THE

ARITHMETICAL ILLUSTRATOR,

Containing

EXPLANATIONS OF THE RULES AND

MOST INTRICATE PARTS OF

ARITHMETIC,

Together with

SOLUTIONS OF THE MOST ABSTRUSE QUESTIONS

THROUGHOUT THE

American Tutor's Assistant.

In a dialogue between a Tutor and his Pupil.

BY WILLIAM DOUGLASS.

PHILADELPHIA:

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1809.

DISTRICT OF NEW-JERSEY, TO WIT:

BE IT REMEMBERED, that on the fifth day of October, in the thirty-third year of the Independence of the United States of America, William Douglass, of the said district, hath deposited in this office the title of a book, the right whereof he claims as author, in the words following, *to wit:*

" The Arithmetical Illustrator, containing explanations
" of the rules and most intricate parts of Arithmetic,
" together with solutions of the most abstruse ques-
" tions throughout the American Tutor's Assistant.
" In a dialogue between a tutor and his pupil. By
" WILLIAM DOUGLASS."

In conformity to the act of the Congress of the United States, entitled, " An act for the encouragement of learning, by securing the copies of maps, charts and books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to the act, entitled, " An act supplementary to an act, entitled an Act for the encouragement of learning by securing the copies of maps, charts and books, to the authors and proprietors of such copies, during the times therein mentioned, and extending the benefits thereof, to the arts of designing, engraving, and etching historical and other prints."

ROBERT BOGGS, *Clk.*
of the Dist. of N. Jersey.

Israel McDowning

PREFACE.

THE Author of the following treatise having observed at sundry times with a degree of pain, that the greater part of those who study Arithmetic, and a considerable number who teach it, are contented with following the rules laid down in the Assistant, without endeavouring to investigate the principles on which they are founded: he has therefore spared no pains in endeavouring to elucidate the rules and other intricate parts of Arithmetic, in as simple, explicit, and concise a manner as the nature of the subject would admit; and he now offers the production of his labours to the public, in order to assist the learner in his endeavours to understand the fundamental principles of Arithmetic; as much depends on a competent knowledge of this art, in order to qualify the student to enter into the study of the various branches of the mathematics. The Author is therefore of opinion, that those who have but an imperfect knowledge of numbers, and wish to improve themselves, will find this treatise of considerable assistance to them in acquiring a necessary knowledge of this useful science. Wherefore he deems it unnecessary to say much in giving credit to his work, but submits the decision of its merit or demerit to the judgment of the impartial reader.

The principal intent of the work, is not to show the student how to work Arithmetic, but the reason why it must be wrought according to the rules laid down; and therefore, since the rules are not inserted, it will be necessary for the student to have an Assistant by him that he may turn to the rules occasionally; for he must have the rule or the substance of it in his mind, as he traces the explanation, otherwise he will understand but little about it. The American Tutor's Assistant will be the best; but any other, whose rules are similar to those in the American Tutor's Assistant, will do nearly as well.

It will also be necessary for the student to get a perfect knowledge of the signs or characters used in Arithmetic, previous to the studying this; for the Author, for the sake of brevity, has made considerable use of them.

Several eminent teachers having of late adopted a new method of stating the rule of three, the Author thought it proper to insert it, and explain it; and he recommends it to teachers and others as abundantly preferable to the common way of stating; nevertheless, he has treated largely on the old way; and such questions in other parts of the work as he has had occasion to state, he has stated agreeably to it; under a persuasion that the greater part of those, for whose use his work is intended, will understand them the better.

But few, it is presumed, will make any objection to the work, on account of its being set forth in form of a dialogue; nay, the Author is of opinion that it will be found to be a considerable addition to it: but he submits the whole to the judgment of the public; and seeing it is next to impossible for the first impression of a work like this to be without errors, he trusts that the *candid will not* censure him on this head.

THE

ARITHMETICAL ILLUSTRATOR, &c.

NUMERATION.

Pupil. Having a desire to understand the fundamental principles of arithmetic, I am inclined to ask some questions concerning it.

Tutor. That is right: I am well pleased whenever I see one of my scholars desirous of gaining a perfect knowledge of what he studies. I will endeavour to answer all thy questions in as simple a manner as possible; so begin.

P. Well; in the first place, please to give me an illustration of Numeration, for I think I do not understand it fully.

T. In the first place then, we may observe, that there are but ten figures or characters used to express numbers, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; and that these may be placed so as to represent any number that we can name or conceive. But though there are ten figures, there are but nine of them that have any value, for the 0 signifies nothing, and is, therefore, called a nought or a cipher: but, although it signifies nothing when it is by itself, yet, when it is annexed to a significant figure, it increases the value of it tenfold: as, 50 express ten times as many as 5; and 500, ten times as many as 50, &c.

Each of the nine significant figures has two values; one of which is known by its form, as 4 is known by its form to be four; 5, five, &c.; the other, by its place, which is exhibited in the following number; viz. 666,666,666,666. Every one of these figures is known by its form to represent the same thing, to wit, six, simply; but in the manner they are placed they represent different kinds of sixes. The first at the right hand represents six ones, or units; the next represents six tens, or sixty units; that is, ten times as many as the first; therefore, the first two at the right hand represent sixty-six. The third represents six hundreds; the fourth, six thousands; the fifth, sixty thousands, &c.; so that the whole number, 666,666,666,666, represents 666 thousands of millions, 666 millions, 666 thousands, 666 units, or ones.

P. That is tolerably plain; but what is meant by billions, trillions, quadrillions, &c.?

T. A billion is a million of millions, and a trillion is a million of billions, or a million of millions of millions; that is, a billion exceeds a million just as much as a million exceeds one, or in other words, there are just as many millions in one billion as there are ones in one million; and as many billions in one trillion as there are millions in one billion, and so on with the other denominations. Otherwise, a billion is the square of a million, a trillion, the cube of a million, &c. that is, a million multiplied by a million equal a billion, &c.

SIMPLE AND COMPOUND ADDITION.

Pupil. Addition is so simple that I think I can explain it myself.

Tutor. Well; go on with it. But in the first place, tell me what the word *sum* means.

P. The word sum in addition, signifies the total or answer; for instance, 25, 30, and 45 added together make 100, which is called the sum of those three numbers. The reason we must place units under units, tens under tens, &c. as the rule directs, is manifest; for we may know, it would not do to put units under tens, and add them together, and call them all units, or all tens. We may likewise know, that, in compound addition, it would not do to set pence under shillings, and add them together, and call them all pence, or shillings; or grains under pennyweights, and add them, and call them grains or dwts.

SIMPLE AND COMPOUND SUBSTRACTION.

Tutor. Tell me what is meant by the terms, minuend, subtrahend, and difference.

Pupil. The minuend is the greater of the two given numbers; that is, the upper number, when they are stated. The subtrahend is the less of the two given numbers; that is, the under number. The difference is the same as the remainder; and it shows how much greater the minuend is than the subtrahend; that is, it is the answer, or number required.

THE RULE EXPLAINED.

T. What is the reason that working by the rule gives the difference or answer?

P. When each figure in the minuend is greater than the one in the subtrahend which stands under it, it is tolerably plain; for, if we had 456 to take from 987, we should take the 6 units from the 7 units, the 5 tens from the 8 tens, and the 4 hundreds from the 9 hundreds; which would leave 531, the difference between the units, between the tens, and between the hundreds; and therefore, the whole difference. But, when some of the figures in the minuend are less than those that stand under them, I cannot tell why we borrow ten, &c.

T. We borrow ten, because ten units make one ten, ten tens make one hundred, &c.; and, by adding ten to the upper number, we make it 10 too great; and therefore, we should take from the next figure in the upper number, (which is equal to the 10 we added,) to make it the same it was at first: but, instead of doing that, (it being troublesome to perform,) we add 1 to the next figure in the lower number, which makes it likewise 10 too great; and therefore, the difference must be right. Compound substraction being founded upon the same *principles,* if this is understood, that must be manifest.

SIMPLE MULTIPLICATION.

Pupil. I observe there are three cases in multiplication of integers; what is the reason that working by the rule in each case will bring the answer?

Tutor. Tell me first, what is meant by the terms, multiplicand, multiplier, product, and factors?*

P. The multiplicand and multiplier are the numbers which are to be multiplied together. Which ever we set uppermost is the multiplicand, and the under one the multiplier. The factors are the same as the multiplicand and multiplier; and the product is what the factors make, when they are multiplied together: for instance, 8 multiplied by 6 make 48, which is the product of the factors 6 and 8.

THE RULE TO CASE 1ST EXPLAINED.

T. Let us now, for example, multiply 24 by 6.

Now, since multiplication is the taking of any number as many times as there are units in another number, if we take this 24 6 times we shall have the product, or number required: And, in working by the rule, we first take the 4 units 6 times, which make 24 units or 4 units and 2 tens; we then take the 2 tens 6 times, which make 12 tens; to which add the 2 tens and 4 units and we have 144 as above; which must be 6 times 24. We may see by this, that multiplication is a short way of performing several additions; for, since 6 24's are 144, if we were to sit down the 24 6 times and add them together, they would make 144.

$$\begin{array}{r} 24 \\ 6 \\ \hline 144 \end{array}$$

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* Some may think it unnecessary for me to say so much concerning the terms, seeing they are explained in common systems of arithmetic; but, I find, that scholars as frequently err through the means of their not understanding the terms as any thing else. A schoolmaster, who had taught school about forty years, asked me some time ago to show him how to do the last example in multiplication of integers in the A. T.'s Assistant. In doing which, I found that his ignorance consisted in his not understanding the *terms*.

THE RULE TO CASE 2D EXPLAINED.

Let us here multiply 72 by 36. Now, since 6 times 6 make 36, we first take the 72 6 times, then, we take 6 times the 72 6 times, which must make 36 times 72; for, since there are 6 72's in the 152, the 912, which are 6 of the 152 put together, must have 36 72's in it.

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 72
  6
---
152
  6
---
912
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THE RULE TO CASE 3D EXPLAINED.

Let us now multiply 39 by 29. Here, we first take the 39 9 times; then we take it twice; but, by putting the first figure of this line under the tens place, we make it 20 times the 39; therefore by adding the two lines together, we get 29 times 39. When there are more figures in the multiplicand and multiplier, the operation depends on the same principles.

```
  39
  29
----
 351
 78
----
1131
```

THE PROOF EXPLAINED.

P. What is the reason that multiplying the factors together, when they are changed will prove the work?

T. The reason is this; when we multiply two numbers together, we may make either of them the multiplicand or multiplier, which we may see by the multiplication table; for, for example, 7 times 8 make 56, and 8 times 7 make 56; therefore, if we multiply two numbers together, then change the factors and multiply them together; again, if the products are alike it proves the work to be right.

We may also observe, that, when we have several numbers to multiply together, they will make the same product. Let us place them as we will; for, $3\times4\times5=60$, $3\times5\times4=60$, $4\times5\times3=60$, $4\times3\times5=60$, $5\times3\times4=60$, and $5\times4\times3=60$. Thus, we see, three numbers *may be multiplied* together six different ways.

SIMPLE DIVISION.

Pupil. Seeing I have had to tell the meaning of the terms so far, I will endeavour to tell what those mean, that are used in division. The dividend is the number to be divided; the divisor is the number to divide by; the quotient is the number found by dividing; and the remainder is a part of the dividend, which is left after division, and it must always be less than the divisor. Now please to explain the rules to me.

THE RULE TO SHORT DIVISION EXPLAINED.

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4)2691
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  672-3
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Tutor. Let us, for example, divide 2691 by 4, in doing of which we shall have to find how often the 2691 contains the 4, and the remainder, since division is the finding how often one number contains another, &c.; and, in working by the rule, we first find that the 26 hundreds contain the 4, 6 hundred times, and 2 hundreds are left. Now the 2 hundreds will contain the 4, but they will not contain it a hundred times; therefore we conceive the 2 to be prefixed to the 9, and they both make 29 tens, which we find to contain the 4, 7 tens of times, or 70 times, and 1 ten is left, which prefixed to the 1 unit, makes 11 units, and contains the 4, 2 units of times, and 3 are left. Whence it is plain.

THE PROOF EXPLAINED.

Division is the reverse of multiplication; for, if we multiply two numbers together, then divide the product by one of these numbers, the quotient will be the other. From which it is manifest, that the divisor and quotient correspond with the factors in multiplication, and the dividend with the product; and therefore, if we multiply the divisor and quotient together, they will of course produce the dividend, when there is no remainder, and when there is one, it being part of the dividend, must of course be added in.

The reason shown, why dividing by two numbers will produce the same quotient as to divide by their product. Since division is the reverse of multiplication, and since it has been shown, that to multiply by two numbers will produce the same product, or to multiply by the product of these numbers; consequently, to divide by two numbers will produce the same quotient, as to divide by their product.

P. This is plain; but what is the reason that in this kind of division, the last remainder must be multiplied by the first divisor, and the first remainder added in, to make the true remainder.

T. The reason is this: The first remainder is part of the first dividend, and therefore, it is part of the true remainder. The second remainder is part of the second dividend, which was produced by dividing by the first divisor; and therefore the last remainder must be multiplied by the first divisor to bring it back to its original state, to which the first remainder must of course be added.

THE RULE TO LONG DIVISION EXPLAINED.

Long division is performed just as short division, only in short division, the multiplication and substraction are performed by the head; and in long division, for the assistance of the memory, they are performed by figures.

For example, let us divide 1826 by 29. Now, although we can tell, for the most part, by exercising our judgment, how many times the divisor is contained in the containing figures of the dividend; as in this case we judge the 29 to be contained 6 times in the 182; yet we cannot tell the remainder: but since the 182 must at least be as great as 6 times 29, to contain the 29 6 times by multiplying the 29 by the 6, and substracting the product from the 182, we must of course get the remainder.

```
29)1826(62
   174
   ---
    86
    58
    --
    28
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The reason shown why we cut the noughts off from the right of the divisor, and as many figures from the dividend I may observe here; that a very little attention will make it manifest; that if we divide any part of a dividend by the same part of the divisor, it will produce the same quotient as to divide the whole dividend by the whole divisor; for instance, to divide half the dividend by half the divisor, will produce the same quotient as to divide the whole dividend by the whole divisor. Now, by cutting off a nought from the right of the divisor, and a nought or a figure from the right of the dividend, we divide each of them by 10, that is, we get the tenth part of each; and the tenth part of the dividend divided by the tenth part of the divisor, gives the same as the whole dividend divided by the whole divisor, &c.

P. Is there not a short way to multiply by 9, 99, 999, 9999, &c.?

T. Yes; and it is performed in the following manner: Annex as many ciphers to the multiplicand as there are figures in the multiplier, and substract the multiplicand from this number, and the difference will be the product required.

It may be explained thus: In this case we want to take the 271, 99 times; but by annexing two ciphers to the 271, we take it 100 times; therefore we take it one time too much which must be substracted from it.

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 271    27100
  99      271
----    -----
2439    26829
2439
-----
26829
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There is likewise a short method of dividing by 9, 99, &c.; but it is mostly as troublesome as the common way, and it would take so much work to show how to do it, and explain it, that it had better be omitted.

PROOF BY REJECTING THE NINES, EXPLAINED.

P. I know the method of proving addition, multiplication, and division, by casting out the nines; but I do not understand it.

T. It depends upon a property of the number 9, which, except 3, belongs to no other figure whatever; viz. that any number divided by 9 will leave the same remainder as the sum of its figures divided by 9, which may be shown thus: if any figure by itself, or with one or more *ciphers* annexed, be divided by 9, the remainder will be equal to the figure taken in its simple value. Thus, if 6, 60, 600, 6000, &c. be divided by 9, the remainder in each case will be 6; for, 9 will go into 6 no times, and 6 off; into 60, 6 times and 6 off; into 600, 66 times and 6 off; and into 6000, 666 times and 6 off, &c. If I had taken instead of 6, 8, 7, 5, 4, 3, 2 or 1, the same reasoning would have applied. From this it is manifest, that if we divide any number by 9, then add together the figures of this number, and divide the sum by 9, the remainders will be equal; for if we take any number O's, 654, we can resolve it into its constituent parts, thus: the number 654 may be resolved into the numbers 600, 50 and 4; and if each of these be divided by 9, the remainders will be 6, 5 and 4; and since these figures are the same as those in the given number, if we divide their sums by 9, the remainders will of course be equal. Wherefore, the remainder arising from a number divided by 9 is found by adding the figures of said number, and casting out the 9's.

Now, since the sum or total in addition contains all the several numbers, if the excess above the 9's in it, be equal to the excess above the 9's in all the numbers, the work must be right; but from what is said above, the excess above the 9's in all the numbers, and in their sum, is found by adding the figures together, and rejecting the 9's. Whence the reason of the proof of addition is manifest.

In multiplication, if both, or either of the factors be an even number of 9's, the product must be an even number of 9's; and hence this part of the rule is obvious, for O will, in this case, be in the top and bottom of the cross: But if neither of the factors has an even number of 9's, then the product of the multiplicand by the 9's in the multiplier, must be an even number of 9's; and the product of the 9's in the multiplicand by the excess above the 9's in the multiplier must be an even number of 9's; but these products together with the product of the excess above the 9's in one factor by that of the other, make the whole product. Therefore, if we multiply the excess above the 9's in one factor by the excess above the 9's in the other, the excess above the 9's in the product will be equal to the excess above the 9's in the whole product; but this same excess is found by casting the 9's out of the factors. Whence it is plain.

Division being the reverse of multiplication, similar reasons will of course apply in it.

P. Some prove long division by addition, how is that performed?

T. Add the remainder and the several subtrahends together, as they stand in the operation, and the sum will equal the dividend. The reason of this is obvious; for, since each subtrahend was taken from the dividend, or part of it, and since the remainder is all that is left of the dividend, we must of course add all the subtrahends to the remainder to produce the dividend.

P. This method of proof and that by the cross, are very short; what is the reason they are not inserted in Assistants?

T. The reason is, because they are not to be entirely depended upon; for the work will sometimes prove by them when it is wrong. It, however, will never fail to prove when it is right. Jeremiah Paul has inserted these methods in his arithmetic, and has likewise observed that they are not to be entirely depended upon. They may be used by such as are well versed in numbers; but they are by no means suitable for beginners.

P. How is it that they are not to be entirely depended upon?

T. Why the proof by the cross is not to be depended upon in multiplication, may be shown thus: If we make two mistakes in multiplying, such that one will balance the other, the work will be manifestly wrong, yet it will prove as in the subjoined example.

26	1
26	8×8
——	1
146	
62	
——	
766	

Again, if in multiplying by one or more of the figures of the multiplier, we should set the first figure of the product in a wrong place, the work will be very erroneous, and yet it will prove.

132	0
102	6×3
——	0
264	
132	
——	
1584	

P. I have heard it said, that numbers may be considered either abstractedly or applicately; what is meant by that?

T. When we consider numbers as numbers simply, without applying them to any thing, they are then taken in an abstracted sense; as, 1, 2, 3, 4, &c.; but when they are applied to some thing, they are taken in an applicate sense; as, 1 pound, 2 dollars, 3 men, 4 days, &c. In addition and subtraction, all the numbers must be considered as numbers simply, or all applied to the same kind of articles. In multiplication both of the factors may be considered abstractedly; but they cannot both be considered applicately; for we cannot multiply 7 pounds by 5 men.

P. I should think it might be done; and I imagine I can do it.

T. Well, do it. I should like to see it done.

P. 5 times 7 make 35. Does not that do it?

T. That is taken both numbers in an abstracted sense.

P. I will try it again. 5 times 7 pounds make 35 pounds.

T. The 5 is taken this time in an abstracted sense. It is not worth while to try it any more: for it cannot be done. The nearest that we can come to it is, to say 5 men times 7 pounds make 35 men pounds, or men and pounds; which is nonsense.

In division, the divisor must always be considered in an abstracted sense.

P. I should like now to know the reason why, in subtraction of time according to the calendar, we are directed to borrow as many days as there are in the month of the subtrahend.

T. The best reason that I can give, is this:

The days of the subtrahend are part of the month of the subtrahend, and we must of course borrow the number of which they are a part, from which to subtract them. This we do in other kinds of subtraction.

COMPOUND MULTIPLICATION.

Pupil. I think I understand the reason of the rules to all the cases in compound multiplication, except the last. Please to explain that to me.

Tutor. Tell me the meaning of the word integer first; and likewise explain the rule to case first.

P. Let us then for example, multiply 5s. 7½d. by 6. The rule says, multiply the price of an integer by the quantity, and the product will be the answer. Now, we always multiply the price of a single article by the number of articlēs; therefore an integer in this case is a single article, as 1 man, 1 day, 1 year, &c. Now, to multiply the 5s. 7½d. by 6; we say, 6 times *a* ½d. make 6 halfpennies; and since there are 2 halfpennies in a penny, we must of course divide the 6 by 2 to find the number of pence they contain, which is 3. Then we say, 6 times 7 make 42, that is 42 pence, and the 3 pence added to them make 45 pence; and since there are 12 pence in a shilling, we must divide the 45 by 12, to find the number of shillings they contain, which is 3, and 9d. over. Again, we say 6 times 5 make 30, and the 3 we carry make 33, that is 33s.; and since 20 shillings make a pound, we must divide the 33 by 20, to find the pounds they contain.

£	*s.*	*d.*
	5	7½
		6
1	13	9

RULE TO CASE 4TH* EXPLAINED.

T. Let us now multiply 6½d. by 176.

In working by the rule, we first take the 6½d. 10 times; then we take 10 times the 6½d. 10 times, which make 100 times the 6½d. Then we take the 5s. 5d. 7 times, which make 70 times the 6½d. for the 5s. 5d. are 10 times the 6½d. Then we take the 6½d. 6 times; and the 100 times, 70 times, and 6 times added together make 176 times the 6½d.

£	s.	*d.*
		6½×6
		10
	5	5×7
		10
2	14	2
1	17	11
	3	3
4	15	4

COMPOUND DIVISION.

Pupil. I think I see clearly into all the rules in *compound division.*

* *In the* American Tutor's Assistant.

Tutor. Since that is the case, take an example in the last case and explain it, and we will let that suffice.

RULE TO CASE 3D EXPLAINED.

P. Let us then divide 36*l.* 16*s.* 3*d.* by 19: In doing which, we find that the 36*l.* contain the 19, 1 time, (which is 1*l.*) and there are 17*l.* over. Now, since 20 shillings make a pound, we must of course multiply the 17*l.* by 20, to reduce them to shillings, and add in the 16s. and find the number of times that the whole contains the 19, (which will of course be shillings), and the number of shillings that are over; which must of course be multiplied by 12 to reduce them to pence, and the 3d added in, &c.

```
    L.  s. d. L. s. d.
19)36  16 3(1 18 9
   19
   --
   17
    20
   ---
19)356
   19
   ---
   166
   152
   ---
    14
    12
   ---
19)171
   171
```

REDUCTION.

Tutor. Tell me what is meant by reducing great names to small, and small names to great; or reduction descending and reduction ascending, and explain the rule.

Pupil. Let us then for example reduce 6*l.* 15*s.* $7\frac{3}{4}$*d.* to farthings, and the farthings back again to pounds. Since 20 shillings make a pound, every one of the pounds must contain 20s.; and therefore, we must multiply the 6*l.* by 20 to reduce them to shillings, and since the 15 are shillings they must of course be added in: Again, since 12 pence make a shilling, each of the 135 must contain 12d. and therefore we must multiply the

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 L.  s.  d.
 6  15  7¾
 20
 ---
 135 shillings
 12
 ----
 1627 pence
 4
 ----
 6511 farthings
```

135s. by 12 to reduce them to pence, and add in the 7*d*. Lastly, since 4 farthings make a penny, each of the 1627 must contain 4 farthings; and therefore the 1627*d*. must be multiplied by 4 to reduce them to farthings, and the 3 farthings added in.

Whence it is manifest, that there are 6511 farthings in 6*l*. 15*s*. $7\frac{3}{4}$*d*.

This is called reduction descending; or reducing from a great name to a small: a pound being the great name, and a farthing the small.

Let us now reduce the 6511 farthings back again to pounds. Since there are 4 farthings in a penny, there must be as many pence in the 6511 farthings as there are 4's; and therefore, we must divide them by 4 to reduce them to pence. For similar reasons the pence must be divided by 12 to reduce them to shillings, and the shillings by 20 to reduce them to pounds.

$$\begin{array}{r} 4)6511 \text{ qrs.} \\ \hline 12)1627\frac{3}{4} \\ \hline 2,0)135\ \ 7\frac{3}{4} \\ \hline L.\ 6\ \ 15\ \ 7\frac{3}{4} \end{array}$$

This is called reduction ascending; or reducing from a small name to a great.

THE RULES FOR REDUCING DOLLARS, PISTOLES, GUINEAS, &C. TO POUNDS, EXPLAINED.

T. What is the reason that multiplying by 3 and dividing by 8 will reduce dollars to pounds?

P. There are 3 half crowns in a dollar, and 8 half crowns in a pound; therefore, if the dollars be multiplied by 3, they will be reduced to half crowns; and if these half crowns be divided by 8, they will of course be reduced to pounds. Please to explain to me now the rules for reducing pistoles, guineas, &c. to pounds.

T. It is manifest that if we multiply any number of pistoles, guineas, moidores, &c. by the number of pence in one of them, and divide the product by the number of pence in a pound, it will reduce them to pounds. Now there are 330 pence in a French pistole, and 240 pence in a pound; therefore, if we were to multiply any number of French pistoles by 330, and divide the product by 240, it would reduce them to pounds. Again it is manifest, that if we were to multiply any number of pistoles by any part of the 330, and divide by the same part of the 240, it would produce the same number as if we had multiplied by the whole of the former, and divided by the whole of the latter. But, if we take the thirtieth part of each of the numbers 330, 240, that is, divide each by 30, we shall get 11 and 8; therefore if we multiply any number of French pistoles by 11 and divide by 8, it will reduce them to pounds: which is the rule.

A Spanish pistole$=$336 pence, and a pound$=$240 pence; and $336\div48=7$, $240\div48=5$; therefore, $\times7\div5$ will reduce Spanish pistoles to pounds: which is the rule. The rest of the rules are formed in a similar manner.

P. To reduce dollars to French crowns, we are directed to deduct $\frac{1}{11}$; crowns to dollars, to add $\frac{1}{10}$; cents to pence, to deduct $\frac{1}{10}$; pence to cents, to add $\frac{1}{9}$. I do not understand these rules.

T. 11 dollars make 10 French crowns, and $\frac{1}{11}$ of $11=1$, and $11-1=10=$the number of crowns in 11 dollars. And $\frac{1}{10}$ of $10=1$, and $10+1=11=$the number of dollars in 10 crowns. Again, 10 cents make 9 pence; and $\frac{1}{10}$ of $10=1$, and $10-1=9=$the number of pence in 10 cents. And $\frac{1}{9}$ of $9=1$, and $9+1=10=$the number of cents in 9 pence.

TO REDUCE POUNDS, SHILLINGS, AND PENCE, TO DOLLARS AND CENTS.

P. Can there not be a convenient rule given for reducing pounds, shillings, and pence, to dollars and cents?

T. The most convenient rule that I know of is this: Multiply by 8, and divide the product by 3, the shillings remaining more, $\frac{2}{3}$ thereof, will be cents; to which 1 must be added for the pence, if they be above 7. For example, let us reduce 36*l*. 13*s*. 8½*d*. to dollars and cents.

		L. *s.* *d.*		
		36 13 8½		
		8	3)49	
			16 3	
	3)293	9 8	16 3	
1 cent is added for the 8 pence.	$ 97	82 6	81 6	

This rule may be explained thus: A shilling is 20 times less in value than a pound, and a cent 100 times less in value than a dollar. But in this case, each shilling remaining is 60 times less in value than a dollar, (a dollar being produced by dividing the pounds by 3) and $\frac{2}{3}$ of 60 are 40, and 60 and 40 make 100, that is, 100 times less in value than a dollar, the same as a cent. In regard to the pence remaining, any number of pence greater than 7, added $\frac{2}{3}$ thereof, will make a shilling or upwards, and a shilling in this case becomes a cent.

TO REDUCE DOLLARS AND CENTS, TO POUNDS, SHILLINGS AND PENCE.

Dollars and cents may be reduced to pounds, &c. thus: Multiply by 3, and divide by 8; the remainder prefix to the cents, adding to that number its half. This last sum, excepting the unit figure, divide by 6, for the shillings, and the remainder by 5 for pence.

For example, let us reduce 25 dolls. 68 cents, to pounds.

Dolls. Cts.		
25 68	½)504	
3	252	
8)77 04	6,5)75,6	
l. 9 12 7	12 7	

This may be explained thus: The dollars remaining, being prefixed to the cents, are considered together with the cents, as cents: and each of these cents, being undivided by 8, is 800 times less in value than a pound; and $\frac{1}{2}$ of 800 is 400, and $800+400=1200$ times less in value than a pound; and without the unit figure it will be 120; and $120\div6=20$ times less in value than a pound, the same as a shilling. The remainder being prefixed to the unit figure, becomes 10 times less in value than the 120; and, being undivided by the 6, each of them is 60 times less in value than a shilling; and 60÷5=12 times less in value than a shilling, the same as a penny.

THE RULE OF THREE DIRECT.

Pupil. I do not understand the Rule of Three perfectly; and I have been told that a perfect knowledge of it cannot be attained, without a previous knowledge of the doctrine of proportion.

Tutor. There can be no knowledge scarcely of the Rule of Three attained, much less a perfect knowledge, without a knowledge of proportion, for the Rule of Three is the rule of proportion; and therefore, a knowledge of proportion must be essential to the understanding of it. Wherefore, previous to my undertaking to illustrate the Rule of Three, it will be necessary for me to say something respecting proportion.

Proportion may be defined thus: Proportion is a comparative relation that subsists among four numbers; that is, when the first of four numbers contains the second, or some part of the second, as often as the third contains the fourth, or some part of the fourth; then the first bears the same proportion to the second that the third does to fourth, and the numbers are proportionals; for this is what constitutes proportion. It may be more fully explained by a few examples, thus: As 8··2::12··3; that is, as 8 is to 2 so is 12 to 3. Here the 8 contains the 2, 4 times, and the 12 contains the 3, 4 times; which shews that the four number are proportionals; that is, it shews that 8 bears the same proportion to 2, that 12 does to 3.

Again, as 6··9::8··12. Here, 6 contains $\frac{1}{3}$ of 9 twice, and 8 contains $\frac{1}{3}$ of 12 twice. Wherefore those four numbers are proportionals.

Any four numbers that are proportionals, may be arranged several ways, and still be proportionals. Thus,

As 3··9::4··12. The 3 contains $\frac{1}{3}$ of 9 once, and the 4 contains $\frac{1}{3}$ of 12 once.

As 3··4::9··12. The 3 contains $\frac{1}{4}$ of 4, 3 times, and the 9 contains $\frac{1}{4}$ of 12, 3 times.

As 4··3::12··9. The 4 contains $\frac{1}{3}$ of 3, 4 times, and the 12 contains $\frac{1}{3}$ of 9, 4 times.

As 4··12::3··9. The 4 contains $\frac{1}{3}$ of 12 once, and the 3 contains $\frac{1}{3}$ of 9 once.

As 9··3::12··4. The 9 contains 3, 3 times, and the 12 contains 4, 3 times.

As 9··12::3··4. The 9 contains $\frac{1}{4}$ of 12, 3 times, and the 3 contains $\frac{1}{4}$ of 4, 3 times.

As 12··9::4··3. The 12 contains $\frac{1}{3}$ of 9, 4 times, and the 4 contains $\frac{1}{3}$ of 3, 4 times.

As 12··4::9··3. The 12 contains 4, 3 times, and the 9 contains 3, 3 times.

If we examine the above proportionals closely, we shall find, that, notwithstanding they are arranged eight different ways, there are but four changes of proportion ; for, if 3 bear the same proportion to 9 that 4 does to 12, then 4 bears the same proportion to 12 that 3 does to 9.

The two outside numbers, or the first and fourth, are called extremes : And the two inside numbers, or second and third, are called means. For example, as 9··6::12··8. The 9 and 8 are called the extremes ; and, the 6 and 12 the means. Now the nature of proportionals is such, that the product of the extremes, is equal to the product of the means. Ths may be seen by examining the example above ; for, 9 times 8 make 72, which is the product of the extremes ; and, 6 times 12 make 72, which is the product of the means. Wherefore, if any three of these numbers be given, we can find the fourth ; for it is well known, that, if we divide the product of any two numbers by one of the numbers, the quotient will be the other ; and therefore, when one of the extremes is required, if we divide the product of the extremes by the given extreme, the quotient will be the required extreme. But the product of the extremes is equal to the product of the means : therefore, if we divide the product of the means by the given extreme, the quotient will be the required extreme. Thus, $6\times12=72$, and $72\div9=8$, and $72\div8=9$. Again, when one of the means is required, the product of the extremes divided by the given mean will give the required mean. Thus, $9\times8=72$, and $72\div6=12$, and $72\div12=6$.

Having, as I imagine, said enough respecting proportion, *to make it appear* tolerably plain, I shall now endeavour to apply it to the Rule of Three. But in the first place I may observe, that, since it takes four numbers to constitute proportion, it would be more proper, in my opinion, to call the rule of proportion, the rule of four than the rule of three, notwithstanding there are but three numbers mentioned in it; for we have always to take the answer into consideration together with the three given numbers to form a proportion.

THE RULES FOR STATING AND WORKING THE RULE OF THREE, EXPLAINED.

Let us now take the proportionals which we have used in explaining the extremes and means, and form them into four examples of the Rule of Three, and illustrate the rules for stating and working them.

1st Example. If 9 dollars will buy 6 yards of cloth, how many yards will 12 dollars buy?

Here, the 9 and 12 are the two similar terms, for they are both dollars; that is both of one name, which is what is meant by the similar terms. The 6 is of the same name with the answer; to wit, yards; and therefore it must be put in the second place. The term on which the demand lies may generally be known by its following the words, How many? How much? What cost? &c. as in this example, we may easily see that the demand lies on the 12; for it is demanded, or asked, how many yards 12 dollars will buy at the rate of 6 yards for 9 dollars; and therefore the 12 must be put in the third place, and the 9 in the first of course.

Dolls. Yds. Dolls.
As 9 ·· 6 :: 12
6
—
9)72
—
Answer.....8 *Yards*.

The rule for working an example after it is stated, may be explained thus: In direct proportion, the required term, or answer, is always the last extreme; the second and third terms, the means, and the first term, the given extreme. Now the rule directs us to multiply the second and third terms together, and divide the product by the first term for the answer. But agreeably to what has been said in proportion, the product of the means divided by the given extreme will give the required extreme: Therefore, the product of the second and third terms divided by the first term will give the answer.

2ND EXAMPLE. How many yards of cloth will 9 dollars buy, at the rate of 8 yards for 12 dollars?

Here the demand lies on the 9, and the 8 is of the same name with the answer; and therefore the question must be stated thus.

Dolls.	*Yds.*	*Dolls.*	*Yds.*
As 12 ··	8 ::	9 ··	6

I believe there are many that can work the rule of three tolerably well, yet, when they see an example stated like this, with all four of the numbers in a line, they do not know what it means. They do not consider that the fourth number is the answer; and that the example has been wrought, and the answer placed in a line with the other numbers to complete the proportion; and that the rest of the work is left out.

CONTRACTION AND CANCELLING EXPLAINED.

Pupil. I have paid great attention to what has been said, and have gained considerable information; but there are still some things in the rule of three, which I do not understand; one of which is the method of contraction.

Tutor. Contraction is very simple, and so exceedingly useful in the single and double rule of three, that I am surprised that there are so few teachers that teach their scholars it. But the reason probably is, because they do not understand it themselves. Wherefore I will take an example and endeavour to explain it.

3RD EXAMPLE. If 6 yards of cloth cost 9 dollars, what will 8 yards cost?

	Yds.	*Dolls.*	*Yds.*
As	6 ··	9 ::	8
	2	3	4
			—
			Answer....12

In this example, I perceive that the 6, the dividing term, will not go into either of the others without remainder, neither will either of the others go into the 6; but I find that 3 will go into the 6 and 9 without remainder; and therefore, I divide the 6 and 9 by 3, and cancel *them, or* cross them out. Then I divide the 2 into the 8, cancel the 2 and 8, and multiply the figures together which are not cancelled for the answer.

Pupil. Where did the 3 come from by which the 6 and 9 were divided?

Tutor. It was hardly worth while to ask that question; for, agreeably to the rule, we are at liberty to make use of any number whatever, that will divide the first term and one of the others without remainder.

The ground work of contraction has been already shown, for, it has been shown in Reduction, that if we multiply by any part of a number, and divide by the same part of another number, it will produce the same quotient, as to multiply by the whole of the one, and divide by the whole of the other. Therefore, if we multiply the 8 by the 3 and divide by the 2, it will produce the same quotient, as to multiply the 8 by the 9 and divide by the 6. Again, it will produce the same number to multiply the 4 by the 3, and not divide it by any thing, as to multiply the 8 by 3, and divide by 2.

THE RULE FOR WORKING THE RULE OF THREE EXPLAINED IN A SIMPLE MANNER.

I will now explain the rule for working the rule of three in a very simple manner, by the help of the following example.

4th Example. How many dollars will 6 yards of cloth cost, if 8 yards can be purchased for 12 dollars?

	yds.	dols.		yds.	dols.
As	8	12	: :	6	9

Since the 12 dollars are the price of 8 yards, $\frac{1}{8}$ of the 12 dollars must be the price of 1 yard, and the price of 1 yard multiplied by the 6 yards will of course give the price of the 6 yards, which is what we want to find. But the 12 dollars are 8 times the price of 1 yard; and therefore, the 12 dollars multiplied by 6 will give 8 times the price of the 6 yards, which must of course be divided by the 8 to give the price of the 6 yards.

DIRECT PROPORTION EXPLAINED.

Pupil. I had like to have forgotten to ask for an explanation of direct proportion. The assistant says, that direct proportion is, when more requires more ; or, when less requires less. What is meant by more requiring more, and less requiring less ?

Tutor. In direct proportion, the first term always bears the same proportion to the second, that the third does to the fourth ; and when this is the case, the first term always bears the same proportion to the third that the second does to the fourth : and in a stating of the rule of three, on account of the term which is of the same name with the answer being put in the second place, we have to compare the first and third terms together, and the second and fourth, or answer. Now, the meaning of more requiring more, and less requiring less may be shown thus. When the third term is greater than the 1st and the answer, according to the nature of the question, is required to be greater than the second ; then it is greater requiring greater, which is the same as more requiring more. Again, when the third term is less than the first, and the answer, according to the nature of the question, is required to be less than the second, then it is less requiring less. To explain it more clearly, let us examine the following statements.

yds. s. yds.
As 2 . . 4 : : 8
s. yds. s.
As 16 . . 8 : : 4

In the first statement it is easily to be seen, that the 8 yards will cost more than the two yards ; that is, 8 yards are more than 2 yards, and require more than 4 shillings to buy them ; so that, it is more requiring more. In the second statement the 4 shillings being less than the 16, it is manifest that they will buy a less number of yards ; and therefore, it is less requiring less ; for 4 shillings are less than 16 shillings, and require a less number for yards than 8.

INVERSE PROPORTION.

Pupil. Inverse proportion is defined to be, when more requires less, or, when less requires more ; and direct proportion has been shown to be, when more requires more, or, when less requires less: From which it appears, that, when they are both alike, that is, both more, or both less, the question is direct; but when they are different, that is, one more and the other less, the question is inverse. But, notwithstanding this is tolerably plain, I am still at a loss, in many cases, to determine whether the question is direct or inverse: and therefore, I should like the subject further illustrated, by a few examples.

Tutor. To determine whether the question is direct or inverse, depends entirely upon the judgment; but, notwithstanding it is exceedingly puzzling to scholars, it is not difficult to be understood. The reason that so few scholars get to understand it, is, because they go to work without considering whether the question belongs to direct or inverse proportion; and after having wrought it one way, if it does not bring the answer, they try it another; and so, after several trials, they will perhaps get the answer. By this means they get such an imperfect knowledge of the rule of three, that, if a question were given to them without the answer, they could not find the answer to a certainty, and therefore, the learning of the rule of three is of very little use to them.

The meaning of more requiring less, and less requiring more, may be shown thus: When the third term is greater than the first, and the answer, according to the nature of the question, is required to be less than the second, then it is more requiring less. Again, when the third term is less than the first, and the answer, according to the nature of the question, is required to be greater than the second, then it is less requiring more. To explain it more fully we will take an example or two.

Example. If 5 men can do a piece of work in 24 days, how many days will it take 8 men to do it?

Now, it is manifest, that 8 men can do a piece of work in less time than 5 men; and therefore it is more requiring less; for

men,	days,	men.
If 5 .	. 24 :	: 8

8 men are more than 5 men, and they require less than 24 days to do a piece of work. Whence it is inverse.

ANOTHER EXAMPLE.

If a footman perform a journey in 3 days, when the days are 16 hours long, how many days will he require of 12 hours long to perform the same?

Here it is manifest, that when the days are but 12 hours long it will take more days for a person to perform a journey, than when the days are 16 hours long; and therefore, it is less requiring more; for 12 hours are less than 16 hours, and they require more than 3 days. Whence this is also inverse.

	hours.		*days.*		*hours.*
If	16	..	3	: :	12

THE RULE EXPLAINED.

Pupil. What is the reason, that muitiplying the first and second terms together, and dividing the product by the third term, will bring the answer?

Tutor. In inverse proportion, the first term bears the same proportion to the third that the answer does to the second, and this being the case, the third term must bear the same proportion to the first that the second does to the answer. Therefore, if we invert the first and third terms, that is, put the first term in the third place, and the third in the first place, the proportion will be direct; and then, if we multiply the second and third terms together and divide by the first, it will bring the answer. But this would be doing the same as to multiply the first and second together, and divide by the third, without inverting the first and third. Whence it is plain.

Pupil. In some questions of the rule of three, there are four numbers mentioned, one of which is not to be used in the stating, how are we to know to a certainty which this number is?

Tutor. The superfluous number, or the number which is not to be used in the stating, is always mentioned twice. For instance in the 11th example of *Application to the rule of three*, in the American Tutor's Assistant, the 48 shillings are mentioned twice; for they are first mentioned, and then it says 'for the same money;' which is the same as mentioning the 48 the second time. Therefore, the 48 is the number that is not to be used.

Pupil. Some of the questions in Application, in the American Tutor's Assistant, are very hard. Please to explain a few of the hardest, to wit, the 16th, 18th, 19th, 22nd, and 24th.

Tutor. We state the 16th example in the first place, thus: As 1*l.* 7*s.* 10*d.*. 4 E. E. : : 118*l.* 17*s.* 7½*d.* which gives 341 E. E. 3*qr.* 1*na.* Then since there are 33 ells Flemish 1*qr.* 2*na.* in a piece, we say, as 33 E. F. 1*qr.* 2*na.*. 1 *p.* : : 341 E. E. 3*qr.* 1*na.* to the answer. And, since the 33 are ells Flemish, and the 341 ells English, it is manifest, that we must multiply the 33 by 3, and the 341 by 5, to reduce them both to quarters. Whence the work is plain.

18th Example. Here, the height of the pole bears the same proportion to the length of the shadow of the pole, that the height of the steeple does to the length of the shadow of the steeple. Therefore, we state the question thus: As 50*ft.* 11 *in.*.. 98*ft.* 6 *in.* :: 300*ft.* 8 *in.* to the length of the shadow of the steeple, which is 581*ft.* 7 *in.* Now, since the width of the river is required, and since the steeple stands 20*ft.* 6*in.* from the river and its shadow extends 30 *ft.* 3 *in.* beyond the river, if we add the 20*ft.* 6 *in.* and 30 *ft.* 3 *in.* together and subtract them from 581*ft.* 7 *in.* the length of the shadow, we shall of course get the width of the river.

19th Example. It is manifest, that a board of 1*ft.* wide and 20*ft.* long, contains just 20 square feet; and therefore, we state the question thus: As 1*ft.* . 20*ft.* :: 7½ *in.* 32*ft* the answer: that is, if 1 foot wide requires 20 feet long, 7½ inches wide will require 32 feet long.

22nd Example. Many teachers direct their scholars to work this question in the following manner.

					10
	M.	M.	M.		20
As	10 ..	80 : :	150	Inverse	40
		3)16	15		80
			3		
		5 - 20		*Answer*	150

This method, notwithstanding it is short, is, in my opinion, very improper. For, if the numbers do not bear a proportion to each other similar to the proportion that 10 bears to 20, 20 to 40, &c. it will not do.* For instance, if the numbers had been 10, 30, 50, & 70, instead of 10, 20, 40, and 80, this method would not have produced the answer.

To work this question by the rule of three, the proper method is as follows.

	Cis.	*M.*	*Cis.*	
As	15	80	1	direct
		3)16		
		5 20		*answer*

	M.	*C.*	*M.*	*C.*
As	80 ..	1 : :	80 ..	1
	40	1 : :	80	2
	20	1 : :	80	4
	10	1 : :	80	8
				15

This method may be explained thus: since it takes the last pipe 80 minutes to fill the cistern, I suppose all the pipes to run 80 minutes, and find how many cisterns each will fill in that time, which I add together. Then, since all the pipes running together 80 minutes, will fill 15 cisterns, it is manifest, that the above stating will show how long it will take them to fill one cistern.

If I had supposed all the pipes to run together any other number of minutes, instead of the 80, it would have answered the purpose; the 80 was only used for conveniency.

24th Example. Here, we first say, as 1 sec. . 1142 ft.: : 60 sec.. . 68520 ft. the number of feet that sound flies in a minute.

* That is, the numbers are not in a geometrical progression, *this method will not* produce the answer.

Then, since a person in health has 75 pulsations in a minute, we say, as 75 puls. . .68520 ft.::6 puls ..$5481\frac{3}{5}$ ft. the answer.

THE DOUBLE RULE OF THREE.

Pupil. How is it that this is called the double rule of three, seeing it contains but five numbers ?

Tutor. The reason that it is called the double rule of three, is, because it contains two statings of the single rule of three : for notwithstanding it contains but five terms, it can be wrought by two statings of the single rule of three ; and this is the way that we are told to work it, to prove the work.

Pupil. I should like to hear this matter explained a little further; but perhaps I had better get a more perfect knowledge of the manner of working by one stating, in the first place. Therefore, please to illustrate the manner of stating the question.

THE RULE FOR STATING EXPLAINED.

Tutor. To explain this we will take an example.

EXAMPLE. If 12 oxen in 16 days eat 20 acres of grass ; how many acres will serve 24 oxen 48 days ?

Oxen.		Oxen.
If 12	Acres	24
days	20	days
16		48

In this question, 12 oxen are supposed in 16 days to eat 20 acres of grass ; and therefore, the 12, 16, and 20 are the terms of the supposition. Now, without looking at the answer,* it is easy to determine which of these numbers is of the same name with the answer ; for the question says, How many acres will serve 24 oxen 48 days ? Wherefore, it is manifest that acres are required, and that the 24 and 48 are the terms on which the demand lies, and therefore, agreeably to the

* The student should always be taught to state the question without looking at the answer, in both the single, and double Rule of Three ; for, if he does not know how to do this, how is it possible for him to find the answer to a certainty, when it is not given ?

rule, the 12 and 16 must be put in the first place, the 20 in the second, and the 24 and 48 in the third.

Pupil. To determine whether the question is direct or inverse, we are directed to consider the upper, and lower pair of extremes, each separately with the middle term, as a stating of the single Rule of Three. This is very dark to me.

Tutor. In the first place it is to be observed, that here, as in the single Rule of Three, Direct Proportion is, when more requires more, or less requires less; and Inverse Proportion is, when more requires less, or less requires more.

Now, let us consider whether the before mentioned question is direct, or inverse. To do this, we consider the 12, 20, and 24, and the 16, 20, and 48 as two distinct statings of the single Rule of Three, thus: If 12 oxen eat 20 acres of grass, how many acres will 24 oxen eat in the same time? that is, will they eat more or less? it is manifest they will eat more in the same time, and we are to allow them the same time; for we are not to take the 16 and 48 days at all into consideration in this stating. Wherefore, this is more requiring more; for 24 oxen are more than 12 oxen, and they require more than 20 acres. Whence this stating is direct. Again, if in 16 days 20 acres of grass be eaten by any number of oxen, (the 12 and 24 oxen are not to be taken into consideration in this stating) how many acres will the same number of oxen eat in 48 days? that is, will they eat more, or less? It is manifest that they will eat more, for they have more time to do it. Wherefore, this is also more requiring more; and therefore, the question is direct.

Oxen.		Oxen.
If 12	Acres	24
days	20	days
16		48

We will take another example and explain the matter further.

Example. If a footman travel 240 miles in 12 days, when the days are 12 hours long; how many days will he require to travel 720 miles when the days are 16 hours long?

	miles.		miles.
	If 240	days	720
	hours	12	hours
	16 $\overline{12}$		16 $\overline{12}$

Here in the first place, since a person can travel 240 miles in 12 days, we consider how many days it will take him to taavel 720 miles; and the sense clearly dictates that it will take more; and therefore, it is more requiring more. Whence the first stating is direct. In the second stating, since, when the days are 12 hours long, it takes a person 12 days to perform a journey, it is plain to be seen, that it will not take him so many days to perform the same journey when the days are 16 hours long; and therefore, it is more requiring less: which shows it to be inverse. Whence this question is inverse.

Pupil. I should like now to see a question wrought by two statings of the single Rule of Three, and the work explained.

Tutor. Let us then take the first of the before mentioned examples, and work it by two statings.

	Oxen.	Acres.	Oxen.
As	$\overline{12}$	. . 20 : :	$\overline{24}$
		2	2
		40 acres.	

This stating shows, that, if 12 oxen in 16 days will eat 20 acres of grass, 24 oxen will eat 40 acres in the same time; and, since in 16 days 24 oxen will eat 40 acres, it is manifest, that the second stating shows, the number of acres that the same oxen will eat in 48 days: which must be the true answer to the question.

	Days.	Acres.	Days.
As	$\overline{16}$	. . 40 : :	$\overline{48}$
		3	3
		120 *Acres, answ.*	

THE RULE EXPLAINED.

From what has been said the rule for working the double Rule of Three may be explained, thus: It has been shown in Multiplication and Division, that to multiply by two numbers anddivide by other two, will produce the same number, as to multiply by the product of the former two and divide by the product of the latter two. But, in working by the single Rule of Three, we multiply by the last two terms, and divide by the first two; and, in working by the double Rule of Three, we multiply by the product of the last two terms, and divide by the product of the first two. Whence the results must be the same.

Pupil. The note given in the Assistant to show how to contract and cancel the double Rule of Three, is so very dark that I cannot understand it. Can it not be worded in a plainer manner?

Tutor. I should word it thus: If either of the first two terms, or both, will divide any of the last three, or if any of the last three will divide either of the first two, or if any other number will divide one of the first two and one of the last three without remainder; the operation may be contracted by cancelling such terms and using their quotients in their stead.

Pupil. Please to do the 8th question in Inverse Proportion in the American Tutor's Asistant, by cancelling.

$\overline{30}$	$\overline{15}$		$\overline{30}$	$\overline{15}$
$\overline{2}$				
	s	D		
$\overline{3}$		C	s.	d.
			$\overline{13}$	$\overline{4}$
			6	8
	s		s	
$\overline{6}$	$\overline{9}$		6	93
				20 days

Tutor. In the first place we may observe, that there are three statings of the single Rule of Three in this question; each of which must be considered separately to determine whether it is direct, or inverse. The first is manifestly inverse; for 30 men will require less days than 15 men to eat any quantity of bread. The second is direct; for 13*s.* 4*d.* worth of bread will last any number of men a longer time than 3*s.* worth. Again, the third is inverse; for when wheat is sold at 6*s.* per bushel, any number of shillings worth will last a number of men a longer time, than when wheat is sold at 9*s.* per bushel. Wherefore, the 15 and 30 must be inverted, as also the

9 and 6.

Now, to cancel the statement, I first observe that the 15 will go into the 30 twice without remainder; and therefore I set down the 2 and cancel, or cross out, the 15 and 30. Then I divide the 2 into the 13*s*. 4*d*. and cancel the 2 and 13*s*. 4*d*. Again, I perceive that the 3 will go into the 9, 3 times; so I divide and cancel the 3 and 9. Lastly, I cancel the 6*s*. and 6 days, for being both alike they destroy each other. Now, all the numbers being canceled except the 6*s*. 8*d*. and the 3 below, I multiply them together for the answer. But 3 times 6*s*. 8*d*. make 20*s*; and therefore, it may be thought that the 20 are shillings; but in this case they become days, for the middle term is days, and the result is always of the same name as the middle term.

Pupil. Please to show me now how to state the last question in Application. The reason that I cannot state it is, because it has but three terms.

Tutor. This question is hardly suitable for this place; for the scholar having never studied interest, does not know what is meant by rate per cent.; and therefore he cannot supply the terms that are understood. The rate per cent. is the interest of 100*L*. for a year; and the 86*L*. 17*s*. 4*d*. mentioned in the question are the interest and principal added together of 86*L*. for 8 months; and therefore, 88*L*. 17*s*. 4*d*.—86*L*.=2*L*. 17*s*. 4*d*.=the interest of 86*L*. for 8 months. Wherefore the question may be worded in the following manner, which will make the stating very simple.

A usurer put out 86*L*. to receive interest for the same; and when it had continued 8 months, he received 2*L*. 17*s*. 4*d*. interest; what is the interest of 100*L*. for 12 months at that rate?

THE SINGLE RULE OF THREE THE NEW WAY.

Pupil. Do not some Teachers direct their scholars to work the single and double rule of three by rules different from those in the Assistant?

Tutor. Yes; there are several eminent Teachers within my knowledge, who have adopted a method of working, or rather of stating those rules, which is different from that in common use, and is preferable to it; for, it is abundantly more scientific; and is much easier for scholars to learn, and further, it destroys Inverse Proportion.

The common way of stating the Rule of Three is manifestly improper; for, when four numbers are proportionals, the first bears the same proportion to the second, that the third does to the fourth; which does not hold good in a common statement of the Rule of Three, unless we consider the numbers abstractedly, which would be improper when they are applicate. For instance, if we say, As 3 men..4 days::6 men..8 days, (which is the common way of stating the Rule of Three) we may easily discover an impropriety; for it is the same as to say, that 3 men bear the same proportion to 4 days that 6 men bear to 8 days; which is absurd; for men can bear no proportion to days. But if we say, As 3 men..6 men::4 days..8 days, (which is the new, or proper way of stating the Rule of Three) we can discover no impropriety in it; for it is manifest, that 3 men bear the same proportion to 6 men that 4 days bear to 8 days.

To know how to state questions the new way observe the following

GENERAL RULE.

Write that number for the third term, which is of the same kind with the answer or number sought. Consider from the nature of the question whether this third term is greater or less than the answer; and if greater, write the greater of the other two given numbers for the first term and the less for the second; but if less, write the less for the first term and the greater for the second.

The question being stated the answer is found by this

GENERAL RULE.

Reduce the first and second terms to the same denomination; (if necessary) then multiply the second and third terms together and divide by the first; the quotient will be the answer in the denomination of the third term.

The rule for stating the question may be illustrated by a few examples.

Example. If 3 yards of cloth cost 12 shillings, how much will 5 yards cost?

In stating this question, it is easily perceived that the answer required is money; and therefore the 12*s.* which is also money, must be written for the third term, or put in the third place. Again, since this third term is the price of 3 yards, it is evident, that it is less than the answer or price of 5 yards; consequently 3 the less of the other two given numbers must be put in the first place, and the 5 in the second. Wherefore the stating will stand thus:

	yds.	*yds.*	*s.*
As	3 . .	5 : :	12
		4	4

20*s. answer.*

Another Example. If 8 men mow 12 acres of grass in a certain time; how many men will it take to mow 9 acres in the same time?

In this example we see the answer required is men; and therefore the 8 men must be the third term; and, since this third term is the number of men that mow 12 acres, it is of course greater than the answer or number of men that mow 9 acres; therefore 12 the greater of the other two given numbers must be put in the first place and the 9 in the second, thus: As 12 *A.*.9 *A.*::8 *M.*.6 *M* the answer.

A third Example. If 5 men can finish a piece of work in 24 days; how many men must be employed to do the same piece of work in 15 days?

Here we see the answer required is men; and therefore the 5 men must be the third term; and, seeing this third term is the number of men that must be employed to do a piece of work in 24 days, and the answer the number of men that must be employed to do the same piece of work in 15 days, it is manifest that the third term is less than the answer; and therefore the 15 the less of the other two given terms must be put in the first place, and the 24 in the second, thus: As 15 d..24d::5 M..8 M.

This question is one of those that are called inverse; but it may be shown to be theoretically of the same nature with those that are called direct.

It is evident from the question that 5 men can do fifteen times this work in 24×15, or 360 days; and that the number of men required, must do twenty four times the same work in 15×24, or 360 days. Wherefore the question amounts to this;

If 5 men can perform a piece of work 15 times in a certain number of days; how many men will it take to do that piece of work in the same number of days?

It may be worded in a more simple manner, thus:

If 5 men can perform 15 pieces of work in 360 days; how many men will it take to do 24 such pieces of work in the same time?

THE DOUBLE RULE OF THREE THE NEW WAY.

To know how to state questions in this rule the new way, observe this

GENERAL RULE.

Set that term in the fifth place which is of the same kind with the answer; then take either pair of the similar terms, and form a stating with them and the fifth term as in the single Rule of Three, observing the directions there given; then take the other pair of similar terms, and form another stating with them and the fifth term.

The question being stated the answer is found by this

GENERAL RULE.

Reduce the similar terms to the same denomination; then multiply the first term and the one that stands under it together for a divisor, and the other three for a dividend;

the quotient will be the answer in the denomination of the fifth term.

The rule for stating may be illustrated by an example.

EXAMPLE. If 4 reapers have 12 dols. for 3 days work; how many men will earn 48 dols. in 16 days?

In this example the answer required is men; and therefore the 4 reapers must be the fifth term; and, since this term is the number of men which earn 12 dols. it is of course less than the number of men that earn 48; consequently, the 12 the less of the two similar terms 12 dols. and 48 dols. must be put in the first place and the 48 in the second. Again, since the fifth term is the number of men that 3 days require, it is of course greater than the number of men that 16 days require to do the same work; and therefore, the 16 the greater of the other two similar terms, must be put in the first place and e 3 in the second.

dolls.	*dolls.*	
12	48	Men
days.	*days.*	4
16	3	

PRACTICE.

Pupil. I can work Practice very readily after I get the parts taken right; but I find the taking of the parts to be very difficult.

Tutor. What is meant by an aliquot part?

Pupil. The parts mentioned in the tables, I expect, are aliquot parts; that is, 1 farthing and a halfpenny are the aliquot parts of a penny. Again, 1*d.* $1\frac{1}{2}$*d.* 2*d.* &c. are the aliquot parts of a shilling, &c.

Tutor. It is true those are aliquot parts; but this is not giving the definition of an aliquot part. An aliquot part of any number, is such a part as will exactly measure it, or divide it without a remainder. Now the $\frac{1}{2}$ and the $\frac{1}{4}$ of a penny, or 2 and 4 will divide 4 farthings without remainder; and therefore, a halfpenny, and a farthing are the aliquot parts of a penny. Again, 1*d*, $1\frac{1}{2}$*d*, 2*d*, 3*d*, 4*d*, and 6*d*, are the aliquot parts of a shilling; for 1, $1\frac{1}{2}$, 2, 3, 4, and 6 will each of them divide 12*d*. without remainder.

THE RULE TO CASE 1ST EXPLAINED.

The rule to case 1st says, take such aliquot part or parts of the given quantity, as the price is of a penny, for the answer in pence. Now the given quantity is the given number, or number of articles, as ℔ gallons, yards, &c. and the given price is the price of a single article, and the answer is the price of all the articles together. Wherefore, to show that working by the rule will bring the answer, we will take the following example.

What will 4712 *lbs.* amount to, at $\frac{3}{4}$ per *lb.*?

		lbs.	
$\frac{1}{2}$	$\frac{1}{2}$	4712	
$\frac{1}{4}$	$\frac{1}{4}$	2356	
		1178	
	2	3534	
	20	29,4	6
		L. 14...14...6	

		lbs.	
$\frac{1}{2}$	$\frac{1}{2}$	4712	
$\frac{1}{4}$	$\frac{1}{2}$	2356	
		1178	
	2	3534	
	20	29,4	6
		L. 14...14...6	

Now the rule directs us to take the same part or parts of the given number that the price is of a penny; which I have done in the first of the above operations; and that it brings the answer in pence, may be shown thus: If the price had been a penny, it is manifest, that the 4712 ℔ would have amounted to just 4712 pence; and therefore, by taking ½ of the 4712 lbs. we get the number of pence to which they amount at a halfpenny a piece; and by taking ¼ of them, we get the number of pence to which they amount at a farthing a piece, and by adding these two numbers together, we get, of course, the number of pence to which they amount at three farthings a piece. Whence the 3534 are pence; that is, the answer in pence.

The second operation is different from the first; for, instead of taking ¼ of the top number, I have taken ½ of the second number, which must be equal to ¼ of the top number, because the second number is ½ of the top number. The reason that I took ½ of the second number is, because 1 farthing is the ½ of a halfpenny, and the second number was produced by a halfpenny.

This method is called, taking parts of parts; and when ever it will apply it should be used, for it is much the better way.

THE RULE TO CASE 2ND EXPLAINED.

From what has been said, the rule to case 2nd, as also chief of the following rules in practice, must be manifest; for, it has been shown, that if we take the same part, or parts of the given number that the price is of a penny, it will give the answer in pence; and consequently, if we take the same part, or parts of the given number that the price is of a shilling, it will give the answer in shillings. Therefore, all I have to do here is, to explain a little the manner of taking parts; to do which, I will work the following example several ways.

EXAMPLE.

What will 6100 yards amount to at $5\frac{3}{4}d$. per *yard*?

1st way.

d.		yards.
4	$\frac{1}{3}$	6100
d. 1	$\frac{1}{4}$	2033.4
		508.4
$\frac{1}{2}$	$\frac{1}{2}$	254.2
$\frac{1}{4}$	$\frac{1}{2}$	127.1
	2,0	292,2.11
		L. 146. 2.11

2nd way.

d.		yards.
4	$\frac{1}{3}$	6100
d. 1		2033.4
	$\frac{1}{4}$	508.4
		454.2
		127.1
	2,0	292,2.11
		L. 146. 2.11

3d way.

	yards.
	6100
	1525
	162.6
	508.4
	127.1
2,0	292,2.11
	L. 146.2..11

4th way.

		yards.
	$\frac{1}{4}$	6100
		1525
		1316.8
		254.2
		127.1
	2,0	292,2.11
		L. 146.2.11

5th way.

d.		yards.
3	¼	6100
2	⅙	6525
¾	¼	1016.8
		381.3
	2,0	292.2 11
		L. 146.2..11

6th way.

d.		yards.
4	⅓	6100
½	⅛	2033.4
¼	⅙	762.6
		127.1
	2,0	292,2 11
		L. 146.2..11

Thus we see an example wrought six different ways.

In taking the parts in the first of these ways, I say, 4 d. are ⅓ of a shilling, and 1 d. is $\frac{1}{12}$ of a shilling; and, since they are both parts of a shilling, I divide them both into the top number. Then, I say, a halfpenny is ½ of 1 d. and divide it into the number that was formed by the 1d. Lastly, I say, 1 farthing is ½ of a halfpenny, and divide it into the n[illegible]t was formed by the halfpenny.

In the second way, instead of saying 1d. is 1-12 of a shilling, I say, 1d. [illegible] ¼ of 4 d.; which must of course bring the same thin[illegible]

In the third w[illegible]y, 3d. are ¼ of a shilling; 1½d. ½ of 3d.; 1 d. ⅓ of 3d.; and 1 farthing, ¼ of a 1d.

In the 4th way, I say, 2d. are ⅙ of a shilling; a halfpenny ¼ of 2 d. &c.

In the 5th way, I say, 3 farthings are ¼ of 3 d. and divide it into the number that was formed by the 3 d.

In the 6th way, I say, 1½ d. is ⅛ of a shilling; and 1 farthing ⅙ of 1½ d.

When the parts may be taken several different ways, we should endeavour to choose the shortest.

Pupil. The Assistant says, When the complement of the given price, in any case, is an aliquot part, deduct the said aliquot part of the given quantity therefrom, and the remainder will be the answer of the same denomination with the integer, of which the divisor is a part.

I do not understand this; for I know not the meaning of the word complement.

Tutor. The complement is the difference between the price and the integer of which it is a part; and it is found thus: when the price is farthings subtract it from a penny; when it is pence, from a shilling; and when it is shillings, from pounds. The remainder in each case is the complement.

To explain it further, let us find the amount of 3120 gallons at $10\frac{1}{2}$d. per gallon, by this method.

d		*gallons.*
$1\frac{1}{2}$	$\frac{1}{8}$	3120
		390
	2,0	273,0
		L. 136..10

By subtracting the $10\frac{1}{2}$d. from a shilling, we get $1\frac{1}{2}$d. which is the complement. That is, we find that the price of each gallon lacks but $1\frac{1}{2}$d. of being a shilling. Therefore, since at a shilling per gallon, the amount in shillings would be equal to the given number, if we find the amount at $1\frac{1}{2}$d. per gallon, and subtract it from the given number, the remainder must be the amount in shillings, at $10\frac{1}{2}$ d. per gallon.

Pupil. The second rule to case 4th says, If the price be even shillings, multiply by half the price, and double the first figure of the product for shillings, the rest of the product will be pounds. This is a very short way of finding the answer; but I do not understand the reason of it.

Tutor. It may be explained thus: By multiplying by half the number of shillings, we get half the answer in shillings; which, being divided by 10, instead of 20, will of course give the answer in pounds. But, by doubling the first figure at the right, for shillings we take one figure from the number, which is the same as dividing by 10; and therefore, the remaining figures must be the answer in pounds, and the figure that we double, half the shillings remaining, which shows that it must be doubled.

Pupil. I think I understand Practice now tolerably well except the last case.

THE RULE TO CASE 7TH EXPLAINED.

Tutor. We may explain the last case very easily by the help of an example. Therefore, we will take the following, viz.

What is the value of 12C. 2 qrs. 14 lbs. at 3 *L.* 14*s.* per C. ?

qrs.		*L.*	*S.*
2	$\frac{1}{2}$	3 ..	14
			12
lb.		44 ..	8
14	$\frac{1}{4}$	1 .	17
			9..3
		L. 46 ..	14..3

We first multiply the 3 *L.* 14 *s.* by the 12; that is, we multiply the price by the integers of the quantity, as the rule directs; which must give the value of the 12 C. And, since the 3 *L.* 14*s.* are the value of 1 C., $\frac{1}{2}$ of the 3 *L.* 14 *s.* must be the value of 2 *qrs.* and $\frac{1}{4}$ of this half, must be the value of 14 ℔; for 14 ℔ are 1-4 of 2 qrs. Wherefore, by adding these values together, we get the whole value; or the value of the 12 C. 2 qrs. 14 ℔.

TARE AND TRETT.

Pupil. The rules to case 1st and 2nd in Tare and Trett, are so simple that I think I can explain them myself.

Tutor. I am glad to hear that; so go on with them.

THE RULE TO CASE 1ST EXPLAINED.

Pupil. Case 1st is very plain, for since the tare is the weight of the barrel, box, bag, &c. or whatever contains the goods; and the gross, the weight of the goods together what contains them; and the neat, the weight of the goods alone, it is manifest, that we must subtract the tare from the gross to get the neat.

THE RULE TO CASE 2ND EXPLAINED.

In the 2nd case, the given tare is the weight of one of the articles that contain the goods: and therefore, it must be multiplied by the number of articles to give the whole tare; and this must of course be subtracted from the gross to get the neat.

THE RULES TO CASE 3D EXPLAINED.

Tutor. Let us now explain the rules to case 3d by the help of the following example.

Sold 10 casks of Alum, weighing 48 C. 3 qrs. 12 lbs. gross; tare 14 lb. per C. wt.; what is the neat weight?

℔	*C.*	*qrs.*	*lb.*	
$14 = \frac{1}{8}$)	48 ..	3 ..	12	gross,
	6 ..	0 ..	12	tare,
	42 ..	3 ..	0	neat.

The first rule tells us to deduct from the gross such aliquot part or parts of it, as the tare is of a C. wt. and the remainder will be the neat. Now the tare in this example is 14 ℔ per C. wt. that is, there are 14 ℔ tare in every C. wt. gross; and therefore, since 14 ℔ are $\frac{1}{8}$ of C. wt. if we take $\frac{1}{8}$ of the gross, it will of course give the whole tare.

The second rule is simply the rule of Three, thus: As 112 ℔ gross are to 14 ℔ tare, so are 5444 ℔ gross to the whole tare; to find which, we multiply the 5444 by the 14, and divide by the 112; which is the same as working *by the rule.*

THE RULE TO CASE 4TH EXPLAINED.

The suttle is the same as the neat in the preceding cases, and found the same way. And, since there are 4 ℔ trett in every 104 ℔ suttle, and since, 4 are $\frac{1}{26}$ of 104, if we divide 104 ℔ suttle by 26 it will give the trett they contain; and therefore, if we divide the whole suttle by 26 it will give the whole trett; which being subtracted from the suttle gives the answer.

SIMPLE INTEREST.

Pupil. I can work the most of the examples in Interest by following the directions of the rules; but I do not think that I understand the ground-work of a single rule.

Tutor. That is frequently the case with scholars; they hobble along in some manner through Interest; and when they get through, they know but little about it. The reason of this is, I conceive, because they have not a perfect knowledge of the meaning of Interest. There are some, who have gone through Interest, that, if they were asked the meaning of the words, per cent. per annum, they could not tell. Therefore, before I go to explaining the rules, it will be necessary for me to illustrate the meaning of Interest; which I will do by the help of the following example; viz.

What is the interest of 220 *L.* for 2 years at 6 per cent. per annum?

To give an idea of the meaning of this question, I may first observe, that per cent. means per hundred, and per annum, per year; and that, since Interest is defined to be, a consideration allowed for the use of money, if I were to let a person have 100 *L.* for a year at 6 per cent. at the end of the year, he would have to give me 6 *L.* for the use of the 100 *L.*; that is, he would have to give me 6 *L.* interest for the use of the 100 *L.* 1 year; which 6 L. are called the rate per cent.; and therefore, the rate per cent. is always the interest of 100 *L.* for a year. Wherefore, it is manifest, that if I were to let a person have any sum of money for any time, at 6 per cent., the interest that he would have to give me, must be at the rate of 6 *L.* on 100 for a year.

THE RULE TO CASE 1ST EXPLAINED.

From what has been said, the rule to case 1st may easily be explained; thus: since it has be shown that the rate per cent. is the interest of 100 *L.* for a year, it is manifest, that let the principal be what it will, it must bear the same proportion to its interest for a year, that 100 *L.* bear to the rate per cent. Wherefore, to find the interest of the 220 *L.* mentioned in the example, we must say, As 100 *L.* principal are to 6 *L.* interest, so are 220 *L.* principal to the interest of the same for a year; to find which we multiply the 220 by 6 and didive by the 100; that is, we multiply the principal by the rate per cent. and divide by 100 as the rule directs. When the time is more years than one, it is plain that the interest for one year must be multiplied by the number of years to get the whole interest.

THE RULE TO CASE 2ND EXPLAINED.

This case is nearly the same as case 1st; for here we multiply the principal by the pounds of the rate per cent. as before. Therefore, the only difference is this: In this case the rate per cent. is $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a pound more than even pounds; and therefore, we have to take parts of the principal for the parts as in practice; by which means we must get the true interest.

Pupil. Paul's Assistant says, 'If the interest be required in Federal money, multiply the principal by 16, and separate the product of the pounds for the interest in dollars and cents, at 6 per cent.' This is a very convenient rule; but I can not see into it.

Tutor. Let us see if we cannot unravel it by the help of the following example.

Example. What is the interest, in federal money, of 987*L.* 18*s.* 11*d.* for a year, at 6 per cent.?

L.	*S.*	*D.*	
987	18	11	
			4×4=16
3951	15	8	
158,07	2	8	
158 *dols.* 7 *cts.*			

If we examine this closely we shall find that it is very simple; for, if the interest had been required in pounds, we should have multiplied by the 6 per cent. &c. But 6*L.* are equal to 16 dollars. Therefore, by multiplying by 16, and cutting off two figures at the right, we must get the interest in dollars.

Perhaps it would not be amiss for me to remark, before we go any farther, though it seems too simple to need a remark, that the rate per cent. is not confined to pounds; for, if 100*L.* bring 6 pound interest, 100 dollars will bring 6 dollars interest, 100 cents, 6 cents, &c.

Pupil. There is another rule in Paul's Assistant which I should like to see explained. It runs thus: If the principal be in dollars, and the interest required in pounds at 6 per cent. multiply by $2\frac{1}{4}$ and divide by 100.

Tutor. This is the reverse of the other and equally simple; for, if the interest had been required in dollars, we should have multiplied by 6 and divided by 100. But 6 dollars are 2*L.* 5*s.* that is, $2\frac{1}{4}$ pounds. Therefore, if we multiply by $2\frac{1}{4}$, and divide by 100, it must give the interest in pounds.

Pupil. Bennett's Assistant says, 'When the principal is dollars, multiply by the rate per cent. the product will be the interest for one year, in cents.'

This rule is so plain that I think I can explain it myself. To do which, I have only to observe, that, if we multiply the principal in dollars by the rate per cent. and divide by 100, we know we shall get the interest in dollars. But, if we multiply these dollars by 100, we shall reduce them to cents. Therefore, if we omit dividing by 100, we must have the interest in cents.

There is another rule in Bennett's Assistant which I will undertake to explain. It runs thus: 'When the principal is dollars and cents, multiply by the rate per cent. and separate one figure to the right of the product, (as a remainder or fraction) the left will be the interest for one year in mills.

The principal here being dollars and cents, we may consider it altogether as cents. But, if we multiply the principal in cents, by the rate per cent. and divide by 100, we shall get the interest in cents. Therefore, since 10 mills make a cent, dividing by 10 instead of 100, will of course give the interest in mills; and separating one figure to the right is dividing by 10.

THE RULES TO CASE 3D EXPLAINED.

Seeing I have undertaken to explain rules myself, I will try my hand upon those to case 3d; the first of which says,

As the months, weeks, or days in a year,
Are to the interest of the given sum for a year;
So are the months, weeks, or days in the time given,
To the interest required.

I think I now understand proportion well enough to see, that the interest of the given sum for a year, must bear the same proportion to 12, 52, or 365, that the answer, or interest required bears to the given number of months, weeks, or days. Therefore, if we first find the interest of the given sum for a year; then, make a stating of the rule of three according to the nature of the question, we must get the answer.

The second rule tells us to take the aliquot parts of the yearly interest, for the given parts of a year; that is, we must first find the interest of the given sum for a year; then take the same part or parts of this interest that the given time is of a year; which must of course *give the answer.*

THE RULE AND NOTE IN THE A. T.'S ASSISTANT, PAGES 87 AND 88, EXPLAINED.

Please to explain to me now, the rule and note in the American Tutor's Assistant, pages 87 and 88; for they appear to me, especially the note, to be very abstruse.

Tutor. The rule is very plain: for the interest of 100*L.* at 6 per cent. for a year, or 12 months, we know is equal to half the number of months; and if so, the interest of the same for any other number of months must be equal to half the number of months. Therefore, multiplying by half the number of months and dividing by 100, must give the interest at 6 per cent.

With respect to the note, we may observe, that, by allowing 30 days to a month, the interest accumulates too fast; for 12 times 30 make but 360 instead of 365; and, therefore, we make the interest for a year due 5 days too soon; and 5 days are $\frac{1}{73}$ part of a year, for 5 will go 73 times in 365. Therefore, we make the interest due $\frac{1}{73}$ part of the right time too soon; and if so, we must of course make it $\frac{1}{73}$d part of itself too great; and consequently if we divide it by 73, we shall get the part to be taken from it to reduce it to the true interest. But, if we divide by 72 instead of 73, it will make but a trifling difference; and therefore, we will call the 73 for conveniency 72. Now the pounds with the units excepted are ten times greater than the interest that should be divided by 72; and, therefore, they must be divided by 720 to give the part to be deducted. But, 3, 12, and 20 multiplied together make 720. Therefore, if we divide by 3, 12, and 20, we shall get the part in pounds to be deducted. Whence dividing by 3 must give the pence.

Pupil. There are rules given in several Assistants for finding the interest of any sum as computed at the banks, at 6 per cent. In what manner do they compute interest at the banks?

Tutor. I will endeavour to show the manner, and explain the rules as they are given in Bennett's Assistant, by the help of the following example, viz.

What is the interest of 1542 dollars, for 90 days, at 6 per cent.?

To show how to work this, I may first observe, that interest is calculated at the banks after the rate of 360, instead of 365 days to the year; by which means 100 dollars in 60 days gain 1 dollar interest; and therefore, the first rule may be explained by working the example by the double rule of three, thus:

$$\left.\begin{array}{c}\textit{Dols.}\\ \text{If } 100\\ \textit{Days.}\\ 60\end{array}\right\} \begin{array}{c}\textit{Dol.}\\ 1\end{array} \left\{\begin{array}{c}\textit{Dols.}\\ 1542\\ \textit{Days.}\\ 90\end{array}\right.$$

Now we know, that, if we multiply the 1542 by the 90 and divide by 6000, the product of the 100 by the 60, we shall get the interest in dollars. Therefore, since 1000 mills make a dollar, if we divide by 6, instead of 6000, we shall get the interest in mills. But multiplying the 1542 by the 90 and dividing by 6, is doing just as the first rule directs. Whence the first rule must be plain.

The second rule says, if the principal be any number of dollars, the interest for 60 days, at 6 per cent. will be exactly the same number of cents; and for the time more or less than 60 take aliquot parts.

To make this rule clear, I need only observe, that it has been shown that 100 dollars in 60 days gain 1 dollar, or 100 cents interest; that is, 100 dollars principal gain 100 cents interest; and, therefore, the interest of any other number of dollars must be the same number of cents.

The third rule is the same in effect as the first, for if we divide the 90, in the statement, by 6, and multiply the 1542 by the quotient, it must produce the same number as to multiply the 1542 by the 90 and divide by the 6.

Pupil. With regard to Insurance, Commission, and Brokage, I may observe, that Insurance and Commission are plain; but Brokage seems somewhat dark.

Tutor. To find the brokage, the rule says, divide the sum by 100, and take such aliquot parts of the quotient, as the brokage is of a pound.

By dividing the sum by 100, we get the brokagé of it at 1*L.* per cent. and by taking the same part or parts of this that the brokage is of a pound, we must get the brokage required.

Pupil. The last three cases in interest appear very dark to me, and not only so; but it seems as if I could never learn to work them without applying to the rules.

Tutor. That is mostly the way with scholars, they make out to work through these cases by the assistance of the rules and the teacher; and after they have gone several times through them, they cannot do questions in them without the help of the book. Such scholars ought to consider that they may some time have occasion to work such questions when they have no assistant by them. For this reason, they should try to learn to perform the operation from the nature or sense of the case, without looking at the rule.

CASE 5TH EXPLAINED.

EXAMPLE. What principal, at interest for nine years, at 5 per cent. per annum, will amount to 725*L.*?

Let us now see if we cannot work this example without applying to the rule. The amount, time, and rate per cent. we see are given, to find the principal. Now it is manifest, that the amount of any principal must bear the same proportion to its principal, that the amount of another principal does to its principal. Wherefore, since 100*L.* added to the rate per cent. are the amount of 100*L.* for 1 year, if we multiply the rate per cent. by the time, and add the product to 100*L.* it will give the amount of 100*L.* at the rate and time given; and this amount must be to its principal, 100*L.* as the given amount is to the required principal. Whence, to work the example, we must first multiply the 5 by the 9, and add the product to 100, and then make a stating of the rule of three, thus: As 145*L.* amount are to 100*L.* principal, so are 725*L.* amount to the principal required.

CASE 6TH EXPLAINED.

EXAMPLE. At what rate per cent. per annum, will 500*L*. amount to 725*L*. in 9 years?

In this case the amount, time, and principal are given to find the rate per cent. and it is manifest, that the given principal must bear the same proportion to its interest for the whole time, that 100*L*. principal do to their interest for the same time; and this last interest divided by the time must give the rate per cent. seeing the rate per cent. is the interest of 100*L*. for a year. Wherefore, to work the example, we must first subtract the principal from the amount, which will give the interest for the whole time. Then we must make a stating of the rule of three, thus: As 500*L*. principal are to 225*L*. interest, so are 100*L*. principal to the interest of the same for 9 years; which, being divided by 9 must give the rate per cent.

CASE 7TH EXPLAINED.

EXAMPLE. In what time will 500*L*. amount to 725*L*. at 5 per cent. per annum?

We may observe here, that the principal, amount, and rate per cent. are given, to find the time; and that, since the interest of the given principal for 1 year, must be multiplied by the time to give the whole interest, the whole interest must of course be divided by the interest for 1 year to give the time. Whence, to work the example, we must find the interest of the 500*L*. for a year, and divide it into the whole interest, and it will give the time.

COMPOUND INTEREST.

Pupil. I believe I understand compound interest *pretty well*; *for* it is performed by several operations of simple interest. That is, we first find the interest of the given principal for a year, and add it to the given principal, and call this sum the principal for the second year. Then we find the interest of this last principal for a year, and add it and the principal together, and call the sum the principal for the third year, &c. making as many operations of simple interest as there are years given.

REBATE, OR DISCOUNT.

Tutor. What is the meaning of rebate, or discount?

Pupil. The assistant defines it to be, an abatement for the payment of money before due, by accepting so much as would amount to the whole debt at the time payable, at a given rate. But this definition is beyond my comprehension.

Tutor. I thought so, for it is beyond the comprehension of many; and therefore, I will endeavour to explain it. To do which, we will suppose a person to buy goods at a vendue to the amount of 106 dollars, the conditions being such as allow a year's credit; but having money by him, he is willing to pay the cash, provided the seller will allow him discount at 6 per cent. that is, provided he will take, instead of 106 dollars, such a sum, as, being put out to interest for a year at 6 per cent. will amount to 106 dollars. But 100 dollars will amount to 106 in a year at 6 per cent.; and therefore, 100 dollars are the present worth of 106 dollars at 6 per cent. for a year; that is, they are the sum that the seller should receive at present instead of 106 dollars at the end of the year; for it will be just as profitable to him to receive 100 dollars now as 106 at the year's end, seeing 100 dollars will gain 6 dollars interest in a year.

From what has been said, it appears that the whole debt is the amount of the present worth at the rate and time given; and the rebate, or discount the interest of the same; for 106 dollars are the amount of 100 dollars for a year at 6 per cent and 6 dollars are the interest of the same. Wherefore, to find the present worth, we must first find the amount of 100 at the rate and time given; and then say, as this amount is to 100 dollars, or pounds, its present worth, so is the whole debt to its present worth; which is the same in effect as the rule, and also as case 5th in simple interest; and there it must be plain.

Some people take the discount to be the interest of the sum due; but they are under a great mistake, for it is the interest of the present worth, as has been shown. Such people, for instance, take the discount of 100 dollars for a year at 6 per cent. to be 6 dollars; and if so, the present worth must be 94 dollars. But the interest of 94 dollars for a year at 6 per cent. is less than 6 dollars; and therefore, the creditor will lose by allowing 6 dollars discount. Whence, the discount, in this case, ought to be less than 6 dollars; that is, it should be 5 dollars 66 cents.

EQUATION.

Pupil. Equation is very simple in its operation; but I cannot see why the sum of the products of the several payments by their times divided by the sum of the payments, gives the equated time.

THE RULE EXPLAINED.

Tutor. The reason of it may be shown, thus: The products of two or more payments by their times, are to one another as the interests of the payments; for the interests of the payments for the same time, are of course to one another as the payments; and therefore, the products of these interests by their times must be to one another as the products of the payments by the same times. But the interest of the sum of the payments at the equated time must be equal to the sum of the interests of the several payments at their respective times, that neither party may sustain loss. Therefore, the product of the sum of the payments by the equated time, is equal to the sum of the products of the several payments by their times. Whence, the sum of the products of the several payments by their times divided by the sum of the payments, will give the equated time.

Notwithstanding equation appears tolerably simple, there has been considerable controversy among Mathematicians concerning it. Some, and I believe the greater number, maintain that the rule which I have just explained, does not give the true equated time. Others hold that it does, and endeavour to prove it. Nicholas Pike in speaking of it, says, "This rule is founded upon a supposition, that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument; that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due; but this cannot be the case; for though by keeping a debt after it is due, there is gained the interest of it for that time; yet by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest."

From these words of Pike, it appears to me, that the occasion of the dispute is this, to wit, those who maintain that the rule is false, calculate whether either of the parties gains or loses by the equated time at the equated time, and they find that the payer gains a trifle, and the receiver loses as much. But those who hold that the rule is true, calculate whether the equated time occasions any gain or loss at the time of the last payment; and they find that it occasions neither; and it appears to me that the latter is the proper method of computation; for when we consider the case at the several payments, to make a comparison with the equated time, we find how the parties are circumstanced at the time of the last payment; and therefore, if the equated time causes them to be circumstanced in the same way at the same time, (which it does) I think we may safely conclude that it is the true equated time.

BARTER.

Pupil. As the rule for working barter directs us to work by the rule of three, direct, or inverse, or by practice, as the tenor of the question may require, there seems nothing here that is necessary to be explained; for I think I understand these rules now pretty well.

Tutor. Though that be the case, I may remark, that, when a question may be wrought more ways than one, we should endeavour to choose the shortest way. For instance, the first example in barter in the American Tutor's Assistant, is wrought two ways; by the rule of three direct, and inverse, and as the latter is much the shorter method, it is to be preferred. The second example may likewise be wrought by the rule of three inverse; as also by the 1st, 2nd, 6th, 7th, and 8th, in Bennett's Assistant. Practice should be used wherever it will apply, for it is mostly shorter than the rule of three.

Pupil. Please to show me how to work the last question in this rule in A. T.'s Assistant.*

Tutor. The way to work it, is this: $3L.\times12\frac{1}{2}=37L.$ 10*s*, the value of A's hops at the barter price. Again, 63 gallons at 5*s* per gallon$=15L.$ 15*s*. the value of B's hogshead of wine at the cash price, which is to be raised in proportion to A's demand, thus: As $2L.$ 16*s*..$3L.$::$15L.$

* The 13th example in this rule in Bennett's Assistant is similar to this.

15s..16L. 17s. 6d, the value of B's wine at the barter price. Whence 37L. 10s.=16L. 17s. 6d.=20L. 12s. 6d. the answer.

LOSS AND GAIN.

Pupil. We are directed to work loss and gain by the rule of three, or by practice, as the nature of the question may require; but it is very puzzling to me, notwithstanding I understand these rules. I am very much at a loss to work, much more to understand, such questions as have gain or loss per cent. mentioned in them; and therefore, if one of these were explained, perhaps it would cast some light upon the rest; for instance, the 3rd in the A. T.'s Assistant, which says, "hats bought at 4s. a piece, and sold at 4s. 9d.; what is the gain per cent.?

Tutor. Since per cent. means per 100, it is manifest here, that we are to find how much will be gained by selling as many hats as cost 100L. at the rate of 9d. gain on one that cost 4s. Wherefore, the question would be plain, and easily stated, if it were worded in the following manner, viz. If there be 9d. gained by selling a hat that cost 4s. how much will be gained by selling as many as cost 100L.?

Pupil. I begin to see into it now; yet, if the 6th and 7th questions were explained, I think it would make it still plainer: which are as follow:

6. Bought 60 reams of paper, at 15s. per ream, what is lost in the whole quantity, at 4 per cent.?

7. Sold 500 penknives, at 15d. a piece, and 9 per cent. loss; what is lost in the whole number?

I can work the first of these questions so as to get the answer, but not the second; which is a mystery to me; for I think the second ought to be worked in the same manner as the first.

Tutor. It is a great mistake to imagine that the second must be wrought in the same manner as the first; and it is a mistake which many labour under; and therefore, I will endeavour to rectify it: To do which, I may first observe, that the difference in the nature of the two questions, depends solely upon the words, bought, and sold; for the first says, bought 60 reams, &c. and the second, sold 500 penknives, &c. Now the gain or loss is always reckoned on the prime, or first cost; that is, on the sum for which the articles were bought, and not on that for which they were sold. But the value of the 60 reams of paper at 15*s.* per ream, is the first cost of the paper; and therefore, to find the loss on it, at 4 per cent. we must of course say, as 100*L.*4*s*::45*s*..1*L.*16*s.* But the first cost of the penknives is not known; for their amount at 15*d.* a piece, is that for which they were sold; nevertheless, since the loss is at 9 per cent. it is known, that as many penknives as cost 100*L.* must be sold for 91*L.* that 9*L.* may be lost. Therefore, the 91*L.* the sum of the sale of a certain number of penknives, must bear the same proportion to the 100*L.* the sum for which they were bought, that the sum of the sale of the 500 bears to that for which they were bought; and if so, the 91*L.* must bear the same proportion to the 9*L.* loss, that the sum of the sale of the 500 penknives bears to the whole loss. Whence, as 91*L.*.9*L.*::31*L.* 5*s.*.3*L.* 1*s.* 9*d.* $\frac{3}{4}$.

Pupil. I understand these questions now tolerably well; but there are some others that I do not, viz. the 13th, 14th, 23d, and 24th.

Tutor. In the 13th example, or question, we find, by subtracting the 18*L.* from the 25*L.* that 18*L.* gain 7*L.* in 4 months; and it is plain, that it is required, at that rate, to find how much 100*L.* will gain in 12 months. Whence, the double rule of three will apply.

In the 14th example, we first find by practice, that the value of the 300*lb.* at 4*s.* 2*d.* per *lb.* is 62*L.* 10*s.* the first cost. Then we find by the same rule, that the value of the 300 *lb.* at 5*s.* per *lb.* is 75*L.* the amount of the sale; but since 8 months credit are allowed, we must find the present worth of the 75*L.* for 8 months at 6 per cent. thus: as 104*L.*.100*L.*::75*L.*.72*L.* 2*s.* 3$\frac{1}{4}$*d.* the present worth of the sale. Therefore, 72*L*. 2*s*. 3½*d*.—62*L*. 16*s* = 9*L*. 12*s*. 3½*d*. the whole gain.

To find the gain per cent. per annum, we take the 62*L*. 10*s*. from the 75*L*. which shows, that the 62*L*. 10*s*. in 8 months, gain 12*L*. 10*s*.; and then, since it is required at that rate, to find how much 100*L*. will gain in 12 months, the stating becomes simple.

In the 23d example, it is manifest, that the first cost of the 1 ℔ of pepper must be 12½*d*. since there are 2*d*. lost by selling it for 10½*d*; and therefore, to find the loss on 100*L*. we must say, as 12½*d*..2*d*::100*L*. 16*L*.

In working the 24th example, many people make use of about 5 times as many figures as are necessary. The shortest way of working it, and of course the best, is the following.

As	100*L*..	125*L*::	16*s*.	As	28*s*..	12*lb*::	20*s*.
	4	5	4			4	4
		4					—
		—					80 *lb*.
		20*s*.					

The reason that these two statings bring the answer may be shown, thus: Since each cask of raisins cost 16*s*. and was sold so as to gain 25 per cent. it is manifest that the first stating must give the number of shillings for which each cask was sold, to wit, 20*s*. Again, since each cask was sold for 20*s*. and 112 ℔ for 28*s*. it is clear, that the second stating must give the weight of each cask.

FELLOWSHIP.

Pupil. The first case of fellowship is very simple; for it is plain to me, that each man's share of the gain or loss, must bear the same proportions to his share of the stock, that the whole gain or loss bears to the whole stock; and therefore, the rule is plain.

I should like now to see the rule to case 2d explained.

THE RULES TO CASE 2D EXPLAINED.

Tutor. That may readily be done by the help of an example.

EXAMPLE. A. and B. traded jointly; A. put in 200*L.* for 18 months; and B. 100*L.* for 12 months; they gained 60*L.*; what must each have?

A. 200×18=3600. As 4800..60::3600..45. A's.
B. 100×12=1200. As 4800..60::1200..15. B's.

Sum, 4,800

Now, to show that this way of working gives each man's share, I may observe, that, if A's stock had been 18 times 200*L.* for 1 month, instead of 200*L.* for 18 months, and B's 12 times 100 for the same time, instead of 100 for 12 months, it is manifest, that their stocks would have been the same that their products are now; and therefore, agreeably to case 1st, the statements would have been the same that they are now. But it is plain, that 18 times 200*L.* will gain the same in 1 month, that 200*L.* will in 18 months, and that 12 times 100*L.* will gain the same in 1 month, that 100*L.* will in 12 months. Whence it is manifest, that the sum of the products is to the whole gain or loss, as each man's product is to his share of the gain or loss.

Pupil. I should like now to see the last question in fellowship in the A. T's Assistant solved; for the solution of it is entirely out of my reach.

Tutor. It is solved thus: 456+431+375=1262, and 1262÷2=631=the whole gain. The reason that the 1262 must be divided by 2 to get the whole gain, is manifest, for A. B. and C's gains are each of them mentioned twice in the foregoing numbers. Now, since the sum of C. and B's shares is 431, C. and A's 375, A. and B's 456, if we take each of these from the whole gain, we shall have each man's share of the gain. To find the price of B's cloth, we say, as A's gain is to the product of his stock by his time, so is B's gain to the product of his stock by his time; that is, as 200..600::256..768, the product of B's stock by his time; and since B's time is 8 months, 168÷8=96*L.*=B's stock, or the price of his cloth; and 96*L.*÷160=12*s.*=the price per yard. Again, to find the price of C's wheat, we say, as 200..600::175..525; and 525÷7=75*L.* and 75*L.*÷240=6*s.* 3*d.*=the price per bushel.

EXCHANGE.

Pupil. I can work some of the examples in exchange by the help of the tables; but I understand but very little about them; the reason of which is, perhaps, because I do not understand fully the meaning of exchange, and of the terms par and agio.

Tutor. The assistant tells the meaning of exchange; as also of these terms; but perhaps I can simplify them a little; but I shall first make some observations respecting money.

There are two kinds of money, viz. real and imaginary. Real money is a piece of metal coined by the authority of the state, and current at a certain price, by virtue of the said authority, or of its own intrinsic value. For instance, an eagle, an half eagle, a dollar, a half dollar, a quarter dollar, a cent, or any other piece of money, is real money.

Imaginary money is a denomination, or name, used to express a sum of money of which there is no real species; that is, it is a name used to express a sum of money of which there is no piece of metal of the same name; for instance, a pound is an imaginary sum, for there is no piece of money that is called a pound. A shilling is also an imaginary sum in this country, for we have no piece of money that is called a shilling. The same may be said with respect to a penny and a farthing. A pound in England is also an imaginary sum, and so is a livre in France.

Real money is the same in all countries and states; but imaginary money is quite different in some countries and states, from what it is in some others. Wherefore, it is manifest, that there would be no such thing as exchanging of money, if there were no imaginary money in use; and if so, how easily the United States might make the 1st case in exchange become useless, by laying aside entirely the use of pounds, shillings, &c.

Par of exchange is the intrinsic value of the money in one country compared with that of another; as one pound sterling is equal to 33 shillings and 4 pence Pennsylvania currency. To make it more plain, suppose a merchant of Philadelphia in exchanging with a merchant of London, were to allow him 33 shillings and 4 pence Pennsylvania currency for each pound sterling; the exchange would then be at par.

Agio is a term used to signify the difference, in some countries, between bank and current money; for bank money is considered better than current money in some countries. The agio is always at so much per cent.; and therefore, it is manifest, that the agio must be added to 100, of any kind of current money, to make it equal to 100 of bank money of the same kind. Wherefore, the 13th question of foreign exchange in the A. T's Assistant, must be stated thus: As 104 florins..100 florins:: 4376 florins..4207 florins, 13 stivers, 13 pennings, the answer in florins; and since 6 florins make a pound Flemish, the 4207 florins must be divided by 6 to reduce them to pounds.

Again, to work the 19th question, we must say, as 1*L.* sterling..33*s.* 10*d.* Flemish::290*L.* 11*s.* 10*d.* sterling.. 491*L.* 11*s.* 8*d.* Flemish; and as 100*L.*..104*L.* 10*s*::491*L.* 11*s.* 8*d.*..513*L.* 14*s.* 1*d.* the answer.

THE RULES TO CASE 1ST EXPLAINED.

I will now say something respecting the rules; the first of which says,

> As dollars rate from state to state,
> Make other kinds proportionate.

This is very simple; for, for instance, if we have any sum Pennsylvania currency to reduce to New-York currency, since a dollar in Pennsylvania is 7*s.* 6*d.* and in New-York 8*s.* it is manifest, that we must say, as 7*s.* 6*d.* are to 8*s.* so is the given sum to its equal in York money.

To explain the second rule, I must explain the table, which may be done thus: To reduce New-England currency to Pennsylvania currency, the table says, add one 4th. Now, since there are 4 eighteen pences in a dollar in New-England, (a dollar there being 6*s.*) and 5 in a dollar in Pennsylvania, if we add one 4th of the 4 eighteen pences to themselves, it will make 5 eighteen pences; that is, it will reduce a dollar New-England currency to a dollar Pennsylvania currency; and since the rule holds good in a dollar, it will of course hold good in any other sum. Again, if we take one 5th of 5 eighteen pences from themselves, we shall have 4 eighteen pences left; that is, we shall reduce Pennsylvania currency to New-England currency, as the table directs. The rest of the rules in the table are founded upon similar principles; and therefore, they are not hard to be understood.

The rule to case 2nd says, the various operations, in the exchanging of monies, are performed by the single rule of three, or by practice; therefore, it needs no explanation; however, I may observe, that when the exchange is at so much per cent. if the answer, from the nature of the question, is required to be greater than the given number, the question may be wrought by either the rule of three or practice; but practice is to be preferred. But if the nature of the question requires the answer to be less than the given number, it can be wrought by the rule of three only.

F 2

Pupil. Exchange appears plain to me now, all but the 23d and 24th questions in the A. T.'s Assistant.

Tutor. The 23d question is wrought thus: As 400 reas..52*d*::1 mill-rea..130*d*.; and as 34*s*. 3*d*..1*L*::130*d*.. $75.\frac{125}{137}d$. the exact answer, which is something more than $75\frac{3}{4}d$.

In the 24th question, we first find the commission on the 1200 crowns, at $\frac{1}{2}$ per cent. to be 6 crowns; then we say, as 55*d*..1206*C*::56*d*.$1184\frac{13}{28}C$. inversely; and 1200*C*. $-1184\frac{13}{28}C.=15\frac{15}{28}C$. the answer, which is a little more than $15\frac{1}{2}C$.

VULGAR FRACTIONS.

Pupil. The assistant says, that a vulgar fraction is a part or parts of an integer, and is denoted thus: $\frac{1}{8}$, one eighth; $\frac{7}{8}$, seven eighths. The upper number is called the numerator, and shows the part, or parts, expressed by the fraction; the lower number is called the denominator, and denotes the number of such parts contained in a unit. This definition is so very dark, that I know but very little more about the meaning of a fraction now than I did before I saw it.

Tutor. It may be made more plain by supposing a single article of any kind, for instance, a yard of cloth, to be divided into any number of equal parts, we will say 8; and by supposing any number of those parts to be taken, we will say 5; for, in this case, the 8 becomes the lower number, or denominator, and shows the number of parts the yard is divided into; and the 5 becomes the upper number, or numerator, and shows the number of parts that are taken; and therefore, the fraction must be expressed thus, $\frac{5}{8}$, that is, five eighths, or five of the 8 parts which the yard was supposed to be divided into; and, since we have supposed $\frac{5}{8}$ of the yard to be taken, there must be $\frac{3}{8}$ left; for, since 5 and 3 make 8, $\frac{5}{8}$ and $\frac{3}{8}$ *must make* $\frac{8}{8}$, which is the whole.

To make it still more plain, let us suppose a shilling to be divided into 12 equal parts; then, since there are 12 pence in a shilling, each of those parts is a penny; and therefore, 1 penny is one twelfth of a shilling; 2 pence, two twelfths, &c. that is, 1 penny is $\frac{1}{12}$ of a shilling; 2 pence, $\frac{2}{12}$; 3 pence, $\frac{3}{12}$, &c.

Fractions are frequently considered abstractedly; that is, without being applied to any article; for, we can conceive a unit or 1 to be divided into any number of equal parts, as well as a yard or shilling; and therefore, any fraction may be called a fraction of a unit or 1; for instance, $\frac{5}{8}$ may be called $\frac{5}{8}$ of a unit or 1, as well as $\frac{5}{8}$ of a yard.

A proper fraction is always less than 1 or unity, and to be so, the numerator must of course be less than the denominator, for when they are equal, the fraction is equal to 1; for then the numerator expresses the same number of parts as the denominator, which is the whole unit. Wherefore, $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{7}{7}$, $\frac{8}{8}$, $\frac{9}{9}$, &c. are all equal to one another; for, from what has been said, each of them is equal to 1. Whence, if the numerator of a fraction be greater than the denominator, the fraction is then greater than 1; and therefore, properly speaking, it is not a fraction; but, being expressed in a fractional form, it is called an improper fraction.

Pupil. I think I understand the meaning of fractions now tolerably well; but I do not know why they are called vulgar fractions.

Tutor. The word vulgar signifies common; therefore, vulgar fractions are common fractions; and are so called to distinguish them from decimal fractions; for their denominators are, as it were, common to all numbers; that is, the denominators of vulgar fractions may consist of any numbers whatever; but the denominators of decimal fractions are confined to tens as will be shown in its proper place.

REDUCTION OF VULGAR FRACTIONS.

CASE 1ST EXPLAINED.

Pupil. To reduce a fraction to its lowest terms, we are directed to find the common measure, and to divide both terms by it. I do not understand this; for I know not what is meant by the common measure, nor by reducing a fraction to its lowest terms.

Tutor. In the first place it should be known that a fraction may be represented an infinite number of different ways without altering the value of it: For instance. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, &c. are all equal to one another; for they are each of them equal to $\frac{1}{2}$, because the numerator of each is equal to $\frac{1}{2}$ its denominator; and each of them may be deduced to $\frac{1}{2}$, (which is its lowest terms) by dividing both terms by the greatest number that will divide them both without remainder; and this number is called the common measure.

THE RULES EXPLAINED.

To show that the common measure may be found by working by the rule, let us take the following example, viz. Reduce $\frac{24}{60}$ to its lowest terms.

24)60(2   12)$\frac{24}{60}=\frac{2}{5}$ lowest terms.
48
—
Common measure, 12)24(2
24

Now it is plain to be seen, that the common measure, in this example, cannot be greater than 24, for it must divide the 24 without remainder; and since 24 will divide itself without remainder, if it would divide the 60 without remainder, it would be the common measure; *but we find,* by dividing it into the 60, that it leaves *a remainder of 12;* and therefore, since the 60 consists of

24's and 12 over, the greatest number that will divide the 24 and 60 without remainder, must also divide the 24 and 12 without remainder; wherefore, the common measure cannot be greater than 12. But 12 will divide itself and the 24 without remainder; and therefore, it must be the common measure. Whence the rule is plain.

It may now be observed, that it is plain, from the above example, that it does not alter the value of a fraction to divide both terms by the same number; for $\frac{2}{5}$ are equal to $\frac{24}{60}$, and the $\frac{2}{5}$ were produced by dividing both terms of the $\frac{24}{60}$ by 12.

Pupil. It is plain to me, that dividing both terms of a fraction by the same number does not alter its value, provided $\frac{2}{5}$ and $\frac{24}{60}$ are equal to each other; but I cannot see that they are.

Tutor. There has been enough said to show that they are equal; however, it may be proved, thus: Since in one of the fractions the unit is divided into 5 equal parts, and into 60 in the other, each of the parts expressed by the 60 must be 12 times less than one of those expressed by the 5; and therefore, the 2 in the $\frac{2}{5}$ must be equal to the 24 of the parts expressed by the 60; which proves the fractions to be equal.

Now, since it does not alter the value of a fraction to divide both terms by the same number, it is manifest, that it cannot alter the value to multiply both terms by the same number.

In reducing a fraction to its lowest terms by the second rule, we are at liberty to use any numbers whatever that will divide both terms without remainder; and when we get the fraction so that no number will divide both terms without remainder, it is in its lowest terms. The $\frac{24}{60}$ may be reduced by this rule thus:

$$\overset{6)}{\frac{24}{60}} = \overset{2)}{\frac{4}{10}} = \frac{2}{5}$$

THE RULE TO CASE 2ND EXPLAINED.

To explain the rule to case 2nd. let us reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{7}{8}$ to a common denominator.

$2\times4\times3\times8=192$ ⎫
$3\times3\times3\times8=216$ ⎬ Numerators.
$1\times3\times4\times8=\ 96$ ⎪
$7\times3\times4\times3=252$ ⎭
$3\times4\times3\times8=288$ Common denominator.

By placing each of these new numerators over the common denominator, we have $\frac{192}{288}$, $\frac{216}{288}$, $\frac{96}{288}$, and $\frac{252}{288}$, which are equal to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{7}{8}$; that is, $\frac{192}{288}=\frac{2}{3}$, $\frac{216}{288}=\frac{3}{4}$, &c. which may easily be proved; that is, it may easily be proved, that, reducing fractions to a common denominator by the rule, does not alter their value; for it has been shown, that multiplying both terms of a fraction by the same number does not alter its value, and if so, multiplying both terms by the same numbers cannot alter its value. But, by examining the operation, we shall find that each numerator and its denominator are multiplied by the same numbers; for the numerator 2, is multiplied by 4, 3, and 8; and its denominator 3, is also multiplied by 4, 3, and 8; therefore, $\frac{192}{288}$ is equal to $\frac{2}{3}$. The same may be shown respecting the rest, and therefore, the rule is plain.

There is another method to reduce fractions to a common denominator; which is as follows:

If any two, or more of the denominators can be divided by any number without remainder, divide them by it; then divide the quotients, and the undivided denominators in like manner, until they all become so that no number will divide two of them without remainder. Multiply the divisors, quotients, and the undivided numbers continually for a common multiple. Then, divide the multiple by each of the denominators, and multiply these quotients severally by the correspondent numerators for the numerators required; under which, place the multiple, for the least possible common denominator.

Let us now take the same fractions that we had before, and reduce them to a common denominator by this method.

$$4)\frac{2}{3},\ \frac{3}{4},\ \frac{1}{3},\ \frac{7}{8} \qquad 4\times3\times2=24 \text{ Common multiple.}$$

$$3)3\ \ 1\ \ 3\ \ 2$$

$$1\,.\,1\,.\,1\,.\,2 \qquad 24\div3,4,3,\text{and } 8=8,\ 6,\ 8 \text{ and } 3 \Big\} \text{ Multiply.}$$

$$2\ \ 3\ \ 1\ \ \ 7$$

$$16\ \ 18\ \ 8\ \ \ 21 \text{ numerators}$$

The common multiple is the least possible common denominator; and therefore, by placing each of the new numerators over it, we have $\frac{16}{24}$, $\frac{18}{24}$, $\frac{8}{24}$, and $\frac{21}{24}$, which are equal to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{7}{8}$; for both terms of each fraction are multiplied by the same number; which may be shown thus: The numerators 2, 3, 1, and 7 are multiplied, we see, by 8, 6, 8, and 3; and these numbers were produced by dividing the 24 by the denominators 3, 4, 3, and 8; and therefore, if we multiply each of the denominators by the same one of those numbers that we multiplied its numerator by, it will produce 24. Whence it is plain.

THE 3D AND 4TH CASES EXPLAINED.

The 4th case being the reverse of the 3d. by reducing an example, for instance $14\frac{5}{8}$, to an improper fraction, and back again to a mixed number, we may explain both.

Since the $\frac{5}{8}$ denote a unite divided into 8 equal parts, and 5 of them taken, each one of the 14 contains 8 such parts; and therefore, by multiplying the 14 by 8, and adding in the 5, as the rule directs, we get $\frac{117}{8}$, the number of eighths in $14\frac{5}{8}$.

Again, since the $\frac{117}{8}$ were produced by multiplying the whole number of the $14\frac{5}{8}$ by 8 adding in the 5, it is manifest, that if we divide the 117 by the 8, it will reduce them back to $14\frac{5}{8}$; that is, it will reduce the improper fraction $\frac{117}{8}$ to a mixed number agreeably to the rule.

THE RULE TO CASE 5TH EXPLAINED.

To reduce a compound fraction to a single one, or, in other words, to reduce the fraction of a fraction to the fraction of a unit, the rule directs us to multiply all the numerators together for a new numerator and all the denominators for a new denominator. To explain this let us reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a single fraction, thus: $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$; that is, $\frac{6}{12}$ of a unit; wherefore, $\frac{2}{3}$ of $\frac{3}{4}$ of a unit, are $\frac{6}{12}$ of a unit according to the rule; and that it is so may be shown thus: The $\frac{3}{4}$ denote a unit divided into 4 equal parts and 3 of them taken, and, seeing the $\frac{2}{3}$ are $\frac{2}{3}$ of the $\frac{3}{4}$, they denote each of the 3 in the $\frac{3}{4}$ divided into 3 equal parts and 2 of each 3 taken. But by dividing each of the 3 into 3 equal parts, the $\frac{3}{4}$ become $\frac{9}{12}$, and by taking 2 of each 3 in the 9 we get $\frac{6}{12}$ as above; and $\frac{6}{12}=\frac{1}{2}$. Whence $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$.

To simplify it still more, $\frac{2}{3}$ of $\frac{3}{4}$ may be shown to be $\frac{1}{2}$, thus: $\frac{1}{3}$ of $\frac{3}{4}$ is manifestly $\frac{1}{4}$; and if so, $\frac{2}{3}$ of $\frac{3}{4}$ must be $\frac{2}{4}$, which are $\frac{1}{2}$.

Examples in this case may frequently be very much contracted; to show which let us reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{1}{6}$ of $\frac{5}{7}$ to a single fraction. To do this by the rule the work will stand thus; $\frac{3}{4}\times\frac{2}{3}\times\frac{3}{5}\times\frac{1}{6}\times\frac{5}{7}=\frac{90}{2520}$, and $10)\frac{90}{2520}=9)\frac{9}{252}=\frac{1}{28}$. By contraction the work will stand thus:

$$\underset{2}{\frac{3}{4}}\times\frac{2}{3}\times\frac{3}{5}\times\underset{2}{\frac{1}{6}}\times\frac{5}{7}=\frac{1}{28}.$$

This must be plain, for it is manifest, that like numerators and denominators may be cancelled; for if we do not cancel them, we shall multiply both terms of the fraction by the same numbers which will not alter the value; and therefore, it will not alter the value to cancel them, and omit multiplying by them. The rest of the work must be plain from what has been said respecting cancelling.

Contraction in this case not only shortens the multiplication, but saves us the trouble of reducing the fraction to its lowest terms; for, if we cancel the numbers as much as they can be, we shall never fail of getting the fraction in its lowest terms.

THE 6TH AND 7TH CASES EXPLAINED.

The 7th case is the reverse of the 6th; and therefore, we may explain them both, by reducing a fraction agreeably to the 6th, and back again to its first state, agreeably to the 7th. Let us therefore reduce $\frac{5}{7}$ of a penny to the fraction of a pound. Here we are to find what part of a pound $\frac{5}{7}$ of a penny are; and since a penny is $\frac{1}{12}$ of a shilling, and a shilling $\frac{1}{20}$ of a pound, $\frac{5}{7}$ of a penny must of course be, $\frac{5}{7}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound; which must be reduced to a single fraction, thus:

$$\frac{5}{7}\times\frac{1}{12}\times\frac{1}{\underset{4}{20}}=\frac{1}{336} \text{ of a pound;}$$

That is, $\frac{1}{336}$ of a pound is equal to $\frac{5}{7}$ of a penny. If the operation had not been contracted, it would have produced $\frac{5}{1680}$ of a pound, instead of $\frac{1}{366}$; therefore, let us reduce $\frac{5}{1680}$ of a pound to the fraction of a penny. Here we are to tell what part, or parts of a penny $\frac{5}{1680}$ of a pound are; and, since pounds multiplied by 20 and 12 give pence, parts of a pound multiplied by 20 and 12, must of course give parts of a penny; and therefore, if we multiply the numerator 5 by 20 and 12, and set the product over the denominator 1680, we shall have $\frac{1200}{1680}$ of a penny, which are equal to $\frac{5}{7}$ of a penny.

We may observe here, that, since the 7th case is the reverse of the 6th, and since the $\frac{5}{1680}$ were produced by multiplying the denominator of the $\frac{5}{7}$ by 12 and 20, one would think that the 1680 should be divided by 20 and 12 to reduce the $\frac{5}{1680}$ of a pound back to $\frac{5}{7}$ of a penny; and this method will answer in this case and some others, and will bring the fraction in its lowest terms; but in some cases the denominator will not divide without remainder, in which it is manifest, that this method will not apply; and therefore, we use the other because it will apply in all cases.

From what has been said, it appears, that, to divide the denominator of a fraction by any number has the same effect as to multiply its numerator by the same number; that is, if we have any fraction to multiply by any number, for instance $\frac{2}{9}$ by 3, we may either multiply the numerator by the number, or divide the denominator; for if we multiply the numerator 2 by the 3, the fraction will become $\frac{6}{9}$, which are manifestly 3 times as great as $\frac{2}{9}$. Again, if we divide the denominator 9 by the 3, the fraction will become $\frac{2}{3}$; which are manifestly equal to $\frac{6}{9}$; and therefore, the $\frac{2}{3}$ are also 3 times as great as the $\frac{2}{9}$. Whence, by dividing the denominator of a fraction by a number, we multiply the fraction by that number; and this is a shorter way than to multiply the numerator; but it will not always apply on account of remainders.

THE RULE TO CASE 8TH EXPLAINED.

To reduce the value or quantity of a fraction, to the known parts of an integer, the rule says, multiply the numerator by the common parts of the integer, and divide by the denominator. To explain this, let us reduce $\frac{7}{8}$ of a pound to their proper value.

Here we are to find how much money $\frac{7}{8}$ of a pound are; and, since they are less than a pound their value must of course be expressed in shillings, or shillings and pence; and, since the $\frac{7}{8}$ are parts of a pound, the 7 must of course be multiplied by 20 to reduce them to parts of a shilling; and, seeing it takes 8 of those parts to make a shilling, they must consequently be divided by 8 to reduce them to shillings; and the remainder, if any, (being parts of a shillings,) must be multiplied by 12 to reduce it to parts of a penny, which must be divided by 8 as before to reduce them to pence. Wherefore, the work will stand, thus: $\frac{7}{8}$ of $\frac{20}{1} = \frac{140}{8} =$ 17*s*. 6*d*; or by contraction, $\frac{7}{3}$ of $\frac{20}{1} = \frac{140}{3} =$ 17*s*. 6*d*; or, by contraction, $\frac{7}{\not{8}_{2}}$ of $\frac{\not{20}^{5}}{1} = \frac{35}{2} =$ 17*s*. 6*d*.

THE RULE TO CASE 9TH EXPLAINED.

The 9th case is the reverse of the 8th; and therefore, we will reduce the result of the above operation back again; that is, we will reduce the 17*s*. 6*d*. to the fraction of a pound.

Pence being the lowest term mentioned in this example, we must reduce the 17*s*. 6*d*. to pence for a numerator, agreeably to the rule; and a pound being the integer, we must reduce it also to pence for a denominator; and seeing this denominator will be the parts of a pound, and' the numerator the same kind of parts, it is manifest, that the fraction will be the fraction of a pound; and it will be $\frac{210}{240}$; for there are 210 pence in 17*s.* 6*d.* and 240 in a pound; and $30)\frac{210}{240}=\frac{7}{8}$ of a pound.

I need not take the pains to explain the last three cases of reduction; for the first two are very simple; and they are all so useless, that they are left out of most systems of arithmetic.

ADDITION OF VULGAR FRACTIONS.

Pupil. I have heard many say, that addition is the hardest rule in vulgar fractions; and I am of the same opinion; for it seems very hard to me; and the reason of it is, because most of the examples require different methods of working.

Tutor. There are many, it is true, who find a great deal of difficulty in working this rule; but the reason of it is, because they have but a very imperfect knowledge of reduction of fractions; which is, as it were, the foundation of addition; for there is scarcely an example but that requires some preparation before the fractions can be added, and this preparation is performed by reduction.

Pupil. Please to work a few examples, and illustrate them a little, by showing how to prepare them by reduction.

Tutor. Let us then, in the first place, add $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$ together.

Now, since the value of a fraction is expressed by its numerator, in adding fractions, we must of course add the numerators together; but it will not do to add the numerators 7, 11, and 4 together; for they are different *kinds of parts*, on account of their denominators being different: therefore, before we can add the $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$ together, we must reduce them to a common denominator by case 2nd, of reduction; which being done the shortest way, the fractions will be $\frac{126}{180}$, $\frac{165}{180}$, and $\frac{80}{180}$; and now, the denominators being all equal, the numerators are the same kind of parts; and therefore, by adding them together, we shall get $\frac{371}{180}$, the sum of the fractions $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$; and, by dividing the numerator of the $\frac{371}{180}$ by the denominator, we find that the $\frac{371}{180}$ are equal to 2 $\frac{11}{180}$, the answer.

Let us now add $\frac{2}{4}$ of $\frac{7}{8}$, and $\frac{4}{6}$ of $\frac{19}{20}$ together.

Here, the $\frac{2}{4}$ of $\frac{7}{8}$, and of $\frac{4}{6}$ of $\frac{19}{20}$, being compound fractions, must in the first place be reduced to single ones by case 5th of reduction, thus:

$$\tfrac{2}{4}\times\tfrac{7}{8}=\tfrac{7}{16}, \text{ and } \tfrac{4}{6}\times\tfrac{15}{20}=\tfrac{19}{30}.$$

Now, the $\frac{7}{16}$ and $\frac{49}{30}$ being reduced to a common denominator, and the numerators added together, we shall have the answer.

Let us next add $12\frac{1}{2}$, $3\frac{2}{3}$, and $4\frac{3}{4}$ together.

Here, we first reduce the $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator, and add them together; then we add what they make to the sum of the 12, 3, and 4, which gives the answer.

We will now show how to add $\frac{4}{7}$ of a ton to $\frac{9}{10}$ of C. wt.

Here we first reduce the $\frac{4}{7}$ of a ton to their proper value, by case 8th of reduction: then we reduce the $\frac{9}{10}$ of C. wt. to their proper value, by the same case; and these two values, added together, give the answer.

SUBTRACTION OF VULGAR FRACTIONS.

Pupil. The rule to subtraction says, prepare the fractions as in addition, &c. I suppose it means by that, that *we are to prepare them* by reduction.

Tutor. Yes; that is what it means; and since that is the case, subtraction must be plain to such as understand addition; however, I may perhaps cast a little more light upon it, by working an example or two.

EXAMPLE. From $\frac{5}{8}$ take $\frac{3}{5}$ of $\frac{5}{8}$.

In this example we first reduce the compound fraction $\frac{3}{5}$ of $\frac{5}{8}$ to a single one, thus: $\frac{3}{5}\times\frac{5}{8}=\frac{3}{8}$. Then we reduce the $\frac{3}{8}$ and $\frac{5}{8}$ to a common denominator, and subtract the less numerator from the greater, which gives the difference or answer.

Let us now, from $96\frac{1}{3}$ take $14\frac{3}{7}$.

Here, by reducing the $\frac{1}{3}$ and $\frac{3}{7}$ to a common denominator, the $96\frac{1}{3}$ and $14\frac{3}{7}$ become $96\frac{7}{21}$, and $14\frac{9}{21}$. Now, to subtract, we set the $14\frac{9}{21}$ under the $96\frac{7}{21}$, and say, 9 from 7 we cannot, but 9 from 21 and 11 are left, and 7 make 18; that is $\frac{18}{21}$ or $\frac{6}{7}$. The reason that we borrow 21 is manifest; for, since there are 21 such parts in a unit as the 9 expresses, by borrowing 21 we borrow a unit or 1; and therefore, we must carry one to the 4, and then subtract as usual.

MULTIPLICATION OF VULGAR FRACTIONS.

Pupil. It appears, that multiplying by a fraction makes the number less instead of greater; for I have observed that the product is always less than the multiplicand; which is a mystery to me, for I thought that multiplication always increased a number.

Tutor. It may be shown, without applying to fractions, that multiplication does not always increase the number, for, if we multiply by 1, we know it does not alter the number; but, if we multiply by any number greater than 1, it then increases it; and therefore, if we multiply by any number less than 1, it must decrease it, in the same proportion as the multiplier is less than 1;

if so, dividing by a fraction must increase the number in the same proportion as the divisor is less than 1: and therefore, $\frac{1}{2}$ divided by $\frac{1}{2}$ makes 1; or, in other words, $\frac{1}{2}$ will go into $\frac{1}{2}$, 1 time. To make it more plain, it may be shown that $\frac{1}{2}$ divided by $\frac{1}{2}$ makes 1, thus: When the divisor and dividend are equal, we know that the divisor will go into the dividend just 1 time; for instance, 8 will go into 8, 1 time; 6 into 6, 1 time, &c. and therefore, $\frac{1}{2}$ will go into $\frac{1}{2}$, 1 time.

THE RULE EXPLAINED.

Since division is the reverse of multiplication, and since, in multiplying fractions, we multiply the numerators together for the numerator of the product, and the denominators for the denominator, if we divide the numerator of the dividend by the numerator of the divisor, it must give the numerator of the quotient, and the denominator of the dividend by the denominator of the divisor, the denominator of the quotient; but this can but seldom be done on account of remainders; and therefore, we have to work as the rule directs, which is the same thing in effect; for it has been shown, that it has the same effect on a fraction to multiply its numerator by any number, as to divide its denominator by the same; and if so, to multiply the denominator by any number must have the same effect, as to divide the numerator by the same; and therefore, multiplying the numerator of the dividend by the denominator of the divisor, as the rule directs, must have the same effect, as to divide the denominator of the dividend by the same; and to multiply the denominator of the dividend by the numerator of the divisor, must have the same effect, as to divide the numerator of the dividend by the same. Wherefore, working by the rule must give the quotient.

Now, since, in dividing fractions by the rule, we multiply the numerator of the dividend by the denominator of the divisor, &c. if we invert the divisor, (that is, set the denominator in the place of the numerator, and the numerator in the place of the denominator) and then work as in multiplication, we shall multiply them in the same manner as if we had worked by the rule; and therefore, the handiest way to divide fractions is to invert the divisor, and proceed as in multiplication, remembering to contract and cancel as much as possible.

THE SINGLE RULE OF THREE IN VULGAR FRACTIONS.

Pupil. The rules, in both direct and inverse proportion, being the same here as in whole numbers, need no explanation; however, some of the examples are so hard, that I cannot work them, to wit, the 15th, 19th, and 21st, in direct proportion in the A. T.'s Assistant.

Tutor. Those three examples puzzle many; but, notwithstanding that is the case, they appear very simple to such as understand clearly the meaning of them. Before I work them, I may observe, that, after an example is stated, the best way of working it, is to invert the dividing term, as the second rule directs, and multiply the three terms together for the fractional answer. The reason that this method will bring the answer, must be manifest, from what has been said in division.

The 15th example says, if $\frac{1}{3}$ ℔ less by $\frac{1}{6}$, cost $13\frac{1}{5}d.$ what cost 14 ℔ less by $\frac{1}{5}$ of 2 ℔?

This must be wrought thus: $\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$; $13\frac{1}{5}d=\frac{66}{5}$; and $\frac{1}{5}$ of $\frac{2}{1}=\frac{2}{5}$; and $\frac{14}{1}-\frac{2}{5}=\frac{68}{5}$; and therefore, its $\frac{1}{6}$ ℔ : $\frac{66}{5}d.$ :: $\frac{68}{5}$ ℔ to the answer; which is found thus: $\frac{6}{1}\times\frac{66}{5}\times\frac{68}{5}=\frac{26928}{25}=4L.\ 9s.\ \frac{93}{25}d.$

The 19th example says, if $3\frac{1}{2}$ times $3\frac{1}{2}$ ℔ cost $1\frac{1}{2}$ time $1\frac{1}{2}L.$ what is the value of $\frac{1}{2}$ of $\frac{1}{3}$ of $12\frac{1}{4}$ ℔?

Here, the first term is the $3\frac{1}{2}$ times $3\frac{1}{2}$ ℔. the second, the $1\frac{1}{2}$ time $1\frac{1}{2}L.$ &c. but we need not take the pains to reduce these fractions to single ones, and state the question; for, after reducing the mixed numbers to improper fractions, we may invert the first two fractions, and multiply them all together for the fractional answer, thus :

$$\frac{2}{7}\times\frac{2}{7}\times\frac{3}{2}\times\frac{3}{2}\times\frac{1}{2}\times\frac{1}{3}\times\frac{7}{\frac{49}{4}}=\frac{3}{8}=7s.\ 6d.\ \text{the answer.}$$

The 21st example says, a person having $\frac{4}{5}$ of a ship, sells $\frac{2}{3}$ of his share for 319*L.* what is the proportional worth of the whole vessel ?

Since the person possessed $\frac{4}{5}$ of a ship, and sold $\frac{2}{3}$ of his share, he must have sold $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$; and therefore, as $\frac{8}{15} : \frac{319}{1}L. :: \frac{1}{1}$ to the answer, that is, as $\frac{8}{15}$ of a ship are to 319*L.* so is a whole ship to its value ; which is found thus : $\frac{15}{8}\times\frac{319}{1}\times\frac{1}{1}=\frac{4785}{8}=598L.\ 2s.\ 6d.$ the answer.

THE DOUBLE RULE OF THREE IN VULGAR FRACTIONS.

Pupil. There seems to be nothing in this rule that needs an explanation ; nevertheless, many of the examples appear very hard to me.

Tutor. They appear hard to many ; but the reason of it is, because they have not a perfect knowledge of the foregoing rules. If, in working an example, we first reduce the mixed numbers to improper fractions ; then state the question as directed in whole numbers, and reduce the correspondent terms to the same denomination, or to a fraction of the same name ; then transpose the inverse terms, if the question belong to inverse proportion ; and then invert the dividing terms, or the two terms which stand in the first place, and multiply them all together, we cannot fail of getting the fractional answer.

DECIMAL FRACTIONS.

Pupil. The assistant says, that a decimal fraction is a part or parts of a unit, denoted by a point prefixed to a figure, thus: .4, .45, .456; the first figure after the point, denotes so many tenths of a unit; the second, so many hundredths of a unit, or tenths of one tenth; which are equal to, and read as, $\frac{4}{10}$, $\frac{45}{100}$, $\frac{456}{1000}$.

I think I understand from this definition, something of the meaning of a decimal fraction; but I should like to see a little more light thrown upon the subject; however, in the first place, please to tell me the reason that fractions of this kind are called decimal fractions.

Tutor. The word decimal signifies numbered by tens, and these fractions are numbered by tens; and for that reason, they are called decimal fractions.

To illustrate decimals a little, it may be observed, that decimals decrease, counting from left to right, in the same proportion that whole numbers increase, counting from the right to left; that is, in a ten fold proportion; for, as it takes ten units to make one ten, ten tens to make one hundred, &c. so it takes ten tenths to make one unit, ten hundredths to make one tenth, ten thousandths to make one hundredth, &c. And, since the first figure after the point, is so many tenths; the second, so many hundredths, &c .4 are 4 tenths; .45, 4 tenths and 5 hundredths, or 45 hundredths; .456, 4 tenths, 5 hundredths and 6 thousandths, or 456 thousandths, &c. and therefore, decimals may be expressed in the form of vulgar fractions, thus: $.4=\frac{4}{10}$; $.45=\frac{45}{100}$; $.456=\frac{456}{1000}$; $.4567=\frac{4567}{10000}$, &c. and from this, it is manifest, that the denominator of a decimal, is always 1 with just as many ciphers annexed to it as there are figures in the decimal; for, if we examine the above fractions, we shall find, that each denominator contains just as many ciphers as there are figures in the decimal. It does not alter the value of a decimal to annex ciphers to it; for, if .5. .50, and .500

are all equal to one another; because, they are equal to $\frac{5}{10}$, $\frac{50}{100}$, and $\frac{500}{1000}$, which are known to be equal to one another, from what has been said in vulgar fractions; but, ciphers prefixed to decimals, decrease their value in a ten fold proportion; for, .5 are ten times as great as .05; .05, ten times as great as .005, &c.; because, the first 5 are $\frac{5}{10}$; the second 5, being removed into the place of hundredths, are $\frac{5}{100}$; and the third 5, being in the place of thousandths, are $\frac{5}{1000}$.

ADDITION AND SUBTRACTION OF DECIMALS.

Pupil. Addition and Subtraction of Decimals are very simple; for they are wrought just like whole numbers, only, some care is necessary in placing the points; and that is not difficult, for, it is manifest, that they must be placed under one another; because, by that means the tenths come under the tenths, the hundredths under the hundredths, &c. which is the way that they ought to be placed of course; and, by placing the units, or the first figure at the right of the whole numbers, under one another, the points will come under one another, and all the figures will come right.

Tutor. I perceive that addition and subtraction are well understood; therefore, we will leave them, and proceed to multiplication.

MULTIPLICATION OF DECIMALS.

Pupil. It is very plain to me, from what has been said, that decimals may be multiplied together as whole numbers; but why there should be just as many decimal places, or figures, pointed off in the product, as there are both factors, is what I cannot see.

THE RULE EXPLAINED.

Tutor. That the product must have just as many decimal places as there are in both factors, may easily be shown; for, since the denominator of a decimal is always 1 with a certain number of ciphers annexed, it is manifest, that the product of two denominators will have just as many ciphers as there are in both denominators. But it has already been shown, that the number of places in a decimal is always equal to the number of ciphers in its denominator; and if so, the number of places in the product of two decimals, must be equal to the number of ciphers in the product of their denominators; but the number of ciphers in the product of two denominators is equal to the number of ciphers in both denominators, and the number of ciphers in both denominators is equal to the number of places in both decimals; and therefore, the number of places in the product of two decimals, must be equal to the number of places in both decimals, or both factors. To make it more plain, let us multiply .31 by .12. Now. 31×.12=372; but, since there are four decimal places in the two factors .31 and .12, the product is .0372; for, .31 and .12 are equal to $\frac{31}{100}\times\frac{12}{100}=$ and $\frac{31}{100}\times\frac{12}{100}=\frac{372}{10000}$; and, since the number of places in a decimal is equal to the number of ciphers in its denominator, it is manifest, that the $\frac{372}{10000}$, expressed decimally, must be .0372; for the denominator has four ciphers.

CONTRACTED MULTIPLICATION OF DECIMALS EXPLAINED.

To illustrate the method of contracting multiplication of decimals, let us first multiply two whole numbers together, (for instance, 654 by 123 ;) first by the common method ; then by the method of inversion, thus :

654	654	
123	321	Multiplier inverted.
1962	65400	
1308	13080	
654	1962	
80442	80442	

Here, we see, by inverting the multiplier, and setting the units figure under the units of the multiplicand, and multiplying as the note directs, we get the true product ; and, by examining the operation, it will appear plain ; for, the 1 in the multiplier is a hundred ; the 2, tens ; and the 3, units ; and therefore, in multiplying by the 1, the first figure should be set in the hundreds place ; by the 2 in the tens place ; and by the 3 in the units place : and we see they come just so.

Let us now multiply 6.54 by 1.23, by the method of contraction ; first, so as to retain all the decimal places ; secondly, so as to retain them all but one ; and thirdly, so as to retain them all but two ; thus :

6.5400	6.540	6.54
32 1	3.21	32.1
65400	6540	654
13080	1308	131
1962	196	20
8.0442	8.044	8.05

The number of decimal places in both factors being four, to retain all the decimal places in the product, the units place of the multiplier, agreeably to the note, must be set under the fourth place from the point; and, by setting it so, we see that the figures are brought into the same position, or situation, as they are in the inverted operation of whole numbers; and they bring the same figures in the product; and therefore, it must be right. Now, since by placing the units figure under the fourth place from the point, we get four places in the product; if we place it under the third place, we shall of course get three places in the product; and if under the second, two, &c. as in the second and third operations; and therefore, the reason that the units figure must be placed as the note directs, is manifest. The reason we carry one for the first five, in multiplying by the figures that stand at the right of the multiplying figure may easily be perceived; for, if we carry the tens only, the excess above the tens will be lost; but, by rejecting this excess when it is less than five, and by carrying one, or calling it ten, when it is five, or above, we gain in most cases about as much as we lose; and therefore, we get the true product nearly.

DIVISION OF DECIMALS.

THE RULE EXPLAINED.

Pupil. I think, from what has been said in multiplication, that I can show the reason myself, why the quotient must have just as many decimal places as the dividend has more than the divisor; for, since the dividend corresponds with the product in multiplication, and the divisor and quotient with the factors; and, since the product must have as many decimal places as both factors, the dividend must of course have as many as the divisor and quotient ; and therefore, the quotient must have as many as the dividend has more than the divisor.

CONTRACTED DIVISION OF DECIMALS EXPLAINED.

Tutor. That is a very good explanation, therefore, I will go on to contracted division ; but, since it is the reverse of contracted multiplication, it does not need much explanation ; however, to illustrate it a little, let us divide 165.6995001296 by 3.141592, so as to have four places of decimals in the quotient ; and prove the work by contracted multiplication, thus :

```
3.14159|2)165.6995001296(52.7438
          157.0796
          --------
     3.1415)86199
            62832
            -----
      3.141)23367
            21991
            -----
       3.14)1376
            1257
            ----
        3.1)119
             94
            ---
          3)25
            --
             0
```

3.141592
8347.25
1570796
62832
21991
1257
94
25
Proof, 165.6995

The first thing we have to do in this kind of division, is to consider how many figures the quotient must have, to serve the present purpose; and this is not hard to be ascertained; for the number of decimal figures is always given, and those of the whole number may easily be found, by conceiving the whole number only of the divisor to be divided into the whole number of the dividend. By this method we find that the above example must have six quotient figures; and by taking the same number at the left of the divisor; and dividing the same number of times, we of course get the number of quotient figures required. Wherefore, by examining the operation and the proof, the whole of the work will doubtless appear plain, to such as understand contracted multiplication; for each quotient figure is multiplied into the same number as its corresponding figure in the multiplier.

REDUCTION OF DECIMALS.

CASE 1ST EXPLAINED.

Pupil. What is the reason, that, to reduce a vulgar fraction to a decimal, we must annex ciphers to the numerator, and divide by the denominator?

Tutor. It is founded on direct proportion; for, if we reduce $\frac{3}{4}$ to a decimal by the rule, the work will stand thus:

$$4)3.00 \quad ; \quad .75$$

which is the same as this; as 4..3::100

$$\begin{array}{r} 100 \\ \hline 4)300 \\ \hline .75 \end{array}$$

That this stating reduces the fraction to a decimal, is manifest; for the denominator of the given fraction, is of course to its numerator as the denominator of the decimal is to its numerator, or the decimal required.

CASE 2ND EXPLAINED.

To explain case 2nd let us reduce 10*s.* 9*d.* $\frac{1}{4}$ to the decimal of a pound; that is, let us find what decimal part or parts of a pound, 10*s.* 9*d.* $\frac{1}{4}$ are; to do which, by the first rule, we first find by case 9th in reduction of vulgar fractions, that the 10*s.* 9*d.* $\frac{1}{4}$ are $\frac{517}{960}$ of a pound; and then, by reducing the $\frac{517}{960}$ to a decimal by the foregoing case, we find, that the 10*s.* 9*d.* $\frac{1}{4}$ are .5385416+of a pound. Whence, the first rule is plain.

We will now reduce the same quantity to a decimal by the second rule, thus:

4	1 *qr.*
12	9.25*d.*
20	10.770833*s.*
	.5385416+of a pound.

Here, in the first place, we divide the 4 into the 1 farthing, conceiving ciphers to be annexed to the 1, which reduces the 1 farthing to the decimal of a penny. Then, since 12 pence make a shilling, 12 decimal parts of a penny, must make 1 decimal part of a shilling; therefore, dividing the 9.25*d.* by 12, must reduce them to the decimal of a shilling; and, for a similar reason, dividing the 10.770833*s.* by 20, must reduce them to the decimal of a pound.

CASE 3D EXPLAINED.

To explain case 3d, let us reduce .87615 of a pound to their proper value.

Here we are to find the number of shillings, pence, &c. that there are in .87615 of a pound; which are found thus:

```
     .87615
         20
  ---------
  s.17.52300
         12
  ---------
   d.6.27600
          4
  ---------
 qr.1.10400   Answer 17s. 6d. 1.104 qr.
```

The .87615 being parts of a pound, multiplying by 20, reduces them to parts of a shilling; and, since ten tenths of a shilling make a shilling, it is clear, that the 17 are shillings; for each of them is equal to ten tenths of a shilling. Again, multiplying the .52300 by 12, reduces them to parts of a penny; and therefore, the 6 are pence, &c.

THE METHOD OF FINDING THE VALUE OF A DECIMAL BY INSPECTION, EXPLAINED.

Let us now reduce the .87615 to their proper value by inspection; to do which, we are directed to double the first figure after the point for shillings, adding one, if the *second be 5, or* upwards, &c. which may be illustrated, thus: The first figure after the point being tenths of a pound, 10 such parts as it expresses, make a pound, or 20 shillings; and therefore, each tenth must be 2 shillings; and, since 5 in the second place are equal to half a tenth, they must be 1 shilling; and again, since 5 in the second place make a shilling, it takes 50 of those parts which we call farthings, to make a shilling. But, 48 farthings make a shilling; and therefore, these parts must be 50, to be two too many, and 25, to be one too many; but they are better than $1\frac{1}{2}$ too many, when above 36, and better than $\frac{1}{2}$ when above 12. Wherefore, by deducting one from the farthings when they are above 12, and two when above 36, we get the true value nearly. Whence, the 8 in the above decimal, are 16*s.* and the 5 in the 7, 1, which makes 17; and the 7 being 2 above 5, the parts that become farthings, are 26; but, being above 12, we call them 25, or 6*d.* $\frac{1}{4}$; and therefore, the whole value is 17*s.* 6*d.* $\frac{1}{4}$.

It is not necessary for me to say any thing respecting the single and double rule of three in decimals; for they are wrought exactly as those rules in whole numbers; only regard is to be had to placing the point.

Pupil. Notwithstanding that is the case, I should like to see the last question in the double rule in the A. T.'s Assistant, explained: which says, 'a cellar which is 22.5 feet long, 17.3 wide and 10.25 deep, being dug in $2\frac{1}{2}$ days, by 6 men, working 12.3 hours a day; how many days, of 8.2 hours, should 9 men take to excavate one which measures 45 by 34.6 by 12.3 feet?

Tutor. I should work it thus :

Long.		Long.
22.5		45. direct.
		2
Wide.		Wide.
17.3		34.6 direct.
		2
Depth		Deep.
10.25	Days.	12.3 direct.
4.1	2.5	3
Men.		Men.
6		9 inverse.
39		6
Hours.		Hours.
12.3		8.2 inverse.
8.2		12.3
4.1		4.1 $6 \times 2 = 12$ days, the answer.

The beauty of contracting and cancelling, is abundantly to be seen in this example ; for, all the numbers are destroyed, except the 6 and 2 which are multiplied together for the answer : whereas, to work the question the common way, it takes about 278 figures.

INVOLUTION AND EVOLUTION.

Pupil. Involution ; or the raising of power, is very simple ; for it is performed by simple multipli cation.

Tutor. Notwithstanding it is so simple in its operation many scholars go over it without understanding the *meaning of it.*

Pupil. I think I understand it clearly.

Tutor. Well; since that is the case, tell me what is meant by a power?

Pupil. A power is the product arising from multiplying any given number into itself continually a certain number of times; for instance, 36 are the second power, or square of 6; for $6\times6=36$; and 216 are the third power, or cube of 6; because $6\times6\times6=216$. Again, the fourth power of 3, is 81; because $3\times3\times3\times3=81$. The second power, or square of 9, is also 81; because $9\times9=81$, and so on.

Tutor. What is the reason that the raising of powers is called involution.

Pupil. I find now that I do not understand this subject as well as I thought I did; for I do not know the reason of that.

Tutor. The raising of a number to any power, is called involving it to that power; for which reason, the raising of powers is called involution.

The number denoting the power is called the index, or the exponent of that power; for instance, the index of the second power, is 2; because the second power is denoted by 2; the index of the third power 3; because the third power is denoted by 3; the index of the fourth power 4, &c.

A number may be involved, or raised to a high power, with less work than to multiply it continually by itself; for, if two or more powers be multiplied together, their product is that power whose index is the sum of the indices, or indexes of the factors. For instance, if it be required to raise 5 to the sixth power, instead of multiplying by 5 continually, the work may be performed thus: $5\times5=25$, the 2d power of 5; and $25\times25=625$, the 4th power of 5; that is, the 2d power multiplied by the 2d power, gives the 4th power; because the sum of the indexes of the two second powers, is 4. Again, $625\times25=15625$, the 6th power of 5; that is, the 4th power multiplied by the 2d power, gives the 6th power; because 4 and 2 make 6.

Pupil. The assistant says, that the root of any number, or power, is such a number, as being multiplied into itself a certain number of times, will produce that number. Please to tell me the reason that this number is called the root.

Tutor. One of the meanings of the word root is, original or first cause; and, since a power originates from the number that we call the root, or is produced by it, that number is the original cause of the power; and for that reason, it is called the root of the power.

Evolution, or the extracting of a root, is the finding of such a number from a given number or power, as, being multiplied into itself a certain number of times, will produce the number or power from which it was extracted. For instance, the extracting of the 4th root of 81, is the finding of such a number, as being involuted to the 4th power, will equal 81; and this number is 3, and it is the fourth root of 81.

Pupil. I observe that the second power of a number is called the square of the number, and the third power the cube of it. Please to tell me the meaning of a square, and also of a cube.

Tutor. A square is a four sided figure, whose sides are all equal to one another, and whose corners are right angles. For instance, any thing in the shape of a chest-lid, door, window, &c. if it be just as wide as it is long, is a square; because the corners of such like things are right angles; but none of these things are squares, unless they are just as wide as they are long.

To illustrate the matter more fully, suppose a field of corn to be planted in straight rows both ways, and just as many hills in a row one way as the other; then it is a square field of corn; and, by multiplying the number of hills in a row one way by the number of hills in a row the other way; or, which is the same thing, by squaring the number of hills in one row, we shall get the whole number of hills in the field; which is the square of the number in one row; and the number in one row is the root of the whole number. Again, if a field be 40 perches long and 40 wide, it is a square field; and therefore, $40\times40=1600$, the number of square perches that the field contains; that is, a field that is 40 perches *square, contains* just 1600 square perches, or little squares, that are each just a perch long and a perch wide; and, since there are 160 such little squares or square perches in an acre, if we divide the 1600 by 160 it will give 10, the number of acres in the field.

A cube is a square solid body; that is, it is a solid body whose length, breadth, and thickness, or depth, are all equal. For instance, a box, block of wood, or any thing of the kind, whose length, breadth, and depth are all equal, is a cube. To make it more plain, suppose a block of wood to be 6 inches long, 6 inches wide, and 6 inches deep; then it is a cube, or cubic block; and, by multiplying the length, breadth, and depth together; or, which is the same thing, by cubing one of the sides, we shall get the number of cubic inches that the block contains; thus, $6\times6\times6=216$, which shows, that a block that is 6 inches every way, contains 216 cubic inches, or little cubes, each an inch every way. Whence, a cubic box, that is 6 inches long, 6 inches wide, and 6 inches deep in the inside, will hold just 216 little cubic blocks that are each an inch long, an inch wide, and an inch deep.

THE SQUARE ROOT.

Pupil. The extraction of the square root, (if I understand it right) is the finding of such a number from a given number, as, being multiplied by itself, will produce the given number.

Tutor. Yes, that is the sense of it; and to illustrate it, it must in the first place be observed, that, if any number be divided into any two parts, the sum of the squares of the two parts together with twice the product of the two parts, is equal to the square of the whole number. For instance, the square of 6 is 36; and if the 6 be divided into the two parts 4 and 2, the square of 4 will be 16, and the square of 2 will be 4, and 4 added to 16 make 20 ; and the product of 4 by 2 is 8, and twice the product 16, and 16 added to 20 make 36, the same as the square of the whole number. Again, if the 6 be divided into the two parts 5 and 1, the square of 5 will be 25, and the square of 1 will be 1, which makes 26 ; and twice the product of 5 by 1 is 10, and 10 added to 26 make 36, as before. The same principle will hold good in any number whatever, and it is the foundation of the rule for extracting the square root, which I shall endeavour to show.

THE RULE EXPLAINED.

The first thing to be done in explaining the rule, is to show the reason that pointing the given number into periods of two figures each, shows the number of figures that the root will contain ; which may be done thus : The square of a number cannot have more than twice as many figures as its root, nor less than twice as many lacking one. For instance, the square of 99 is 9801, and it is the greatest square that can be produced from a number consisting of two figures ; and it contains, we see, just twice as many figures as the 99, its root; again, the square of 10 is 100, and it is the least square that can be produced from a number consisting of two figures ; and it lacks but one, we see, of containing twice as many figures as the 10, its root. If we had taken 999 and 100, or any two numbers similar to them, instead of the 99 and 10, the same principle would have held good. Wherefore, it is manifest, that the number of periods of two figures each in any number, is equal to the number of figures in its root ; and if there be one figure more than equal periods, it must make a period by itself.

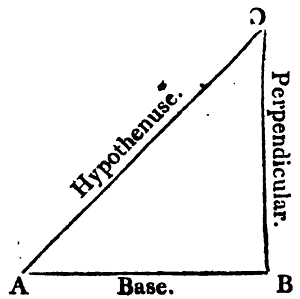
To illustrate the remaining part of the rule, let us extract the square root of 625 ; first by the common method, and then by retaining such ciphers in the operation as are understood ; thus :

625(25	625(25
4	400
45)225	40)225
225	5)225
0	45) 0

In working this example, we divide the root, which consists of two figures, into two parts ; to wit, 20 and 5 ; for we first find the tens figures, and then the units. But it has been shown, that, if a number be divided into any two parts, the squares of the two parts together with twice the product of the two parts, is equal to the square of the whole number. Therefore, the squares of the two parts, 20 and 5, together with twice the product of the 20 by the 5, must be equal to the 625 ; because the 625 are the square of 25, the whole root ; and, in working by the rule, we take the squares of those two parts together with twice their product from the 625 ; which may be seen by examining the second of the above operations ; (which is the same in effect as the first) for, 400 are first taken from the 625, which 400 are the square of 20, the first part ; and therefore, the remainder 225 must be equal to twice the product of the 20 by the 5 together with the square of 5. But by doubling the 20 for a divisor, and multiplying it together with the 5 by the 5, we manifestly get twice the product of the 20 by the 5 together with the square of the 5. Wherefore, the rule must be plain when the root contains but two figures ; and when it contains three or more, the whole operation depends upon the same principle ; for, after finding the first two figures of the root, we take them together as the first part, and the third figure becomes the second part, &c.

Pupil. The assistant says something about finding the sides of a right angled triangle ; but I do not understand it; and I expect the reason is, because I do not know what a triangle is.

Tutor. Any three sided figure is a triangle; but a right angled triangle, is a three sided figure that has one right angle; that is, that has one of its corners of the shape of a corner of a square.



For instance, the angle at B in the figure A B C is a right angle; and therefore, the three sided figure A. B. C. is a right angled triangle. The longest side is called the hypothenuse, and the side that stands straight up, the perpendicular, and the other side the base. Now, in any right angled triangle, the square of the hypothenuse or longest side, is equal to the sum of the squares of the other two sides; and therefore, it is manifest, that, if any two of the sides be given, we can find the third side by the square root.

THE CUBE ROOT.

Pupil. The extraction of the cube root, (if I understand it right) is the finding of such a number from a given number, as, being cubed, or raised to the third power, *will produce* the given number.

Tutor. Yes, that is the sense of it; and to illustrate it, it must in the first place be observed, that, if any number be divided into any two parts, the sum of the cubes of the two parts together with the sum of the products of 3 times the square of each part multiplied by the other part, will be equal to the cube of the whole number. For instance, the cube of 5 is 125; and, if we divide the 5 into the two parts 3 and 2, the cube of the 3 will be 27, and the cube of the 2 will be 8: and 27 added to 8 make 35; and 3 times the square of 3 multiplied by 2 make 54; and 3 times the square of 2 multiplied by 3 make 36; and $36+54+35=125$, the same as the cube of the whole number. Again, if we divide the 5 into the two parts 4, and 1, the cube of the 4 will be 64, the cube of the 1 will be 1; and $64+1=65$; and 3 times the square of 4 multiplied by $1=48$; and 3 times the square of 1 multiplied by $4=12$; and $12+48+65=125$, as before. The same principle will hold good in any number whatever; and it is the foundation of the rule for extracting the cube root; which I shall endeavour to show.

THE RULE EXPLAINED.

The first thing that is to be done in explaining the rule, is to show the reason that the given number must be pointed into periods of three figures each, to show the number of figures that the root will have. But this must be manifest already to an observant person, from what has been said in the square root; and therefore, I think it unnecessary to enter into the investigation of it. But, to explain the rest of the rule, let us extract the cube root 18576; first by the common method, and then by retaining such ciphers in the operation as are understood, and by multiplying separately each of the numbers that are added together to complete the divisor, by the units figure, thus:

```
       .  .
      17576(26
       8
      ----
    1236)9576
     360)9576
    ---------
    1596    0
```

```
  .  .
 17576(26
  8000
  ----
1200)9576    7200 the product of 3 times the square of
                  20 by 6.
  36)9576     216 the cube of 6, the second part.
  -----
 360)   0    2160 the product of 3 times the square of
 ----        ----      6 by 20.
1596         9576
```

It may be observed here, as in the square root, that, in working the example, we divide the root into two parts, to wit, 20 and 6. But it has been shown, that, if a number be divided into any two parts, the sum of the cubes of the two parts together with the sum of the products of 3 times the square of each part multiplied by the other part, will be equal to the cube of the whole number. Therefore, the sum of the cubes of the two parts, 20 and 6, together with 3 times the square of 20 multiplied by 6, and 3 times the square of 6 multiplied by 20, must be equal to the cube of the whole root; that is, to the 17576; and, in working by the rule, we take those two cubes and products from the 17576; which may be seen by examining the second operation; (which is the same in effect as the first) for, we first take 8000 from the 17576, and the 8000 are the cube of 20, the first part. Then, by multiplying the square of 20 by 3 for the defective divisor, (the *0 here being* 20) and, by multiplying this defective divisor by the 6, we get 7200, the product of 3 times the square of the first part multiplied by the second part. Next, in multiplying the square of 6 by the 6, we get 216 the cube of 6, the second part; and it is manifest, that, in working by the rule, the square of 6 is multiplied by 6. Lastly, by multiplying the 2 by the 6, and this product by 30, as the rule directs, it is manifest, that we get 3 times the product of the 20 by the 6, and this being multiplied by the 6, gives 2160, the product of 3 times the second part multiplied by the first part. Wherefore, the sum of the three numbers, 1200, 216, and 2160 must be equal to the remainder of the 17576 after the 8000 are taken from the 17576, agreeably to what has been said; and we see it is. Whence, the rule must be plain when the root contains but two figures; and when it contains three or more, the whole operation depends upon the same principle; for, after finding the first two figures of the root, we take them together as the first part, and the third figure becomes the second part, &c.

I deem it unnecessary for me to undertake to investigate the general rule for extracting the roots of all powers; for it is very abstruse, and the powers which are intended to be extracted by it, are of very little use.

ARITHMETICAL PROGRESSION.

Pupil. The Assistant says, that arithmetical progression is a rank, or series of numbers, which increase or decrease by a common difference. This seems dark; for it does not tell the meaning of a series of numbers.

Tutor. The word series signifies succession; and therefore, any rank or row of numbers, that succeed, or follow one another in a regular order, is a series of numbers; and when the terms of the series increase or decrease regularly by a common difference; that is, when the difference between the first and second terms is equal to the difference between second and third, &c. then it is a series of numbers in arithmetical progression. For instance, the numbers, 4, 7, 10, 13, 16, 19, 22, &c. are an increasing series in arithmetical progression; because they increase regularly by the continual addition of the common difference 3; that is, the 7 are 3 greater than the 4; the 10, three greater than the 7; the 13, 3 greater than the 10, &c. The first and last terms are called the extremes; and in any series of numbers in arithmetical progression, the sum of the two extremes will equal the sum of any two terms that are so situated, that one of them is just as far from the first term as the other is from the last; and if the number of terms be odd, the sum of the extremes will be equal to twice the middle term. For instance, in the series, 4, 7, 10, 13, 16, 19, 22, we see, that the sum of the two extremes 4 and 22 is equal to the sum of the two terms 7 and 19, which are equally distant from the extremes; as also to the two terms 10 and 16, &c. for, $4+22=26$; and $7+19=26$; and $10+16=26$; and $13+13=26$.

TO CASE 1ST EXPLAINED.

From what has been said, case 1st may easily be explained; for, since the second term exceeds the first by once the common difference; the third by twice; the fourth by three times, &c. the last term must of course exceed the first by as many times the common difference, as there are terms except one. Therefore, it is manifest, that if we multiply the number of terms, less than 1, or except 1, by the common difference, it will show how much the last term exceeds the first; and if the first term be added to this, it will of course give the last term.

Again, to find the sum of the series; since the sum of the two extremes is equal to the sum of any two terms equally distant from the extremes, the number of such sums in the sum of the series must be equal to half the number of terms; that is, all the terms added together, *which is the* sum of the series, must be equal to the sum of the extremes multiplied by half the number of terms. But, to multiply by half the number of terms will produce the same number as to multiply by the whole number of terms and take half the product. Wherefore, the rule must be plain.

CASE 2D EXPLAINED

In case 2d we see that the two extremes and the number of terms are given to find the common difference. Now, since it has been shown, that the last term exceeds the first term by the product of the common difference by the number of terms less than 1, if we take the first term from the last, it is manifest, that the remainder must be the product of the common difference by the number of terms less than 1; and therefore, if we divide this remainder by the number of terms less than 1, it must of course give the common difference.

GEOMETRICAL PROGRESSION.

Pupil. The Assistant says, that geometrical progression is a series of numbers, increasing or decreasing by one continual multiplier or divisor, called the ratio. This is not plain, for I do not understand the meaning of ratio.

Tutor. The word ratio has more meanings than one; but strictly speaking, it signifies proportion; for instance, if the first of four numbers bear the same proportion to the second, that the third does to the fourth; then the first is said to have the same ratio to the second that the third has to the fourth. But, in geometrical progression, the ratio is the number by the continual multiplication or division, of which the terms of the series increase or decrease; for instance, the numbers, 2, 6, 18, 54, &c. increase by multiplying continually by 3; for, $2\times3=6$; $6\times3=18$; $18\times3=54$, &c. and therefore, in this case, the ratio is 3.

In any series of numbers, in geometrical progression, the product of the two extremes, is equal to the product of any two means, or terms, that are equally distant from the extremes. For instance, in the series, 2, 6, 18, 54, we see, that $54\times2=108$; and $18\times6=108$. If the number of terms be odd, the product of the two extremes is equal to the square of the middle term; for, if the terms be, 2, 6, 18, then, $18\times2=36$; and, $6\times6=36$.

THE RULE EXPLAINED.

Pupil. In the American Tutor's Assistant, the rule for finding the last term, says, "Multiply the first term into such a power of the ratio, as is indicated by the number of terms less than one, and the product will be the last term." This is so dark that I cannot work by it; much less understand the ground work of it.

Tutor. It may be worded in a more simple manner, thus:

Involve the ratio to a power equal to the number of terms, less than 1; and multiply this power by the first term, for the last term.

This rule is the same in sense as the one above; and it may be explained, thus: Since the second term is formed by multiplying the first by the ratio; and the third, by multiplying the second by the ratio, &c. it is manifest, that the second term is equal to the first multiplied once by the ratio, and the third equal to the first multiplied twice by the ratio; and the fourth equal to the first multiplied 3 times by the ratio, &c. Therefore, the last term is equal to the first multiplied by the ratio as many times as there are terms, less than 1. But, multiplying the the first term by the ratio as many times as there are terms, less than 1, will manifestly produce the same number, as to multiply the ratio by itself as many times as there are terms, less than 1; and then multiply by the first term. But, multiplying the ratio by itself as many times as there are terms, less than 1, is the same thing as involving the ratio to a power equal to the number of terms, less than 1; and therefore, the rule is plain.

To find the sum of the series, or the sum of all the terms, we are directed to multiply the last term by the ratio, and from the product to subtract the first term, and divide the remainder by the ratio less than one. To investigate this, it must in the first place be observed, that, in any increasing series of numbers in geometrical progression, the last term less than the first, or the difference between the last term and the first, is always equal to the sum of all the terms except the last, multiplied by the ratio less than 1; and therefore, if we multiply the sum of all the terms except the last by the ratio less than 1, and add the first term to the product, it will give the last term; and, since it is manifest, that the sum of all the terms must be 1 time the sum of all the terms except the last, greater than the last, it is plain, that, if we multiply the sum of all the terms except the last, by the ratio, and add the first term to the product, it will give the sum of all the terms. Wherefore, if we subtract the first term from the last, and divide the remainder by the ratio less than 1, it will give the sum of all the terms except the last; and if we multiply this by the ratio, and add the first term to the product, it will give the sum of all the terms, agreeably to what has been said. But it must be manifest, to an observant person, that this is the same in effect as working by the rule; and therefore, I need investigate it no further.

SIMPLE INTEREST—BY DECIMALS.

Pupil. I can work examples in this rule tolerably well by following the directions of the rules; but I cannot see the reason why we must work as the rules direct.

Tutor. I should think that might easily be seen by a person who has a knowledge of interest; for, by taking particular notice of what is given and what is required, the sense will dictate what must be done to find what is required; which I shall endeavour to show.

CASE 1ST EXPLAINED.

In case 1st we see, that the principal, time, and ratio are given, to find the interest and amount; and, since the ratio is the interest of 1 pound, or dollar, for 1 year, it is clear, that if we multiply it by the principal, it will give the interest of the whole principal for 1 year; and this, being multiplied by the time, will of course give the interest required; which being added to the principal, will give the amount.

CASE 2D EXPLAINED.

In case 2d we see, that the amount, time, and ratio are given, to find the principal; and, since the ratio is the interest of 1 pound for 1 year, if we multiply it by the time, it will of course give the interest of 1 pound for the whole time; and if we add 1 to this interest, it will give the amount of 1 pound principal for the whole time; and, it is manifest, that this amount divided into the given amount, will give the principal required.

CASE 3D EXPLAINED.

In case 3d the amount, principal, and time are given, to find the rate per cent. or ratio; and, since the amount is the sum of the principal and interest, if we subtract the principal from it, the remainder will be the interest; and, since it was shown in case 1st, that the principal, time, and ratio, must be multiplied together to give the interest, it is manifest, that, if we divide the interest by the product of the principal and time, it will give the ratio.

CASE 4TH EXPLAINED.

In case 4th the amount, principal, and rate per cent. are given to find the time; and, since the principal, time, and ratio, must be multiplied together to give the interest, it is plain, that, if we divide the interest by the product of the ratio and principal, it will give the time.

COMPOUND INTEREST—BY DECIMALS.

Pupil. It seems that the ratio in compound interest, is a different thing from what it is in simple interest; for, in simple interest, it is the interest of 1 pound, or dollar, for 1 year; but, in compound interest, according to the Assistant, it is the amount of 1 pound, or dollar, for 1 year. This appears tolerably plain, as also the method of finding the ratio; but the tables seem dark.

Tutor. The ratio involved to any given time, is the amount of 1 pound, or dollar, for that time; that is, the square of the ratio, is the amount of 1 pound for 2 years; the cube of the ratio, the amount of 1 pound for 3 years; the fourth power, for 4 years, &c. Therefore, by the raising of the ratio, table II. is calculated, for it shows the amount of 1 pound, for any number of years from 1 to 50; which shows that the ratio is involved to the 50th power, and every power from 1 to 50, retained and placed in the table in a regular order.

Pupil. I cannot see how it can be, that involving the ratio to any time, will give the amount of 1 pound for that time.

Tutor. That is not difficult to be seen; for, since the ratio is the amount of 1 pound for 1 year, if we find the interest of the ratio for 1 year, and add it to the ratio, it will give the amount of 1 pound for 2 years, agreeably to the definition of compound interest; and, if we find the interest of this amount for 1 year, and add it to the amount, it will give the amount of 1 pound for 3 years, &c. But this is the same in effect as involving the ratio; for, the interest of 1.06 for 1 year, is .0636; and 1.06+.0636= 1.1236, the amount of 1 pound for 2 years; and it is the same as the square of the ratio; for 1.06×1.06=1.1236. Again, the interest of 1.1236 for 1 year is .067416; and 1.1236×.067416=1.191016, the amount of 1 pound for 3 years, the same as the cube of the ratio; for 1.06× 1.06×1.06=1.191016. Whence it is plain.

Pupil. The assistant signifies, that the amount of 1 pound for 1 quarter of a year, is the 4th root of the ratio; for 2 quarters, the square root; and for 3 quarters, the product of the square and 4th roots. This is beyond my comprehension.

Tutor. It may easily be made to appear plain; for, we know that the 4th power of the amount of 1 pound for 1 year, is the amount of 1 pound for 4 years; and if so, the 4th power of the amount of 1 pound for 1 quarter of a year, must be the amount of 1 pound for 4 quarters; that is, it must be the ratio; and therefore, the 4th root of the ratio must be the amount of 1 pound for 1 quarter of a year. For a similar reason, the square root is the amount for 2 quarters, &c.

By extracting the square and 4th roots of the ratio, &c. table I. is calculated; for the numbers in the third column are the amounts of 1 pound for 3 quarters of a year, and are found by multiplying the square root of the ratio by the 4th root; and the numbers in the fourth column are the amounts of 1 pound for a half of a year; and those in the fifth, for 1 quarter of a year; the former of which are found by extracting the square root of the ratio; and the latter, the 4th root.

CASE 1ST EXPLAINED.

Now, to explain the rules, we may observe, that in case 1st the principal, time, and rate are given, to find the amount; and, since the ratio is the amount of 1 pound for 1 year, if we involve it to the time, it will give the *amount of 1 pound* for the whole time, as has been shown;

and if this amount be multiplied by the principal, it will of course give the amount required.

Pupil. How are we to involve the ratio to the time, when there are years and months, or years, months, and days given? For instance, the last example in this case in the A. T.'s Assistant, says, "What is the amount at compound interest of 259*L.* 10*s.* 6*d.* for 3 years, 7 months, and 15 days, at $4\frac{1}{2}$ per cent. per annum?

Tutor. The ratio is involved to 3 years, 7 months, and 15 days, or, the amount of 1 pound for 3 years, 7 months, and 15 days, is found thus: The amount of 1 pound for 3 years is found in table II. to be 1.141166; and the amount of 1 pound for a half of a year, is found in table I. to be 1.022252; and therefore, $1.141166 \times 1.022252 = 1.1665602$, the amount of 1 pound for 3 years and 6 months. Again, the simple interest of 1 pound for 1 month, is found in table I. to be .00375, and half of it .001875, the interest for 15 days; and, $1.1665602 + .00375 + .001875 = 1.1721852$, the amount of 1 pound for 3 years, 7 months, and 15 days nearly. I say nearly, because, to get the exact amount, instead of adding the simple interest of 1 pound for 1 month and a half, we should multiply the amount for 3 years and 6 months by the 12th root of the ratio; which would give the amount for 3 years and 7 months; and by multiplying this amount by the 24th root of the ratio, we should get the exact amount of 1 pound for 3 years, 7 months, and 15 days; which would differ but very little from the foregoing amount.

CASE 2D EXPLAINED.

In case 2d, the amount, rate, and time, are given, to find the principal: and, since it has been shown, that the ratio involved to the time must be multiplied by the principal to give the amount, it is manifest, that, if we divide the amount by the ratio involved to, it will give the principal.

CASE 3D EXPLAINED.

In case 3d, the principal, rate, and amount, are given, to find the time: and, since the ratio involved to the time multiplied by the principal gives the amount, it is plain, that the amount divided by the principal will give the ratio involved to the time; and, therefore, involving the ratio till it equals the quotient must give the time.

CASE 4TH EXPLAINED.

In case 4th, the principal, amount, and time, are given, to find the rate of interest, or ratio: and, since it has been shown, that, dividing the amount by the principal, gives the ratio involved to the time, it is manifest, that, if we extract such a root of the quotient as is indicated by the time, it will be the ratio.

ANNUITIES AT COMPOUND INTEREST.

Pupil. The assistant says, that an annuity is a sum of money payable yearly, half yearly, or quarterly, for a number of years, during life, or for ever; and may draw interest if it remain unpaid after it becomes due. I cannot understand clearly the meaning of an annuity from this definition.

Tutor. The definition is plain; nevertheless, the following observations may perhaps make it appear more so; to wit, if a person have a yearly income of any value, it is called an annuity; because it becomes due annually, or yearly; and this income may continue a certain number of years, during the life of the person, or for ever; and the payments may be made either yearly, half yearly, or quarterly, according to contract; and if they are not paid *as they become due*, they of course draw interest.

Pupil. What is the ratio in annuities?

Tutor. The same that it is in compound interest; to wit, the amount of 1 pound, or dollar, for 1 year, at the given rate.

Pupil. The tables and rules appear exceedingly dark.

Tutor. I do not wonder at their appearing dark; for they are so very abstruse, and depend so much upon algebra, that it would be almost impossible to illustrate them so as to make them appear plain to such as have never studied algebra; therefore, I shall not undertake to explain them fully; but shall make some remarks, which may probably cast a little light upon them.

Pupil. I should like to know what the numbers are in table III, table IV, and table V, or what they represent.

Tutor. The numbers in table III, are the amounts of 1 pound annuity at different rates per cent., and for different times; and they are found by taking a unit from the ratio involved to the time, and dividing the remainder by the ratio less a unit. For instance, the number in the table under 6 per cent. and opposite 6 years, is 6.975318; and it is found thus: The ratio involved to the 6th power is 1.4185191; and 1.4185191 less a unit =.4185191; and 1.06 less a unit =.06; and .4185191÷.06=6.975318, which is the same as the tabular number.

RULE TO CASE 1ST EXPLAINED.

From what has been said, the rule to case 1st may easily be explained; for it has been shown, that, if we take a unit from the ratio for a divisor, and from the ratio involved to the time for a dividend, the quotient will be the amount of 1 pound annuity for the given time; and if so, the quotient multiplied by the annuity, will consequently be the amount required, for yearly payments; and, when the payments are half yearly or quarterly, we are directed to multiply the amount for yearly payments by the proper number in table V, for the true amount: from which it appears, that the numbers in table V, are the amounts of 1 pound annuity for 1 year at half yearly and quarterly payments; for the amount of 1 pound for 1 year at yearly payments, is 1 pound, and 1 pound multiplied by one of the tabular numbers, will give the tabular number.

RULE TO CASE 2D EXPLAINED.

The rule to case 2d may be worded thus:

Take a unit from the ratio involved to the time, and divide the remainder by the ratio less 1; the amount divided by the quotient will be the annuity.

Although this differs widely in words from the rule in the Assistant, it is manifestly the same in effect; and it may be explained thus: Since it has been shown, that, by taking a unit from the ratio involved to the time and dividing the remainder by the ratio less 1, we get the amount of 1 pound annuity for the given time; and that this multiplied by the annuity gives the amount, the amount divided by this must give the annuity.

RULE TO CASE 3D EXPLAINED.

The rule to case 3d may be worded thus:

Divide the amount by the annuity, multiply the quotient by the ratio less 1, and add a unit to the product, then involve the ratio up to the sum; and the power to which it is raised will indicate the time.

This is manifestly the same in effect as the rule in the Assistant, and it may easily be explained; for, it has been shown, that the amount of 1 pound annuity for the given time, must be multiplied by the annuity to give the amount; and if so, the amount divided by the annuity must give the amount of 1 pound for the given time; and, since this amount is found by taking a unit from the ratio involved to the time, and dividing the remainder by the ratio less 1, it is manifest, that, if we multiply it by the ratio less 1, and add one to the product, the sum will be the ratio involved to the time; and, if we involve the ratio up to this sum, the power to which it is raised will of course indicate the time.

Before we go any further, it may be best to observe, that the numbers in table IV, are the present worths of 1 pound annuity for any number of years from 1 to 40; and they are found, by dividing 1 pound annuity by the *ratio involved* to the time, and subtracting the quotient from 1 pound, and dividing the remainder by the ratio less 1. For instance, the number under 6 per cent., and opposite 5 years, is 4.21236; and it is the present worth of 1 pound annuity for 5 years at 6 per cent. and is found thus: The ratio involved to the 5th power, is 1.3382256; and 1 ÷ 13382256 = .74725809; and 1 — .74725809 = .2527419; and .2527419 ÷ .06 = 4.21236; which is the same as the tabular number.

RULE TO CASE 4TH EXPLAINED.

The rule to case 4th is exactly the same as the one by which table IV is calculated; and, since the numbers in the table are the present worths of 1 pound annuity, it follows of course, that, if we work any other annuity by the rule, it will produce the present worth of said annuity.

RULE TO CASE 5TH EXPLAINED.

The rule to case 5th, as it stands in the Assistant, is very difficult to explain; therefore, we will make another, thus:

Divide 1 by the ratio involved to the time, and subtract the quotient from 1, and divide the remainder by the ratio less 1; then divide the present by the quotient, and the last quotient will be the annuity.

This rule may easily be explained from what has been said; for it has been shown, that, to divide 1 by the ratio involved to the time, and subtract the quotient from 1, and divide the remainder by the ratio less 1, will give the present worth of 1 pound annuity for the given time; and if so, this present worth divided in the given present worth, must give the annuity required.

RULE TO CASE 6TH EXPLAINED.

The rule to case 6th may be explained, by tracing the rule to case 4th backwards; for, the rule to case 4th says, "Divide the annuity by the ratio involved to the time, and subtract the quotient from the annuity; divide the remainder by the ratio less 1, and the quotient will be the present worth:" therefore, if we multiply the present worth by the ratio less 1, and subtract the product from the annuity, the remainder must be the same as the quotient produced by dividing the annuity by the ratio involved to the time; and, therefore, if we divide the annuity by this remainder, the quotient must be the ratio involved to the time; and, therefore, if the ratio be involved till it equals the quotient, the power to which it is raised must indicate the time.

I might continue on through annuities in this manner; but I think it unnecessary; therefore, I shall relinquish the subject.

ALLIGATION.

Pupil. The first case of alligation is much more simple than the others; but I do not understand it fully; and, as for the others, I do not even know how to work them; the reason of which is, because I do not understand how to link the numbers.

CASE 1ST EXPLAINED.

Tutor. To explain case 1st, let us take the following example; to wit,

If 6 gallons of wine at 67 cts. per gallon, 7 at 80 cts., and 5 at 120 cts., be mixed together; what is one gallon of this mixture worth?

Now, it is manifest, that the whole number of gallons must bear the same proportion to the whole value of the wine, that one gallon does to its value; and, by multiplying each number of gallons by the price per gallon, and adding the products together, we get the whole value of the wine, or the value of all the wine; and, therefore, to get the value of one gallon, we must, of course, say, As the whole number of gallons is to the whole value of the wine, so is one gallon to its value. Wherefore, $6\times67=$

402, 7×80=560, and 5×120=600; and 402+560+600 =1562, the value of all the wine. Again, 5+7+6=18, the whole number of gallons. Whence, as 18 g..1562 cts. :: 1 g..86.77 cts.+ the answer.

CASE 2D EXPLAINED.

Previously to explaining the ground-work of case 2d, it may not be unnecessary for me to make the following observations respecting the manner of linking the numbers; viz.

1. If there be two rates or prices given, beside the mean rate, one greater and one less than the mean rate, they must be linked together, and the less one must be taken from the mean rate, and the remainder set opposite the greater; and the mean rate must be taken from the greater, and the remainder set opposite the less.

2. If there be three or more rates, and but one of them less than the mean rate, it must be linked with each of the others; or, if there be but one greater than the mean rate, it must be linked with each of the others; and, in each case, the difference between the one rate and the mean rate must be set opposite to each of the others, and the difference between the mean rate and each of the others must be set opposite to the one rate.

3. If all of the given rates be greater, or less than the mean rate, each of them must be linked with a cipher; and the difference between each of them and the mean rate must be set opposite to the cipher; and the difference between the mean rate and the cipher, or the mean rate itself, must be set opposite to each of the other rates.

4. If there be four rates given beside the mean rate, two greater and two less than the mean rate, they may be linked seven different ways, which will produce seven different answers; but one of the most simple ways of linking them is, to link one of the two which are less than the mean rate, with one of those that are greater, and the other with the other.

5. If there be five rates given, or more, beside the mean rate, each one which is less than the mean rate, must be linked with one or more that is greater; and, if

we wish to get one of the most simple answers, we must link each rate with as few as the nature of the case will admit.

Now, the reason that working is directed above, will give the number of simples, that is, gallons, bushels, &c. that must be taken, at their respective rates, to make a compound at any proposed price, is this : By linking the less rate with the greater, and placing the difference between the less and the mean rate opposite the greater, and the difference between the greater and the mean rate opposite the less, the quantities resulting are such, that there is precisely as much gained by one quantity as there is lost by the other ; and therefore, the gain and loss, upon the whole, are equal, and are exactly the proposed rate.

In like manner, let the number of rates be what it may, and with how many soever each one is linked, since it is always a less with a greater than the mean rate, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

To make this more plain, we will take the following example ; viz.

How much wheat at 12*s*. per bushel, and rye at 7*s*. will it take to make a mixture worth 9*s*. per bushel ?

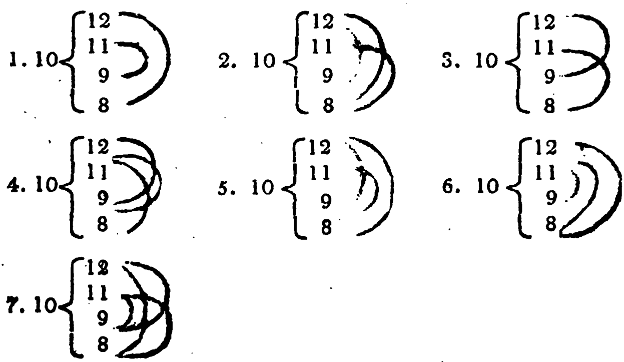
		s.		
Mean rate	*s*. 9	12	- - 2 at 12*s*.	Answer.
		7	- - 3 at 7*s*.	

Here, by taking 2 bushels at 12 shillings per bushel, 6*s*. are gained ; because 2 bushels at 12*s*. per bushel, amount to 6*s*. more than they do at 9*s*. per bushel. Again, by taking 3 bushels at 7*s*. per bushel, 6*s*. are lost ; because 3 bushels at 7*s*. per bushel, amount to 6*s*. less than they do at 9*s*. per bushel. Wherefore, if 2 bushels of wheat at 12*s*. per bushel, be mixed with 3 bushels of rye at 7*s*. per bushel, the mixture will be worth 9*s*. per bushel.

Pupil. I should like now to see the rates in the following example linked seven different ways.

Example. A druggist has several sorts of tea, viz. at 12*s*. per *lb*. at 11*s*. at 9*s*. and at 8*s*. how much of each sort must be taken to be sold at 10*s*. per *lb*. ?

Tutor. To produce the answers in the order in which they stand in the Assistant, the rates must be linked thus:



Pupil. Since four rates, two greater and two less than the mean rate, will admit of seven answers, will not five or more rates admit of still more answers?

Tutor. Yes; five rates, two greater and three less, or two less and three greater, than the mean rate, will admit of twenty-five answers.

CASE 3D EXPLAINED.

Pupil. In case 3d, the rates are to be linked together, it seems, and the differences taken in the same manner as in case 2d; but after this is done, we have to make several statings of the rule of three to get the answer, the reason of which seems somewhat dark.

Tutor. It may easily be explained by the help of the following example; viz.

A farmer has 50 bushels of wheat at 12*s*. per bushel, which he would mix with rye at 7*s*. and oats at 3*s*., how much rye, and how much oats must he take to mix with the 50 bushels of wheat, that he may sell the mixture at 10*s*. per bushel?

10	12	7+3=10. As 10..2::50..10 of rye.	Answ.
	7	- - 2. As 10..2::50..10 of oats.	
	3	- - 2.	

By linking the rates together, and taking the differences, we find, that, if we mix 2 bushels of rye at 7*s*. per bushel, and 2 of oats at 3*s*. with 10 bushels of wheat at 12*s*. the mixture will be worth 10*s*. per bushel. But, agreeably to the question, we are to find the number of bushels of each sort that must be mixed with 50 bushels of wheat, that the mixture may be worth 10*s*. per bushel; and, since 2 bushels of each sort mixed with 10 bushels of wheat, make such a mixture; it is plain, that the 50 bushels must bear the same proportion to the number of bushels of each sort that are to be mixed with the 50 bushels, that 10 bushels bear to 2; and therefore, as 10..2::50..10, the number of bushels required.

I shall not take the pains to explain case 4th, because, if case 3d be fully understood, it must be manifest.

POSITION.

Pupil. Seeing we have to use supposed numbers in working questions in this rule, would it not be more proper to call it supposition than position?

Tutor. I think it would; for I can see but very little reason for calling it position, since position signifies situation, principle laid down, &c. It is sometimes called the rule of false, or false position; and there appears to be some reason for calling it false position; because it is wrought in some measure upon false principles; that is, the calculations are made on false numbers; but in the *conclusion*, the number sought is determined.

There are many hard intricate questions, to which no rule in Arithmetic will apply except this; and, therefore, this rule may be said to be the last resort in Arithmetic.

SINGLE POSITION.

THE RULE EXPLAINED.

All questions which require only one supposed number, belong to single position: and to show how to work such questions we will take the following example, viz.

A person having about him a certain number of dollars, said, that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 171; how many dollars had he?

Now, according to the rule, we are in the first place to suppose a number, and work with it, according to the tenor of the question; and therefore, we will suppose 60; that is, we will suppose that the person had 60 dollars about him; and, to work with this 60 according to the tenor of the question, it is plain, that we must take the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of it, and add them together, thus: The $\frac{1}{3}$ of $60=20$, $\frac{1}{4}$ of $60=15$, $\frac{1}{5}$ of $60=12$, and $\frac{1}{6}$ of $60=10$; and $20+15+12+10=57$. Now, it is manifest, that if we had supposed the right number, and had taken the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of it, and added them together, we should have got 171, instead of 57; and it is clear, in questions like this, that the result of one supposed number must be to the supposed number as the result of another supposed number is to that supposed number; and, therefore, since 57 are the result of 60, and 171 the result of the right number, as 57..60::171..180, the answer.

Pupil. If any other number had been supposed instead of 60, would it not have answered the same purpose?

Tutor. Yes; but 60 was made choice of, because it is the most convenient number; for it is the least number that will divide by 3, 4, 5, and 6, without remainder.

Pupil. How are we to determine whether the question belongs to single or double position?

Tutor. Suppose two numbers, and work with each according to the tenor of the question; then multiply each supposed number by the result of the other, and if the products be equal, the question belongs to single position, otherwise not.

DOUBLE POSITION.

Pupil. Double Position appears very dark to me.

Tutor. It appears dark to most beginners, and I do not much wonder at it; for it is one of the most difficult rules in Arithmetic to understand; however, I will endeavour to cast a little light on it by working the following example, viz.

Divide 100*L.* so that B may have twice as much as A, wanting 8*L.* and C three times as much, wanting 15*L.* what is each man's share?

Let us first suppose A's share to be 10*L.* then, agreeably to the question, $10\times2-8=12=$B's share; and, $10\times3-15=15=$C's share; and $10+12+15=37=$the sum of the shares according to the supposition. But the sum of the shares is 100, agreeably to the question; and, therefore, $100-37=63=$the error too little; that is, the 63 shows that the result is 63 too little. Let us next suppose A's share to be 15*L.* then $15\times2-8=22=$B's share; and $15\times3-15=30=$C's share; and $15+22+30=67=$ the sum of the shares according to the supposition; and, since the true sum of the shares is 100, $100-67=33=$ the error, too little; that is, the 33 show that the result is 33 too little. Now, since the errors are alike; that is, since each error shows that the result of its supposed *number* is too little, we must multiply each supposed number by the error of the other, and take the difference of the products for a dividend, and the difference of the errors for a divisor; thus: $10\times33=330$; and $15\times63=945$; and $945-330=615=$the difference of the products; and $63-33=30=$the difference of the errors; and therefore, $615\div30=20L.10s.=$A's share; and $20L.10s.\times2-8=33L.=$B's share; and $20L.10s.\times3-15=46L.10s.=$C's share.

THE FOUNDATION OF THE RULE.

The rule is founded on this supposition; to wit, that the first error is to the second, as the difference between the true number and first supposed number is to the difference between the true number and the second supposed number: when this is not the case, the exact answer to the question cannot be found by this rule.

That the rule is true according to the supposition, may be demonstrated by algebra;* but not by common arithmetic; or, at least, not without considerable difficulty.

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* It may be demonstrated by algebra, thus:

Let a = the first supposed number, b = the second, r = the first error, s = the second, and x = the number required. Then, according to the supposition on which the rule is founded, $r..s::x-a..x-b$; from which we get $rx-rb=sx-sa$, and, by transposition, $rx-sx=rb-sb$, and, by division, $x=\frac{rb-sa}{r-s}=$the number required; and, if r and s be both negative, the theorem will be the same; and, if one of them be negative, x will be equal to $\frac{rb+sa}{r+s}$, which is the rule.

PERMUTATION.

Pupil. What is the reason that this rule is called permutation?

Tutor. The word permutation signifies changing of an exchange, and this rule teaches to find the number of changes or variations in position that a given number of things may undergo, for which reason it is called permutation.

Pupil. Permutation and Combination are neither of them of much use; are they?

Tutor. No; they are not of much use; nevertheless, they help to show the power and curious properties of numbers, to know which, is, at least, satisfactory.

It is, for instance, very curious, that multiplying 1 by 2, that product by 3, that product by 4, &c. to the given number (as the rule directs), will produce the number of changes, of which the given number is susceptible: but that it is so, may be shown thus: It is manifest, that one thing can have but one position, for it cannot change with itself. Two things can have two positions or situations; thus, *ab*, *ba*, are two different positions of two letters. Three things can have six different positions; for instance, the letters *a*, *b*, and *c*, can be placed six different ways, thus, *abc*, *acb*, *bac*, *bca*, *cab*, and *cba*. Four things can be placed twenty-four different ways, or they can have twenty-four different positions; for instance, the letters *a*, *b*, *c*, and *d*, may be placed twenty-four different ways, thus, *abcd*, *abdc*, *acbd*, *acdb*, *adbc*, *adcb*, *bacd*, *badc*, *bcad*, *bcda*, *bdac*, *bdca*, *cabd*, *cadb*, *cbad*, *cbda*, *cdab*, *cdba*, *dabc*, *dacb*, *dbac*, *dbca*, *dcab*, and *dcba*. In the same manner it may be shown, that five things can have 120 different positions, &c. and if we find by the rule the several numbers of positions that 1, 2, 3, 4, and 5 things are susceptible of, we shall find them to be, 1, 2, 6, 24, and 120, the same as those above.

COMBINATION.

There is a material difference between Permutation and Combination, yet they have some similarity. In Permutation, *ab*, and *ba*, are two different positions of two letters; but in combination, although they are two letters combined two different ways, yet they are but one combination of two letters.

To illustrate this rule a little, let us see how many different combinations of 2, 3, 4, 5, and 6 things respectively there are in 6. It is manifest, that there can be no combination of 1 thing; but there are 15 combinations of 2 in 6; found by the rule, thus: $\frac{6\times5}{1\times2}=\frac{30}{2}=15$. It may appear strange to some that there are so many combinations of 2 things in 6; but, that it is so, may be shown in the following manner, viz. Let the letters *a*, *b*, *c*, *d*, *e*, and *f*, be the 6 things out of which the combinations are to be made; then it is manifest, that each letter may be combined with each of the others; for *a* can be combined with each of the others thus, *ab*, *ac*, *ad*, *ae*, and *af*; *b* with each of the others thus, *ba*, *bc*, *bd*, *be*, and *bf*, &c.; but, according to this, there will be 5 combinations for each letter, which will make 30; but this is double the number of combinations, for every 2 letters are combined twice; as, *ab*, and *ba*, and it has been shown, that *ab* and *ba* are but one combination of two letters; and, therefore, the number of combinations of 2 in 6 is 15.

The number of combinations of three things in six is 20; and it is found thus: $\frac{6\times5\times4}{1\times2\times3}=\frac{120}{6}=20.$* In the

* Cancelling will apply here as in Multiplication of Vulgar Fractions.

same manner, the number of combinations of four in six, is found to be 15; those of five, six; and of six, one.

Thus we see there are more combinations of three things in six, than there are of any other number; and in any other case, the number of combinations will be the greatest when the number to be combined is equal to half the number out of which the combinations are to be made: and, when the number to be combined is one less than the said half, the number of combinations is the same as when it is one greater; and when two less, the same as when two greater, &c.

DUODECIMALS.

Pupil. What is the reason, that feet, inches, seconds, &c. are called duodecimals?

Tutor. Duodecimal signifies counted by twelves, and since 12 thirds make 1 second, 12 seconds 1 inch, and 12 inches 1 foot; inches, seconds, thirds, &c. are counted by twelves: and, therefore, they are properly called duodecimals.

Pupil. Addition and subtraction of duodecimals need no explanation, for they are wrought just like compound Addition and Subtraction; but Multiplication appears somewhat dark.

MULTIPLICATION OF DUODECIMALS.

Tutor. Multiplication of Duodecimals used to be called Cross Multiplication, and it is called so yet by some; but Arithmeticians have discovered of late a way of placing the numbers so as to supersede the necessity of multiplying crosswise; which is much better than the *old way; and* therefore, this rule should no longer be *called Cross* Multiplication.

CASE 1ST EXPLAINED.

To explain case 1st, let us multiply 12 feet, 9 inches, and 6 seconds, by 4 feet, 5 inches, and 7 seconds.

Now, the rule directs us to set the feet of the multiplier under the lowest denomination of the multiplicand, and in multiplying carry 1 for every 12, &c. which being easily performed, there is nothing for me to do but to show the reason that this way of working produces the product.

F.	I.	''	'''	''''
12	9	6		
		4	5	7
	7	5	6	6
5	11	11	6	
51	2	0		
57	9	5	0	6

In the first place, then, that the explanation may be the more easily performed, we will begin with the feet of the multipliers, not in multiplying, but in considering the work; but, since it has been shown, that two applicate numbers cannot be multiplied together, we must consider the inches and seconds of the multiplier as a fraction of a foot, and the feet as an abstract whole number; nevertheless, I shall make use of the words, feet multiplied by seconds, inches multiplied by seconds, &c. for the sake of simplicity.

Now, to explain the work, if we consider the 4 feet as 4 simply, then the product of the 4 and the 6 seconds, must be seconds; and, therefore, in multiplying by the 4, the first figure that is set down should come in the column of seconds, and we see it does. Again, since seconds multiplied by feet give seconds, and since inches are twelfths of a foot, seconds multiplied by inches, must give twelfths of a second, which are thirds; and, therefore, in multiplying by the 5, the first figure that is set down should come in the column of thirds, and we see it does.

Lastly, since seconds multiplied by inches give thirds, and since seconds are twelfths of inches, seconds multiplied by seconds, must give twelfths of thirds, which is fourths; and, therefore, in multiplying by the 7, the first figure that is set down should come in the column of fourths, and we see it does. The first figure at the right in each of the three lines which form the product, being thus proved to be right, the others must of course be right; and therefore, the product is right.

PROMISCUOUS QUESTIONS.

Pupil. I think I have a tolerably good knowledge of Arithmetic now; nevertheless, I should like to see some of the hardest of the promiscuous questions in the A. T.'s Assistant, wrought, before we conclude; for some of them are very hard.

Tutor. Very well; I will select out some of the most difficult ones, and state them, and make some observations on some of them.

QUESTION 8TH.

A stationer sold quills at 10*s*. 6*d*. a thousand, by which he cleared $\frac{1}{3}$ of the money; but growing scarce, raised them to 12*s*. a thousand: what did he clear per cent. by the latter price?

Here, $\frac{1}{3}$ of 10*s*. 6*d*. = 3*s*. 6*d*. and 10*s*. 6*d*. — 3*s*. 6*d*. = 7*s*. = the prime or first cost of a thousand; and 12*s*. — 7*s*. = 5*s*. = the gain per thousand by the latter price. Wherefore, as 7*s*. ·· 100*l*. :: 5*s*. ·· 71*l*. 8*s*. 6*d*. $\frac{6}{7}$, the answer.

QUESTION 13TH.

A person being asked the hour of the day, said, the *time past noon* is equal to $\frac{4}{5}$ of the time till midnight: *what was the time?*

Many teachers direct their scholars to work this question by Position; but it may be done more elegantly, and with abundantly less work, thus: Conceive the time past noon to be divided into four equal parts; then, according to the question, the time till midnight is equal to five such parts; and, consequently, the time from noon to midnight is equal to nine such parts. Wherefore, as 9··4::12h.·· 5h. 20m. the answer.

QUESTION 20TH.

Sold goods for 63*l.* and by so doing lost 17 per cent. whereas I ought, in dealing, to have cleared 20 per cent. then how much under their just value were they sold?

It is manifest, that as many goods as cost 100*l.* must be sold for 83*l.* to make the loss 17 per cent, and to make the gain 20 per cent. they must be sold for 120*l.* and, therefore, as 83*l.*··120*l.*::63*l.*··91*l.*1*s.*8$\frac{20}{83}$*d.* and 91*l.*1*s.*8$\frac{20}{83}$*d.* —63*l.*=28*l.*1*s.*8$\frac{20}{83}$*d.* the answer.

QUESTION 23D.

A merchant who hired a clerk for 50*l.* per annum, payable quarterly, has (agreeably to a subsequent contract) retained the young man's salary in trade for 11 years and an half, on conditions of allowing him 6 per cent. compound interest on the several payments as they become due; how much has he now in the merchant's hands?

This question is wrought by case 1st of Annuities; but not by table III; for that table will not apply on account of the half year. Wherefore it must be wrought by the first part of the rule to case 1st, thus: 1.06—1=.06, the divisor; and 1.8982985×1.029563=1.954417, the ratio involved to the time; and therefore, 1.954417—1=954417, and .954417÷.06=15.90695, 15.90695×50×1.022257=813*l.*1*s.* the answer.

QUESTION 24TH.

In what time will 20*l.* a-year raise a stock of 167,877*l.* compound interest being computed at 6 per cent. per annum?

This is wrought by case 3d, annuities, thus: 167.877 × .06 ÷ 20 + 1 = 1.503631, which is the ratio involved to the time; and, therefore, if we involve the ratio up to this number, the power to which it is raised will indicate the time; or, if we look in table II, under 6 per cent. we shall find a number nearly equal to this, and opposite to it, 7 years, which is the answer.

QUESTION 25TH.

Which would be preferable, an annual rent of 365*l.* clear for 12 years, to be received in quarterly payments, or 3000*l.* in hand, reckoning interest at 5 per cent.?

This is wrought by case 4th of annuities, thus: 8.86325 × 365 × 1.018559 — 3000 = 295*l.* 2*s.* 5$\frac{3}{4}$*d.* the answer.

QUESTION 26TH.

When $\frac{1}{2}$ of the members of an assembly + 15 were met, there were $\frac{1}{3}$ + 10 absent; how many did that branch of the legislature consist of?

Many work this question by Position; but it may be wrought with much less work, thus: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$, which, according to the question, is the whole assembly all to 15 + 10 = 25; and, therefore, $\frac{1}{6}$ of the whole assembly is 25; and if so, 6 times 25 must be the whole assembly. Whence, as 1··25::6··150, the answer.

QUESTION 27TH.

Which is the most advantageous, a term of 15 years in an estate of 500*l.* per annum? or the reversion of the same estate for ever after the expiration of that time; interest reckoned at 5 per cent.?

This is wrought by case 4th of Annuities, and by case 1st of Perpetuities in reversion, thus: 10.37965 × 500 = 5189.825 = the present worth of 500*l.* annuity. Again, 2.0789281 × .05 = .103946405, and 500 ÷ .103946405 = 4810.171, and 5189.825 — 4810.171 = 379.654 = 379*l.* 13*s.* 1*d.* the answer.

QUESTION 28TH.

What annuity, now commencing to continue 19 years, may be purchased for a bill of 2000*l.* payable 3 years hence; reckoning compound interest at 6 per cent.?

This is wrought by case 2d of Compound Interest, and by case 5th of Annuities, thus: 2000÷1.191016=1679.239 = the present worth. Again, 1679.239÷11.15811 = 150.494=150*l.* 9*s.* 10½*d.* the answer.

QUESTION 29TH.

For a certain lease to continue 7 years, *A* offers 650*l.* down, and 200*l.* per annum: *B* would give 150*l.* in hand, and 300*l.* a-year: pray which is the better bid, supposing interest at 5 per cent.?

This is wrought by case 4th of Annuities, thus: 5.78637×200+650=1807*l.* 5*s.* 5¾*d.*=*A*'s bid. Again, 5.78637× 300 + 150 = 1885*l.* 18*s.* 2½*d.*=*B*'s bid. Whence, 1885*l.* 18*s.* 2½*d.* — 1807*l.* 5*s.* 5¾*d.* = 78*l.* 12*s.* 8¾*d.* the answer.

QUESTION 32D.

In what time will any sum of money double itself at 6 per cent. simple interest?

It is evident, that it will take one sum just as long to double itself as another; and, therefore, all we have to do is to find the time that it will take 100*l.* principal to gain 100*l.* interest; to do which, it is manifest, that we must say, as 6*l.*∶1*y.*∷100*l.*∶16*y.*8*mo.* the answer.

QUESTION 33D.

In what time will money be doubled at 6 per cent. compound interest?

Here, we are to find the time that it will take 100*l.* to amount to 200*l.* which must be done by case 3d of Compound Interest, thus: 200÷100=2=the ratio involved to the time; and we find, by the table, that the time corresponding with this number is between 11 and 12 years; and, therefore, it must be found by the note under the rule, thus: 2.012964—1.8982985 =.1138979, and 2—1.8982985=.1017015; and, as .1138979∶1*y.*∷.1017015∶.8929*y.* and .8929+11=11.8929*y.* the answer.

QUESTION 34TH.

A person in Philadelphia, about 70 years of age, has an income of 10*l.* per annum for ever, which is not sufficient for his subsistence, and he is become too infirm to labour; he therefore offers his perpetuity for an insured maintenance during his life; how much a-year would that be for his board, &c. according to the probability of life annuities; reckoning interest at 5 per cent.?

This is wrought by case 1st of Perpetuities, and by Life Annuities, thus: $10 \div .05 = 200 =$ the present worth of the perpetuity, which, divided by the proper number in table VI, will give the annuity; thus, $200 \div 5.77 = 34.66$, the answer.

QUESTION 42D.

Take the aliquot parts $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ successively one from the other out of 6*s.* $9\frac{1}{7}$*d.* and give their sum.

This question is worded in such an abstruse, or rather unintelligible manner, that it appears to me to be next to impossible for a person to know what is to be done from the words of it. To make it explicit, it should, in my opinion, be worded in the following manner, viz.

Take the $\frac{1}{3}$ of 6*s.* $9\frac{1}{7}$*d.*, the $\frac{1}{3}$ of $\frac{1}{4}$ of it, the $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of it, and the $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of it, and give their sum

The shortest way of working this question is this:

$$\frac{1}{3} = \frac{1}{3}$$
$$\frac{1}{3} \text{ of } \frac{1}{4} = \frac{1}{12}$$
$$\frac{1}{12} \text{ of } \frac{1}{5} = \frac{1}{60}$$
$$\frac{1}{60} \text{ of } \frac{1}{6} = \frac{1}{360}$$

These 4 fractions being reduced to a common denominator, by the most convenient method, and added together, will become $\frac{120}{360} + \frac{30}{360} + \frac{6}{360} + \frac{1}{360} = \frac{157}{360}$, and $\frac{157}{360}$ of 6*s.* $9\frac{1}{7}$*d.* or $\frac{157}{360}$ of $\frac{568}{7} = \frac{11147}{315} = 2s. 11\frac{1}{4}d. \frac{173}{315}$, the answer.

QUESTION 44TH.

E can mow an acre of grass $7\frac{1}{3}$ hours, and *F*. in $8\frac{4}{5}$ hours; in what time would they mow an acre both of them working together?

There are several ways of working questions like this; but the most convenient way, I think, is this: Multiply one man's time by the other's for a dividend, and add them together for a divisor; the quotient will be the time required; thus, $7\frac{1}{3}\times8\frac{4}{5}=\frac{22}{3}\times\frac{44}{5}=\frac{968}{15}$ = the dividend; and, $\frac{22}{3}+\frac{44}{5}=\frac{242}{15}$ = the divisor; and, therefore, $968\div242=4$ the answer.

QUESTION 45TH.

In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40 cherries; how many trees does the orchard contain?

This is another of those questions which many work by Double Position, for the want of a knowledge of a better way.

It is evident, that if we add all the fractions together, and subtract their sum from a unit, the remainder will be the same part of the orchard, that the sum of 60 and 40 is. Wherefore the question may be wrought thus: $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, and $\frac{1}{1}-\frac{11}{12}=\frac{1}{12}$; and, therefore, as $\frac{1}{12}\cdot\cdot100::\frac{1}{1}\cdot\cdot1200$, or, as $1\cdot\cdot100::12\cdot\cdot1200$, the answer.

QUESTION 46TH.

A person who was possessed of $\frac{2}{5}$ of a vessel, sold $\frac{5}{8}$ of his interest for 375*l.* what was the ship worth at that rate?

Here, we say, as $\frac{5}{8}$ of $\frac{2}{5}\cdot\cdot\frac{375}{1}::\frac{1}{1}\cdot\cdot1500$; or, $\frac{8}{5}\times\frac{5}{2}\times\frac{375}{1}\times\frac{1}{1}=1500$*l.* the answer.

QUESTION 47TH.

If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{4}$ of $\frac{7}{8}$ of $\frac{12}{15}$ of the cargo, *valued at 1000l.* what did both ship and cargo cost?

Here, we say, as $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$..$\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{13}{13}$ of $\frac{1000}{1}$ :: $\frac{1}{1}$..

$\frac{98000}{117}$, or, $\frac{7}{5}\times\frac{8}{3}\times\frac{5}{4}\times\frac{2}{9}\times\frac{7}{8}\times\frac{13}{13}\times\frac{1000}{1}\times\frac{1}{1}=\frac{98000}{117}$ = 837*l.* 12*s.* 1*d.* $\frac{25}{39}$ = the value of the ship; and, therefore, 837*l.* 12*s.* 1*d.* $\frac{25}{39}$ + 1000*l.* = 1837*l.* 12*s.* 1*d.* $\frac{25}{39}$, the answer.

QUESTION 48TH.

A younger brother received 1560*l.* which was just $\frac{7}{12}$ of the elder brother's fortune; and $5\frac{3}{8}$ times the elder's money was $\frac{2}{3}$ as much again as the father was worth; what was his estate valued at?

Here, we say, as $\frac{7}{12}$..$\frac{1560}{1}$ :: $\frac{1}{1}$..$\frac{18720}{7}$, which is the elder brother's fortune. Again, $\frac{18720}{7}\times5\frac{3}{8}=\frac{100620}{7}$, which, agreeably to the question, is $\frac{2}{3}$ of twice the father's estate; and if so, it must be $\frac{4}{3}$ of his estate; and, therefore, $\frac{3}{4}$ of it must be his estate. Whence, $\frac{100620}{7}\times\frac{3}{4}=\frac{75465}{7}$ = 10780*l.* $\frac{5}{7}$, the true answer.

The answer to this question in the Assistant is erroneous.

QUESTION 49TH.

A gentleman left his son a fortune; $\frac{5}{16}$ of which he spent in 3 months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only 537*l.* left; what did his father bequeath him?

Here, $\frac{16}{16}-\frac{5}{16}=\frac{11}{16}$, and $\frac{3}{4}$ of $\frac{5}{6}$ of $=\frac{11}{16}\frac{55}{128}$, and $\frac{11}{16}-\frac{55}{128}=\frac{33}{128}$; and as $\frac{33}{128}$..$\frac{537}{1}$ :: $\frac{1}{1}$..$\frac{68736}{33}$ = 2082*l.* 18*s.* 2*d.* $\frac{2}{11}$; or, as 33··537::128··2082*l.* 18*s.* 2*d.* $\frac{2}{11}$, the answer.

THE END.