range("o3").Select

ActiveCell.Value = bid15

If n >= 16 Then GoTo row23:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 And bid1 < bid13 And bid1 < bid14 And bid1 < bid15 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 And bid2 < bid15 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 And bid3 < bid15 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 And bid4 < bid15 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 And bid5 < bid15 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 And bid6 < bid15 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 And bid7 < bid15 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 And bid8 < bid15 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 And bid9 < bid15 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 And bid10 < bid15 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

```
Range("j3").Select
```

```
ActiveCell.Value = payoff10
```

```
GoTo row100:
```

```
End If
```

```
If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5  
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <  
bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 And bid11 < bid15 Then
```

```
    payoff11 = bid11 - cost11
```

```
    Sheets("sheet4").Select
```

```
    Range("k3").Select
```

```
    ActiveCell.Value = payoff11
```

```
    GoTo row100:
```

```
End If
```

```
If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5  
And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <  
bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 And bid12 < bid15 Then
```

```
    payoff12 = bid12 - cost12
```

```
    Sheets("sheet4").Select
```

```
    Range("l3").Select
```

```
    ActiveCell.Value = payoff12
```

```
    GoTo row100:
```

```
End If
```

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 And bid13 < bid15 Then

payoff13 = bid13 - cost13

Sheets("sheet4").Select

Range("m3").Select

ActiveCell.Value = payoff13

GoTo row100:

End If

If bid14 < bid1 And bid14 < bid2 And bid14 < bid3 And bid14 < bid4 And bid14 < bid5
And bid14 < bid6 And bid14 < bid7 And bid14 < bid8 And bid14 < bid9 And bid14 <
bid10 And bid14 < bid11 And bid14 < bid12 And bid14 < bid13 And bid14 < bid15 Then

payoff14 = bid14 - cost14

Sheets("sheet4").Select

Range("n3").Select

ActiveCell.Value = payoff14

GoTo row100:

Else

payoff15 = bid15 - cost15

Sheets("sheet4").Select

Range("o3").Select

ActiveCell.Value = payoff15

GoTo row100:

End If

row23:

Sheets("sheet2").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " cost16"

Sheets("sheet3").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " bid16"

Sheets("sheet4").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " payoff16"

'computation of cost16

rand16 = Rnd()

If rand16 < ((m - k) / (h - k)) Then

cost16 = ((rand16 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost16 = h - ((1 - rand16) * (h - k) * (h - m)) ^ 0.5

End If

'cost16 to sheet2

Sheets("sheet2").Select

Range("p3").Select

ActiveCell.Value = cost16

'computation of bid16

If cost16 < m Then

bid16b = n * k + (n - 1) * cost16

bid16c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost16 * (n - 1) * k)

bid16 = (bid16b + (bid16b ^ 2 + 2 * bid16c) ^ 0.5) / (2 * bid1a)

Else

bid16 = (h + (2 * (n - 1) * cost16)) / (2 * bid1a)

End If

'bid16 to sheet3

Sheets("sheet3").Select

Range("p3").Select

ActiveCell.Value = bid16

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And

bid1 < bid12 And bid1 < bid13 And bid1 < bid14 And bid1 < bid15 And bid1 < bid16

Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 And bid2 < bid15 And bid2 < bid16

Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 And bid3 < bid15 And bid3 < bid16

Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 And bid4 < bid15 And bid4 < bid16

Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 And bid5 < bid15 And bid5 < bid16

Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 And bid6 < bid15 And bid6 < bid16

Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 And bid7 < bid15 And bid7 < bid16

Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 And bid8 < bid15 And bid8 < bid16

Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 And bid9 < bid15 And bid9 < bid16

Then

 payoff9 = bid9 - cost9

 Sheets("sheet4").Select

 Range("i3").Select

 ActiveCell.Value = payoff9

 GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 And bid10 < bid15 And
bid10 < bid16 Then

 payoff10 = bid10 - cost10

 Sheets("sheet4").Select

 Range("j3").Select

 ActiveCell.Value = payoff10

 GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <

bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 And bid11 < bid15 And
bid11 < bid16 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5
And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <
bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 And bid12 < bid15 And
bid12 < bid16 Then

payoff12 = bid12 - cost12

Sheets("sheet4").Select

Range("l3").Select

ActiveCell.Value = payoff12

GoTo row100:

End If

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 And bid13 < bid15 And
bid13 < bid16 Then

payoff13 = bid13 - cost13

```

Sheets("sheet4").Select
Range("m3").Select
ActiveCell.Value = payoff13
GoTo row100:
End If
If bid14 < bid1 And bid14 < bid2 And bid14 < bid3 And bid14 < bid4 And bid14 < bid5
And bid14 < bid6 And bid14 < bid7 And bid14 < bid8 And bid14 < bid9 And bid14 <
bid10 And bid14 < bid11 And bid14 < bid12 And bid14 < bid13 And bid14 < bid15 And
bid14 < bid16 Then
    payoff14 = bid14 - cost14
    Sheets("sheet4").Select
    Range("n3").Select
    ActiveCell.Value = payoff14
    GoTo row100:
End If
If bid15 < bid1 And bid15 < bid2 And bid15 < bid3 And bid15 < bid4 And bid15 < bid5
And bid15 < bid6 And bid15 < bid7 And bid15 < bid8 And bid15 < bid9 And bid15 <
bid10 And bid15 < bid11 And bid15 < bid12 And bid15 < bid13 And bid15 < bid14 And
bid15 < bid16 Then
    payoff15 = bid15 - cost15
    Sheets("sheet4").Select
    Range("o3").Select
    ActiveCell.Value = payoff15

```

GoTo row100:

Else

payoff16 = bid16 - cost16

Sheets("sheet4").Select

Range("p3").Select

ActiveCell.Value = payoff16

GoTo row100:

End If

row100: 'collecting the results of the game in a new row

Sheets("sheet2").Select

Range("a4").Select

Selection.EntireRow.Insert

Range("a3:x3").Select

Selection.Copy

Range("a6").Select

Selection.PasteSpecial Paste:=xlValues, operation:=xlNone

Application.CutCopyMode = False

Range("a3:x3").Select

Selection.Clear

Sheets("sheet3").Select

Range("a4").Select

Selection.EntireRow.Insert

Range("a3:x3").Select

```
Selection.Copy
Range("a6").Select
Selection.PasteSpecial Paste:=xlValues, operation:=xlNone
Application.CutCopyMode = False
Range("a3:x3").Select
Selection.Clear
Sheets("sheet4").Select
Range("a4").Select
Selection.EntireRow.Insert
Range("a3:x3").Select
Selection.Copy
Range("a6").Select
Selection.PasteSpecial Paste:=xlValues, operation:=xlNone
Application.CutCopyMode = False
Range("a3:x3").Select
Selection.Clear

Next
'end signal
For I = 1 To 3 'loop 3 times.
Beep 'sound a tone.
Next
End Sub
```


APPENDIX C

RESULTS OF THE COMPUTER SIMULATIONS

A. UNIFORM COST DISTRIBUTION

1. Two-Bidder Bidding Game

Scenario 1. Both bidders used equilibrium strategy.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave. Exp. Profit | |
|------------|-----------------|---------|----------------|---------|--------------|---------|------------------|---------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.34764 | 0.34379 | 44 | 56 | 15.2961 | 19.2524 | 0.15296 | 0.19252 |
| 200 | 0.3365 | 0.32693 | 92 | 108 | 30.9579 | 35.308 | 0.15479 | 0.17654 |
| 300 | 0.33524 | 0.3373 | 144 | 156 | 48.2748 | 52.6193 | 0.16092 | 0.1754 |
| 400 | 0.33433 | 0.33199 | 202 | 198 | 67.5337 | 65.7344 | 0.16883 | 0.16434 |
| 500 | 0.33716 | 0.33486 | 239 | 261 | 80.5803 | 87.3994 | 0.16116 | 0.1748 |
| 600 | 0.33828 | 0.33584 | 289 | 311 | 97.7642 | 104.447 | 0.16294 | 0.17408 |
| 700 | 0.33779 | 0.33627 | 334 | 366 | 112.823 | 123.077 | 0.16118 | 0.17582 |
| 800 | 0.33656 | 0.3337 | 385 | 415 | 129.575 | 138.486 | 0.16197 | 0.17311 |
| 900 | 0.33654 | 0.336 | 435 | 465 | 146.394 | 156.242 | 0.16266 | 0.1736 |
| 1000 | 0.33346 | 0.33578 | 480 | 520 | 160.061 | 174.604 | 0.16006 | 0.1746 |
| 1100 | 0.32966 | 0.33415 | 521 | 579 | 171.751 | 193.471 | 0.15614 | 0.17588 |
| 1200 | 0.33086 | 0.33339 | 576 | 624 | 190.578 | 208.036 | 0.15881 | 0.17336 |
| 1300 | 0.33128 | 0.33411 | 637 | 663 | 211.025 | 221.512 | 0.16233 | 0.17039 |
| 1400 | 0.33158 | 0.33237 | 682 | 718 | 226.135 | 238.644 | 0.16153 | 0.17046 |
| 1500 | 0.33274 | 0.33028 | 731 | 769 | 243.236 | 253.983 | 0.16216 | 0.16932 |
| 1600 | 0.33395 | 0.33071 | 780 | 820 | 260.48 | 271.182 | 0.1628 | 0.16949 |
| 1700 | 0.33452 | 0.3313 | 826 | 874 | 276.315 | 289.555 | 0.16254 | 0.17033 |
| 1800 | 0.33299 | 0.33111 | 872 | 928 | 290.369 | 307.267 | 0.16132 | 0.1707 |
| 1900 | 0.33349 | 0.33235 | 928 | 972 | 309.479 | 323.041 | 0.16288 | 0.17002 |
| 2000 | 0.33326 | 0.33288 | 982 | 1018 | 327.263 | 338.872 | 0.16363 | 0.16944 |

Table 1

Scenario 2. Bidder1 underbid by 0.2 while bidder2 used equilibrium strategy.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave. Exp. Profit | |
|------------|-----------------|---------|----------------|---------|--------------|---------|------------------|---------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.08862 | 0.38873 | 77 | 23 | 6.824 | 8.94082 | 0.06824 | 0.08941 |
| 200 | 0.08651 | 0.40146 | 159 | 41 | 13.7548 | 16.46 | 0.06877 | 0.0823 |
| 300 | 0.087 | 0.39823 | 239 | 61 | 20.792 | 24.2921 | 0.06931 | 0.08097 |
| 400 | 0.08602 | 0.39419 | 318 | 82 | 27.3553 | 32.3235 | 0.06839 | 0.08081 |
| 500 | 0.08643 | 0.39843 | 396 | 104 | 34.2254 | 41.4365 | 0.06845 | 0.08287 |
| 600 | 0.08903 | 0.39825 | 468 | 132 | 41.6643 | 52.5689 | 0.06944 | 0.08761 |
| 700 | 0.08518 | 0.39929 | 553 | 147 | 47.1067 | 58.6958 | 0.0673 | 0.08385 |
| 800 | 0.08239 | 0.40089 | 637 | 163 | 52.4801 | 65.3448 | 0.0656 | 0.08168 |
| 900 | 0.08197 | 0.40151 | 724 | 176 | 59.3467 | 70.6651 | 0.06594 | 0.07852 |
| 1000 | 0.08459 | 0.4002 | 807 | 193 | 68.2648 | 77.2383 | 0.06826 | 0.07724 |
| 1100 | 0.08464 | 0.39952 | 885 | 215 | 74.9079 | 85.8959 | 0.0681 | 0.07809 |
| 1200 | 0.08703 | 0.3999 | 971 | 229 | 84.5068 | 91.5763 | 0.07042 | 0.07631 |
| 1300 | 0.08366 | 0.39966 | 1052 | 248 | 88.0134 | 99.1149 | 0.0677 | 0.07624 |
| 1400 | 0.08614 | 0.40073 | 1139 | 261 | 98.114 | 104.59 | 0.07008 | 0.07471 |
| 1500 | 0.0856 | 0.40128 | 1221 | 279 | 104.514 | 111.956 | 0.06968 | 0.07464 |

Table2

Scenario 3. Both bidders did not use equilibrium strategy and underbid by 0.1.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave. Exp. Profit | |
|------------|-----------------|---------|----------------|---------|--------------|---------|------------------|---------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.2245 | 0.2025 | 50 | 50 | 11.227 | 10.126 | 0.1123 | 0.1013 |
| 200 | 0.2272 | 0.1958 | 98 | 102 | 22.263 | 19.971 | 0.1113 | 0.0999 |
| 300 | 0.2296 | 0.2018 | 148 | 152 | 33.981 | 30.68 | 0.1133 | 0.1023 |
| 400 | 0.2295 | 0.2168 | 190 | 210 | 43.604 | 45.53 | 0.109 | 0.1138 |
| 500 | 0.2319 | 0.2194 | 244 | 256 | 56.58 | 56.175 | 0.1132 | 0.1124 |
| 600 | 0.226 | 0.2202 | 299 | 301 | 67.579 | 66.281 | 0.1126 | 0.1105 |
| 700 | 0.2274 | 0.2215 | 350 | 350 | 79.593 | 77.532 | 0.1137 | 0.1108 |
| 800 | 0.2276 | 0.223 | 405 | 395 | 92.184 | 88.093 | 0.1152 | 0.1101 |
| 900 | 0.2297 | 0.2274 | 452 | 448 | 103.84 | 101.85 | 0.1154 | 0.1132 |
| 1000 | 0.2301 | 0.2277 | 501 | 499 | 115.28 | 113.64 | 0.1153 | 0.1136 |
| 1100 | 0.231 | 0.2308 | 546 | 554 | 126.12 | 127.86 | 0.1147 | 0.1162 |
| 1200 | 0.2299 | 0.2309 | 590 | 610 | 135.61 | 140.84 | 0.113 | 0.1174 |
| 1300 | 0.2318 | 0.233 | 640 | 660 | 148.35 | 153.79 | 0.1141 | 0.1183 |
| 1400 | 0.2302 | 0.2349 | 695 | 705 | 159.96 | 165.63 | 0.1143 | 0.1183 |
| 1500 | 0.2306 | 0.2316 | 742 | 758 | 171.08 | 175.54 | 0.1141 | 0.117 |

Table 3

2. Three-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Ave. Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|----------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.2498 | 0.251 | 0.2405 | 37 | 23 | 40 | 9.2416 | 5.7734 | 9.6207 | 0.0924 | 0.0577 | 0.0962 |
| 200 | 0.251 | 0.2509 | 0.2536 | 70 | 55 | 75 | 17.568 | 13.798 | 19.018 | 0.0878 | 0.069 | 0.0951 |
| 300 | 0.2532 | 0.2517 | 0.2537 | 104 | 92 | 104 | 26.33 | 23.156 | 26.382 | 0.0878 | 0.0772 | 0.0879 |
| 400 | 0.2517 | 0.2509 | 0.2504 | 135 | 125 | 140 | 33.98 | 31.361 | 35.06 | 0.0849 | 0.0784 | 0.0876 |
| 500 | 0.2549 | 0.254 | 0.2459 | 164 | 156 | 180 | 41.81 | 39.63 | 44.266 | 0.0836 | 0.0793 | 0.0885 |
| 600 | 0.2558 | 0.2517 | 0.2465 | 196 | 185 | 219 | 50.13 | 46.572 | 53.98 | 0.0836 | 0.0776 | 0.09 |
| 700 | 0.2554 | 0.2488 | 0.2476 | 226 | 224 | 250 | 57.727 | 55.721 | 61.905 | 0.0825 | 0.0796 | 0.0884 |
| 800 | 0.2537 | 0.2488 | 0.2472 | 265 | 253 | 282 | 67.22 | 62.946 | 69.719 | 0.084 | 0.0787 | 0.0871 |
| 900 | 0.2539 | 0.249 | 0.2465 | 306 | 288 | 306 | 77.703 | 71.719 | 75.414 | 0.0863 | 0.0797 | 0.0838 |
| 1000 | 0.2533 | 0.249 | 0.2483 | 338 | 321 | 341 | 85.628 | 79.944 | 84.678 | 0.0856 | 0.0799 | 0.0847 |
| 1100 | 0.2541 | 0.2481 | 0.2492 | 372 | 355 | 373 | 94.537 | 88.082 | 92.951 | 0.0859 | 0.0801 | 0.0845 |
| 1200 | 0.2541 | 0.2485 | 0.2485 | 412 | 383 | 405 | 104.68 | 95.189 | 100.66 | 0.0872 | 0.0793 | 0.0839 |
| 1300 | 0.252 | 0.2492 | 0.2498 | 451 | 413 | 436 | 113.67 | 102.91 | 108.93 | 0.0874 | 0.0792 | 0.0838 |
| 1400 | 0.2519 | 0.2501 | 0.2494 | 490 | 449 | 461 | 123.44 | 112.32 | 114.97 | 0.0882 | 0.0802 | 0.0821 |
| 1500 | 0.2513 | 0.2498 | 0.2499 | 524 | 486 | 490 | 131.7 | 121.42 | 122.45 | 0.0878 | 0.0809 | 0.0816 |
| 1600 | 0.2514 | 0.2482 | 0.2507 | 557 | 519 | 524 | 140.05 | 128.81 | 131.39 | 0.0875 | 0.0805 | 0.0821 |
| 1700 | 0.2515 | 0.2487 | 0.2501 | 582 | 562 | 556 | 146.39 | 139.76 | 139.06 | 0.0861 | 0.0822 | 0.0818 |
| 1800 | 0.251 | 0.2493 | 0.251 | 612 | 597 | 591 | 153.6 | 148.85 | 148.36 | 0.0853 | 0.0827 | 0.0824 |
| 1900 | 0.2501 | 0.25 | 0.2514 | 643 | 623 | 634 | 160.82 | 155.77 | 159.37 | 0.0846 | 0.082 | 0.0839 |
| 2000 | 0.25 | 0.2494 | 0.2513 | 683 | 652 | 665 | 170.72 | 162.6 | 167.13 | 0.0854 | 0.0813 | 0.0836 |

Table 4

Scenario 2. Bidder1 underbid by 0.1 while others used equilibrium strategy.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Average Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|-------------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.1293 | 0.2497 | 0.2543 | 42 | 26 | 32 | 5.4323 | 6.4931 | 8.1382 | 0.0543 | 0.0649 | 0.0814 |
| 200 | 0.1261 | 0.249 | 0.2563 | 89 | 57 | 54 | 11.222 | 14.193 | 13.839 | 0.0561 | 0.071 | 0.0692 |
| 300 | 0.1293 | 0.2494 | 0.2545 | 128 | 90 | 82 | 16.546 | 22.446 | 20.867 | 0.0552 | 0.0748 | 0.0696 |
| 400 | 0.1319 | 0.2519 | 0.2542 | 172 | 116 | 112 | 22.694 | 29.216 | 28.47 | 0.0567 | 0.073 | 0.0712 |
| 500 | 0.1311 | 0.2512 | 0.2537 | 213 | 149 | 138 | 27.914 | 37.433 | 35.004 | 0.0558 | 0.0749 | 0.07 |
| 600 | 0.1318 | 0.2512 | 0.2553 | 260 | 175 | 165 | 34.275 | 43.959 | 42.12 | 0.0571 | 0.0733 | 0.0702 |
| 700 | 0.1307 | 0.2507 | 0.2584 | 302 | 201 | 197 | 39.477 | 50.383 | 50.914 | 0.0564 | 0.072 | 0.0727 |
| 800 | 0.1301 | 0.2501 | 0.2579 | 352 | 228 | 220 | 45.786 | 57.023 | 56.729 | 0.0572 | 0.0713 | 0.0709 |
| 900 | 0.1307 | 0.254 | 0.2586 | 402 | 256 | 242 | 52.557 | 65.016 | 62.585 | 0.0584 | 0.0722 | 0.0695 |
| 1000 | 0.1292 | 0.2565 | 0.2588 | 439 | 285 | 276 | 56.707 | 73.111 | 71.431 | 0.0567 | 0.0731 | 0.0714 |
| 1100 | 0.1302 | 0.2579 | 0.2598 | 492 | 307 | 301 | 64.076 | 79.169 | 78.189 | 0.0583 | 0.072 | 0.0711 |
| 1200 | 0.1278 | 0.2572 | 0.2586 | 544 | 331 | 325 | 69.55 | 85.141 | 84.057 | 0.058 | 0.071 | 0.07 |
| 1300 | 0.1294 | 0.2568 | 0.2589 | 583 | 363 | 354 | 75.466 | 93.212 | 91.633 | 0.0581 | 0.0717 | 0.0705 |
| 1400 | 0.1296 | 0.2559 | 0.2584 | 632 | 386 | 382 | 81.933 | 98.773 | 98.723 | 0.0585 | 0.0706 | 0.0705 |
| 1500 | 0.1302 | 0.2578 | 0.2595 | 680 | 412 | 408 | 88.502 | 106.2 | 105.87 | 0.059 | 0.0708 | 0.0706 |

Table 5

Scenario 3. Bidders "1" and "2" underbid by 0.1 and 0.15 respectively, while others used equilibrium strategy.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Average Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|-------------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.1208 | 0.0815 | 0.2891 | 32 | 51 | 17 | 3.8649 | 4.1544 | 4.9149 | 0.0386 | 0.0415 | 0.0491 |
| 200 | 0.141 | 0.0976 | 0.2812 | 72 | 92 | 36 | 10.151 | 8.9808 | 10.124 | 0.0508 | 0.0449 | 0.0506 |
| 300 | 0.1425 | 0.0953 | 0.2715 | 112 | 134 | 54 | 15.962 | 12.774 | 14.659 | 0.0532 | 0.0426 | 0.0489 |
| 400 | 0.1435 | 0.0925 | 0.2671 | 149 | 185 | 66 | 21.381 | 17.106 | 17.63 | 0.0535 | 0.0428 | 0.0441 |
| 500 | 0.1457 | 0.092 | 0.2652 | 184 | 232 | 84 | 26.8 | 21.35 | 22.273 | 0.0536 | 0.0427 | 0.0445 |
| 600 | 0.1463 | 0.0912 | 0.2656 | 218 | 284 | 98 | 31.892 | 25.888 | 26.029 | 0.0532 | 0.0431 | 0.0434 |
| 700 | 0.1438 | 0.0904 | 0.2689 | 260 | 319 | 121 | 37.38 | 28.827 | 32.535 | 0.0534 | 0.0412 | 0.0465 |
| 800 | 0.1445 | 0.0894 | 0.2654 | 295 | 361 | 144 | 42.628 | 32.269 | 38.212 | 0.0533 | 0.0403 | 0.0478 |
| 900 | 0.1437 | 0.0898 | 0.2643 | 333 | 404 | 163 | 47.861 | 36.282 | 43.083 | 0.0532 | 0.0403 | 0.0479 |
| 1000 | 0.144 | 0.0893 | 0.2634 | 369 | 457 | 174 | 53.121 | 40.823 | 45.84 | 0.0531 | 0.0408 | 0.0458 |
| 1100 | 0.1447 | 0.0882 | 0.2643 | 402 | 510 | 188 | 58.189 | 44.973 | 49.68 | 0.0529 | 0.0409 | 0.0452 |
| 1200 | 0.1456 | 0.087 | 0.2656 | 441 | 555 | 204 | 64.193 | 48.277 | 54.175 | 0.0535 | 0.0402 | 0.0451 |
| 1300 | 0.1456 | 0.0875 | 0.2641 | 476 | 600 | 224 | 69.319 | 52.518 | 59.157 | 0.0533 | 0.0404 | 0.0455 |
| 1400 | 0.1444 | 0.0866 | 0.2637 | 508 | 649 | 243 | 73.347 | 56.211 | 64.072 | 0.0524 | 0.0402 | 0.0458 |
| 1500 | 0.1441 | 0.0857 | 0.2645 | 541 | 703 | 256 | 77.961 | 60.255 | 67.722 | 0.052 | 0.0402 | 0.0451 |

Table 6

3. Five-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.16454 | 0.16666 | 0.16219 | 0.16679 | 0.14863 | 23 | 24 | 18 | 20 | 15 |
| 200 | 0.16428 | 0.16866 | 0.16528 | 0.16976 | 0.16478 | 42 | 44 | 29 | 44 | 41 |
| 300 | 0.16326 | 0.16577 | 0.16625 | 0.16859 | 0.16719 | 60 | 71 | 48 | 69 | 52 |
| 400 | 0.16242 | 0.16581 | 0.16628 | 0.17011 | 0.16849 | 78 | 95 | 63 | 92 | 72 |
| 500 | 0.16003 | 0.16653 | 0.16856 | 0.17027 | 0.16734 | 94 | 114 | 91 | 108 | 93 |
| 600 | 0.16297 | 0.16639 | 0.16682 | 0.16953 | 0.16519 | 116 | 134 | 109 | 128 | 113 |
| 700 | 0.16416 | 0.16618 | 0.16642 | 0.16798 | 0.16598 | 131 | 157 | 123 | 156 | 133 |
| 800 | 0.16533 | 0.16724 | 0.16768 | 0.16829 | 0.16704 | 142 | 176 | 143 | 179 | 160 |
| 900 | 0.16544 | 0.16813 | 0.16729 | 0.16763 | 0.16794 | 155 | 202 | 167 | 198 | 178 |
| 1000 | 0.16481 | 0.16728 | 0.16799 | 0.16717 | 0.16784 | 181 | 220 | 186 | 219 | 194 |
| 1100 | 0.16429 | 0.16714 | 0.1677 | 0.16772 | 0.16637 | 202 | 237 | 205 | 239 | 217 |
| 1200 | 0.16452 | 0.16698 | 0.1681 | 0.16658 | 0.16665 | 222 | 254 | 229 | 259 | 236 |
| 1300 | 0.16376 | 0.16536 | 0.16885 | 0.16611 | 0.16682 | 239 | 277 | 249 | 277 | 258 |
| 1400 | 0.16377 | 0.16549 | 0.1686 | 0.16555 | 0.16746 | 257 | 296 | 275 | 290 | 282 |
| 1500 | 0.16384 | 0.16578 | 0.16823 | 0.16564 | 0.16781 | 277 | 313 | 294 | 310 | 306 |

Table 7

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 3.78435 | 3.99975 | 2.91939 | 3.33584 | 2.22947 | 0.03784 | 0.04 | 0.02919 | 0.03336 | 0.02229 |
| 200 | 6.8999 | 7.42123 | 4.79326 | 7.46951 | 6.75599 | 0.0345 | 0.03711 | 0.02397 | 0.03735 | 0.03378 |
| 300 | 9.79589 | 11.7695 | 7.98008 | 11.633 | 8.69383 | 0.03265 | 0.03923 | 0.0266 | 0.03878 | 0.02898 |
| 400 | 12.6686 | 15.7516 | 10.4759 | 15.6505 | 12.1314 | 0.03167 | 0.03938 | 0.02619 | 0.03913 | 0.03033 |
| 500 | 15.0424 | 18.9843 | 15.3387 | 18.3891 | 15.5623 | 0.03008 | 0.03797 | 0.03068 | 0.03678 | 0.03112 |
| 600 | 18.9044 | 22.2961 | 18.1839 | 21.6998 | 18.6661 | 0.03151 | 0.03716 | 0.03031 | 0.03617 | 0.03111 |
| 700 | 21.5047 | 26.0895 | 20.4691 | 26.2051 | 22.0758 | 0.03072 | 0.03727 | 0.02924 | 0.03744 | 0.03154 |
| 800 | 23.4768 | 29.4339 | 23.978 | 30.1234 | 26.7261 | 0.02935 | 0.03679 | 0.02997 | 0.03765 | 0.03341 |
| 900 | 25.6432 | 33.9621 | 27.9375 | 33.1909 | 29.8928 | 0.02849 | 0.03774 | 0.03104 | 0.03688 | 0.03321 |
| 1000 | 29.8303 | 36.8021 | 31.2462 | 36.6109 | 32.5607 | 0.02983 | 0.0368 | 0.03125 | 0.03661 | 0.03256 |
| 1100 | 33.1857 | 39.6125 | 34.379 | 40.0854 | 36.1022 | 0.03017 | 0.03601 | 0.03125 | 0.03644 | 0.03282 |
| 1200 | 36.523 | 42.4118 | 38.4945 | 43.1446 | 39.3303 | 0.03044 | 0.03534 | 0.03208 | 0.03595 | 0.03278 |
| 1300 | 39.1395 | 45.8051 | 42.0424 | 46.0134 | 43.039 | 0.03011 | 0.03523 | 0.03234 | 0.03539 | 0.03311 |
| 1400 | 42.0897 | 48.9861 | 46.3654 | 48.009 | 47.2229 | 0.03006 | 0.03499 | 0.03312 | 0.03429 | 0.03373 |
| 1500 | 45.383 | 51.889 | 49.4582 | 51.3496 | 51.3498 | 0.03026 | 0.03459 | 0.03297 | 0.03423 | 0.03423 |

Table 8

Scenario 2. Bidder1 underbid by 0.09 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.06662 | 0.17163 | 0.17003 | 0.17628 | 0.16915 | 28 | 24 | 9 | 24 | 15 |
| 200 | 0.06989 | 0.17244 | 0.16765 | 0.17623 | 0.17016 | 60 | 43 | 35 | 38 | 24 |
| 300 | 0.07082 | 0.17034 | 0.16696 | 0.17415 | 0.16936 | 103 | 56 | 47 | 54 | 40 |
| 400 | 0.07035 | 0.16842 | 0.16934 | 0.17014 | 0.16941 | 131 | 68 | 62 | 81 | 58 |
| 500 | 0.07073 | 0.16872 | 0.16992 | 0.16733 | 0.16788 | 159 | 93 | 79 | 100 | 69 |
| 600 | 0.07168 | 0.16837 | 0.16862 | 0.16918 | 0.16675 | 194 | 107 | 98 | 116 | 85 |
| 700 | 0.07025 | 0.16775 | 0.16787 | 0.16809 | 0.16647 | 226 | 122 | 112 | 137 | 103 |
| 800 | 0.07072 | 0.16788 | 0.16867 | 0.16895 | 0.16595 | 249 | 141 | 132 | 154 | 124 |
| 900 | 0.07063 | 0.16812 | 0.1685 | 0.16932 | 0.16621 | 281 | 161 | 146 | 171 | 141 |
| 1000 | 0.07025 | 0.16908 | 0.16885 | 0.16975 | 0.16656 | 309 | 181 | 162 | 192 | 156 |
| 1100 | 0.07056 | 0.16932 | 0.16916 | 0.16919 | 0.16647 | 333 | 201 | 175 | 216 | 175 |
| 1200 | 0.07043 | 0.16962 | 0.16974 | 0.16872 | 0.16617 | 359 | 219 | 189 | 242 | 191 |
| 1300 | 0.07031 | 0.16934 | 0.1697 | 0.16821 | 0.16634 | 390 | 235 | 206 | 256 | 213 |
| 1400 | 0.07078 | 0.16962 | 0.16917 | 0.16864 | 0.16708 | 416 | 251 | 232 | 275 | 226 |
| 1500 | 0.07034 | 0.17009 | 0.16929 | 0.16867 | 0.16774 | 446 | 272 | 249 | 292 | 241 |

Table 9

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.86533 | 4.11905 | 1.5303 | 4.23083 | 2.53728 | 0.01865 | 0.04119 | 0.0153 | 0.04231 | 0.02537 |
| 200 | 4.19321 | 7.415 | 5.86764 | 6.69662 | 4.08382 | 0.02097 | 0.03708 | 0.02934 | 0.03348 | 0.02042 |
| 300 | 7.29441 | 9.53898 | 7.84725 | 9.4039 | 6.77447 | 0.02431 | 0.0318 | 0.02616 | 0.03135 | 0.02258 |
| 400 | 9.21626 | 11.4524 | 10.4992 | 13.7811 | 9.82578 | 0.02304 | 0.02863 | 0.02625 | 0.03445 | 0.02456 |
| 500 | 11.2464 | 15.6913 | 13.4236 | 16.7329 | 11.5839 | 0.02249 | 0.03138 | 0.02685 | 0.03347 | 0.02317 |
| 600 | 13.9068 | 18.0155 | 16.525 | 19.6243 | 14.1736 | 0.02318 | 0.03003 | 0.02754 | 0.03271 | 0.02362 |
| 700 | 15.8768 | 20.4652 | 18.8009 | 23.029 | 17.1465 | 0.02268 | 0.02924 | 0.02686 | 0.0329 | 0.02449 |
| 800 | 17.6103 | 23.6715 | 22.2644 | 26.0181 | 20.5779 | 0.02201 | 0.02959 | 0.02783 | 0.03252 | 0.02572 |
| 900 | 19.8471 | 27.0666 | 24.6006 | 28.9534 | 23.4349 | 0.02205 | 0.03007 | 0.02733 | 0.03217 | 0.02604 |
| 1000 | 21.7079 | 30.6035 | 27.3544 | 32.5928 | 25.9828 | 0.02171 | 0.0306 | 0.02735 | 0.03259 | 0.02598 |
| 1100 | 23.4981 | 34.0336 | 29.6034 | 36.5454 | 29.1322 | 0.02136 | 0.03094 | 0.02691 | 0.03322 | 0.02648 |
| 1200 | 25.2851 | 37.1476 | 32.0818 | 40.8302 | 31.7392 | 0.02107 | 0.03096 | 0.02673 | 0.03403 | 0.02645 |
| 1300 | 27.4214 | 39.7955 | 34.9583 | 43.0621 | 35.43 | 0.02109 | 0.03061 | 0.02689 | 0.03312 | 0.02725 |
| 1400 | 29.4465 | 42.5757 | 39.2472 | 46.3758 | 37.7599 | 0.02103 | 0.03041 | 0.02803 | 0.03313 | 0.02697 |
| 1500 | 31.3722 | 46.2656 | 42.152 | 49.2516 | 40.4257 | 0.02091 | 0.03084 | 0.0281 | 0.03283 | 0.02695 |

Table 10

Scenario 3. Bidders “1” and “2” underbid by 0.02 while others used

equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.0401 | 0.0465 | 0.1724 | 0.1725 | 0.1584 | 33 | 31 | 14 | 13 | 9 |
| 200 | 0.0379 | 0.0416 | 0.1713 | 0.1719 | 0.1691 | 69 | 63 | 27 | 26 | 15 |
| 300 | 0.0392 | 0.0424 | 0.1743 | 0.1675 | 0.1695 | 103 | 93 | 45 | 38 | 21 |
| 400 | 0.0361 | 0.0409 | 0.1715 | 0.167 | 0.1704 | 133 | 123 | 60 | 50 | 34 |
| 500 | 0.0347 | 0.0412 | 0.1721 | 0.168 | 0.1668 | 164 | 147 | 76 | 66 | 47 |
| 600 | 0.0359 | 0.0405 | 0.1727 | 0.1691 | 0.1688 | 194 | 174 | 89 | 84 | 59 |
| 700 | 0.0359 | 0.0413 | 0.1727 | 0.1695 | 0.1675 | 222 | 198 | 102 | 100 | 78 |
| 800 | 0.0362 | 0.0402 | 0.1719 | 0.1677 | 0.1684 | 254 | 224 | 116 | 112 | 94 |
| 900 | 0.0377 | 0.0407 | 0.1726 | 0.1668 | 0.1693 | 280 | 259 | 132 | 126 | 103 |
| 1000 | 0.0382 | 0.0409 | 0.1735 | 0.1659 | 0.1692 | 312 | 282 | 149 | 142 | 115 |
| 1100 | 0.0386 | 0.0414 | 0.1737 | 0.1667 | 0.1696 | 339 | 315 | 159 | 155 | 132 |
| 1200 | 0.0382 | 0.0405 | 0.1734 | 0.1668 | 0.1695 | 365 | 348 | 175 | 171 | 141 |
| 1300 | 0.0377 | 0.0405 | 0.1739 | 0.1668 | 0.1695 | 390 | 378 | 186 | 191 | 155 |
| 1400 | 0.0374 | 0.0408 | 0.174 | 0.1672 | 0.1705 | 416 | 404 | 195 | 206 | 179 |
| 1500 | 0.0378 | 0.0407 | 0.1739 | 0.1676 | 0.1704 | 449 | 436 | 207 | 219 | 189 |

Table 11

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.3238 | 1.4415 | 2.4138 | 2.2427 | 1.426 | 0.0132 | 0.0144 | 0.0241 | 0.0224 | 0.0143 |
| 200 | 2.617 | 2.6196 | 4.6238 | 4.4683 | 2.5366 | 0.0131 | 0.0131 | 0.0231 | 0.0223 | 0.0127 |
| 300 | 4.0397 | 3.943 | 7.8443 | 6.3651 | 3.5598 | 0.0135 | 0.0131 | 0.0261 | 0.0212 | 0.0119 |
| 400 | 4.7984 | 5.0304 | 10.289 | 8.3486 | 5.7927 | 0.012 | 0.0126 | 0.0257 | 0.0209 | 0.0145 |
| 500 | 5.6849 | 6.0596 | 13.08 | 11.086 | 7.8381 | 0.0114 | 0.0121 | 0.0262 | 0.0222 | 0.0157 |
| 600 | 6.9729 | 7.0388 | 15.372 | 14.205 | 9.96 | 0.0116 | 0.0117 | 0.0256 | 0.0237 | 0.0166 |
| 700 | 7.9641 | 8.1723 | 17.619 | 16.945 | 13.062 | 0.0114 | 0.0117 | 0.0252 | 0.0242 | 0.0187 |
| 800 | 9.1951 | 8.9943 | 19.939 | 18.781 | 15.834 | 0.0115 | 0.0112 | 0.0249 | 0.0235 | 0.0198 |
| 900 | 10.543 | 10.552 | 22.78 | 21.021 | 17.438 | 0.0117 | 0.0117 | 0.0253 | 0.0234 | 0.0194 |
| 1000 | 11.928 | 11.535 | 25.846 | 23.552 | 19.452 | 0.0119 | 0.0115 | 0.0258 | 0.0236 | 0.0195 |
| 1100 | 13.089 | 13.04 | 27.614 | 25.843 | 22.39 | 0.0119 | 0.0119 | 0.0251 | 0.0235 | 0.0204 |
| 1200 | 13.945 | 14.103 | 30.352 | 28.516 | 23.898 | 0.0116 | 0.0118 | 0.0253 | 0.0238 | 0.0199 |
| 1300 | 14.705 | 15.319 | 32.348 | 31.851 | 26.28 | 0.0113 | 0.0118 | 0.0249 | 0.0245 | 0.0202 |
| 1400 | 15.56 | 16.483 | 33.92 | 34.439 | 30.528 | 0.0111 | 0.0118 | 0.0242 | 0.0246 | 0.0218 |
| 1500 | 16.992 | 17.737 | 36.004 | 36.711 | 32.21 | 0.0113 | 0.0118 | 0.024 | 0.0245 | 0.0215 |

Table 12

Scenario 4. Bidders “1” and “2” underbid by 0.12 and 0.1 while others used

equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.03643 | 0.05744 | 0.16193 | 0.17741 | 0.17095 | 27 | 26 | 12 | 15 | 20 |
| 200 | 0.03835 | 0.05811 | 0.15915 | 0.17442 | 0.17133 | 67 | 52 | 23 | 28 | 30 |
| 300 | 0.03658 | 0.05538 | 0.16932 | 0.17086 | 0.16876 | 100 | 73 | 40 | 40 | 47 |
| 400 | 0.03551 | 0.05884 | 0.16984 | 0.17093 | 0.17146 | 126 | 102 | 55 | 56 | 61 |
| 500 | 0.037 | 0.05921 | 0.17139 | 0.16949 | 0.16925 | 162 | 126 | 71 | 66 | 75 |
| 600 | 0.03765 | 0.05914 | 0.16879 | 0.17093 | 0.16926 | 192 | 151 | 86 | 82 | 89 |
| 700 | 0.03853 | 0.05805 | 0.16947 | 0.1708 | 0.1674 | 222 | 175 | 96 | 100 | 107 |
| 800 | 0.03876 | 0.05844 | 0.16828 | 0.17028 | 0.16738 | 254 | 206 | 108 | 112 | 120 |
| 900 | 0.03837 | 0.05973 | 0.1689 | 0.17135 | 0.16791 | 286 | 233 | 123 | 125 | 133 |
| 1000 | 0.03819 | 0.06077 | 0.16959 | 0.17109 | 0.16783 | 312 | 263 | 143 | 137 | 145 |
| 1100 | 0.03838 | 0.06081 | 0.16839 | 0.17055 | 0.16755 | 336 | 293 | 158 | 156 | 157 |
| 1200 | 0.03836 | 0.06074 | 0.16743 | 0.16996 | 0.16707 | 364 | 329 | 170 | 166 | 171 |
| 1300 | 0.03828 | 0.06148 | 0.16815 | 0.17076 | 0.1684 | 389 | 360 | 183 | 181 | 187 |
| 1400 | 0.03794 | 0.06174 | 0.16842 | 0.17058 | 0.1691 | 420 | 388 | 198 | 193 | 201 |
| 1500 | 0.03781 | 0.06149 | 0.16858 | 0.16997 | 0.16899 | 448 | 414 | 211 | 209 | 218 |

Table 13

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.98357 | 1.49353 | 1.94317 | 2.66117 | 3.41902 | 0.00984 | 0.01494 | 0.01943 | 0.02661 | 0.03419 |
| 200 | 2.56972 | 3.02163 | 3.66044 | 4.88385 | 5.13989 | 0.01285 | 0.01511 | 0.0183 | 0.02442 | 0.0257 |
| 300 | 3.65787 | 4.04251 | 6.77262 | 6.83429 | 7.93155 | 0.01219 | 0.01348 | 0.02258 | 0.02278 | 0.02644 |
| 400 | 4.47372 | 6.00132 | 9.34147 | 9.57231 | 10.4589 | 0.01118 | 0.015 | 0.02335 | 0.02393 | 0.02615 |
| 500 | 5.99464 | 7.46092 | 12.1687 | 11.1864 | 12.6938 | 0.01199 | 0.01492 | 0.02434 | 0.02237 | 0.02539 |
| 600 | 7.22792 | 8.93027 | 14.5162 | 14.0162 | 15.0637 | 0.01205 | 0.01488 | 0.02419 | 0.02336 | 0.02511 |
| 700 | 8.55258 | 10.1589 | 16.2691 | 17.0796 | 17.9118 | 0.01222 | 0.01451 | 0.02324 | 0.0244 | 0.02559 |
| 800 | 9.84624 | 12.0397 | 18.1744 | 19.0711 | 20.0857 | 0.01231 | 0.01505 | 0.02272 | 0.02384 | 0.02511 |
| 900 | 10.9743 | 13.9162 | 20.7746 | 21.4188 | 22.3324 | 0.01219 | 0.01546 | 0.02308 | 0.0238 | 0.02481 |
| 1000 | 11.9163 | 15.9819 | 24.251 | 23.439 | 24.335 | 0.01192 | 0.01598 | 0.02425 | 0.02344 | 0.02434 |
| 1100 | 12.894 | 17.8186 | 26.6053 | 26.6054 | 26.3049 | 0.01172 | 0.0162 | 0.02419 | 0.02419 | 0.02391 |
| 1200 | 13.964 | 19.9833 | 28.4625 | 28.2132 | 28.5691 | 0.01164 | 0.01665 | 0.02372 | 0.02351 | 0.02381 |
| 1300 | 14.8911 | 22.1316 | 30.7718 | 30.9067 | 31.4907 | 0.01145 | 0.01702 | 0.02367 | 0.02377 | 0.02422 |
| 1400 | 15.9334 | 23.9558 | 33.3479 | 32.9222 | 33.9893 | 0.01138 | 0.01711 | 0.02382 | 0.02352 | 0.02428 |
| 1500 | 16.9386 | 25.4564 | 35.57 | 35.5231 | 36.8405 | 0.01129 | 0.01697 | 0.02371 | 0.02368 | 0.02456 |

Table 14

4. Ten-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.09112 | 0.09513 | 0.09277 | 0.08994 | 0.09006 | 0.09359 | 0.09605 | 0.08883 | 0.09575 | 0.09416 |
| 200 | 0.09293 | 0.09383 | 0.09253 | 0.08893 | 0.08917 | 0.09301 | 0.09231 | 0.0906 | 0.09409 | 0.09204 |
| 300 | 0.09073 | 0.09231 | 0.09183 | 0.09026 | 0.09108 | 0.09319 | 0.09291 | 0.08972 | 0.09291 | 0.09213 |
| 400 | 0.09113 | 0.09248 | 0.09146 | 0.0912 | 0.09054 | 0.09162 | 0.09195 | 0.09003 | 0.09274 | 0.09095 |
| 500 | 0.09044 | 0.0911 | 0.09131 | 0.09097 | 0.09059 | 0.09203 | 0.09134 | 0.09072 | 0.09238 | 0.09069 |
| 600 | 0.09031 | 0.091 | 0.09096 | 0.09122 | 0.09134 | 0.09126 | 0.09172 | 0.09106 | 0.09246 | 0.09055 |
| 700 | 0.09064 | 0.09087 | 0.09094 | 0.09137 | 0.09112 | 0.09169 | 0.09185 | 0.09104 | 0.09196 | 0.08951 |
| 800 | 0.09087 | 0.08979 | 0.09078 | 0.09138 | 0.0908 | 0.09167 | 0.09219 | 0.09123 | 0.09209 | 0.08976 |
| 900 | 0.09029 | 0.09 | 0.09076 | 0.09113 | 0.09112 | 0.09112 | 0.09196 | 0.09171 | 0.09178 | 0.08987 |
| 1000 | 0.09053 | 0.09067 | 0.09086 | 0.09085 | 0.09075 | 0.09142 | 0.09207 | 0.09149 | 0.0918 | 0.09006 |
| 1100 | 0.09063 | 0.09107 | 0.091 | 0.091 | 0.09098 | 0.091 | 0.09228 | 0.09176 | 0.09185 | 0.09006 |
| 1200 | 0.0908 | 0.0906 | 0.09101 | 0.09054 | 0.09089 | 0.09087 | 0.09198 | 0.09151 | 0.0916 | 0.0904 |
| 1300 | 0.09109 | 0.09057 | 0.09092 | 0.09065 | 0.09045 | 0.09077 | 0.092 | 0.09169 | 0.0915 | 0.09062 |
| 1400 | 0.09062 | 0.09083 | 0.09073 | 0.09086 | 0.09036 | 0.09075 | 0.09192 | 0.09169 | 0.09138 | 0.09093 |
| 1500 | 0.09072 | 0.0909 | 0.09068 | 0.0907 | 0.09014 | 0.09103 | 0.09188 | 0.09135 | 0.09148 | 0.09093 |

Table 15

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 4 | 7 | 15 | 13 | 10 | 12 | 12 | 9 | 9 | 9 |
| 200 | 20 | 18 | 33 | 18 | 15 | 21 | 23 | 20 | 18 | 14 |
| 300 | 26 | 28 | 44 | 29 | 23 | 31 | 38 | 25 | 32 | 24 |
| 400 | 39 | 36 | 50 | 35 | 31 | 46 | 50 | 40 | 41 | 32 |
| 500 | 49 | 46 | 53 | 45 | 36 | 58 | 62 | 59 | 53 | 39 |
| 600 | 58 | 52 | 64 | 57 | 47 | 68 | 72 | 67 | 63 | 52 |
| 700 | 65 | 60 | 76 | 71 | 53 | 78 | 80 | 78 | 77 | 62 |
| 800 | 74 | 68 | 90 | 79 | 59 | 88 | 90 | 89 | 89 | 74 |
| 900 | 83 | 75 | 104 | 92 | 66 | 100 | 102 | 99 | 99 | 80 |
| 1000 | 100 | 86 | 112 | 98 | 79 | 111 | 110 | 107 | 105 | 92 |
| 1100 | 108 | 100 | 117 | 106 | 90 | 124 | 115 | 121 | 118 | 101 |
| 1200 | 118 | 112 | 123 | 124 | 105 | 132 | 126 | 125 | 126 | 109 |
| 1300 | 128 | 121 | 134 | 130 | 117 | 145 | 138 | 136 | 135 | 116 |
| 1400 | 138 | 136 | 143 | 141 | 127 | 151 | 147 | 144 | 146 | 127 |
| 1500 | 146 | 146 | 151 | 147 | 140 | 162 | 157 | 156 | 156 | 139 |

Table 16

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.36447 | 0.66594 | 1.39152 | 1.16916 | 0.90062 | 1.12302 | 1.15263 | 0.79944 | 0.86173 | 0.84748 |
| 200 | 1.85865 | 1.68889 | 3.05351 | 1.60077 | 1.33759 | 1.95321 | 2.12315 | 1.81191 | 1.69357 | 1.28859 |
| 300 | 2.359 | 2.58456 | 4.04047 | 2.6176 | 2.09487 | 2.889 | 3.53074 | 2.24301 | 2.97322 | 2.21122 |
| 400 | 3.55404 | 3.32931 | 4.57305 | 3.19207 | 2.80681 | 4.21455 | 4.59773 | 3.60111 | 3.80216 | 2.91044 |
| 500 | 4.43143 | 4.1904 | 4.83954 | 4.09343 | 3.26114 | 5.33768 | 5.66285 | 5.35238 | 4.89608 | 3.53674 |
| 600 | 5.23824 | 4.73194 | 5.82116 | 5.19937 | 4.29282 | 6.20598 | 6.60383 | 6.1012 | 5.82523 | 4.70863 |
| 700 | 5.89172 | 5.45218 | 6.91164 | 6.48759 | 4.82919 | 7.1518 | 7.34794 | 7.10141 | 7.08082 | 5.54992 |
| 800 | 6.72469 | 6.10603 | 8.17038 | 7.21896 | 5.35732 | 8.06681 | 8.29743 | 8.11988 | 8.19575 | 6.64225 |
| 900 | 7.49384 | 6.74991 | 9.43952 | 8.3841 | 6.01359 | 9.11161 | 9.38034 | 9.07973 | 9.08594 | 7.18936 |
| 1000 | 9.05273 | 7.79752 | 10.1761 | 8.90353 | 7.16935 | 10.1471 | 10.1281 | 9.78926 | 9.63874 | 8.28518 |
| 1100 | 9.78817 | 9.10718 | 10.6469 | 9.64636 | 8.18797 | 11.2845 | 10.6117 | 11.1035 | 10.8383 | 9.09649 |
| 1200 | 10.7149 | 10.1476 | 11.1938 | 11.2271 | 9.5439 | 11.9948 | 11.5901 | 11.439 | 11.5421 | 9.85373 |
| 1300 | 11.6596 | 10.9591 | 12.1827 | 11.7844 | 10.5822 | 13.1619 | 12.696 | 12.4697 | 12.3525 | 10.5117 |
| 1400 | 12.5058 | 12.3523 | 12.9742 | 12.8115 | 11.4753 | 13.7026 | 13.5125 | 13.2031 | 13.3417 | 11.5482 |
| 1500 | 13.2448 | 13.2716 | 13.6921 | 13.3329 | 12.6196 | 14.7461 | 14.4247 | 14.2506 | 14.2712 | 12.6389 |

Table 17

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00364 | 0.00666 | 0.01392 | 0.01169 | 0.00901 | 0.01123 | 0.01153 | 0.00799 | 0.00862 | 0.00847 |
| 200 | 0.00929 | 0.00844 | 0.01527 | 0.008 | 0.00669 | 0.00977 | 0.01062 | 0.00906 | 0.00847 | 0.00644 |
| 300 | 0.00786 | 0.00862 | 0.01347 | 0.00873 | 0.00698 | 0.00963 | 0.01177 | 0.00748 | 0.00991 | 0.00737 |
| 400 | 0.00889 | 0.00832 | 0.01143 | 0.00798 | 0.00702 | 0.01054 | 0.01149 | 0.009 | 0.00951 | 0.00728 |
| 500 | 0.00886 | 0.00838 | 0.00968 | 0.00819 | 0.00652 | 0.01068 | 0.01133 | 0.0107 | 0.00979 | 0.00707 |
| 600 | 0.00873 | 0.00789 | 0.0097 | 0.00867 | 0.00715 | 0.01034 | 0.01101 | 0.01017 | 0.00971 | 0.00785 |
| 700 | 0.00842 | 0.00779 | 0.00987 | 0.00927 | 0.0069 | 0.01022 | 0.0105 | 0.01014 | 0.01012 | 0.00793 |
| 800 | 0.00841 | 0.00763 | 0.01021 | 0.00902 | 0.0067 | 0.01008 | 0.01037 | 0.01015 | 0.01024 | 0.0083 |
| 900 | 0.00833 | 0.0075 | 0.01049 | 0.00932 | 0.00668 | 0.01012 | 0.01042 | 0.01009 | 0.0101 | 0.00799 |
| 1000 | 0.00905 | 0.0078 | 0.01018 | 0.0089 | 0.00717 | 0.01015 | 0.01013 | 0.00979 | 0.00964 | 0.00829 |
| 1100 | 0.0089 | 0.00828 | 0.00968 | 0.00877 | 0.00744 | 0.01026 | 0.00965 | 0.01009 | 0.00985 | 0.00827 |
| 1200 | 0.00893 | 0.00846 | 0.00933 | 0.00936 | 0.00795 | 0.01 | 0.00966 | 0.00953 | 0.00962 | 0.00821 |
| 1300 | 0.00897 | 0.00843 | 0.00937 | 0.00906 | 0.00814 | 0.01012 | 0.00977 | 0.00959 | 0.0095 | 0.00809 |
| 1400 | 0.00893 | 0.00882 | 0.00927 | 0.00915 | 0.0082 | 0.00979 | 0.00965 | 0.00943 | 0.00953 | 0.00825 |
| 1500 | 0.00883 | 0.00885 | 0.00913 | 0.00889 | 0.00841 | 0.00983 | 0.00962 | 0.0095 | 0.00951 | 0.00843 |

Table 18

Scenario 2. Bidder1 underbid by 0.0825 while others used equilibrium

strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00514 | 0.09302 | 0.09079 | 0.08799 | 0.08817 | 0.09169 | 0.09593 | 0.09138 | 0.09197 | 0.09342 |
| 200 | 0.00556 | 0.09198 | 0.0918 | 0.08977 | 0.09045 | 0.09098 | 0.09227 | 0.09086 | 0.09054 | 0.09195 |
| 300 | 0.00626 | 0.0922 | 0.09096 | 0.09052 | 0.09039 | 0.09062 | 0.09317 | 0.09185 | 0.09212 | 0.09238 |
| 400 | 0.00646 | 0.09209 | 0.08904 | 0.0908 | 0.09035 | 0.09163 | 0.09308 | 0.09202 | 0.09194 | 0.09299 |
| 500 | 0.00691 | 0.0918 | 0.08943 | 0.08893 | 0.09127 | 0.09159 | 0.09222 | 0.09158 | 0.09157 | 0.09238 |
| 600 | 0.00679 | 0.09056 | 0.08903 | 0.08979 | 0.09046 | 0.09199 | 0.09158 | 0.09116 | 0.09181 | 0.09199 |
| 700 | 0.00669 | 0.09045 | 0.08957 | 0.09026 | 0.09115 | 0.09132 | 0.09133 | 0.09111 | 0.0913 | 0.09234 |
| 800 | 0.0066 | 0.0909 | 0.08945 | 0.0897 | 0.09075 | 0.09107 | 0.09126 | 0.09113 | 0.09191 | 0.09232 |
| 900 | 0.00631 | 0.09131 | 0.08964 | 0.08976 | 0.09122 | 0.09098 | 0.09108 | 0.09073 | 0.09185 | 0.0921 |
| 1000 | 0.0061 | 0.09141 | 0.08971 | 0.09028 | 0.09134 | 0.09065 | 0.09115 | 0.09087 | 0.09189 | 0.09166 |
| 1100 | 0.00613 | 0.09149 | 0.08945 | 0.09 | 0.09128 | 0.0911 | 0.09117 | 0.09083 | 0.09194 | 0.09138 |
| 1200 | 0.00585 | 0.09163 | 0.08972 | 0.08994 | 0.09123 | 0.09123 | 0.09147 | 0.09087 | 0.0916 | 0.09148 |
| 1300 | 0.00573 | 0.09173 | 0.08947 | 0.09007 | 0.09125 | 0.091 | 0.09143 | 0.09095 | 0.09159 | 0.09146 |
| 1400 | 0.0057 | 0.09175 | 0.08909 | 0.0903 | 0.09145 | 0.09108 | 0.09131 | 0.09108 | 0.0918 | 0.09147 |
| 1500 | 0.0057 | 0.0917 | 0.08923 | 0.09059 | 0.09109 | 0.09121 | 0.09135 | 0.09114 | 0.09185 | 0.09125 |

Table 19

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 16 | 6 | 14 | 12 | 8 | 8 | 5 | 16 | 8 | 7 |
| 200 | 39 | 20 | 18 | 16 | 15 | 18 | 19 | 21 | 13 | 21 |
| 300 | 67 | 27 | 23 | 23 | 30 | 23 | 27 | 27 | 24 | 29 |
| 400 | 89 | 33 | 31 | 29 | 39 | 35 | 36 | 35 | 36 | 37 |
| 500 | 107 | 41 | 44 | 40 | 46 | 45 | 47 | 44 | 41 | 45 |
| 600 | 127 | 54 | 52 | 47 | 58 | 48 | 54 | 54 | 50 | 56 |
| 700 | 144 | 66 | 64 | 55 | 67 | 58 | 61 | 58 | 62 | 65 |
| 800 | 164 | 76 | 69 | 68 | 76 | 65 | 72 | 64 | 74 | 72 |
| 900 | 184 | 88 | 77 | 79 | 82 | 77 | 80 | 71 | 84 | 78 |
| 1000 | 201 | 99 | 84 | 89 | 88 | 86 | 88 | 84 | 90 | 91 |
| 1100 | 218 | 106 | 92 | 100 | 99 | 95 | 97 | 93 | 98 | 102 |
| 1200 | 241 | 114 | 105 | 107 | 105 | 104 | 106 | 96 | 107 | 115 |
| 1300 | 263 | 120 | 117 | 114 | 114 | 115 | 113 | 108 | 118 | 118 |
| 1400 | 280 | 133 | 125 | 122 | 124 | 121 | 121 | 118 | 131 | 125 |
| 1500 | 296 | 143 | 134 | 127 | 134 | 138 | 131 | 124 | 140 | 133 |

Table 20

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.08227 | 0.55815 | 1.27104 | 1.05591 | 0.70533 | 0.73352 | 0.47967 | 1.46209 | 0.73574 | 0.65395 |
| 200 | 0.2168 | 1.83969 | 1.65233 | 1.43626 | 1.35675 | 1.63769 | 1.7532 | 1.90813 | 1.177 | 1.93085 |
| 300 | 0.41948 | 2.48947 | 2.09202 | 2.08201 | 2.71156 | 2.08419 | 2.51553 | 2.48002 | 2.21092 | 2.67904 |
| 400 | 0.57537 | 3.03899 | 2.76039 | 2.63334 | 3.52349 | 3.20692 | 3.35073 | 3.22073 | 3.30999 | 3.44077 |
| 500 | 0.7399 | 3.76385 | 3.93479 | 3.55736 | 4.19862 | 4.1216 | 4.33455 | 4.02967 | 3.75436 | 4.15711 |
| 600 | 0.86196 | 4.89001 | 4.62954 | 4.22018 | 5.24678 | 4.41557 | 4.94537 | 4.92287 | 4.59046 | 5.15165 |
| 700 | 0.96295 | 5.96987 | 5.73229 | 4.96409 | 6.10724 | 5.29641 | 5.57137 | 5.28447 | 5.66082 | 6.00224 |
| 800 | 1.08181 | 6.90815 | 6.17194 | 6.09935 | 6.89696 | 5.91981 | 6.57098 | 5.83244 | 6.80157 | 6.64673 |
| 900 | 1.16102 | 8.03528 | 6.90199 | 7.09119 | 7.48035 | 7.0056 | 7.28668 | 6.44167 | 7.71524 | 7.18398 |
| 1000 | 1.22677 | 9.0494 | 7.53529 | 8.03523 | 8.03804 | 7.7961 | 8.02141 | 7.63322 | 8.27021 | 8.34094 |
| 1100 | 1.33619 | 9.6978 | 8.22963 | 8.99963 | 9.03629 | 8.65438 | 8.84384 | 8.44676 | 9.01027 | 9.32058 |
| 1200 | 1.40924 | 10.4462 | 9.42101 | 9.62364 | 9.57904 | 9.48842 | 9.69541 | 8.72344 | 9.80094 | 10.5199 |
| 1300 | 1.50682 | 11.0071 | 10.4675 | 10.2685 | 10.403 | 10.4653 | 10.3315 | 9.82239 | 10.8077 | 10.7927 |
| 1400 | 1.59586 | 12.2033 | 11.1363 | 11.017 | 11.3399 | 11.0212 | 11.0486 | 10.7472 | 12.026 | 11.4333 |
| 1500 | 1.68729 | 13.1132 | 11.9566 | 11.5053 | 12.2065 | 12.5871 | 11.9666 | 11.3013 | 12.8588 | 12.1364 |

Table 21

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00082 | 0.00558 | 0.01271 | 0.01056 | 0.00705 | 0.00734 | 0.0048 | 0.01462 | 0.00736 | 0.00654 |
| 200 | 0.00108 | 0.0092 | 0.00826 | 0.00718 | 0.00678 | 0.00819 | 0.00877 | 0.00954 | 0.00588 | 0.00965 |
| 300 | 0.0014 | 0.0083 | 0.00697 | 0.00694 | 0.00904 | 0.00695 | 0.00839 | 0.00827 | 0.00737 | 0.00893 |
| 400 | 0.00144 | 0.0076 | 0.0069 | 0.00658 | 0.00881 | 0.00802 | 0.00838 | 0.00805 | 0.00827 | 0.0086 |
| 500 | 0.00148 | 0.00753 | 0.00787 | 0.00711 | 0.0084 | 0.00824 | 0.00867 | 0.00806 | 0.00751 | 0.00831 |
| 600 | 0.00144 | 0.00815 | 0.00772 | 0.00703 | 0.00874 | 0.00736 | 0.00824 | 0.0082 | 0.00765 | 0.00859 |
| 700 | 0.00138 | 0.00853 | 0.00819 | 0.00709 | 0.00872 | 0.00757 | 0.00796 | 0.00755 | 0.00809 | 0.00857 |
| 800 | 0.00135 | 0.00864 | 0.00771 | 0.00762 | 0.00862 | 0.0074 | 0.00821 | 0.00729 | 0.0085 | 0.00831 |
| 900 | 0.00129 | 0.00893 | 0.00767 | 0.00788 | 0.00831 | 0.00778 | 0.0081 | 0.00716 | 0.00857 | 0.00798 |
| 1000 | 0.00123 | 0.00905 | 0.00754 | 0.00804 | 0.00804 | 0.0078 | 0.00802 | 0.00763 | 0.00827 | 0.00834 |
| 1100 | 0.00121 | 0.00882 | 0.00748 | 0.00818 | 0.00821 | 0.00787 | 0.00804 | 0.00768 | 0.00819 | 0.00847 |
| 1200 | 0.00117 | 0.00871 | 0.00785 | 0.00802 | 0.00798 | 0.00791 | 0.00808 | 0.00727 | 0.00817 | 0.00877 |
| 1300 | 0.00116 | 0.00847 | 0.00805 | 0.0079 | 0.008 | 0.00805 | 0.00795 | 0.00756 | 0.00831 | 0.0083 |
| 1400 | 0.00114 | 0.00872 | 0.00795 | 0.00787 | 0.0081 | 0.00787 | 0.00789 | 0.00768 | 0.00859 | 0.00817 |
| 1500 | 0.00112 | 0.00874 | 0.00797 | 0.00767 | 0.00814 | 0.00839 | 0.00798 | 0.00753 | 0.00857 | 0.00809 |

Table 22

Scenario 3. Bidders “1” and “2” underbid by 0.0825 while others used

equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0069 | 0.00508 | 0.08713 | 0.09506 | 0.0893 | 0.09447 | 0.09273 | 0.09156 | 0.09101 | 0.08796 |
| 200 | 0.00631 | 0.00699 | 0.0894 | 0.09528 | 0.09013 | 0.0895 | 0.0925 | 0.09322 | 0.08887 | 0.09069 |
| 300 | 0.00566 | 0.00538 | 0.09056 | 0.0946 | 0.09155 | 0.08877 | 0.09193 | 0.09229 | 0.08911 | 0.09041 |
| 400 | 0.00619 | 0.00632 | 0.09124 | 0.09303 | 0.09134 | 0.08973 | 0.0912 | 0.0905 | 0.08962 | 0.09098 |
| 500 | 0.00644 | 0.00542 | 0.09155 | 0.09318 | 0.09039 | 0.08971 | 0.09106 | 0.09141 | 0.08811 | 0.08934 |
| 600 | 0.00635 | 0.00562 | 0.09071 | 0.09291 | 0.09099 | 0.08972 | 0.09021 | 0.09106 | 0.08837 | 0.0898 |
| 700 | 0.00551 | 0.00597 | 0.09085 | 0.09235 | 0.09108 | 0.08994 | 0.09001 | 0.09116 | 0.08899 | 0.09044 |
| 800 | 0.00524 | 0.00563 | 0.09123 | 0.09236 | 0.09148 | 0.08994 | 0.09014 | 0.08999 | 0.08914 | 0.0906 |
| 900 | 0.0054 | 0.00529 | 0.09094 | 0.09243 | 0.09121 | 0.08989 | 0.08983 | 0.08946 | 0.08906 | 0.0908 |
| 1000 | 0.00542 | 0.00549 | 0.09114 | 0.09246 | 0.09135 | 0.09019 | 0.08997 | 0.08945 | 0.08852 | 0.09066 |
| 1100 | 0.00544 | 0.00597 | 0.0909 | 0.09199 | 0.09119 | 0.09034 | 0.09029 | 0.08968 | 0.08875 | 0.09103 |
| 1200 | 0.00518 | 0.0059 | 0.09127 | 0.09199 | 0.09062 | 0.09063 | 0.09017 | 0.09004 | 0.08922 | 0.09116 |
| 1300 | 0.0054 | 0.00604 | 0.09133 | 0.09189 | 0.09079 | 0.09083 | 0.0904 | 0.09026 | 0.08912 | 0.09117 |
| 1400 | 0.0053 | 0.00587 | 0.09157 | 0.09138 | 0.09068 | 0.09114 | 0.09059 | 0.09005 | 0.08925 | 0.0914 |
| 1500 | 0.00553 | 0.00592 | 0.0914 | 0.09109 | 0.09081 | 0.09103 | 0.09052 | 0.09017 | 0.08919 | 0.09117 |

Table 23

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 14 | 11 | 13 | 9 | 9 | 11 | 11 | 10 | 4 | 8 |
| 200 | 29 | 24 | 19 | 19 | 14 | 25 | 21 | 17 | 12 | 20 |
| 300 | 52 | 38 | 31 | 29 | 21 | 33 | 27 | 25 | 19 | 25 |
| 400 | 62 | 60 | 39 | 40 | 31 | 43 | 34 | 33 | 23 | 35 |
| 500 | 80 | 72 | 50 | 51 | 37 | 52 | 44 | 44 | 30 | 40 |
| 600 | 99 | 93 | 54 | 58 | 42 | 67 | 50 | 50 | 36 | 51 |
| 700 | 121 | 110 | 62 | 65 | 48 | 74 | 61 | 52 | 47 | 60 |
| 800 | 134 | 133 | 71 | 73 | 53 | 80 | 70 | 62 | 54 | 70 |
| 900 | 154 | 154 | 74 | 78 | 62 | 85 | 83 | 71 | 63 | 76 |
| 1000 | 171 | 172 | 82 | 80 | 74 | 95 | 93 | 81 | 69 | 83 |
| 1100 | 193 | 189 | 92 | 92 | 80 | 100 | 99 | 90 | 74 | 91 |
| 1200 | 214 | 201 | 102 | 102 | 92 | 108 | 103 | 98 | 83 | 97 |
| 1300 | 229 | 224 | 112 | 107 | 98 | 117 | 115 | 105 | 91 | 102 |
| 1400 | 245 | 244 | 126 | 120 | 104 | 124 | 119 | 110 | 100 | 108 |
| 1500 | 264 | 259 | 135 | 130 | 114 | 133 | 128 | 117 | 109 | 111 |

Table 24

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.09658 | 0.0559 | 1.13263 | 0.85553 | 0.80368 | 1.03914 | 1.02007 | 0.91562 | 0.36403 | 0.70368 |
| 200 | 0.18295 | 0.1677 | 1.69859 | 1.81037 | 1.26181 | 2.23741 | 1.94247 | 1.58467 | 1.06643 | 1.81385 |
| 300 | 0.29439 | 0.20431 | 2.80727 | 2.74341 | 1.92257 | 2.92945 | 2.48223 | 2.30737 | 1.69313 | 2.26036 |
| 400 | 0.38381 | 0.37924 | 3.55817 | 3.72134 | 2.83163 | 3.85857 | 3.10085 | 2.98647 | 2.06136 | 3.18438 |
| 500 | 0.51543 | 0.39029 | 4.57772 | 4.75219 | 3.34436 | 4.66516 | 4.00676 | 4.02223 | 2.64321 | 3.57366 |
| 600 | 0.62906 | 0.5229 | 4.89855 | 5.38859 | 3.82165 | 6.01145 | 4.5106 | 4.55298 | 3.18118 | 4.57999 |
| 700 | 0.66642 | 0.65634 | 5.63294 | 6.00272 | 4.37197 | 6.6556 | 5.49034 | 4.74017 | 4.18233 | 5.4264 |
| 800 | 0.70277 | 0.74877 | 6.47719 | 6.7423 | 4.84855 | 7.19507 | 6.30998 | 5.57959 | 4.81357 | 6.3419 |
| 900 | 0.83192 | 0.81442 | 6.72959 | 7.2098 | 5.65485 | 7.64059 | 7.45624 | 6.35173 | 5.61103 | 6.90056 |
| 1000 | 0.92757 | 0.94454 | 7.47348 | 7.39663 | 6.76023 | 8.56834 | 8.36761 | 7.24546 | 6.10775 | 7.52453 |
| 1100 | 1.04945 | 1.12836 | 8.3629 | 8.46345 | 7.29519 | 9.03396 | 8.93866 | 8.07154 | 6.56717 | 8.28415 |
| 1200 | 1.10751 | 1.18626 | 9.30985 | 9.38258 | 8.33709 | 9.7885 | 9.28735 | 8.8241 | 7.40553 | 8.84255 |
| 1300 | 1.23628 | 1.35282 | 10.2292 | 9.83216 | 8.8973 | 10.6271 | 10.396 | 9.47734 | 8.10997 | 9.2994 |
| 1400 | 1.29956 | 1.43238 | 11.538 | 10.9661 | 9.43076 | 11.3019 | 10.7808 | 9.90596 | 8.92511 | 9.87132 |
| 1500 | 1.4591 | 1.53433 | 12.3395 | 11.8423 | 10.3519 | 12.1077 | 11.5871 | 10.5495 | 9.72215 | 10.1197 |

Table 25

| # of games | Average Expected profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00097 | 0.00056 | 0.01133 | 0.00856 | 0.00804 | 0.01039 | 0.0102 | 0.00916 | 0.00364 | 0.00704 |
| 200 | 0.00091 | 0.00084 | 0.00849 | 0.00905 | 0.00631 | 0.01119 | 0.00971 | 0.00792 | 0.00533 | 0.00907 |
| 300 | 0.00098 | 0.00068 | 0.00936 | 0.00914 | 0.00641 | 0.00976 | 0.00827 | 0.00769 | 0.00564 | 0.00753 |
| 400 | 0.00096 | 0.00095 | 0.0089 | 0.0093 | 0.00708 | 0.00965 | 0.00775 | 0.00747 | 0.00515 | 0.00796 |
| 500 | 0.00103 | 0.00078 | 0.00916 | 0.0095 | 0.00669 | 0.00933 | 0.00801 | 0.00804 | 0.00529 | 0.00715 |
| 600 | 0.00105 | 0.00087 | 0.00816 | 0.00898 | 0.00637 | 0.01002 | 0.00752 | 0.00759 | 0.0053 | 0.00763 |
| 700 | 0.00095 | 0.00094 | 0.00805 | 0.00858 | 0.00625 | 0.00951 | 0.00784 | 0.00677 | 0.00597 | 0.00775 |
| 800 | 0.00088 | 0.00094 | 0.0081 | 0.00843 | 0.00606 | 0.00899 | 0.00789 | 0.00697 | 0.00602 | 0.00793 |
| 900 | 0.00092 | 0.0009 | 0.00748 | 0.00801 | 0.00628 | 0.00849 | 0.00828 | 0.00706 | 0.00623 | 0.00767 |
| 1000 | 0.00093 | 0.00094 | 0.00747 | 0.0074 | 0.00676 | 0.00857 | 0.00837 | 0.00725 | 0.00611 | 0.00752 |
| 1100 | 0.00095 | 0.00103 | 0.0076 | 0.00769 | 0.00663 | 0.00821 | 0.00813 | 0.00734 | 0.00597 | 0.00753 |
| 1200 | 0.00092 | 0.00099 | 0.00776 | 0.00782 | 0.00695 | 0.00816 | 0.00774 | 0.00735 | 0.00617 | 0.00737 |
| 1300 | 0.00095 | 0.00104 | 0.00787 | 0.00756 | 0.00684 | 0.00817 | 0.008 | 0.00729 | 0.00624 | 0.00715 |
| 1400 | 0.00093 | 0.00102 | 0.00824 | 0.00783 | 0.00674 | 0.00807 | 0.0077 | 0.00708 | 0.00638 | 0.00705 |
| 1500 | 0.00097 | 0.00102 | 0.00823 | 0.00789 | 0.0069 | 0.00807 | 0.00772 | 0.00703 | 0.00648 | 0.00675 |

Table 26

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.0825, 0.075, 0.0675

and 0.06 respectively while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00438 | 0.01511 | 0.02549 | 0.03139 | 0.09044 | 0.09141 | 0.0904 | 0.09346 | 0.09334 | 0.09358 |
| 200 | 0.00531 | 0.01504 | 0.02509 | 0.03153 | 0.09143 | 0.09182 | 0.09034 | 0.09366 | 0.09419 | 0.09193 |
| 300 | 0.0057 | 0.01487 | 0.02396 | 0.0315 | 0.0899 | 0.09286 | 0.09107 | 0.09282 | 0.09311 | 0.09154 |
| 400 | 0.0058 | 0.01475 | 0.02309 | 0.03118 | 0.09076 | 0.09159 | 0.09056 | 0.09125 | 0.0918 | 0.09113 |
| 500 | 0.00641 | 0.01522 | 0.02346 | 0.03042 | 0.09005 | 0.09229 | 0.09167 | 0.09095 | 0.09156 | 0.09123 |
| 600 | 0.00659 | 0.0154 | 0.02348 | 0.0298 | 0.0902 | 0.09289 | 0.09134 | 0.09132 | 0.09094 | 0.09016 |
| 700 | 0.00722 | 0.01477 | 0.02328 | 0.02954 | 0.09041 | 0.09268 | 0.09115 | 0.09169 | 0.09064 | 0.09119 |
| 800 | 0.00735 | 0.01475 | 0.02315 | 0.0297 | 0.09024 | 0.09178 | 0.09099 | 0.09221 | 0.09063 | 0.09139 |
| 900 | 0.00724 | 0.01446 | 0.02297 | 0.02963 | 0.09068 | 0.09198 | 0.09114 | 0.09206 | 0.09042 | 0.09168 |
| 1000 | 0.00708 | 0.01478 | 0.02316 | 0.02964 | 0.09132 | 0.09212 | 0.09149 | 0.09242 | 0.09042 | 0.09152 |
| 1100 | 0.00658 | 0.01468 | 0.02282 | 0.02978 | 0.09141 | 0.09219 | 0.0916 | 0.09264 | 0.09059 | 0.0914 |
| 1200 | 0.00647 | 0.0147 | 0.02279 | 0.02967 | 0.09085 | 0.09215 | 0.09156 | 0.09277 | 0.09113 | 0.09135 |
| 1300 | 0.00641 | 0.0146 | 0.02274 | 0.02991 | 0.09093 | 0.09192 | 0.09112 | 0.09292 | 0.09064 | 0.09142 |
| 1400 | 0.00652 | 0.01461 | 0.02284 | 0.03008 | 0.09088 | 0.09189 | 0.09126 | 0.09278 | 0.09069 | 0.09157 |
| 1500 | 0.0063 | 0.0148 | 0.0229 | 0.03004 | 0.09083 | 0.09225 | 0.09116 | 0.09277 | 0.09064 | 0.09173 |

Table 27

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 17 | 13 | 19 | 11 | 6 | 4 | 7 | 8 | 7 | 8 |
| 200 | 34 | 34 | 30 | 24 | 11 | 13 | 14 | 14 | 10 | 16 |
| 300 | 52 | 50 | 48 | 32 | 14 | 23 | 22 | 20 | 16 | 23 |
| 400 | 72 | 66 | 60 | 43 | 20 | 29 | 30 | 28 | 21 | 31 |
| 500 | 91 | 87 | 75 | 50 | 28 | 36 | 36 | 34 | 29 | 34 |
| 600 | 106 | 98 | 90 | 67 | 37 | 47 | 45 | 36 | 34 | 40 |
| 700 | 119 | 117 | 104 | 82 | 44 | 55 | 48 | 42 | 40 | 49 |
| 800 | 133 | 133 | 113 | 99 | 51 | 63 | 56 | 49 | 46 | 57 |
| 900 | 146 | 147 | 127 | 115 | 57 | 68 | 62 | 57 | 57 | 64 |
| 1000 | 162 | 159 | 144 | 128 | 66 | 72 | 71 | 64 | 62 | 72 |
| 1100 | 183 | 174 | 159 | 139 | 68 | 75 | 81 | 69 | 71 | 81 |
| 1200 | 194 | 185 | 171 | 157 | 75 | 83 | 89 | 74 | 78 | 94 |
| 1300 | 206 | 199 | 189 | 170 | 81 | 88 | 103 | 81 | 82 | 101 |
| 1400 | 218 | 217 | 201 | 188 | 90 | 100 | 111 | 85 | 85 | 105 |
| 1500 | 233 | 237 | 215 | 200 | 97 | 109 | 121 | 93 | 88 | 107 |

Table 28

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.07439 | 0.19648 | 0.48422 | 0.34525 | 0.54263 | 0.36564 | 0.6328 | 0.74772 | 0.65339 | 0.74866 |
| 200 | 0.18063 | 0.51132 | 0.7526 | 0.75671 | 1.00576 | 1.1937 | 1.26479 | 1.31117 | 0.94192 | 1.47084 |
| 300 | 0.29617 | 0.7434 | 1.1502 | 1.00801 | 1.25853 | 2.1358 | 2.00348 | 1.85648 | 1.4898 | 2.10546 |
| 400 | 0.41783 | 0.97346 | 1.38517 | 1.34073 | 1.81517 | 2.65622 | 2.71694 | 2.55498 | 1.92773 | 2.82496 |
| 500 | 0.58352 | 1.32379 | 1.75984 | 1.52112 | 2.52148 | 3.32242 | 3.30009 | 3.09222 | 2.65533 | 3.10198 |
| 600 | 0.69871 | 1.50964 | 2.11354 | 1.99679 | 3.33729 | 4.36588 | 4.11028 | 3.28768 | 3.09212 | 3.60628 |
| 700 | 0.85964 | 1.7284 | 2.42152 | 2.42201 | 3.97815 | 5.09724 | 4.37543 | 3.85107 | 3.62554 | 4.46824 |
| 800 | 0.97816 | 1.96132 | 2.61539 | 2.94001 | 4.60218 | 5.78207 | 5.09556 | 4.51851 | 4.16914 | 5.20947 |
| 900 | 1.05636 | 2.1251 | 2.91662 | 3.40688 | 5.16878 | 6.25492 | 5.65067 | 5.24715 | 5.15374 | 5.86763 |
| 1000 | 1.14669 | 2.35022 | 3.33472 | 3.79456 | 6.02686 | 6.63269 | 6.49585 | 5.91476 | 5.60589 | 6.58973 |
| 1100 | 1.20407 | 2.55394 | 3.62889 | 4.13939 | 6.21596 | 6.91459 | 7.41957 | 6.39214 | 6.43161 | 7.40303 |
| 1200 | 1.25503 | 2.7186 | 3.89789 | 4.65837 | 6.81354 | 7.64863 | 8.14909 | 6.86484 | 7.1079 | 8.58648 |
| 1300 | 1.32103 | 2.90502 | 4.29724 | 5.08494 | 7.36513 | 8.08872 | 9.38583 | 7.52654 | 7.43223 | 9.23342 |
| 1400 | 1.4216 | 3.17035 | 4.59082 | 5.65596 | 8.17886 | 9.18902 | 10.1296 | 7.88653 | 7.70834 | 9.61527 |
| 1500 | 1.46744 | 3.50842 | 4.92302 | 6.00835 | 8.81069 | 10.0552 | 11.0308 | 8.62794 | 7.97675 | 9.81493 |

Table 29

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00074 | 0.00196 | 0.00484 | 0.00345 | 0.00543 | 0.00366 | 0.00633 | 0.00748 | 0.00653 | 0.00749 |
| 200 | 0.0009 | 0.00256 | 0.00376 | 0.00378 | 0.00503 | 0.00597 | 0.00632 | 0.00656 | 0.00471 | 0.00735 |
| 300 | 0.00099 | 0.00248 | 0.00383 | 0.00336 | 0.0042 | 0.00712 | 0.00668 | 0.00619 | 0.00497 | 0.00702 |
| 400 | 0.00104 | 0.00243 | 0.00346 | 0.00335 | 0.00454 | 0.00664 | 0.00679 | 0.00639 | 0.00482 | 0.00706 |
| 500 | 0.00117 | 0.00265 | 0.00352 | 0.00304 | 0.00504 | 0.00664 | 0.0066 | 0.00618 | 0.00531 | 0.0062 |
| 600 | 0.00116 | 0.00252 | 0.00352 | 0.00333 | 0.00556 | 0.00728 | 0.00685 | 0.00548 | 0.00515 | 0.00601 |
| 700 | 0.00123 | 0.00247 | 0.00346 | 0.00346 | 0.00568 | 0.00728 | 0.00625 | 0.0055 | 0.00518 | 0.00638 |
| 800 | 0.00122 | 0.00245 | 0.00327 | 0.00368 | 0.00575 | 0.00723 | 0.00637 | 0.00565 | 0.00521 | 0.00651 |
| 900 | 0.00117 | 0.00236 | 0.00324 | 0.00379 | 0.00574 | 0.00695 | 0.00628 | 0.00583 | 0.00573 | 0.00652 |
| 1000 | 0.00115 | 0.00235 | 0.00333 | 0.00379 | 0.00603 | 0.00663 | 0.0065 | 0.00591 | 0.00561 | 0.00659 |
| 1100 | 0.00109 | 0.00232 | 0.0033 | 0.00376 | 0.00565 | 0.00629 | 0.00675 | 0.00581 | 0.00585 | 0.00673 |
| 1200 | 0.00105 | 0.00227 | 0.00325 | 0.00388 | 0.00568 | 0.00637 | 0.00679 | 0.00572 | 0.00592 | 0.00716 |
| 1300 | 0.00102 | 0.00223 | 0.00331 | 0.00391 | 0.00567 | 0.00622 | 0.00722 | 0.00579 | 0.00572 | 0.0071 |
| 1400 | 0.00102 | 0.00226 | 0.00328 | 0.00404 | 0.00584 | 0.00656 | 0.00724 | 0.00563 | 0.00551 | 0.00687 |
| 1500 | 0.00098 | 0.00234 | 0.00328 | 0.00401 | 0.00587 | 0.0067 | 0.00735 | 0.00575 | 0.00532 | 0.00654 |

Table 30

5. Fifteen-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.06284 | 0.06461 | 0.06259 | 0.06113 | 0.06264 | 0.06279 | 0.06301 | 0.06448 | 0.06302 | 0.06396 | 0.06213 | 0.06176 | 0.06092 | 0.06144 | 0.06119 |
| 200 | 0.0631 | 0.06389 | 0.06326 | 0.06265 | 0.06219 | 0.06221 | 0.06379 | 0.06374 | 0.06412 | 0.06208 | 0.06129 | 0.06091 | 0.06227 | 0.06264 | 0.06152 |
| 300 | 0.06285 | 0.06281 | 0.06352 | 0.06211 | 0.0623 | 0.06167 | 0.06398 | 0.06345 | 0.06406 | 0.0629 | 0.06296 | 0.06153 | 0.06288 | 0.06329 | 0.06187 |
| 400 | 0.0632 | 0.06284 | 0.06365 | 0.06197 | 0.06173 | 0.06064 | 0.06346 | 0.06344 | 0.06415 | 0.0626 | 0.06336 | 0.06213 | 0.0628 | 0.06328 | 0.06201 |
| 500 | 0.06316 | 0.06256 | 0.0636 | 0.06228 | 0.06212 | 0.06097 | 0.06346 | 0.063 | 0.06433 | 0.06289 | 0.06332 | 0.06194 | 0.06238 | 0.06297 | 0.06193 |
| 600 | 0.06273 | 0.06249 | 0.0632 | 0.06209 | 0.06182 | 0.06121 | 0.06309 | 0.06313 | 0.06452 | 0.063 | 0.06337 | 0.06214 | 0.06268 | 0.06275 | 0.06213 |
| 700 | 0.06314 | 0.06259 | 0.06257 | 0.0616 | 0.06157 | 0.06128 | 0.06319 | 0.063 | 0.06415 | 0.06327 | 0.06312 | 0.06199 | 0.0626 | 0.06249 | 0.06223 |
| 800 | 0.06277 | 0.06244 | 0.06258 | 0.06158 | 0.06164 | 0.06175 | 0.06337 | 0.0632 | 0.06409 | 0.06325 | 0.06252 | 0.06215 | 0.06245 | 0.06268 | 0.06239 |
| 900 | 0.06282 | 0.06249 | 0.06244 | 0.06164 | 0.06182 | 0.06197 | 0.06334 | 0.06307 | 0.06401 | 0.06331 | 0.06252 | 0.0622 | 0.06243 | 0.0629 | 0.06258 |
| 1000 | 0.06278 | 0.06228 | 0.06253 | 0.0616 | 0.06203 | 0.06227 | 0.06322 | 0.06304 | 0.0639 | 0.06331 | 0.06236 | 0.06211 | 0.06236 | 0.06282 | 0.06243 |
| 1100 | 0.06287 | 0.06248 | 0.0625 | 0.06179 | 0.06223 | 0.06236 | 0.06323 | 0.06308 | 0.06371 | 0.06308 | 0.06232 | 0.06211 | 0.06217 | 0.06283 | 0.06219 |
| 1200 | 0.06282 | 0.06245 | 0.0624 | 0.06184 | 0.06217 | 0.06242 | 0.0631 | 0.06275 | 0.06371 | 0.06308 | 0.06245 | 0.06231 | 0.06199 | 0.06289 | 0.06215 |
| 1300 | 0.06266 | 0.06256 | 0.06245 | 0.06196 | 0.06227 | 0.06241 | 0.0629 | 0.06284 | 0.06366 | 0.06313 | 0.06264 | 0.06234 | 0.06174 | 0.063 | 0.06195 |
| 1400 | 0.06274 | 0.0626 | 0.06257 | 0.06209 | 0.06214 | 0.06242 | 0.06298 | 0.06281 | 0.06346 | 0.06303 | 0.06236 | 0.06248 | 0.06187 | 0.06294 | 0.06209 |
| 1500 | 0.06237 | 0.06256 | 0.06263 | 0.06216 | 0.06218 | 0.06244 | 0.0629 | 0.0629 | 0.0635 | 0.06306 | 0.06227 | 0.06234 | 0.06191 | 0.06289 | 0.06201 |

Table 31

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 6 | 7 | 7 | 7 | 8 | 7 | 10 | 6 | 5 | 5 | 5 | 8 | 5 | 8 | 6 |
| 200 | 11 | 13 | 16 | 15 | 16 | 12 | 17 | 12 | 14 | 12 | 7 | 12 | 13 | 17 | 13 |
| 300 | 18 | 19 | 25 | 20 | 23 | 18 | 24 | 18 | 21 | 17 | 16 | 17 | 20 | 25 | 19 |
| 400 | 21 | 25 | 30 | 27 | 28 | 31 | 33 | 26 | 34 | 22 | 22 | 22 | 28 | 28 | 23 |
| 500 | 30 | 27 | 40 | 36 | 35 | 38 | 35 | 35 | 39 | 32 | 25 | 30 | 34 | 34 | 30 |
| 600 | 38 | 38 | 46 | 45 | 40 | 45 | 41 | 38 | 48 | 42 | 28 | 36 | 41 | 42 | 32 |
| 700 | 46 | 45 | 55 | 54 | 47 | 53 | 48 | 45 | 57 | 48 | 32 | 41 | 42 | 51 | 36 |
| 800 | 55 | 48 | 67 | 59 | 54 | 64 | 53 | 50 | 60 | 54 | 41 | 47 | 46 | 58 | 44 |
| 900 | 62 | 58 | 78 | 63 | 61 | 69 | 63 | 58 | 66 | 57 | 48 | 52 | 51 | 64 | 50 |
| 1000 | 70 | 62 | 85 | 68 | 68 | 77 | 69 | 67 | 74 | 68 | 52 | 56 | 59 | 69 | 56 |
| 1100 | 74 | 67 | 96 | 74 | 72 | 86 | 78 | 72 | 81 | 73 | 59 | 60 | 68 | 76 | 64 |
| 1200 | 83 | 73 | 101 | 80 | 79 | 93 | 87 | 79 | 81 | 81 | 65 | 67 | 76 | 85 | 70 |
| 1300 | 91 | 79 | 106 | 83 | 87 | 97 | 96 | 91 | 86 | 92 | 71 | 70 | 86 | 89 | 76 |
| 1400 | 97 | 84 | 114 | 88 | 91 | 103 | 100 | 97 | 91 | 101 | 85 | 79 | 95 | 94 | 81 |
| 1500 | 104 | 94 | 121 | 95 | 103 | 112 | 105 | 100 | 95 | 104 | 90 | 86 | 102 | 101 | 88 |

Table 32

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.37703 | 0.45225 | 0.43812 | 0.42792 | 0.50109 | 0.43952 | 0.6301 | 0.38886 | 0.31512 | 0.31978 | 0.31065 | 0.49405 | 0.30458 | 0.49152 | 0.36715 |
| 200 | 0.69407 | 0.83058 | 1.01209 | 0.93974 | 0.99503 | 0.74653 | 1.08441 | 0.76492 | 0.89771 | 0.745 | 0.42901 | 0.73093 | 0.80954 | 1.06492 | 0.79981 |
| 300 | 1.13136 | 1.19333 | 1.58801 | 1.24226 | 1.43281 | 1.11007 | 1.53554 | 1.14202 | 1.34521 | 1.06926 | 1.00742 | 1.04608 | 1.25766 | 1.58232 | 1.17545 |
| 400 | 1.32721 | 1.57107 | 1.90944 | 1.67328 | 1.72846 | 1.87993 | 2.09415 | 1.64945 | 2.18113 | 1.37709 | 1.39385 | 1.36688 | 1.7584 | 1.77185 | 1.42628 |
| 500 | 1.89478 | 1.68903 | 2.5439 | 2.24191 | 2.17417 | 2.31681 | 2.22093 | 2.20509 | 2.50893 | 2.01244 | 1.58303 | 1.85808 | 2.12081 | 2.14111 | 1.85785 |
| 600 | 2.38378 | 2.37453 | 2.907 | 2.79409 | 2.47261 | 2.75428 | 2.58676 | 2.39881 | 3.09708 | 2.64608 | 1.77434 | 2.23714 | 2.57007 | 2.63546 | 1.98803 |
| 700 | 2.90429 | 2.81674 | 3.44125 | 3.32642 | 2.89392 | 3.24791 | 3.03295 | 2.83513 | 3.65672 | 3.03719 | 2.01987 | 2.54168 | 2.62911 | 3.18719 | 2.24042 |
| 800 | 3.45222 | 2.99727 | 4.19261 | 3.63338 | 3.32857 | 3.95217 | 3.35846 | 3.15991 | 3.84515 | 3.41563 | 2.56338 | 2.92127 | 2.8728 | 3.63544 | 2.74501 |
| 900 | 3.89474 | 3.6242 | 4.86997 | 3.88348 | 3.77087 | 4.27593 | 3.99059 | 3.65806 | 4.22468 | 3.60882 | 3.00099 | 3.23463 | 3.18368 | 4.02576 | 3.12912 |
| 1000 | 4.3946 | 3.86141 | 5.31535 | 4.1887 | 4.21817 | 4.79498 | 4.36236 | 4.22399 | 4.72891 | 4.30503 | 3.24287 | 3.47815 | 3.67937 | 4.33462 | 3.49598 |
| 1100 | 4.6522 | 4.18596 | 5.99989 | 4.57264 | 4.48049 | 5.36307 | 4.93226 | 4.5416 | 5.16061 | 4.60504 | 3.67689 | 3.72639 | 4.22723 | 4.77542 | 3.98006 |
| 1200 | 5.21413 | 4.55886 | 6.30258 | 4.94759 | 4.91146 | 5.80519 | 5.49007 | 4.95751 | 5.16061 | 5.10963 | 4.05937 | 4.17469 | 4.71112 | 5.3453 | 4.35061 |
| 1300 | 5.70223 | 4.94192 | 6.6195 | 5.14262 | 5.41729 | 6.05381 | 6.0385 | 5.71844 | 5.47513 | 5.80761 | 4.44715 | 4.36371 | 5.30976 | 5.60678 | 4.70801 |
| 1400 | 6.08585 | 5.25851 | 7.13275 | 5.46391 | 5.65445 | 6.4295 | 6.29763 | 6.09267 | 5.77441 | 6.36573 | 5.30038 | 4.93585 | 5.87747 | 5.91681 | 5.02892 |
| 1500 | 6.48661 | 5.88043 | 7.57841 | 5.90476 | 6.40428 | 6.99315 | 6.60414 | 6.28976 | 6.03212 | 6.55817 | 5.60392 | 5.36106 | 6.31508 | 6.35177 | 5.45699 |

Table 33

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.00377 | 0.00452 | 0.00438 | 0.00428 | 0.00501 | 0.0044 | 0.0063 | 0.00387 | 0.00315 | 0.0032 | 0.00311 | 0.00494 | 0.00305 | 0.00492 | 0.00367 |
| 200 | 0.00347 | 0.00415 | 0.00506 | 0.0047 | 0.00498 | 0.00373 | 0.00542 | 0.00382 | 0.00449 | 0.00373 | 0.00215 | 0.00365 | 0.00405 | 0.00532 | 0.004 |
| 300 | 0.00377 | 0.00398 | 0.00529 | 0.00414 | 0.00478 | 0.0037 | 0.00512 | 0.00381 | 0.00448 | 0.00356 | 0.00336 | 0.00349 | 0.00419 | 0.00527 | 0.00392 |
| 400 | 0.00332 | 0.00393 | 0.00477 | 0.00418 | 0.00432 | 0.0047 | 0.00524 | 0.00412 | 0.00545 | 0.00344 | 0.00348 | 0.00342 | 0.0044 | 0.00443 | 0.00357 |
| 500 | 0.00379 | 0.00338 | 0.00509 | 0.00448 | 0.00435 | 0.00463 | 0.00444 | 0.00441 | 0.00502 | 0.00402 | 0.00317 | 0.00372 | 0.00424 | 0.00428 | 0.00372 |
| 600 | 0.00397 | 0.00396 | 0.00485 | 0.00466 | 0.00412 | 0.00459 | 0.00431 | 0.004 | 0.00516 | 0.00441 | 0.00296 | 0.00373 | 0.00428 | 0.00439 | 0.00331 |
| 700 | 0.00415 | 0.00402 | 0.00492 | 0.00475 | 0.00413 | 0.00464 | 0.00433 | 0.00405 | 0.00522 | 0.00434 | 0.00289 | 0.00363 | 0.00376 | 0.00455 | 0.0032 |
| 800 | 0.00432 | 0.00375 | 0.00524 | 0.00454 | 0.00416 | 0.00494 | 0.0042 | 0.00395 | 0.00481 | 0.00427 | 0.0032 | 0.00365 | 0.00359 | 0.00454 | 0.00343 |
| 900 | 0.00433 | 0.00403 | 0.00541 | 0.00431 | 0.00419 | 0.00475 | 0.00443 | 0.00406 | 0.00469 | 0.00401 | 0.00333 | 0.00359 | 0.00354 | 0.00447 | 0.00348 |
| 1000 | 0.00439 | 0.00386 | 0.00532 | 0.00419 | 0.00422 | 0.00479 | 0.00436 | 0.00422 | 0.00473 | 0.00431 | 0.00324 | 0.00348 | 0.00368 | 0.00433 | 0.0035 |
| 1100 | 0.00423 | 0.00381 | 0.00545 | 0.00416 | 0.00407 | 0.00488 | 0.00448 | 0.00413 | 0.00469 | 0.00419 | 0.00334 | 0.00339 | 0.00384 | 0.00434 | 0.00362 |
| 1200 | 0.00435 | 0.0038 | 0.00525 | 0.00412 | 0.00409 | 0.00484 | 0.00458 | 0.00413 | 0.0043 | 0.00426 | 0.00338 | 0.00348 | 0.00393 | 0.00445 | 0.00363 |
| 1300 | 0.00439 | 0.0038 | 0.00509 | 0.00396 | 0.00417 | 0.00466 | 0.00465 | 0.0044 | 0.00421 | 0.00447 | 0.00342 | 0.00336 | 0.00408 | 0.00431 | 0.00362 |
| 1400 | 0.00435 | 0.00376 | 0.00509 | 0.0039 | 0.00404 | 0.00459 | 0.0045 | 0.00435 | 0.00412 | 0.00455 | 0.00379 | 0.00353 | 0.0042 | 0.00423 | 0.00359 |
| 1500 | 0.00432 | 0.00392 | 0.00505 | 0.00394 | 0.00427 | 0.00466 | 0.0044 | 0.00419 | 0.00402 | 0.00437 | 0.00374 | 0.00357 | 0.00421 | 0.00423 | 0.00364 |

Table 34

Scenario 2. Bidder1 underbid by 0.075 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.01134 | 0.06322 | 0.05986 | 0.06169 | 0.06163 | 0.06171 | 0.06277 | 0.06175 | 0.062 | 0.06271 | 0.06504 | 0.06405 | 0.06377 | 0.06418 | 0.06417 |
| 200 | 0.01178 | 0.06307 | 0.06182 | 0.06256 | 0.06107 | 0.06181 | 0.06322 | 0.06194 | 0.0617 | 0.06201 | 0.0649 | 0.06404 | 0.06303 | 0.06319 | 0.063 |
| 300 | 0.01187 | 0.0628 | 0.0627 | 0.06182 | 0.06165 | 0.06167 | 0.06287 | 0.0616 | 0.06144 | 0.06139 | 0.06355 | 0.06367 | 0.06359 | 0.06369 | 0.06253 |
| 400 | 0.01206 | 0.06302 | 0.06271 | 0.06236 | 0.0624 | 0.0611 | 0.06295 | 0.06136 | 0.06139 | 0.06191 | 0.06346 | 0.06392 | 0.06317 | 0.06236 | 0.06256 |
| 500 | 0.01127 | 0.06294 | 0.06326 | 0.06227 | 0.06255 | 0.06179 | 0.06308 | 0.06063 | 0.06159 | 0.06205 | 0.06333 | 0.06369 | 0.06243 | 0.06306 | 0.06284 |
| 600 | 0.01135 | 0.06268 | 0.06316 | 0.06239 | 0.06281 | 0.06224 | 0.06306 | 0.06092 | 0.06182 | 0.06142 | 0.06304 | 0.06332 | 0.06266 | 0.06319 | 0.0629 |
| 700 | 0.01165 | 0.0628 | 0.06335 | 0.06226 | 0.06301 | 0.06226 | 0.06286 | 0.06103 | 0.06183 | 0.06112 | 0.06265 | 0.06334 | 0.06275 | 0.06308 | 0.06274 |
| 800 | 0.01156 | 0.06242 | 0.06343 | 0.06218 | 0.06299 | 0.06218 | 0.06303 | 0.06137 | 0.06145 | 0.06163 | 0.06243 | 0.06344 | 0.06245 | 0.06327 | 0.06282 |
| 900 | 0.01151 | 0.06243 | 0.06319 | 0.06232 | 0.06305 | 0.06235 | 0.06304 | 0.06163 | 0.0614 | 0.06162 | 0.06229 | 0.06325 | 0.06233 | 0.06328 | 0.06289 |
| 1000 | 0.01166 | 0.06259 | 0.06323 | 0.06218 | 0.06302 | 0.06229 | 0.06303 | 0.06188 | 0.06146 | 0.06144 | 0.06233 | 0.0631 | 0.06214 | 0.06317 | 0.06246 |
| 1100 | 0.01163 | 0.06268 | 0.06314 | 0.06218 | 0.06298 | 0.0624 | 0.06308 | 0.06176 | 0.06134 | 0.06158 | 0.06224 | 0.06313 | 0.06219 | 0.06295 | 0.06254 |
| 1200 | 0.01161 | 0.06292 | 0.06324 | 0.06223 | 0.06296 | 0.06244 | 0.0631 | 0.06162 | 0.06155 | 0.06172 | 0.06219 | 0.06312 | 0.06237 | 0.06311 | 0.06263 |
| 1300 | 0.01164 | 0.06277 | 0.06321 | 0.06215 | 0.06298 | 0.06238 | 0.06318 | 0.06169 | 0.06152 | 0.06177 | 0.06217 | 0.06322 | 0.06247 | 0.06298 | 0.0627 |
| 1400 | 0.0117 | 0.06283 | 0.06308 | 0.06202 | 0.06308 | 0.06256 | 0.06301 | 0.06178 | 0.06154 | 0.06197 | 0.0623 | 0.0632 | 0.06234 | 0.06298 | 0.06279 |
| 1500 | 0.01161 | 0.06286 | 0.06309 | 0.06206 | 0.06304 | 0.06252 | 0.06295 | 0.06179 | 0.0616 | 0.06183 | 0.0624 | 0.06321 | 0.06242 | 0.06276 | 0.06288 |

Table 35

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 12 | 9 | 4 | 4 | 7 | 9 | 2 | 7 | 9 | 4 | 6 | 8 | 8 | 5 | 6 |
| 200 | 27 | 16 | 9 | 9 | 12 | 14 | 8 | 16 | 17 | 13 | 10 | 14 | 12 | 9 | 14 |
| 300 | 36 | 24 | 14 | 18 | 17 | 21 | 16 | 21 | 23 | 19 | 19 | 21 | 19 | 13 | 19 |
| 400 | 45 | 30 | 21 | 21 | 24 | 34 | 25 | 26 | 32 | 30 | 23 | 25 | 24 | 16 | 24 |
| 500 | 59 | 35 | 27 | 25 | 36 | 40 | 29 | 32 | 38 | 35 | 28 | 27 | 34 | 26 | 29 |
| 600 | 68 | 42 | 29 | 31 | 44 | 54 | 33 | 36 | 45 | 43 | 34 | 33 | 40 | 29 | 39 |
| 700 | 79 | 46 | 31 | 38 | 50 | 60 | 42 | 43 | 55 | 47 | 43 | 38 | 45 | 36 | 47 |
| 800 | 90 | 52 | 36 | 40 | 52 | 65 | 52 | 51 | 62 | 54 | 49 | 47 | 54 | 42 | 54 |
| 900 | 99 | 58 | 44 | 42 | 65 | 72 | 58 | 58 | 66 | 61 | 55 | 59 | 60 | 44 | 59 |
| 1000 | 109 | 64 | 49 | 47 | 73 | 80 | 63 | 68 | 70 | 68 | 63 | 63 | 70 | 48 | 65 |
| 1100 | 132 | 72 | 51 | 52 | 80 | 85 | 65 | 75 | 77 | 75 | 75 | 67 | 72 | 54 | 68 |
| 1200 | 145 | 78 | 61 | 55 | 84 | 91 | 70 | 81 | 83 | 83 | 81 | 69 | 83 | 60 | 76 |
| 1300 | 159 | 85 | 71 | 62 | 89 | 95 | 80 | 85 | 84 | 88 | 90 | 73 | 90 | 67 | 82 |
| 1400 | 168 | 89 | 79 | 65 | 95 | 104 | 83 | 87 | 89 | 97 | 94 | 80 | 99 | 79 | 92 |
| 1500 | 179 | 97 | 84 | 69 | 101 | 109 | 90 | 97 | 96 | 102 | 100 | 90 | 102 | 84 | 100 |

Table 36

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.13613 | 0.56894 | 0.23944 | 0.24678 | 0.43139 | 0.55337 | 0.12534 | 0.43226 | 0.558 | 0.25086 | 0.39024 | 0.51244 | 0.51017 | 0.3209 | 0.38503 |
| 200 | 0.31808 | 1.00911 | 0.55641 | 0.56301 | 0.73285 | 0.8654 | 0.50576 | 0.99103 | 1.04888 | 0.80617 | 0.64903 | 0.89662 | 0.75634 | 0.56869 | 0.88199 |
| 300 | 0.42745 | 1.50721 | 0.87778 | 1.11282 | 1.04808 | 1.29502 | 1.00588 | 1.29357 | 1.41307 | 1.16642 | 1.2074 | 1.33714 | 1.20817 | 0.82798 | 1.18808 |
| 400 | 0.54252 | 1.8905 | 1.31697 | 1.30958 | 1.49765 | 2.07746 | 1.57381 | 1.59529 | 1.96464 | 1.85723 | 1.45966 | 1.59797 | 1.5161 | 0.99772 | 1.50145 |
| 500 | 0.66473 | 2.20296 | 1.70803 | 1.55669 | 2.2517 | 2.47173 | 1.8293 | 1.94026 | 2.34027 | 2.17187 | 1.77322 | 1.71955 | 2.12273 | 1.63959 | 1.82242 |
| 600 | 0.77171 | 2.63249 | 1.8317 | 1.93404 | 2.76347 | 3.36073 | 2.08114 | 2.19322 | 2.7818 | 2.6409 | 2.14337 | 2.08951 | 2.50652 | 1.83247 | 2.45294 |
| 700 | 0.92032 | 2.88883 | 1.96374 | 2.3657 | 3.15059 | 3.73587 | 2.64016 | 2.62448 | 3.40082 | 2.87259 | 2.6939 | 2.40677 | 2.82388 | 2.27079 | 2.94863 |
| 800 | 1.03998 | 3.24568 | 2.2834 | 2.48708 | 3.27536 | 4.04163 | 3.27739 | 3.13 | 3.81018 | 3.3279 | 3.0592 | 2.98164 | 3.37236 | 2.65743 | 3.39238 |
| 900 | 1.13991 | 3.62071 | 2.78032 | 2.61752 | 4.09849 | 4.48904 | 3.6564 | 3.57427 | 4.05257 | 3.75864 | 3.42617 | 3.73181 | 3.73956 | 2.78446 | 3.71063 |
| 1000 | 1.27136 | 4.00583 | 3.09808 | 2.92239 | 4.60046 | 4.98287 | 3.97085 | 4.20768 | 4.30187 | 4.17784 | 3.92702 | 3.97515 | 4.34982 | 3.03227 | 4.06007 |
| 1100 | 1.53512 | 4.51319 | 3.21999 | 3.23314 | 5.03838 | 5.30382 | 4.09997 | 4.63232 | 4.72319 | 4.61847 | 4.66782 | 4.22954 | 4.4774 | 3.39943 | 4.2525 |
| 1200 | 1.68365 | 4.90796 | 3.85757 | 3.42278 | 5.28836 | 5.68161 | 4.4172 | 4.99148 | 5.10874 | 5.12292 | 5.03755 | 4.35538 | 5.1766 | 3.78674 | 4.76006 |
| 1300 | 1.85138 | 5.33582 | 4.48814 | 3.85302 | 5.60555 | 5.92618 | 5.05404 | 5.24372 | 5.16788 | 5.43597 | 5.59522 | 4.61502 | 5.62271 | 4.21949 | 5.14156 |
| 1400 | 1.96511 | 5.59154 | 4.98324 | 4.03135 | 5.9924 | 6.50661 | 5.22945 | 5.37483 | 5.47737 | 6.01073 | 5.85617 | 5.05635 | 6.17201 | 4.97512 | 5.77687 |
| 1500 | 2.07798 | 6.09721 | 5.29944 | 4.28189 | 6.36677 | 6.81471 | 5.66514 | 5.99345 | 5.91407 | 6.30696 | 6.24025 | 5.68861 | 6.36726 | 5.27144 | 6.28848 |

Table 37

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.00136 | 0.00569 | 0.00239 | 0.00247 | 0.00431 | 0.00555 | 0.00126 | 0.00432 | 0.00558 | 0.00251 | 0.0039 | 0.00512 | 0.0051 | 0.00321 | 0.00385 |
| 200 | 0.00159 | 0.00505 | 0.00278 | 0.00282 | 0.00366 | 0.00433 | 0.00253 | 0.00496 | 0.00524 | 0.00403 | 0.00325 | 0.00448 | 0.00378 | 0.00284 | 0.00441 |
| 300 | 0.00142 | 0.00502 | 0.00293 | 0.00371 | 0.00349 | 0.00432 | 0.00335 | 0.00431 | 0.00471 | 0.00389 | 0.00402 | 0.00446 | 0.00403 | 0.00276 | 0.00396 |
| 400 | 0.00136 | 0.00473 | 0.00329 | 0.00327 | 0.00374 | 0.00519 | 0.00393 | 0.00399 | 0.00491 | 0.00464 | 0.00365 | 0.00399 | 0.00379 | 0.00249 | 0.00375 |
| 500 | 0.00133 | 0.00441 | 0.00342 | 0.00311 | 0.0045 | 0.00494 | 0.00366 | 0.00388 | 0.00468 | 0.00434 | 0.00355 | 0.00344 | 0.00425 | 0.00328 | 0.00364 |
| 600 | 0.00129 | 0.00439 | 0.00305 | 0.00322 | 0.00461 | 0.0056 | 0.00347 | 0.00366 | 0.00464 | 0.0044 | 0.00357 | 0.00348 | 0.00418 | 0.00305 | 0.00409 |
| 700 | 0.00131 | 0.00413 | 0.00281 | 0.00338 | 0.0045 | 0.00534 | 0.00377 | 0.00375 | 0.00486 | 0.0041 | 0.00385 | 0.00344 | 0.00403 | 0.00324 | 0.00421 |
| 800 | 0.0013 | 0.00406 | 0.00285 | 0.00311 | 0.00409 | 0.00505 | 0.0041 | 0.00391 | 0.00476 | 0.00416 | 0.00382 | 0.00373 | 0.00422 | 0.00332 | 0.00424 |
| 900 | 0.00127 | 0.00402 | 0.00309 | 0.00291 | 0.00455 | 0.00499 | 0.00406 | 0.00397 | 0.0045 | 0.00418 | 0.00381 | 0.00415 | 0.00416 | 0.00309 | 0.00412 |
| 1000 | 0.00127 | 0.00401 | 0.0031 | 0.00292 | 0.0046 | 0.00498 | 0.00397 | 0.00421 | 0.0043 | 0.00418 | 0.00393 | 0.00398 | 0.00435 | 0.00303 | 0.00406 |
| 1100 | 0.0014 | 0.0041 | 0.00293 | 0.00294 | 0.00458 | 0.00482 | 0.00373 | 0.00421 | 0.00429 | 0.0042 | 0.00424 | 0.00385 | 0.00407 | 0.00309 | 0.00387 |
| 1200 | 0.0014 | 0.00409 | 0.00321 | 0.00285 | 0.00441 | 0.00473 | 0.00368 | 0.00416 | 0.00426 | 0.00427 | 0.0042 | 0.00363 | 0.00431 | 0.00316 | 0.00397 |
| 1300 | 0.00142 | 0.0041 | 0.00345 | 0.00296 | 0.00431 | 0.00456 | 0.00389 | 0.00403 | 0.00398 | 0.00418 | 0.0043 | 0.00355 | 0.00433 | 0.00325 | 0.00396 |
| 1400 | 0.0014 | 0.00399 | 0.00356 | 0.00288 | 0.00428 | 0.00465 | 0.00374 | 0.00384 | 0.00391 | 0.00429 | 0.00418 | 0.00361 | 0.00441 | 0.00355 | 0.00413 |
| 1500 | 0.00139 | 0.00406 | 0.00353 | 0.00285 | 0.00424 | 0.00454 | 0.00378 | 0.004 | 0.00394 | 0.0042 | 0.00416 | 0.00379 | 0.00424 | 0.00351 | 0.00419 |

Table 38

Scenario 3. Bidders “1”, “2” and “3” underbid by 0.055 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0062 | 0.0076 | 0.0065 | 0.0656 | 0.0652 | 0.0629 | 0.0601 | 0.0606 | 0.0622 | 0.0602 | 0.0639 | 0.0603 | 0.0578 | 0.0621 | 0.0641 |
| 200 | 0.0058 | 0.0076 | 0.0058 | 0.0638 | 0.0648 | 0.0639 | 0.062 | 0.0626 | 0.0636 | 0.0613 | 0.0632 | 0.0606 | 0.0605 | 0.0623 | 0.0626 |
| 300 | 0.0055 | 0.0073 | 0.0065 | 0.0628 | 0.0647 | 0.0635 | 0.0625 | 0.062 | 0.0632 | 0.062 | 0.0638 | 0.0606 | 0.0614 | 0.0621 | 0.0631 |
| 400 | 0.0055 | 0.0074 | 0.0062 | 0.0628 | 0.0644 | 0.0641 | 0.0627 | 0.0623 | 0.0633 | 0.0619 | 0.064 | 0.0612 | 0.0614 | 0.0624 | 0.0631 |
| 500 | 0.0059 | 0.0071 | 0.0063 | 0.0625 | 0.0642 | 0.0642 | 0.0628 | 0.062 | 0.0637 | 0.0623 | 0.064 | 0.0606 | 0.0621 | 0.0623 | 0.0629 |
| 600 | 0.0058 | 0.0068 | 0.0063 | 0.0627 | 0.0635 | 0.0641 | 0.0632 | 0.0623 | 0.0639 | 0.0625 | 0.064 | 0.0607 | 0.0624 | 0.0625 | 0.0625 |
| 700 | 0.0059 | 0.0066 | 0.0064 | 0.0626 | 0.0637 | 0.0643 | 0.0634 | 0.0623 | 0.0638 | 0.0623 | 0.064 | 0.0606 | 0.0626 | 0.0621 | 0.0629 |
| 800 | 0.0057 | 0.0066 | 0.0067 | 0.0626 | 0.0636 | 0.0645 | 0.0629 | 0.062 | 0.0635 | 0.0623 | 0.0638 | 0.0609 | 0.0624 | 0.062 | 0.0628 |
| 900 | 0.0057 | 0.0066 | 0.0063 | 0.0627 | 0.0636 | 0.0641 | 0.0624 | 0.062 | 0.0631 | 0.0625 | 0.0639 | 0.0611 | 0.0624 | 0.062 | 0.0628 |
| 1000 | 0.0058 | 0.0067 | 0.0064 | 0.0625 | 0.0636 | 0.0641 | 0.0625 | 0.0623 | 0.0627 | 0.0623 | 0.0639 | 0.0612 | 0.0625 | 0.0624 | 0.063 |
| 1100 | 0.0057 | 0.0066 | 0.0064 | 0.0626 | 0.0634 | 0.064 | 0.0625 | 0.0624 | 0.0629 | 0.0624 | 0.0639 | 0.0612 | 0.0628 | 0.0625 | 0.0628 |
| 1200 | 0.0059 | 0.0068 | 0.0064 | 0.0625 | 0.0632 | 0.0641 | 0.0625 | 0.0621 | 0.063 | 0.0625 | 0.0638 | 0.0612 | 0.0627 | 0.0625 | 0.0628 |
| 1300 | 0.0058 | 0.0067 | 0.0065 | 0.0626 | 0.0632 | 0.064 | 0.0626 | 0.0618 | 0.0628 | 0.0626 | 0.0638 | 0.0612 | 0.0627 | 0.0624 | 0.0628 |
| 1400 | 0.0058 | 0.0066 | 0.0065 | 0.0625 | 0.0632 | 0.064 | 0.0626 | 0.062 | 0.0629 | 0.0626 | 0.0636 | 0.0613 | 0.0627 | 0.0625 | 0.0629 |
| 1500 | 0.0057 | 0.0066 | 0.0065 | 0.0625 | 0.0631 | 0.0638 | 0.0626 | 0.0617 | 0.063 | 0.0624 | 0.0635 | 0.0615 | 0.0629 | 0.0626 | 0.0629 |

Table 39

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 14 | 13 | 9 | 6 | 7 | 4 | 5 | 3 | 4 | 6 | 7 | 4 | 4 | 8 | 6 |
| 200 | 27 | 22 | 19 | 15 | 13 | 11 | 12 | 5 | 8 | 14 | 13 | 11 | 7 | 10 | 13 |
| 300 | 40 | 36 | 29 | 19 | 17 | 15 | 19 | 11 | 10 | 20 | 19 | 20 | 11 | 17 | 17 |
| 400 | 52 | 45 | 41 | 24 | 23 | 22 | 21 | 19 | 16 | 27 | 23 | 25 | 15 | 26 | 21 |
| 500 | 63 | 52 | 51 | 34 | 28 | 27 | 31 | 22 | 18 | 31 | 28 | 35 | 27 | 28 | 25 |
| 600 | 77 | 62 | 57 | 43 | 33 | 34 | 40 | 25 | 23 | 38 | 33 | 39 | 33 | 32 | 31 |
| 700 | 88 | 71 | 77 | 47 | 36 | 39 | 43 | 31 | 28 | 42 | 37 | 45 | 40 | 36 | 40 |
| 800 | 96 | 84 | 88 | 54 | 44 | 47 | 49 | 33 | 32 | 46 | 40 | 55 | 44 | 39 | 49 |
| 900 | 107 | 91 | 98 | 59 | 52 | 57 | 56 | 37 | 41 | 57 | 42 | 60 | 49 | 40 | 54 |
| 1000 | 119 | 103 | 109 | 65 | 56 | 63 | 59 | 43 | 51 | 62 | 46 | 62 | 54 | 49 | 59 |
| 1100 | 131 | 115 | 124 | 67 | 64 | 71 | 62 | 44 | 58 | 70 | 49 | 63 | 62 | 53 | 67 |
| 1200 | 144 | 125 | 136 | 82 | 71 | 74 | 65 | 48 | 62 | 76 | 54 | 63 | 68 | 56 | 76 |
| 1300 | 158 | 135 | 152 | 87 | 76 | 82 | 67 | 52 | 68 | 80 | 59 | 70 | 70 | 62 | 82 |
| 1400 | 167 | 154 | 160 | 93 | 81 | 84 | 69 | 60 | 72 | 85 | 68 | 78 | 71 | 71 | 87 |
| 1500 | 176 | 163 | 168 | 99 | 86 | 88 | 70 | 67 | 79 | 91 | 76 | 84 | 76 | 82 | 95 |

Table 40

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0873 | 0.0987 | 0.0589 | 0.3939 | 0.4562 | 0.2515 | 0.3007 | 0.1817 | 0.2489 | 0.3612 | 0.4476 | 0.2412 | 0.2312 | 0.4966 | 0.3846 |
| 200 | 0.1566 | 0.1664 | 0.1105 | 0.9569 | 0.8427 | 0.7033 | 0.7438 | 0.3129 | 0.5088 | 0.8579 | 0.8213 | 0.6668 | 0.4236 | 0.6233 | 0.8137 |
| 300 | 0.2185 | 0.2636 | 0.1895 | 1.1941 | 1.1006 | 0.9524 | 1.1873 | 0.6823 | 0.6324 | 1.2398 | 1.2128 | 1.212 | 0.6753 | 1.0561 | 1.0723 |
| 400 | 0.2874 | 0.3324 | 0.254 | 1.5064 | 1.481 | 1.4095 | 1.3169 | 1.1842 | 1.0133 | 1.6711 | 1.4711 | 1.5311 | 0.9214 | 1.6225 | 1.3243 |
| 500 | 0.3689 | 0.3694 | 0.3206 | 2.1257 | 1.7971 | 1.7341 | 1.947 | 1.3643 | 1.1463 | 1.9314 | 1.7912 | 2.1222 | 1.6768 | 1.744 | 1.5729 |
| 600 | 0.4502 | 0.419 | 0.3586 | 2.6967 | 2.0959 | 2.1793 | 2.5296 | 1.5571 | 1.4696 | 2.3738 | 2.112 | 2.3664 | 2.0596 | 2.0015 | 1.9362 |
| 700 | 0.5153 | 0.4672 | 0.4932 | 2.941 | 2.2923 | 2.5077 | 2.7256 | 1.9311 | 1.7861 | 2.617 | 2.3684 | 2.729 | 2.5036 | 2.2342 | 2.5148 |
| 800 | 0.5437 | 0.5523 | 0.5888 | 3.3806 | 2.7978 | 3.0329 | 3.0797 | 2.0451 | 2.0319 | 2.8674 | 2.5524 | 3.3499 | 2.7438 | 2.4172 | 3.0756 |
| 900 | 0.6103 | 0.6015 | 0.6216 | 3.7001 | 3.3048 | 3.6536 | 3.495 | 2.2947 | 2.5872 | 3.5605 | 2.6839 | 3.6637 | 3.0561 | 2.4806 | 3.3921 |
| 1000 | 0.6872 | 0.6928 | 0.7003 | 4.0638 | 3.5589 | 4.0414 | 3.6852 | 2.68 | 3.1979 | 3.8647 | 2.9412 | 3.7951 | 3.3763 | 3.0571 | 3.7148 |
| 1100 | 0.7461 | 0.7644 | 0.7915 | 4.1945 | 4.0602 | 4.5463 | 3.8773 | 2.7443 | 3.6468 | 4.3701 | 3.1287 | 3.8536 | 3.8907 | 3.3099 | 4.21 |
| 1200 | 0.8529 | 0.8467 | 0.8637 | 5.1255 | 4.487 | 4.7402 | 4.0649 | 2.9793 | 3.9071 | 4.7513 | 3.4453 | 3.8536 | 4.2628 | 3.5 | 4.7726 |
| 1300 | 0.9156 | 0.9026 | 0.9949 | 5.443 | 4.8001 | 5.2469 | 4.1935 | 3.2121 | 4.2733 | 5.0102 | 3.765 | 4.2855 | 4.3894 | 3.871 | 5.1477 |
| 1400 | 0.974 | 1.0135 | 1.0462 | 5.8137 | 5.1221 | 5.3775 | 4.3183 | 3.7184 | 4.529 | 5.3189 | 4.3262 | 4.7841 | 4.455 | 4.4365 | 5.471 |
| 1500 | 1.0074 | 1.0798 | 1.0914 | 6.1872 | 5.4229 | 5.6154 | 4.3818 | 4.1325 | 4.9732 | 5.6753 | 4.8255 | 5.1678 | 4.7795 | 5.1327 | 5.9767 |

Table 41

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0009 | 0.001 | 0.0006 | 0.0039 | 0.0046 | 0.0025 | 0.003 | 0.0018 | 0.0025 | 0.0036 | 0.0045 | 0.0024 | 0.0023 | 0.004966 | 0.003846 |
| 200 | 0.0008 | 0.0008 | 0.0006 | 0.0048 | 0.0042 | 0.0035 | 0.0037 | 0.0016 | 0.0025 | 0.0043 | 0.0041 | 0.0033 | 0.0021 | 0.003117 | 0.004068 |
| 300 | 0.0007 | 0.0009 | 0.0006 | 0.004 | 0.0037 | 0.0032 | 0.004 | 0.0023 | 0.0021 | 0.0041 | 0.004 | 0.004 | 0.0023 | 0.00352 | 0.003574 |
| 400 | 0.0007 | 0.0008 | 0.0006 | 0.0038 | 0.0037 | 0.0035 | 0.0033 | 0.003 | 0.0025 | 0.0042 | 0.0037 | 0.0038 | 0.0023 | 0.004056 | 0.003311 |
| 500 | 0.0007 | 0.0007 | 0.0006 | 0.0043 | 0.0036 | 0.0035 | 0.0039 | 0.0027 | 0.0023 | 0.0039 | 0.0036 | 0.0042 | 0.0034 | 0.003488 | 0.003146 |
| 600 | 0.0008 | 0.0007 | 0.0006 | 0.0045 | 0.0035 | 0.0036 | 0.0042 | 0.0026 | 0.0024 | 0.004 | 0.0035 | 0.0039 | 0.0034 | 0.003336 | 0.003227 |
| 700 | 0.0007 | 0.0007 | 0.0007 | 0.0042 | 0.0033 | 0.0036 | 0.0039 | 0.0028 | 0.0026 | 0.0037 | 0.0034 | 0.0039 | 0.0036 | 0.003192 | 0.003593 |
| 800 | 0.0007 | 0.0007 | 0.0007 | 0.0042 | 0.0035 | 0.0038 | 0.0038 | 0.0026 | 0.0025 | 0.0036 | 0.0032 | 0.0042 | 0.0034 | 0.003022 | 0.003844 |
| 900 | 0.0007 | 0.0007 | 0.0007 | 0.0041 | 0.0037 | 0.0041 | 0.0039 | 0.0025 | 0.0029 | 0.004 | 0.003 | 0.0041 | 0.0034 | 0.002756 | 0.003769 |
| 1000 | 0.0007 | 0.0007 | 0.0007 | 0.0041 | 0.0036 | 0.004 | 0.0037 | 0.0027 | 0.0032 | 0.0039 | 0.0029 | 0.0038 | 0.0034 | 0.003057 | 0.003715 |
| 1100 | 0.0007 | 0.0007 | 0.0007 | 0.0038 | 0.0037 | 0.0041 | 0.0035 | 0.0025 | 0.0033 | 0.004 | 0.0028 | 0.0035 | 0.0035 | 0.003009 | 0.003827 |
| 1200 | 0.0007 | 0.0007 | 0.0007 | 0.0043 | 0.0037 | 0.004 | 0.0034 | 0.0025 | 0.0033 | 0.004 | 0.0029 | 0.0032 | 0.0036 | 0.002917 | 0.003977 |
| 1300 | 0.0007 | 0.0007 | 0.0008 | 0.0042 | 0.0037 | 0.004 | 0.0032 | 0.0025 | 0.0033 | 0.0039 | 0.0029 | 0.0033 | 0.0034 | 0.002978 | 0.00396 |
| 1400 | 0.0007 | 0.0007 | 0.0007 | 0.0042 | 0.0037 | 0.0038 | 0.0031 | 0.0027 | 0.0032 | 0.0038 | 0.0031 | 0.0034 | 0.0032 | 0.003169 | 0.003908 |
| 1500 | 0.0007 | 0.0007 | 0.0007 | 0.0041 | 0.0036 | 0.0037 | 0.0029 | 0.0028 | 0.0033 | 0.0038 | 0.0032 | 0.0034 | 0.0032 | 0.003422 | 0.003984 |

Table 42

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.055, 0.05, 0.045 and 0.04 respectively while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.00361 | 0.01086 | 0.0181 | 0.02178 | 0.06058 | 0.06402 | 0.06454 | 0.06011 | 0.06285 | 0.06323 | 0.06263 | 0.06174 | 0.06112 | 0.06228 | 0.06248 |
| 200 | 0.00397 | 0.01142 | 0.01729 | 0.02226 | 0.06093 | 0.06357 | 0.0635 | 0.06209 | 0.06374 | 0.06401 | 0.06337 | 0.0619 | 0.0615 | 0.06138 | 0.06273 |
| 300 | 0.00465 | 0.01159 | 0.01633 | 0.02154 | 0.06182 | 0.06384 | 0.06311 | 0.06253 | 0.06356 | 0.06352 | 0.06319 | 0.06152 | 0.06161 | 0.06276 | 0.06234 |
| 400 | 0.00529 | 0.01164 | 0.01635 | 0.02192 | 0.06194 | 0.06327 | 0.06315 | 0.06272 | 0.06377 | 0.06201 | 0.06316 | 0.06213 | 0.06223 | 0.063 | 0.06224 |
| 500 | 0.00563 | 0.01173 | 0.01631 | 0.02201 | 0.06217 | 0.06339 | 0.06334 | 0.06289 | 0.06307 | 0.06232 | 0.06343 | 0.0618 | 0.06248 | 0.06269 | 0.06245 |
| 600 | 0.0056 | 0.01195 | 0.01634 | 0.02224 | 0.06236 | 0.0634 | 0.06343 | 0.06236 | 0.06291 | 0.06236 | 0.06343 | 0.06203 | 0.06224 | 0.06232 | 0.06255 |
| 700 | 0.00542 | 0.01144 | 0.01621 | 0.02191 | 0.06256 | 0.06329 | 0.06343 | 0.06258 | 0.06239 | 0.06244 | 0.06283 | 0.06189 | 0.06237 | 0.0616 | 0.06256 |
| 800 | 0.00577 | 0.01166 | 0.01648 | 0.02203 | 0.06244 | 0.06297 | 0.06344 | 0.06269 | 0.06261 | 0.06222 | 0.06289 | 0.06198 | 0.06211 | 0.06161 | 0.06265 |
| 900 | 0.00566 | 0.01172 | 0.01637 | 0.02211 | 0.06237 | 0.06303 | 0.06338 | 0.06274 | 0.0626 | 0.06234 | 0.06308 | 0.06233 | 0.0621 | 0.06157 | 0.06242 |
| 1000 | 0.00598 | 0.01144 | 0.01667 | 0.02228 | 0.06214 | 0.06296 | 0.06319 | 0.06281 | 0.06257 | 0.06212 | 0.0628 | 0.06225 | 0.06202 | 0.06167 | 0.06251 |
| 1100 | 0.00588 | 0.01139 | 0.01674 | 0.02217 | 0.06218 | 0.06281 | 0.0633 | 0.06299 | 0.06284 | 0.06217 | 0.063 | 0.06211 | 0.06196 | 0.06186 | 0.06243 |
| 1200 | 0.00584 | 0.01145 | 0.0168 | 0.02224 | 0.06221 | 0.06287 | 0.06327 | 0.06302 | 0.06284 | 0.06221 | 0.06277 | 0.06229 | 0.06199 | 0.06188 | 0.06238 |
| 1300 | 0.00596 | 0.01154 | 0.0168 | 0.02217 | 0.06239 | 0.06287 | 0.06329 | 0.06303 | 0.06271 | 0.06236 | 0.06271 | 0.06232 | 0.06197 | 0.06179 | 0.06241 |
| 1400 | 0.00604 | 0.01153 | 0.01695 | 0.02218 | 0.06254 | 0.0631 | 0.06332 | 0.06289 | 0.06269 | 0.06233 | 0.06278 | 0.06222 | 0.06191 | 0.06181 | 0.06268 |
| 1500 | 0.00605 | 0.01152 | 0.01689 | 0.02202 | 0.06255 | 0.06313 | 0.06331 | 0.06273 | 0.06275 | 0.06246 | 0.06282 | 0.06228 | 0.06192 | 0.06189 | 0.0627 |

Table 43

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 12 | 10 | 9 | 6 | 6 | 5 | 4 | 3 | 7 | 8 | 5 | 3 | 6 | 4 | 12 |
| 200 | 26 | 20 | 21 | 11 | 14 | 17 | 7 | 10 | 10 | 13 | 9 | 6 | 11 | 8 | 17 |
| 300 | 34 | 31 | 27 | 24 | 19 | 22 | 11 | 15 | 15 | 17 | 17 | 9 | 17 | 15 | 27 |
| 400 | 41 | 42 | 33 | 34 | 25 | 26 | 18 | 21 | 19 | 28 | 25 | 14 | 22 | 21 | 31 |
| 500 | 45 | 52 | 44 | 45 | 27 | 33 | 25 | 30 | 26 | 35 | 29 | 20 | 28 | 25 | 36 |
| 600 | 53 | 64 | 56 | 56 | 30 | 38 | 27 | 36 | 29 | 41 | 29 | 27 | 34 | 38 | 42 |
| 700 | 67 | 76 | 69 | 62 | 32 | 41 | 32 | 39 | 34 | 49 | 34 | 32 | 43 | 44 | 46 |
| 800 | 78 | 84 | 81 | 75 | 35 | 50 | 36 | 45 | 39 | 56 | 39 | 34 | 50 | 51 | 47 |
| 900 | 91 | 92 | 95 | 83 | 37 | 59 | 40 | 54 | 42 | 63 | 43 | 40 | 55 | 55 | 51 |
| 1000 | 105 | 100 | 107 | 96 | 43 | 64 | 45 | 59 | 47 | 69 | 48 | 45 | 60 | 58 | 54 |
| 1100 | 120 | 108 | 121 | 104 | 47 | 73 | 48 | 63 | 51 | 73 | 53 | 51 | 66 | 64 | 58 |
| 1200 | 127 | 118 | 132 | 114 | 58 | 77 | 53 | 70 | 57 | 77 | 59 | 57 | 70 | 68 | 63 |
| 1300 | 140 | 129 | 147 | 128 | 63 | 84 | 58 | 72 | 60 | 80 | 60 | 66 | 71 | 74 | 68 |
| 1400 | 149 | 142 | 155 | 144 | 67 | 93 | 64 | 77 | 61 | 82 | 63 | 72 | 75 | 77 | 79 |
| 1500 | 156 | 155 | 162 | 156 | 69 | 97 | 67 | 89 | 67 | 88 | 73 | 81 | 79 | 81 | 80 |

Table 44

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.04337 | 0.1086 | 0.16288 | 0.13066 | 0.36349 | 0.32009 | 0.25816 | 0.18034 | 0.43992 | 0.50587 | 0.31314 | 0.18521 | 0.3667 | 0.24912 | 0.74979 |
| 200 | 0.10313 | 0.2285 | 0.36304 | 0.24483 | 0.85307 | 1.08067 | 0.44453 | 0.62092 | 0.63736 | 0.83218 | 0.57035 | 0.37142 | 0.67654 | 0.49101 | 1.06642 |
| 300 | 0.1582 | 0.35936 | 0.44098 | 0.51699 | 1.1745 | 1.40448 | 0.69419 | 0.93798 | 0.95334 | 1.07986 | 1.07421 | 0.5537 | 1.0474 | 0.94135 | 1.6833 |
| 400 | 0.21685 | 0.48905 | 0.5397 | 0.74519 | 1.54855 | 1.64507 | 1.13663 | 1.31705 | 1.2117 | 1.73625 | 1.57895 | 0.86982 | 1.36902 | 1.32309 | 1.92947 |
| 500 | 0.25352 | 0.61006 | 0.71761 | 0.99024 | 1.67851 | 2.09189 | 1.58358 | 1.88677 | 1.63979 | 2.18136 | 1.83958 | 1.23607 | 1.74942 | 1.56732 | 2.24815 |
| 600 | 0.29698 | 0.76495 | 0.91523 | 1.24551 | 1.87071 | 2.40901 | 1.71255 | 2.24505 | 1.82434 | 2.55695 | 1.83958 | 1.67482 | 2.1162 | 2.36821 | 2.6269 |
| 700 | 0.36298 | 0.86912 | 1.11847 | 1.3585 | 2.00201 | 2.59498 | 2.02986 | 2.44063 | 2.12124 | 3.05967 | 2.13624 | 1.98059 | 2.68195 | 2.71038 | 2.87795 |
| 800 | 0.44995 | 0.9793 | 1.33501 | 1.6521 | 2.18529 | 3.14864 | 2.28383 | 2.82094 | 2.44168 | 3.48418 | 2.45273 | 2.10716 | 3.1057 | 3.1419 | 2.94435 |
| 900 | 0.5152 | 1.07807 | 1.55537 | 1.83532 | 2.30757 | 3.71902 | 2.53507 | 3.38788 | 2.62899 | 3.92737 | 2.71248 | 2.49303 | 3.41534 | 3.38658 | 3.18341 |
| 1000 | 0.62826 | 1.14357 | 1.78413 | 2.13883 | 2.6722 | 4.02925 | 2.84364 | 3.70584 | 2.94078 | 4.28657 | 3.01453 | 2.80146 | 3.72103 | 3.57709 | 3.37536 |
| 1100 | 0.70508 | 1.2299 | 2.0252 | 2.30615 | 2.92264 | 4.58548 | 3.03829 | 3.96822 | 3.20479 | 4.53864 | 3.33885 | 3.1677 | 4.08907 | 3.95894 | 3.62073 |
| 1200 | 0.74141 | 1.35052 | 2.21726 | 2.53562 | 3.60802 | 4.84126 | 3.35357 | 4.41143 | 3.5816 | 4.7903 | 3.70352 | 3.55068 | 4.33919 | 4.20751 | 3.92977 |
| 1300 | 0.83375 | 1.48862 | 2.46987 | 2.83771 | 3.93032 | 5.28112 | 3.67109 | 4.53836 | 3.76238 | 4.98851 | 3.76276 | 4.11292 | 4.40018 | 4.57261 | 4.24376 |
| 1400 | 0.90024 | 1.63771 | 2.62772 | 3.19336 | 4.19018 | 5.86839 | 4.05236 | 4.84216 | 3.82436 | 5.11139 | 3.95526 | 4.48012 | 4.64309 | 4.75944 | 4.95161 |
| 1500 | 0.9441 | 1.78621 | 2.73697 | 3.43539 | 4.31565 | 6.12357 | 4.24164 | 5.58314 | 4.20418 | 5.49658 | 4.58613 | 5.04455 | 4.89204 | 5.01345 | 5.01629 |

Table 45

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.00043 | 0.00109 | 0.00163 | 0.00131 | 0.00363 | 0.0032 | 0.00258 | 0.0018 | 0.0044 | 0.00506 | 0.00313 | 0.00185 | 0.00367 | 0.00249 | 0.00075 |
| 200 | 0.00052 | 0.00114 | 0.00182 | 0.00122 | 0.00427 | 0.0054 | 0.00222 | 0.0031 | 0.00319 | 0.00416 | 0.00285 | 0.00186 | 0.00338 | 0.00246 | 0.00533 |
| 300 | 0.00053 | 0.0012 | 0.00147 | 0.00172 | 0.00391 | 0.00468 | 0.00231 | 0.00313 | 0.00318 | 0.0036 | 0.00358 | 0.00185 | 0.00349 | 0.00314 | 0.00561 |
| 400 | 0.00054 | 0.00122 | 0.00135 | 0.00186 | 0.00387 | 0.00411 | 0.00284 | 0.00329 | 0.00303 | 0.00434 | 0.00395 | 0.00217 | 0.00342 | 0.00331 | 0.00482 |
| 500 | 0.00051 | 0.00122 | 0.00144 | 0.00198 | 0.00336 | 0.00418 | 0.00317 | 0.00377 | 0.00328 | 0.00436 | 0.00368 | 0.00247 | 0.0035 | 0.00313 | 0.0045 |
| 600 | 0.00049 | 0.00127 | 0.00153 | 0.00208 | 0.00312 | 0.00402 | 0.00285 | 0.00374 | 0.00304 | 0.00426 | 0.00307 | 0.00279 | 0.00353 | 0.00395 | 0.00438 |
| 700 | 0.00052 | 0.00124 | 0.0016 | 0.00194 | 0.00286 | 0.00371 | 0.0029 | 0.00349 | 0.00303 | 0.00437 | 0.00305 | 0.00283 | 0.00383 | 0.00387 | 0.00411 |
| 800 | 0.00056 | 0.00122 | 0.00167 | 0.00207 | 0.00273 | 0.00394 | 0.00285 | 0.00353 | 0.00305 | 0.00436 | 0.00307 | 0.00263 | 0.00388 | 0.00393 | 0.00368 |
| 900 | 0.00057 | 0.0012 | 0.00173 | 0.00204 | 0.00256 | 0.00413 | 0.00282 | 0.00376 | 0.00292 | 0.00436 | 0.00301 | 0.00277 | 0.00379 | 0.00376 | 0.00354 |
| 1000 | 0.00063 | 0.00114 | 0.00178 | 0.00214 | 0.00267 | 0.00403 | 0.00284 | 0.00371 | 0.00294 | 0.00429 | 0.00301 | 0.0028 | 0.00372 | 0.00358 | 0.00338 |
| 1100 | 0.00064 | 0.00112 | 0.00184 | 0.0021 | 0.00266 | 0.00417 | 0.00276 | 0.00361 | 0.00291 | 0.00413 | 0.00304 | 0.00288 | 0.00372 | 0.0036 | 0.00329 |
| 1200 | 0.00062 | 0.00113 | 0.00185 | 0.00211 | 0.00301 | 0.00403 | 0.00279 | 0.00368 | 0.00298 | 0.00399 | 0.00309 | 0.00296 | 0.00362 | 0.00351 | 0.00327 |
| 1300 | 0.00064 | 0.00115 | 0.0019 | 0.00218 | 0.00302 | 0.00406 | 0.00282 | 0.00349 | 0.00289 | 0.00384 | 0.00289 | 0.00316 | 0.00338 | 0.00352 | 0.00326 |
| 1400 | 0.00064 | 0.00117 | 0.00188 | 0.00228 | 0.00299 | 0.00419 | 0.00289 | 0.00346 | 0.00273 | 0.00365 | 0.00283 | 0.0032 | 0.00332 | 0.0034 | 0.00354 |
| 1500 | 0.00063 | 0.00119 | 0.00182 | 0.00229 | 0.00288 | 0.00408 | 0.00283 | 0.00372 | 0.0028 | 0.00366 | 0.00306 | 0.00336 | 0.00326 | 0.00334 | 0.00334 |

Table 46

B. TRIANGULAR COST DISTRIBUTION

1. Two-Bidder Bidding Game

Scenario 1. Both bidders used equilibrium strategy.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave. Exp. Profit | |
|------------|-----------------|----------|----------------|---------|--------------|----------|------------------|----------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.206219 | 0.238089 | 49 | 51 | 10.10471 | 12.14254 | 0.101047 | 0.121425 |
| 200 | 0.214211 | 0.238282 | 92 | 108 | 19.70744 | 25.73446 | 0.098537 | 0.128672 |
| 300 | 0.221497 | 0.228089 | 131 | 169 | 29.01608 | 38.5471 | 0.09672 | 0.12849 |
| 400 | 0.226884 | 0.221775 | 179 | 221 | 40.61228 | 49.01231 | 0.101531 | 0.122531 |
| 500 | 0.223304 | 0.221598 | 224 | 276 | 50.02017 | 61.16102 | 0.10004 | 0.122322 |
| 600 | 0.223888 | 0.220807 | 278 | 322 | 62.2409 | 71.09995 | 0.103735 | 0.1185 |
| 700 | 0.224465 | 0.222031 | 336 | 364 | 75.42034 | 80.81943 | 0.107743 | 0.115456 |
| 800 | 0.224813 | 0.220863 | 378 | 422 | 84.97923 | 93.20432 | 0.106224 | 0.116505 |
| 900 | 0.223285 | 0.221015 | 429 | 471 | 95.78906 | 104.0983 | 0.106432 | 0.115665 |
| 1000 | 0.223027 | 0.221912 | 479 | 521 | 106.8301 | 115.6162 | 0.10683 | 0.115616 |
| 1100 | 0.221668 | 0.222563 | 535 | 565 | 118.5924 | 125.7482 | 0.107811 | 0.114317 |
| 1200 | 0.221597 | 0.221951 | 575 | 625 | 127.4181 | 138.7192 | 0.106182 | 0.115599 |
| 1300 | 0.221923 | 0.22262 | 621 | 679 | 137.8139 | 151.1589 | 0.106011 | 0.116276 |
| 1400 | 0.222089 | 0.223333 | 669 | 731 | 148.5776 | 163.2562 | 0.106127 | 0.116612 |
| 1500 | 0.220204 | 0.223635 | 725 | 775 | 159.6482 | 173.3172 | 0.106432 | 0.115545 |
| 1600 | 0.220878 | 0.223417 | 782 | 818 | 172.7268 | 182.7552 | 0.107954 | 0.114222 |
| 1700 | 0.220718 | 0.222405 | 827 | 873 | 182.5334 | 194.1595 | 0.107373 | 0.114211 |
| 1800 | 0.221144 | 0.223524 | 871 | 929 | 192.6167 | 207.6536 | 0.107009 | 0.115363 |
| 1900 | 0.221703 | 0.22439 | 931 | 969 | 206.4058 | 217.4339 | 0.108635 | 0.114439 |
| 2000 | 0.221375 | 0.224752 | 976 | 1024 | 216.0625 | 230.1461 | 0.108031 | 0.115073 |
| 2100 | 0.221022 | 0.224139 | 1023 | 1077 | 226.1053 | 241.3976 | 0.107669 | 0.114951 |
| 2200 | 0.221547 | 0.224441 | 1074 | 1126 | 237.9419 | 252.7204 | 0.108155 | 0.114873 |
| 2300 | 0.222452 | 0.224167 | 1130 | 1170 | 251.3706 | 262.2757 | 0.109292 | 0.114033 |
| 2400 | 0.222262 | 0.224398 | 1178 | 1222 | 261.8242 | 274.2143 | 0.109093 | 0.114256 |
| 2500 | 0.222308 | 0.223598 | 1231 | 1269 | 273.6609 | 283.7462 | 0.109464 | 0.113498 |

Table 47

Scenario 2. Bidder1 underbid by 0.15 while bidder2 used equilibrium strategy.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave. Exp. Profit | |
|------------|-----------------|----------|----------------|---------|--------------|----------|------------------|----------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.062809 | 0.256952 | 68 | 32 | 4.271004 | 8.222466 | 0.04271 | 0.082225 |
| 200 | 0.045651 | 0.252716 | 143 | 57 | 6.528056 | 14.40484 | 0.03264 | 0.072024 |
| 300 | 0.042422 | 0.251165 | 209 | 91 | 8.866159 | 22.85599 | 0.029554 | 0.076187 |
| 400 | 0.043888 | 0.248662 | 288 | 112 | 12.63972 | 27.85014 | 0.031599 | 0.069625 |
| 500 | 0.046166 | 0.245824 | 359 | 141 | 16.57369 | 34.66119 | 0.033147 | 0.069322 |
| 600 | 0.048138 | 0.245034 | 433 | 167 | 20.84361 | 40.92069 | 0.034739 | 0.068201 |
| 700 | 0.045984 | 0.245149 | 498 | 202 | 22.90004 | 49.52015 | 0.032714 | 0.070743 |
| 800 | 0.046905 | 0.244969 | 572 | 228 | 26.82973 | 55.85291 | 0.033537 | 0.069816 |
| 900 | 0.046433 | 0.244144 | 646 | 254 | 29.99544 | 62.01265 | 0.033328 | 0.068903 |
| 1000 | 0.04572 | 0.24415 | 707 | 293 | 32.32439 | 71.53586 | 0.032324 | 0.071536 |
| 1100 | 0.045926 | 0.244085 | 778 | 322 | 35.73045 | 78.59541 | 0.032482 | 0.07145 |
| 1200 | 0.046346 | 0.24425 | 849 | 351 | 39.34792 | 85.73181 | 0.03279 | 0.071443 |
| 1300 | 0.048115 | 0.244711 | 924 | 376 | 44.45839 | 92.01122 | 0.034199 | 0.070778 |
| 1400 | 0.048381 | 0.244242 | 992 | 408 | 47.99432 | 99.65085 | 0.034282 | 0.071179 |
| 1500 | 0.048138 | 0.243496 | 1068 | 432 | 51.41143 | 105.1904 | 0.034274 | 0.070127 |

Table 48

Scenario 3. Both bidders did not use equilibrium strategy and underbid by 0.1.

| # of games | Expected Profit | | # of Games Won | | Total Profit | | Ave.Exp. Profit | |
|------------|-----------------|----------|----------------|---------|--------------|----------|-----------------|----------|
| | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 | Bidder1 | Bidder2 |
| 100 | 0.113678 | 0.112292 | 47 | 53 | 5.342864 | 5.951497 | 0.053429 | 0.059515 |
| 200 | 0.12326 | 0.121317 | 98 | 102 | 12.07945 | 12.37434 | 0.060397 | 0.061872 |
| 300 | 0.129273 | 0.120065 | 154 | 146 | 19.90808 | 17.52956 | 0.06636 | 0.058432 |
| 400 | 0.126543 | 0.122567 | 202 | 198 | 25.56173 | 24.26822 | 0.063904 | 0.060671 |
| 500 | 0.125876 | 0.118776 | 255 | 245 | 32.09846 | 29.10015 | 0.064197 | 0.0582 |
| 600 | 0.127644 | 0.121315 | 299 | 301 | 38.16551 | 36.5157 | 0.063609 | 0.06086 |
| 700 | 0.127994 | 0.120436 | 351 | 349 | 44.92607 | 42.03215 | 0.06418 | 0.060046 |
| 800 | 0.125206 | 0.120689 | 393 | 407 | 49.20592 | 49.12057 | 0.061507 | 0.061401 |
| 900 | 0.125597 | 0.121691 | 451 | 449 | 56.64446 | 54.63906 | 0.062938 | 0.06071 |
| 1000 | 0.123275 | 0.120888 | 501 | 499 | 61.76094 | 60.32298 | 0.061761 | 0.060323 |
| 1100 | 0.122718 | 0.120584 | 547 | 553 | 67.12702 | 66.68284 | 0.061025 | 0.060621 |
| 1200 | 0.122694 | 0.121073 | 597 | 603 | 73.24847 | 73.00698 | 0.06104 | 0.060839 |
| 1300 | 0.12317 | 0.121394 | 654 | 646 | 80.55314 | 78.42025 | 0.061964 | 0.060323 |
| 1400 | 0.121684 | 0.122553 | 705 | 695 | 85.78711 | 85.17436 | 0.061277 | 0.060839 |
| 1500 | 0.121015 | 0.122933 | 755 | 745 | 91.36622 | 91.58526 | 0.060911 | 0.061057 |
| 1600 | 0.121948 | 0.121706 | 808 | 792 | 98.53395 | 96.39112 | 0.061584 | 0.060244 |
| 1700 | 0.122473 | 0.122582 | 860 | 840 | 105.3271 | 102.9691 | 0.061957 | 0.06057 |
| 1800 | 0.121631 | 0.122916 | 914 | 886 | 111.1707 | 108.9035 | 0.061762 | 0.060502 |
| 1900 | 0.121133 | 0.122171 | 968 | 932 | 117.2568 | 113.8637 | 0.061714 | 0.059928 |
| 2000 | 0.121426 | 0.122314 | 1019 | 981 | 123.7332 | 119.9902 | 0.061867 | 0.059995 |
| 2100 | 0.120975 | 0.122718 | 1072 | 1028 | 129.6853 | 126.1544 | 0.061755 | 0.060074 |
| 2200 | 0.121425 | 0.123081 | 1122 | 1078 | 136.2389 | 132.6812 | 0.061927 | 0.06031 |
| 2300 | 0.120909 | 0.123714 | 1160 | 1140 | 140.255 | 141.0338 | 0.06098 | 0.061319 |
| 2400 | 0.120884 | 0.123577 | 1210 | 1190 | 146.2696 | 147.0561 | 0.060946 | 0.061273 |
| 2500 | 0.121266 | 0.123949 | 1259 | 1241 | 152.6738 | 153.8211 | 0.06107 | 0.061528 |

Table 49

2. Three-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Average Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|-------------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.14 | 0.1468 | 0.1514 | 35 | 30 | 35 | 4.9012 | 4.4051 | 5.299 | 0.049 | 0.0441 | 0.053 |
| 200 | 0.1491 | 0.1463 | 0.1554 | 74 | 61 | 65 | 11.032 | 8.9265 | 10.099 | 0.0552 | 0.0446 | 0.0505 |
| 300 | 0.1469 | 0.1485 | 0.1535 | 113 | 93 | 94 | 16.595 | 13.807 | 14.433 | 0.0553 | 0.046 | 0.0481 |
| 400 | 0.1452 | 0.1479 | 0.15 | 146 | 128 | 126 | 21.205 | 18.936 | 18.901 | 0.053 | 0.0473 | 0.0473 |
| 500 | 0.1465 | 0.147 | 0.1471 | 172 | 170 | 158 | 25.206 | 24.993 | 23.237 | 0.0504 | 0.05 | 0.0465 |
| 600 | 0.1486 | 0.1478 | 0.1477 | 215 | 200 | 185 | 31.958 | 29.551 | 27.321 | 0.0533 | 0.0493 | 0.0455 |
| 700 | 0.1486 | 0.147 | 0.1485 | 241 | 231 | 228 | 35.825 | 33.954 | 33.865 | 0.0512 | 0.0485 | 0.0484 |
| 800 | 0.1495 | 0.1481 | 0.1497 | 280 | 256 | 264 | 41.862 | 37.917 | 39.527 | 0.0523 | 0.0474 | 0.0494 |
| 900 | 0.1504 | 0.1488 | 0.1501 | 310 | 287 | 303 | 46.618 | 42.698 | 45.493 | 0.0518 | 0.0474 | 0.0505 |
| 1000 | 0.1499 | 0.1491 | 0.1506 | 339 | 325 | 336 | 50.822 | 48.467 | 50.613 | 0.0508 | 0.0485 | 0.0506 |
| 1100 | 0.1499 | 0.1488 | 0.1508 | 380 | 353 | 367 | 56.955 | 52.519 | 55.351 | 0.0518 | 0.0477 | 0.0503 |
| 1200 | 0.1486 | 0.1495 | 0.1504 | 414 | 374 | 412 | 61.525 | 55.924 | 61.975 | 0.0513 | 0.0466 | 0.0516 |
| 1300 | 0.1472 | 0.1492 | 0.1501 | 449 | 407 | 444 | 66.084 | 60.724 | 66.64 | 0.0508 | 0.0467 | 0.0513 |
| 1400 | 0.1479 | 0.1501 | 0.15 | 479 | 445 | 476 | 70.853 | 66.777 | 71.378 | 0.0506 | 0.0477 | 0.051 |
| 1500 | 0.1473 | 0.1495 | 0.15 | 510 | 479 | 511 | 75.134 | 71.621 | 76.653 | 0.0501 | 0.0477 | 0.0511 |
| 1600 | 0.1479 | 0.1495 | 0.1498 | 551 | 509 | 540 | 81.481 | 76.085 | 80.877 | 0.0509 | 0.0476 | 0.0505 |
| 1700 | 0.1483 | 0.149 | 0.1501 | 581 | 537 | 582 | 86.177 | 80.022 | 87.363 | 0.0507 | 0.0471 | 0.0514 |
| 1800 | 0.1484 | 0.1493 | 0.1505 | 618 | 561 | 621 | 91.697 | 83.757 | 93.472 | 0.0509 | 0.0465 | 0.0519 |
| 1900 | 0.1491 | 0.1496 | 0.1508 | 642 | 598 | 660 | 95.72 | 89.475 | 99.503 | 0.0504 | 0.0471 | 0.0524 |
| 2000 | 0.1488 | 0.1496 | 0.1511 | 675 | 635 | 690 | 100.41 | 94.983 | 104.24 | 0.0502 | 0.0475 | 0.0521 |

Table 50

Scenario 2. Bidder1 underbid by 0.1 while others used equilibrium strategy.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Average Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|-------------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.0454 | 0.1467 | 0.1594 | 47 | 29 | 24 | 2.1341 | 4.2538 | 3.8264 | 0.0213 | 0.0425 | 0.0383 |
| 200 | 0.0428 | 0.1491 | 0.1548 | 92 | 48 | 60 | 3.9393 | 7.1574 | 9.2883 | 0.0197 | 0.0358 | 0.0464 |
| 300 | 0.0452 | 0.1541 | 0.1508 | 136 | 78 | 86 | 6.1407 | 12.017 | 12.967 | 0.0205 | 0.0401 | 0.0432 |
| 400 | 0.0424 | 0.1519 | 0.151 | 177 | 106 | 117 | 7.4977 | 16.1 | 17.672 | 0.0187 | 0.0403 | 0.0442 |
| 500 | 0.0414 | 0.1534 | 0.1512 | 229 | 132 | 139 | 9.4821 | 20.25 | 21.02 | 0.019 | 0.0405 | 0.042 |
| 600 | 0.0407 | 0.1521 | 0.1521 | 273 | 161 | 166 | 11.098 | 24.485 | 25.254 | 0.0185 | 0.0408 | 0.0421 |
| 700 | 0.0426 | 0.1512 | 0.1534 | 322 | 187 | 191 | 13.703 | 28.274 | 29.295 | 0.0196 | 0.0404 | 0.0419 |
| 800 | 0.0413 | 0.153 | 0.1533 | 380 | 206 | 214 | 15.696 | 31.527 | 32.813 | 0.0196 | 0.0394 | 0.041 |
| 900 | 0.0402 | 0.1524 | 0.1528 | 428 | 233 | 239 | 17.184 | 35.505 | 36.511 | 0.0191 | 0.0395 | 0.0406 |
| 1000 | 0.0399 | 0.1526 | 0.151 | 467 | 268 | 265 | 18.626 | 40.904 | 40.021 | 0.0186 | 0.0409 | 0.04 |
| 1100 | 0.0407 | 0.1526 | 0.152 | 520 | 296 | 284 | 21.161 | 45.175 | 43.166 | 0.0192 | 0.0411 | 0.0392 |
| 1200 | 0.0405 | 0.1516 | 0.153 | 559 | 323 | 318 | 22.638 | 48.961 | 48.669 | 0.0189 | 0.0408 | 0.0406 |
| 1300 | 0.0414 | 0.1522 | 0.1534 | 607 | 345 | 348 | 25.13 | 52.521 | 53.38 | 0.0193 | 0.0404 | 0.0411 |
| 1400 | 0.0419 | 0.1526 | 0.1536 | 646 | 373 | 381 | 27.054 | 56.934 | 58.53 | 0.0193 | 0.0407 | 0.0418 |
| 1500 | 0.0416 | 0.1529 | 0.1541 | 687 | 405 | 408 | 28.554 | 61.925 | 62.879 | 0.019 | 0.0413 | 0.0419 |

Table 51

Scenario 3. Bidders “1” and “2” underbid by 0.1 and 0.12 respectively.

| # of games | Expected Profit | | | Number of Games Won | | | Total Profit | | | Average Expected Profit | | |
|------------|-----------------|---------|---------|---------------------|---------|---------|--------------|---------|---------|-------------------------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 | Bidder1 | Bidder2 | Bidder3 |
| 100 | 0.0465 | 0.0272 | 0.1513 | 27 | 47 | 26 | 1.2558 | 1.2775 | 3.9342 | 0.0126 | 0.0128 | 0.0393 |
| 200 | 0.0514 | 0.0226 | 0.1559 | 61 | 88 | 51 | 3.1362 | 1.9853 | 7.9525 | 0.0157 | 0.0099 | 0.0398 |
| 300 | 0.0527 | 0.02 | 0.1581 | 98 | 120 | 82 | 5.1683 | 2.3989 | 12.968 | 0.0172 | 0.008 | 0.0432 |
| 400 | 0.0516 | 0.0229 | 0.1587 | 134 | 158 | 108 | 6.9093 | 3.6177 | 17.139 | 0.0173 | 0.009 | 0.0428 |
| 500 | 0.0488 | 0.0244 | 0.1587 | 169 | 201 | 130 | 8.2412 | 4.9023 | 20.635 | 0.0165 | 0.0098 | 0.0413 |
| 600 | 0.0496 | 0.0223 | 0.1581 | 207 | 245 | 148 | 10.276 | 5.4653 | 23.401 | 0.0171 | 0.0091 | 0.039 |
| 700 | 0.049 | 0.024 | 0.1583 | 238 | 294 | 168 | 11.667 | 7.0605 | 26.591 | 0.0167 | 0.0101 | 0.038 |
| 800 | 0.048 | 0.0228 | 0.1573 | 285 | 331 | 184 | 13.679 | 7.5342 | 28.945 | 0.0171 | 0.0094 | 0.0362 |
| 900 | 0.048 | 0.0237 | 0.1579 | 320 | 378 | 202 | 15.369 | 8.9754 | 31.902 | 0.0171 | 0.01 | 0.0354 |
| 1000 | 0.0481 | 0.0241 | 0.1575 | 357 | 414 | 229 | 17.156 | 9.9641 | 36.077 | 0.0172 | 0.01 | 0.0361 |
| 1100 | 0.0471 | 0.0229 | 0.1567 | 397 | 456 | 247 | 18.712 | 10.462 | 38.703 | 0.017 | 0.0095 | 0.0352 |
| 1200 | 0.0463 | 0.0233 | 0.1582 | 429 | 502 | 269 | 19.867 | 11.687 | 42.562 | 0.0166 | 0.0097 | 0.0355 |
| 1300 | 0.0466 | 0.0236 | 0.1575 | 466 | 543 | 291 | 21.709 | 12.84 | 45.841 | 0.0167 | 0.0099 | 0.0353 |
| 1400 | 0.0469 | 0.0237 | 0.1577 | 505 | 591 | 304 | 23.705 | 14.011 | 47.947 | 0.0169 | 0.01 | 0.0342 |
| 1500 | 0.0469 | 0.0237 | 0.1578 | 535 | 639 | 326 | 25.093 | 15.152 | 51.438 | 0.0167 | 0.0101 | 0.0343 |
| 1600 | 0.0461 | 0.0237 | 0.1573 | 573 | 681 | 346 | 26.395 | 16.116 | 54.424 | 0.0165 | 0.0101 | 0.034 |
| 1700 | 0.0456 | 0.0234 | 0.1575 | 611 | 716 | 373 | 27.877 | 16.759 | 58.73 | 0.0164 | 0.0099 | 0.0345 |
| 1800 | 0.0461 | 0.0236 | 0.1574 | 643 | 763 | 394 | 29.635 | 17.99 | 62.018 | 0.0165 | 0.01 | 0.0345 |
| 1900 | 0.0467 | 0.0237 | 0.1569 | 681 | 804 | 415 | 31.798 | 19.074 | 65.118 | 0.0167 | 0.01 | 0.0343 |
| 2000 | 0.0469 | 0.0239 | 0.1573 | 720 | 851 | 429 | 33.771 | 20.352 | 67.483 | 0.0169 | 0.0102 | 0.0337 |

Table 52

3. Five-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.08749 | 0.09988 | 0.08972 | 0.08912 | 0.08875 | 15 | 15 | 23 | 25 | 22 |
| 200 | 0.09069 | 0.09436 | 0.09066 | 0.08947 | 0.09082 | 33 | 42 | 40 | 36 | 49 |
| 300 | 0.09114 | 0.09304 | 0.08912 | 0.09072 | 0.09182 | 54 | 60 | 64 | 53 | 69 |
| 400 | 0.09091 | 0.09187 | 0.08984 | 0.09145 | 0.09122 | 75 | 79 | 80 | 80 | 86 |
| 500 | 0.09193 | 0.09233 | 0.09091 | 0.0921 | 0.0911 | 103 | 96 | 95 | 102 | 104 |
| 600 | 0.09242 | 0.09207 | 0.09159 | 0.09272 | 0.0919 | 121 | 119 | 117 | 123 | 120 |
| 700 | 0.09259 | 0.09162 | 0.09119 | 0.09293 | 0.09108 | 140 | 142 | 136 | 150 | 132 |
| 800 | 0.093 | 0.09141 | 0.09153 | 0.09219 | 0.09193 | 158 | 161 | 153 | 171 | 157 |
| 900 | 0.09308 | 0.09191 | 0.09097 | 0.09184 | 0.09189 | 174 | 193 | 172 | 186 | 175 |
| 1000 | 0.09271 | 0.0922 | 0.09133 | 0.0914 | 0.09166 | 187 | 217 | 185 | 211 | 200 |
| 1100 | 0.0922 | 0.09201 | 0.09114 | 0.09156 | 0.09124 | 207 | 232 | 205 | 235 | 221 |
| 1200 | 0.09179 | 0.09208 | 0.09112 | 0.09221 | 0.09101 | 223 | 248 | 228 | 254 | 247 |
| 1300 | 0.09218 | 0.09218 | 0.0908 | 0.0927 | 0.09128 | 241 | 264 | 253 | 275 | 267 |
| 1400 | 0.09201 | 0.09252 | 0.09098 | 0.09276 | 0.0916 | 260 | 288 | 276 | 291 | 285 |
| 1500 | 0.09229 | 0.09242 | 0.09119 | 0.09274 | 0.09183 | 279 | 310 | 297 | 311 | 303 |

Table 53

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.31239 | 1.4982 | 2.06353 | 2.22795 | 1.9524 | 0.01312 | 0.01498 | 0.02064 | 0.02228 | 0.01952 |
| 200 | 2.99263 | 3.96305 | 3.62641 | 3.22092 | 4.45015 | 0.01496 | 0.01982 | 0.01813 | 0.0161 | 0.02225 |
| 300 | 4.92172 | 5.58251 | 5.70388 | 4.80836 | 6.33535 | 0.01641 | 0.01861 | 0.01901 | 0.01603 | 0.02112 |
| 400 | 6.81839 | 7.25786 | 7.18723 | 7.31595 | 7.84471 | 0.01705 | 0.01814 | 0.01797 | 0.01829 | 0.01961 |
| 500 | 9.46874 | 8.86324 | 8.63624 | 9.394 | 9.47491 | 0.01894 | 0.01773 | 0.01727 | 0.01879 | 0.01895 |
| 600 | 11.1831 | 10.9568 | 10.7161 | 11.4043 | 11.0286 | 0.01864 | 0.01826 | 0.01786 | 0.01901 | 0.01838 |
| 700 | 12.962 | 13.0094 | 12.4024 | 13.9388 | 12.0229 | 0.01852 | 0.01858 | 0.01772 | 0.01991 | 0.01718 |
| 800 | 14.6941 | 14.7163 | 14.0046 | 15.7645 | 14.4335 | 0.01837 | 0.0184 | 0.01751 | 0.01971 | 0.01804 |
| 900 | 16.1953 | 17.738 | 15.6474 | 17.0824 | 16.081 | 0.01799 | 0.01971 | 0.01739 | 0.01898 | 0.01787 |
| 1000 | 17.3369 | 20.0068 | 16.8958 | 19.2861 | 18.3316 | 0.01734 | 0.02001 | 0.0169 | 0.01929 | 0.01833 |
| 1100 | 19.0863 | 21.3458 | 18.6837 | 21.5154 | 20.1631 | 0.01735 | 0.01941 | 0.01699 | 0.01956 | 0.01833 |
| 1200 | 20.4695 | 22.8358 | 20.7747 | 23.4201 | 22.4783 | 0.01706 | 0.01903 | 0.01731 | 0.01952 | 0.01873 |
| 1300 | 22.2152 | 24.3365 | 22.9722 | 25.4933 | 24.3709 | 0.01709 | 0.01872 | 0.01767 | 0.01961 | 0.01875 |
| 1400 | 23.9232 | 26.6467 | 25.1104 | 26.9925 | 26.107 | 0.01709 | 0.01903 | 0.01794 | 0.01928 | 0.01865 |
| 1500 | 25.749 | 28.6512 | 27.083 | 28.8432 | 27.8236 | 0.01717 | 0.0191 | 0.01806 | 0.01923 | 0.01855 |

Table 54

Scenario 2. Bidder1 underbid by 0.05 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.04421 | 0.09189 | 0.09409 | 0.09356 | 0.09471 | 30 | 10 | 19 | 16 | 25 |
| 200 | 0.0402 | 0.09197 | 0.09405 | 0.09558 | 0.09412 | 60 | 32 | 32 | 32 | 44 |
| 300 | 0.03931 | 0.091 | 0.09468 | 0.0967 | 0.09561 | 91 | 43 | 59 | 48 | 59 |
| 400 | 0.03921 | 0.09139 | 0.09439 | 0.09708 | 0.09486 | 121 | 56 | 80 | 70 | 73 |
| 500 | 0.03991 | 0.0924 | 0.09421 | 0.09613 | 0.09473 | 153 | 68 | 99 | 83 | 97 |
| 600 | 0.04001 | 0.09305 | 0.0939 | 0.09602 | 0.09366 | 178 | 92 | 117 | 100 | 113 |
| 700 | 0.03954 | 0.09377 | 0.09422 | 0.09533 | 0.09413 | 202 | 110 | 135 | 117 | 136 |
| 800 | 0.03992 | 0.09342 | 0.09442 | 0.09456 | 0.09438 | 227 | 129 | 153 | 133 | 158 |
| 900 | 0.03993 | 0.09268 | 0.09455 | 0.09397 | 0.09399 | 248 | 155 | 166 | 153 | 178 |
| 1000 | 0.04028 | 0.09278 | 0.09415 | 0.09391 | 0.09371 | 276 | 178 | 178 | 176 | 192 |
| 1100 | 0.04022 | 0.09247 | 0.09395 | 0.09392 | 0.09334 | 309 | 195 | 190 | 195 | 211 |
| 1200 | 0.04026 | 0.09232 | 0.09388 | 0.09381 | 0.0937 | 331 | 209 | 215 | 213 | 232 |
| 1300 | 0.04046 | 0.09232 | 0.0939 | 0.09372 | 0.09322 | 359 | 228 | 228 | 230 | 255 |
| 1400 | 0.04062 | 0.09252 | 0.09393 | 0.09332 | 0.09326 | 383 | 250 | 250 | 244 | 273 |
| 1500 | 0.04068 | 0.09272 | 0.09401 | 0.09315 | 0.09358 | 402 | 273 | 269 | 270 | 286 |

Table 55

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.32642 | 0.91892 | 1.78778 | 1.49689 | 2.36768 | 0.01326 | 0.00919 | 0.01788 | 0.01497 | 0.02368 |
| 200 | 2.41209 | 2.94303 | 3.00973 | 3.0584 | 4.1414 | 0.01206 | 0.01472 | 0.01505 | 0.01529 | 0.02071 |
| 300 | 3.57749 | 3.91316 | 5.58639 | 4.64137 | 5.64104 | 0.01192 | 0.01304 | 0.01862 | 0.01547 | 0.0188 |
| 400 | 4.74467 | 5.11773 | 7.55149 | 6.79533 | 6.92446 | 0.01186 | 0.01279 | 0.01888 | 0.01699 | 0.01731 |
| 500 | 6.10665 | 6.28286 | 9.32679 | 7.97881 | 9.1889 | 0.01221 | 0.01257 | 0.01865 | 0.01596 | 0.01838 |
| 600 | 7.12091 | 8.56039 | 10.9868 | 9.60236 | 10.5832 | 0.01187 | 0.01427 | 0.01831 | 0.016 | 0.01764 |
| 700 | 7.98719 | 10.3145 | 12.7194 | 11.1532 | 12.8018 | 0.01141 | 0.01474 | 0.01817 | 0.01593 | 0.01829 |
| 800 | 9.06094 | 12.0515 | 14.4462 | 12.5765 | 14.912 | 0.01133 | 0.01506 | 0.01806 | 0.01572 | 0.01864 |
| 900 | 9.90244 | 14.3655 | 15.6957 | 14.377 | 16.7295 | 0.011 | 0.01596 | 0.01744 | 0.01597 | 0.01859 |
| 1000 | 11.1185 | 16.5152 | 16.7589 | 16.528 | 17.9917 | 0.01112 | 0.01652 | 0.01676 | 0.01653 | 0.01799 |
| 1100 | 12.4287 | 18.0318 | 17.8513 | 18.3153 | 19.6943 | 0.0113 | 0.01639 | 0.01623 | 0.01665 | 0.0179 |
| 1200 | 13.3257 | 19.2948 | 20.1846 | 19.9812 | 21.739 | 0.0111 | 0.01608 | 0.01682 | 0.01665 | 0.01812 |
| 1300 | 14.5249 | 21.0483 | 21.4086 | 21.5563 | 23.7717 | 0.01117 | 0.01619 | 0.01647 | 0.01658 | 0.01829 |
| 1400 | 15.5561 | 23.1292 | 23.4823 | 22.7698 | 25.4607 | 0.01111 | 0.01652 | 0.01677 | 0.01626 | 0.01819 |
| 1500 | 16.3539 | 25.3116 | 25.2892 | 25.1509 | 26.7629 | 0.0109 | 0.01687 | 0.01686 | 0.01677 | 0.01784 |

Table 56

Scenario 3. Bidders “1” and “2” underbid by 0.05 while others used

equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.04182 | 0.04036 | 0.088 | 0.09277 | 0.09275 | 27 | 29 | 20 | 11 | 13 |
| 200 | 0.03974 | 0.04107 | 0.08954 | 0.09311 | 0.09657 | 49 | 53 | 33 | 29 | 36 |
| 300 | 0.03946 | 0.04074 | 0.09098 | 0.09213 | 0.09617 | 78 | 80 | 49 | 42 | 51 |
| 400 | 0.03927 | 0.04122 | 0.09188 | 0.09232 | 0.09596 | 106 | 104 | 69 | 59 | 62 |
| 500 | 0.03935 | 0.04016 | 0.09139 | 0.09209 | 0.09636 | 127 | 134 | 83 | 79 | 77 |
| 600 | 0.03903 | 0.03931 | 0.0911 | 0.0908 | 0.09563 | 155 | 153 | 98 | 94 | 100 |
| 700 | 0.03934 | 0.03983 | 0.09116 | 0.09162 | 0.09517 | 175 | 168 | 120 | 114 | 123 |
| 800 | 0.03969 | 0.04073 | 0.0923 | 0.09217 | 0.09458 | 192 | 195 | 141 | 135 | 137 |
| 900 | 0.0403 | 0.04085 | 0.09292 | 0.09221 | 0.09454 | 213 | 216 | 161 | 155 | 155 |
| 1000 | 0.0405 | 0.04059 | 0.09299 | 0.09215 | 0.09436 | 244 | 241 | 179 | 168 | 168 |
| 1100 | 0.04056 | 0.04073 | 0.09256 | 0.09218 | 0.09368 | 270 | 258 | 196 | 186 | 190 |
| 1200 | 0.04049 | 0.04052 | 0.09215 | 0.09219 | 0.09389 | 285 | 279 | 224 | 202 | 210 |
| 1300 | 0.04055 | 0.04099 | 0.09218 | 0.09242 | 0.09421 | 305 | 312 | 239 | 212 | 232 |
| 1400 | 0.04099 | 0.04085 | 0.0922 | 0.09223 | 0.09391 | 327 | 338 | 255 | 226 | 254 |
| 1500 | 0.04058 | 0.04106 | 0.09227 | 0.09219 | 0.09371 | 351 | 359 | 272 | 242 | 276 |

Table 57

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.12904 | 1.1705 | 1.76 | 1.02048 | 1.20575 | 0.01129 | 0.0117 | 0.0176 | 0.0102 | 0.01206 |
| 200 | 1.94749 | 2.1767 | 2.95472 | 2.7002 | 3.47652 | 0.00974 | 0.01088 | 0.01477 | 0.0135 | 0.01738 |
| 300 | 3.0778 | 3.25927 | 4.45789 | 3.86953 | 4.90488 | 0.01026 | 0.01086 | 0.01486 | 0.0129 | 0.01635 |
| 400 | 4.16245 | 4.28717 | 6.34 | 5.44716 | 5.94929 | 0.01041 | 0.01072 | 0.01585 | 0.01362 | 0.01487 |
| 500 | 4.99722 | 5.38145 | 7.58527 | 7.27489 | 7.42006 | 0.00999 | 0.01076 | 0.01517 | 0.01455 | 0.01484 |
| 600 | 6.05031 | 6.01496 | 8.9278 | 8.5356 | 9.56324 | 0.01008 | 0.01002 | 0.01488 | 0.01423 | 0.01594 |
| 700 | 6.88429 | 6.69195 | 10.9387 | 10.4452 | 11.7063 | 0.00983 | 0.00956 | 0.01563 | 0.01492 | 0.01672 |
| 800 | 7.62075 | 7.9431 | 13.014 | 12.4435 | 12.9572 | 0.00953 | 0.00993 | 0.01627 | 0.01555 | 0.0162 |
| 900 | 8.58449 | 8.82439 | 14.9603 | 14.2931 | 14.653 | 0.00954 | 0.0098 | 0.01662 | 0.01588 | 0.01628 |
| 1000 | 9.88266 | 9.78181 | 16.6443 | 15.4819 | 15.853 | 0.00988 | 0.00978 | 0.01664 | 0.01548 | 0.01585 |
| 1100 | 10.9506 | 10.5072 | 18.1409 | 17.1449 | 17.7996 | 0.00996 | 0.00955 | 0.01649 | 0.01559 | 0.01618 |
| 1200 | 11.5403 | 11.304 | 20.6423 | 18.6217 | 19.7171 | 0.00962 | 0.00942 | 0.0172 | 0.01552 | 0.01643 |
| 1300 | 12.3685 | 12.7889 | 22.0318 | 19.5933 | 21.8564 | 0.00951 | 0.00984 | 0.01695 | 0.01507 | 0.01681 |
| 1400 | 13.4044 | 13.8081 | 23.5105 | 20.8441 | 23.8539 | 0.00957 | 0.00986 | 0.01679 | 0.01489 | 0.01704 |
| 1500 | 14.2425 | 14.7406 | 25.0985 | 22.3112 | 25.8637 | 0.0095 | 0.00983 | 0.01673 | 0.01487 | 0.01724 |

Table 58

Scenario 4. Bidders “1” and “2” underbid by 0.05 and 0.075 respectively while others used equilibrium strategy.

| # of games | Expected Profit | | | | | Number of Games Won | | | | |
|------------|-----------------|---------|---------|---------|---------|---------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 0.04674 | 0.01812 | 0.09244 | 0.09663 | 0.09607 | 23 | 21 | 18 | 18 | 20 |
| 200 | 0.04745 | 0.01615 | 0.09205 | 0.09375 | 0.09433 | 47 | 45 | 35 | 34 | 39 |
| 300 | 0.04605 | 0.01695 | 0.09205 | 0.09358 | 0.09387 | 69 | 74 | 50 | 52 | 55 |
| 400 | 0.04549 | 0.01678 | 0.09322 | 0.09323 | 0.0948 | 98 | 103 | 64 | 69 | 66 |
| 500 | 0.04366 | 0.0169 | 0.09281 | 0.09297 | 0.09354 | 115 | 128 | 83 | 90 | 84 |
| 600 | 0.04315 | 0.01636 | 0.09278 | 0.09343 | 0.09352 | 135 | 162 | 96 | 99 | 108 |
| 700 | 0.04304 | 0.01631 | 0.09118 | 0.09336 | 0.09353 | 162 | 184 | 114 | 114 | 126 |
| 800 | 0.04235 | 0.01569 | 0.0917 | 0.09339 | 0.09326 | 186 | 210 | 128 | 138 | 138 |
| 900 | 0.04258 | 0.01595 | 0.09214 | 0.09333 | 0.09312 | 220 | 235 | 139 | 154 | 152 |
| 1000 | 0.04268 | 0.01605 | 0.09241 | 0.09372 | 0.0934 | 240 | 262 | 160 | 172 | 166 |
| 1100 | 0.04271 | 0.01578 | 0.0922 | 0.09374 | 0.09267 | 259 | 292 | 175 | 199 | 175 |
| 1200 | 0.04299 | 0.01542 | 0.09233 | 0.09339 | 0.09318 | 280 | 320 | 187 | 215 | 198 |
| 1300 | 0.04292 | 0.01548 | 0.09201 | 0.09318 | 0.09315 | 301 | 359 | 203 | 224 | 213 |
| 1400 | 0.04268 | 0.01557 | 0.0922 | 0.09289 | 0.09282 | 316 | 390 | 214 | 242 | 238 |
| 1500 | 0.0423 | 0.01542 | 0.09206 | 0.09289 | 0.09245 | 341 | 410 | 231 | 266 | 252 |
| 1600 | 0.04209 | 0.01544 | 0.09215 | 0.09321 | 0.09213 | 359 | 431 | 251 | 281 | 278 |
| 1700 | 0.04221 | 0.01556 | 0.09185 | 0.09359 | 0.09224 | 385 | 448 | 273 | 302 | 292 |
| 1800 | 0.04209 | 0.0158 | 0.09205 | 0.09362 | 0.09236 | 406 | 480 | 293 | 316 | 305 |
| 1900 | 0.04228 | 0.0158 | 0.09213 | 0.09362 | 0.09244 | 431 | 506 | 310 | 333 | 320 |
| 2000 | 0.04225 | 0.01572 | 0.09188 | 0.09362 | 0.09246 | 458 | 538 | 328 | 343 | 333 |
| 2100 | 0.04202 | 0.01575 | 0.09189 | 0.0936 | 0.09287 | 480 | 564 | 341 | 361 | 354 |
| 2200 | 0.04184 | 0.01574 | 0.09194 | 0.09347 | 0.09296 | 507 | 594 | 356 | 374 | 369 |
| 2300 | 0.04177 | 0.0157 | 0.09206 | 0.09346 | 0.09302 | 534 | 621 | 375 | 390 | 380 |
| 2400 | 0.0417 | 0.01549 | 0.09195 | 0.09336 | 0.09321 | 555 | 652 | 388 | 410 | 395 |
| 2500 | 0.04151 | 0.01513 | 0.09196 | 0.09316 | 0.09321 | 582 | 674 | 402 | 424 | 418 |

Table 59

| # of games | Total Profit | | | | | Average Expected Profit | | | | |
|------------|--------------|---------|---------|---------|---------|-------------------------|---------|---------|---------|---------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 |
| 100 | 1.075 | 0.3805 | 1.664 | 1.7394 | 1.9214 | 0.0107 | 0.0038 | 0.0166 | 0.0174 | 0.0192 |
| 200 | 2.23 | 0.7269 | 3.2216 | 3.1875 | 3.6791 | 0.0111 | 0.0036 | 0.0161 | 0.0159 | 0.0184 |
| 300 | 3.1777 | 1.2542 | 4.6025 | 4.866 | 5.1628 | 0.0106 | 0.0042 | 0.0153 | 0.0162 | 0.0172 |
| 400 | 4.4579 | 1.7284 | 5.9661 | 6.4331 | 6.2567 | 0.0111 | 0.0043 | 0.0149 | 0.0161 | 0.0156 |
| 500 | 5.0208 | 2.1638 | 7.7036 | 8.3677 | 7.857 | 0.01 | 0.0043 | 0.0154 | 0.0167 | 0.0157 |
| 600 | 5.8259 | 2.6507 | 8.9067 | 9.2495 | 10.1 | 0.0097 | 0.0044 | 0.0148 | 0.0154 | 0.0168 |
| 700 | 6.973 | 3.0006 | 10.395 | 10.643 | 11.785 | 0.01 | 0.0043 | 0.0148 | 0.0152 | 0.0168 |
| 800 | 7.8774 | 3.2954 | 11.738 | 12.888 | 12.869 | 0.0098 | 0.0041 | 0.0147 | 0.0161 | 0.0161 |
| 900 | 9.3682 | 3.7474 | 12.807 | 14.372 | 14.154 | 0.0104 | 0.0042 | 0.0142 | 0.016 | 0.0157 |
| 1000 | 10.243 | 4.2043 | 14.785 | 16.12 | 15.505 | 0.0102 | 0.0042 | 0.0148 | 0.0161 | 0.0155 |
| 1100 | 11.062 | 4.6091 | 16.135 | 18.655 | 16.218 | 0.0101 | 0.0042 | 0.0147 | 0.017 | 0.0147 |
| 1200 | 12.037 | 4.9337 | 17.266 | 20.078 | 18.45 | 0.01 | 0.0041 | 0.0144 | 0.0167 | 0.0154 |
| 1300 | 12.919 | 5.5556 | 18.677 | 20.872 | 19.84 | 0.0099 | 0.0043 | 0.0144 | 0.0161 | 0.0153 |
| 1400 | 13.488 | 6.073 | 19.73 | 22.48 | 22.091 | 0.0096 | 0.0043 | 0.0141 | 0.0161 | 0.0158 |
| 1500 | 14.424 | 6.3237 | 21.267 | 24.71 | 23.297 | 0.0096 | 0.0042 | 0.0142 | 0.0165 | 0.0155 |
| 1600 | 15.111 | 6.6525 | 23.131 | 26.192 | 25.612 | 0.0094 | 0.0042 | 0.0145 | 0.0164 | 0.016 |
| 1700 | 16.253 | 6.9694 | 25.076 | 28.265 | 26.934 | 0.0096 | 0.0041 | 0.0148 | 0.0166 | 0.0158 |
| 1800 | 17.091 | 7.5816 | 26.971 | 29.584 | 28.17 | 0.0095 | 0.0042 | 0.015 | 0.0164 | 0.0156 |
| 1900 | 18.222 | 7.9954 | 28.559 | 31.175 | 29.582 | 0.0096 | 0.0042 | 0.015 | 0.0164 | 0.0156 |
| 2000 | 19.351 | 8.4585 | 30.138 | 32.11 | 30.788 | 0.0097 | 0.0042 | 0.0151 | 0.0161 | 0.0154 |
| 2100 | 20.169 | 8.882 | 31.333 | 33.79 | 32.874 | 0.0096 | 0.0042 | 0.0149 | 0.0161 | 0.0157 |
| 2200 | 21.213 | 9.3467 | 32.73 | 34.96 | 34.303 | 0.0096 | 0.0042 | 0.0149 | 0.0159 | 0.0156 |
| 2300 | 22.307 | 9.749 | 34.521 | 36.45 | 35.347 | 0.0097 | 0.0042 | 0.015 | 0.0158 | 0.0154 |
| 2400 | 23.142 | 10.101 | 35.676 | 38.278 | 36.818 | 0.0096 | 0.0042 | 0.0149 | 0.0159 | 0.0153 |
| 2500 | 24.156 | 10.194 | 36.97 | 39.499 | 38.961 | 0.0097 | 0.0041 | 0.0148 | 0.0158 | 0.0156 |

Table 60

4. Ten-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.05026 | 0.04687 | 0.04906 | 0.04925 | 0.04982 | 0.04808 | 0.04671 | 0.04911 | 0.04737 | 0.04993 |
| 200 | 0.04791 | 0.04597 | 0.04851 | 0.0481 | 0.04933 | 0.04775 | 0.04745 | 0.04825 | 0.04707 | 0.04938 |
| 300 | 0.04808 | 0.04694 | 0.04849 | 0.04783 | 0.04937 | 0.04743 | 0.04838 | 0.04786 | 0.04694 | 0.04899 |
| 400 | 0.04804 | 0.0471 | 0.04762 | 0.04784 | 0.04914 | 0.04709 | 0.04823 | 0.0478 | 0.04762 | 0.04851 |
| 500 | 0.04847 | 0.04723 | 0.04772 | 0.04777 | 0.04895 | 0.04723 | 0.04824 | 0.04806 | 0.04763 | 0.0482 |
| 600 | 0.04827 | 0.04735 | 0.04794 | 0.04781 | 0.0489 | 0.0474 | 0.04805 | 0.04825 | 0.04723 | 0.04842 |
| 700 | 0.04818 | 0.04775 | 0.04799 | 0.04817 | 0.04894 | 0.04773 | 0.04798 | 0.04817 | 0.0475 | 0.0486 |
| 800 | 0.04776 | 0.0478 | 0.04767 | 0.0481 | 0.04876 | 0.04778 | 0.04811 | 0.04817 | 0.04758 | 0.0485 |
| 900 | 0.0478 | 0.04747 | 0.04763 | 0.0482 | 0.04882 | 0.04794 | 0.04779 | 0.04802 | 0.04773 | 0.04866 |
| 1000 | 0.04775 | 0.04758 | 0.0473 | 0.04825 | 0.04868 | 0.04752 | 0.04799 | 0.04804 | 0.04776 | 0.04871 |
| 1100 | 0.04769 | 0.04772 | 0.04731 | 0.04844 | 0.04869 | 0.04764 | 0.04809 | 0.04808 | 0.04782 | 0.04864 |
| 1200 | 0.04776 | 0.04784 | 0.04738 | 0.04835 | 0.04865 | 0.0477 | 0.04788 | 0.04818 | 0.04795 | 0.04861 |
| 1300 | 0.04773 | 0.04783 | 0.04759 | 0.04809 | 0.04851 | 0.04774 | 0.04779 | 0.04807 | 0.04798 | 0.04821 |
| 1400 | 0.04758 | 0.04787 | 0.04772 | 0.04814 | 0.0485 | 0.04778 | 0.0478 | 0.0478 | 0.04778 | 0.04824 |
| 1500 | 0.04758 | 0.04792 | 0.04777 | 0.04816 | 0.04843 | 0.04784 | 0.04788 | 0.04783 | 0.04792 | 0.04814 |

Table 61

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 3 | 10 | 13 | 11 | 8 | 9 | 11 | 10 | 11 | 14 |
| 200 | 14 | 22 | 23 | 19 | 17 | 22 | 22 | 18 | 19 | 24 |
| 300 | 25 | 30 | 34 | 28 | 24 | 35 | 30 | 32 | 26 | 36 |
| 400 | 33 | 40 | 47 | 34 | 31 | 46 | 40 | 44 | 40 | 45 |
| 500 | 44 | 48 | 62 | 45 | 38 | 54 | 53 | 52 | 51 | 53 |
| 600 | 61 | 60 | 68 | 53 | 46 | 66 | 61 | 63 | 57 | 65 |
| 700 | 72 | 69 | 84 | 63 | 51 | 81 | 70 | 71 | 66 | 73 |
| 800 | 84 | 78 | 94 | 67 | 65 | 94 | 79 | 81 | 76 | 82 |
| 900 | 94 | 87 | 104 | 75 | 76 | 108 | 89 | 88 | 86 | 93 |
| 1000 | 107 | 100 | 113 | 85 | 79 | 122 | 99 | 103 | 93 | 99 |
| 1100 | 117 | 113 | 122 | 97 | 87 | 132 | 107 | 116 | 97 | 112 |
| 1200 | 126 | 128 | 130 | 110 | 97 | 139 | 117 | 126 | 106 | 121 |
| 1300 | 136 | 136 | 139 | 121 | 105 | 153 | 128 | 137 | 113 | 132 |
| 1400 | 153 | 145 | 150 | 132 | 115 | 161 | 137 | 143 | 120 | 144 |
| 1500 | 161 | 153 | 155 | 145 | 130 | 171 | 144 | 152 | 133 | 156 |

Table 62

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.15079 | 0.4687 | 0.63781 | 0.54174 | 0.39859 | 0.43274 | 0.51377 | 0.4911 | 0.52107 | 0.69908 |
| 200 | 0.67075 | 1.01141 | 1.11579 | 0.91394 | 0.83867 | 1.05043 | 1.04391 | 0.86841 | 0.89432 | 1.18501 |
| 300 | 1.20193 | 1.40814 | 1.64852 | 1.33925 | 1.18478 | 1.65995 | 1.45142 | 1.53138 | 1.22045 | 1.76347 |
| 400 | 1.58536 | 1.88405 | 2.23817 | 1.62651 | 1.5232 | 2.16622 | 1.92932 | 2.10319 | 1.90461 | 2.18315 |
| 500 | 2.13288 | 2.26682 | 2.95892 | 2.14976 | 1.86006 | 2.55046 | 2.55657 | 2.49924 | 2.42891 | 2.55475 |
| 600 | 2.94422 | 2.84113 | 3.25965 | 2.53367 | 2.24959 | 3.12811 | 2.9312 | 3.03944 | 2.69183 | 3.14708 |
| 700 | 3.46909 | 3.29466 | 4.03112 | 3.03458 | 2.49581 | 3.86584 | 3.35857 | 3.42031 | 3.13471 | 3.54814 |
| 800 | 4.01204 | 3.72815 | 4.48109 | 3.22297 | 3.16916 | 4.49151 | 3.80043 | 3.902 | 3.61583 | 3.97693 |
| 900 | 4.49303 | 4.12965 | 4.95399 | 3.61473 | 3.71014 | 5.17737 | 4.25317 | 4.22602 | 4.10521 | 4.52576 |
| 1000 | 5.10881 | 4.75789 | 5.34487 | 4.10115 | 3.84593 | 5.79802 | 4.75128 | 4.94776 | 4.44183 | 4.8218 |
| 1100 | 5.58027 | 5.39242 | 5.77149 | 4.69851 | 4.23619 | 6.28816 | 5.14514 | 5.57671 | 4.63856 | 5.44733 |
| 1200 | 6.01733 | 6.12402 | 6.15977 | 5.31874 | 4.71887 | 6.63022 | 5.60246 | 6.07045 | 5.08234 | 5.88167 |
| 1300 | 6.49161 | 6.50554 | 6.615 | 5.81876 | 5.09331 | 7.30392 | 6.1175 | 6.5852 | 5.42129 | 6.36337 |
| 1400 | 7.28042 | 6.94089 | 7.15868 | 6.35497 | 5.57765 | 7.69201 | 6.5487 | 6.83476 | 5.73393 | 6.94706 |
| 1500 | 7.66067 | 7.3325 | 7.4039 | 6.98389 | 6.29654 | 8.17982 | 6.89477 | 7.27088 | 6.37304 | 7.51016 |

Table 63

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00151 | 0.00469 | 0.00638 | 0.00542 | 0.00399 | 0.00433 | 0.00514 | 0.00491 | 0.00521 | 0.00699 |
| 200 | 0.00335 | 0.00506 | 0.00558 | 0.00457 | 0.00419 | 0.00525 | 0.00522 | 0.00434 | 0.00447 | 0.00593 |
| 300 | 0.00401 | 0.00469 | 0.0055 | 0.00446 | 0.00395 | 0.00553 | 0.00484 | 0.0051 | 0.00407 | 0.00588 |
| 400 | 0.00396 | 0.00471 | 0.0056 | 0.00407 | 0.00381 | 0.00542 | 0.00482 | 0.00526 | 0.00476 | 0.00546 |
| 500 | 0.00427 | 0.00453 | 0.00592 | 0.0043 | 0.00372 | 0.0051 | 0.00511 | 0.005 | 0.00486 | 0.00511 |
| 600 | 0.00491 | 0.00474 | 0.00543 | 0.00422 | 0.00375 | 0.00521 | 0.00489 | 0.00507 | 0.00449 | 0.00525 |
| 700 | 0.00496 | 0.00471 | 0.00576 | 0.00434 | 0.00357 | 0.00552 | 0.0048 | 0.00489 | 0.00448 | 0.00507 |
| 800 | 0.00502 | 0.00466 | 0.0056 | 0.00403 | 0.00396 | 0.00561 | 0.00475 | 0.00488 | 0.00452 | 0.00497 |
| 900 | 0.00499 | 0.00459 | 0.0055 | 0.00402 | 0.00412 | 0.00575 | 0.00473 | 0.0047 | 0.00456 | 0.00503 |
| 1000 | 0.00511 | 0.00476 | 0.00534 | 0.0041 | 0.00385 | 0.0058 | 0.00475 | 0.00495 | 0.00444 | 0.00482 |
| 1100 | 0.00507 | 0.0049 | 0.00525 | 0.00427 | 0.00385 | 0.00572 | 0.00468 | 0.00507 | 0.00422 | 0.00495 |
| 1200 | 0.00501 | 0.0051 | 0.00513 | 0.00443 | 0.00393 | 0.00553 | 0.00467 | 0.00506 | 0.00424 | 0.0049 |
| 1300 | 0.00499 | 0.005 | 0.00509 | 0.00448 | 0.00392 | 0.00562 | 0.00471 | 0.00507 | 0.00417 | 0.00489 |
| 1400 | 0.0052 | 0.00496 | 0.00511 | 0.00454 | 0.00398 | 0.00549 | 0.00468 | 0.00488 | 0.0041 | 0.00496 |
| 1500 | 0.00511 | 0.00489 | 0.00494 | 0.00466 | 0.0042 | 0.00545 | 0.0046 | 0.00485 | 0.00425 | 0.00501 |

Table 64

Scenario 2. Bidder1 underbid by 0.025 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0217 | 0.0507 | 0.0494 | 0.0483 | 0.0481 | 0.047 | 0.0479 | 0.0472 | 0.0484 | 0.049 |
| 200 | 0.0217 | 0.0489 | 0.0486 | 0.0486 | 0.0466 | 0.0473 | 0.0479 | 0.0474 | 0.0486 | 0.0483 |
| 300 | 0.0212 | 0.0492 | 0.0478 | 0.0479 | 0.0475 | 0.0465 | 0.0478 | 0.0469 | 0.0487 | 0.0469 |
| 400 | 0.022 | 0.0488 | 0.0479 | 0.0483 | 0.0475 | 0.047 | 0.0477 | 0.0474 | 0.0485 | 0.0472 |
| 500 | 0.022 | 0.0488 | 0.0479 | 0.0482 | 0.0473 | 0.0476 | 0.0481 | 0.0476 | 0.0483 | 0.0477 |
| 600 | 0.0221 | 0.0488 | 0.0478 | 0.0483 | 0.0473 | 0.0471 | 0.0478 | 0.0477 | 0.0484 | 0.048 |
| 700 | 0.0221 | 0.0487 | 0.0475 | 0.0486 | 0.0473 | 0.0471 | 0.0478 | 0.0477 | 0.048 | 0.0478 |
| 800 | 0.0223 | 0.0484 | 0.0474 | 0.0486 | 0.047 | 0.0472 | 0.048 | 0.0473 | 0.0481 | 0.0477 |
| 900 | 0.0223 | 0.0483 | 0.0474 | 0.0485 | 0.0468 | 0.0474 | 0.048 | 0.0476 | 0.0483 | 0.0479 |
| 1000 | 0.0225 | 0.0483 | 0.0477 | 0.0484 | 0.0467 | 0.0476 | 0.0479 | 0.0475 | 0.0482 | 0.0478 |
| 1100 | 0.0225 | 0.0484 | 0.0476 | 0.0483 | 0.0468 | 0.0477 | 0.048 | 0.0475 | 0.0482 | 0.0477 |
| 1200 | 0.0223 | 0.0485 | 0.0476 | 0.0483 | 0.0469 | 0.0477 | 0.0481 | 0.0476 | 0.0483 | 0.0478 |
| 1300 | 0.0224 | 0.0484 | 0.0477 | 0.048 | 0.0469 | 0.0476 | 0.048 | 0.0476 | 0.0479 | 0.0477 |
| 1400 | 0.0225 | 0.0485 | 0.0478 | 0.0481 | 0.0472 | 0.0476 | 0.048 | 0.0477 | 0.0478 | 0.0477 |
| 1500 | 0.0225 | 0.0485 | 0.0479 | 0.0483 | 0.0472 | 0.0476 | 0.048 | 0.0477 | 0.0478 | 0.0476 |
| 1600 | 0.0224 | 0.0483 | 0.048 | 0.0482 | 0.0473 | 0.0477 | 0.048 | 0.0476 | 0.0478 | 0.0476 |
| 1700 | 0.0225 | 0.0483 | 0.048 | 0.0483 | 0.0472 | 0.0476 | 0.0477 | 0.0477 | 0.0478 | 0.0476 |
| 1800 | 0.0226 | 0.0483 | 0.0479 | 0.0483 | 0.0472 | 0.0476 | 0.0476 | 0.0477 | 0.0479 | 0.0477 |
| 1900 | 0.0225 | 0.0484 | 0.0478 | 0.0483 | 0.0472 | 0.0477 | 0.0477 | 0.0478 | 0.0479 | 0.0476 |
| 2000 | 0.0224 | 0.0484 | 0.0477 | 0.0484 | 0.0472 | 0.0477 | 0.0477 | 0.0479 | 0.048 | 0.0477 |
| 2100 | 0.0225 | 0.0483 | 0.0477 | 0.0485 | 0.0473 | 0.0477 | 0.0477 | 0.0479 | 0.0479 | 0.0477 |
| 2200 | 0.0226 | 0.0484 | 0.0479 | 0.0485 | 0.0474 | 0.0476 | 0.0475 | 0.048 | 0.048 | 0.0476 |
| 2300 | 0.0225 | 0.0484 | 0.0479 | 0.0484 | 0.0474 | 0.0476 | 0.0477 | 0.048 | 0.048 | 0.0476 |
| 2400 | 0.0225 | 0.0484 | 0.048 | 0.0484 | 0.0474 | 0.0476 | 0.0478 | 0.048 | 0.0479 | 0.0477 |
| 2500 | 0.0225 | 0.0484 | 0.0481 | 0.0483 | 0.0474 | 0.0477 | 0.0478 | 0.048 | 0.048 | 0.0477 |

Table 65

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 15 | 14 | 7 | 10 | 6 | 10 | 11 | 8 | 10 | 9 |
| 200 | 25 | 28 | 13 | 16 | 16 | 22 | 22 | 18 | 21 | 19 |
| 300 | 36 | 41 | 22 | 25 | 35 | 35 | 29 | 22 | 27 | 28 |
| 400 | 45 | 51 | 38 | 39 | 42 | 42 | 39 | 30 | 39 | 35 |
| 500 | 53 | 66 | 47 | 54 | 58 | 50 | 46 | 32 | 48 | 46 |
| 600 | 64 | 75 | 55 | 64 | 64 | 70 | 57 | 40 | 55 | 56 |
| 700 | 75 | 84 | 71 | 77 | 70 | 80 | 65 | 50 | 64 | 64 |
| 800 | 88 | 93 | 85 | 87 | 77 | 88 | 69 | 63 | 76 | 74 |
| 900 | 101 | 107 | 93 | 97 | 84 | 99 | 79 | 74 | 82 | 84 |
| 1000 | 112 | 116 | 105 | 108 | 94 | 109 | 85 | 85 | 94 | 92 |
| 1100 | 120 | 124 | 113 | 123 | 101 | 124 | 97 | 93 | 104 | 101 |
| 1200 | 130 | 133 | 118 | 132 | 113 | 135 | 111 | 100 | 116 | 112 |
| 1300 | 144 | 138 | 126 | 145 | 119 | 145 | 122 | 114 | 127 | 120 |
| 1400 | 159 | 151 | 136 | 151 | 133 | 155 | 127 | 123 | 134 | 131 |
| 1500 | 178 | 160 | 146 | 162 | 140 | 169 | 131 | 132 | 141 | 141 |
| 1600 | 195 | 165 | 153 | 170 | 149 | 182 | 138 | 139 | 157 | 152 |
| 1700 | 206 | 172 | 163 | 177 | 165 | 194 | 148 | 146 | 168 | 161 |
| 1800 | 218 | 178 | 179 | 189 | 172 | 206 | 157 | 155 | 174 | 172 |
| 1900 | 227 | 188 | 192 | 197 | 189 | 216 | 164 | 166 | 182 | 179 |
| 2000 | 235 | 196 | 204 | 211 | 197 | 225 | 174 | 175 | 197 | 186 |
| 2100 | 245 | 208 | 212 | 226 | 210 | 229 | 180 | 184 | 205 | 201 |
| 2200 | 257 | 213 | 222 | 235 | 216 | 238 | 193 | 198 | 220 | 208 |
| 2300 | 265 | 228 | 235 | 245 | 223 | 248 | 205 | 207 | 229 | 215 |
| 2400 | 276 | 238 | 244 | 249 | 233 | 259 | 219 | 218 | 238 | 226 |
| 2500 | 289 | 246 | 257 | 255 | 244 | 270 | 229 | 226 | 246 | 238 |

Table 66

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.3252 | 0.7101 | 0.3456 | 0.4828 | 0.2889 | 0.47 | 0.5274 | 0.3779 | 0.484 | 0.4409 |
| 200 | 0.5418 | 1.3688 | 0.6314 | 0.7774 | 0.7451 | 1.0417 | 1.0531 | 0.8524 | 1.0197 | 0.917 |
| 300 | 0.7647 | 2.0162 | 1.0508 | 1.1968 | 1.664 | 1.6284 | 1.385 | 1.0314 | 1.3143 | 1.3142 |
| 400 | 0.9903 | 2.4872 | 1.8188 | 1.8827 | 1.9937 | 1.976 | 1.8622 | 1.4217 | 1.8899 | 1.6521 |
| 500 | 1.1665 | 3.2196 | 2.2519 | 2.6004 | 2.7447 | 2.3804 | 2.2107 | 1.5224 | 2.3201 | 2.1941 |
| 600 | 1.4152 | 3.6569 | 2.6297 | 3.0931 | 3.0268 | 3.3003 | 2.7218 | 1.9075 | 2.6632 | 2.6859 |
| 700 | 1.6568 | 4.0934 | 3.376 | 3.7438 | 3.3123 | 3.7692 | 3.1079 | 2.3841 | 3.0716 | 3.0565 |
| 800 | 1.9617 | 4.4979 | 4.0272 | 4.2296 | 3.6161 | 4.1525 | 3.3095 | 2.9784 | 3.6586 | 3.5308 |
| 900 | 2.2565 | 5.1641 | 4.4109 | 4.7027 | 3.9282 | 4.693 | 3.79 | 3.5209 | 3.9626 | 4.0257 |
| 1000 | 2.5205 | 5.6029 | 5.0042 | 5.2256 | 4.3928 | 5.1862 | 4.0703 | 4.0382 | 4.5313 | 4.401 |
| 1100 | 2.6953 | 5.9997 | 5.382 | 5.936 | 4.7301 | 5.9183 | 4.6528 | 4.4207 | 5.0124 | 4.821 |
| 1200 | 2.9012 | 6.4525 | 5.615 | 6.3691 | 5.2963 | 6.4429 | 5.3356 | 4.7587 | 5.6063 | 5.3497 |
| 1300 | 3.2297 | 6.6856 | 6.0123 | 6.9659 | 5.577 | 6.9054 | 5.8584 | 5.4236 | 6.0865 | 5.7283 |
| 1400 | 3.5791 | 7.3168 | 6.5073 | 7.258 | 6.2721 | 7.3733 | 6.1007 | 5.8692 | 6.4063 | 6.2494 |
| 1500 | 4.0011 | 7.7628 | 6.9986 | 7.8166 | 6.6119 | 8.0409 | 6.293 | 6.2929 | 6.7457 | 6.7108 |
| 1600 | 4.372 | 7.9771 | 7.3455 | 8.2025 | 7.0534 | 8.678 | 6.6245 | 6.6109 | 7.5113 | 7.2307 |
| 1700 | 4.6287 | 8.3098 | 7.8207 | 8.5468 | 7.7884 | 9.2265 | 7.0657 | 6.9613 | 8.0355 | 7.663 |
| 1800 | 4.9233 | 8.5942 | 8.5704 | 9.1217 | 8.1113 | 9.8048 | 7.4765 | 7.3946 | 8.33 | 8.1996 |
| 1900 | 5.1088 | 9.0943 | 9.1807 | 9.5111 | 8.9199 | 10.3 | 7.819 | 7.9405 | 8.7265 | 8.5292 |
| 2000 | 5.2698 | 9.4777 | 9.7404 | 10.217 | 9.2893 | 10.736 | 8.297 | 8.3835 | 9.4467 | 8.8703 |
| 2100 | 5.514 | 10.057 | 10.122 | 10.956 | 9.9334 | 10.925 | 8.5882 | 8.8097 | 9.829 | 9.5834 |
| 2200 | 5.8054 | 10.307 | 10.629 | 11.393 | 10.235 | 11.329 | 9.1767 | 9.5034 | 10.566 | 9.9025 |
| 2300 | 5.9633 | 11.04 | 11.259 | 11.856 | 10.573 | 11.815 | 9.7797 | 9.9395 | 10.988 | 10.24 |
| 2400 | 6.2043 | 11.518 | 11.716 | 12.044 | 11.042 | 12.341 | 10.463 | 10.473 | 11.397 | 10.779 |
| 2500 | 6.5026 | 11.9 | 12.349 | 12.322 | 11.557 | 12.876 | 10.943 | 10.859 | 11.798 | 11.356 |

Table 67

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0033 | 0.0071 | 0.0035 | 0.0048 | 0.0029 | 0.0047 | 0.0053 | 0.0038 | 0.0048 | 0.0044 |
| 200 | 0.0027 | 0.0068 | 0.0032 | 0.0039 | 0.0037 | 0.0052 | 0.0053 | 0.0043 | 0.0051 | 0.0046 |
| 300 | 0.0025 | 0.0067 | 0.0035 | 0.004 | 0.0055 | 0.0054 | 0.0046 | 0.0034 | 0.0044 | 0.0044 |
| 400 | 0.0025 | 0.0062 | 0.0045 | 0.0047 | 0.005 | 0.0049 | 0.0047 | 0.0036 | 0.0047 | 0.0041 |
| 500 | 0.0023 | 0.0064 | 0.0045 | 0.0052 | 0.0055 | 0.0048 | 0.0044 | 0.003 | 0.0046 | 0.0044 |
| 600 | 0.0024 | 0.0061 | 0.0044 | 0.0052 | 0.005 | 0.0055 | 0.0045 | 0.0032 | 0.0044 | 0.0045 |
| 700 | 0.0024 | 0.0058 | 0.0048 | 0.0053 | 0.0047 | 0.0054 | 0.0044 | 0.0034 | 0.0044 | 0.0044 |
| 800 | 0.0025 | 0.0056 | 0.005 | 0.0053 | 0.0045 | 0.0052 | 0.0041 | 0.0037 | 0.0046 | 0.0044 |
| 900 | 0.0025 | 0.0057 | 0.0049 | 0.0052 | 0.0044 | 0.0052 | 0.0042 | 0.0039 | 0.0044 | 0.0045 |
| 1000 | 0.0025 | 0.0056 | 0.005 | 0.0052 | 0.0044 | 0.0052 | 0.0041 | 0.004 | 0.0045 | 0.0044 |
| 1100 | 0.0025 | 0.0055 | 0.0049 | 0.0054 | 0.0043 | 0.0054 | 0.0042 | 0.004 | 0.0046 | 0.0044 |
| 1200 | 0.0024 | 0.0054 | 0.0047 | 0.0053 | 0.0044 | 0.0054 | 0.0044 | 0.004 | 0.0047 | 0.0045 |
| 1300 | 0.0025 | 0.0051 | 0.0046 | 0.0054 | 0.0043 | 0.0053 | 0.0045 | 0.0042 | 0.0047 | 0.0044 |
| 1400 | 0.0026 | 0.0052 | 0.0046 | 0.0052 | 0.0045 | 0.0053 | 0.0044 | 0.0042 | 0.0046 | 0.0045 |
| 1500 | 0.0027 | 0.0052 | 0.0047 | 0.0052 | 0.0044 | 0.0054 | 0.0042 | 0.0042 | 0.0045 | 0.0045 |
| 1600 | 0.0027 | 0.005 | 0.0046 | 0.0051 | 0.0044 | 0.0054 | 0.0041 | 0.0041 | 0.0047 | 0.0045 |
| 1700 | 0.0027 | 0.0049 | 0.0046 | 0.005 | 0.0046 | 0.0054 | 0.0042 | 0.0041 | 0.0047 | 0.0045 |
| 1800 | 0.0027 | 0.0048 | 0.0048 | 0.0051 | 0.0045 | 0.0054 | 0.0042 | 0.0041 | 0.0046 | 0.0046 |
| 1900 | 0.0027 | 0.0048 | 0.0048 | 0.005 | 0.0047 | 0.0054 | 0.0041 | 0.0042 | 0.0046 | 0.0045 |
| 2000 | 0.0026 | 0.0047 | 0.0049 | 0.0051 | 0.0046 | 0.0054 | 0.0041 | 0.0042 | 0.0047 | 0.0044 |
| 2100 | 0.0026 | 0.0048 | 0.0048 | 0.0052 | 0.0047 | 0.0052 | 0.0041 | 0.0042 | 0.0047 | 0.0046 |
| 2200 | 0.0026 | 0.0047 | 0.0048 | 0.0052 | 0.0047 | 0.0051 | 0.0042 | 0.0043 | 0.0048 | 0.0045 |
| 2300 | 0.0026 | 0.0048 | 0.0049 | 0.0052 | 0.0046 | 0.0051 | 0.0043 | 0.0043 | 0.0048 | 0.0045 |
| 2400 | 0.0026 | 0.0048 | 0.0049 | 0.005 | 0.0046 | 0.0051 | 0.0044 | 0.0044 | 0.0047 | 0.0045 |
| 2500 | 0.0026 | 0.0048 | 0.0049 | 0.0049 | 0.0046 | 0.0052 | 0.0044 | 0.0043 | 0.0047 | 0.0045 |

Table 68

Scenario 3. Bidders “1” and “2” underbid by 0.025 while others used

equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.02212 | 0.02373 | 0.0487 | 0.04898 | 0.04677 | 0.04714 | 0.04748 | 0.04981 | 0.04578 | 0.04711 |
| 200 | 0.02207 | 0.02381 | 0.04921 | 0.04882 | 0.04797 | 0.04861 | 0.04696 | 0.04608 | 0.04735 | 0.04866 |
| 300 | 0.02203 | 0.02328 | 0.04903 | 0.04859 | 0.04752 | 0.04817 | 0.04762 | 0.04588 | 0.04626 | 0.04945 |
| 400 | 0.02199 | 0.02298 | 0.04873 | 0.04901 | 0.04763 | 0.04793 | 0.04801 | 0.04686 | 0.04673 | 0.04955 |
| 500 | 0.02202 | 0.02291 | 0.04886 | 0.04915 | 0.04775 | 0.048 | 0.04788 | 0.04694 | 0.04698 | 0.04901 |
| 600 | 0.02221 | 0.02302 | 0.04853 | 0.04893 | 0.04787 | 0.04827 | 0.04792 | 0.04724 | 0.04711 | 0.04875 |
| 700 | 0.02252 | 0.02285 | 0.04854 | 0.04893 | 0.0474 | 0.04813 | 0.04795 | 0.04763 | 0.04707 | 0.04831 |
| 800 | 0.0222 | 0.02259 | 0.0484 | 0.04866 | 0.0473 | 0.04784 | 0.04791 | 0.0479 | 0.0475 | 0.04763 |
| 900 | 0.02237 | 0.02287 | 0.04826 | 0.04872 | 0.04741 | 0.04787 | 0.04804 | 0.0478 | 0.04752 | 0.04761 |
| 1000 | 0.02255 | 0.02265 | 0.04837 | 0.04875 | 0.0472 | 0.0478 | 0.04805 | 0.04791 | 0.04768 | 0.04773 |
| 1100 | 0.02244 | 0.02273 | 0.04801 | 0.0488 | 0.04731 | 0.04779 | 0.04809 | 0.04796 | 0.04773 | 0.04773 |
| 1200 | 0.02234 | 0.023 | 0.04818 | 0.04861 | 0.04743 | 0.04798 | 0.04811 | 0.04813 | 0.04788 | 0.04792 |
| 1300 | 0.02246 | 0.02301 | 0.04827 | 0.04866 | 0.04725 | 0.04801 | 0.04806 | 0.0479 | 0.04758 | 0.04791 |
| 1400 | 0.02247 | 0.02297 | 0.04822 | 0.04879 | 0.04738 | 0.048 | 0.04798 | 0.04787 | 0.04757 | 0.04771 |
| 1500 | 0.02258 | 0.02307 | 0.04816 | 0.04889 | 0.04735 | 0.048 | 0.04802 | 0.04801 | 0.04759 | 0.0478 |

Table 69

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 11 | 15 | 8 | 14 | 11 | 6 | 13 | 7 | 8 | 7 |
| 200 | 22 | 23 | 18 | 21 | 24 | 19 | 18 | 23 | 14 | 18 |
| 300 | 34 | 31 | 28 | 36 | 35 | 28 | 31 | 29 | 21 | 27 |
| 400 | 44 | 39 | 35 | 45 | 46 | 40 | 43 | 40 | 35 | 33 |
| 500 | 57 | 49 | 47 | 55 | 51 | 48 | 51 | 47 | 48 | 47 |
| 600 | 72 | 56 | 58 | 68 | 61 | 55 | 59 | 63 | 56 | 52 |
| 700 | 84 | 64 | 67 | 80 | 71 | 66 | 67 | 75 | 61 | 65 |
| 800 | 100 | 73 | 73 | 93 | 73 | 78 | 72 | 85 | 74 | 79 |
| 900 | 112 | 92 | 83 | 99 | 86 | 87 | 78 | 95 | 80 | 88 |
| 1000 | 122 | 103 | 93 | 106 | 98 | 93 | 94 | 108 | 89 | 94 |
| 1100 | 139 | 117 | 110 | 114 | 105 | 95 | 102 | 120 | 95 | 103 |
| 1200 | 147 | 133 | 119 | 123 | 111 | 102 | 110 | 139 | 106 | 110 |
| 1300 | 157 | 143 | 132 | 131 | 117 | 114 | 121 | 148 | 116 | 121 |
| 1400 | 169 | 150 | 144 | 142 | 128 | 126 | 128 | 157 | 123 | 133 |
| 1500 | 184 | 161 | 152 | 149 | 132 | 136 | 141 | 170 | 131 | 144 |

Table 70

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.24331 | 0.35598 | 0.3896 | 0.68573 | 0.51442 | 0.28283 | 0.61725 | 0.34869 | 0.36623 | 0.32979 |
| 200 | 0.4856 | 0.54767 | 0.88579 | 1.02517 | 1.15136 | 0.92353 | 0.84526 | 1.05989 | 0.66284 | 0.87587 |
| 300 | 0.74908 | 0.72159 | 1.37281 | 1.74907 | 1.66318 | 1.34871 | 1.47631 | 1.33042 | 0.97153 | 1.33509 |
| 400 | 0.96776 | 0.89614 | 1.70571 | 2.20533 | 2.19088 | 1.91717 | 2.06425 | 1.87429 | 1.63549 | 1.63505 |
| 500 | 1.25502 | 1.12266 | 2.29643 | 2.70325 | 2.43527 | 2.304 | 2.44196 | 2.20628 | 2.25522 | 2.30336 |
| 600 | 1.59923 | 1.28921 | 2.81496 | 3.32723 | 2.91996 | 2.65465 | 2.82752 | 2.97594 | 2.63821 | 2.53521 |
| 700 | 1.89195 | 1.46243 | 3.25206 | 3.91403 | 3.36545 | 3.17633 | 3.21252 | 3.57201 | 2.87118 | 3.14025 |
| 800 | 2.21981 | 1.64926 | 3.53285 | 4.52538 | 3.45279 | 3.73164 | 3.44953 | 4.07183 | 3.51474 | 3.76287 |
| 900 | 2.50595 | 2.10389 | 4.00517 | 4.82356 | 4.07686 | 4.1649 | 3.74683 | 4.54129 | 3.80147 | 4.18985 |
| 1000 | 2.75127 | 2.33287 | 4.49838 | 5.167 | 4.62512 | 4.44522 | 4.51633 | 5.17454 | 4.24391 | 4.48682 |
| 1100 | 3.11969 | 2.65933 | 5.28077 | 5.56285 | 4.96761 | 4.53987 | 4.90521 | 5.75562 | 4.53427 | 4.91599 |
| 1200 | 3.28444 | 3.05956 | 5.73341 | 5.97865 | 5.26508 | 4.89431 | 5.29241 | 6.68969 | 5.07568 | 5.27066 |
| 1300 | 3.5257 | 3.29036 | 6.37126 | 6.37396 | 5.52846 | 5.4737 | 5.81544 | 7.08965 | 5.51965 | 5.79725 |
| 1400 | 3.79716 | 3.44557 | 6.94326 | 6.92853 | 6.06497 | 6.04738 | 6.14142 | 7.5158 | 5.85118 | 6.34563 |
| 1500 | 4.1538 | 3.71366 | 7.31958 | 7.28469 | 6.24978 | 6.52806 | 6.77046 | 8.16089 | 6.234 | 6.88334 |

Table 71

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.00243 | 0.00356 | 0.0039 | 0.00686 | 0.00514 | 0.00283 | 0.00617 | 0.00349 | 0.00366 | 0.0033 |
| 200 | 0.00243 | 0.00274 | 0.00443 | 0.00513 | 0.00576 | 0.00462 | 0.00423 | 0.0053 | 0.00331 | 0.00438 |
| 300 | 0.0025 | 0.00241 | 0.00458 | 0.00583 | 0.00554 | 0.0045 | 0.00492 | 0.00443 | 0.00324 | 0.00445 |
| 400 | 0.00242 | 0.00224 | 0.00426 | 0.00551 | 0.00548 | 0.00479 | 0.00516 | 0.00469 | 0.00409 | 0.00409 |
| 500 | 0.00251 | 0.00225 | 0.00459 | 0.00541 | 0.00487 | 0.00461 | 0.00488 | 0.00441 | 0.00451 | 0.00461 |
| 600 | 0.00267 | 0.00215 | 0.00469 | 0.00555 | 0.00487 | 0.00442 | 0.00471 | 0.00496 | 0.0044 | 0.00423 |
| 700 | 0.0027 | 0.00209 | 0.00465 | 0.00559 | 0.00481 | 0.00454 | 0.00459 | 0.0051 | 0.0041 | 0.00449 |
| 800 | 0.00277 | 0.00206 | 0.00442 | 0.00566 | 0.00432 | 0.00466 | 0.00431 | 0.00509 | 0.00439 | 0.0047 |
| 900 | 0.00278 | 0.00234 | 0.00445 | 0.00536 | 0.00453 | 0.00463 | 0.00416 | 0.00505 | 0.00422 | 0.00466 |
| 1000 | 0.00275 | 0.00233 | 0.0045 | 0.00517 | 0.00463 | 0.00445 | 0.00452 | 0.00517 | 0.00424 | 0.00449 |
| 1100 | 0.00284 | 0.00242 | 0.0048 | 0.00506 | 0.00452 | 0.00413 | 0.00446 | 0.00523 | 0.00412 | 0.00447 |
| 1200 | 0.00274 | 0.00255 | 0.00478 | 0.00498 | 0.00439 | 0.00408 | 0.00441 | 0.00557 | 0.00423 | 0.00439 |
| 1300 | 0.00271 | 0.00253 | 0.0049 | 0.0049 | 0.00425 | 0.00421 | 0.00447 | 0.00545 | 0.00425 | 0.00446 |
| 1400 | 0.00271 | 0.00246 | 0.00496 | 0.00495 | 0.00433 | 0.00432 | 0.00439 | 0.00537 | 0.00418 | 0.00453 |
| 1500 | 0.00277 | 0.00248 | 0.00488 | 0.00486 | 0.00417 | 0.00435 | 0.00451 | 0.00544 | 0.00416 | 0.00459 |

Table 72

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.03, 0.025, 0.02 and

0.015 respectively while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0082 | 0.0149 | 0.0152 | 0.022 | 0.0484 | 0.0499 | 0.0496 | 0.0493 | 0.0475 | 0.0477 |
| 200 | 0.0082 | 0.0136 | 0.016 | 0.0228 | 0.0478 | 0.0472 | 0.0496 | 0.0492 | 0.0481 | 0.0456 |
| 300 | 0.0076 | 0.0133 | 0.0173 | 0.0231 | 0.0483 | 0.047 | 0.0472 | 0.049 | 0.048 | 0.0466 |
| 400 | 0.0079 | 0.0128 | 0.0172 | 0.023 | 0.0484 | 0.047 | 0.0481 | 0.0487 | 0.0484 | 0.0471 |
| 500 | 0.0079 | 0.0128 | 0.0169 | 0.0231 | 0.0484 | 0.047 | 0.0477 | 0.0486 | 0.0482 | 0.0476 |
| 600 | 0.0078 | 0.0131 | 0.0169 | 0.0227 | 0.0484 | 0.0472 | 0.048 | 0.0489 | 0.0482 | 0.047 |
| 700 | 0.0079 | 0.0129 | 0.0171 | 0.0226 | 0.0485 | 0.0477 | 0.0483 | 0.0488 | 0.0482 | 0.0473 |
| 800 | 0.0078 | 0.0129 | 0.017 | 0.0228 | 0.0487 | 0.0477 | 0.0483 | 0.0489 | 0.0484 | 0.0473 |
| 900 | 0.0077 | 0.013 | 0.017 | 0.0225 | 0.0484 | 0.0479 | 0.0485 | 0.0489 | 0.0483 | 0.0475 |
| 1000 | 0.0077 | 0.0128 | 0.0169 | 0.0226 | 0.048 | 0.0478 | 0.0483 | 0.0488 | 0.0482 | 0.0474 |
| 1100 | 0.0078 | 0.0129 | 0.017 | 0.0225 | 0.0479 | 0.0479 | 0.0484 | 0.0487 | 0.0482 | 0.0475 |
| 1200 | 0.0078 | 0.0128 | 0.0171 | 0.0225 | 0.0479 | 0.048 | 0.0483 | 0.0487 | 0.0482 | 0.0475 |
| 1300 | 0.0077 | 0.013 | 0.0172 | 0.0224 | 0.0479 | 0.0481 | 0.0482 | 0.0486 | 0.0482 | 0.0475 |
| 1400 | 0.0075 | 0.013 | 0.0173 | 0.0224 | 0.048 | 0.0481 | 0.0482 | 0.0486 | 0.0481 | 0.0474 |
| 1500 | 0.0074 | 0.013 | 0.0174 | 0.0224 | 0.0481 | 0.0482 | 0.0479 | 0.0487 | 0.0481 | 0.0474 |
| 1600 | 0.0073 | 0.0131 | 0.0174 | 0.0225 | 0.0481 | 0.0483 | 0.048 | 0.0486 | 0.0481 | 0.0473 |
| 1700 | 0.0074 | 0.013 | 0.0174 | 0.0226 | 0.0481 | 0.0482 | 0.0478 | 0.0487 | 0.0482 | 0.0473 |
| 1800 | 0.0074 | 0.0129 | 0.0174 | 0.0224 | 0.0479 | 0.048 | 0.0477 | 0.0485 | 0.0481 | 0.0471 |
| 1900 | 0.0074 | 0.013 | 0.0175 | 0.0225 | 0.0477 | 0.048 | 0.0478 | 0.0486 | 0.0482 | 0.0473 |
| 2000 | 0.0074 | 0.0129 | 0.0174 | 0.0225 | 0.0477 | 0.0481 | 0.0479 | 0.0484 | 0.0482 | 0.0473 |

Table 73

| # of games | Number of Games Won | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 7 | 11 | 15 | 11 | 10 | 10 | 6 | 15 | 5 | 10 |
| 200 | 22 | 25 | 23 | 24 | 16 | 18 | 14 | 23 | 16 | 19 |
| 300 | 32 | 43 | 36 | 35 | 25 | 27 | 21 | 29 | 26 | 26 |
| 400 | 40 | 57 | 47 | 41 | 34 | 34 | 31 | 42 | 36 | 38 |
| 500 | 52 | 68 | 64 | 50 | 38 | 44 | 42 | 52 | 47 | 43 |
| 600 | 68 | 80 | 79 | 60 | 42 | 52 | 53 | 59 | 55 | 52 |
| 700 | 77 | 93 | 92 | 68 | 49 | 61 | 67 | 67 | 69 | 57 |
| 800 | 93 | 106 | 105 | 77 | 55 | 76 | 73 | 72 | 78 | 65 |
| 900 | 110 | 118 | 117 | 84 | 71 | 85 | 84 | 78 | 86 | 67 |
| 1000 | 125 | 137 | 129 | 95 | 83 | 92 | 88 | 82 | 93 | 76 |
| 1100 | 139 | 149 | 143 | 109 | 90 | 104 | 94 | 94 | 98 | 80 |
| 1200 | 151 | 161 | 155 | 126 | 95 | 109 | 102 | 104 | 109 | 88 |
| 1300 | 161 | 174 | 168 | 134 | 107 | 118 | 109 | 117 | 115 | 97 |
| 1400 | 181 | 181 | 179 | 144 | 121 | 128 | 116 | 121 | 126 | 103 |
| 1500 | 201 | 195 | 185 | 156 | 131 | 137 | 120 | 127 | 133 | 115 |
| 1600 | 215 | 210 | 202 | 169 | 135 | 144 | 124 | 135 | 142 | 124 |
| 1700 | 225 | 224 | 213 | 176 | 142 | 154 | 139 | 145 | 152 | 130 |
| 1800 | 239 | 235 | 232 | 185 | 145 | 165 | 147 | 154 | 160 | 138 |
| 1900 | 250 | 255 | 235 | 200 | 159 | 176 | 154 | 161 | 166 | 144 |
| 2000 | 265 | 273 | 244 | 206 | 166 | 181 | 165 | 168 | 180 | 152 |

Table 74

| # of games | Total Profit | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0573 | 0.1639 | 0.2282 | 0.2416 | 0.4837 | 0.4988 | 0.2973 | 0.7399 | 0.2376 | 0.4774 |
| 200 | 0.1795 | 0.3398 | 0.3688 | 0.5468 | 0.7651 | 0.8502 | 0.6947 | 1.1314 | 0.7699 | 0.8655 |
| 300 | 0.2446 | 0.5729 | 0.6244 | 0.8086 | 1.208 | 1.2691 | 0.9914 | 1.4207 | 1.249 | 1.2108 |
| 400 | 0.3148 | 0.7275 | 0.8063 | 0.945 | 1.6456 | 1.5979 | 1.4897 | 2.0459 | 1.7411 | 1.7914 |
| 500 | 0.411 | 0.8681 | 1.0833 | 1.1554 | 1.8378 | 2.0672 | 2.0049 | 2.5295 | 2.2653 | 2.0449 |
| 600 | 0.5273 | 1.0449 | 1.3385 | 1.3649 | 2.0328 | 2.4532 | 2.5422 | 2.8828 | 2.6517 | 2.4442 |
| 700 | 0.606 | 1.1976 | 1.5771 | 1.536 | 2.3743 | 2.9091 | 3.2364 | 3.2723 | 3.3269 | 2.6967 |
| 800 | 0.7282 | 1.3654 | 1.7808 | 1.7523 | 2.6761 | 3.6278 | 3.5295 | 3.5211 | 3.7738 | 3.0774 |
| 900 | 0.8452 | 1.5333 | 1.9847 | 1.8923 | 3.4358 | 4.068 | 4.0741 | 3.8108 | 4.1546 | 3.1794 |
| 1000 | 0.9587 | 1.7545 | 2.1758 | 2.1503 | 3.9857 | 4.4017 | 4.2535 | 4.002 | 4.4862 | 3.6047 |
| 1100 | 1.0906 | 1.9154 | 2.436 | 2.457 | 4.3113 | 4.977 | 4.5508 | 4.5732 | 4.7201 | 3.7971 |
| 1200 | 1.1722 | 2.0667 | 2.6498 | 2.8404 | 4.5538 | 5.2305 | 4.9311 | 5.0645 | 5.2541 | 4.1835 |
| 1300 | 1.2348 | 2.2678 | 2.8897 | 2.9959 | 5.1273 | 5.6795 | 5.2534 | 5.6898 | 5.5403 | 4.6117 |
| 1400 | 1.3578 | 2.3604 | 3.0952 | 3.2268 | 5.8035 | 6.1622 | 5.5878 | 5.8856 | 6.0622 | 4.8814 |
| 1500 | 1.4857 | 2.5406 | 3.2198 | 3.4992 | 6.3051 | 6.6027 | 5.7517 | 6.18 | 6.3968 | 5.451 |
| 1600 | 1.5725 | 2.7434 | 3.5155 | 3.8005 | 6.4965 | 6.951 | 5.949 | 6.5631 | 6.8297 | 5.8683 |
| 1700 | 1.6575 | 2.9125 | 3.6983 | 3.9721 | 6.8242 | 7.4179 | 6.6387 | 7.062 | 7.3214 | 6.1503 |
| 1800 | 1.7591 | 3.0343 | 4.0452 | 4.1413 | 6.9431 | 7.9245 | 7.0067 | 7.4717 | 7.696 | 6.5066 |
| 1900 | 1.8505 | 3.3144 | 4.1048 | 4.5068 | 7.5811 | 8.4559 | 7.3549 | 7.8192 | 7.9965 | 6.8099 |
| 2000 | 1.9694 | 3.5223 | 4.2502 | 4.6414 | 7.9129 | 8.7005 | 7.8983 | 8.1379 | 8.682 | 7.1935 |

Table 75

| # of games | Average Expected Profit | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 |
| 100 | 0.0006 | 0.0016 | 0.0023 | 0.0024 | 0.0048 | 0.005 | 0.003 | 0.0074 | 0.0024 | 0.0048 |
| 200 | 0.0009 | 0.0017 | 0.0018 | 0.0027 | 0.0038 | 0.0043 | 0.0035 | 0.0057 | 0.0038 | 0.0043 |
| 300 | 0.0008 | 0.0019 | 0.0021 | 0.0027 | 0.004 | 0.0042 | 0.0033 | 0.0047 | 0.0042 | 0.004 |
| 400 | 0.0008 | 0.0018 | 0.002 | 0.0024 | 0.0041 | 0.004 | 0.0037 | 0.0051 | 0.0044 | 0.0045 |
| 500 | 0.0008 | 0.0017 | 0.0022 | 0.0023 | 0.0037 | 0.0041 | 0.004 | 0.0051 | 0.0045 | 0.0041 |
| 600 | 0.0009 | 0.0017 | 0.0022 | 0.0023 | 0.0034 | 0.0041 | 0.0042 | 0.0048 | 0.0044 | 0.0041 |
| 700 | 0.0009 | 0.0017 | 0.0023 | 0.0022 | 0.0034 | 0.0042 | 0.0046 | 0.0047 | 0.0048 | 0.0039 |
| 800 | 0.0009 | 0.0017 | 0.0022 | 0.0022 | 0.0033 | 0.0045 | 0.0044 | 0.0044 | 0.0047 | 0.0038 |
| 900 | 0.0009 | 0.0017 | 0.0022 | 0.0021 | 0.0038 | 0.0045 | 0.0045 | 0.0042 | 0.0046 | 0.0035 |
| 1000 | 0.001 | 0.0018 | 0.0022 | 0.0022 | 0.004 | 0.0044 | 0.0043 | 0.004 | 0.0045 | 0.0036 |
| 1100 | 0.001 | 0.0017 | 0.0022 | 0.0022 | 0.0039 | 0.0045 | 0.0041 | 0.0042 | 0.0043 | 0.0035 |
| 1200 | 0.001 | 0.0017 | 0.0022 | 0.0024 | 0.0038 | 0.0044 | 0.0041 | 0.0042 | 0.0044 | 0.0035 |
| 1300 | 0.0009 | 0.0017 | 0.0022 | 0.0023 | 0.0039 | 0.0044 | 0.004 | 0.0044 | 0.0043 | 0.0035 |
| 1400 | 0.001 | 0.0017 | 0.0022 | 0.0023 | 0.0041 | 0.0044 | 0.004 | 0.0042 | 0.0043 | 0.0035 |
| 1500 | 0.001 | 0.0017 | 0.0021 | 0.0023 | 0.0042 | 0.0044 | 0.0038 | 0.0041 | 0.0043 | 0.0036 |
| 1600 | 0.001 | 0.0017 | 0.0022 | 0.0024 | 0.0041 | 0.0043 | 0.0037 | 0.0041 | 0.0043 | 0.0037 |
| 1700 | 0.001 | 0.0017 | 0.0022 | 0.0023 | 0.004 | 0.0044 | 0.0039 | 0.0042 | 0.0043 | 0.0036 |
| 1800 | 0.001 | 0.0017 | 0.0022 | 0.0023 | 0.0039 | 0.0044 | 0.0039 | 0.0042 | 0.0043 | 0.0036 |
| 1900 | 0.001 | 0.0017 | 0.0022 | 0.0024 | 0.004 | 0.0045 | 0.0039 | 0.0041 | 0.0042 | 0.0036 |
| 2000 | 0.001 | 0.0018 | 0.0021 | 0.0023 | 0.004 | 0.0044 | 0.0039 | 0.0041 | 0.0043 | 0.0036 |

Table 76

5. Fifteen-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0335 | 0.0318 | 0.0321 | 0.0328 | 0.0326 | 0.0327 | 0.0329 | 0.0331 | 0.0304 | 0.0326 | 0.0329 | 0.0338 | 0.0323 | 0.0319 | 0.0333 |
| 200 | 0.0326 | 0.0323 | 0.0321 | 0.0322 | 0.0323 | 0.0329 | 0.0327 | 0.0303 | 0.0299 | 0.0323 | 0.0333 | 0.033 | 0.0326 | 0.0319 | 0.0322 |
| 300 | 0.0327 | 0.0321 | 0.0325 | 0.0322 | 0.0318 | 0.0328 | 0.0323 | 0.0316 | 0.0304 | 0.0326 | 0.0331 | 0.0329 | 0.0322 | 0.032 | 0.0323 |
| 400 | 0.0328 | 0.0319 | 0.0327 | 0.0321 | 0.0319 | 0.0328 | 0.0324 | 0.032 | 0.0306 | 0.0328 | 0.0328 | 0.0327 | 0.032 | 0.032 | 0.0325 |
| 500 | 0.0327 | 0.0319 | 0.0324 | 0.0321 | 0.0318 | 0.0325 | 0.0322 | 0.0321 | 0.0309 | 0.0328 | 0.0327 | 0.0325 | 0.0325 | 0.0321 | 0.0323 |
| 600 | 0.0325 | 0.0319 | 0.0324 | 0.0323 | 0.0319 | 0.0326 | 0.0323 | 0.0322 | 0.0313 | 0.0328 | 0.0326 | 0.0326 | 0.0326 | 0.0318 | 0.0323 |
| 700 | 0.0326 | 0.0318 | 0.0321 | 0.0323 | 0.0321 | 0.0325 | 0.0323 | 0.0322 | 0.0313 | 0.0325 | 0.0326 | 0.0325 | 0.0325 | 0.0319 | 0.0322 |
| 800 | 0.0326 | 0.0318 | 0.0322 | 0.0324 | 0.0322 | 0.0325 | 0.0323 | 0.0321 | 0.0316 | 0.0325 | 0.0326 | 0.0326 | 0.0326 | 0.0319 | 0.0322 |
| 900 | 0.0327 | 0.0319 | 0.0322 | 0.0325 | 0.0323 | 0.0324 | 0.0325 | 0.0321 | 0.0318 | 0.0326 | 0.0326 | 0.0325 | 0.0325 | 0.032 | 0.0322 |
| 1000 | 0.0325 | 0.0319 | 0.0321 | 0.0325 | 0.0323 | 0.0324 | 0.0326 | 0.0321 | 0.0319 | 0.0327 | 0.0326 | 0.0325 | 0.0324 | 0.032 | 0.0323 |
| 1100 | 0.0325 | 0.0319 | 0.0322 | 0.0326 | 0.0323 | 0.0324 | 0.0325 | 0.0323 | 0.0319 | 0.0327 | 0.0325 | 0.0325 | 0.0322 | 0.032 | 0.0324 |
| 1200 | 0.0325 | 0.0319 | 0.0322 | 0.0325 | 0.0324 | 0.0322 | 0.0325 | 0.0323 | 0.032 | 0.0327 | 0.0324 | 0.0323 | 0.0323 | 0.0321 | 0.0324 |
| 1300 | 0.0325 | 0.0319 | 0.0323 | 0.0326 | 0.0323 | 0.0323 | 0.0326 | 0.0323 | 0.0321 | 0.0328 | 0.0324 | 0.0324 | 0.0324 | 0.0321 | 0.0324 |
| 1400 | 0.0326 | 0.0319 | 0.0323 | 0.0325 | 0.0323 | 0.0322 | 0.0326 | 0.0322 | 0.0321 | 0.0325 | 0.0323 | 0.0323 | 0.0324 | 0.032 | 0.0324 |
| 1500 | 0.0326 | 0.0319 | 0.0323 | 0.0324 | 0.0324 | 0.0323 | 0.0326 | 0.0322 | 0.0321 | 0.0325 | 0.0322 | 0.0323 | 0.0324 | 0.032 | 0.0324 |

Table 77

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 6 | 15 | 5 | 7 | 5 | 7 | 6 | 3 | 9 | 6 | 5 | 4 | 6 | 7 | 9 |
| 200 | 11 | 22 | 13 | 16 | 9 | 9 | 12 | 6 | 15 | 13 | 12 | 13 | 13 | 15 | 21 |
| 300 | 15 | 38 | 19 | 23 | 15 | 20 | 18 | 14 | 20 | 19 | 20 | 18 | 17 | 20 | 24 |
| 400 | 21 | 47 | 28 | 27 | 22 | 23 | 25 | 22 | 22 | 24 | 29 | 26 | 23 | 27 | 34 |
| 500 | 25 | 56 | 32 | 34 | 34 | 31 | 28 | 26 | 29 | 32 | 36 | 32 | 30 | 32 | 43 |
| 600 | 30 | 59 | 44 | 42 | 40 | 37 | 37 | 37 | 37 | 38 | 41 | 36 | 33 | 40 | 49 |
| 700 | 37 | 66 | 61 | 46 | 48 | 41 | 41 | 44 | 42 | 47 | 46 | 41 | 37 | 47 | 56 |
| 800 | 40 | 75 | 66 | 51 | 56 | 47 | 47 | 49 | 51 | 54 | 51 | 49 | 46 | 53 | 65 |
| 900 | 44 | 79 | 72 | 57 | 60 | 55 | 54 | 59 | 56 | 58 | 60 | 55 | 54 | 62 | 75 |
| 1000 | 52 | 88 | 82 | 64 | 64 | 63 | 60 | 67 | 63 | 70 | 66 | 57 | 59 | 67 | 78 |
| 1100 | 55 | 97 | 86 | 73 | 73 | 69 | 67 | 75 | 64 | 80 | 74 | 65 | 64 | 72 | 86 |
| 1200 | 63 | 103 | 91 | 83 | 76 | 74 | 73 | 81 | 73 | 89 | 82 | 72 | 70 | 76 | 94 |
| 1300 | 67 | 114 | 98 | 90 | 87 | 84 | 78 | 89 | 81 | 92 | 88 | 76 | 78 | 80 | 98 |
| 1400 | 77 | 122 | 103 | 93 | 88 | 92 | 84 | 93 | 92 | 103 | 94 | 80 | 83 | 86 | 110 |
| 1500 | 82 | 130 | 108 | 102 | 96 | 101 | 90 | 99 | 97 | 111 | 100 | 82 | 90 | 89 | 123 |

Table 78

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.2009 | 0.4774 | 0.1605 | 0.2297 | 0.163 | 0.2291 | 0.1974 | 0.0992 | 0.2734 | 0.1956 | 0.1646 | 0.1352 | 0.1935 | 0.2233 | 0.2972 |
| 200 | 0.3587 | 0.7114 | 0.4167 | 0.5144 | 0.2907 | 0.2961 | 0.392 | 0.1819 | 0.4481 | 0.4204 | 0.4001 | 0.4293 | 0.4239 | 0.479 | 0.6764 |
| 300 | 0.4901 | 1.2192 | 0.6174 | 0.7398 | 0.4772 | 0.6554 | 0.582 | 0.4419 | 0.6076 | 0.6197 | 0.6626 | 0.5921 | 0.5467 | 0.6409 | 0.7742 |
| 400 | 0.6896 | 1.497 | 0.9142 | 0.8655 | 0.701 | 0.7536 | 0.8088 | 0.7033 | 0.6734 | 0.7873 | 0.9514 | 0.8494 | 0.7371 | 0.8645 | 1.1067 |
| 500 | 0.817 | 1.7859 | 1.0376 | 1.0929 | 1.0812 | 1.0071 | 0.901 | 0.834 | 0.8975 | 1.0487 | 1.1781 | 1.0412 | 0.9735 | 1.0257 | 1.3887 |
| 600 | 0.9761 | 1.8794 | 1.4236 | 1.3562 | 1.276 | 1.2049 | 1.1943 | 1.1927 | 1.1569 | 1.246 | 1.3368 | 1.1721 | 1.0744 | 1.2716 | 1.5846 |
| 700 | 1.206 | 2.1009 | 1.9589 | 1.4877 | 1.5403 | 1.3331 | 1.3255 | 1.4154 | 1.3164 | 1.5266 | 1.4979 | 1.3328 | 1.2023 | 1.5004 | 1.8042 |
| 800 | 1.3057 | 2.3847 | 2.1256 | 1.6518 | 1.8015 | 1.5264 | 1.5182 | 1.5726 | 1.6122 | 1.7561 | 1.6609 | 1.5959 | 1.5 | 1.6927 | 2.0933 |
| 900 | 1.4397 | 2.5171 | 2.3171 | 1.8525 | 1.9355 | 1.7809 | 1.7528 | 1.8931 | 1.7791 | 1.8916 | 1.956 | 1.7869 | 1.7557 | 1.9843 | 2.4174 |
| 1000 | 1.6893 | 2.8055 | 2.6331 | 2.0789 | 2.0674 | 2.0435 | 1.9543 | 2.1531 | 2.0071 | 2.2889 | 2.1502 | 1.8546 | 1.91 | 2.1448 | 2.5162 |
| 1100 | 1.7891 | 3.0973 | 2.7661 | 2.3762 | 2.3607 | 2.2354 | 2.1797 | 2.4189 | 2.0415 | 2.6138 | 2.4018 | 2.1119 | 2.0633 | 2.307 | 2.7825 |
| 1200 | 2.0491 | 3.2853 | 2.9293 | 2.7009 | 2.4595 | 2.3852 | 2.3757 | 2.6177 | 2.3359 | 2.913 | 2.653 | 2.3272 | 2.261 | 2.4363 | 3.0416 |
| 1300 | 2.1772 | 3.6393 | 3.1608 | 2.9304 | 2.8063 | 2.7139 | 2.541 | 2.8725 | 2.599 | 3.0132 | 2.8498 | 2.4617 | 2.5303 | 2.5649 | 3.1719 |
| 1400 | 2.5075 | 3.888 | 3.3238 | 3.018 | 2.8406 | 2.966 | 2.738 | 2.9961 | 2.9554 | 3.3499 | 3.0349 | 2.5855 | 2.6892 | 2.754 | 3.5652 |
| 1500 | 2.6708 | 4.1422 | 3.4835 | 3.3099 | 3.1072 | 3.2633 | 2.9335 | 3.1886 | 3.1143 | 3.6105 | 3.2205 | 2.6471 | 2.9164 | 2.8502 | 3.9867 |

Table 79

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.002 | 0.0048 | 0.0016 | 0.0023 | 0.0016 | 0.0023 | 0.002 | 0.001 | 0.0027 | 0.002 | 0.0016 | 0.0014 | 0.0019 | 0.0022 | 0.003 |
| 200 | 0.0018 | 0.0036 | 0.0021 | 0.0026 | 0.0015 | 0.0015 | 0.002 | 0.0009 | 0.0022 | 0.0021 | 0.002 | 0.0021 | 0.0021 | 0.0024 | 0.0034 |
| 300 | 0.0016 | 0.0041 | 0.0021 | 0.0025 | 0.0016 | 0.0022 | 0.0019 | 0.0015 | 0.002 | 0.0021 | 0.0022 | 0.002 | 0.0018 | 0.0021 | 0.0026 |
| 400 | 0.0017 | 0.0037 | 0.0023 | 0.0022 | 0.0018 | 0.0019 | 0.002 | 0.0018 | 0.0017 | 0.002 | 0.0024 | 0.0021 | 0.0018 | 0.0022 | 0.0028 |
| 500 | 0.0016 | 0.0036 | 0.0021 | 0.0022 | 0.0022 | 0.002 | 0.0018 | 0.0017 | 0.0018 | 0.0021 | 0.0024 | 0.0021 | 0.0019 | 0.0021 | 0.0028 |
| 600 | 0.0016 | 0.0031 | 0.0024 | 0.0023 | 0.0021 | 0.002 | 0.002 | 0.002 | 0.0019 | 0.0021 | 0.0022 | 0.002 | 0.0018 | 0.0021 | 0.0026 |
| 700 | 0.0017 | 0.003 | 0.0028 | 0.0021 | 0.0022 | 0.0019 | 0.0019 | 0.002 | 0.0019 | 0.0022 | 0.0021 | 0.0019 | 0.0017 | 0.0021 | 0.0026 |
| 800 | 0.0016 | 0.003 | 0.0027 | 0.0021 | 0.0023 | 0.0019 | 0.0019 | 0.002 | 0.002 | 0.0022 | 0.0021 | 0.002 | 0.0019 | 0.0021 | 0.0026 |
| 900 | 0.0016 | 0.0028 | 0.0026 | 0.0021 | 0.0022 | 0.002 | 0.0019 | 0.0021 | 0.002 | 0.0021 | 0.0022 | 0.002 | 0.002 | 0.0022 | 0.0027 |
| 1000 | 0.0017 | 0.0028 | 0.0026 | 0.0021 | 0.0021 | 0.002 | 0.002 | 0.0022 | 0.002 | 0.0023 | 0.0022 | 0.0019 | 0.0019 | 0.0021 | 0.0025 |
| 1100 | 0.0016 | 0.0028 | 0.0025 | 0.0022 | 0.0021 | 0.002 | 0.002 | 0.0022 | 0.0019 | 0.0024 | 0.0022 | 0.0019 | 0.0019 | 0.0021 | 0.0025 |
| 1200 | 0.0017 | 0.0027 | 0.0024 | 0.0023 | 0.002 | 0.002 | 0.002 | 0.0022 | 0.0019 | 0.0024 | 0.0022 | 0.0019 | 0.0019 | 0.002 | 0.0025 |
| 1300 | 0.0017 | 0.0028 | 0.0024 | 0.0023 | 0.0022 | 0.0021 | 0.002 | 0.0022 | 0.002 | 0.0023 | 0.0022 | 0.0019 | 0.0019 | 0.002 | 0.0024 |
| 1400 | 0.0018 | 0.0028 | 0.0024 | 0.0022 | 0.002 | 0.0021 | 0.002 | 0.0021 | 0.0021 | 0.0024 | 0.0022 | 0.0018 | 0.0019 | 0.002 | 0.0025 |
| 1500 | 0.0018 | 0.0028 | 0.0023 | 0.0022 | 0.0021 | 0.0022 | 0.002 | 0.0021 | 0.0021 | 0.0024 | 0.0021 | 0.0018 | 0.0019 | 0.0019 | 0.0027 |

Table 80

Scenario 2. Bidder1 underbid by 0.015 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.01851 | 0.03183 | 0.03209 | 0.03269 | 0.03259 | 0.03273 | 0.03291 | 0.03307 | 0.03037 | 0.03261 | 0.03292 | 0.03381 | 0.03189 | 0.0319 | 0.03293 |
| 200 | 0.01774 | 0.03234 | 0.03206 | 0.03206 | 0.0323 | 0.0329 | 0.03267 | 0.03032 | 0.02987 | 0.03234 | 0.03334 | 0.03302 | 0.03248 | 0.03193 | 0.03213 |
| 300 | 0.01777 | 0.03208 | 0.03252 | 0.0321 | 0.03182 | 0.03277 | 0.03233 | 0.03157 | 0.03038 | 0.03262 | 0.03313 | 0.0329 | 0.03204 | 0.03205 | 0.03219 |
| 400 | 0.01788 | 0.03185 | 0.03268 | 0.032 | 0.03186 | 0.03276 | 0.03235 | 0.03197 | 0.03061 | 0.0328 | 0.03281 | 0.03267 | 0.03196 | 0.03202 | 0.03251 |
| 500 | 0.01773 | 0.03189 | 0.03244 | 0.0321 | 0.0318 | 0.03249 | 0.03218 | 0.03208 | 0.03095 | 0.03277 | 0.03272 | 0.03254 | 0.03239 | 0.03203 | 0.03226 |
| 600 | 0.01745 | 0.03185 | 0.03242 | 0.0323 | 0.0319 | 0.03256 | 0.03228 | 0.03224 | 0.03127 | 0.03279 | 0.03261 | 0.03256 | 0.03251 | 0.03176 | 0.03231 |
| 700 | 0.01754 | 0.03183 | 0.03215 | 0.03236 | 0.03205 | 0.03249 | 0.03233 | 0.03217 | 0.03134 | 0.03248 | 0.03256 | 0.03251 | 0.03245 | 0.0319 | 0.03219 |
| 800 | 0.01749 | 0.03184 | 0.03224 | 0.0324 | 0.03214 | 0.03246 | 0.0323 | 0.03209 | 0.03161 | 0.03252 | 0.03257 | 0.03257 | 0.03258 | 0.03192 | 0.03218 |
| 900 | 0.01757 | 0.03191 | 0.03221 | 0.03252 | 0.03223 | 0.03236 | 0.03246 | 0.03209 | 0.03177 | 0.03261 | 0.0326 | 0.03249 | 0.03248 | 0.03199 | 0.03221 |
| 1000 | 0.01739 | 0.03192 | 0.03213 | 0.0325 | 0.03228 | 0.03242 | 0.03257 | 0.03214 | 0.03186 | 0.0327 | 0.03258 | 0.03254 | 0.03234 | 0.032 | 0.03224 |
| 1100 | 0.01742 | 0.03197 | 0.03219 | 0.03257 | 0.03232 | 0.03238 | 0.03253 | 0.03225 | 0.0319 | 0.03268 | 0.03246 | 0.03249 | 0.03221 | 0.03203 | 0.03234 |
| 1200 | 0.01737 | 0.03193 | 0.03221 | 0.03255 | 0.03234 | 0.03222 | 0.03254 | 0.03232 | 0.03205 | 0.03274 | 0.03235 | 0.03232 | 0.03227 | 0.03205 | 0.03234 |
| 1300 | 0.01737 | 0.03195 | 0.03228 | 0.03257 | 0.03224 | 0.03229 | 0.03258 | 0.03228 | 0.03212 | 0.03276 | 0.03238 | 0.03238 | 0.03242 | 0.03205 | 0.03235 |
| 1400 | 0.01733 | 0.03193 | 0.0323 | 0.03246 | 0.03226 | 0.03223 | 0.03259 | 0.03228 | 0.03215 | 0.03253 | 0.03229 | 0.03231 | 0.03238 | 0.03201 | 0.0324 |
| 1500 | 0.01735 | 0.03192 | 0.03228 | 0.03246 | 0.03235 | 0.0323 | 0.03259 | 0.03226 | 0.03213 | 0.03253 | 0.0322 | 0.03227 | 0.03239 | 0.03201 | 0.0324 |

Table 81

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 9 | 15 | 5 | 6 | 5 | 7 | 6 | 3 | 9 | 6 | 5 | 4 | 5 | 7 | 8 |
| 200 | 15 | 22 | 12 | 15 | 9 | 9 | 12 | 6 | 15 | 13 | 12 | 13 | 12 | 15 | 20 |
| 300 | 19 | 38 | 18 | 22 | 15 | 20 | 18 | 14 | 20 | 19 | 20 | 18 | 16 | 20 | 23 |
| 400 | 25 | 47 | 27 | 26 | 22 | 23 | 25 | 22 | 22 | 24 | 29 | 26 | 22 | 27 | 33 |
| 500 | 30 | 56 | 31 | 33 | 34 | 31 | 28 | 26 | 29 | 32 | 36 | 32 | 29 | 31 | 42 |
| 600 | 38 | 59 | 41 | 40 | 40 | 37 | 37 | 37 | 37 | 38 | 41 | 36 | 32 | 39 | 48 |
| 700 | 47 | 66 | 58 | 44 | 47 | 40 | 41 | 44 | 42 | 47 | 46 | 41 | 36 | 46 | 55 |
| 800 | 51 | 74 | 63 | 49 | 55 | 46 | 47 | 49 | 51 | 54 | 51 | 49 | 45 | 52 | 64 |
| 900 | 55 | 78 | 69 | 55 | 59 | 54 | 54 | 59 | 56 | 58 | 60 | 55 | 53 | 61 | 74 |
| 1000 | 64 | 87 | 78 | 62 | 63 | 62 | 60 | 67 | 63 | 70 | 66 | 57 | 58 | 66 | 77 |
| 1100 | 68 | 96 | 82 | 71 | 72 | 68 | 67 | 75 | 64 | 79 | 74 | 65 | 63 | 71 | 85 |
| 1200 | 77 | 102 | 87 | 81 | 75 | 73 | 73 | 81 | 72 | 88 | 82 | 72 | 69 | 75 | 93 |
| 1300 | 83 | 113 | 94 | 88 | 86 | 83 | 78 | 89 | 79 | 91 | 88 | 75 | 77 | 79 | 97 |
| 1400 | 95 | 120 | 99 | 91 | 87 | 91 | 84 | 92 | 90 | 102 | 94 | 79 | 82 | 85 | 109 |
| 1500 | 100 | 128 | 104 | 100 | 95 | 100 | 90 | 98 | 95 | 110 | 100 | 81 | 89 | 88 | 122 |

Table 82

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.16655 | 0.47743 | 0.16047 | 0.19616 | 0.16295 | 0.2291 | 0.19743 | 0.09921 | 0.27336 | 0.19563 | 0.16461 | 0.13524 | 0.15946 | 0.22328 | 0.26348 |
| 200 | 0.2661 | 0.71141 | 0.38472 | 0.48086 | 0.29069 | 0.29611 | 0.39202 | 0.18194 | 0.44807 | 0.4204 | 0.40013 | 0.42927 | 0.38978 | 0.479 | 0.64267 |
| 300 | 0.33755 | 1.21919 | 0.58539 | 0.70623 | 0.47723 | 0.65539 | 0.58197 | 0.44192 | 0.60759 | 0.61973 | 0.66256 | 0.59213 | 0.51264 | 0.64094 | 0.7404 |
| 400 | 0.44702 | 1.497 | 0.88226 | 0.83193 | 0.70101 | 0.75357 | 0.80876 | 0.70332 | 0.67342 | 0.78726 | 0.95139 | 0.84943 | 0.70301 | 0.86447 | 1.07293 |
| 500 | 0.53181 | 1.78589 | 1.00566 | 1.05928 | 1.0812 | 1.00713 | 0.90102 | 0.83398 | 0.89754 | 1.04872 | 1.17808 | 1.04117 | 0.93943 | 0.99278 | 1.35492 |
| 600 | 0.66299 | 1.87942 | 1.32928 | 1.29212 | 1.27601 | 1.20488 | 1.19427 | 1.19271 | 1.15692 | 1.24603 | 1.33685 | 1.17213 | 1.0403 | 1.23868 | 1.55082 |
| 700 | 0.82456 | 2.10087 | 1.86454 | 1.4237 | 1.50639 | 1.29965 | 1.32555 | 1.41539 | 1.31645 | 1.52659 | 1.49786 | 1.33282 | 1.16823 | 1.46746 | 1.77049 |
| 800 | 0.89222 | 2.35633 | 2.03131 | 1.58776 | 1.76756 | 1.49293 | 1.51816 | 1.57261 | 1.61225 | 1.7561 | 1.66091 | 1.59589 | 1.46588 | 1.65979 | 2.05957 |
| 900 | 0.96617 | 2.48877 | 2.22277 | 1.78846 | 1.9016 | 1.74741 | 1.7528 | 1.89305 | 1.77914 | 1.89164 | 1.956 | 1.78691 | 1.72164 | 1.95138 | 2.3836 |
| 1000 | 1.11269 | 2.77711 | 2.50646 | 2.01491 | 2.03342 | 2.01003 | 1.95434 | 2.15305 | 2.00714 | 2.28888 | 2.15018 | 1.8546 | 1.87587 | 2.11191 | 2.48245 |
| 1100 | 1.18424 | 3.0689 | 2.6395 | 2.31218 | 2.32674 | 2.20192 | 2.17965 | 2.41893 | 2.04146 | 2.58179 | 2.40183 | 2.11194 | 2.0292 | 2.27409 | 2.7487 |
| 1200 | 1.33749 | 3.2569 | 2.80265 | 2.63688 | 2.42561 | 2.35175 | 2.37571 | 2.6177 | 2.30724 | 2.88102 | 2.65296 | 2.32718 | 2.22692 | 2.40339 | 3.00787 |
| 1300 | 1.44182 | 3.6109 | 3.03414 | 2.86639 | 2.77236 | 2.68043 | 2.54102 | 2.87253 | 2.53709 | 2.98123 | 2.84979 | 2.42835 | 2.49622 | 2.53201 | 3.13812 |
| 1400 | 1.64671 | 3.83127 | 3.19721 | 2.95397 | 2.80667 | 2.93251 | 2.73797 | 2.9695 | 2.89348 | 3.31794 | 3.03492 | 2.55216 | 2.65508 | 2.72105 | 3.5314 |
| 1500 | 1.735 | 4.08555 | 3.35686 | 3.24584 | 3.07331 | 3.22983 | 2.93347 | 3.16197 | 3.05242 | 3.57853 | 3.22046 | 2.61378 | 2.88232 | 2.81727 | 3.95297 |

Table 83

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.00167 | 0.00477 | 0.0016 | 0.00196 | 0.00163 | 0.00229 | 0.00197 | 0.00099 | 0.00273 | 0.00196 | 0.00165 | 0.00135 | 0.00159 | 0.00223 | 0.00263 |
| 200 | 0.00133 | 0.00356 | 0.00192 | 0.0024 | 0.00145 | 0.00148 | 0.00196 | 0.00091 | 0.00224 | 0.0021 | 0.002 | 0.00215 | 0.00195 | 0.0024 | 0.00321 |
| 300 | 0.00113 | 0.00406 | 0.00195 | 0.00235 | 0.00159 | 0.00218 | 0.00194 | 0.00147 | 0.00203 | 0.00207 | 0.00221 | 0.00197 | 0.00171 | 0.00214 | 0.00247 |
| 400 | 0.00112 | 0.00374 | 0.00221 | 0.00208 | 0.00175 | 0.00188 | 0.00202 | 0.00176 | 0.00168 | 0.00197 | 0.00238 | 0.00212 | 0.00176 | 0.00216 | 0.00268 |
| 500 | 0.00106 | 0.00357 | 0.00201 | 0.00212 | 0.00216 | 0.00201 | 0.0018 | 0.00167 | 0.0018 | 0.0021 | 0.00236 | 0.00208 | 0.00188 | 0.00199 | 0.00271 |
| 600 | 0.0011 | 0.00313 | 0.00222 | 0.00215 | 0.00213 | 0.00201 | 0.00199 | 0.00199 | 0.00193 | 0.00208 | 0.00223 | 0.00195 | 0.00173 | 0.00206 | 0.00258 |
| 700 | 0.00118 | 0.003 | 0.00266 | 0.00203 | 0.00215 | 0.00186 | 0.00189 | 0.00202 | 0.00188 | 0.00218 | 0.00214 | 0.0019 | 0.00167 | 0.0021 | 0.00253 |
| 800 | 0.00112 | 0.00295 | 0.00254 | 0.00198 | 0.00221 | 0.00187 | 0.0019 | 0.00197 | 0.00202 | 0.0022 | 0.00208 | 0.00199 | 0.00183 | 0.00207 | 0.00257 |
| 900 | 0.00107 | 0.00277 | 0.00247 | 0.00199 | 0.00211 | 0.00194 | 0.00195 | 0.0021 | 0.00198 | 0.0021 | 0.00217 | 0.00199 | 0.00191 | 0.00217 | 0.00265 |
| 1000 | 0.00111 | 0.00278 | 0.00251 | 0.00201 | 0.00203 | 0.00201 | 0.00195 | 0.00215 | 0.00201 | 0.00229 | 0.00215 | 0.00185 | 0.00188 | 0.00211 | 0.00248 |
| 1100 | 0.00108 | 0.00279 | 0.0024 | 0.0021 | 0.00212 | 0.002 | 0.00198 | 0.0022 | 0.00186 | 0.00235 | 0.00218 | 0.00192 | 0.00184 | 0.00207 | 0.0025 |
| 1200 | 0.00111 | 0.00271 | 0.00234 | 0.0022 | 0.00202 | 0.00196 | 0.00198 | 0.00218 | 0.00192 | 0.0024 | 0.00221 | 0.00194 | 0.00186 | 0.002 | 0.00251 |
| 1300 | 0.00111 | 0.00278 | 0.00233 | 0.0022 | 0.00213 | 0.00206 | 0.00195 | 0.00221 | 0.00195 | 0.00229 | 0.00219 | 0.00187 | 0.00192 | 0.00195 | 0.00241 |
| 1400 | 0.00118 | 0.00274 | 0.00228 | 0.00211 | 0.002 | 0.00209 | 0.00196 | 0.00212 | 0.00207 | 0.00237 | 0.00217 | 0.00182 | 0.0019 | 0.00194 | 0.00252 |
| 1500 | 0.00116 | 0.00272 | 0.00224 | 0.00216 | 0.00205 | 0.00215 | 0.00196 | 0.00211 | 0.00203 | 0.00239 | 0.00215 | 0.00174 | 0.00192 | 0.00188 | 0.00264 |

Table 84

Scenario 3. Bidders “1”, “2” and “3” underbid by 0.02 while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0124 | 0.0125 | 0.0106 | 0.0317 | 0.0318 | 0.0331 | 0.0333 | 0.0324 | 0.0319 | 0.0323 | 0.0322 | 0.033 | 0.0307 | 0.0329 | 0.0317 |
| 200 | 0.0124 | 0.0125 | 0.0118 | 0.0329 | 0.0322 | 0.0319 | 0.0334 | 0.032 | 0.0324 | 0.0332 | 0.0322 | 0.0331 | 0.0323 | 0.0331 | 0.032 |
| 300 | 0.0125 | 0.0125 | 0.0117 | 0.0329 | 0.0319 | 0.0317 | 0.0328 | 0.0324 | 0.0327 | 0.0327 | 0.0321 | 0.0329 | 0.0324 | 0.033 | 0.0322 |
| 400 | 0.0124 | 0.0124 | 0.0118 | 0.0329 | 0.032 | 0.0319 | 0.0328 | 0.0325 | 0.0328 | 0.0329 | 0.0324 | 0.0328 | 0.0326 | 0.0328 | 0.0321 |
| 500 | 0.0123 | 0.0123 | 0.012 | 0.033 | 0.032 | 0.0316 | 0.0324 | 0.0326 | 0.0329 | 0.033 | 0.0322 | 0.0325 | 0.0324 | 0.0326 | 0.0322 |
| 600 | 0.0123 | 0.0123 | 0.0119 | 0.0329 | 0.032 | 0.0318 | 0.0324 | 0.0325 | 0.0328 | 0.033 | 0.0321 | 0.0326 | 0.0324 | 0.0327 | 0.0323 |
| 700 | 0.0123 | 0.0122 | 0.0119 | 0.0327 | 0.0321 | 0.032 | 0.0325 | 0.0327 | 0.0327 | 0.033 | 0.032 | 0.0326 | 0.0324 | 0.0327 | 0.0322 |
| 800 | 0.0123 | 0.0121 | 0.012 | 0.0328 | 0.0321 | 0.032 | 0.0324 | 0.0327 | 0.0324 | 0.0329 | 0.0319 | 0.0323 | 0.0324 | 0.0325 | 0.0322 |
| 900 | 0.0123 | 0.0121 | 0.0118 | 0.0327 | 0.0321 | 0.032 | 0.0324 | 0.0328 | 0.0326 | 0.0329 | 0.0321 | 0.0325 | 0.0323 | 0.0325 | 0.0323 |
| 1000 | 0.0123 | 0.0121 | 0.0119 | 0.0327 | 0.0322 | 0.0321 | 0.0325 | 0.0327 | 0.0325 | 0.0328 | 0.0321 | 0.0325 | 0.0321 | 0.0325 | 0.0322 |
| 1100 | 0.0123 | 0.0121 | 0.012 | 0.0328 | 0.0322 | 0.0322 | 0.0325 | 0.0326 | 0.0324 | 0.0329 | 0.032 | 0.0325 | 0.0322 | 0.0325 | 0.0322 |
| 1200 | 0.0124 | 0.0121 | 0.0119 | 0.0328 | 0.0322 | 0.0321 | 0.0325 | 0.0325 | 0.0325 | 0.0328 | 0.032 | 0.0325 | 0.0321 | 0.0324 | 0.0322 |
| 1300 | 0.0124 | 0.0122 | 0.012 | 0.0327 | 0.0323 | 0.0322 | 0.0324 | 0.0326 | 0.0324 | 0.0327 | 0.032 | 0.0326 | 0.0322 | 0.0324 | 0.0322 |
| 1400 | 0.0124 | 0.0123 | 0.012 | 0.0326 | 0.0323 | 0.0323 | 0.0325 | 0.0326 | 0.0324 | 0.0326 | 0.0321 | 0.0325 | 0.0322 | 0.0324 | 0.0322 |
| 1500 | 0.0125 | 0.0122 | 0.012 | 0.0326 | 0.0323 | 0.0322 | 0.0325 | 0.0326 | 0.0324 | 0.0326 | 0.032 | 0.0322 | 0.0322 | 0.0325 | 0.0322 |
| 1600 | 0.0125 | 0.0123 | 0.0121 | 0.0325 | 0.0324 | 0.0322 | 0.0325 | 0.0325 | 0.0324 | 0.0327 | 0.032 | 0.0322 | 0.0322 | 0.0325 | 0.0323 |
| 1700 | 0.0125 | 0.0122 | 0.012 | 0.0325 | 0.0324 | 0.0323 | 0.0326 | 0.0325 | 0.0325 | 0.0326 | 0.0321 | 0.0322 | 0.0322 | 0.0324 | 0.0323 |
| 1800 | 0.0125 | 0.0122 | 0.0121 | 0.0325 | 0.0324 | 0.0324 | 0.0326 | 0.0324 | 0.0325 | 0.0327 | 0.0321 | 0.0322 | 0.0322 | 0.0324 | 0.0324 |
| 1900 | 0.0125 | 0.0122 | 0.0121 | 0.0325 | 0.0324 | 0.0324 | 0.0325 | 0.0324 | 0.0326 | 0.0327 | 0.0321 | 0.0323 | 0.0322 | 0.0323 | 0.0324 |
| 2000 | 0.0125 | 0.0122 | 0.0121 | 0.0325 | 0.0325 | 0.0323 | 0.0325 | 0.0324 | 0.0325 | 0.0326 | 0.0321 | 0.0323 | 0.0323 | 0.0323 | 0.0324 |

Table 85

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 8 | 11 | 6 | 5 | 7 | 7 | 7 | 6 | 4 | 2 | 5 | 5 | 4 | 8 | 15 |
| 200 | 17 | 18 | 12 | 15 | 11 | 17 | 13 | 14 | 11 | 7 | 9 | 10 | 9 | 17 | 20 |
| 300 | 23 | 25 | 22 | 24 | 17 | 19 | 20 | 20 | 19 | 12 | 17 | 16 | 18 | 25 | 23 |
| 400 | 31 | 35 | 34 | 26 | 23 | 23 | 27 | 25 | 22 | 18 | 23 | 22 | 23 | 34 | 34 |
| 500 | 42 | 42 | 44 | 31 | 30 | 29 | 35 | 29 | 27 | 23 | 29 | 28 | 29 | 38 | 44 |
| 600 | 54 | 48 | 55 | 39 | 33 | 34 | 38 | 35 | 37 | 30 | 35 | 36 | 38 | 41 | 47 |
| 700 | 62 | 55 | 60 | 47 | 41 | 41 | 44 | 42 | 47 | 36 | 41 | 42 | 45 | 44 | 53 |
| 800 | 70 | 63 | 71 | 53 | 41 | 50 | 48 | 49 | 56 | 41 | 42 | 47 | 52 | 53 | 64 |
| 900 | 78 | 72 | 79 | 57 | 43 | 54 | 55 | 57 | 62 | 45 | 51 | 54 | 60 | 63 | 70 |
| 1000 | 87 | 79 | 87 | 59 | 51 | 63 | 63 | 62 | 65 | 46 | 61 | 63 | 64 | 70 | 80 |
| 1100 | 95 | 86 | 98 | 65 | 55 | 70 | 72 | 71 | 70 | 50 | 68 | 65 | 70 | 77 | 88 |
| 1200 | 100 | 96 | 107 | 72 | 61 | 74 | 77 | 80 | 77 | 60 | 76 | 69 | 76 | 83 | 92 |
| 1300 | 112 | 107 | 117 | 78 | 66 | 81 | 83 | 85 | 84 | 73 | 77 | 73 | 79 | 89 | 96 |
| 1400 | 125 | 120 | 124 | 82 | 70 | 86 | 92 | 89 | 91 | 78 | 81 | 82 | 86 | 94 | 100 |
| 1500 | 133 | 133 | 130 | 86 | 76 | 88 | 96 | 97 | 96 | 85 | 88 | 91 | 94 | 102 | 105 |
| 1600 | 143 | 137 | 141 | 89 | 84 | 96 | 99 | 108 | 106 | 92 | 91 | 99 | 96 | 107 | 112 |
| 1700 | 152 | 141 | 148 | 97 | 91 | 106 | 106 | 116 | 113 | 97 | 95 | 105 | 100 | 114 | 119 |
| 1800 | 165 | 147 | 160 | 104 | 95 | 113 | 110 | 123 | 120 | 106 | 101 | 109 | 109 | 118 | 120 |
| 1900 | 175 | 157 | 164 | 110 | 100 | 119 | 116 | 127 | 128 | 111 | 107 | 113 | 119 | 125 | 129 |
| 2000 | 182 | 165 | 173 | 119 | 108 | 121 | 123 | 132 | 135 | 119 | 111 | 125 | 121 | 133 | 133 |

Table 86

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0995 | 0.1379 | 0.0638 | 0.1586 | 0.2226 | 0.2314 | 0.2332 | 0.1946 | 0.1276 | 0.0645 | 0.1608 | 0.1652 | 0.1228 | 0.2635 | 0.4757 |
| 200 | 0.2116 | 0.2243 | 0.1418 | 0.4928 | 0.3547 | 0.5416 | 0.4341 | 0.4478 | 0.3561 | 0.2326 | 0.2895 | 0.331 | 0.2909 | 0.5629 | 0.6397 |
| 300 | 0.288 | 0.3126 | 0.2583 | 0.7899 | 0.5427 | 0.6028 | 0.657 | 0.6476 | 0.6216 | 0.3923 | 0.5461 | 0.526 | 0.5839 | 0.8261 | 0.7397 |
| 400 | 0.3848 | 0.433 | 0.4024 | 0.8566 | 0.737 | 0.7331 | 0.8861 | 0.8113 | 0.7214 | 0.5922 | 0.7446 | 0.721 | 0.7499 | 1.1142 | 1.0915 |
| 500 | 0.5157 | 0.5146 | 0.5262 | 1.0228 | 0.9596 | 0.9163 | 1.1354 | 0.9452 | 0.8891 | 0.7585 | 0.9326 | 0.9098 | 0.9387 | 1.2401 | 1.4163 |
| 600 | 0.6644 | 0.5884 | 0.6562 | 1.2835 | 1.0563 | 1.0805 | 1.2301 | 1.137 | 1.2141 | 0.9891 | 1.1242 | 1.1737 | 1.2317 | 1.3407 | 1.5165 |
| 700 | 0.7612 | 0.6689 | 0.7137 | 1.5391 | 1.3163 | 1.3101 | 1.4301 | 1.3714 | 1.5386 | 1.1874 | 1.3118 | 1.3692 | 1.4586 | 1.4396 | 1.7064 |
| 800 | 0.858 | 0.7642 | 0.8532 | 1.7374 | 1.3163 | 1.5981 | 1.5558 | 1.6031 | 1.8161 | 1.3493 | 1.3404 | 1.5179 | 1.6833 | 1.7246 | 2.0622 |
| 900 | 0.9562 | 0.8684 | 0.9349 | 1.8636 | 1.3795 | 1.7289 | 1.7842 | 1.867 | 2.0185 | 1.4798 | 1.6375 | 1.7531 | 1.9385 | 2.0496 | 2.263 |
| 1000 | 1.0716 | 0.9547 | 1.039 | 1.9315 | 1.641 | 2.0215 | 2.0445 | 2.0252 | 2.1101 | 1.5095 | 1.9596 | 2.0499 | 2.0558 | 2.2737 | 2.5747 |
| 1100 | 1.1732 | 1.0429 | 1.1733 | 2.1347 | 1.7734 | 2.2532 | 2.3419 | 2.3116 | 2.2701 | 1.6431 | 2.1774 | 2.1143 | 2.2518 | 2.5019 | 2.8332 |
| 1200 | 1.2359 | 1.1663 | 1.2774 | 2.3631 | 1.9646 | 2.3736 | 2.5005 | 2.5984 | 2.4993 | 1.9706 | 2.4342 | 2.2455 | 2.4422 | 2.6873 | 2.9621 |
| 1300 | 1.3844 | 1.3072 | 1.4051 | 2.5493 | 2.1295 | 2.6071 | 2.6913 | 2.7687 | 2.7212 | 2.39 | 2.4657 | 2.3783 | 2.5417 | 2.8821 | 3.0941 |
| 1400 | 1.5541 | 1.4705 | 1.4833 | 2.6757 | 2.2602 | 2.7756 | 2.9875 | 2.9004 | 2.9462 | 2.5452 | 2.5965 | 2.6677 | 2.7679 | 3.0472 | 3.2244 |
| 1500 | 1.6599 | 1.6251 | 1.5607 | 2.802 | 2.4533 | 2.8371 | 3.1195 | 3.1577 | 3.1081 | 2.7749 | 2.8185 | 2.9326 | 3.0234 | 3.3114 | 3.3842 |
| 1600 | 1.7859 | 1.6803 | 1.6995 | 2.894 | 2.7201 | 3.091 | 3.2205 | 3.5066 | 3.436 | 3.0055 | 2.9165 | 3.1854 | 3.087 | 3.4745 | 3.6159 |
| 1700 | 1.9007 | 1.7191 | 1.7796 | 3.149 | 2.9488 | 3.4222 | 3.4516 | 3.7681 | 3.6724 | 3.1654 | 3.0482 | 3.3832 | 3.2172 | 3.6978 | 3.8478 |
| 1800 | 2.0572 | 1.7907 | 1.938 | 3.3794 | 3.0811 | 3.6574 | 3.5815 | 3.9904 | 3.9049 | 3.4664 | 3.243 | 3.5148 | 3.5138 | 3.8193 | 3.8823 |
| 1900 | 2.1861 | 1.9137 | 1.9906 | 3.5745 | 3.2425 | 3.8506 | 3.7722 | 4.1118 | 4.1681 | 3.6247 | 3.4362 | 3.6501 | 3.8373 | 4.0333 | 4.1809 |
| 2000 | 2.277 | 2.0129 | 2.0946 | 3.8675 | 3.5074 | 3.9058 | 4.0014 | 4.2785 | 4.3863 | 3.8763 | 3.5663 | 4.0318 | 3.9047 | 4.2906 | 4.3072 |

Table 87

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.001 | 0.0014 | 0.0006 | 0.0016 | 0.0022 | 0.0023 | 0.0023 | 0.0019 | 0.0013 | 0.0006 | 0.0016 | 0.0017 | 0.0012 | 0.0026 | 0.0048 |
| 200 | 0.0011 | 0.0011 | 0.0007 | 0.0025 | 0.0018 | 0.0027 | 0.0022 | 0.0022 | 0.0018 | 0.0012 | 0.0014 | 0.0017 | 0.0015 | 0.0028 | 0.0032 |
| 300 | 0.001 | 0.001 | 0.0009 | 0.0026 | 0.0018 | 0.002 | 0.0022 | 0.0022 | 0.0021 | 0.0013 | 0.0018 | 0.0018 | 0.0019 | 0.0028 | 0.0025 |
| 400 | 0.001 | 0.0011 | 0.001 | 0.0021 | 0.0018 | 0.0018 | 0.0022 | 0.002 | 0.0018 | 0.0015 | 0.0019 | 0.0018 | 0.0019 | 0.0028 | 0.0027 |
| 500 | 0.001 | 0.001 | 0.0011 | 0.002 | 0.0019 | 0.0018 | 0.0023 | 0.0019 | 0.0018 | 0.0015 | 0.0019 | 0.0018 | 0.0019 | 0.0025 | 0.0028 |
| 600 | 0.0011 | 0.001 | 0.0011 | 0.0021 | 0.0018 | 0.0018 | 0.0021 | 0.0019 | 0.002 | 0.0016 | 0.0019 | 0.002 | 0.0021 | 0.0022 | 0.0025 |
| 700 | 0.0011 | 0.001 | 0.001 | 0.0022 | 0.0019 | 0.0019 | 0.002 | 0.002 | 0.0022 | 0.0017 | 0.0019 | 0.002 | 0.0021 | 0.0021 | 0.0024 |
| 800 | 0.0011 | 0.001 | 0.0011 | 0.0022 | 0.0016 | 0.002 | 0.0019 | 0.002 | 0.0023 | 0.0017 | 0.0017 | 0.0019 | 0.0021 | 0.0022 | 0.0026 |
| 900 | 0.0011 | 0.001 | 0.001 | 0.0021 | 0.0015 | 0.0019 | 0.002 | 0.0021 | 0.0022 | 0.0016 | 0.0018 | 0.0019 | 0.0022 | 0.0023 | 0.0025 |
| 1000 | 0.0011 | 0.001 | 0.001 | 0.0019 | 0.0016 | 0.002 | 0.002 | 0.002 | 0.0021 | 0.0015 | 0.002 | 0.002 | 0.0021 | 0.0023 | 0.0026 |
| 1100 | 0.0011 | 0.0009 | 0.0011 | 0.0019 | 0.0016 | 0.002 | 0.0021 | 0.0021 | 0.0021 | 0.0015 | 0.002 | 0.0019 | 0.002 | 0.0023 | 0.0026 |
| 1200 | 0.001 | 0.001 | 0.0011 | 0.002 | 0.0016 | 0.002 | 0.0021 | 0.0022 | 0.0021 | 0.0016 | 0.002 | 0.0019 | 0.002 | 0.0022 | 0.0025 |
| 1300 | 0.0011 | 0.001 | 0.0011 | 0.002 | 0.0016 | 0.002 | 0.0021 | 0.0021 | 0.0021 | 0.0018 | 0.0019 | 0.0018 | 0.002 | 0.0022 | 0.0024 |
| 1400 | 0.0011 | 0.0011 | 0.0011 | 0.0019 | 0.0016 | 0.002 | 0.0021 | 0.0021 | 0.0021 | 0.0018 | 0.0019 | 0.0019 | 0.002 | 0.0022 | 0.0023 |
| 1500 | 0.0011 | 0.0011 | 0.001 | 0.0019 | 0.0016 | 0.0019 | 0.0021 | 0.0021 | 0.0021 | 0.0018 | 0.0019 | 0.002 | 0.002 | 0.0022 | 0.0023 |
| 1600 | 0.0011 | 0.0011 | 0.0011 | 0.0018 | 0.0017 | 0.0019 | 0.002 | 0.0022 | 0.0021 | 0.0019 | 0.0018 | 0.002 | 0.0019 | 0.0022 | 0.0023 |
| 1700 | 0.0011 | 0.001 | 0.001 | 0.0019 | 0.0017 | 0.002 | 0.002 | 0.0022 | 0.0022 | 0.0019 | 0.0018 | 0.002 | 0.0019 | 0.0022 | 0.0023 |
| 1800 | 0.0011 | 0.001 | 0.0011 | 0.0019 | 0.0017 | 0.002 | 0.002 | 0.0022 | 0.0022 | 0.0019 | 0.0018 | 0.002 | 0.002 | 0.0021 | 0.0022 |
| 1900 | 0.0012 | 0.001 | 0.001 | 0.0019 | 0.0017 | 0.002 | 0.002 | 0.0022 | 0.0022 | 0.0019 | 0.0018 | 0.0019 | 0.002 | 0.0021 | 0.0022 |
| 2000 | 0.0011 | 0.001 | 0.001 | 0.0019 | 0.0018 | 0.002 | 0.002 | 0.0021 | 0.0022 | 0.0019 | 0.0018 | 0.002 | 0.002 | 0.0021 | 0.0022 |

Table 88

Scenario 4. Bidders “1”, “2”, “3”, “4” and “5” underbid by 0.025, 0.0225, 0.02, 0.0175 and 0.015 respectively while others used equilibrium strategy.

| # of games | Expected Profit | | | | | | | | | | | | | | |
|------------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0072 | 0.0101 | 0.0112 | 0.0145 | 0.0136 | 0.032 | 0.0334 | 0.0317 | 0.0313 | 0.0309 | 0.0326 | 0.0325 | 0.0336 | 0.0312 | 0.0332 |
| 200 | 0.0075 | 0.0097 | 0.012 | 0.0148 | 0.0166 | 0.0322 | 0.033 | 0.0318 | 0.0319 | 0.0315 | 0.0326 | 0.0327 | 0.0326 | 0.0324 | 0.0333 |
| 300 | 0.0077 | 0.0101 | 0.0118 | 0.0147 | 0.0169 | 0.0323 | 0.033 | 0.0321 | 0.0322 | 0.0314 | 0.0323 | 0.0325 | 0.033 | 0.0321 | 0.0331 |
| 400 | 0.0077 | 0.0098 | 0.0119 | 0.0145 | 0.0172 | 0.0324 | 0.0329 | 0.0321 | 0.0323 | 0.0318 | 0.0324 | 0.0326 | 0.0319 | 0.0322 | 0.0332 |
| 500 | 0.0077 | 0.0098 | 0.0119 | 0.0144 | 0.0173 | 0.0326 | 0.0327 | 0.0323 | 0.0323 | 0.0319 | 0.0325 | 0.0326 | 0.032 | 0.0323 | 0.0331 |
| 600 | 0.0075 | 0.0097 | 0.012 | 0.0146 | 0.0173 | 0.0327 | 0.0327 | 0.0323 | 0.0321 | 0.0321 | 0.0326 | 0.0326 | 0.0323 | 0.0325 | 0.0329 |
| 700 | 0.0076 | 0.0097 | 0.0119 | 0.0146 | 0.0173 | 0.0324 | 0.0326 | 0.0322 | 0.0319 | 0.0321 | 0.0326 | 0.0325 | 0.0323 | 0.0324 | 0.033 |
| 800 | 0.0075 | 0.0098 | 0.012 | 0.0145 | 0.0173 | 0.0325 | 0.0326 | 0.0322 | 0.0321 | 0.0321 | 0.0327 | 0.0325 | 0.0323 | 0.0324 | 0.0329 |
| 900 | 0.0074 | 0.0097 | 0.0118 | 0.0146 | 0.0175 | 0.0324 | 0.0326 | 0.0321 | 0.0321 | 0.0322 | 0.0327 | 0.0326 | 0.0322 | 0.0324 | 0.0327 |
| 1000 | 0.0074 | 0.0097 | 0.0117 | 0.0145 | 0.0175 | 0.0324 | 0.0327 | 0.0322 | 0.0322 | 0.0323 | 0.0325 | 0.0325 | 0.0323 | 0.0325 | 0.0325 |
| 1100 | 0.0075 | 0.0097 | 0.0117 | 0.0146 | 0.0175 | 0.0324 | 0.0327 | 0.0323 | 0.032 | 0.0323 | 0.0325 | 0.0326 | 0.0322 | 0.0325 | 0.0325 |
| 1200 | 0.0075 | 0.0098 | 0.0118 | 0.0145 | 0.0175 | 0.0324 | 0.0326 | 0.0321 | 0.0318 | 0.0323 | 0.0326 | 0.0326 | 0.0323 | 0.0324 | 0.0324 |
| 1300 | 0.0075 | 0.0096 | 0.0119 | 0.0145 | 0.0173 | 0.0325 | 0.0326 | 0.0321 | 0.0318 | 0.0323 | 0.0327 | 0.0326 | 0.0323 | 0.0324 | 0.0324 |
| 1400 | 0.0075 | 0.0095 | 0.012 | 0.0145 | 0.0172 | 0.0325 | 0.0325 | 0.0322 | 0.0318 | 0.0324 | 0.0326 | 0.0326 | 0.0322 | 0.0324 | 0.0325 |
| 1500 | 0.0075 | 0.0096 | 0.012 | 0.0145 | 0.0172 | 0.0324 | 0.0325 | 0.0322 | 0.0318 | 0.0324 | 0.0326 | 0.0325 | 0.0323 | 0.0324 | 0.0324 |
| 1600 | 0.0075 | 0.0095 | 0.012 | 0.0146 | 0.0172 | 0.0325 | 0.0325 | 0.0322 | 0.0319 | 0.0324 | 0.0326 | 0.0325 | 0.0324 | 0.0323 | 0.0324 |
| 1700 | 0.0074 | 0.0095 | 0.0119 | 0.0146 | 0.0173 | 0.0324 | 0.0325 | 0.0322 | 0.0319 | 0.0323 | 0.0326 | 0.0325 | 0.0324 | 0.0323 | 0.0323 |
| 1800 | 0.0074 | 0.0095 | 0.012 | 0.0146 | 0.0173 | 0.0324 | 0.0325 | 0.0322 | 0.032 | 0.0324 | 0.0326 | 0.0325 | 0.0324 | 0.0323 | 0.0323 |
| 1900 | 0.0075 | 0.0095 | 0.012 | 0.0147 | 0.0173 | 0.0324 | 0.0325 | 0.0322 | 0.032 | 0.0324 | 0.0326 | 0.0325 | 0.0324 | 0.0323 | 0.0323 |
| 2000 | 0.0074 | 0.0095 | 0.012 | 0.0147 | 0.0173 | 0.0324 | 0.0326 | 0.0322 | 0.0321 | 0.0324 | 0.0326 | 0.0325 | 0.0323 | 0.0323 | 0.0323 |

Table 89

| # of games | Number of Games Won | | | | | | | | | | | | | | |
|------------|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 9 | 9 | 3 | 7 | 2 | 4 | 4 | 9 | 4 | 7 | 13 | 7 | 5 | 6 | 11 |
| 200 | 17 | 16 | 12 | 14 | 8 | 8 | 13 | 16 | 9 | 16 | 15 | 17 | 10 | 13 | 16 |
| 300 | 28 | 24 | 14 | 18 | 17 | 10 | 18 | 25 | 14 | 20 | 21 | 25 | 15 | 23 | 28 |
| 400 | 36 | 39 | 20 | 26 | 21 | 12 | 26 | 30 | 22 | 25 | 26 | 37 | 22 | 26 | 32 |
| 500 | 47 | 47 | 26 | 37 | 23 | 17 | 32 | 34 | 29 | 31 | 29 | 45 | 32 | 34 | 37 |
| 600 | 54 | 52 | 32 | 43 | 28 | 25 | 38 | 40 | 37 | 37 | 37 | 51 | 43 | 39 | 44 |
| 700 | 62 | 62 | 38 | 47 | 34 | 33 | 45 | 46 | 44 | 47 | 42 | 57 | 47 | 47 | 49 |
| 800 | 70 | 68 | 47 | 51 | 41 | 40 | 54 | 51 | 50 | 54 | 49 | 61 | 56 | 54 | 54 |
| 900 | 78 | 78 | 56 | 55 | 48 | 42 | 59 | 57 | 60 | 58 | 55 | 70 | 62 | 59 | 63 |
| 1000 | 85 | 85 | 61 | 67 | 54 | 47 | 64 | 67 | 68 | 62 | 61 | 75 | 65 | 67 | 72 |
| 1100 | 95 | 100 | 68 | 74 | 59 | 54 | 69 | 70 | 77 | 68 | 66 | 79 | 69 | 73 | 79 |
| 1200 | 101 | 108 | 76 | 83 | 64 | 56 | 74 | 73 | 83 | 74 | 73 | 87 | 76 | 81 | 91 |
| 1300 | 105 | 126 | 82 | 94 | 74 | 67 | 77 | 79 | 87 | 80 | 79 | 91 | 79 | 86 | 94 |
| 1400 | 111 | 135 | 92 | 100 | 82 | 70 | 82 | 87 | 89 | 84 | 88 | 99 | 85 | 92 | 104 |
| 1500 | 116 | 146 | 96 | 109 | 94 | 77 | 85 | 91 | 96 | 92 | 94 | 104 | 92 | 95 | 113 |
| 1600 | 122 | 151 | 111 | 122 | 100 | 83 | 94 | 99 | 103 | 97 | 97 | 108 | 94 | 101 | 118 |
| 1700 | 131 | 159 | 128 | 126 | 108 | 86 | 98 | 106 | 107 | 106 | 102 | 113 | 98 | 108 | 124 |
| 1800 | 135 | 169 | 133 | 132 | 117 | 92 | 104 | 109 | 116 | 113 | 107 | 119 | 107 | 114 | 133 |
| 1900 | 140 | 174 | 141 | 138 | 122 | 99 | 111 | 119 | 120 | 117 | 115 | 125 | 114 | 123 | 142 |
| 2000 | 149 | 186 | 153 | 146 | 127 | 105 | 116 | 126 | 127 | 126 | 120 | 127 | 119 | 128 | 145 |

Table 90

| # of games | Total Profit | | | | | | | | | | | | | | |
|------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0652 | 0.0913 | 0.0337 | 0.1014 | 0.0272 | 0.128 | 0.1337 | 0.2857 | 0.125 | 0.2165 | 0.4233 | 0.2273 | 0.1682 | 0.1873 | 0.3657 |
| 200 | 0.1276 | 0.156 | 0.1441 | 0.2071 | 0.1331 | 0.2575 | 0.4287 | 0.5092 | 0.287 | 0.5045 | 0.4896 | 0.5555 | 0.3264 | 0.4206 | 0.532 |
| 300 | 0.2161 | 0.2417 | 0.1655 | 0.2653 | 0.2873 | 0.323 | 0.594 | 0.8033 | 0.4508 | 0.629 | 0.6783 | 0.8132 | 0.4943 | 0.7373 | 0.9262 |
| 400 | 0.2779 | 0.3818 | 0.238 | 0.3776 | 0.3609 | 0.389 | 0.8548 | 0.9642 | 0.7112 | 0.795 | 0.8413 | 1.2055 | 0.702 | 0.8371 | 1.0611 |
| 500 | 0.361 | 0.4585 | 0.3101 | 0.5311 | 0.3988 | 0.5547 | 1.0461 | 1.0982 | 0.9354 | 0.9898 | 0.9429 | 1.4687 | 1.0239 | 1.0991 | 1.2255 |
| 600 | 0.4047 | 0.5054 | 0.3827 | 0.6262 | 0.4845 | 0.8169 | 1.2412 | 1.293 | 1.1873 | 1.1889 | 1.2058 | 1.6614 | 1.3874 | 1.2669 | 1.4494 |
| 700 | 0.4714 | 0.5988 | 0.452 | 0.6865 | 0.5891 | 1.0708 | 1.4651 | 1.4789 | 1.4054 | 1.5095 | 1.3707 | 1.8543 | 1.516 | 1.5245 | 1.6148 |
| 800 | 0.5263 | 0.6633 | 0.5626 | 0.7409 | 0.7108 | 1.2984 | 1.758 | 1.6397 | 1.6027 | 1.7333 | 1.6033 | 1.9844 | 1.8115 | 1.7522 | 1.7741 |
| 900 | 0.5789 | 0.7564 | 0.6607 | 0.8032 | 0.8378 | 1.3609 | 1.9236 | 1.8274 | 1.9248 | 1.8661 | 1.7981 | 2.2801 | 1.9975 | 1.9114 | 2.0597 |
| 1000 | 0.6326 | 0.827 | 0.7123 | 0.9724 | 0.9425 | 1.5207 | 2.0902 | 2.1586 | 2.1873 | 2.0002 | 1.9798 | 2.4389 | 2.097 | 2.1803 | 2.3371 |
| 1100 | 0.7155 | 0.9669 | 0.7979 | 1.0796 | 1.0304 | 1.7498 | 2.2533 | 2.2578 | 2.4607 | 2.1958 | 2.1445 | 2.5742 | 2.2246 | 2.3718 | 2.5666 |
| 1200 | 0.7551 | 1.054 | 0.8954 | 1.2068 | 1.117 | 1.8168 | 2.4144 | 2.3405 | 2.6354 | 2.3866 | 2.38 | 2.834 | 2.4549 | 2.6275 | 2.9458 |
| 1300 | 0.7865 | 1.2112 | 0.976 | 1.3674 | 1.2773 | 2.1761 | 2.5086 | 2.5358 | 2.7666 | 2.5859 | 2.5817 | 2.9661 | 2.5484 | 2.7895 | 3.0436 |
| 1400 | 0.836 | 1.2865 | 1.1065 | 1.4485 | 1.4144 | 2.2743 | 2.6677 | 2.7972 | 2.8324 | 2.7208 | 2.8705 | 3.2234 | 2.7388 | 2.985 | 3.3761 |
| 1500 | 0.8708 | 1.3946 | 1.1499 | 1.5835 | 1.6146 | 2.4964 | 2.76 | 2.9278 | 3.0565 | 2.9823 | 3.0631 | 3.3815 | 2.9752 | 3.0794 | 3.6581 |
| 1600 | 0.9096 | 1.4396 | 1.3312 | 1.7784 | 1.7194 | 2.6942 | 3.0532 | 3.19 | 3.2843 | 3.1465 | 3.1596 | 3.5124 | 3.0431 | 3.2639 | 3.8201 |
| 1700 | 0.9754 | 1.5103 | 1.5265 | 1.84 | 1.8637 | 2.7889 | 3.1845 | 3.4127 | 3.4155 | 3.4271 | 3.3206 | 3.6731 | 3.171 | 3.4927 | 4.0099 |
| 1800 | 1.0049 | 1.6011 | 1.5933 | 1.9315 | 2.0238 | 2.9822 | 3.3771 | 3.5063 | 3.7113 | 3.6566 | 3.4837 | 3.8697 | 3.4687 | 3.685 | 4.2989 |
| 1900 | 1.0464 | 1.655 | 1.6898 | 2.0272 | 2.1114 | 3.2045 | 3.6118 | 3.8267 | 3.8441 | 3.7921 | 3.7498 | 4.0607 | 3.6911 | 3.9766 | 4.5896 |
| 2000 | 1.1038 | 1.7695 | 1.8318 | 2.1479 | 2.2021 | 3.4011 | 3.7798 | 4.0537 | 4.0721 | 4.0867 | 3.9122 | 4.1284 | 3.8453 | 4.1372 | 4.6884 |

Table 9I

| # of games | Average Expected Profit | | | | | | | | | | | | | | |
|------------|-------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|
| | Bidder1 | Bidder2 | Bidder3 | Bidder4 | Bidder5 | Bidder6 | Bidder7 | Bidder8 | Bidder9 | Bidder10 | Bidder11 | Bidder12 | Bidder13 | Bidder14 | Bidder15 |
| 100 | 0.0007 | 0.0009 | 0.0003 | 0.001 | 0.0003 | 0.0013 | 0.0013 | 0.0029 | 0.0013 | 0.0022 | 0.0042 | 0.0023 | 0.0017 | 0.0019 | 0.0037 |
| 200 | 0.0006 | 0.0008 | 0.0007 | 0.001 | 0.0007 | 0.0013 | 0.0021 | 0.0025 | 0.0014 | 0.0025 | 0.0024 | 0.0028 | 0.0016 | 0.0021 | 0.0027 |
| 300 | 0.0007 | 0.0008 | 0.0006 | 0.0009 | 0.001 | 0.0011 | 0.002 | 0.0027 | 0.0015 | 0.0021 | 0.0023 | 0.0027 | 0.0016 | 0.0025 | 0.0031 |
| 400 | 0.0007 | 0.001 | 0.0006 | 0.0009 | 0.0009 | 0.001 | 0.0021 | 0.0024 | 0.0018 | 0.002 | 0.0021 | 0.003 | 0.0018 | 0.0021 | 0.0027 |
| 500 | 0.0007 | 0.0009 | 0.0006 | 0.0011 | 0.0008 | 0.0011 | 0.0021 | 0.0022 | 0.0019 | 0.002 | 0.0019 | 0.0029 | 0.002 | 0.0022 | 0.0025 |
| 600 | 0.0007 | 0.0008 | 0.0006 | 0.001 | 0.0008 | 0.0014 | 0.0021 | 0.0022 | 0.002 | 0.002 | 0.002 | 0.0028 | 0.0023 | 0.0021 | 0.0024 |
| 700 | 0.0007 | 0.0009 | 0.0006 | 0.001 | 0.0008 | 0.0015 | 0.0021 | 0.0021 | 0.002 | 0.0022 | 0.002 | 0.0026 | 0.0022 | 0.0022 | 0.0023 |
| 800 | 0.0007 | 0.0008 | 0.0007 | 0.0009 | 0.0009 | 0.0016 | 0.0022 | 0.002 | 0.002 | 0.0022 | 0.002 | 0.0025 | 0.0023 | 0.0022 | 0.0022 |
| 900 | 0.0006 | 0.0008 | 0.0007 | 0.0009 | 0.0009 | 0.0015 | 0.0021 | 0.002 | 0.0021 | 0.0021 | 0.002 | 0.0025 | 0.0022 | 0.0021 | 0.0023 |
| 1000 | 0.0006 | 0.0008 | 0.0007 | 0.001 | 0.0009 | 0.0015 | 0.0021 | 0.0022 | 0.0022 | 0.002 | 0.002 | 0.0024 | 0.0021 | 0.0022 | 0.0023 |
| 1100 | 0.0007 | 0.0009 | 0.0007 | 0.001 | 0.0009 | 0.0016 | 0.002 | 0.0021 | 0.0022 | 0.002 | 0.0019 | 0.0023 | 0.002 | 0.0022 | 0.0023 |
| 1200 | 0.0006 | 0.0009 | 0.0007 | 0.001 | 0.0009 | 0.0015 | 0.002 | 0.002 | 0.0022 | 0.002 | 0.002 | 0.0024 | 0.002 | 0.0022 | 0.0025 |
| 1300 | 0.0006 | 0.0009 | 0.0008 | 0.0011 | 0.001 | 0.0017 | 0.0019 | 0.002 | 0.0021 | 0.002 | 0.002 | 0.0023 | 0.002 | 0.0021 | 0.0023 |
| 1400 | 0.0006 | 0.0009 | 0.0008 | 0.001 | 0.001 | 0.0016 | 0.0019 | 0.002 | 0.002 | 0.0019 | 0.0021 | 0.0023 | 0.002 | 0.0021 | 0.0024 |
| 1500 | 0.0006 | 0.0009 | 0.0008 | 0.0011 | 0.0011 | 0.0017 | 0.0018 | 0.002 | 0.002 | 0.002 | 0.002 | 0.0023 | 0.002 | 0.0021 | 0.0024 |
| 1600 | 0.0006 | 0.0009 | 0.0008 | 0.0011 | 0.0011 | 0.0017 | 0.0019 | 0.002 | 0.0021 | 0.002 | 0.002 | 0.0022 | 0.0019 | 0.002 | 0.0024 |
| 1700 | 0.0006 | 0.0009 | 0.0009 | 0.0011 | 0.0011 | 0.0016 | 0.0019 | 0.002 | 0.002 | 0.002 | 0.002 | 0.0022 | 0.0019 | 0.0021 | 0.0024 |
| 1800 | 0.0006 | 0.0009 | 0.0009 | 0.0011 | 0.0011 | 0.0017 | 0.0019 | 0.0019 | 0.0021 | 0.002 | 0.0019 | 0.0021 | 0.0019 | 0.002 | 0.0024 |
| 1900 | 0.0006 | 0.0009 | 0.0009 | 0.0011 | 0.0011 | 0.0017 | 0.0019 | 0.002 | 0.002 | 0.002 | 0.002 | 0.0021 | 0.0019 | 0.0021 | 0.0024 |
| 2000 | 0.0006 | 0.0009 | 0.0009 | 0.0011 | 0.0011 | 0.0017 | 0.0019 | 0.002 | 0.002 | 0.002 | 0.002 | 0.0021 | 0.0019 | 0.0021 | 0.0023 |

Table 92

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