Wm. D. Harris

Arithmetical Magazine;

OR,

Mercantile Accountant,

ADAPTED TO

THE COMMERCE

OF

THE UNITED STATES OF AMERICA:

CONTAINING

SOME USEFUL IMPROVEMENTS

IN

Mercantile Arithmetic:

A NEW SYSTEM OF EXCHANGE,

IN WHICH THE REAL AND IMAGINARY CURRENCIES OF THE DIFFERENT COMMERCIAL STATES IN THE WORLD ARE REDUCED TO

THE FEDERAL STANDARD.

WITH

CALCULATIONS ON THE MECHANICAL POWERS.

BY W. M. FINLAY.

New-York:

PRINTED BY G. F. HOPKINS, FOR THE AUTHOR, AND SOLD BY THE PRINCIPAL BOOKSELLERS IN THE UNITED STATES.

1803.

TO

THE COMMERCIAL INTERESTS

OF THE

United States of America,

The active Supporters of Trade and Agriculture,

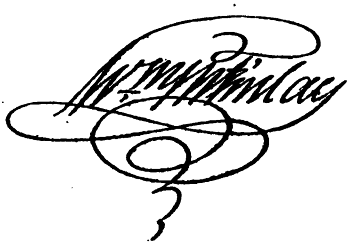
THIS SYSTEM

OF

MERCANTILE ARITHMETIC

IS INSCRIBED WITH DEFERENCE,

BY THEIR OBEDIENT SERVANT,



NEW-YORK, *June* 22, 1803.

INTRODUCTION.

ARITHMETIC is a science of such general utility, that a perfect knowledge of it is indispensably necessary to the farmer and mechanic, as well as to the merchant; its importance in education is great, as its application is useful; it is the foundation of the useful arts, the introduction to philosophical investigation; by it the merchant regulates his commerce, and by it the artisan or philosopher ascertains the certainty of theoretical researches.

To acquire expedition and exactitude in accounts are of the greatest importance; they are at once both elegant and useful accomplishments: a system of arithmetic should therefore exhibit the plainest methods of calculation, together with every useful abbreviation that numbers are capable of; the rules should be perspicuous, plain and intelligent; and every part, tending to explanation, expressed in the simplest language possible.

Aukwardness and *embarrassment* are the general attendants of youth on quitting the school for the counting house. This arises solely from the difference between the theory of the school and the practice of this office. Care should be taken to remove this, by reducing the mode of instruction, in Schools, to a similarity of the transaction of business.

Books, with new names, have been multiplied; but the compilers have candidly confessed, that the mercantile parts have been copied chiefly from foreign authors: this by no means answers for American commerce; it only tends to lead the student into error; and the accountant must form a new theory for himself, before he can regulate his own, or his employer's concerns.

The commerce of America is now co-extensive with the globe: a knowledge of all foreign monies, weights and measures are therefore necessary, to form the complete *American merchant*. No pains or labour have been spared in collecting the best information on this head; and every part of mercantile arithmetic is explained in such a manner, as to convey perfect ideas of the transaction of business, together with furnishing means for the application of *speculative genius*, well aware that youth should be exercised in the abstruse and scientific, as well as in the practical parts of arithmetic: by this means they acquire a mode of reasoning which gives them a promptitude and exactness in calculation, which cannot be practised by any who have not properly studied the power of numbers.

The part on *Exchange*, is only an extract from a system designed as a *Commercial Dictionary* (now preparing for the press) in which the money, weights, measures, productions, manufactures, commerce, laws of trade, trading and banking companies of foreign nations, are considered with respect to the commerce of the U. States. The other parts of arithmetic are treated of in an extensive manner; the prolix methods hitherto used are superseded by such useful and plain abbreviations, particularly in interest and proportion, as will render it of the utmost utility in the school and the counting house.

In a land like *America*, where Commerce explores her ample rivers to their sources; where there is not a farmer but is more or less concerned in trade; where, at a word, the nation, in its most remote parts, is *commercial:* to such a people, whose interests and advantages connect them in mutual dependence, and reciprocal intercourse, a genuine system of mercantile arithmetic should be acceptable.

Though the Editor has taken all possible pains to preserve the work free from error; yet he would remind the reader of a truth, that no man is infallible in accounts; he hopes, therefore, that those which have imperceptibly crept in, will be excused by the impartial reader.

Relying on the *merits* of the work, he does not court the protection of individual recommendation, but offers it to the consideration of the *candid*—CERTAIN OF SUCCESS, *if merit marks its pages;* or, on the contrary, DESERVED NEGLECT, *though partial interest might patronise it.*

CONTENTS.

Arithmetical Magazine.

PART I.

Definitions.

Q. WHAT is arithmetic?

A. Arithmetic is that part of the mathematics which explains the powers and properties of numbers.

Q. What is a number?

A. One or more quantities, answering the question "how many?"

Q. Why do you consider arithmetic as an art or science?

A. From theory and practice.

Q. What is theoretical arithmetic?

A. Theoretical arithmetic considers the nature and power of numbers; demonstrates the reason of practical operations; and, in this sense, arithmetic is a science.

Q. What is practical arithmetic?

A. Practical arithmetic applies the usefulness of numbers to the regulation of business;—in this sense arithmetic is an art.

Q. What is the nature of arithmetical calculation?

A. From numbers that are given to find others that are required.

Q. Which are the fundamental rules of arithmetic?

A. These five, viz.—Notation, Addition, Subtraction, Multiplication, and Division.

Notation.

Q. WHAT is notation?

A. Notation, or numeration, teaches to read and write numbers according to their true value.

Q. How is a number expressed?

A. All numbers are expressed by the different disposition of the following ten characters, called figures:

	Cypher.	One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.
viz.	0	1	2	3	4	5	6	7	8	9.

Q. But how can all numbers be expressed by these figures?

A. Every figure standing alone will express its own value, as 7 expresses seven, 9 nine; but, beside their simple value there expressed, they receive a new value from the place they occupy in numbers expressed by several figures—for the value of the places increase in a tenfold proportion infinite. The first place is that on the right hand, the next towards the left, &c.

Thus: 4 Four.
40 Forty, or ten times four.
400 Four hundred, or one hundred times four, &c.

and so on continually, every figure to the left hand having ten times the value it would have in the next place towards the right; hence it follows, that every ten of the value of the lower place is *equal to one of the next higher.* But, as the frequent repetition of tens would occasion confusion, different terms are used to express the local value of the figures, by the following table.

TABLE I.

C. of 1000 quadrillions.	X. of thousands of q.	thousands of q.	hundreds of q.	tens of q.	quadrillion.	C. of 1000 of trillions.	X. of thousands of trillions.	thousands of t.	hundreds of t.	tens of t.	trillions.
7	3	5	8	4	6	7	9	6	3	0	4
5th.						4th.					

C. of thousands of billions.	X. of thousands of billions.	thousands of b.	hundreds of b.	tens of b.	billions.	C. of thousands of millions.	X. of thousands of millions.	thousands of m.	hundreds of m.	tens of m.	millions.	C. of thousands.	X. of thousands.	thousands.	hundreds.	tens.	units.
8	4	7	2	5	6	3	9	8	0	3	2	6	4	8	3	8	7
3rd.						2nd.						1st period.					

Every round of six figures is called a period, and the table may yet be continued further, by having a name for the highest place of each succeeding period. A period may be subdivided into members of three figures each.

The first period may be called . . . Units.
2nd . . . Millions.
3d . . . Billions.
4th . . . Trillions.
5th . . . Quadrillions.
6th . . . Quintillions.
7th . . . Sextillions.
8th . . . Septillions.
9th . . . Octillions.

Of the Cypher.

The *Cypher*, or, as it is usually called, *nought*, is of itself of no signification, and put to the left hand of a figure alters not its value.

Thus 06
006
0006 only expresses 6, because it is still in unit's place; but, *being put to the right hand*, it increases the value of the figure in a tenfold proportion to the place it occupies.

Thus .... 60 Sixty.
600 Six hundred.
6000 Six thousand.

These definitions are sufficient. For the table of numeration, the teacher may direct the pupil to Table I. and show him how it is to be read.

Roman Notation.

The *Romans* made use of the following seven letters of their alphabet as *numeral characters*, viz:—

I.	V.	X.	L.	C.	D.	M.
1	5	10	50	100	500	1000.

The different disposition of which is found sufficient to express any number.

These characters do not increase or decrease in value according to their distance from unit's place, but each numeral represents only its simple value, however variously situated. Thus,

X *represents ten.* XX *two ten's, or* 20.
XXX *three ten's, or* 30, &c.

When a numeral of less value follows, or is placed to the right of one of greater value, they are added. Thus,

VI=6. XV=15. LVI=56. LX=60. &c.

When a numeral of less value precedes, or is placed to the left of one of greater value, the less is subtracted from the greater. Thus,

IV=4. IX=9. XL=40. LD=450.

The numerals I and C combined thus IↃ are valued 500, and for every C added the value is increased ten times. Thus,

IↃ=500. IↃↃ=5000. IↃↃↃ=50000.

The numerals C and Ↄ combined with I, thus, CIↃ is valued 1000, and for every C prefixed and subjoined the value is increased ten times. Thus,

CIↃ=1000. CCIↃↃ=10000. CCCIↃↃↃ=100000, &c.

A line, or dash, drawn above any numeral increases its value one thousand times. Thus,

$\overline{V}$ *is valued* 5000. $\overline{X}$=10000. $\overline{C}$=100000. $\overline{M}$=1000000, &c.

Addition.

Q. WHAT is addition?

A. Addition is the *collecting of several numbers into one sum,* which shall be *equal* to all the numbers taken together.

Q. How is addition performed?

A. Let the numbers be so placed that each figure may stand directly underneath, or in a perpendicular line with, the figures of the same value; i. e. *units under units, tens under tens, hundreds under hundreds,* &c.

Then draw a line beneath them, and begin the addition in *unit's place,* set the unit figure of this amount in unit's place, and carry *one* for every *ten* in the amount to the *next column, (i. e. one at* 10, *two at* 20, *three at* 30, *four at* 40, *&c.)* proceed through each place in the same manner, and set down the sum of the last column.

Thus	475
	326
	843
	792
	2436

EXAMPLES.

$ 47632	$ 1234	℔ 123456	Yds. 3865
12345	5678	78912	472
67891	9123	34567	384
23456	4567	89123	796
78912	1234	4567	8654
34567	5678	8912	27
89123	1234	3456	97634
45678	5678	7891	4284
91234	1234	234	328
56789	9876	567	65
12345	1234	891	4
67891	5678	234	307
23456	1234	56	263
		7	45

5. Bought 8 casks indigo, No. 1 weighing 210℔; No. 2, 196℔; No. 3, 265℔; No. 4, 145℔; No. 5, 179℔; No. 6, 135℔; No. 7, 117℔; and No. 8, 135℔; neat. How many pounds must I pay for? *Answer,* 1382℔.

6. If from the Creation to the flood was 1650 years, from the flood to the calling of Abraham 427, from that time to the founding of the temple 1010, from that to the foundation of Rome 266, from that to the christian era 752, and since that 1803 years. How long is it since the creation? *Ans.* 5908 years.

7. How many strokes does a well regulated clock strike in a week? *Ans.* 1092.

8. What day of the year was the 17th of August, 1798? *Ans.* 239th.

9. How many days in a year? *Ans.* 365.

Note.—*Thirty days hath September,*
April, June, and November;
February hath twenty-eight alone;
And all the rest thirty-one.
Every 4th year is called bissextile, or leap year, and then the month of February has 29 days.

10. If from New-York to Philadelphia be 95 miles, thence to Brandywine 30, thence to Gunpowder ferry 64, to Patapsco ferry 20, to Annapolis 30, to Portroyal 64, to Williamsburg 84, to Nansemond 45, to Bell's ferry 68, to New-river ferry 127, to George-town 145, and thence to Charleston 106. How many miles from New-York to Charleston? *Ans.* 768.

11. What year of the world was the universal deluge in? See Genesis vii. 5, 6. *Ans.* 1650.

12. On settling with sundries, I find I owe for house-rent 250 dollars, to the baker 35, butcher 84, brewer 18, taylor 39, mercer 47, merchant 375, for storage 64, sundries 33. I want to know my entire debt. *Ans.* 945 dols.

To prove addition, add all the columns upward for the first and downward for the second addition; and if there be no difference in the amounts, the sum may be said to be right.

The character of addition is + which signifies *plus*, or more.

Table of Addition and Subtraction.

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

To use this table for addition, *take either of the two figures* you have to add on the left hand, and *underneath the other*, in a line *opposite the first*, is the sum of both.

For subtraction, *take the figure* you have to subtract on the left, and *opposite to it* find the number you are to subtract it from; in a line above that number you will *find the remainder at the top.*

Though this table of *addition and subtraction is simple* in itself, it is of the *greatest consequence* to the learner; as nothing will facilitate him more than to have a perfect knowledge of the *sum and difference* of two numbers. It will save the *awkward custom* learners have of counting on their fingers, and drawing lines on their slates.

Subtraction.

Q. WHAT is subtraction?

A. Subtraction shows how *to separate one sum from another*, and show the *difference between them.*

Q. How is subtraction performed?

A. Place the number to be subtracted beneath the other, *units under units*, *tens under tens*, &c. If the lower figures to be subtracted be less than their correspondent figures, the difference is had immediately. But if the higher figures be less than their correspondent figures, you must add 10 to each as you proceed, and subtract the *lower figure from this increase*, carrying one to the next figure on the left, &c.

From	643852
Take	274312
Difference	369540

Thus the difference of 2, 1, and 3, and 2, 5, 8, is found immediately to be 540; but 4, the next figure, being greater than 3, I add 10 to 3=13, and subtracting 4 I find 9 difference: carrying 1 to 7=8, being more than 4, I increase 4 by 10=14, and find the difference 6: carrying 1 to 2=3, deducted from 6, leaves 3 the difference, viz. 369540.

To prove subtraction, add the difference to the less number. The sum should be equal to the greater.

The character of subtraction is −, which signifies *minus*, or less.

EXAMPLES.

1. From 64753048004		2. From 10000004300
Take 23009058306		Take 9000004345
Rem.		
Proof.		

3. From 380046385	4. From 238565479
Take 154796	Take 4567899
Rem.	
Proof.	

5. Bought 2000 yards of linen for £466, and sold 1476 yards for £369. How many yards have I left, and how much do I want to make up the first cost?

Ans. 524 yards left. Wanting £97.

6. What number added to 376 will make 432. *Ans.* 56.

7. If my age be 29 years this present year (1803), in what year was I born? *Ans.* 1774.

8. In 4 bags were £500; in the first 34, in the second 42, in the third 19. What was the content of the fourth?

Ans. £405.

9. Five notable events took place in 215 years time, viz. 1st. the invention of the mariner's compass; 2nd. gunpowder; 3d. printing; 4th. America; 5th. the reformation. The last was brought about Anno 1517; the third 77 years before; the second 42 years after the first; and the fourth 148 years after the second. In what year did each happen?

Ans. Compass 1302. Gunpowder 1344. Printing 1440. America 1492. Reformation 1517.

10. If a merchant owes $1000, and pays in cash 280, likewise his draft on Peter Paywell for 156; how much does he still owe? *Ans.* $564.

11. The earth's main distance from the sun is computed at 95173000 miles; that of Venus 68891486. When they are both on the same side of the sun, they are are in *perigee;* and when on opposite sides, they are in *apogee.* How much farther are they in apogee from each other than in perigee?

Ans. Venus is 137782972 miles farther from the earth in apogee than when in perigee.

12. A. borrowed from B. $6038, and paid him as follows:—First payment 374; second 465; third 875; and fourth 3335. I demand how much is owing of the debt? *Ans.* $989.

Multiplication.

Q. WHAT is multiplication?

A. Multiplication is *a compendious method of performing many additions.*—It consists of three parts, viz. the *multiplicand,* or number to be multiplied. The *multiplier,* or number to be multiplied by. The *number produced* by these factors is called the *Product.*—The *Product* contains the *multiplicand* as often as the *multiplier contains units,* &c.

Q. Why do you call multiplication a compendious method of addition?

A. Suppose I receive from 4 men 6 dollars each, viz. A. B. C. and D. *I would place them in the order of addition, and find their sum.*

Viz. A. 6
B. 6
C. 6
D. 6

24 Sum, or 6 told 4 times.

But as this is tedious, I take a more compendious method, and say 4 times 6 make 24.

The character of multiplication is ×.

TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

USE.—*Opposite the multiplier,* on the left, and *directly* under the figure to be *multiplied,* is the product. Thus, opposite 6, on the left, and under 8, is the product 48, &c.

RULE.—*Multiply the first figure of the multiplicand by the first of the multiplier. If the product be less than* 10, *set it down in unit's place; but if more than* 10 *or ten's, set down the excess only, and carry one for every* 10 *to the product of the next figure,* *and so on. Only observe, that the first figure of the product of every multiplying figure must stand directly under itself.*

EXAMPLES.

Multiply	476385	7643846
By	6	7
Product	2858310	
Multiply	6483728	43567849
By	8	9
Product		
Multiply	374297	486438
By	10	11
Product		
Multiply	8486737	386547
By	12	12
Product		

Secondly.—When the multiplicand and multiplier are mixed with cyphers.

Multiply as before, still observing the place and value of the cypher in the multiplier.

```
      406030 - - - 4
       60304 - - - 4
    --------
     1624120
   12180900
 24361800
 -----------
 24485233120 - - - 7
```

To prove multiplication, multiply the multiplier by the multiplicand: if the products be the same, the work is right.

Otherwise, *Cast the nines out of the multiplicand and multiplier, setting their different excesses after each other. Multiply these excesses together, cast the nines from their product, and set the excess opposite the product. Then cast the nines from the product; and if the excess agrees with the opposite figure, the work is right.*

10.	68404679003	11.	64480047960
	80907064		700422600
12.	987600004	13.	860090000
	479800		708009

14. 3790006009
    56007890

15. 848000658000
    300605400

When the multiplier is such, that the product of any two figures will measure it, multiply the given number by one of the figures, and the product found by the other will be the answer.

EXAMPLES.

16. Multiply 654 by 35
I find 7 x 5=35
Therefore, 654
5
―――
3270
7
―――
Ans. 22890

17. 721 by 64
8 x 8=64 therefore 8
―――
5768
8
―――
Ans. 46144

18. Multiply 476043 by 63

19. 4860325 by 84

20. Multiply 457654 by 27

21. 860004 by 96

When the multiplier is 10, 100, 1000, *or unity with any number of cyphers, the number is multiplied by prefixing the number of cyphers in the multiplier to the multiplicand.*

22. Multiply 3764
By 1000 *Ans.* 3764,000

23. Multiply 4896
By 10 *Ans.* 4896,0

24. Multiply 649638
By 10000 *Ans.* 649638,0000

Simple Division.

Q. What is Divison?

A. Division is the *separating of any quantity or sum* into any number of parts *required. Or the finding how often one number is contained in another.*

Q. What are these numbers called?

A. That which divides is called the *divisor:* the number to be divided, the *dividend:* the third *number found* by these is called the *quotient, which shows how often the divisor is contained in the dividend:* and that which remains, after the division is performed, is called the *remainder, which is always of the same name with the dividend.*

Q. Why is division called a compendious method of subtraction?

A. That division is a short method of subtraction I prove thus: Suppose 24 dollars were to be divided between 6 men equally 24

First, I would give each man one, viz. . . .	6
Remains to be distributed . .	18
Secondly, I would give each man one . . .	6
Remains . . .	12
Thirdly, I would give each man one . . .	6
Remains . . .	6
Fourthly, I would give each man one . . .	6
Remains . . .	0

Now, as I have given each man one 4 times, each man must have 4 dollars; but this being too tedious, I take a quicker method, and try how often 6 is contained in 24.

6)24(4 times, or 4 dollars to each man.

RULE.—Set the divisor to the *left hand of the dividend*, and try how often the divisor is contained in the highest figure of the dividend; but if the said figure be less than the divisor, take the two first figures, and try how often the divisor is contained therein, *noting the multiplying figure for the quotient*; the product of which subtract from the said figures; to the remainder bring down the next figure, which put in *unit's place: this number becomes the dividend. Then proceed as before, till you have all the figures in the dividend brought down. The numbers thus found are called the quotient.*

The character of division is $\div$.

EXAMPLE.

```
7)8654(1236
  7···
  —
  16
  14
  ——
   25
   21
   ——
    44
    42
    ——
     2
```

Thus I try how often 7 is contained in 8654. *I say 7 in* 8 *once. One I place in the quotient, and put 7 under* 8, *which I subtract; remains* 1, *to which I bring down* 6=16, *in which 7 is contained twice, which I make my second quotient figure, putting the product under* 16, *viz.* 14, *and subtract as before; remains* 2, *to which I bring down* 5=25, &c.

Q. Suppose the divisor contains more figures than one?

A. Compare the highest figure in the divisor with the highest figure in the dividend, and see how often it is contained, making an allowance for what may be carried from the foregoing figure.—Or, if the highest figure in the *dividend* is less than the highest in the *divisor*, compare the *highest figure* in the divisor with the *two highest* in the dividend, making an allowance, as above. *When this is found, proceed, as before taught, by multiplying the divisor from unit's place, putting the multiplying figure as first or highest figure in the quotient, and the product under the leading figures in the dividend. Subtract, and proceed as before taught.*

```
856543)4276546746(4992
       3426172···
       -------
        8503747
        7708887
        -------
         7948604          6
         7708887         4X6
         -------          6
          2397176
          1713086
          -------
Remainder  684090
```

8)76543865	9)78846325
11)4567890123	12)456789101112
15)461400043	25)865432568
37)12345678	39)432123456
64)846543263	87)456386543
99)9876785432	376)456789123
4564)8465738046	9876)97865438547
398654)4685465474	7896547)84654765484

904630476)45678904325646

730050963)678912345678943

4005906405)84567986543256 43

987564321)456789386547964

376500004)800000000000

60000004)600000000000000

To prove division, multiply the quotient and divisor together, adding in the remainder. If the product agrees with the dividend, the work is right.

The most expeditious method is by casting the nines out of the different numbers, as follows:—

Make a ×. *Cast the nines out of the divisor—set the excess on the left. Cast the nines from the quotient—set the excess on the* *right. Multiply these figures together, cast the nines from the product, and carry the excess to the remainder; from which cast the nines, and set the excess at the top. Then cast the nines from the dividend, and set the excess at the bottom. If the top and bottom figures agree, the work is right.*

Short Division.

To *divide by* 12, or any figure under, in a line, try how often the divisor is contained in the first part of the dividend as before, *making the subtraction in your mind*, and carry the difference to the next figure, *setting no figure down but in the quotient.*

EXAMPLE.

6)45643854

7607309

Here I try how often 6 *is contained in* 45. *I find* 7 *times, which I place under the second figure in that part of the dividend, and carry* 3 (the remainder) *to* 6=36, *which, divided as before, quotes* 6, *no remainder. Then* 6 *in* 4 *nought times; but take* 4 *to* 3=43, *divided by* 6=7 *times*, &c.

If a divisor be such, that the product of any two figures will measure it, divide the dividend by either of these figures, and the quotient found by the other figure. This last quotient will be the answer sought.

EXAMPLE.

Divide 476543 by 35. Now 5 times 7=35.

Therefore: 7)476543

68077+4

5)13615+18

Here I divide mentally by 7, *and find* 4 *the remainder; then I divide by* 5, *the next figure, and find my quotient with* 2 *remainder. Then multiply the first divisor by this last remainder, and add in the first remainder, which equals* 18, *the full remainder at the end of the division.*

In dividing by 10, 100, 1000, &c. *cut off as many figures from the right of the dividend as the divisor contains units*, and the work is *done*. If I divide 47634 by 100, I point off 34 for the two cyphers in 100, which 34 is the remainder.

100)476..34	7)43065465
3)678943	8)43867654
4)56789463	9)479893256
5)476548042	10)475485473
6)74685473	12)4784376504

Problems resulting from the foregoing Rules.

1. Having the sum of two numbers, and one of them given to find the other; *subtract the given number from the given sum. The remainder is the number required.*

Let 475 be the sum and 144 the number known.—Required the other.

475 Sum.
144 Given number.
331 Required.

2. Having the greater of two numbers, and the difference of it and a less, given to find the less. *Subtract one from the other.*

475 Greater.
331 Difference.
144 Less required.

3. Having the less, and difference given to find the greater. *Add them together.*

144 Less.
331 Difference.
475 Greater.

4. Having the product of two numbers, and one of them given to find the other. *Divide the product by the given number, the quotient will be the number required.*

Let the product of two numbers be 120, and 8 the number given, required the other.

8)120
15 Number sought.

5. Having the dividend and quotient to find the divisor. *Divide the dividend by the quotient.*

COROLLARY.—*Hence we find another method of proving division.*

6. Having the divisor and quotient given to find the dividend. *Multiply them together.*

Questions to exercise the foregoing Rules.

By an application of the foregoing problems, the following questions may be *elegantly solved.*

1. What is the difference, and what is the sum of six dozen dozen and half a dozen dozen? *Ans.* 936 sum, 792 difference.

2. The remainder of a division sum is 432, the quotient is 423, the divisor is the sum of both and 19 more: What is the dividend? *Ans.* 200934.

3. There is a certain number which, being divided by 7, the quotient multiplied by 3, that product divided by 5, from the quotient subtract 20, to the remainder add 30, and half the last sum shall be 35. *Ans.* 700.

4. What number is that which, being added to 9709, will make 10901? *Ans.* 1192.

5. A sheep-fold was robbed three nights successively, the first night half the sheep were stolen, and half a sheep more; the second night half the remainder were taken, and half a sheep more; the third night they took half of what were left, and half a sheep more; by which time they were reduced to 20: How many were there at first? *Ans.* 167.

6. What number must I multiply by 7, that the product may be 623? *Ans.* 89.

7. The product of two numbers is 31383450, and one of them 4050: The other factor is required. *Ans.* 7749.

8. What number deducted from the 26th part of 2262 will leave the 87th part of the same? *Ans.* 61.

9. There are two numbers, the greater is 73 times 109, and their difference is 17 times 28: What is their sum and product? *Ans.* Sum 15438. Product 59526317.

Numbers of divers Denominations.

NUMBERS of *divers denominations* are those whereby we express the sundry divisions of *Money*, *Weight*, and *Measure*.

The *following weights and measures*, as established in the U. S. of America, are agreeable to the *standard weights and measures* preserved in the Exchequer of Great Britain, under the care of an officer.

Troy Weight.

gr. 24 grains	make	1 pennyweight, dwt.
20 dwt.	make	1 ounce . . . . . oz.
12 oz.	make	1 pound . . . . . ℔.

grs.	dwt.		
24	=1	oz.	
480	=20	=1	℔.
5760	=240	=12	=1

By Troy weight is weighed gold, silver, jewels, and liquors.

The standard for the gold coin of the United States of America is 11 *parts of pure gold melted with one of alloy, which is of the same fineness of British gold.*

The standard for the silver coin of the United States is 1485 *parts of pure silver, and* 179 *parts of alloy, which must be wholly of copper.*

Money of the United States.

10 Mills	make	1 Cent	*Real monies.*
10 Cents		1 Dime	
10 Dimes		1 Dollar	
10 Dollars		1 Eagle	

Weight of the federal coins.

		dwts.	grs.
Gold.	Eagle . .	11	6
	Half do. .	5	15
	Quarter do.	2	19½

		dwts.	grs.
Silver.	Dollar .	17	8
	Half do.	8	16
	Quarter do.	4	8
	Dime . .	1	17⅗
	Half do.		20⅘

		dwts.	grs.
Copper.	Cent . .	11	
	Half do.	5	12

Current Money of Great Britain and Ireland.

	Current in Ireland, at
4 farthings, qrs. make one penny.	
12 pence 1 shilling	£. 0 1 1
20s. 1 pound, £.	1 1 8
21s. 1 guinea, ...	1 2 9
5s. 1 crown, ...	5 5
2s. 6d. half do.	2 8½

Guineas, half-guineas, crowns, half-crowns, shillings, sixpences, and *pence,* are the real coins of Great Britain; the £. or pound, is *imaginary.* Their characters are, £. *s. d.*

Jeweller's Weight.

1 ounce	makes	152 carats.
1 carat		4 grains.

And these grains are again divided into $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, &c. parts. The above are the divisions of an ounce Troy.

Apothecaries Weight.

20 grains, grs.	make	1 scruple, ℈
3 scruples		1 dram, ʒ
8 drams		1 ounce, ℥
12 ounces		1 pound, ℔

Apothecaries do not use these denominations now, as they find avordupoise weight more convenient.

Avoirdupoise Weight

is used in weighing merchandize of all kinds. Its denominations are as follow:

16 drams, drs.	make	1 ounce . . . oz.
16 ounces		1 pound . . . ℔.
28 ℔		1 quarter . . qr.
112 ℔ or 4 qrs.		1 hundred . . cwt.
20 cwt.		1 ton . . . . T.

Long Measure

is used in measuring distances; likewise to take the dimensions, or the length, breadth, and thickness of all bodies: its denominations are;

3 barleycorns	make	1 inch . . . . *i.*
12 inches		1 foot . . . . *f.*
3 feet		1 yard . . . . *yd.*
$5\frac{1}{2}$ yds. $16\frac{1}{2}$ feet		1 pole or perch Eng. *p.*
40 poles		1 furlong . . . . . *fr.*
8 furlongs		1 mile . . . . *m.*
3 miles		1 league . . . . . . *L.*
$69\frac{1}{2}$ English, or 60 geographic miles		1 degree . . . . . °.
360 degrees		1 great circle of the globe or sphere.

$7\frac{92}{100}$ inches	make	1 link.
25 links		1 pole.
100 links		1 chain.
10 chains		1 furlong.
8 furlongs		1 mile.

Inches.		Feet.								
12	=	1		Yd.						
36	=	3	=	1		Pole.				
198	=	$16\frac{1}{2}$	=	$5\frac{1}{2}$		1		Furlong.		
7920	=	660	=	220	=	40	=	1		Mile.
63360	=	5280	=	1760	=	320	=	8	=	1

Inches.		Link.								
$7\frac{92}{100}$	=	1		Pole.						
198	=	25	=	1		Ch.				
792	=	100	=	4	=	1		Furlong.		
7920	=	1000	=	40	=	10	=	1		Mile.
63360	=	8000	=	320	=	80	=	8	=	1

4840 sq. yards = 1 acre.

Square Measure

is used to find the content of surfaces: its denominations are:

144 square inches	make	1 square foot.
9 —— feet		1 —— yard.
$30\frac{1}{4}$ —— yards		1 —— pole.
40 —— poles		1 —— rood.
4 —— roods		1 acre.

Otherwise.—$62\frac{7264}{10000}$ square inches	make	1 square link.
265 square links		1 do. pole.
100000 do. do.		1 acre.

Solid or Cubic Measure.

1728 solid inches	make	1 cubic foot.
27 —— feet		1 yard.
40 feet of round, or 50 feet square timber		1 ton.
128 feet, or 8 feet long, 4 feet broad, and 4 feet thick		1 cord for fuel.

Cloth Measure.

4 nails	make	1 quarter . . . qr.
4 quarters		1 yard . . . . yd.
3 ——		1 ell Flemish . E. F.
5 ——		1 ell English . E. E.
6 ——		1 ell French . E. F.

Wine Measure.

2 pints	make	1 quart.
4 quarts		1 gallon.
42 gallons		1 tierce.
63 gallons		1 hogshead.
126 galls. or 2 hhds.		1 pipe or butt.
252 galls. or 2 pipes		1 ton.
84 gallons		1 puncheon.
10 do.		1 anker of brandy.
18 do.		1 rundlet.

This measure was originally founded on the ℔ Troy, as it was ordered that 8℔ of wheat gathered from the middle of the ear, and well dried, should make a gallon.

Note.—One pint of this measure is equal to 1℔ Troy, and contains $28\frac{7}{8}$ cubic inches, or 231 solid inches to the gallon of wine, brandy, cider, &c. according to the standard of the British exchequer. The wine gallon is ascertained to be equal to 8 ℔. 1.4 oz. avoirdupoise. A cylindrical vessel 7 inches in diameter, and 6 inches deep, is found to answer almost exactly to a gallon of the above measure, and is appointed to be a legal wine gallon.

Pints.	Quart.	Gall.	Tierce.	hhd.	Punch.	Pipe.	Ton.
2	1						
8	4	1					
336	168	42	1				
504	252	63	1½	1			
672	336	84	2	1⅓	1		
1008	504	126	3	2	1½	1	
2016	1008	252	6	4	3	2	1

Winchester Measure.

2 pints make 1 quart.
4 quarts make 1 gallon.

Ale:
8 gallons make 1 firkin
16 do. or 2 firkins make 1 kilderkin
32 do. or 2 kilderkins make 1 barrel
48 do. or 1½ barrels make 1 hhd.

Beer:
9 do. make 1 firkin
18 do. or 2 firkins make 1 kilderkin
36 do. or 2 kilderkins make 1 barrel
54 do. or 1½ barrels make 1 hhd.
72 do. or 1⅓ hhds. make 1 puncheon
108 do. or 1½ punch. make 1 butt

A gallon of this measure contains 282 cubic inches, and is equal to 10lb 3 oz. avoirdupoise.

Beer.

Pints.	Quart.	Gal.	Firkin.	Kilderk.	Bar.	Hhd.	Punch.	Butt.
2	1							
8	4	1						
72	36	9	1					
144	72	18	2	1				
288	144	36	4	2	1			
432	216	54	6	3	1½	1		
576	288	72	8	4	2	1⅓	1	
864	432	108	12	6	3	2	1½	1

Ale.

Pints.	Quart.	Gall.	Firkin.	Kilderk.	Bar.	Hhd.
2	1					
8	4	1				
64	32	8	1			
128	64	16	2	1		
256	128	32	4	2	1	
384	192	48	6	3	1½	1

Dry Measure.

2 pints		make	1 quart.
2 quarts		make	1 pottle.
4 quarts		make	1 gallon.
2 gallons		make	1 peck.
8	do. or 4 pecks	make	1 bushel.
64	do. or 8 bushels	make	1 quarter.
288	do. or 36 bushels	make	1 chaldron.
320	do. or 5 quarters	make	1 wey.
640	do. or 2 weys	make	1 last.

The standard for dry measure is a Winchester bushel, 18½ inches wide at top and bottom, and 8 inches deep. One gallon of this contains 268⅘ cubic inches, and is less than an ale gallon by 13⅕ solid inches.

Time.

60 seconds	make	1 minute.
60 minutes	make	1 hour.
24 hours	make	1 day.
7 days	make	1 week.
4 weeks	make	1 month.

This is called domestic time.

13 months, 1 day, 6 hours, or 365 days, 6 hours, make one circle or julian year. But by the Calendar the year is divided into 12 parts, called months, as follows:

January hath	31 days.	July hath	31 days.
February	28	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

The odd 6 hours make every 4th year one day more, and this odd day is added to the month of *February*. This year is called bissextile, or leap-year. The reason is, the solar year of 365 days being short of the tropical year by nearly 6 hours.

365 days, 6 hours, 9 minutes, 14 seconds, make one periodical year, or the time of the earth's departure from any fixed point in the heavens, till its return to the same.

365 days, 5 hours, 48 minutes, 57 seconds, make one tropical year. The Ecliptic having a retrogade motion, the sun will arrive at the Equinox, or first point of Aries, before his revolution is completed, and this space of time is called a tropical year.

Motion.

			Characters.
60 seconds	″ make	1 prime minute,	′
60 minutes, or miles	make	1 degree,	°
30 degrees	make	1 sign,	s.
12 signs, or 360 degrees	make	1 great circle of the sphere,	

called the Zodiac, in which are marked the twelve signs:

The sun enters

Aries	♈	March 20	Libra.	♎	Sept. 22
Taurus . . .	♉	April 19	Scorpio . . .	♏	Oct. 22
Gemini . . .	♊	May 20	Sagittarius . .	♐	Nov. 21
Cancer . . .	♋	June 21	Capricorn . .	♑	Dec. 21
Leo	♌	July 22	Aquaries . . .	♒	Jan. 19
Virgo	♍	August 22	Pisces	♓	Feb. 18

The Zodiac is a circle which cuts the Equator obliquely, and in which the above 12 signs are represented, in the middle of which is supposed another circle, called the Ecliptic, from which the sun never deviates in his annual course, and in which he advances 30 degrees each month.

Miscellaneous.

220℔ make a barrel of beef or pork.
196℔ —— —— flour.
100℔ —— —— gun-powder.
120℔ —— seam of glass.
16℔ —— stone of feathers.
112℔ —— quintal of fish.
14℔ —— stone of shot.
19½ cwt. —— fodder of lead.
6 feet —— fathom.
4 inches make a hand.
12 articles — dozen.
5 dozen — roll of parchment.
12 do. — small groce.
12 groce, or 144 dozen, make a great groce.
24 sheets make a quire of paper.
20 quires — ream do.
2 reams — bundle do. &c.

Compound Addition.

RULE.—*Place all the numbers so that those which are of the same name may stand directly underneath each other; then begin with the lowest denomination, and add them into one sum, divide this sum by that number which one of the next higher contains of the name added, the remainder set down under the name added, and carry your quotient figure to the next denomination, which add up as before, and so proceed from denomination to denomination, till all is added.*

PENCE TABLE.

	s.	d.		s.	d.
12 pence is	1	0	130 pence is	10	10
20	1	8	140	11	8
30	2	6	150	12	6
40	3	4	160	13	4
50	4	2	170	14	2
60	5	0	180	15	0
70	5	10	190	15	10
80	6	8	200	16	8
90	7	6	210	17	6
100	8	4	220	18	4
110	9	2	230	19	2
120	10	0	240	20	0

or £1 currency.

¼ one farthing.
½ two farthings, or one halfpenny.
¾ three farthings.

I consider the above rule as the most *expeditious*, and of course the most *perfect* for addition *Nothing makes a clerk appear more aukward than dotting or marking a sum in addition.* For proof, add it the contrary way.

$	C.		$	C.
6438	72		47	14
364	56		38	75
796	50		46	84
463	25		672	85
796	84		8	72
647	26		8	96
36	22		54	87

The addition of dollars and cents being the same as whole numbers, any further examples would be useless. The character of dollars is $, *of cents* c; *but the character of cents is still understood, as* $500.50, *five hundred dollars fifty cents.*

£.	s.	d.
3	14	6
2	16	8
7	19	6

£.	s.	d.
375	16	8
27	12	6½
64	18	6

£.	s.	d.
743	14	8¼
222	12	2½
333	13	3

£.	s.	d.
4	15	11
3	16	9
7	16	4
3	14	6
7	19	8
73	19	4½

£.	s.	d.
72	19	3¼
6	17	9¼
8	18	8¼
3	19	7½
6	15	5
20	7	6

£.	s.	d.
676	16	6½
444	14	4
100	10	10¼
8	18	8
4	19	9¾
345	17	6½

£.	s.	d.
44	14	4¼
55	15	5¾
67	13	6½
88	16	4½
33	17	9
44	16	3½
17	12	8

£.	s.	d.
33	0	8½
83	19	9¼
64	16	9½
33	14	5
86	19	5¼
64	14	6¼
20	14	8

£.	s.	d.
464	14	9½
375	16	8¼
43	14	8
49	19	9½
64	19	9
37	17	11¾
77	19	7¼

NOTE.—Some point their pence at 120 or 10 shillings, and proceed with the excess to the end of the column, setting down the excess of the number above even shillings, and carrying the even shillings more 10, for the point before mentioned, which they likewise add in the same manner, only observe to set down under shillings the figure in unit's place, and carry as before to the tens, which divide by 2, because every 2 ten shillings make 1 pound; the half found carry to £, and if one remains, set it to the left hand in shillings place.

Troy Weight.

℔	oz.	dwt.	gr.
10	14	16	22
3	10	18	16
14	4	12	23

℔	oz.	dwt.	gr.
4	10	16	18
2	9	18	22
3	7	19	20

Avoirdupoise Weight.

cwt.	qr.	℔	oz.	dr.
33	3	22	14	15
74	1	15	12	10
38	1	12	8	9

T.	cwt.	qr.	℔	oz.	dr.
22	14	3	27	14	16
18	19	3	26	15	15
17	16	2	22	12	13

Liquid Measure.

T.	hhd.	gal.
14	2	13
16	1	54
73	3	22
14	1	19
18	0	62
33	1	17
14	3	02

hhd.	gal.	qt.	pt.
43	32	3	1
13	18	1	1
12	37	2	0
14	19	1	0
12	2	3	1
19	19	1	0
20	39	1	1

Compound Subtraction.

RULE.—*Place the denominations under each other, and begin the subtraction in the lowest place, subtracting it from the number of the same denomination above it (if it be greater;) but if the lower number be greatest, then subtract it from that number which one of the next higher contains of the denomination you subtract, and to the remainder add the upper number, the sum is the true remainder required; then carry one to the next denomination, and proceed as before, till all be subtracted.*

From	$ 7438650	54
Take	238675	60
Rem.		

$ 76543865	72
67386	94

	£.	s.	d.
From	17964	14	8
Take	2986	17	6¼
Rem.			

£.	s.	d.
37	14	9¼
21	19	9¾

	£.	s.	d.
From	10000	0	0
Take	0	0	0¼
Rem.			

£.	s.	d.
1000	0	0
999	19	11¼

	£.	s.	d.
From	1009	09	09¼
Take	990	19	10½
Rem.			

£.	s.	d.
100	1	1¼
99	19	9½

Avoirdupoise Weight.

	cwt.	qr.	lb	oz.	dr.
From	340	3	14	12	13
Take	109	3	17	12	14
Rem.					

	T.	cwt.	qr.	lb	oz.	dr.
From	22	18	1	3	9	14
Take	1	19	2	27	15	15
Rem.						

Troy Weight.

	lb	oz.	dwt.	gr.
From	12	8	14	12
Take	8	10	14	23
Rem.				

	lb	oz.	dwt.	gr.
From	2	3	5	4
Take	1	10	16	18
Rem.				

Cloth Measure.

	yds.	qr.	n.
From	18	1	3
Take	14	2	1
Rem.			

	yds.	qr.	n.
From	74	1	1
Take	36	1	2
Rem.			

Long Measure.

	L.	m.	fr.	p.
From	69	2	4	30
Take	20	1	5	37
Rem.				

	m.	fr.	p.
From	74	3	20
Take	2	4	36
Rem.			

	yds.	ft.	in.	bar.
From	14	2	8	1
Take	9	2	10	2
Rem.				

Liquid Measure.

	hhd.	gal.	qt.
From	34	36	1
Take	25	47	2
Rem.			

	T.	hhd.	gal.	qt.	pt.
From	45	0	26	1	1
Take	27	3	37	2	1
Rem.					

Time.

	Y.	m.	d.	h.	m.	s.
From	64	8	20	12	45	32
Take	25	9	28	14	56	38
Rem.						

The month is esteemed 30 *days.*

Use of Compound Addition and Subtraction.

New-York, Jan. 1st, 1803.

Cash, *Dr. to Sundries.*

To Peter Paywell, received on account		$	450	50
Timothy Cash	in full		75	44
John Moore	do.		68	05
Thomas Trueman	do.		86	84
Edward Ireland	do.		78	86
George Gray	do.		75	14
Henry Pike	do.		64	16
Nicholas Ray	do.		88	88

I demand the amount received. *Ans.*

New-York, 2d Jan. 1803.

Sundries *Dr. to Cash, paid the following*:

I. A. Merchant	$	654	75
Taylor		25	40
Brewer		22	25
Baker		76	75
Fuel		35	25
Grocer		22	
Chandler		30	25
Sundries		45	
Teas		400	50
Cheese		74	27
Flour		484	

What is the amount paid? *Ans.*

3. A gentleman going to the country orders his creditors to send in their bills, which were as follows, viz.—Brewer £41 10*s.* butcher £112 0*s.* 6*d.* baker £24, tallow-chandler £13 8*s.* taylor £37 9*s.* 9*d.* draper £74 13*s.* 6*d.* coach-maker £214 16*s.* 8*d.* wine-merchant £68 11*s.* 10*d.* confectioner £16 2*s.* rent £50, servants' wages £46 14*s.* 6*d.* and he would carry with him £50 to defray his expenses: For what sum must he draw on his banker to defray these bills? *Ans.* £

4. Bought 6 bags hops, viz.—No. 1, weighing 2 *cwt.* 2 *qrs.* 10℔, No. 2, 2 *cwt.* 1 *qr.* 16℔, No. 3, 2 *cwt.* 0 *qr.* 24℔, No. 4, 2 *cwt.* 3 *qrs.* No. 5, 2 *cwt.* 1 *qr.* 12℔, and No. 6, 2 *cwt.* 1 *qr.* 16℔: How many cwt. are in the whole? *Ans.*

5. In a gentleman's service of plate, were 14 dishes, weighing 16℔ 10 *oz.* 13 *dwts.* 36 plates, wt. 35℔ 10 *oz.* 11 *dwts.* 6 salts, 2℔ 8 *oz.* knives and forks, 6℔ 11 *oz.* 9 *dwts.* 4 salvers, 9℔ 5 *oz.* 4 *dwts.* cups and tankards, 22℔ 0 *oz.* 18 *dwts.* 22 *grs.* tea-kettle,

4℔ 6 *oz.* 9 *dwts.* 17 *grs.* lamp, 5℔ 4 *oz.* 16 *dwts.* 23 *grs.* What was the entire weight? *Ans.*

6. A merchant's clerk receives of sundries, viz —Of A. $259 and half a crown; of B. $172.50; of C. $656.97; of D. $485.84; of E. $1754.30; of F. $00.75; of G. $808.08; of H. $694.84; and of K. $9.99: How much had he to settle with his empolyer for? *Ans.*

7. A merchant brings 600 salt ox hides, wt. 561 *cwt.* 0 *qrs.* 2℔, of which he sells 250 hides, weighing 239 *cwt.* 3 *qrs.* 25℔: How many hides has he left, and what do they weigh?

Ans. 350 hides, 321 *cwt.* 0 *qrs.* 5℔.

8. A store-keeper bought a piece of cloth, containing 42 yards, for £22 10*s.* of which he sells 27 yards, for £15 15*s*: How many yards has he left, and what do they stand him in?

Ans. 15 yards, £6 15.

9. A merchant bought 209 casks butter, wt. 400 *cwt.* 2 *qrs.* 14℔; shipped off 173 casks, weighing 213 *cwt.* 2 *qrs.* 27℔: How many casks has he left, and what is their weight?

Ans. 36 casks, wt. 186 *cwt.* 3 *qrs.* 15℔.

10. What five numbers of dollars and cents, all different, will make 100 dollars? *Ans.*

11. Paid A. B. in full for E. F's. draft on me for £75 as follows, viz.—Richard Drawer's note for £7 12*s.* 6*d.* John Johnson's, £5; an assignment on Robert Dealer for £17 13*s.* 9½*d.* in bank notes £40; the rest I made up in cash: How much was it? *Ans.* £4 18*s.* 8½*d.*

12. A trader failing, was indebted to A. £71 12*s.* 6*d.* to B. £34 9*s.* 9*d.* to C. £16 18*s.* 8*d.* to D. £44, to E. £66 7*s.* 6*d.* to F. £11 2*s.* 3*d.* to G. £19 19*s.* to H. £20. At the time of his disaster he had in cash £3 13*s.* 6*d.* in wares £23 10*s.* in furniture £13 8*s.* 6*d.* in plate £7 18*s.* 5*d.* in a tenement £56 15*s.* in good book-debts £87 13*s.* 10*d.* Suppose these matters are faithfully delivered, what will be the amount of loss suffered by the creditors? *Ans.* £91 10*s.* 5*d.*

13. Bought 5 hogsheads of sugar, weight as follows:

No. 1.	7 *cwt.*	1 *qr.*	18℔	Tare 3 *qrs.*	12℔
2.	6	3	24	3	4
3.	6	2	14	2	25
4.	7	0	5	3	8
5.	7	3	26	3	19

What is the neat weight? *Ans.* 31 *cwt.* 3 *qrs.* 19℔.

14. Bought from A. 6 *cwt.* 3 *qrs.* 14℔ sugar, for £27 14*s.* 8½*d.* from B. 8 *cwt.* 1 *qr.* 27℔, for £34 4*s.* 8¼*d.* Sold 7 *cwt.* 2 *qrs.* 27℔ for £38 14*s.* 9½*d.* I demand what quantity remains, and what it stands me in? *Ans.* 7 *cwt.* 2 *qrs.* 14℔ = £23 4*s.* 7¼*d.*

Compound Multiplication.

This rule, for its *expedition and certainty in calculation*, is esteemed *equal*, if not *superior*, to any other made use of: therefore a student, who wishes to become an *expert accountant*, cannot study it too closely.

CASE I.

As multiplication is a compendious method of performing addition, multiply the price by the number of the article; the amount will be the same as if you had set down the price as often as the multiplier contains units. For instance,

What cost 6 yards of cloth at £1 14s. 7½d. per yard?

	£.	s.	d.		£.	s.	d.
By addition.	1	14	7½	By multiplication.	1	14	7½
	1	14	7½				6
	1	14	7½				
	1	14	7½	£.	10	7	9
	1	14	7½				
	1	14	7½				
£.	10	7	9				

General Rule.—*Multiply the lowest denomination, and find how many of the next higher is contained therein, which add to the product of the next higher denomination, setting down the remainder under its proper name. Thus,*

I say 6 times ½ equal 3 pence (no remainder,) which I keep in mind to carry; then 6 times 7 equal 42, and 3 I carry are 45 pence, equal three shillings, and 9 pence over; the odd pence I set under pence, and carry the even shillings, viz. 3; then 6 times 4 make 24, and 3 I carry are 27; the 7 I set in unit's place in shillings, and carry 2; then 6 times 1 are 6, and 2 I carry are 8, which 8 I divide by 2, (because twice 10 shillings is equal to £1) and find 4, to carry, without a remainder; then 6 times 1 are 6, and 4 I carry are £10, equal to £10 7s. 9d.

	£.	s.	d.				£.	s.	d.
Multiply	1	16	4½	by 4.	. . .	*Ans.*	7	5	6
	2	17	9¼	5.	. . .		14	8	10¼
	3	8	10½	6.	. . .		20	13	3
	1	15	6	7.	. . .		12	8	6
	2	18	4	8.	. . .		23	6	8
	6	8	11	9.	. . .		58	0	3
	1	19	7½	11.	. . .		21	15	10½
	3	2	8¼	12.	. . .		37	12	3

	cwt.	qrs.	lb.				cwt.	qrs.	lb.
Multiply	4	3	14	by 5.	. . .	*Ans.*	24	1	14
	6	2	25	8.	. . .		53	3	4

CASE II.

When the multiplier is greater than 12, but can be measured by the product of any two numbers less than 12.

RULE.—*Multiply the price by one of the composite numbers, and that product found by the other; this last product found will be the answer.*

EXAMPLE.

What is the price of 21 yards of broad-cloth, at £2 16*s.* 8½*d.* per yard?

I find that 3 *multiplied by* 7 *is equal to* 21.

	£.	s.	d.	
Therefore,	2	16	8½	
			3	first composite number.
	8	10	1½	price of 3.
			7	
	59	10	10½	price of 7 times 3=21

	£.	s.	d.		£.	s.	d.
What cost 15 yards of linen, at	0	3	9½	per yard? *Ans.*	2	16	10½
16	0	4	8½	. . .	3	15	4
18	0	8	6½	. . .	7	13	9
21 . . . baize,	0	17	4½	. . .	18	4	10½
44	1	10	8	. . .	67	9	4
56	4	16	3½	. . .	269	12	4
64	3	17	8	. . .	248	10	8
121	1	13	6	. . .	202	13	6
72	0	18	3	. . .	65	14	0
144	0	4	9	. . .	34	4	0
48	0	6	8½	. . .	16	2	0

	cwt.	qr.	lb.			cwt.	qrs.	lb.
Multiply	3	1	4	by 28	*Ans.*	92	0	0
	0	3	14	by 36		31	2	0

CASE III.

When the multiplier is not a composite of two numbers less than 12, but exceeds such composite number by a number not greater than 12.

RULE.—*For the composite number next less than the given multiplier, find the product; then multiply the given multiplicand by the overplus—add these products together, the sum is the product required.*

EXAMPLE.

What cost 86 *cwt.* of sugar, at £3 15*s.* 6*d.* per cwt.?

Now 7 multiplied by 12 make 84+2=86.

	£.	s.	d.	
Therefore,	3	15	6×2	
			12	
	45	6	0	value of 12.
			7	
	317	2	0	value of 7 times 12, or 84.
	7	11	0	value of 2, overplus, or twice the multiplicand.
Ans.	£324	13	0	value of 86.

	£.	s.	d.		£.	s.	d.
What cost 31 yds. of brocade, at	3	13	4	per yard? *Ans.*	113	13	4
23	1	11	6	. . .	36	4	6
47	0	16	6½	. . .	38	17	5½
68	1	18	10	. . .	132	0	8
75	1	2	9	. . .	85	6	3
112	0	16	8	. . .	93	6	8
139		5	5	. . .	37	12	11
155	1	15	4½	. . .	274	3	1½

Use of Compound Multiplication.

1. Bought a piece of broad-cloth, containing 24 yards, at 15*s.* 3*d.* per yard: What is the amount? *Ans.* £18 6*s.*

2. What cost a chest of tea, weight neat 98℔, at 5*s.* 6*d.* per ℔? *Ans.* £26 19*s.*

3. Bought 39 cwt. of butter, at £1 5*s.* 6*d.* per cwt. What is the amount? *Ans.* £49 14*s.* 6*d.*

4. What cost 1 cwt. of sugar, at 7½*d.* per ℔? *Ans.* £3 10*s.*

5. Bought a hogshead of wine, at 5*s.* 4*d.* per gallon: What is the amount? *Ans.* £16 16*s.*

6. What is the rent of 120 acres of land, at 14*s.* 6*d.* per acre? *Ans.* £87.

7. A merchant sends his clerk to the bank to receive cash for a note of £75, which he receives in the following species, viz.

					£.	s.	d.
27 at	£1	2*s.*	9*d.*	- - - - - - - -			
23	1	9	1	- - - - - - -			
40	0	5	5	- - - - - - -			
Change				- - - - - - - - - - -			2
					£. 75	0	0

8. Paid John Atkinson William Steele's Bill on me for £100, in the following species:

60 guineas, at £1	2*s.*	9*d.* - - - - -	£.	*s.*	*d.*
21 moidores,	1	9	3 - - - - -			
1 do.	0	19	6 - - - - -			
Change - - - - - - - - - -					1	3
				£. 100	0	0

9. Received from John Taylor, for Paul Roberts' draft in my favour for £150, as follows:

36	pieces, at £1	18*s.*	10*d.* - - - - -	£.	*s.*	*d.*
11	do.	1	9	1 - - - - -			
153	do.	0	5	5 - - - - -			
20	do.	1	2	9 - - - - -			
					£. 150	1	8
				Returned		1	8
					£. 150	0	0

10. Received from Messrs. Brown, Talbot and co. in full, for tobacco sold them, £143 12*s.* 9*d.* in the following species: viz.

29 at	£1	9*s.*	3*d.* - - - - - - - - -	£.	*s.*	*d.*
13	3	17	8 - - - - - - - - -			
28	0	18	1 - - - - - - - - -			
16	0	18	1 - - - - - - - - -			
115½	0	1	1 - - - - - - - - -			
16½	0	5	5 - - - - - - - - -			
				£. 143	12	9

11. Bought from George Nugent the following: amount £250 0*s.* 1*d.* viz.

19 at	£3	17*s.*	8*d.* - - - - - - - - -	£.	*s.*	*d.*
25	3	17	6 - - - - - - - - -			
14	1	18	10 - - - - - - - - -			
20	1	9	3 - - - - - - - - -			
83	0	5	5 - - - - - - - - -			
8	0	1	1 - - - - - - - - -			
				£. 250	0	1

12. Bought from Thomas Salter, viz.

17 yards serge, at	3*s.*	6*d.*	- - - - -	£. *s.* *d.*
24 do.	2	9	- - - - -	
37 do. linen,	6	10	- - - - -	
49 do. cloth,	18	9	- - - - -	
44 do. cambrick,	24	6	- - - - -	
64 do. lawn,	5	7	- - - - -	
				£. 136 9 5

13. Received from Thomas James £327, for my bill of £300 sterling, on C. Bird of London, at 9 per cent. as follows:

13 at	£3	17*s.*	8*d.*	- - - - - - - - -	£. *s.* *d.*
25	3	17	6	- - - - - - - - -	
14	1	18	10	- - - - - - - - -	
29	0	19	6	- - - - - - - - -	
27	0	19	4	- - - - - - - - -	
21	1	2	9	- - - - - - - - -	
7	1	9	3	- - - - - - - - -	
9	1	9	1	- - - - - - - - -	
16	1	8	11	- - - - - - - - -	
26	0	18	3	- - - - - - - - -	
3	0	9	10	- - - - - - - - -	
2	0	9	8	- - - - - - - - -	
1	0	9	2	- - - - - - - - -	
1	0	9	0	- - - - - - - - -	
2	0	4	11	- - - - - - - - -	
1	0	4	9	- - - - - - - - -	
					£. 327 1 4
				Returned	1 4
					£. 327 0 0

14. Passed a bond for £114 10*s.* When the interest came to £19, paid off 40 guineas, and gave a fresh bond for the remainder: by the time there was £13 4*s.* 8*d.* due on the second bond, paid off 24 moidores, 2 guineas, and 6*s.* 8*d.* took up the old bond and passed a new one for the remainder; the principal ran on till there was £9 11*s.* 3*d.* interest due, when the bond was cancelled: How much was paid at settlement? *Ans.* £73 1*s.* 9*d.*

Guinea £1 2*s.* 9*d.* Moidore £1 9*s.* 3*d.*

Compound Division.

Division, being the reverse of Multiplication, may be called a concise method of performing many subtractions, or of separating any given sum into any given number of parts required, according to the nature of the question.

CASE I.

To divide a number of divers denominations by 12, or a number less than 12.

RULE.—*Divide the highest denomination; multiply the remainder, if any, by that number which one of the same denomination contains of the next lower; to that product add the next lower number, and divide as before. Thus proceed through each denomination.*

EXAMPLE.

Divide £866 16*s.* 6*d.* between 12 men equally.

```
      £.   s. d.  £. s. d.
   12)866  16 .6 (72  4  8½ each man. Ans.
      84
      ——
       26
       24
       ——
Rem. £. 2
Multiply by 20 the shillings in a £. and add 16s.
            ——
     Divide 56s. (4s.
             48
             ——
       Rem.  8s.
 Multiply by 12 the pence in a shilling, and add 6d.
             ——
     Divide 102d. (8d.
             96
             ——
        Rem. 6d.
  Multiply by 4 the farthings in a penny.
             ——
      Divide 24 (½
             24
             ——
```

This question, being worked at large, is a sufficient direction for the performance of all questions whose divisors are whole numbers; *but division may be much abridged if the reduction and subtraction be performed mentally, for all divisors at and under* 12. The foregoing question would stand thus:

	£.	s.	d.
12)	866	16	6
£.	72	4	$8\frac{1}{2}$ each.

	£.	s.	d.			£.	s.	d.
Divide	74	8	9	between 3 men equally.	*Ans.*	24	16	3
	44	18	9	- - - - 4 - - - - - - - -		11	4	$8\frac{1}{4}$
	38	12	6	- - - - 5 - - - - - - - -		7	14	6
	44	2	4	- - - - 6 - - - - - - - -		7	7	$0\frac{2}{3}$
	666	16	6	- - - - 7 - - - - - - - -		95	5	$2\frac{4}{7}$
	28	18	8	- - - - 8 - - - - - - - -		3	12	4
	22	12	2	- - - - 9 - - - - - - - -		2	10	$2\frac{8}{9}$
	18	18	6	- - - - 10 - - - - - - - -		1	17	$10\frac{1}{5}$
	12	14	7	- - - - 11 - - - - - - - -		1	3	$1\frac{8}{11}$
	1	19	9	- - - - 12 - - - - - - - -		0	3	$3\frac{3}{4}$
	cwt.	*qrs.*	lb			*cwt.*	*qrs.*	lb
Divide	10	3	14	- - - 7 - - - - - - - -		1	2	6
	yds.	*qrs.*	*n.*			*yds.*	*qrs.*	*n.*
Divide	17	3	2	- - - 6 - - - - - - - -		2	3	$3\frac{2}{3}$

CASE II.

To divide by a composite number greater than 12.

RULE.—*Divide by one component part, and that quotient divide by the other, the last quotient will be the answer.*

EXAMPLE.

Divide £865 16*s.* 4*d.* between 42 men equally.

In comparing the divisor I find $6\times7=42$.

	£.	s.	d.
Therefore 7)	865	16	4
6)	123	13	$9\frac{1}{7}$
£.	20	12	$3\frac{22}{42}$ each.

In dividing by 7, I find 1 penny remaining, which I call $\frac{1}{7}$; then, by dividing by 6, I find 3 pence remaining; with this remainder I multiply the denominator of the fraction, or first divisor, and add in the numerator, or first remainder thus, $3\times7+1=22$, for a numerator; then multiply the two divisors together, $6\times7=42$, for a denominator, which I place as a fraction, or part of a penny, $\frac{22}{42}$.

	£.	s.	d.			£.	s.	d.
Divide	29	15	0	between 21 men.	*Ans.*	1	8	4
	27	16	0	- - - 32		0	17	$4\frac{1}{2}$
	67	9	4	- - - 44		1	10	8
	269	12	4	- - - 56		4	16	$3\frac{1}{2}$
	248	10	8	- - - 64		3	17	8
	202	13	6	- - - 121		1	13	6
	65	14	0	- - - 72		0	18	3
	34	4	0	- - - 144		0	4	9

The following, not being composite numbers, must be performed by long division.

	£.	s.	d.			£.	s.	d.
Divide	113	13	4	between 31 men.	*Ans.*	3	13	4
	38	17	$5\frac{1}{2}$	- - 47		0	16	$6\frac{1}{2}$
	132	0	8	- - 68		1	18	10
	85	6	3	- - 75		1	2	9
	740	16	7	- - 100		7	8	2

CASE III.

If the given quantity consists of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, multiply the given quantity, or divisor, by 4, adding to the product 1 for $\frac{1}{4}$, 2 for $\frac{1}{2}$, 3 for $\frac{3}{4}$, &c. and it will give the divisor; then multiply the dividend by 4, and divide by your new-found divisor, the quotient is the answer sought.

EXAMPLE.

Suppose I purchase $7\frac{1}{2}$ acres of ground for £454 18*s.* 6*d.* What is that per acre?

```
              £.   s.  d.
7½ for       454  18  6
4                      4
—            ———————————
   ⎧6)1819  14  0
30 ⎨ ————————————
   ⎩5)303    5  8
     ———————————
     £. 60  13  1⅗ per acre.
```

Otherwise, multiply the whole number by the denominator, the fractional part, and add in the numerator for a divisor; multiply the dividend by the said denominator, and divide as before.

What is the value of 1 yard of cloth, if I pay £45 14*s.* 6*d.* for 7$\frac{1}{5}$ yards?

		£.	s.	d.
	7$\frac{1}{5}$	45	14	6
	5			5
36 Div.	6)	228	12	6
	6)	38	2	1
		£. 6	7	0$\frac{1}{6}$ *Ans.*

Suppose a person in trade to clear £1035 9*s.* 0$\frac{1}{4}$*d.* in 10$\frac{1}{2}$ years, what was his yearly increase of fortune? *Ans.* £98 12*s.* 3$\frac{1}{2}$*d.*

Suppose a merchant clears £518 8*s.* 0$\frac{1}{4}$*d.* in 8$\frac{3}{4}$ years, what is his yearly profit? *Ans.* £59 4*s.* 11*d.*

Use of Compound Division.

1. If a piece of broadcloth, containing 24 yards, cost £18 6*s.* what did it cost per yard? *Ans.* 15*s.* 3*d.*

2. Bought 39 cwt. of butter for £49 14*s.* 6*d.* what is it per cwt.? *Ans.* £1 5*s.* 6*d.*

3. If a hogshead of wine cost £16 16*s.* what is it per gallon? *Ans.* 5*s.* 4*d.*

4. If 27 cwt. of sugar cost £47 13*s.* 9*d.* what cost 1 cwt? *Ans.* £1 15*s* 3$\frac{2}{3}$*d.*

5. If 1 cwt. of sugar cost £3 10*s.* what is that per ℔? *Ans.* 7$\frac{1}{2}$*d.*

6. If I hold 120 acres of land for £87 yearly rent, what is that per acre? *Ans.* 14*s.* 6*d.*

7. If 35 persons spend £6 14*s.* 2*d.* what has each to pay? *Ans.* 3*s* 10*d.*

8. A man dying left an estate worth £1000, between his wife and three sons, viz. to his wife $\frac{1}{3}$, to the eldest son $\frac{1}{4}$, and the remainder to the second and third share alike: What is each person's part? *Ans.* wife £333 6*s.* 8*d.* eldest son £250, each of the others £208 6*s.* 8*d.*

9. Three merchants, A. B. and C. have a ship in Co. A. has $\frac{5}{8}$, B. $\frac{1}{4}$, and C. $\frac{1}{8}$; they receive £228 6*s.* 8*d.* freight: It is required to divide it among the owners, according to their respective shares? *Ans.* A. £143 0*s.* 5*d.* B. £57 4*s.* 2*d.* C. £28 12*s.* 1*d.*

10. Four merchants have a ship in company, A. owns $\frac{11}{32}$, B. $\frac{1}{4}$, C. $\frac{7}{32}$, and D. $\frac{6}{32}$; they charter her to the West-Indies at £22 10*s.* 8*d.* per month; she performs the voyage in 18 months: How much does each man receive? *Ans.* A. £139 8*s.* 6*d.* B. £101 8*s.* C. £88 14*s.* 6*d.* and D. £76 1*s.*

11. A father left his five sons an estate, value £500; five bills, each £48 10*s*. 6*d*. He ordered £20 to be laid out in funeral expenses, and his debts to be paid, amounting to £164; the remainder he leaves thus: to his eldest $\frac{1}{3}$, and to the other 4 equal shares: How much must each son have?

Ans. Eldest £186 4*s*. 2*d*. the others £93 2*s*. 1*d*. each.

12. A privateer having taken a prize worth £1025, it is divided into 100 shares, of which the captain is to have 11, two lieutenants each 5, twelve midshipmen each 2 shares, and the remainder is to be divided among the sailors, who are 120 in number: What is the share of each? *Ans.* Capt. £112 15*s*. lieut. £51 5*s*. midshipman £20 10*s*. a sailor £4 13*s*. $4\frac{1}{2}$*d*.

13. Ten pounds a quarter is allowed to 5 auditors of a fire-office; they attend 7 times in a quarter, and the absentee's money is always divided equally among such as do attend: A. and B. on these occasions never miss; C. and D. are twice in a quarter absent, and E. only once: At payment what had each to receive? *Ans.* A. $2\frac{1}{3}$, B. $2\frac{1}{3}$, C. $1\frac{2}{3}$, D. $1\frac{2}{3}$, and E. 2.

Reduction

is the changing of one denomination, or name to that of an other without altering the value.

Q. How many kinds of Reduction are there?

A. Two: *First,* great names are brought to small by *Multiplication: This is called Reduction Descending.*

Second. Small names are brought to great by *Division: This is called Reduction Ascending.*

Reduction descending and ascending, mutually prove each other, &c.

CASE I.

Money.

To reduce a member of a higher denomination to that of a lower.

Rule.—*Multiply the given number by that number which one of the greater name contains of the less.*

For instance, £64 may be reduced *to shillings* by multiplying by 20, because 20 shillings make £1, *viz.* 1280 *shillings, which* may likewise be reduced *to pence,* by multiplying by 12, because 12 pence make 1 shilling, viz. 15360*d*. These may be reduced to *farthings,* by multiplying by 4, because 4 farthings make 1*d*, viz. 61440 *farthings,* &c.

£
64
20
1280
12
15360
4
61440

Q. But when there are divers denominations?

A. Multiply as before, only observe, as you multiply, to add in the odd number of the name you multiply by. Thus,

Reduce £23 15 6½ to half pence.

Here I say nought multiplies nought, but *adds* 5, that is, the 5 in *unit's place of shillings*, then twice 3 = 6 more 1, in tens place of *shillings* make 7, and twice 2 = 4, viz. 475 shillings. *Second*—Twelve times 5 are 60 more 6*d.* equal 66, &c. viz. 5706 pence. *Third*—Twice 6 are 12 more 1 halfpenny make 13, &c. viz. 11413 halfpence.

£.	s.	d.
23	15	6½
20		
475 shillings.		
12		
5706 pence.		
2		
11413 halfpence.		

CASE II.

To reduce a number of a lower denomination to a higher.

RULE.—*Divide the given number by that which makes one of the next higher, and so on continually, till the highest name is arrived at. The remainder is always of the name of the dividend.*

EXAMPLE.

Reduce 11413 halfpence into pounds.

First—I divide by 2, because 2 halfpence equal 1 penny, and find 1 remainder which I note ½, viz. 5706½ pence. *Second*—These I divide by 12, because 12 pence make 1 shilling, and find 6½ pence remainder which I note, viz. 475s. 6½. *Lastly*—I divide by 20, because 20 shillings make £1 and find 15 remainder, viz. £23 15*s.* 6½*d.*

```
 2)11413
 -------
12)5706½
 -------
20)475 6½
 -------
 £. 23 15 6½
```

1. In $545 how many cents? *Ans.* 54500.
2. In $25 how many mills? 25000.
3. In £476 how many farthings? 456960.
4. In £49 18*s.* how many shillings? 998.
5. In £36 17*s.* 9*d.* how many pence? 8853.
6. In £375 17*s.* 10¾*d.* how many farthings? 360859.
7. In £38 17*s.* 11½*d.* how many halfpence? 18671.
8. In 54500 cents how many dollars? 545.
9. In 25000 mills how many dollars? 25.
10. In 456960 farthings how many pounds? 476.

11. In 998*s.* how many pounds? *Ans.* 49 18*s.*
12. In 8733 pence how many pounds? £36 17*s.* 9*d.*
13. In 360859 farthings how many pounds? £375 17*s.* 10¼*d.*
14. In 23471 halfpence how many pounds? £48 17*s.* 11½*d.*

Troy Weight.

EXAMPLE.

℔ *oz. dwt.*
In 4 2 18 how many pennyweights?
12
—
50
20
—
1018
═

1. In 354℔ how many grains? *Ans.* 2039040.
2. In 7℔ 2 oz. 15 dwts. how many pennyweights? 1735.
3. In 482℔ 7 oz. 13 dwts. how many grains? 2779992.
4. In 2039040 grains how many ℔? 354.
5. In 1735 dwts. how many ℔? 7℔ 2 oz. 15 dwts.
6. In 2779992 grains how many ℔? 482℔ 7 oz. 13 dwts.

Avoirdupoise Weight.

EXAMPLES.

cwt. qr. ℔
In 14 2 14 how many ℔?
4
—
58 quarters.
28
—
468
117
—
1638℔ *Ans.*
═

In 1638℔ how many cwt?
28)1624
—
14
4)58 14
—
14 2 14
═

1. In 17 cwt. 2 qrs. 12℔ how many ℔? *Ans.* 3092.
2. In 24 ton, 14 cwt. 3 qrs. 15℔ how many ℔? 55427.
3. In 18℔ 10 oz. 8 drs. how many drams? 4776.
4. In 24 tons, 17 cwt. 3 qrs. 17℔ 5 oz. 14 drs. how many drs? *Ans.* 14275934.
5. In 3092℔ how many cwt.? 17 cwt. 2 qrs. 12℔.
6. In 55427℔ how many tons? 24 t. 14 cwt. 3 qrs. 15℔.
7. In 4776 drs. how many ℔? 18℔ 10 oz. 8 drs.
8. In 14275934 drs. how many tons? *Ans.* 24 t. 17 cwt. 3 qrs. 17℔ 5 oz. 14 drs.

But as 112℔ makes 1 cwt. we may multiply thereby for ℔, or divide for cwt.

If there be odd quarters and pounds, add in 28 for 1 qr. 56℔ for 2 qrs. 84℔ for 3 qrs. and the odd pound, if any, placing units under units, &c.

EXAMPLE.

```
          ℔  qr. oz.
Reduce  45   3   17  to pounds.
       112
      ----
        90
       495
        84=3 quarters.
        17 odd pound.
      ----
Ans.  5141℔
```

If a remainder results after a division, it will be pounds, which divide by 28, for quarters, and the remainder will be pounds.

EXAMPLE.

```
              ℔  qr. oz.
112)5141(45   3  17
    448
    ---
     661
     560
     ---
  28)101 remains 3.
      84
     ---
      17 second remainder.
```

But if a number be added to itself, it is the same as if it were multiplied by 2; and the other figures of 112 being 1. A number is multiplied by 112 when it is put down 4 times, thus, put the number under itself, units under units, &c. Second—*put the same number one place removed to the left hand (which is the product of ten's place in the multiplier.)* Third—*put the same number one place more to the left, which gives the product of hundred's place in the multiplier.*

EXAMPLE.

```
        cwt. qr. ℔.
Reduce  45   3   17 to pounds.
        45
       45.
      45..
        84 the pounds in 3 qrs.
        17℔
      ----
      5141℔
```

```
  45 }
  45 } as if multiplied by   2
 45.        = 45 × by  10
45..        = 45 × by 100
----                   ---
5040        = 45 × by 112
  84        = 3 qrs.
  17 odd pounds.
----
5141
```

I put 45 beneath itself, which, when added=45 × 2. *Second*—I remove 45 one place to the left=45 × 10. *Third*—I remove 45 one place more to the left=45 × 100, &c. then add 84 for 3 qrs. &c.

In 12 cwt.	1 qr.	13℔	how many pounds?	*Ans.* 1385
275	3	27	- - - - - - - - - - - - - -	30911
75	2	27	- - - - - - - - - - - - - -	8483
72	1	8	- - - - - - - - - - - - - -	8100

Cloth Measure.

EXAMPLE.

In 24 yards, 3 quarters, 3 nails, how many nails?

	24 *yds.* 3 *qr.* 3 *n.*
	4
	99
	4
Ans.	399

1. In 75 yds. 1 qr. 3 n. how many nails? *Ans.* 1207
2. In 720 yds. how many ells English? 576
3. In 675 ells Flemish, how many ells English? 405
4. In 1207 nails, how many yards? 75 yds. 1 qr. 3 n.
5. In 576 ells English, how many yards? 720
6. In 405 ells English, how many ells Flemish? 675

Liquid Measure.

1. In 65 ton, 2 hhds. 7 galls. how many gallons? *Ans.* 16513
2. In 27 ton, 3 hhds, 53 galls. 5 pints, how many pints? *Ans.* 56373
3. In 16513 galls. how many ton? 65 t. 2 hhds. 7 galls.
4. In 56373 pints how many ton? 27 t. 3 hhds. 53 galls. 5 pints.

Questions for Exercise.

1. In £3400, York currency, how many crowns, dollars and cents, and of each an equal number? *Ans.* 4028 of each, and 92 cents over.

2. In 12 bags of wool, each weighing $2\frac{1}{2}$ cwt. how many stone? *Ans.* 240.

3. How many strokes does a regular clock strike in a year? *Ans.* 56940.

4. How many minutes since the commencement of the christian æra, allowing it to be —— years? *Ans.*

5. If from Newark to Philadelphia be 86 miles, I want to know how many barleycorns will reach between the two places? *Ans.* 16346880.

6. How often will a chariot-wheel, 18 feet 4 inches in circumference, turn round in running from Elizabethtown to Brunswick, supposing the distance to be 20 miles? *Ans.* 5760 times.

7. Admit a ship's cargo to be 250 pipes, 130 hhds. 150 quarter-casks, ($\frac{1}{2}$ hhds.) How many gallons were in all, and allowing every pint to be a ℔, what was the weight? *Ans.* 44415 galls. 158 ton, 12 cwt. 2 qrs.

8. The great wheel of a coach is 18 feet 6 inches, the small wheel 14 feet 4 inches in circumference; I demand how many

times oftener will the small wheel turn than the large one, in running from Newark to Springfield, if the distance be 9½ miles?

Ans. 788 times oftener.

9. Sold five packs of wool,

No. 1 weighing	5 cwt.	1 qr.	14 lb
2	5	3	14
3	6	2	04
4	4	3	26
5	6	1	12

I want to know how many stone are therein. *Ans.* 204.

10. How many boxes, each to weigh 12 lb, may be filled out of a hogshead of tobacco, containing 7½ cwt. *Ans.* 70.

11. Received from Jamaica 56 hhds. of sugar, each 12 cwt. 1 qr. 10 lb (100 lb being their cwt.) How many cwt. of 112 lb?

Ans. 617 cwt. 2 qrs.

12. Imported from Rotterdam 46 bales of cloth, each containing 24 pieces, and each piece 42 ells Flemish: How many yards are therein? *Ans.* 34776.

13. In 150000 crusades, each 400 reas, 1000 reas to a millrea of 5*s.* 6*d.* sterling: How many pounds sterling? *Ans.* £16500.

14. How many minutes since the christian æra to this day, being the — day of ——, 18 ? *Ans.*

15. The globe of the earth is 360 degrees in circumference, each degree 60 geographical miles: this body turns on its axis in 24 hours: I demand the number of miles on the equator that shall equal 15 minutes. *Ans.* 225 miles.

Arithmetical Signs and Abbreviations.

SIGN of Equality =.

This sign placed between two numerical expressions denotes an equality between them.

Sign of Addition + (*plus*).

This sign placed between two numbers, shows that they must be added,

As, $4+8=12$, *or* 4 *more* 8 *equals* 12.

Sign of Subtraction − (*minus*).

This sign placed between two numbers, denotes that the latter must be subtracted from the former.

As, $12-8=4$, *or* 12 *less* 8 *equals* 4.

Sign of Multiplication ×.

This sign placed between two numbers, shows that they must be multiplied.

As, $4\times 12=48$, *or* 4 *times* 12 *equals* 48.

Sign of Division ÷.

This sign placed between two numbers, shows that the former is to be divided by the latter.

As, $48\div 12=4$, *or* 48 *divided by* 12 *equals* 4.

Or, if two numbers are placed fractionally, it denotes the number above the line, to be divided by that below.

As, $\frac{48}{12}=4$, *or* 48, *divided by* 12 *equals* 4.

Sign of Proportion : :: :

These points placed between numbers, show that the two last are in the same proportion to each other as the two first.

4 : 8 :: 12 : 24, *as* 4 *is to* 8 *so is* 12 *to* 24.

Or, as 24 *is to* 12 *so is* 8 *to* 4.

Sign of Extraction of the Square Root $\sqrt{}$, or $\sqrt[2]{}$.

This sign placed before any number, shows its Square Root is required.

As, $\sqrt{64}=8$, *or the sq. r. of* 64 *is* 8.

Sign of Extraction of the Cube Root $\sqrt[3]{}$.

This sign placed before any number, shows its Cube Root is required.

As, $\sqrt[3]{512}=8$, *or the Cube Root of* 512 *is* 8.

In giving Theorems or Solutions.

Known numbers, or denominations, are generally represented (in abbreviation) by letters or symbols, expressive of the import of the numbers or denominations used.

$$\textit{Thus, } \frac{a-p}{tp} = r$$

Explanation.—a *less* p, *divided by the rectangle of* t *and* p, *equals* r.

Note.—*Two or more letters or symbols placed together, without a separation, are to be multiplied together.*

$$\frac{tr-p}{yxa} = n$$

The rectangle of t *and* r *less* p, *divided by the continued product of* y, x *and* a, *equals* n.

A letter, or symbol with a letter or figure above it at the right hand, shows that it must be involved as often.

Thus, $a,^3 = a\ a\ a$, *or* $12,^3 = 1728$.

That is, the third power of 12 *is required, viz.* 1728. *These figures that denote the power, or root, are called indices, or exponents of that power.*

A line, or vinculum, drawn over given characters, shows how they are to be managed.

$\sqrt[2]{80-16} = 8.$ *The Square Root of* 80 *less* 16 *equals* 8.

The Rule of Three Direct.

Q. What is the Rule of Three Direct?

A. That which teaches, from *three given numbers*, to find a *fourth proportional*, which shall bear such proportion to the *second* as the *first* does to the *third*.

Q. How are these numbers managed?

A. *First*—Let that number which raises a *supposition*, or proposes a *rate*, *price*, or *action*, be put in the *first place*; the *price*, *rate* or *action* in the *second*, and that number on which the *question is* raised, in the *third*: this disposition is called the *statement*.

Second—Multiply the second and third numbers together and divide the product by the first; the Quotient will be the answer required.

Q. How are the places of these numbers distinguished?

A. the *first* is known by the words if, or, as; the *third*, by the demand, *what will*, *what cost*, *how much*, &c. The *second* has no correspondent number given.

Q. How are the numbers managed if composed of divers denominations?

A. Reduce the *first* and *third numbers* to the lowest name mentioned in *either*, and the *second* to its lowest name, proceed as before; the *answer* then found will be of that *name* to which you reduced the *second*.

ELUCIDATION.

First—Suppose I buy 7 yards of cloth for 21 shillings, what cost 2 yards?

Here I find that 7 yards, and 2 yards, are correspondent numbers, or of the same Denomination; and that the other 21 shillings is without any corresponding number—consequently I take 21 shillings for my second number—I see likewise that 7 must be my first number, because it is distinguished from the other two by suppose; *therefore, the third number must be 2, which might have been distinguished from the rest by the words* what cost, *&c. Then stating the question, I say, as 7 yards is to 21s. so is 2 yards to 6 shillings, which 6 shillings is found thus: I multiply 21 the second number by 2s. the third; product = 42; this product I divide by 7 the first number, and find 6 the quotient, viz. 6 shillings, because the second number is shillings.*

Statement.

	yds.		*s.*	*yds.*
as	7	:	21 ::	2
			2	
			7)42	
			Ans. 6 shillings.	

Second.—When there are divers denominations, &c.

EXAMPLE.

What cost 2 cwt. 3 qrs. 14 ℔. sugar, if 3 qrs. cost £6 . 9 . 6 ?

Here I find 3 *qrs. the first number;* 2 *cwt.* 3 *qrs.* 14 ℔ *the third or correspondent number, and* £6 . 9 . 6, *the second, according to rule.—The third number I reduce to* ℔, *which is the lowest name mentioned, viz.* 322 ℔. *The first I reduce likewise to* ℔ *correspondent to the third, viz.* 84 ℔—*The second I reduce to pence, because pence is the lowest name mentioned, viz.* 1554 *pence, which I multiply by the third number,* 322 *and find* 500388 *product, this being divided by* 84 *the first number, quotes* 5957 *pence, the Ans.* = £24 16 . 5.

Statement.

```
 qrs.    £.          cwt.
as 3 : 6.9.6 :: 2.3.14
   28     20           4
   --     --          --
   84    129          11
          12          28
        ----         ---
        1554         322
         322
        ----
        3108
       3108
      4662
      ------
 84)500388      12)5957
    420            ----
    ---         2,0(49,6 5
    803            -------
    756        Ans. £24 16 5
    ---         ===========
     478
     420
     ---
      588
      588
      ===
```

Statement and Proportion.

```
    qrs.    £.        cwt.       £.
    as 3 : 6.9.6 :: 2.3.14 : 24.16.5
```

That is, as 3 *qrs. is to* £6 . 9 . 6, *so is* 2 *cwt.* 3 *qrs.* 14 ℔. *to* £24 16 . 5.

PROOF.—*To prove the Rule of Three Direct, we may use any of the following methods:*

First—Say, as the fourth number found is to the third, so is the second to the first, &c. as in the foregoing Example.

```
  4th. No.  3rd. No.  2nd. No.  as reduced to lowest names.
as 5957  :   322   ::   1554
            1554
            ----
5957)500388(84 first number as reduced.
     47656
     -----
      23828
      23828
      =====
```

Second—Or as the third is to the fourth, so is the first to the second.

3d. No. 4th. No. found. 1st. No. as in lowest names.

```
322   :  5957   ::  84
           84
         ----
  322)500388 (1554 second number as reduced.
      322
      ---
      1783
      1610
      ----
       1738
       1610
       ----
        1288
        1288
        ====
```

Third—The product of the mean or middle numbers will be equal to the product of the extremes.

```
1st. No. 2d. No. 3d. No. 4th. No.
 84  :  1554 :: 322  :  5957
         322              84
        ----            ----
        3108           23828
       3108           47656
      4662           -------
      ------          500388
      500388          ======
      ======
```

In this last Example 84 and 5957 are extremes; 1554 and 322 are mean or middle numbers, and their products are equal; but I would approve of either of the two foregoing methods of proof in preference to this.

CASE I.

The fourth number being always found in the same name with the second, if it is not the highest, reduce it to the highest denomination of its kind when it can be done.

If 7℔ sugar cost 6*s*. 3*d*½. what cost 112℔?

```
     ℔      s. d.      ℔
   as 7  :  6 3½ :: 112
           12
           ——
           75
            2
           ——
          151
          112
         ————
          302
         1661
        —————
      7)16912
        —————
       2)2416 halfpence.
        —————
      12)1208 pence.
        —————
     2,0) 10,0 8
        —————
  Ans. £5:0:8
```

NOTE. *When the price of an integer is given, state the question according to rule and find the Ans. by compound multiplication.*

If 1 Barrel cost £2.5.6, what cost 10 bbls?

```
    Bbl.     £. s. d.   Bbl.
   as 1  :   2.5.6  ::  10
                        10
             ————————
      Ans. £22.15.0
```

3. What cost 128℔ of tea at 7 shillings per ℔? *Ans*. £44.16.0.

4. What cost 139 yards linen at 6*s*. per yard? *Ans*. £41.14.0.

5. What is the price of 178℔ sugar at 17 pence per ℔? *Ans*. £12.12.2.

6. How many dollars must I pay for 150 yards of linen if 10 yards cost $5 50 cents? *Ans*. $82.50.

7. What is the amount of 324 yards lace at $2.50 per yard? *Ans*. $810.

8. What is the value of 120 cwt. at $2.25 per cwt? *Ans*. $270.

9. Bought 24 yards cloth at 15*s.* 3*d.* per yard: What is the amount? *Ans.* £18 . 6 . 0.

10. Bought 39 yards of cloth, at £1 . 5 . 6 per yard: What comes it to? *Ans.* £49 . 14 . 6.

CASE II.

When the first and third are of divers denominations, reduce them to the lowest denomination mentioned in either.

11. What quantity of brandy can I buy for $94.50, at $1.50 per gallon? *Ans.* 63 gallons.

12. At $1.25 the ounce Troy, what quantity will $64 pay for? *Ans.* 51 oz. 4 dwts.

13. If 4 ℔ 3 oz. 4 dwts. silver cost $64, what cost 1 ounce? *Ans.* $1.25.

14. At 3*s.* 4*d.* per pair, what cost 17 dozen 4 pair stockings? *Ans.* £34 . 13 . 4.

15. What cost 17 cwt. 2 qrs. 14 ℔ sugar, at £1 . 6 . 8 per cwt? *Ans.* £23 . 10.

16. At £16 . 16 the hhd. what comes 27 hogsheads 9 gallons to? *Ans.* £456.

CASE III.

When the product of the second and third is divided by the first. If there happens a remainder after the division is ended, and the quotient is not the lowest name of its kind, then multiply the remainder by that number which one of the same denomination contains of the next lower, and divide that product by the first number, noting the quotient in its proper place: Thus proceed till the lowest name be found, or nothing remains.

EXAMPLE.

What quantity of cheese can I buy for £350 . 14, if one ton cost £16 . 16?

as £16 . 6 : 1 t. :: £350 . 14
20 20
———— ————
336 7014 (20 t. 17 cwt. 2 qrs. *Ans.*
672
————
294
20 cwt. 1 ton.
————
5880 (17 cwt.
336
————
2520
2352
————
168
4 qrs. 1 cwt.
————
672 (2 qrs.
672

17. What quantity of wine can I buy for £456, at £16 . 16 per hogshead? *Ans.* 27 hhds. 9 galls.

18. A goldsmith sold a tankard for £10 . 12, at 5*s.* 4*d.* the ounce: I demand the weight. *Ans.* 39 oz. 15 dwts.

19. A butt of wine, containing 234 gallons, was sold for £62 . 8: What was it per gallon? *Ans.* 5*s.* 4.

20. An ingot of silver weighs 36 oz. 10 dwts. What is it worth at 5*s.* per ounce? *Ans.* £9 . 2 . 6.

21. What will the carriage of 17 cwt. 3 qrs. 11 ℔ come to, at the rate of 7*s.* per cwt? *Ans.* £6 . 4 . 11¼.

NOTE.—*If the divisor and dividend have each a cypher or cyphers in their lowest places, cut off an equal number of cyphers from each, and reject them entirely; the remaining significant figures will be divisor and dividend to the end.*

EXAMPLE.

22. If 36 cwt. of merchandize cost $300, how much can I buy for $500?

as $3,00 : 36 cwt. :: $5,00
5

3)180

Ans. 60 cwt.

Here 300 is divisor, 500 multiplier; the cyphers from each leave 3 and 5 in the same proportion to each other as 300 and 500 are, and of course give the same answer.

But if the figures cut off from the dividend be not all, or any of them, cyphers, they must be restored to the remainder, and the same number of cyphers retained in the divisor.

EXAMPLE.

23. If 1 cwt. 3 qrs. 4 ℔ cost $754, what cost 6 cwt. 3 qrs. 14 ℔ tea?

as 1 cwt. 3 qrs. 4 ℔	: $754 ::	6 cwt. 3 qrs. 14 ℔.
	770	84
112		672
84	52780	
4	52780	770
2,00	5805,80	

$2902 1 rem. to which I annex the 80
cut off, = 180, and multiply
by 100

2,00 180,00

90 cents. *Ans.* $2902.90.

24. Bought 9 pieces of broadcloth for £125 . 8 . 4, at 16*s*. 8*d*. per yard: How many yards were therein? *Ans*. 150½ yds.

25. If I sell 21 cwt. 1 qr. 20℔ beef for $150, how much must I charge in proportion for 650 cwt. 2 qrs. 19℔? *Ans*. $4554.68¾.

26. At $12.25 per 100, what cost 124 skeins worsted? *Ans*. $15.31¼.

27. What is the commission on $6540, at 2 per cent? *Ans*. $130.80.

28. What is the exchange between New-York and London, on £545 currency, at 75 per cent? *Ans*. £136 . 5.

NOTE.—*If the first number be greater than the product of the second and third, reduce the product to a lower denomination.*

EXAMPLE.

29. If 17 ton, 12 cwt. iron cost £165, what will 2 cwt. cost?

```
as 17 t. 12 cwt.  :  £165  ::  2 cwt.
    20                  2
   ----               ----
   352                 330 less than the first.
                        20
                      ----
                      6600     (18s. 9d.
                      352
                      ----
                      3080
                      2816
                      ----
                       264
                        12
                      ----
                      3168(9
                      3168
                      ====
```

30. If 25 hhds. of brandy cost $1417.50, what cost one gallon? *Ans*. 90 cents.

31. Suppose 7 cwt. 3 qrs. 14lb. cheese cost $88.20, what cost 1 pound? *Ans*. 10 cents.

32. What is the price of 3lb. of cheese, if 153 cwt. 0 qrs. 16lb. cost $1715.20? *Ans*. 30 cents.

CASE IV.

When any number of bales or packages are given, each containing equal quantities, reduce the content of one to the lowest name, and multiply by the number of packages; or multiply by compound, without reduction, &c.

EXAMPLE.

33. Sold 4 pieces of cloth, each $12\frac{1}{2}$ yards, for $120: What is the price of one yard?

as $12\frac{1}{2}$ yds. : $120 :: 1 yd.
4 pieces.

50

Ans. $2.40

34. If one ounce of silver be worth $1.25, what is the value of 12 ingots, each 6lb. 2 oz. 8 dwts. 8 grs? *Ans.* $1116.25.

35. Bought 6 hhds. of sugar, each 6 cwt. 3 qrs. nt. at 16*s.* per cwt. What came they to? *Ans.* £113 . 8.

36. Bought 14 bags of hops, each weighing 4 cwt. 3 qrs. 14lb. for $585: What do I pay per cwt? *Ans.* $3.

CASE V.

If the given packages, &c. be of unequal weights or measures, add them, and proceed as before.

EXAMPLE.

37. Bought 3 hhds. of brandy, containing 61, $62\frac{1}{2}$, and $63\frac{1}{2}$ gallons, at $1.25 per gallon: What do they amount to?

as 1 gall. : $1.25 :: 61 galls.
$62\frac{1}{2}$
$63\frac{1}{2}$

187
× 1.25

Ans. $233.75

38. If the above was bought at 6*s.* 8*d.* per gallon, what would it amount to? *Ans.* £16 . 16 . 8.

39. Bought 3 pipes of wine, containing $120\frac{1}{2}$, 124, and $126\frac{1}{4}$ gallons, at 5*s.* 6*d.* per gall. What is the amount?
Ans. £102 . 1 . $10\frac{1}{2}$.

40. What is the price of 4 pieces of cloth, containing 23, 24, 25, and 27 yards, at 5*s.* 5*d.* per yard? *Ans.* £26 . 16 . 3.

41. Bought 4 casks butter, the first weighing 10 cwt. 3 qrs. 27lb. second 13 cwt. 0 qrs. 1lb. third 23 cwt. 2 qrs. fourth 19 cwt. 3 qrs. 14lb. nt. at 16*s.* 8*d.* per cwt. What is the amount?
Ans. £56 . 2 . 11.

42. Sold 4 bags wool, containing, viz. No. 1, 4 cwt. 3 qrs. 15lb. No. 2, 5 cwt. 2 qrs. 12lb. No. 3, 7 cwt. 2 qrs. 5lb. No. 4, 6 cwt. 1 qr. 10lb. at 10*s.* 9*d.* per stone (each stone 16lb.) What is the amount? *Ans.* £91.

43. What would the amount be in dollars, if the above was sold at $1.75 per stone? *Ans.* $$289.59\frac{1}{4}$.

44. If the said wool was bought at $1.50 per stone, what is gained on the whole? *Ans.* $42.65⅝.

Contractions.

If the teacher supposes that his pupils understand the foregoing examples, perfectly, they may be taught to *abbreviate or contract* their work by the following methods, which being well understood will render this rule of *more intrinsic value than may appear at first view. If I can solve a question by 5 figures that would otherwise take 50, I save myself much time and labour—likewise by the fewer figures used, there is the less chance of error.*

CASE I.

When the first and third numbers fall under any of the cases of Compound Multiplication or Division; and the second is of divers denominations, we save much trouble by multiplying and dividing as there taught.

EXAMPLE.

45. If 14 ton of tallow cost £338 . 6 . 8 : What cost 17 ton?

Now × 4 × 4 + 1 = 17, and 7 × 2 = 14

as 14 t. : £338 . 6 . 8 :: 17 t.

	£338	6	8	
			4	
	£1253	6	8	*product of* 4.
			4	
	5413	6	8	*do. of* 16, *because* 4 × 4 = 16.
	338	6	8	*do. of* 1.
7 × 2 = 14 *therefore*	2)5751	13	4	*am't. of* 17, *because* 4 × 4 + 1 = 17.
	7)2875	16	8	
Ans.	£410	16*s.*	8*d.*	

46. If a captain's pay for 3 weeks be £6 . 17 . 6 what is his yearly pay? *Ans.* £119 . 13 . 4.

47. At 18*s.* 8*d.* per cwt. what cost 34 ℔? *Ans.* £0 . 5 . 8.

48. If the freight of a ship be £124 . 17 . 6, what must I receive for my $\frac{5}{32}$ parts? *Ans.* £19 . 10 . 2¼.

CASE II.

When the first term is an even part of the second or third: divide that number of which it is a part, and multiply the other by the quotient found, the product will be the answer.

EXAMPLE.

49. If I sell 28 yards of broadcloth for 168 dollars, what must I charge for 240 yards of the same quality?

as 28 yds. : $168 :: 240 yds.

Here I find 28 *contained in* 168 6 *times, which becomes my multiplier, and gives the same answer as if* 168 × 240 ÷ 28, &c.

	———	6
	6	———
		$1440.

50. What cost 86lb sugar if 27lb cost £2 . 14?

Ans. £8 . 1 . 2

51. What is the price of 12 ton 4 cwt. of iron, when 5 cwt. cost $80? *Ans.* $3904.

52. If 9 gallons brandy cost $10.25 what cost 1 hhd?

Ans. $71.75.

53. What cost 1lb silver, if 2 oz. cost 18*s*? *Ans.* £5 . 8.

54. If 16lb. cost $2.25, what cost 1 cwt? *Ans.* $15.75.

CASE III.

When the second or third number is an even part of the first; divide the first thereby, and by the quotient divide the remaining number, the quotient will be the answer.

EXAMPLE.

55. How many cwt. can I buy for $1288 if 5 cwt. cost $70?

as $70 : 5 cwt. :: $1288 (92 cwt. *Ans.*

———	126
14	———
	28
	28

Here I find 5 *contained in* 70 = 14 *times, which* 14 *becomes my divisor, and gives a quotient equal to* 70 *if the dividend were multiplied by* 5, *&c.*

56. If the price of a hhd. rum be £15 . 13 . 9, what is the price of 9 gallons? *Ans.* £2 . 4 . 10.

57. If $56.96 buy 1 cwt. what is the value of 14lb?

Ans. $7.12.

58 Bought 4 bags of wool weighing in all 28 cwt. 3 qrs. 14lb. What is the trett of said wool at 8lb. for every 3 cwt?

Ans. 2 qrs. 21lb.

CASE IV.

When the first and second, or first and third numbers, have a common measure, (that is, when some number will divide any two without a remainder) divide them thereby, and work with the quotients instead of the given numbers.

EXAMPLE.

59. If 63 gallons of wine cost $45, what cost 84 gallons?

as 63 gall. : 45$ :: 84 gall.

21) ———	4	———
3	———	4
	3) 180	
	———	
	$60 *Answer.*	

Here I find 21 *common to* 63 *and* 84, *and their respective quotients, viz.* 3 *and* 4, *are in the same proportion to each other as the original numbers* 63 *and* 84, *which are cancelled. This is so simple, that more need not be said to render this Contraction intelligible.*

60. How much beef can I buy for £100, if 802 cwt. 1 qr. 17lb. cost £650? *Ans.* 123 cwt. 1 qr. 22 lb.

61. If £1 sterling, be worth £1 . 15 New-York currency, what is the value of £100 sterling? *Ans.* £175

62. If 35*s.* New-York currency, be equal to 20*s.* sterling, what is the value of 20*s.* currency? *Ans.* 14*s.* $3\frac{3}{7}$*d.* ster.

63. What is the value of £100 New-York currency in sterling, at 75 per cent. *Ans.* £57 . 2 . $10\frac{2}{7}$

64. What is the value of £45 . 17 . 6 New-York currency in British sterling, at 75 per cent? *Ans.* £26 . 4 . $3\frac{3}{7}$

Questions for Exercise.

65. What must I pay for £1635, deducting 1 per cent. for prompt payment? *Ans.* £1618 . 13.

66. Bought 6 pipes of wine, each containing 121 gallons, at 4*s.* 9*d.* per gallon; and for prompt payment am allowed 1*s.* in the £: What must I pay for said wine, and what am I abated per cent? *Ans.* £163 . 16 .1. *Abatement* 5 *per cent.*

67. If I have owing to me £1000, and compound with my Dr. at 12*s.* 6*d.* in the £: How much must I receive?

Ans. £625.

68. A sets out from a certain place, and goes 12 miles a day— 5 days after, B sets out from the same place, and goes 16 miles a day: In how many days will B overtake A? *Ans.* 15 *days.*

69. If I buy tallow at £35 per ton, how must I sell it a ton to gain by 10 ton as much as one ton cost? *Ans.* £38 . 10.

70. A goldsmith bought of a merchant a wedge of gold which weighed 14℔ 3 oz. 8 dwt. for £514 . 4, what did he pay per oz?

Ans. £3.

71. A draper bought of a merchant 8 packs of cloth, each pack contained 4 parcels, each parcel 10 pieces, each piece 26 yards; he gave at the rate of £4 . 16 for 6 yards: What came the 8 packs to, and what did it stand him in per yard?

Ans. £6656. *per yard* 16*s.*

72. If I buy 100 yards of ribband, at 2 yards for a shilling, and 100 yards do. at 3 yards for a shilling, and sell them again at 2*s.* for 5 yards: Whether do I gain or lose, and how much?

Ans. Lose 3*s.* 4*d.*

73. Bought 45 barrels beef, at 21*s.* per barrel; 16 barrels of which being damaged, I take them on being allowed 4 instead of 3: What must I pay for them? *Ans.* £43 . 1.

74. A merchant bought 5 ton of wine for £285; by the mis-

fortune of a pipe staving, he lost 120 gallons; but, is willing to sell it so as to sustain no loss: How must he sell it per gallon? *Ans.* 5*s.*

75. A gentleman who has an estate of £265 . 19 . 2 yearly rent, would regulate his expenses so as to lay up £68 . 5 per year: How much may he spend per day? *Ans.* 10*s.* 10*d.*

76. Imported from Holland 84 pieces of linen, which cost me £537 . 12, at 4*s.* per ell Flemish: How many yards were there in all, in one piece, and what did it cost per yard?

Ans. 2016 *yards in all,* 24 *do. in* 1 *piece, at* 5*s.* 4*d. per yd.*

77. Two men buy a piece of cloth for 90 dollars, of which A pays 35 dollars, and B takes 11 yards: How many yards were in the piece, and what did it cost per yard?

Ans. 18 *yards, and* $5 *per yd.*

78. A merchant would lay out in spices $560, viz. cloves, at $4.50, mace $7, cinnamon $5.50, and nutmegs $3 per lb, would have an equal quantity of each sort: I demand the q'ty? *Ans.* 28 lb *of each.*

79. A factor bought a certain quantity of broadcloth and drugget which together cost £81; the quantity of broadcloath was 50 yards at 18*s.* per yard; and for every 5 yards of broadcloth he had yards 9 of drugget: How much drugget had he, and what did it cost him per yard? *Ans.* 90 *yds. at* 8*s. per yd.*

80. A debtor owes several persons £1490 . 5 . 10, but compounds to pay them as far as his effects will go, which amount to £931 . 8 . 7¾: what do the creditors receive per £?

Ans. 12*s.* 6*d.*

81. If 2 lb of pepper cost 25 cents, what will 60 lb of cloves come to, if 3 lb of cloves be worth 16 lb of pepper? *Ans.* $40.

82. If in 4 months I spend as much as I gain in 3, how much can I save in the year, if I gain every 6 months £150?

Ans. £75.

83. Suppose a greyhound makes 27 springs while a hare makes 25, and the springs are alike: Now if a hare is 50 springs before the dog, I would know in how many springs he will overtake her? *Ans.* 675.

84. A travels 12 miles a day; 15 days after B sets out after him: How many miles must he travel each day, to overtake A in 60 days? *Ans.* 15 *days.*

85. Bought a certain quantity of cloth at the rate of 6 shillings for every 2 yards, of which I sold a certain quantity at the rate of 18*s.* for every 5 yards, and then I found I had gained as much as 18 had cost: How many yards were sold? *Ans.* 90 *yards.*

86. If 5 and 3 make 10, what will 6 and 8 make? *Ans.* 17½.

87. If the ⅓ of 6 be 3, what will the ¼ of 20 be? *Ans.* 7½.

88. If from a rule of 3 feet long, the shadow 5 be made, what is the steeple's height in yards, that is 90 feet in shade?

Ans. 18 *yards.*

Rule of Three Inverse;

OR,

Inverted Proportion.

Q. *What is Inverse Proportion?*

A. The nature and conditions of Inverted Proportion are such, that the first number must be to the third, as the second is to the fourth; that is, the greater the third is in proportion to the first, the less must the fourth be in proportion to the second.

Q. *How is a question distinguished,* whether it be *Direct* or *Inverse?*

A. *If more do more, or less do less respect,*
It is a question in the *Rule Direct.*
But less requiring more, or greater less,
A Question in the Inverse Rule express.

RULE.—*State the question as in the Rule Direct; multiply the first and second numbers together;* divide by the third: the *quotient* will be the answer of the same name with the *second.*

EXAMPLE.

1. If 12 men make 16 perch of ditch in 4 days, in what time will 18 men finish a like number of perches?

as 12 men : 4 days :: 18 men.
12

18) 48 ($2\frac{2}{3}$ *days answer.*

Here it is evident, that 18 *men will not require so long time to make* 16 *perches, as* 12 *men would do.*

2. There was a certain building raised in 8 months by 120 workmen; but the same being demolished, it is required to be rebuilt in 2 months: How many men must be employed about it? *Ans.* 480 *men.*

3. If 28*s.* will pay for the carriage of an hundred weight 150 miles, how far may 6 cwt. be carried for the same money? *Ans.* 25 *miles.*

4. If for £5 5*s.* I have 14 cwt. carried 136 miles, how many miles may I have 24 cwt. carried foı the same money? *Ans.* $79\frac{1}{3}$ *miles.*

5. If a footman perform a journey in 3 days, when the days are 16 hours long, how many days will he require of 12 hours long to go the same journey in? *Ans.* 4 *days.*

6. How many yards of plush are sufficient to make a cloak of equal magnitude with one which hath in it 4 yards of 7 quarters wide, when the plush is but 3 quarters wide? *Ans.* $9\frac{1}{3}$ *yards of plush.*

7. How many yards of canvas that is ell-wide, will be sufficient to line 20 yards of say, that is three quarters wide?

Ans. 12 *yards.*

8. If a man perform a journey in 6 days, when the day is 8 hours long, in what time will he do it, when the day is 12 hours long? *Ans.* 4 *days.*

9. If I lend my friend £100 for 6 months, (allowing the month to be 30 days) how long ought he to lend me £1000 to requite my kindness? *Ans.* 18 *days.*

10. If 6 mowers can mow a field in 12 days, in what time will 24 mowers do it? *Ans.* 3 *days.*

11. Suppose 800 soldiers were placed in a garrison, and their provisions computed sufficient for 2 months: How many soldiers must depart, that the provisions may serve them 5 months?

Ans. 480 *men.*

12. Admit that I lent to a friend on his occasion £100 for 6 months, and he promised me the like kindness when I desired it; but when I came to request it, he could lend me only £75. The question is, how long I may keep his money to recompense my courtesy to him? *Ans.* 8 *months.*

The Double Rule of Three;

OR,

Compound Proportion.

This rule is called the Double Rule of Three, because questions therein may be solved by two statements.

It is likewise called *Compound Proportion, or the Rule of Five;* because 5 numbers are generally given *to find a 6th required. Three* of these numbers *form a supposition;* on this *supposition,* a question is raised of the *other two,* which, with the number sought, are respectively like *the former three.*

RULE.—*Let either of the two numbers of which the question is raised, be put in the first place, and its correspondent number in the third.* The *second* will be that which has no *correspondent number given.*

Three of the *five given numbers* being thus stated, find a *fourth proportional.*

Put this fourth number found, for the *second* number of the *second statement;* the remaining *number* of which the question is raised in the *third,* and its corresponding number of the same name in *the first place;* the *fourth number resulting* will be the *answer.*

EXAMPLE.

1. If the carriage of 25 ton weight for 16 miles, cost £15 10*s.* what will 40 ton cost for 9 miles?

First—as 25 t. : £15 10 :: 40 t. comes £24 16.
Sec'd—as 16 m. : £24 16 :: 9m. comes £13 19 *Ans.*

Otherwise thus:

First—as 16 m. : £15 10 :: 9 m. comes £8 14 $4\frac{1}{2}$.
Sec'd—as 25 t. : £8 14 $4\frac{1}{2}$ :: 40 t. comes £13 19 *Ans.*

But questions in this rule may be more elegantly solved by one statement—thus:

Place the *three numbers* that form the *supposition*, in this *order*: First, *the number that leads the question;* second, *that which expresses time or distance;* third, *that which denotes gain, loss, or rate*, This done, place the *remaining numbers under their respective denominations;* and under the *odd number*, put a blank——or? because the answer *required* will be of *that name.*

Second.—If the blank falls with the first or second, multiply the fifth by the *first* and *second*, and divide the product by the *third* and *fourth* for the answer.

Third.—If the blank falls with the third, multiply the fifth, fourth, and *third* together, and divide that product by the *first* and *second* for the answer.

This rule answers to *Direct* and *Inverse Proportion*, and is the best given, on account of the *abbreviation it generally admits of.*

The statement of the foregoing question will stand thus:

as 25 t. : 16 m. :: £15 10
40 : 9 :: ——?

Now as the blank falls with the third number, £15 10*s. is multiplied by* 40 *and* 9, *according to rule; then this product is divided by* 25 *and* 16, *which quotes the answer* £13 19.

But this work may be contracted:

5 as 2′5′ t. : 1′6′ m. 2 :: £15 10
8 4′0′ : 9 :: ——?

Here 5 *is common to* 25 *and* 40, *and* 8 *is common to* 8 *and* 16.

5×2=10) 139 10=£15 10×9

£13 19 *Answer.*

2. If £240 in 16 months gain £64, how much will £60 gain in 6 months? *Ans.* £6.

3. A merchant agrees with a carrier to carry 15 cwt. of goods 40 miles for 10 crowns, each crown 65 pence: How much must one pay in proportion to have 6 cwt. carried 32 miles? *Ans.* 17*s.* 4*d.*

4. If 20 cwt. is to be carried 50 miles for £5, how much will 40 cwt. cost if it was to be carried 100 miles? *Ans.* £20.

5. With how many pounds *sterling* could I gain £5 *per annum*, if with £450 I gain in 16 months £30. *Ans.* £100.

6. If £8 is gained in 12 months with £100, with how much money can I gain £8 12*s.* in 5 months? *Ans. with* £258.

7. If £60 in 6 months gain £6, what will £240 gain in 16 months? *Ans.* £64.

8. If 1 pound of thread makes three yards of linen 5 quarters broad, how many pound of thread would be wanted to make a piece of linen 45 yards long, and 1 yard broad? *Ans.* 12 *pound.*

9. If 200℔ of merchandize is carried 40 miles for 3 shillings, how many pounds may be carried 60 miles for £22 14 6?

Ans. 20200℔.

10. If for 3 shillings 200℔ of goods are carried 40 miles, how many miles might 20200℔ be carried for £22 14 6?

Ans. 60 *miles.*

11. If 200℔ are carried 40 miles for 3 shilling, how much must be paid for carrying 20200℔ 60? *Ans.* £22 14 6.

12. If 3℔ of worsted make 10 yards of stuff of 1 yard 2 qrs. broad, how many pounds will be wanted to make a piece 100 yards long and 3 qrs. broad? *Ans.* 15℔.

13. If a footman travel 240 miles in 12 days when the day is 12 hours long, in how many days may he travel 720 miles when the day is 16 hours long? *Ans.* 27 *days.*

Arithmetical Magazine, &c.

PART II.

Vulgar Fractions.

Definitions.

Q. WHAT *is a Fraction?*

A. A part or parts of any whole sum or number.

Q. *How is a fraction expressed?*

A. By two numbers placed one above the other with a line drawn between them, as $\frac{3}{4}$, $\frac{7}{12}$, $\frac{34}{45}$; *the number above the line is called the numerator, because it denotes how many parts the fraction consists of; that below the line, the denominator, because it denominates, or shows how many parts the whole was divided into.*

Q. *What is a proper fraction?*

A. That whose numerator is *less* than its denominator, as $\frac{1}{2}$, $\frac{3}{4}$.

Q. *What is an improper fraction?*

A. That whose numerator is *equal to*, or greater than its denominator, as $\frac{4}{4}$, $\frac{8}{6}$, $\frac{9}{4}$, &c.

Q. *What is a mixed number?*

A. A whole number with a fraction annexed, as $4\frac{3}{4}$, $6\frac{1}{2}$.

Q. *What is a compound fraction?*

A. A *fraction of a fraction* as $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{5}{20}$.

NOTE.—A fraction is said to be in its lowest terms *when it is expressed by the least numbers possible.*

Reduction of Fractions.

PROBLEM I.—*To find the greatest common measure of two given numbers:*

The greater by the less divide,
The less by what remains beside;
The last divisor still again
By what remains, till nought remain:
And what divides and leaveth nought,
Will be the common measure sought.

EXAMPLE.

1. What is the greatest common measure of 112 and 120?

```
112) 120
     112
     ---
       8) 112
          112  therefore 8 is the greatest common measure.
          ===
```

2. What is the greatest common measure of 26 and 62?

Ans. 2.

3. What is the greatest common measure of 279 and 403?

Ans. 31.

APPLICATION.

To reduce a fraction to its lowest terms, divide the numerator and denominator by their greatest common measure.

EXAMPLE.

1. Reduce $\frac{279}{403}$ to its lowest terms? *Ans.* $\frac{9}{13}$.

$$\frac{279}{403} \div 31 = \frac{9}{13}$$

Reduce $\frac{144}{256}$ to its lowest terms? *Ans.* $\frac{9}{16}$

$\frac{91}{117}$ - - - - - - - - - - - - - - - - - $\frac{7}{9}$

$\frac{48}{56}$ - - - - - - - - - - - - - - - - - $\frac{6}{7}$

$\frac{182}{196}$ - - - - - - - - - - - - - - - - - $\frac{13}{14}$

$\frac{216}{240}$ - - - - - - - - - - - - - - - - - $\frac{9}{10}$

Otherwise,

Divide the numerator and denominator by any *number* that will *measure* them; divide their respective quotients in like manner, till the numbers prime to each other.

EXAMPLE.

Reduce $\frac{56}{84}$ to its lowest name. $\frac{56}{84} \div 4 = \frac{14}{21} \div 7 = \frac{2}{3}$ *Answer.*

NOTE.—If the *numerator* and *denominator* be even, an even number may be tried; if they end in 5, or 5 and a cypher, 5 will be common; if they end in cyphers, *cut off the same number from each.*

Reduce $\frac{21}{36}$ to its lowest name? *Ans.* $\frac{7}{12}$

$\frac{23}{46}$ - - - - - - - - - - - - - - - - - $\frac{1}{2}$

$\frac{120}{160}$ - - - - - - - - - - - - - - - - - $\frac{3}{4}$

$\frac{4800}{7650}$ - - - - - - - - - - - - - - - - - $\frac{32}{51}$

$\frac{150}{300}$ - - - - - - - - - - - - - - - - - $\frac{1}{2}$

$\frac{350}{400}$ - - - - - - - - - - - - - - - - - $\frac{7}{8}$

PROBLEM II. To change a fraction to a given denominator:

RULE.—*As the denominator is to its numerator, so is the given denominator to its numerator.*

Otherwise,

Try how often the denominator of the fraction is contained in the given denominator, and multiply the fraction thereby.

EXAMPLE.

Reduce $\frac{2}{3}$ to a fraction whose denominator shall be 15.

$\frac{2}{3}$) 15 (5 then $\frac{2}{3} \times 5 = \frac{10}{15}$ *Answer.*

Reduce $\frac{3}{4}$ to a fraction whose denominator shall be 100.

Answer, $\frac{75}{100}$.

Reduce $\frac{7}{8}$ and $\frac{13}{28}$ to fractions whose denominators shall be 112.

Ans. $\frac{98}{112}$ *and* $\frac{52}{112}$.

Reduce $\frac{5}{8}$ to a fraction whose denominator shall be 1000.

Ans. $\frac{625}{1000}$.

PROBLEM III. To find the least common multiple :

RULE.—Place the *denominators* or *numbers* in a line ; divide by any number which will measure two or more of them, setting down the *undivided numbers* as well as the *respective quotients.* Repeat this as often as you find a number that will divide any *two ;* then multiply all the *divisors* and *quotients* together, the *product* will be the *multiple* required.

EXAMPLE.

What is the least common multiple of 3, 5, 8 and 10 ?

5) 3 5 8 10

2) 3 1 8 2

3 1 4 1 then $5 \times 2 \times 3 \times 4 = 120$ *Answer.*

Here I find 5 *will divide two numbers, viz.* 5 *and* 10 *; their quotients I set under, and bring the undivided numbers into the same line. I find* 2 *will divide two more, viz.* 8 *and* 2 *; I proceed as before, and then I multiply the divisors* 5 *and* 2, *and the quotients* 3 *and* 4, *which produce* 120.

What is the least number which 3, 4, 8 and 12 will measure?

Ans. 24.

What is the least number which 7, 8, 16 and 28 will measure? *Ans.* 112.

What is the least number which 5, 6, 12 and 16 will measure? *Ans.* 240.

APPLICATION.

To reduce given fractions to others, which shall have one common denominator.

RULE.—*Having found the least multiple, or common denominator, divide it by each denominator, these quotients multiply by their respective numerators, the products will be numerators to the common denominator.*

EXAMPLE.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to fractions of a common denominator. By the foregoing problem the common denominator is 12.

Then $\frac{1}{2}$	- - -		$6\times1=\frac{6}{12}$
$\frac{3}{4}$		12	$3\times3=\frac{9}{12}$
$\frac{2}{3}$			$4\times2=\frac{8}{12}$
$\frac{5}{6}$	- - -		$2\times5=\frac{10}{12}$

Reduce $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, to common denominators. *Ans.* $\frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{12}{60}$

$\frac{3}{8}, \frac{4}{9}, \frac{6}{13}$ - - - - - - - - - - - - $\frac{351}{936}, \frac{416}{936}, \frac{432}{936}$

$\frac{2}{5}, \frac{4}{6}, \frac{5}{9}, \frac{7}{10}$ - - - - - - - - - - - $\frac{36}{90}, \frac{60}{90}, \frac{50}{90}, \frac{63}{90}$

$\frac{5}{7}, \frac{3}{8}, \frac{13}{16}, \frac{27}{28}$ - - - - - - - - - - - $\frac{80}{112}, \frac{42}{112}, \frac{91}{112}, \frac{108}{112}$

PROBLEM IV. To reduce a whole number to an improper fraction.

RULE.—*To express a whole number fractionally, subscribe* 1 *under the given number, as* $8=\frac{8}{1}$ $16=\frac{16}{1}$; but if the denominator be given, multiply the *given number* into the *given denominator*, for the numerator to the given denominator.

EXAMPLE.

Reduce 7 to a fraction whose denominator shall be 4?

$7\times4=28$ numerator;

subscribe 4 the given denominator. *Ans.* $\frac{28}{4}$.

Reduce 5 to a fraction whose denominator shall be 19. *Ans.* $\frac{95}{19}$.
Reduce 6 to a fraction whose denominator shall be 16. *Ans.* $\frac{96}{16}$.
Reduce 8 to a fraction whose denominator shall be 18. *Ans.* $\frac{144}{18}$.

PROBLEM V. To reduce an improper fraction to a whole or mixed number:

RULE.—*Divide the numerator by the denominator.*

EXAMPLE.

Reduce $\frac{45}{13}$ to a mixed number?

13) 45 ($3\frac{6}{13}$ *Answer.*
39

$\frac{6}{13}$

Reduce $\frac{25}{7}$ to a proper number. *Ans.* $3\frac{4}{7}$

$\frac{1726}{31}$ - - - - - - - - - - - - - - - - $55\frac{21}{31}$

$\frac{600}{19}$ - - - - - - - - - - - - - - - - $31\frac{11}{19}$

$\frac{87}{8}$ - - - - - - - - - - - - - - - - $10\frac{7}{8}$

$\frac{578}{16}$ - - - - - - - - - - - - - - - - $36\frac{1}{8}$

PROBLEM VI. To reduce a mixed number to an improper fraction:

RULE.—*Multiply the whole number by the denominator of the annexed fraction, and add in the numerator, the product will be the numerator of the improper fraction, under which place the denominator.*

EXAMPLE.

Reduce $4\frac{3}{4}$ to an improper fraction.

$4\frac{3}{4}$

19 numerator.

4 denominator.

Reduce $10\frac{7}{8}$ to an improper fraction. *Ans.* $\frac{87}{8}$

$31\frac{11}{19}$ - - - - - - - - - - - - - - - $\frac{600}{19}$

$36\frac{2}{16}$ - - - - - - - - - - - - - - - $\frac{578}{16}$

PROBLEM 7. To reduce a compound fraction to a simple one:

RULE.—*Multiply all the numerators together for a numerator, and all the denominators for a denominator.*

EXAMPLE.

Reduce $\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ to a simple fraction. *Ans.* $\frac{1}{480}$.

$1 \times 1 \times 1 = 1$ numerator.

$2 \times 12 \times 20 = 480$ denominator.

But if the *numerator of any member measures the denominator of any member, these numbers may be expunged,* and proceed with the remaining members according to rule.

EXAMPLE.

Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{6}{8}$ $\frac{5}{6}$ to a simple fraction.

$\frac{3}{4}\ \frac{4'}{5}\ \frac{6'}{8}\ \frac{5'}{6} = \frac{3}{8}$ *Answer.*

Here I find that 4, 6, 5, *in the upper line, are measured by* 4, 5 *and* 6 *in the lower line. Likewise I see, that to multiply and divide by the same numbers, the result must be the same.*

Reduce $\frac{4}{6}$ of $\frac{7}{9}$ of $\frac{9}{14}$ of $\frac{6}{7}$ of $\frac{14}{16}$ to a simple fraction. *Ans.* $\frac{1}{4}$

How much is the $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{8}$ of 20 bushels of wheat?

Ans. 5 *bushels.*

PROBLEM 8. From less, or known names, to find a fraction of the highest name equal thereto:

RULE.—*Reduce the known or given denominations to the lowest name mentioned for a numerator, and subscribe one of the highest (reduced to the same name) for a denominator.*

EXAMPLE.

Reduce 13*s.* 4*d.* to the fraction of a pound.

13*s.* 4*d.*
12

160 numerator,
and 20 × 12 = 240 denominator, } = £$\frac{2}{3}$ *Answer.*

What part of a pound is 17*s.* 6*d*? - - - - *Ans.* $\frac{7}{8}$
What part of a pound is 17*s.* 9$\frac{1}{2}$*d*? - - - - - $\frac{427}{480}$
What part of a shilling is 3*d*? - - - - - $\frac{1}{4}$
What part of a yard is 3 qrs. 3 n? - - - - - $\frac{15}{16}$
What part of a cwt. is 3 qrs. 14lb? - - - - - $\frac{7}{8}$
What part of a pound is 7 oz. 10 drs? - - - $\frac{61}{128}$
What part of a hogshead is 18 gallons? - - - $\frac{2}{7}$
What part of a day is 4 hours 20 minutes? - - - $\frac{13}{72}$
What part of a pound Troy is 10 oz. 10 dwts. 10 gr? - $\frac{505}{576}$

PROBLEM 9. To find the value of a fraction in known names equal thereto:

RULE.—*Multiply the numerator of the fraction by that number which unity, or one of the same denomination contains of the next lower, and divide by the denominator, the quotient will be the answer of the same name you have multiplied by; proceed with the remainder in the same manner.*

EXAMPLE.

Reduce £0$\frac{2}{3}$ to its proper value. *Ans.* 13*s.* 4*d.*

2 numerator.
20*s.* contained in a £.

3)40

13 4

Here I multiply the numerator 2 by 20, the shillings in a pound.—The product 40 I divide by the denominator, and find 13 shillings and 1 remaining, which I multiply by 12, the pence in a shilling, and dividing as before, I find 4d. viz. 13s. 4d. the answer.

What is the value of £0$\frac{7}{8}$? - - - - *Ans.* 17*s.* 6*d.*
What is the value of £0$\frac{427}{480}$? - - - - 17*s.* 9$\frac{1}{2}$*d.*
What is the value of $\frac{3}{4}$ shilling? - - - - 9*d.*
What is the value of $\frac{15}{16}$ yard? - - - - 3 qrs. 3 n.
What is the value of $\frac{7}{8}$ cwt? - - - - 3 qrs. 14lb.
How much is $\frac{61}{128}$lb. avoirdupoise? - - 7 oz. 10 drs.
How much is $\frac{505}{576}$lb. Troy? - 10 oz. 10 dwts. 10 grs.
What is the value of $\frac{5}{7}$ of a £? - - - 14*s.* 3$\frac{3}{7}$*d.*
What is the value of $\frac{7}{8}$ ell English? - - 4 qrs. 1$\frac{1}{2}$ n.

PROBLEM 10. To reduce a fraction of a lower denomination to a fraction of a higher.

RULE.—*Find that number which unity or one of the higher denomination contains of the lower, and put* 1 *for a numerator; then proceed as directed in Problem* 7, *by reducing the compound fraction to a simple one.*

EXAMPLE.

Reduce $\frac{3}{4}$ of a shilling to the fraction of a pound.

$$\overset{s.}{\tfrac{3}{4}} \text{ of } \overset{£.}{\tfrac{1}{20}} = \overset{£.}{\tfrac{3}{80}} \text{ Ans.}$$

It is easy to conceive, that $\frac{3}{4}$ of a shilling is $\frac{3}{4}$ of $\frac{1}{20}$ of a pound: Therefore, *I multiply the numerators together for a numerator, and the denominators for a denominator.*

What part of a pound is $\frac{1}{2}d$? - - - - -	*Ans.* $\frac{1}{480}$
What part of a pound avoirdupoise is $\frac{3}{4}$ oz? - -	$\frac{3}{64}$
What part of a cwt. is $\frac{7}{8}$ of a pound? - - - -	$\frac{7}{896}$
What part of a yard is $\frac{2}{3}$ of a nail? - - - - -	$\frac{1}{24}$
What part of a pound is $3\frac{1}{2}$ shillings? - - -	$\frac{7}{40}$
What part of an ell English is 3 qrs. 2 nails? - -	$\frac{7}{20}$

PROBLEM 11. To reduce a fraction of a higher denomination to the fraction of a lower.

RULE.—*Subscribe* 1 *under that number which unity or one of the higher contains of the lower, for a denominator; then multiply the numerators for a numerator, and the denominators for a denominator to the required fraction.*

EXAMPLE.

What part of a shilling is $\frac{7}{8}$ of a pound? *Ans.* $\frac{35}{2}$.

$$\overset{£.}{\tfrac{7}{8}} \text{ of } \overset{s.}{\tfrac{20}{1}} = \overset{s.}{\tfrac{140}{8}} \text{ or } \overset{s.}{\tfrac{35}{2}}$$

Here it is manifest, that $\frac{7}{8}$ of a pound is $\frac{7}{8}$ of 20 shillings, or $\frac{20}{1}$. This is so plain that more need not be said.

What part of a shilling is $\frac{2}{3}$ of a pound? - -	*Ans.* $\frac{40}{3}$
What part of a penny is $\frac{1}{180}$ of a pound? - - -	$\frac{4}{3}$
What part of a pound is $\frac{7}{896}$ of a cwt? - - -	$\frac{7}{8}$
What part of a shilling is $\frac{7}{40}$ of a pound? - - -	$\frac{7}{2}$
What part of a nail is $\frac{7}{20}$ of an ell English? - - -	$\frac{7}{2}$

PROBLEM 12. To find what part of a greater any less number is.

RULE.—*Make the less the numerator, and the greater the denominator, of a fraction; then reduce the fraction to its lowest terms.*

EXAMPLE.

What part of 20 is 15?

$$5)\ \tfrac{15}{20} = \tfrac{3}{4} \text{ Ans.}$$

Addition of Vulgar Fractions.

RULE.—Reduce the fractions to *common denominators* (see Reduction, Problem 3); add the *numerators* for a numerator to the *common denominator.*

Second.—If it be an *improper fraction, divide the numerator by the denominator, and take the remainder for a numerator;* then carry the quotient, and proceed as in simple addition.

EXAMPLE.

1. Add $1\frac{3}{4}$, $2\frac{5}{6}$ and $\frac{7}{8}$ into one quantity.

$1\frac{3}{4}$	- - - - - - - -	$6\times3=18$	*Numerators to the common denominator* 24.
$2\frac{5}{6}$	24 com. denominator.	$4\times5=20$	
$\frac{7}{8}$	- - - - - - - -	$3\times7=21$	

$5\frac{11}{24}$ *Ans.* $\frac{59}{24}$ *an improper fraction.* $=2\frac{11}{24}$.

2. Add $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{3}{8}$ together. *Ans.* $1\frac{7}{8}$
3. Add $\frac{12}{36}$, $\frac{11}{18}$, $\frac{7}{12}$, $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$ together. $3\frac{1}{2}$
4. Add $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$ together. $\frac{271}{280}$
5. Bought 5 pieces linen, first contained $40\frac{3}{4}$, second $27\frac{1}{2}$, third $34\frac{7}{8}$, fourth $43\frac{3}{16}$, and fifth $39\frac{1}{4}$ yards: How many yards were in the five pieces? *Ans.* $185\frac{9}{16}$ *yards.*
6. Bought 4 bales spice, No. 1 wt. $150\frac{1}{2}$lb. No. 2, $139\frac{3}{4}$lb. No. 3, $162\frac{2}{3}$lb. No. 4, $170\frac{1}{8}$lb: How much was the entire weight? *Ans.* $623\frac{1}{24}$lb.

Subtraction of Vulgar Fractions.

RULE.—Prepare the fractions as in addition, *then subtract the numerator of the lower fraction from the numerator of the higher;* the remainder take as a numerator to the common denominator.

But if the lower fraction be greater than the upper, *subtract the numerator of the lower fraction from the common denominator, and to the difference add the numerator of the upper fraction, for a numerator to the common denominator;* then carry one to the whole number, and proceed as in common subtraction.

EXAMPLE.

1. From $4\frac{2}{3}$ take $2\frac{4}{5}$. Remains $1\frac{13}{15}$, *Ans.*

$4\frac{2}{3}=\frac{10}{15}$
$2\frac{4}{5}=\frac{12}{15}$
―――
$1\frac{13}{15}$

Here I find $\frac{2}{3}=\frac{10}{15}$, *and* $\frac{4}{5}=\frac{12}{15}$; *then, because the numerator* 12 *to be subtracted is greater than* 10, *the numerator to be subtracted from, I subtract* 12 *from the common denominator* 15, *remains* 3, *to which I add the upper numerator,* $10=13$, *for a numerator to the common denominator, viz.* $\frac{13}{15}$, *and carry* 1 *to* $2=3$, *which I subtract from* 4, *and remains* 1, *viz.* $1\frac{13}{15}$.

2. What is the difference of $\frac{9}{14}$ and $\frac{17}{21}$? *Ans.* $\frac{1}{6}$.
3. What is the difference of $10\frac{7}{8}$ and $12\frac{1}{2}$? *Ans.* $1\frac{5}{8}$.
4. From a piece of cloth, containing $47\frac{3}{4}$ yards, I cut $24\frac{15}{16}$, how much remains? *Ans* $22\frac{13}{16}$.
5. Four bags of money contain £500; in the first was £$130\frac{2}{5}$, in the second £$97\frac{11}{20}$, in the third £$110\frac{7}{12}$: What did the fourth contain? *Ans.* £$161\frac{1}{5}$.

Multiplication of Vulgar Fractions.

RULE.—*Reduce mixed and whole numbers to improper fractions; multiply the numerators together for a numerator, and the denominators for a denominator to the product.*

EXAMPLE.

Multiply $4\frac{3}{7}$ by 4.

$$4\frac{3}{7}$$

$$\frac{31}{7} \quad \frac{4}{1}$$

$$\frac{124}{7} = 17\frac{5}{7} \textit{ Answer.}$$

In this example I reduce $4\frac{3}{7}$ to the *improper fraction* $\frac{31}{7}$, and 4 to an improper fraction, by subscribing 1 under it for a denominator; *I then multiply the numerators*, 31 and 4, for a *numerator*, and the *denominators*, 7 and 1, for a *denominator*, and find the improper fraction $\frac{124}{7} = 17\frac{5}{7}$.

Multiply $8\frac{5}{12}$	by $3\frac{3}{8}$	- - - - - - - - -	*Ans.* $28\frac{39}{96}$
8	$5\frac{2}{3}$	- - - - - - - - -	$45\frac{1}{3}$
$3\frac{3}{4}$	5	- - - - - - - - -	$18\frac{3}{4}$
24	$6\frac{5}{6}$	- - - - - - - - -	164
$8\frac{2}{3}$	$10\frac{4}{5}$	- - - - - - - - -	$93\frac{3}{5}$
24	$\frac{5}{6}$	- - - - - - - - -	20
60	$\frac{2}{15}$	- - - - - - - - -	8
9	$\frac{1}{5}$	- - - - - - - - -	$1\frac{4}{5}$
$19\frac{4}{7}$	$23\frac{9}{11}$	- - - - - - - - -	$466\frac{12}{17}$

NOTE.—Where fractions are to be multiplied, that the *numerator of the one measures the denominator of the other, these equal numbers may be expunged*, and take the remaining figures for the *answer.*

EXAMPLE.

Multiply $\frac{4}{5}$ by $\frac{5}{8}$. $\frac{4}{5} \times \frac{5}{8}' = \frac{4}{8}$ or $\frac{1}{2}$ *Answer.*
This is so manifest it needs no explanation.

To multiply a compound fraction by a simple one.

RULE.—*Multiply the numerators of all the fractions for a numerator, and the denominators for a denominator.*

EXAMPLE.

Multiply $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{4}{5}$. $\frac{1}{2}\ \frac{3}{4}\ \frac{4}{5}' = \frac{3}{10}$ *Answer.*

Use of Multiplication.

1. What is the value of $\frac{3}{16}$ of $\frac{2}{3}$ of a pound? *Ans.* 2*s.* 6*d.*

2. How many superficial feet are in a plank $20\frac{1}{2}$ feet long, and 15 inches broad? *Ans.* $25\frac{5}{8}$.

3. What will $6\frac{1}{4}$lb. indigo come to, at 5*s.* 6*d.* per pound? *Ans.* £1 14 $4\frac{1}{2}$.

4. What will $10\frac{1}{8}$ yards silk come to, at $11\frac{3}{4}$ shillings per yard? *Ans.* £5 18 $11\frac{5}{8}$.

5. How many square yards are in a wainscot 9 feet 8 inches high, and 15 feet 3 inches long? *Ans.* 16 *yds.* $3\frac{5}{12}$ feet.

6. A floor $3\frac{1}{2}$ yards broad, and $5\frac{1}{4}$ yards long, is laid with stones at $5\frac{1}{2}$ shillings the square yard: What did it amount to? *Ans.* £5 1 $0\frac{3}{4}$.

7. What is the value of a plank $20\frac{1}{2}$ feet long, and $1\frac{1}{4}$ feet broad, at $6\frac{1}{2}$*d.* the square foot? *Ans.* £0 13 $10\frac{9}{16}$.

8. How much would the painting of a wainscot room come to, of 16 feet square, and 11 feet high, at $6\frac{1}{2}$*d.* the square yard? *Ans.* £2 2 $4\frac{4}{9}$.

9. How many cubic feet are in 100 stones, 28 inches long, 21 inches broad, and 8 inches thick? *Ans.* $272\frac{2}{9}$.

10. How many cubic feet are in a beam $22\frac{1}{4}$ feet long, 13 inches broad, and 8 inches thick? *Ans.* $16\frac{5}{72}$.

11. How many tiles, 12 inches square, would lay a floor $15\frac{3}{4}$ feet broad, and $17\frac{1}{4}$ feet long? *Ans.* $271\frac{11}{16}$.

12. A marble stone, $6\frac{1}{2}$ feet long, $4\frac{3}{4}$ feet broad, and $2\frac{1}{3}$ feet thick, is sold at 5*s.* 6*d.* the cubic foot: What is the value? *Ans.* £19 16 $2\frac{1}{4}$.

Division of Vulgar Fractions.

General Rule.—*Reduce whole and mixed numbers to improper fractions; multiply the denominator of the divisor into the numerator of the dividend for a numerator; and the denominator of the dividend into the numerator of the divisor for a denominator.*

EXAMPLE.

Divide $\frac{3}{4}$ by $\frac{2}{5}$. divisor $\frac{2}{5} \times \frac{3}{4}$

$\frac{15}{8} = 1\frac{7}{8}$ *Answer.*

Here 5, *the denominator, is multiplied into* 3, *the numerator of the dividend,* = 15, *the numerator; and* 4, *the denominator of the dividend, into* 2, *the numerator of the divisor,* = 8, *the denominator, viz.* $\frac{15}{8}$, *an improper fraction,* = $1\frac{7}{8}$ *Answer.*

When the *denominators* or *numerators* of the factors are *equal,* they may be *expunged,* and place the remaining figures according to rule, for the quotient, &c.

EXAMPLE.

Divide $\frac{6}{9}$ by $\frac{3}{9}$.	Divide $\frac{7}{8}$ by $\frac{7}{19}$.
$\frac{3}{9} \times \frac{6}{9}$	$\frac{7}{19}' \times \frac{7}{8}'$
$\frac{6}{3} = 2$ *Answer.*	$\frac{19}{8} = 2\frac{3}{8}$ *Answer.*

This rule might be rendered universal by reducing fractions either to common denominators, or numerators, when it can be done conveniently.

EXAMPLE.

Divide $\frac{5}{8}$ by $\frac{7}{16}$.

$\frac{5}{8} = \frac{10}{16}$ therefore $\frac{7}{16} \times \frac{10}{16} = \frac{10}{7} = 1\frac{3}{7}$ *Answer.*

Divide	by		*Ans.*	Divide	by		*Ans.*
$\frac{21}{45}$	$\frac{3}{5}$		$\frac{7}{9}$	24	$\frac{5}{8}$		$38\frac{2}{5}$
$\frac{2}{3}$	$\frac{1}{6}$	- -	4	60	$\frac{11}{12}$	- -	$65\frac{5}{11}$
$\frac{1}{6}$	$\frac{2}{3}$	- -	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{5}{8}$	- -	$\frac{6}{13}$
$\frac{15}{16}$	$\frac{5}{8}$	- -	$1\frac{1}{2}$	$\frac{1}{5}$	$1\frac{4}{5}$	- -	$\frac{1}{9}$
$\frac{107}{120}$	$\frac{7}{15}$	- -	$1\frac{51}{56}$	$\frac{7}{8}$	$2\frac{1}{2}$	- -	$\frac{7}{20}$
$\frac{1}{2}$	4	- -	$\frac{1}{8}$	7	$8\frac{5}{8}$	- -	$\frac{56}{69}$
$\frac{2}{5}$	10	- -	$\frac{1}{25}$	$1\frac{1}{9}$	15	- -	$\frac{2}{27}$
4	$\frac{1}{2}$	- -	8	$9\frac{3}{4}$	$13\frac{1}{2}$	- -	$\frac{13}{18}$
10	$\frac{2}{5}$	- -	25	$10\frac{5}{6}$	$4\frac{2}{3}$	- -	$2\frac{9}{28}$

The Rule of Three Direct in Fractions.

This rule is *analogous to the Rule of Three Direct in whole numbers.* For the *first* and *third* numbers or fractions must be of the *same denomination.* If they are of different denominations, *one of them must be reduced to the denomination of the other, &c.*

RULE.—*Multiply the second and third numbers together, and divide by the first, the quotient will be the answer.*

Otherwise,

Multiply the denominator of the first, into the *numerators* of the *second* and *third* numbers for a *numerator;* and the *numerator* of the *first,* into the *denominators* of the *second* and *third* numbers, for a *denominator* of the fourth number required; if the fourth be an improper fraction, reduce it to a mixed number, or known names.

EXAMPLE.

1. What cost $\frac{5}{14}$lb. avoirdupois, if $\frac{4}{7}$ oz. cost $10\frac{1}{2}$ cents?

$\frac{4}{7}$ of $\frac{1}{16} = \frac{4}{112} = \frac{1}{28}$lb.

then as $\frac{1}{28}$lb. : $\frac{21}{2}$ cts. :: $\frac{5}{14}$ to \$1.05 *Answer.*

Due attention being paid to the proportion, much work may be saved; the above proportion 1, 2, 14, multiplied, equals 28, the denominator of the first: therefore, we expunge them. Thus: $\frac{1}{28}$: $\frac{21}{2}$ :: $\frac{5}{14}$ and the answer is found by multiplying 5 and 21, &c.

2. If $\frac{3}{8}$ yard cost £$\frac{15}{32}$, what will $\frac{2}{5}$ yards cost? *Ans.* £0 10

3. What will $\frac{8}{9}$lb. cost, if $\frac{2}{5}$*s.* buy $\frac{7}{8}$lb? *Ans.* £0 4 0$\frac{96}{105}$.
4. If 19$\frac{1}{5}$lb. cost £$\frac{9}{10}$, how many lb. for $\frac{5}{8}$ shillings? $\frac{2}{3}$lb.
5. If $\frac{1}{4}$lb. tea cost 42 cents, what cost $\frac{5}{8}$lb? $1.05
6. If 1lb. tea cost 5$\frac{1}{4}$ shillings, what cost $\frac{7}{8}$lb? £0 4 7$\frac{1}{8}$
7. If $\frac{3}{8}$ yard cost 9*s.* 4$\frac{1}{2}$*d.* what cost $\frac{2}{3}$ yard? £0 16 8

Note.—If the first number be an integer, the answer may be found by *multiplying the price by the numerator, and dividing by the denominator.*

EXAMPLE.

8. If one ton of tallow cost £35, what cost $\frac{3}{4}$ ton?

$$
\begin{array}{r}
£35 \\
3 \\
\hline
4)\ 105 \\
\hline
£26\ \ 5
\end{array}
$$

Answer.

9. If 40 yards of linen cost $30.20, how much must be paid for 6$\frac{1}{2}$ yards? *Ans.* 4.90\frac{3}{4}$
10. If $\frac{4}{7}$ oz. avoirdupois cost 10$\frac{1}{2}$ d. what cost $\frac{5}{14}$lb? £0 8 9
11. What will 1$\frac{1}{2}$ cwt. pepper come to, if 15$\frac{1}{4}$lb. cost 12$\frac{3}{5}$ shillings? *Ans.* £6 16 3$\frac{57}{61}$.
12. What will $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{3}{8}$ less $\frac{1}{3}$lb. come to, if $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{5}{8}$ less $\frac{5}{6}$lb. cost $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$ less 1$\frac{15}{16}$ crowns at 110 cents? *Ans.* 0.20\frac{410}{1008}$.

Exercise.

1. What part of 3*d.* is $\frac{2}{3}$ of 2*d*? *Ans.* $\frac{4}{9}$.
2. By how much must I multiply 13$\frac{2}{3}$, that the product may be 49$\frac{1}{5}$? *Ans.* 3$\frac{3}{5}$.
3. A father dying left his son a fortune, $\frac{3}{16}$ of which he ran out in 6 months, $\frac{2}{3}$ of the remainder held him 12 months longer, at which time he had only £348 left: What sum did the father bequeath him? *Ans.* £1284 18 5$\frac{1}{2}$.
4. A has half of a ship, of which he sells to B $\frac{3}{4}$, and B sells to C $\frac{1}{2}$ his share: What share has each in the ship? *Ans.* A $\frac{2}{16}$, B $\frac{3}{16}$, C $\frac{3}{16}$.
5. A merchant buys 37$\frac{3}{4}$ pieces of cloth, at £23$\frac{1}{2}$ per piece; pays in ready money £235$\frac{1}{4}$; the rest he is to pay in wool, at 7$\frac{2}{3}$ shillings per stone: How many stone of wool must he deliver? *Ans.* 1700$\frac{25}{46}$.
6. Two pieces of cloth contained both together 79$\frac{3}{4}$ yards: How many yards were in each piece, if $\frac{2}{3}$ of the one were as long as $\frac{3}{4}$ of the other? *Ans.* 37$\frac{9}{17}$, *and* 42$\frac{15}{68}$ *yards.*
7. A younger brother received £2200, which was just $\frac{5}{12}$ of his elder brother's fortune, and 3$\frac{1}{8}$ times the elder brother's money was $\frac{1}{2}$ as much again as the father was worth: What was that? *Ans.* £11000.

8. How many stones $1\frac{3}{4}$ feet long, $\frac{2}{3}$ foot broad, and $\frac{3}{4}$ foot thick, are equal to 50 stones $3\frac{1}{3}$ feet long, $2\frac{1}{2}$ feet broad, and $1\frac{1}{5}$ feet thick? *Ans.* $571\frac{3}{7}$.

9. A merchant hath $\frac{3}{4}$ of a ship, and sells $\frac{1}{4}$ of his interest therein for £250: What is the value of the ship at that rate? *Ans.* £1333 6 8.

10. How much will 2 bags of wool come to, No. 1, weight $94\frac{3}{16}$ stone; No. 2, wt. $305\frac{7}{9}$ stone, at 10*s.* $6\frac{2}{3}$*d.* per stone—but $4\frac{2}{3}$ stone of No. 2 are worth but $2\frac{1}{4}$ stone of No. 1? *Ans.* £127 10 $4\frac{59}{84}$.

11. A father divided $\frac{34}{83}$ of his estate to one of his sons, $\frac{34}{83}$ of the residue to another, and the surplus to his relict for life: The childrens' legacies were found to be £257 3 4 different: What money did he leave the widow the use of? *Ans.* £534 2 $7\frac{231}{289}$.

12. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, valued at £12000, what did both ship and cargo stand the owners in? *Ans.* £15223 8 $10\frac{102}{279}$.

13. A man dying gave his eldest son $\frac{2}{3}$ of $\frac{1}{4}$ of his estate; to his second $\frac{1}{5}$ of $\frac{1}{2}$: when they counted, one had £40 more than the other; the remainder was given to the widow and younger children: How much had each?

Ans. first son £100, *second* £60, *widow* £440.

Decimal Fractions.

DECIMALS are a kind of *fractions* that vary in the same proportion, and are managed by the same methods of operation, as *whole numbers* are.

For this purpose, *proper fractions* are supposed to be reducible to others, whose *denominators* shall be 10, 100, 1000, &c. viz. *unity with some number of cyphers annexed, answering to the number of places in the numerator.* Therefore fractions, with such denominators, are called *Decimals, because their denominators consist of even tens:*—such are $\frac{5}{10}$, $\frac{75}{100}$, $\frac{625}{1000}$, &c.

As the denominators of *Decimal Fractions* are 10, 100, 1000, &c. *they need not be expressed,* because the numerator may be made to express the value of the fraction, *by placing a point before it, to distinguish it from a whole number.* Thus, $\frac{5}{10}$ may be written .5; $\frac{75}{100}$=.75; $\frac{625}{1000}$=.625.

We may consider *unity* as a fixed point, from whence whole numbers *infinitely increase;* and decimals *infinitely decrease* towards 0, as in the following Table:

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.		*Tenth parts.*	*Hundredth parts.*	*Thousandth parts.*	*Tens of thousandth parts.*	*Hunds. of thous'dth parts.*	*Millionth parts.*	*Tens of millionth parts.*
1	2	3	4	5	6	7	.	2	3	4	5	6	7	8
Whole Numbers.								*Decimals.*						

From this Table it is manifest that, as whole numbers increase in a *ten-fold proportion* from *unit's place*, so, likewise, decimals decrease in a *ten-fold proportion*, according to their distance from *unity.*

A *cypher*, or *cyphers*, placed at the right hand of a decimal, alter not its value, *because the significant figure occupies the same place from unity;* therefore .5 ; .50 ; .500 are all of the same value, viz. $\frac{5}{10}$.

But *a cypher placed to the left hand of a decimal alters the value,* and decreases the decimal in a *ten-fold proportion*, according to the distance of the significant figure from *unity.*

Thus, .5 is equal to $\frac{5}{10}$ properly $\frac{5}{10}$.
.05 = $\frac{5}{100}$ = $\frac{05}{100}$.
.005 = $\frac{5}{1000}$ = $\frac{005}{1000}$.

By which appears the contrary effects of cyphers on whole numbers and decimals.

Reduction of Decimals.

CASE I.

To reduce a Vulgar Fraction to its equivalent decimal:

RULE.—*Annex a cypher to the numerator, and divide by the denominator, the quotient will be the decimal equal thereto.*

EXAMPLE.

1. Reduce $\frac{4}{5}$ to a decimal.

5) 40 (.8 = to $\frac{4}{5}$ *Answer.*
40

But if a remainder occurs, add cyphers, and continue the operation to as many places as are requisite.

EXAMPLE.

2. Reduce $\frac{1}{27}$ to a decimal.

```
27) 100 (.03703 Answer.
     81
     ---
     190
     189
     ---
      100
       81
      ---
       19
      ===
```

Fractions whose quotient figures recur, are called *Circulating Decimals*, as in the above example; but these shall be treated of in their proper place.—*Five* or *six* decimal places are *sufficient* to express the value of any name.

3. Reduce $\frac{1}{3}$ to its equivalent decimal. *Ans.* .33333
$\frac{11}{15}$ to a decimal. - - - - - - - .73333
$\frac{13}{15}$ to a decimal. - - - - - - - .86666
$\frac{3}{11}$ to a decimal. - - - - - - - .27272
$\frac{11}{14}$ to a decimal. - - - - - - - .78571
$\frac{5}{128}$ to a decimal. - - - - - - - .03906
$\frac{1}{64}$ to a decimal. - - - - - - - .01562

CASE II.

To reduce divers denominations to decimals.:

RULE.—*Add a cypher or cyphers, as before, to the lowest name, and divide by that number which makes one of the higher name given, the quotient will be the decimal required.*

EXAMPLE.

10. Reduce 8 pence to the decimal of a pound currency.

Here I divide 8*d. by* 240, *the pence in a pound, by prefixing cyphers, and find the decimal* .03, *the answer required. Observe, that when the same remainder, or remainders, recur in division, you may drop the performance, and continue the quotient to as many places as are requisite.*

```
240) 800 (.03333 Answer.
     720
     ---
      800
      720
      ---
       80
      ===
```

11. Reduce 11*s*. to the decimal of a pound. *Ans.* .55
12. Reduce 8*s*. to the decimal of a pound. .4
13. Reduce 15*s*. to the decimal of a pound. .75
14. Reduce 8*d*. to the decimal of a shilling. .6666
15. Reduce 5 ounces to the decimal of a lb. Troy. .4166
16. Reduce 6lb. to the decimal of a cwt. .0535714

NOTE.—When the number to be reduced consists of divers names, annex a cypher to the lowest, and divide by that number which makes one of the next higher, for a decimal of that name, to which add the next higher name, and proceed as before, dividing by that number which makes one of the next higher.

EXAMPLE.

17. Reduce 13*s.* 6*d.* to the decimal of a pound.

Here I annex a cypher to the 6d.=60, and divide by 12, the pence in a shilling, and find the decimal .5=6d. to which I add 13=13.5s. and divide by 20, the shillings in a pound, and find the decimal .675£=13s. 6d.

```
12) 60
    ----
20) 13 .5
    -----
£0 .675 Answer.
```

18. Reduce 3 qrs. 2 n. to the decimal of a yard. *Ans.* .875
19. Reduce 17*s.* 6*d.* to the decimal of a pound. .875
20. Reduce 17*s.* 4¼*d.* to the decimal of a pound. .86875
21. Reduce 1 hogshead, 21 gallons, 4 pints, to the decimal of a ton. *Ans.* .335317
22. Reduce 3 qrs. 21lb. to the decimal of a cwt. .9375
23. Reduce 9¾*d.* to the decimal of a shilling. .8125
24. Reduce 9¾*d.* to the decimal of a pound. .040625

CASE III.

To find the value of a decimal in known parts.

RULE.—*Multiply the decimal by that number which one of the higher contains of the less, and from the product point off as many places to the right hand as there are in the given decimal; the figures on the left will be the number of the less denomination, and those on the right the decimal thereof, of which find the value as before.*

EXAMPLE.

25. Find the value of £0 .675.

Here I multiply 675 by 20, the shillings in a pound, and find 13500, from which I point off three places = the number of the given decimal, and there remains 13, which I call shillings; I then multiply the decimal by 12, &c.

```
    .675
     20
   -----
 13 .500
     12
   -----
  6 .000 Answer.
```

26. What is the value of £0.75 ? - - *Ans.* 15*s.*
27. What is the value of .666*s* ? - - 8*d.*
28. What is the value of .0333£ ? - 8*d.*
29. What is the value of .875 yard ? - 3 qrs. 3 n.
30. What is the value of .335317 ton? 1 hhd. 21 g. 4 p.

NOTE.—When any number of *nines* occur in a decimal, they may be expunged, taking care to make the *next figure* to the left hand *one more.*

Addition and Subtraction of Terminate Decimals.

Rule.—*Place the numbers, whether pure or mixed, underneath each other, according to their local value, viz. units under units, tens under tens, tenths under tenths, &c. so that the decimal points be in a perpendicular row; then add or subtract as in simple numbers.*

	hhds.	*yards.*	*miles.*
Add	45.64	3.005	.07
	2.3	4.2	.04
	0.4794	5.1784	.005
	74.006	1.6	.004
	3.104	8.84735	.081
	125.5294		

	Yards.	*Feet.*	*Miles.*	*Leagues.*
From	45.04	8.01	1000.	99.548
Take	2.456	2.22	1.484	9.5693
Rem.	42.584			

Multiplication of Terminate Decimals.

Rule.—*Multiply as if they were whole numbers,* (whether they be pure or mixed) *point off from the product, as many decimal places as there are in the multiplicand and multiplier counted together, &c.*

Multiply	6.42	764.5	837.456
by	8.05	2.76	2.75

It often happens in multiplying decimals by decimals, there will not be as many places in the product as are in the factors; in which case, supply the defect by prefixing cyphers.

Multiply	.2365	.0074	.857	.0005
	.2456	.0007	.007	.0048
	.05808440			

To multiply by 10, 100, 1000, &c. remove the decimal point as many places to the right hand, as the multiplier contains cyphers.

EXAMPLE.

Multiply .354 by 10, 100, 1000, &c.

$.354 \times 10 = 3.54$
$.354 \times 100 = 35.4$
$.354 \times 1000 = 354.$

because $.354 \times 10 = 3.540$, &c.

To multiply terminate decimals, so that the product shall consist of a determinate number of decimal places.

RULE.—Invert the order of the multiplier and let the product of each figure be carried the contrary way; that is, to the right hand. In multiplying, omit those figures that stand to the right of the number of decimal places you choose to preserve, increasing the last figure by the carriage that would arise from the figures omitted; carrying one from 5 to 15, two from 15 to 25, three from 25 to 35, &c. the sum will be a product almost exact.

EXAMPLE.

Let 34.79384 be multiplied by 26.476, so as to preserve 4 decimal places.

```
           34.79384                    34.79384
inverted     67462                        26476
           --------                  -----------
           6958768                    2087|6304
           2087630 .                 24355|688
            139175 . .              139175|36
             24356 . . .           2087630|4
              2088 . . . .         6958768|
           --------                -------------
           921.2017                921.2017|0784
```

Multiply 3.141592 by 52.7438, and preserve but 4 decimal places. *Ans.* 165.6995.

Multiply 384.672158 by 36.8345, and preserve but 4 decimal places. *Ans.* 14169.2065.

Division of Terminate Decimals.

Division of Decimals is performed with the same ease, and in the same manner as whole numbers are.

GENERAL RULE.—Make the number of decimal places in the divisor and dividend alike, by annexing cyphers to which ever has the fewest number, (this by no means lengthens the work) the quotient thus far will be a pure whole number. Add cyphers to the remainder, if any, and continue the division; the quotient then arising will be a decimal, which must consist of as many places as there were cyphers added.

EXAMPLE.

Divide 2.580219 by 5.73.

```
5.730000) 2.580219,0 (.4503
          22920000
          --------
           2882190.0
           28650000
           --------
             17190000
             17190000
             ========
```

Here, after filling the divisor equal to the decimal places of the dividend, I find it will not be contained therein; therefore, I annex a cypher, and the quotient results a decimal.

Otherwise,

If the dividend contains more decimal places than the divisor, count off from the decimal point in the dividend, as many places as there are in the divisor, by placing a point under the figure retained; this call your dividend, the result of which will be a whole number; then bring down the figures cut off for the decimal part.

EXAMPLE.

Divide 7.25406 by .957.

```
.957) 7.25406 (7.58 Answer.
      6699
      ----
       5550
       4785
       ----
        7656
        7656
        ====
```

But if the divisor be not contained in the dividend, extend to the other figures; the quotient thence arising will be a decimal.

EXAMPLE.

Divide .0725406 by .957.

```
.957) .0725406 (0.0758
        6699
        ----
         5550
         4785
         ----
          7656
          7656
          ====
```

Here I take the three first figures, viz. .072 *for my dividend; but I find* .957 *is not contained therein; therefore, I remove the point under* 5, *viz.* .0725 (after putting 0 in unit's place) *and find it still too little, then I put a cypher in the first place of decimals, and remove the point under* 4, *viz.* 07254, *and find it contained* 7 *times.*

4. Divide .30438 by .534	*Ans.*	.57
5. 24.3 by 81 - - - - - - -		.3
6. 649.016928 by 754.32 - - -		.8604

7. Divide 295.75 by 8.45	- - - - - - *Ans.*	35.
3. .4368 by .0078	- - - - - -	56.
9. .0007875 by .525	- - - - - -	.0015
10. 374.86 by 48.25	- - - - - -	7.7691+

When the divisor does not measure the dividend, it is sufficient to keep 5 or 6 decimal places.

To divide by 10, 100, 1000, &c. remove the decimal point in the dividend so many places to the left, as the divisor contains cyphers, and the division is performed.

Divisor	Dividend		Quotient
10	7485	- - - - - - - - -	748.5
100		- - - - - - - - -	74.85
1000		- - - -	7.485
10000		- - - -	.7485
100000		- - - - - - - - -	.07485
1000000		- - - - - - - - -	.007485

Division of decimals may be contracted a little, as follows:

Rule.—Let each remainder be a new dividend, and for each new dividend, point off one figure from the right hand of the divisor, observing at each multiplication to carry the increase of the figures cut off as in multiplication, &c. the quotient will then be pretty nearly exact.

```
384.672158) 14169.206623851   (36.8345
            1154016474
            ----------
             262904188 .
             230803295 .
             ----------
              32100893 . .
              30773772 . .
              --------
               1327121 . . .
               1154016 . . .
               -------
                173105 . . . .
                153869 . . . .
                ------
                 19236 . . . . .
                 19233 . . . . .
                 -----
                     3 . . . . . .
```

Divide 87.076326 by 9.365407. *Ans.* 9.297655.

Let both these examples be wrought by the usual method.

The Rule of Three Direct in Decimals.

The Rule of Three in Decimals, is governed by the same principles, as the Rule of Three Direct in whole numbers: therefore, reduce the fractional parts to decimals of the highest name mentioned. Multiply and divide, as directed in multiplication and division of decimals.

1. Suppose $4\frac{3}{4}$ yards of linen be sold for 6*s.* 3*d.* what is the price of $48\frac{1}{2}$ yards?

First No. $4\frac{3}{4}$=4.75 yards.
Sec'd 6*s.* 3*d.*=£0.3125
Third $48\frac{1}{2}$=48.5 yds.

yds. £ *yds.*
then as 4.75 : .3125 :: 48.5 : to £3.1907.
or £3 3 $9\frac{3}{4}$.

2. If $2\frac{1}{2}$lb. tea cost £2 5*s.* what cost $14\frac{3}{4}$lb? *Ans.* £13 5 6.

3. If 1lb. sugar cost $11\frac{3}{4}$*d.* what cost 4 hhds. each weighing neat 4 cwt. 2 qrs. 14lb. at that rate? *Ans.* £101 8 10.

4. Bought 4 chests of tea, each weighing neat 2 cwt. 3 qrs. 14lb. for £906 10*s*: what did it cost per lb? *Ans.* 14*s.* $0\frac{1}{4}$*d.*

5. If $46\frac{1}{3}$ hogsheads of wine cost £875 13*s.* 4*d.* what cost 1 hogshead? *Ans.* £18 17 11.+

6. Bought sugar at £2 7*s.* 8*d.* per cwt: How must I sell it to gain 5 per cent? *Ans.* £2 10 $6\frac{1}{4}$.+

7. Bought 4 tons $201\frac{1}{2}$ gallons of Florence oil for £240 16*s.* 6*d.*; by misfortune lost $24\frac{1}{2}$ gallons: How must the remainder be sold to sustain no loss? *Ans.* 4*s.* $0\frac{3}{4}$*d.*+

Circulating Decimals.

Note.—The Circulating Decimals may be taught or omitted, as the teacher may think proper.

Those decimals which are produced from vulgar fractions, whose denominators measure their numerators, are called Finite, or Terminate Decimals, because they consist of a finite number of places.

Those decimals (which are produced from vulgar fractions whose denominators do not measure their numerators) in which a figure is repeated continually, or in which the same figures circulate continually, are called Circulating Decimals, because they consist of an infinite number of places.

If one figure only repeats, it is called a Single Repetend, as .3333.

If other figures arise before the figures that circulate, then the decimal is called a Mixed Single Repetend, as .08333.

Such figures as circulate alternately, or every third, fourth, &c. are called Compound Repetends, as .123123123, or .156156, &c. or .454545.

And if other figures arise before those that circulate, then the decimal is called a Mixed Compound Repetend, as .73514514, &c.

As in multiplying and dividing by these in perfect decimals, it frequently requires, that the decimal be extended to a great number of places, to prevent a considerable error resulting from their imperfection: to remedy this, and make the result perfect with less trouble, it will be necessary to consider their generation.

Now, as 9 is contained in 10 once and one remains, unity with cyphers annexed, being divided by 9, *ad infinitum*, the quotient will be $.\dot{1}=\frac{1}{9}$; and since $.\dot{1}$ is a decimal equal to $\frac{1}{9}$, $.\dot{2}$ will equal $\frac{2}{9}$, $.\dot{3}=\frac{3}{9}$, $.\dot{4}=\frac{4}{9}$, $.\dot{5}=\frac{5}{9}$, $.\dot{6}=\frac{6}{9}$, $.\dot{7}=\frac{7}{9}$, $.\dot{8}=\frac{8}{9}$, $.\dot{9}=\frac{9}{9}=1$.

Therefore, every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9, &c. and every compound repetend to a fraction, whose numerator is the repeating figure, and an equal number of nines the denominator.

To find a vulgar fraction equal to a mixed circulate, we must consider the decimal as divisible into its finite and circulating parts: Thus, $1\dot{6}$ is divisible into the finite decimal .1, and the repetend 6; therefore .1 is equal to $\frac{1}{10}$, and $6=\frac{6}{9}$, provided the circulation began after unit's place; but as it began after the place of tenth parts, it is $\frac{6}{9}$ of one of the proceeding place, viz. $\frac{6}{9}$ of $\frac{1}{10}=\frac{6}{90}$; so the mixed circulate $.1\dot{6}=\frac{1}{10}+\frac{6}{90}=$to the vulgar fraction $\frac{15}{90}=\frac{1}{6}$.

Again, the mixed compound repetend $.46\dot{2}\dot{3}$ may be divided into $\frac{46}{100}$, and $\frac{23}{99}$ of $\frac{1}{100}=\frac{23}{9900}$; therefore the decimal $.46\dot{2}\dot{3}$ is equal to $\frac{46}{100}+\frac{23}{9900}=\frac{4577}{9900}$.

To avoid the trouble of writing down unnecessary figures, circulating numbers may be distinguished by putting a point over the repeating figures.

The compound repetend	.123123123, &c.	may be written	$.\dot{1}2\dot{3}$
The single repetend - -	.333333, &c.	- - - - -	$.\dot{3}$
The mixed single rep.	.083333, &c.	- - - - -	$.08\dot{3}$
The mixed comp. rep.	.73514514, &c.	- - - -	$.73\dot{5}1\dot{4}$

Reduction of Circulating Decimals.

PROPOSITION I. To reduce a mixed circulate to a vulgar fraction.

RULE.—From the given mixed circulate deduct the finite part for a numerator, and the denominator of the repetend, with as many cyphers annexed as there are places in the finite part of the decimal, is the denominator.

EXAMPLE.

1. Reduce $.5\dot{7}$ to its equivalent vulgar fraction.

$.5\dot{7}$
5 finite part.

$\frac{52}{90}$ vulgar fraction $= .5\dot{7}$. But if 57 was a whole number, and 7 the repetend, the work would stand thus:

$57.\dot{7}$
57

$\frac{520}{90}$ an improper fraction, equal 57.7777.

2. Reduce $.32\dot{7}$ to a vulgar fraction. - - - *Ans.* $\frac{3245}{9000}$
3. Reduce $.54\dot{6}$ to a vulgar fraction. - - - - $\frac{5412}{9000}$
4. Reduce $3.\dot{8}4\dot{2}$ to its equivalent vulgar fraction. $\frac{3839}{999}$
5. Reduce $42\dot{7}5.8\dot{4}$ to a vulgar fraction. - - - $42\frac{7542}{9999}00$

Similar repetends, which begin at the same place, are said to be conterminious.

Similar repetends may take the form of compound repetends, or compound or mixed circulates, without altering the value.

Dissimilar.	*Made similar.*	*Make the following similar.*	*Ans.*
$.\dot{7}$	$.\dot{7}7777\dot{7}$	$.\dot{1}$	
$.\dot{5}\dot{4}$	$.\dot{5}4545\dot{4}$	$.3\dot{2}$	
$.\dot{4}7\dot{5}$	$.\dot{4}7547\dot{5}$	$4.\dot{2}\dot{7}$	
$.3\dot{2}\dot{4}$	$.3\dot{2}42424\dot{4}$	$2.37\dot{5}$	
$.\dot{5}\dot{9}$	$.\dot{5}9595\dot{9}$	$.04\dot{3}$	
$.\dot{1}$	$.\dot{1}1111\dot{1}$	$.\dot{7}$	
$.32\dot{7}$	$.32\dot{7}7777\dot{7}$	$.\dot{6}$	

Addition and Subtraction of Circulating Decimals.

To add single mixed repetends.

RULE.—Make the repetends similar, then add up the right hand column, adding to the amount one for every nine contained therein—proceed with the rest of the work as in simple addition.

EXAMPLE.

Add $2.\dot{3}$	made similar	$2.33\dot{3}$
$0.3\dot{4}$	- - - -	$0.34\dot{4}$
$0.43\dot{7}$	- - - -	$0.43\dot{7}$
$44.\dot{4}$	- - - -	$44.44\dot{4}$
$2.\dot{8}$	- - - -	$2.88\dot{8}$
$5.\dot{4}$	- - - -	$5.44\dot{4}$
		$55.89\dot{3}$

Sum of the right hand column is 30, *to which I add one for every* 9 *contained therein, viz.* 3=33, *set down* 3 *and carry* 3, *&c.*

To add compound or mixed compound repetends.

RULE.—Make the repetends similar and conterminious; find what amount will arise in the column where all the repetends begin, from which carry to the right hand column as many units as it contains tens; then proceed as in simple addition.

EXAMPLE.

Add $2.\dot{5}\dot{4}$	made similar	2.'545454'
$.0\dot{6}3\dot{4}$	- - - - -	.063463
$.\dot{2}\dot{6}$	- - - - -	.262626
$.\dot{7}$	- - - - -	.777777
$.\dot{4}\dot{5}$	- - - - -	.454545
		4.103868

In the addition of this example, I find the column where all the circulates begin amounts to 30; therefore I carry three to the first column, and proceed as in common addition: put the circulating point or dash over the first and last figures of the circle at the top.

NOTE.—The number of places in the similar numbers must be a multiple of the number in each circle.

Thus, the first decimal circulates in 2 places, the second in 3, the third in 2, the fourth in 1, the fifth in 2.

Therefore, 3) 2, 3, 2, 1, 2

2) 2, 1, 2, 1, 2

1, 1, 1, 1, 1, 1, and $3 \times 2 = 6$ places to be preserved in the foregoing example.

In *Subtraction*, make the numbers similar and conterminious, as before; then, if the first figure to be subtracted be greater than the upper figure, borrow from 9 instead of 10, and proceed as in common subtraction.

EXAMPLE.

From 4.3'	$45.\dot{6}28'$	$.\dot{8}4$	.78'
Take 2.6'	$3.\dot{4}5\dot{7}$	'456'	.257
Rem. 1.6			

Multiplication of Circulating Decimals.

A compendious method of multiplying by any number of nines.

RULE 1.—Write after the multiplicand as many cyphers as there are nines in the multiplier, and from this number subtract the multiplicand, the remainder will be the product.

EXAMPLE.

Multiply 854 by 999.

```
        854000
           854
        ------
        853146 product.
```

Because, a cypher added to the multiplicand would be the same as if multiplied by 10, thus $45 \times 10 = 450$
from which subtract $45 \times 1 = 45$

the remainder is the product of $45 \times 9 = 405$

PROPOSITION. To multiply a finite decimal by a single rep.

RULE 2.—Put a cypher in unit's place, and divide the product of the repetend by 9, the last figure of the quotient will be a repetend, and the quotient will be the true product of that repetend; proceed with the remaining figures as in common multiplication.

EXAMPLE.

```
Multiply      55.324
   by          .342̇
          ----------
       9) 1106480
          ----------
           122942̇  true product of the repetend 2̇
          221296
         165972
         ----------
         18.933102
```

Multiply 6.45 by .73̇ - - - - - - - - - *Ans.* 4.73
.638 .47̇ - - - - - - - - - - .30482

It may happen that the circulate may have many more figures than the finite fraction; therefore, to multiply a circulate by a finite decimal:

RULE 3.—Multiply as in simple numbers, adding as many units to the product of the circulate as the product contains nine; then continue the repetends to unit's place in each line, proceed as taught in addition.

EXAMPLE.

```
Multiply    374.643̇
   by          .456
          ----------
           2247860̇
          1873216,6
         1498573,33
         ----------
         170.837359̇
```

Multiply 8.4$\dot{7}$ by .68 - - - - - - - *Ans.* 5.764$\dot{8}$.
476.05 .08 - - - - - - - - 38.08$\dot{4}$.

The multiplication of single repetends by single repetends is governed by the same rule.

Multiply .1$\dot{6}$ by .$\dot{3}$ - - - - - - - - - *Ans.* .05
7.68$\dot{4}$ .4$\dot{5}$ - - - - - - - - .350069.

To multiply a compound repetend by a finite decimal.

RULE 4.—See what must be carried from the leading figure of the repetend, and add it to the product of unit's place, so proceed through all the places; then fill each line as before with the circulate to unit's place.

Multiply 5$\dot{2}$.43$\dot{6}$
by .54

20$\dot{9}$74$\dot{4}$
26$\dot{2}$181'2

28.$\dot{3}$155$\dot{6}$

To multiply a terminate decimal by a compound repetend.

RULE 5.—Under the product as found by simple multiplication, write the said product towards the right hand, so that there may be as many places left clear towards the left, as there are places in the circulate; repeat this as often as needful, the sum will be the true amount, &c.

EXAMPLE.

Multiply 75.43
by .'75'

37715
52801

565725 first amount by common multiplication.
5657
56

57.1438 true amount.

NOTE.—*Refer to the first section of division of circulating decimals, for an illustration of this rule, page* 88.

Multiply 647.543 by .'456' - - - - *Ans.* 295.575182.
3.842 .'4325' - - - - - 1.6618311.

To multiply a finite decimal by a mixed compound repetend.

RULE 6.—Find the amount as in the last case for the circulate, then multiply for the finite part.

EXAMPLE.

Multiply	4.642
by	.47̇3̇
	13926
	32494
	338866
	3388
	33
	342288 amount of infinite part.
	18568
	2.199088 true amount.

Otherwise, subtract the finite from the infinite part, find the amount by common multiplication; then proceed as directed in rule 5.

EXAMPLE.

	4.642
4.'73' less 4 the finite part =	.469 new multiplier.
	41778
	27852
	18568
	2177098
	21770
	217
	2
	2.199088 as before.

Multiply 35.27 by .6'543'. - - - - *Ans.* 2.3079078.
7.523 25.'54'. - - - - - 192.17844.

To multiply a compound repetend by a compound repetend.

RULE 7.—Subtract the finite from the infinite part of the multiplier—fill each line to unit's place with the repeating figures of each; add, and proceed as directed by rule 5.

Otherwise,

Find the product of the infinite part as per rule 6—filling each line with the circulating figures; then multiply by the finite part, filling its product with the circle to unit's place.

Multiply 3.\`432′ by 2.\`548′ *Ans,* 8.447721.

By rule first.	By rule second.
3.\`432″	3.\`432′
2.\`548′—2=2.\`546	2.\`548′
20\`594′	27\`459′
13\`729′7	13\`729′7
17\`162′16	17\`162′16
6\`864′864	1880974
8738973	1880
8738	1
8	1882856 amt. infinite part.
8.747721 amt.	6\`864′864 amt. finite part.
	8.747721 product as before.

Multiply 4.3\`75 by 2.\`86′. *Ans.* 12.55266.
Multiply 4.\`142857′ by 2.\`428571′. 10.061224489795.
The last may be proved by vulgar fractions $4\frac{1}{7}\times 2\frac{3}{7}=10\frac{3}{49}$.

Division of Circulating Decimals.

CASE I.

A concise method of dividing by 9, 99, 999, &c.

Rule.—Separate the dividend into periods of as many places as there are nines in the divisor, if there be odd figures supply the defect by annexing cyphers.

Then under the left hand figure of the second period, place the left hand figure of the first, and continue the figures of the dividend to the right; thus proceed as often as necessary, the sum will be the quotient required.

Lastly.—Count the number of decimal places in the dividend, with the number of nines in the divisor, for the number of decimal places to be pointed off in the quotient.

EXAMPLE.

Divide 34.86 by 999. *Ans.* .03\`489′.

```
34.8,600,000,
     348 600
         348
------------
.03`489′,4,8,9,4,8
```

ELUCIDATION. Because 9 is contained in 10 once, and one remains, viz. $\frac{1}{9}$—unity with cyphers annexed and divided by 9, willgive the quotient .1111 *ad infinitum*,
for $1\div9=.1+\frac{1}{9}$ of $.1=.01+\frac{1}{9}$ of $.01=.001+\frac{1}{9}$ of $.001=.0001$

Otherwise, $1\div9=.1$
$+\frac{1}{9}$ of $.1=.01$
$+\frac{1}{9}$ of $.01=.001$
$+\frac{1}{9}$ of $.001=.0001$

.1111 &c.

Again, 1 divided by 99 will give the quotient .010101 *ad infinitum*,
for $1\div99=.01+\frac{1}{99}$ of $.01=.0001+\frac{1}{99}$ of $.0001=.000001$, &c.

Otherwise, $1\div99=.01$
$+\frac{1}{99}$ of $.01=.0001$
$+\frac{1}{99}$ of $.0001=.000001$

.010101, &c. *ad infinitum.*

Thus we see by the foregoing elucidation, we have only to separate the dividend into periods of as many places as the divisor contains nines, and place them according to rule, the sum will be the quotient.

CASE II.

To divide a circulating decimal by a terminate number.

RULE.—Divide as in common division, taking care to bring down the recurring figures in due course, till a sufficient number of places are had, or the quotient begins to recur.

EXAMPLE.

Divide 64.'364' by .952

```
.952) 64.'364'   (67.609626
      5712
      ----
       7244
       6664
       ----
        5803
        5712
        ----
         9164
         8568
         ----
          5963
          5712
          ----
           2516
           1904
           ----
            6124
            5712
            ----
             412 remainder.
```

Divide .64′ by .8 - - - - - - - - - - *Ans.* .805′
Divide .‵654′ by .7 - - - - - - - - - - .‵935220′

CASE III.

To divide by a single or mixed repetend.

Rule.—Multiply the Divisor and quotient separately by 9, then divide as before taught.

EXAMPLE.

Divide .5 by .3′

.3′——.5
9 9
— —
13.0) 4.5 (1.5 *Answer.*

Divide .5 by 1.6′ - - - - - - - - - - *Ans.* .3
Divide .6′ by 4.3′ - - - - - - - - - - .‵153846′
Divide .‵27′ by. 8′ - - - - - - - - - - .306‵81′

CASE IV.

To divide a terminate number by a compound or mixed compound repetend.

Rule.—Multiply the terminate by as many nines as the circulate contains places; subtract the finite from the infinite part, if the divisor be mixed, and proceed as in common division.

EXAMPLE.

Divide .75 by 7.‵27′

```
7.‵27′    .7500
    7       75
 ————     ————
 7.20)    .7425    (.103125 Answer.
           720
          ————
           2250
           2160
          ————
            900
            720
          ————
           1800
           1440
          ————
           3600
           3600
          ====
```

Proof.—.75 $=\frac{3}{4}$ and 7.‵27′ $=7\frac{3}{11}$ and $\frac{3}{4}\div7\frac{3}{11}=\frac{33}{320}=$.103125

Divide 4. by .‵142857′ - - - - - - - - *Ans.* 28.
4 by 2.‵142857′ - - - - - - - - 1.86′

CASE V.

To divide a compound repetend by a compound repetend.

RULE.—Make the repetends similar and conterminious, add cyphers, and proceed as in common division.

EXAMPLE.

Divide .'27' by .'714285') '272727' (.3'81'
'1428.57' by '4.28571' - - - - *Ans.* 333.3'

CASE VI.

To divide a mixed compound repetend by a mixed compound repetend.

RULE.—Subtract the finite from the infinite parts, and proceed as in common division.

Divide 8.'571428'	by 2.'142857'	- - - -	*Ans.* 4.
2.'142857'	by 8.'571428'	- - - - -	.25
24.3'306'	11.'198'	- - - - -	2.'172'
2.'428571'	4.'152857'	- - - - -	.5862
.571428	2.'142857'	- - - - -	.26

PART III.

Practice.

PRACTICE *is a compendious method of working the Rule of Three, where one is given for the first number, and divers denominations for the second and third.*

Practice will be found very simple to the person who understands *compound multiplication* and *division*. The only *difficulty* that may occur is, *how to separate the price or quantity into the several parts required.* An attentive perusal of the following *table* will remove that difficulty.

PRACTICE TABLE.

Aliquot parts of £1.

s.	d.	
10	0	is $\frac{1}{2}$
6	8	$\frac{1}{3}$
5		$\frac{1}{4}$
4		$\frac{1}{5}$
3	4	$\frac{1}{6}$
2	6	$\frac{1}{8}$
2		$\frac{1}{10}$
1	8	$\frac{1}{12}$
1		$\frac{1}{20}$
	6	$\frac{1}{40}$
	8	$\frac{1}{30}$
	4	$\frac{1}{60}$
	3	$\frac{1}{80}$
	2	$\frac{1}{120}$

Aliquot parts of 1s.

d.	
6	is $\frac{1}{2}$
4	$\frac{1}{3}$
3	$\frac{1}{4}$
2	$\frac{1}{6}$
$1\frac{1}{2}$	$\frac{1}{8}$
1	$\frac{1}{12}$

Aliquot parts of 1 cwt.

2 qrs.	is $\frac{1}{2}$
1	$\frac{1}{4}$
14 lb.	is $\frac{1}{8}$
16	$\frac{1}{7}$
8	$\frac{1}{14}$
7	$\frac{1}{16}$

Aliquot parts of 1-2 cwt.

$\frac{1}{4}$ or 28 lb.	is $\frac{1}{2}$
14	$\frac{1}{4}$
7	$\frac{1}{8}$
8	$\frac{1}{7}$
4	$\frac{1}{14}$
2	$\frac{1}{28}$

Aliquot parts of 1-4 cwt.

14 lb.	is $\frac{1}{2}$
7	$\frac{1}{4}$
4	$\frac{1}{7}$
$3\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{14}$

The teacher may direct the student how to form tables of aliquot parts, from the aliquot parts of a £, in the same manner of the less parts of a cwt.

Beside the parts expressed in the Table, *aliquant parts* may often occur; in which case the *uneven quantities* must be found in parts of the *even*. This will facilitate the performance in almost every case, because, *the less the divisor is, the greater will be the expedition and certainty.*

CASE I.

When the given price is pounds, or pounds and shillings.

RULE.—*Multiply the given quantity by the pounds, and take parts for the shillings; the sum of the product and several quotients will be the answer.*

EXAMPLE.

What is the price of 35 yards of cloth, at £2 15*s*. per yard?

		35		
		2 15		
s.		70	- - - - - price at £2	
10 =	$\frac{1}{2}$ =	17 10 0	- - - - - - - -	10
5 =	$\frac{1}{4}$ =	8 15 0	- - - - - - - -	5
15		£96 5 0 *Ans.*		

Here I multiply the number of yards by 2, and produce	£70
Second—*Then, because* 10*s*. *is the* $\frac{1}{2}$ *of one pound, I divide* 35 *by* 2, *and find the price of* 35, *at* 10 *shillings*,	17 10
Third—*Likewise, because* 5*s*. *is the* $\frac{1}{4}$ *of one pound, I divide* 35 *by* 4, *and find the price of* 35, *at* 4 *shillings; but, as* 5 *is the half of* 10, *half the price, at* 10 *shillings, would equal the whole price at* 5 *shillings, viz.* £8 15*s*.	8 15
The several prices added, - - - - - - - -	£96 5

What is the price of 178 yards of broadcloth, at £2 4*s*. per yard? *Ans.* £391 12 0.

What cost 327 cwt. of sugar, at £5 12*s*. per cwt? *Ans.* £1831 4*s*.

What cost 275 yds. of silk, at £1 12*s*. per yd?		*Ans.*	£440 0 0
237	at £2 13 0	- - - - - -	628 1 0
124	3 8 0	- - - - - -	421 12 0
57	1 14 0	- - - - - -	96 18 0
22	1 17 0	- - - - - -	40 14 0
745	3 11 0	- - - - - -	2644 15 0
94	6 9 0	- - - - - -	606 6 0
86	5 6 0	- - - - - -	455 16 0
74	1 7 0	- - - - - -	99 18 0
22	3 16 0	- - - - - -	83 12 0
29	1 18 0	- - - - - -	55 2 0
86	4 19 0	- - - - - -	425 14 0

CASE II.

When the price is at pounds, shillings and pence, or shillings and pence:

RULE.—*Proceed, as in the last case, for the higher denominations, then find the parts of the pence in some of the lower parts taken.*

EXAMPLE.

What is the price of 64 yards of silk, at £1 18 9 per yard?

		64 price at £1		
10*s.*	½	32	- - -	0 10
5	½	16	- - -	0 5
2 6*d.*	½	8	- - -	0 2 6
1 3	½	4	- - -	0 1 3
18 9		£124 0 0 *Ans.*		

Here I take 64 *for the price at* 1 *pound* - - - - - - -	64
Then 10*s. is* ½ *of the given quantity for* 10*s.* - - - - - -	32
5*s. is* ½ *of* 10, *therefore half the price for* 10 - -	16
2*s.* 6*d. is* ½ *of* 5, *therefore half the price for* 5 - -	8
And 1*s.* 3*d. is* ½ *of* 2*s.* 6*d. therefore half the price for* 2*s.* 6*d.*	4
18*s.* 9*d. sum of the lower parts* - - - - - -	£124

By inspecting *the last example*, the student may see by what *ease* and *certainty* he may perform his calculations, if he observes the connexion between the several parts: 2, *the divisor used, is easier divided by than the divisors expressing the several parts in the price*, viz. ½, ¼, ⅛, and $\frac{1}{16}$, &c. Thus the student may form *aliquot* from *aliquant* parts with ease.

What is the price of 542 lb. souchong tea, at 7*s.* 9*d.* per lb?

		£. *s. d.*
		542 at 7 9
5*s.*	¼	=135 10 0
2 6*d.*	½	= 67 15 0=half the price for 5*s.*
3	$\frac{1}{10}$	= 6 15 6=$\frac{1}{10}$ of the price for 2*s.* 6*d.*
7 9		£210 0 6

At 15*s.* per cwt. what cost 336 cwt. logwood?		*Ans.*	£252	0 0
What is the price of 713 lb. cassia, at 13*s.* 4*d*?			475	6 8
479 yds. velvet at	17*s.* 6*d*?		419	2 6
95 do. do.	18 4		87	1 8
317	19		301	3 0
72	12 6		45	0 0
88	£4 9 10		395	5 4
27	1 5 6		34	8 6

CASE III.

When the price is pence and parts of pence:

RULE.—*Take parts for the pence in shillings, and for farthings find their parts in the* parts taken; *add the several parts found, and divide by* 20 *for pounds.*

EXAMPLE.

What is the value of 3754 lb. sugar, at 10$\frac{1}{4}$*d.* per pound?

		3754 at 10$\frac{1}{4}$*d.*
6	$\frac{1}{2}$	=1877
3	$\frac{1}{2}$	= 938 6
1	$\frac{1}{3}$	= 312 10
$\frac{1}{4}$	$\frac{1}{4}$	= 78 2$\frac{1}{2}$
	2,0	320,6 6$\frac{1}{2}$
		£160 6 6$\frac{1}{2}$ *Ans.*

What is the price of 604 yds. of black ribbon, at 4$\frac{1}{2}$*d.* per yd? *Ans.* £11 6 6

384 lb. at 5*d*?		- - -	8	0	0
243	5$\frac{3}{4}$	- - -	5	16	5$\frac{1}{4}$
647	7$\frac{1}{4}$	- - -	19	10	10$\frac{3}{4}$
254	9$\frac{3}{4}$	- - -	10	6	4$\frac{1}{2}$
672	11$\frac{1}{4}$	- - -	31	10	
222	11$\frac{3}{4}$	- - -	10	17	4$\frac{1}{2}$
228	$\frac{3}{4}$	- - -		14	3
642	$\frac{1}{4}$	- - -		13	4$\frac{1}{2}$
254	1$\frac{3}{4}$	- - -	1	17	0$\frac{1}{2}$

CASE IV.

If numbers of divers denominations are given, and the price is for one of the higher:

RULE.—*Multiply the price by the higher, and take parts for the lower denominations.*

EXAMPLE.

What cost 7 cwt. 3 qrs. 25 lb. cheese, £2 16 4 per cwt?

	£. *s.* *d.*				
	2 16 4				
	7 3 25				
	19 14 4				
2 qrs. = $\frac{1}{2}$ =	1 8 2		- - -	*half price of* 1 *cwt.*	
1 do. = $\frac{1}{2}$ =	14 1		- - -	*half do. of* 2 *qrs.*	
14 lb. = $\frac{1}{2}$ =	7 0$\frac{1}{2}$	= $\frac{14}{28}$	-	*half do. of* 1 *qr.*	
7 lb. = $\frac{1}{2}$ =	3 6$\frac{1}{4}$	= $\frac{7}{28}$	-	*half do. of* 14 *lb.*	
4 lb. = $\frac{1}{7}$ =	2 0$\frac{1}{7}$	= $\frac{4}{8}$	-	$\frac{1}{7}$ *do. of* 1 *qr.*	
	£22 9 1$\frac{25}{28}$ *Ans.*				

But if the price be of a higher denomination, and the quantity given of a lower:

RULE.—*Take aliquot parts of the higher for the given denominations of the lower; the sum will be the answer.*

EXAMPLE.

At £1 13 4 per cwt. what cost 3 qrs. $17\frac{1}{2}$ lb?

			£. s. d.	
			1 13 4	
2qrs.	$\frac{1}{2}$	=	16 8 =	*price of* 2 *qrs.*
1 do.	$\frac{1}{2}$	=	8 4 =	1 *do.*
14lb.	$\frac{1}{2}$	=	4 2 =	$\frac{1}{2}$ *do.*
$3\frac{1}{2}$do	$\frac{1}{4}$	=	1 $0\frac{1}{2}$=	$\frac{1}{8}$ *do.*
			£1 10 $2\frac{1}{2}$ *Ans.*	

At £0 16 4 per cwt. what cost 14 cwt. 3 qrs? *Ans.* £12 0 11.

What will 193 cwt. 3 qrs. cost, at 17*s.* $9\frac{1}{4}$*d.* per cwt?
Ans. £172 7 $1\frac{1}{4}$.

What will 26 cwt. 2 qrs. 7 lb. cost, at 15*s.* 9*d.* per cwt?
Ans. £20 18 $4\frac{5}{16}$.

NOTE.—It often happens in this case, that the given quantities are large and of course tedious to multiply by:

Therefore, put the quantity for the multiplicand, and the price for the multiplier, taking care to multiply none but the higher denominations of each; then take the remaining parts of the price out of the quantity multiplied, and the lower remaining parts of the quantity given out of the price—the sum will be the answer.

EXAMPLE.

What will 332 cwt. 2 qrs. 22 lb. come to, at 18*s.* 6*d.* per cwt?

	cwt. qrs. lb.		
	332 2 22		
	18*s.*6*d.*		
	2656		
	332		
6*d.* = $\frac{1}{2}$ *of* 332 =	166		
2*qrs.* = $\frac{1}{2}$ *of* 18*s.* 6*d.*	= 9	3	
14*lb.* = $\frac{1}{4}$ *of* 2 *qrs.*	= 2	$3\frac{3}{4}$	= 42
7*lb.* = $\frac{1}{2}$ *of* 44 *lb.*	= 1	$1\frac{7}{8}$	= 49
1*lb.* = $\frac{1}{7}$ *of* 7 *lb.*	=	$1\frac{55}{56}$	= 55
2,0)	615,4	$10\frac{17}{28}$	$\frac{146}{56} = 2\frac{17}{28}$
	£307 14	$10\frac{17}{28}$ *Answer.*	

Or thus,

		332 2 22 at 18*s.* 6*d.*
10*s.*	$\frac{1}{2}$	166
5	$\frac{1}{2}$	83
2 6	$\frac{1}{2}$	41 10
1	$\frac{1}{20}$	16 12
2 *qr.*	$\frac{1}{2}$	9 3
14*lb.*	$\frac{1}{4}$	2 $3\frac{3}{4}$
7	$\frac{1}{2}$	1 $1\frac{7}{8}$
1	$\frac{1}{7}$	$1\frac{55}{56}$
		£307 14 $10\frac{17}{28}$

What cost 203 cwt. 3 qrs. at 16*s.* 5¼*d.* per cwt? *Ans.* £167 9 $1\frac{11}{16}$
2061 cwt. 2 qrs 7 lb. at 16*s.* 6*d*? 1700 15 $9\frac{3}{8}$
106 cwt. 3 qrs. 14 lb. at 9*s.* 4*d*? 49 17 6
306 cwt. 3 qrs. 21 lb. at £1 1*s.* 6*d*? 329 19 $1\frac{7}{8}$
951 cwt. 2 qrs. 27 lb. at 11*s.* 3*d*? 535 7 $1\frac{5}{112}$

If 1 lb. silk cost £3 6*s.* 5*d.* what is the price of 80 lb? 10 oz?
Ans. £267 14 $10\frac{1}{8}$

What will 20 ton 19 cwt. 3 qrs. 14 lb. come to at £19 19*s.* 6*d.* per ton? *Ans.* £419 7 $0\frac{3}{80}$.

What will 7 yds. 3 qrs. 2 na. come to at 6*s.* 9*d.* per yard?
Ans. £2 13 $1\frac{7}{8}$.

If 1 ton cost £21 15*s.* 6*d.* what cost 25 t. 15 cwt. 3 qrs. 14¼ lb?
Ans. £561 13 $3\frac{681}{2240}$.

If 1 cwt. cost 19*s.* 11¾*d.* what cost 11 cwt. 3 qrs. 27½ lb?
Ans. £11 19 $7\frac{119}{128}$.

At £19 19*s.* 11¾*d.* per ton, what cost 19 ton 19 cwt. 3 qrs. 27½ *lb.* *Ans.* £399 19 $5\frac{16641}{17920}$.

Practice performed by Decimals.

RULE.—*Reduce the price to a decimal, and multiply thereby, the product will be pounds, and the decimal of a pound.*

EXAMPLE.

What cost 4 cwt. 3 qrs. 14 lb. at 18 shillings per cwt.?

18*s.* =	=		4 3 14
			.9
			3.6
	2qrs.	½	= .45
	1qr.	½	= .225
	14lb.	½	= .1125
			£4.3875 = £4 7 9 *Answer.*

What cost 256 yards of linen at 6*s.* per yard? *Ans.* £76 16*s.*
54 at £12 10*s.* 6*d*? - - - *Ans.* £676 7*s.*

NOTE.—As 2 shillings is $\frac{1}{10}$ of a £, or, £0.1, the price of any quantity is found by pointing off the unit's place from the rest, for decimal.

What cost 542 yards of ribband at 2 shillings per yard?
£54.2 = £54 4 *Answer.*

Therefore, if the price be less than 2 shillings, find the value of 2 shillings by the above example, and take the aliquot parts of two shillings for the price.

EXAMPLE.

What cost 327 yards of canvas at 1*s.* 9*d.* per yard?

		£	*s.*	*d.*
	32.7 =	32	14	
1*s*	½	16	7	
6*d*	½	8	3	6
3	½	4	1	9
		£28	12	3 *Answer.*

For further examples in decimals, take some of the questions in case the second and fourth.

Bills of Parcels.

James Bates, New-York, October 24th, 1803.

Bt. of Edward Empson,

	£	*s.*	*d.*
27¼ yards linen, at 5*s.* 8*d.* per yard - - - -			
23 do. do. 5*s.* 4*d.* - - - - - - - -			
26½ do. velvet 25*s.* - - - - - - - -			
10¾ lb. hyson skin tea at 14*s.* 6*d.* - - - - - -			
13 lb. green do. at 18*s.* 8*d.* - - - - - - -			
21 lb. sugar at 1*s.* 0½*d.* - - - - - - - -			
	£70	13	4

James Webster,

1803. Bt. of Edward Stewart & Co.

			C.	q.	lb.		£	*s.*	*d.*		£	*s.*	*d.*
May	14.	2 hhds. sugar	17	2	17	at	1	13	10	per			
	16.	3 bbls. raisins	12	1	19		1	14	5				
	24.	1 hhd. tobacco	4	0	12		4	19	4				
June	1.	1 bbl. rice	1	0	15		2	16	4				
	6.	1 bag pepper	1	3	19		3	12	4				
	14.	brimstone	2	1	19		1	19	1				
	24.	4 cakes bees-w.	2	2	12		1	18	4				
July	4.	12 Ch. cheeses	5	2	24		1	17	4				
		45 Glouc'er do.	4	2	10		1	12	6				
		40 Stilton do.	5	1	18			18					
		30 Amer. do.	7	2	0		1	5	6				
		bacon	6	1	17		1	4	8				
											£131	18	11.

Tare and Trett.

Tare is an allowance made by the seller to the buyer, for the weight of the case, cask, &c. in which the goods are packed. This deduction is calculated at so much per cask, or so much per cent. according to the nature of the article.

Trett is an allowance made on some articles on account of waste sustained, and is generally deducted from the suttle.

Suttle is the weight after the tare is deducted.

Neat Weight is the quantity to be settled for after the allowances are made.

The following allowances and tares on goods imported into the United States are established by Act of Congress:

Allowances for draft.

On any quantity of 100 *or* 112 *lb.*		1 *lb.*
above 100 *and under* 200 *lb.*		2
200	300	3
300	1000	4
1000	1800	7
1800 *or upwards,*		9

TARES.

On every whole chest of bohea tea	70 *lb.*
half do. do.	36
quarter do. do.	20
chest of hyson, or other green tea, of 70 *lb. or upwards*	20
box of other tea, between 50 *and* 70 *lb.*	18
if 80	20
from 80 *upwards*	22

On all other boxes of teas according to their actual weight.

On coffee in bags,	2 *pr. ct.*	*On cheese in boxes,*	20 *pr. ct.*
bales,	3	*On chocolate in boxes,*	10
casks,	12	*On cotton in bales,*	2
On sugar in casks,	12	*seroons,*	6
boxes,	15	*On indigo in barrels,*	12
bags or mats,	5	*other casks,*	15
On cocoa in casks,	10	*seroons,*	10
bags,	1	*On pepper in casks,*	12
On pimento in casks,	16	*bales,*	5
bags,	3	*bags,*	2
On cheese in baskets,	10	*On shot in casks,*	3

On all other goods according to the invoice, or actual weight.

CASE I.

OF INVOICE TARE.

RULE.—*Add the gross weight into one sum, and the tare into another; subtract the tare from the gross, the remainder is the neat weight.*

EXAMPLE.

What is the value of 4 casks merchandize, at 5*s*. 6*d*. per lb. number and weight as follows?

	cwt.	qr.	lb.		lb.
No. 1.	4	1	10	tare	36
2.	3	3	2		29
3.	4	0	19		32
4.	4	0	0		35
	16	1	3		123

16
16
16,28
3

1823 *gross.*
123 *tare.*

1691 *neat, at 5s. 6d. per lb. comes to* £465 0 6 *Answer.*

NOTE.—*When the tare is rated on the entire at so many pound per bale, bag, or case, multiply the tare by the number of packages—the product is the tare to be deducted.*

2. What cost 16 hhds. of tallow, wt. gross 86 cwt. 2 qrs. 14 lb. tare 100 lb. per cask, at £3 15 10 per cwt? *Ans.* £274 5 8¼.

3. What must be paid for 14 hhds. of tobacco, gross 89 cwt. 2 qrs. 17 lb. tare 100 lb. per hhd. at 9¾*d*. per pound?
Ans. £351 0 9¾.

4. Sold 4 casks indigo, wt. gross 18 cwt. 2 qrs. tare 37 lb. per cask, at 4*s*. 6*d*. per pound: What is the amount?
Ans. £432 18*s*.

5. Invoice of 12 bbls. ship-bread, shipped on board the Good Intent, of New-York, and consigned to George Loftus, Jamaica, viz.

	cwt.	qr.	lb.		lb.		cwt.	qr.	lb.		lb.
No. 1.	5	2	4	tare	22	No. 7.	4	3	18	tare	19
2.	3	1	14		21	8.	4	2	16		20
3.	4	3	16		20	9.	3	3	19		18
4.	5	0	0		21	10.	4	0	0		21
5.	4	3	18		22	11.	3	3	17		20
6.	5	0	13		20	12.	3	2	24		19

at £2 3 6 per cwt. What does the invoice amount to?
Ans. £112 11 1½.

6. Invoice of 8 hhds. tobacco, received by the Triton, to sell for account of Charles Stewart & Co. Charleston: No. 1, 5 cwt.

3 qrs. 14 lb. tare 96 lb; No. 2, 6 cwt. 7 lb. tare 102 lb; No. 3, 5 cwt. 3 qrs. 21 lb. tare 94 lb; No. 4, 5 cwt. 2 qrs. 14 lb. tare 88 lb; No. 5, 4 cwt. 3 qrs. 21 lb. tare 100 lb; No. 6, 6 cwt. tare 104 lb; No. 7, 4 cwt. 3 qrs. 14 lb. tare 98 lb; No. 8, 6 cwt. 1 qr. tare 100 lb.

Sold the above at 7 cents per pound: How much is the neat weight, and amount to be received?

Ans. Nt. wt. 4321 *lb. at* 7 *cts. per lb. is* $302.47.

CASE II.

When trett is deducted from the gross weight:

RULE.—*Add the gross, as before, then carry the trett under the tare, and subtract the sum of both from the gross.*

EXAMPLE.

7. What is the value of 2 hhds. merchandize, allowing 1 lb. trett per cwt. out of the gross, at £1 10*s*. per cwt. weight as follows?

	cwt.	*qr.*	*lb.*		*qrs.*	*lb.*
No. 1.	10	1	11	tare	3	20
2.	11	0	17		3	14
gross	21	2	0	at 1 lb.	0	$21\frac{1}{2}$ *trett.*
	1	3	$27\frac{1}{2}$		1	3 $27\frac{1}{2}$
	19	2	$0\frac{1}{2}$	*Nt. at* 30*s. comes to* £29 5 $1\frac{17}{28}$ *Ans.*		

8. A merchant has 4 hhds. merchandize, weight as follows: No. 1, 10 cwt. 2 qrs. 4 lb. tare 3 qrs. 4 lb; No. 2, 11 cwt. 10 lb. tare 3 qrs. 10 lb; No. 3, 12 cwt. 1 qr. tare 3 qrs. 14 lb; No. 4, 11 cwt. 2 qrs. 14 lb. tare 3 qrs. 18 lb. trett 1 lb. per cwt. gross, at 8*s*. 6*d*. per cwt. What is the amount? *Ans.* £17 14 $3\frac{75}{112}$.

9. Bought 3 bales merchandize, wt. gross 8 cwt. 1 qr. tare 3 qrs. 17 lb. trett 1 lb. per cwt. What have I to pay, at £1 10 4 per cwt? *Ans.* £11 0 $7\frac{11}{16}$.

CASE III.

When trett is allowed after the tare is deducted, which is generally rated at 4 lb. on every 104 lb. or $\frac{1}{26}$ part.

RULE.—*Deduct the tare—then from the suttle deduct the trett.*

EXAMPLE.

10. What cost 5 packs of wool, gross 21 cwt. 8 lb. tare 28 lb. per pack, trett 8 lb. for every 3 cwt. at £4 0 6 per cwt?

5 *packs,*	21	0	8	*gross.*
5 × 28 =	1	1	0	*tare.*
	19	3	8	*suttle.*
8 lb. = $\frac{1}{42}$ =	0	1	$24\frac{6}{7}$	*trett.*
	19	1	$11\frac{1}{7}$	*Nt. at* £4 0 6 *comes* £77 17 $7\frac{23}{28}$ *Ans.*

11. What cost 12 cwt. 3 qrs. 12 lb. merchandize gross, tare 2 lb. per cwt. trett 4 lb. on 104, at $7.35 per cwt?

Ans. $89.25.

12. What is the value of 45 casks sugar, wt. gross 82 cwt. 3 qrs. 14 lb. tare 28 lb. per cask, trett 2 lb. on 112, at $8.50 per cwt? *Ans.* $595.83.

13. What will 4 hhds. tallow come to, weight as follows: No. 1, 9 cwt. 3 qrs. 24 lb. tare 88 lb; No. 2, 10 cwt. 2 qrs. 16 lb. tare 84 lb; No. 3. 11 cwt. 10 lb. tare 90 lb; No. 4, 9 cwt. 3 qrs. 4 lb. tare 82 lb. trett 2 lb. on 100, at $14.60 per cwt?

Ans. $549.58½.

CASE IV.

When the tare is rated at so much per cent. on the gross:

RULE.—*Multiply the gross by the rate per cent. and divide by* 100 *for the tare; then calculate for the amount as before.*

EXAMPLE.

14. What is the value of 15 hhds. Muscovado sugar, weight gross 68 cwt. 3 qrs. 21 lb. tare 12 per cent. at 14 cents per lb?

cwt. qrs. lb.
68 3 21
68
68
68,84
21

7721 × 12 = 926.52 *tare.*

Then 7721 − 926½ = 6704½, *at* 14 *cents, is* $951.23 *Ans.*

15. Invoice of 12 boxes Havanna sugar, wt. gross 22 cwt. 3 qrs. 14 lb. tare 15 per cent. at 18½ cents per pound.

Ans. $402.89⅛.

New-York, 1*st June*, 1803.

George Thomson,

Bt. of Henry Nugent,

	$.	C.
6 seroons indigo, gross 5 cwt. 3 qrs. 21 lb. tare 10 per cent. at 80 cents per pound - - - - - - - - -		
6 bales coffee, gross 7 cwt. 3 qrs. tare 2 per cent. at 22 cents per pound - - - - - - - - - - - - -		
5 bales cotton, wt. gross 1250 lb. tare 2 per cent. at 25 cents per pound - - - - - - - - - - - - -		
1 cask pepper, wt. gross 3 qrs. 24 lb. tare 12 per cent. at 14 cents per pound - - - - - - - - - - -		

Required the amount. *Ans.* $985.51½.

Barter.

Barter is the exchange of wares according to the prices agreed on.

CASE I.

When a quantity of goods at a given price is given to find the quantity equal thereto at the proposed price:

Rule.—As the price of the quantity given, is to the quantity given, so is the price of the quantity required, to the quantity required.

Note.—From this rule it is evident, that the proportion is *inverse*, and that the fourth number found, will always be of the denomination required, let the second be what it may. The application of this rule dispenses with a second statement, and saves many figures.

EXAMPLE.

Two merchants barter; A has cheese worth 18 cents per lb. B has 300 yards linen at 60 cents the yard: How much cheese must A deliver?

as 6'0' : 300 *yds.* :: 1'8'
10 10 3

3) 3000

1000 *lb. cheese* ÷112=8*c.* 3*q.* 20*lb.* *Answer.*

2. A merchant barters 674 yards linen at 6 shillings per yard, for sugar at 18 pence per pound: How much sugar must he receive? *Ans.* 24 *cwt.* 0 *qrs.* 8 *lb.*

3. A merchant barters coffee worth 30 cents per pound, for 600 lb tea worth 75 cents per pound: How much coffee must he deliver? *Ans.* 13 *cwt.* 1 *qr.* 16 *lb.*

4. B and C barter; B has tea worth $1.25 per pound, C has 14 cwt. of cheese at $20 per cwt: How many pounds of tea must C receive? *Ans.* 224 *lb.*

5. D has 850 yards of linen at 54 cents per yard; E has sugar worth $14 per cwt: How much sugar must D receive? *Ans.* 33½ *cwt.*

6. A has 800 gallons of spirits at $1.25 per gallon; B has tea at 80 cents per pound: How much tea must A receive? *Ans.* 1250 *lb.*

7. C barters 250 yards drugget at 18½*d.* per yard, for pepper at 15*d.* per pound: How many pounds pepper must he receive? *Ans.* 308⅓ *lb.*

8. D barters 100 yards canvas at $9\frac{1}{2}d.$ per yard, for serge at $10\frac{1}{4}d.$ per yard: How many yards serge must he receive?

Ans. $926\frac{14}{41}$ *yards.*

CASE II.

When two quantities at different rates are bartered, and a balance required:

Rule.—Find the price of each article separately, the difference is the sum to be paid or received.

EXAMPLE.

9. A has 224 lb. of tea at $1.25 per pound; B has 13 cwt. of cheese at $20 per cwt: Whether does A pay or receive, and how much?

A	224 *lb. at*	$1.25	*comes*	$280
B	13 *cwt.*	20		260
			A receives	$20 *Answer.*

10. A has 1200 yards of ribband at 2s. $3\frac{1}{4}d.$ B has 3994 lb. of rice at $5\frac{1}{4}d.$ per pound: What is the balance in barter, and which receives it? *Ans.* A *receives* £40 11 $2\frac{1}{2}$ *balance.*

11. Two merchants, A and B, barter; A has 20 cwt. of cheese at 21s. 6d. per cwt. B has 8 pieces linen at £3 14s. per piece: What is the balance in barter, and who receives it?

Ans. A *receives* £8 2s. *balance.*

12. A barters 1600 pound pepper at 17d. per pound, with B for two sorts of goods, the one at 5d. the other at 8d. per pound, to have $\frac{1}{3}$ in money and of each sort of goods an equal quantity: How much money and goods of each sort must he receive?

Ans. £37 15 $6\frac{2}{3}$, *and* $1394\frac{14}{39}$ *lb. of each.*

13. A barters 42 cwt. of hops with B, who gives him £20 in money, and the rest in goods at 5d. per pound: Required the quantity? *Ans.* 17 *cwt.* 3 *qrs.* 4 *lb.*

CASE III.

When goods are enhanced or put above the selling price in barter. To find the proportionate price the other articles should be rated at, to make the barter equal:

Rule.—*As the selling price is to the enhanced price, so is the price of the article required to the price it should be put at, to make the barter equal.*

Note.—This rule is true if goods are bartered value for value: But if one man pays a part in money and the rest in the article agreed on, he is on an equality only for the part of the goods delivered.

EXAMPLE.

A has goods worth 9*d.* per pound, but in barter insists on 11*d.* per pound; B has goods worth 2*s.* 3*d.* per pound: At what rate should he put them at to make the barter equal?

as 9*d.* : 11*d.* :: 27*d.*
3 3

33 *pence per pound*, *Answer* 2*s.* 9*d.*

15. A has linen worth 50 cents per yard, but in barter must have 55 cents; B has cassamere at 2 dollars per yard: But wishing to barter and sustain no loss, he wants to know how he may rate his cassamere? *Ans.* $2.20.

16. C has linen cloth worth 6*s.* but in barter must have 6*s.* 6*d.*; D has cotton at 2*s.* 9*d.* per pound: How must he rate it to make the barter equal? *Ans.* 2*s.* 11$\frac{3}{4}$*d.*

17. A has coffee which he barters with B at 10*d.* per pound more than it cost him, against tea which stands B in 10*s.* but rates it at 12*s.* 6*d.*: I want to know the first cost of the coffee. *Ans.* 3*s.* 4*d.*

18. Two merchants barter; C has 90 cwt. 3 qrs. 14 lb. madder, which cost him 38*s.* 6*d.* per cwt. but rates it at 42*s.* per cwt.; D pays him $\frac{1}{4}$ in ready money and the rest in hemp, which cost 34*s.* 10*d.* per cwt: How much money and hemp is C to receive, and how should D rate his hemp to make the barter equal?

Ans. C *must receive* £47 14 2$\frac{1}{4}$ *in money*, 73 *cwt.* 0 *qrs.* 5$\frac{115}{209}$ *lb. hemp at* 39*s.* 2*d. per cwt.*

Simple Interest.

INTEREST is a *premium* the *borrower* pays the *lender* for the use of *money*, and is generally rated by the 100 for a year.

The sum lent is called the *principal*; the *principal* and *interest*, the *amount*; the *interest* of 1, for 1 year, the *ratio*; and the *premium* agreed on, the *rate per cent.*

INTEREST is of two kinds, *Simple* and *Compound*.

SIMPLE INTEREST is that which arises from the *principal only*, as the interest of 100 for 2 years at 6 per cent. is only 12, viz. 6 due at the end of the *first*, and 6 due at the end of the *second year*. This added to the *principal* make the *amount*=112.

COMPOUND INTEREST is that which arises from *principal* and *interest* at the end of *every term*. Thus, 100 lent out for 2 years at 6 per cent.; the amount for the first is 106, *which becomes a new principal*, on which *interest* must be calculated *for the second year*, viz. $112.36 amount for the second year.

In Simple Interest a *represents the amount.*
r *ratio or rate per cent.*
p *principal.*
t *time.*

CASE I.

When interest is calculated for one or more years:

RULE.—Multiply the *principal* by the *rate per cent.* divide by 100, the quotient will be the interest for 1 year; this multiplied by the given years, gives the interest required.

EXAMPLE.

1. What is the interest of $6724 for 3 years at 5 per cent. per annum?

```
 6724
    5 rate per cent.
 ------
 336.20 interest for 1 year.
      3 years.
 ------
 1008.60 interest for 3 years.
```

NOTE.—The No. 100 need not be expressed, as to point off two places from the product for the two cyphers cut off, answers for cents.

2. What is the interest of $7546, for four years, at 6 per cent. per annum? *Ans.* $1811.04.

3. What is the interest of $471.50 for 6 years, at 7 per cent. per annum? *Ans.* $198.03.

CASE II.

When interest is to be calculated for years and months:

RULE.—*Find the interest for years as before, then take parts of the interest of one year for the months, &c.*

EXAMPLE.

4. What is the interest of $475.50 for 2 years 5 months, at 7 per cent. per annum?

```
            $475.50
                  7
            -------
            33,28.50 interest for 1 year.
                  2
            -------
            66.57          for 2 years.
4 months = 1/3 = 11.095
1 month  = 1/4 =  2.7737
            -------
            80.43|8'7'  Ans. $80.43.
```

NOTE.—It often happens in taking practical parts, that *large fractions occur at the end of each division.* I would, therefore, recommend the above method of *continuing the decimal places, at least, to mills;* and *preserving only two decimal places for cents,* at the end of the performance—will give the answer to a *sufficient degree af exactitude.*

5. What interest must be paid on $7546 for 4 years and 10 months, at 6 per cent. per annum? *Ans.* $2188.34.

6. Lent $3754 for 2 years 7 months, at 6 per cent. per annum: What is the amount at the end of that term? *Ans.* $4335.87.

CASE III.

When interest is to be calculated for months and days:

RULE.—*Take parts out of the interest of a year for the months, and parts of the interest of a month or months, for the days.*

EXAMPLE.

What interest is due on $3000 that has been at use 4 months 14 days, at 6 per cent. per annum?

	3000	
	6	
	180.00 *int.*	*for* 1 *year.*
3 *months* = $\frac{1}{4}$ =	45	*for* 3 *months.*
1 = $\frac{1}{3}$ =	15	*for* 1 *month.*
10 *days* = $\frac{1}{3}$ =	5	*for* 10 *days.*
2 = $\frac{1}{5}$ =	1	*for* 2 *days.*
2 = =	1	*for* 2 *do.*
	$67	*for* 4 *months* 14 *days Ans.*

NOTE.—In calculating interest for months, *the answer may be exact;* but in taking parts of months as valued at 30 days, *there must arise some error,* because 12 × 30 = 360, which is 5 days less than a year; but the error is so trifling, it is esteemed of no consequence in business by many merchants.

In the above example, the interest of 3000 dollars for 14 days, is about 10 cents too much; therefore, in large sums, if perfect exactness be required, deduct the $\frac{1}{73}$ part of the interest for days, the remainder will be the true interest; because 365—360 = 5 and 365 ÷ 5 = 73 or $\frac{1}{73}$ part too much in calculating at 360 days to the year.

8. What interest must be charged on $745 for 3 months 18 days, at 7 per cent. per annum? *Ans.* $15.64.

9. What interest must be paid for the use of $3400 for 6 months 15 days at 7 per cent.? *Ans.* 128.91\frac{2}{3}$.

10. Cast up the interest of $540 for 8 months 18 days, at 5 per cent. per annum? *Ans.* $19.35.

11. What interest have I to pay on $900 for 10 months 21 days, at 7 per cent. per annum? *Ans.* 56.17\frac{1}{2}$.

12. What interest must be paid on $200 for 1 month 25 days, at 6 per cent.? *Ans.* 1.83\frac{1}{3}$.

CASE IV.

When interest is to be calculated for days:

RULE.—*Find the interest of the given sum for a year, then say, as* 365 : *interest found* :: *given days* : *interest required.*

Or thus:

RULE.—*State the question by the double rule of three:*

as 100 } rate per cent. { principal.
365 }   { time   to the interest.

EXAMPLE.

13. What is the interest of $750 for 96 days, at 5 per cent.?

By rule 1st,

```
      750
        5
    -----
    37.50 interest for a year.
```

Then as 365*d.* : 37.50 :: 96*d.* : *to* $9.86 *Answer.*

By rule 2nd,

```
as 100 }  5  { 750
   365 }     {  96
   ---       ------
                72000
                    5
                ------
36500)          360000  ($9.86 Answer.
```

But this performance may be abridged by cancelling such numbers as will admit of contraction, &c.

```
 2'—1'0'0' }  5'  { 7'5'0'—15
73—3'6'5'  }      {  9'6'—48
                  ---------
                  73) 720 ($9.86 Answer.
```

14. What interest will be due on $7480 for 60 days, at 6 per cent. per annum? *Ans.* $73.77.

15. What is the interest of $4850 for 20 days, at 7 per cent. per annum? *Ans.* $18.60.

16. Borrowed $500 for 30 days at 5 per cent.: How much must I pay the lender on settlement? *Ans.* $502.05.

17. Borrowed $1000 with condition to pay what sums I pleased when convenient. At the end of 20 days, I paid off $300;

15 days after, $250 ; 25 days I discharged the entire : What had I to pay if the lender charged interest at 5 per cent. per annum ?

Ans. $455.71.

TABLE I.

Showing the number of days from any day in any month, to the same day in any month following.

	From January	*February*	*March*	*April*	*May*	*June*	*July*	*August*	*September*	*October*	*November*	*December*
To January	365	334	306	275	245	214	184	153	122	92	61	31
February	31	365	337	306	276	245	215	184	153	125	92	62
March	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
August	212	181	153	122	92	61	31	365	334	304	273	243
September	243	212	184	153	123	92	62	31	365	335	304	274
October	273	242	214	183	153	122	92	61	30	365	334	304
November	304	273	245	214	184	153	123	92	61	31	365	335
December	334	303	275	244	214	183	153	122	91	61	30	365

USE.

Required the number of days from the 16th of August to the 24th of May following :

Under August, and opposite May, is - - -	273
From the 16th to the 24th is - - - - - -	8
Ans.	281

Usual method.

August - - - - - - -	15
September - - - - - -	30
October - - - - - - -	31
November - - - - - -	30
December - - - - - -	31
January - - - - - - -	31
February - - - - - -	28
March - - - - - - -	31
April - - - - - - - -	30
May - - - - - - - -	24
	281

In bissextile, or leap-years, if the end of February occurs in calculation, one day must be added.

NOTE.—*The calculation of simple interest, for days, may be much abridged, if we pay due attention to the proportion:* for if certain numbers are to be *multiplied* and *divided by*, it is easy to find a *proportionate mean* that will answer for a *multiplier* or *divisor*, as the nature of the question requires. The proportion of interest for days is this:

As 100 × 365 *days* : *rate per ct.* :: *principal time* : *interest.*

From this statement it is evident, if we multiply 365 by 100, and divide that product by the product of the rate and time, the quotient will be a constant divisor for any sum at that rate and time.

EXAMPLE.

What interest will be due on $750 for 96 days, at 5 per cent.?

First to find a divisor.

365 × 100 = 36500 ÷ 96 × 5 = 76.041 *constant divisor;*

and $750 ÷ 76 = $9.86 *interest required, Answer.*

From hence it appears we have only to calculate a table of divisors, to render interest one of the simplest calculations in arithmetic.

The following table is calculated on this principle.

RULE.—Divide by the number opposite the given days—the quotient will be the interest required.

If the interest required be 5 per cent. subtract $\frac{1}{6}$*; if 7 per cent. add* $\frac{1}{6}$ *to the interest found—the difference, or sum, will be the interest required.*

If the divisor consists of decimals, *subjoin as many cyphers to the principal as will make the decimal places equal to the divisor—the quotient will then be an unmixed whole number:* If a remainder occurs, bring down cyphers till you have found *two places for cents.*

NOTE.—*If a divisor be required for any number of days not expressed in the table, take any number of days that will be a multiple of the number required, and divide the tabular number by the said multiple—the quotient will be the tabular number required.*

EXAMPLE.

Required a divisor for 96 days, at 6 per cent.

96 ÷ 48 = 2.

Then the tabular number for 48 *is* 126.7361 ÷ 2 = 63.368 *Answer.*

Let the student form similar tables for 5 and 7 per cent.

TABLE II.

Constant divisors, from 1 *to* 63 *days, for* 6 *per cent. at* 365 *days to the year.*

Days.		*Days.*		*Days.*	
1	6083.333	27	225.3086	53	114.78
2	3041.6666	28	217.2619	54	112.6543
3	2027.7777	29	209.77	55	110.60606
4	1520.8333	30	202.7777	56	108.6309
5	1216.6666	31	196.2365	57	106.7251
6	1013.8888	32	190.1041	58	104.885
7	869.0476	33	184.3434	59	103.1073
8	760.4166	34	178.9213	60	101.3888
9	675.9259	35	173.8095	61	99.7267
10	608.3333	36	168.9814	62	98.1182
11	553.0303	37	164.4144	63	96.5698
12	506.9444	38	160.0877		
13	467.9487	39	155.9829		
14	434.5238	40	152.0833	*Months.*	
15	405.5555	41	148.3739		
16	380.2083	42	144.8412	1	200.
17	357.8437	43	141.4728	2	100.
18	337.9629	44	138.2575	3	66.'6
19	320.1754	45	135.1851	4	50.
20	304.1666	46	132.2463	5	40.
21	289.6825	47	129.4326	6	33.'3
22	276.5151	48	126.7361	7	28.5714
23	264.4926	49	124.1496	8	25.
24	253.4722	50	121.6666	9	22.'2
25	243.3333	51	119.281	10	20.
26	233.9743	52	116.9871	11	18.1818

CASE V.

We now come to *Practical Interest*, or *Bank Discount.*

The mode in the Banks is to calculate for 30 days to the month, or 360 days to the year; this makes the interest $\frac{1}{73}$ part *more than the true;* but the error is so trifling, that in small sums for a short time it is *not worth correcting: however, this, together with the difference of discount and interest, is esteemed as a compensation for trouble, book-entries, &c.*

Foreign bankers consider the sum expressed in the *bill* or *note*, as a *principal* on which they calculate interest for the given time, and likewise charge $\frac{1}{4}$, $\frac{1}{2}$, and sometimes 1 per cent. for their *trouble*, which is added to the interest before deduction.

The proportion to find the interest for 60 days is this:

as 100 × 360 : 6 *per ct.* :: *principal* × 60 : *interest;*

consequently, $100 \times 360 \div 6 \times 60 = 100$, *a constant divisor* for any sum at 6 per cent. for 60 days: *therefore, the number* 100 *may be rendered a universal calculator at any rate per cent. for any number of days less than a year*—because of 60 days many of the less numbers are *multiples,* or *aliquot parts;* if they are not, it is easy to separate *aliquant* into *aliquot parts.*

2 days $=\frac{1}{30}$, 3 days $=\frac{1}{20}$, 4 days $=\frac{1}{15}$, 5 days $=\frac{1}{12}$, 6 days $=\frac{1}{10}$, 10 days $=\frac{1}{6}$, 12 days $=\frac{1}{5}$, 15 days $=\frac{1}{4}$, 20 days $=\frac{1}{3}$, 30 days $=\frac{1}{2}$ of 60 days, &c.

PROPOSITION I. *If the time given be* 60 *days:*

RULE.—Divide the principal by the number 100, the quotient will be the interest.

EXAMPLE.

What interest must be charged on $475, at 6 per cent. for 60 days?

$475 \div 100 = \$4.75$ *Answer.*

NOTE.—In dividing by 100, it is sufficient to point off two places from the dollars for cents.

PROPOSITION II. *If the time be less than* 60 *days:*

RULE.—Take parts of the interest of 60 days for the given days.

EXAMPLE.

Required the interest of $865 for 35 days, at 6 per cent.

$8.65 *interest for* 60 *days.*

30 *days* $=\frac{1}{2}=4.32$
5 *days* $=\frac{1}{6}=\ .72$

Ans. $5.04 *interest for* 35 *days.*

PROPOSITION III. *If the given days exceed* 60:

RULE.—See how often 60 is contained therein, and note the remainder for days—then multiply the interest of 60 by the number of times contained, and take parts for the odd days, as before; the sum will be the interest required.

EXAMPLE.

Required the interest of $500.50 for 195 days, at 6 per cent.

195÷60=3 *times and* 15 *over.*
5.00.50 *interest for* 60 *days.*
3 *times contained in* 195.

15.015 *interest for* 3 *times* 60 *days.*
15 *days*=$\frac{1}{4}$= 1.25

Ans. 16.26\frac{1}{2}$ *interest for* 195 *days.*

22. What is the interest of $5000 for 60 days, at 6 per cent.? *Ans.* $50.

23. Cast up the interest of $675 for 30 days, at 6 per cent. *Ans.* 3.37\frac{1}{2}$.

24. What is the amount of $500 for 40 days, at 6 per cent.? *Ans.* 503.33\frac{1}{3}$.

25. How much is the interest of $7540 for 30 days, at 6 per cent.? *Ans.* $37.70.

26. How much will $1000 amount to in 45 days, at 6 per cent.? *Ans.* $1007.50.

27. Discounted Thomas Jones' note of $670, which has to remain 36 days at the N. York bank: What credit have I for the same, at 6 per cent. per annum? *Ans.* $665.98.

28. Discounted at the U. S. bank George Bakewell's promissory note, due 75 days hence, for $1000: How much may I draw for, at 6 per cent.? *Ans.* $987.50.

29. Discounted at the Manhattan bank Samuel Kelley's promissory note to Gabriel Lewis, to remain 63 days, for $1500, at 6 per cent.: How much is the credit? *Ans.* $1484.25.

30. Lent at interest, for 6 months, $4500: What is the amount at the end of that term, at 6 per cent.? *Ans.* $4635.

Proposition IV. *If the rate be given at 5 or 7 per cent.:*

Rule.—Find the interest as before; subtract $\frac{1}{6}$ for 5 per cent. or add $\frac{1}{6}$ for 7 per cent.; the remainder, or the sum, will be the interest required in either case.

EXAMPLE.

Required the separate interests of $750, for 76 days, at 5 and 7 per cent.?

7.50 *Interest for* 60 *days.*
12 *days* $=\frac{1}{5}=$ 1.50
4 *days* $=\frac{1}{3}=$ 50

$9.50 *amount interest at* 6 *per cent.*
Deduct $\frac{1}{6}$ *part* = 1.583

Remainder $7.91 *am't interest at* 5 *per cent.*
And $9.50+1.583=$11.08 *am't interest at* 7 *per cent.* } *Ans.*

PROPOSITION 5. *If interest be calculated at* 365 *days to the year:*
RULE.—Cast the interest as before, and deduct $\frac{1}{73}$ part (see note on case 3) the remainder will be the interest required.

EXAMPLE.

Required the interest on $700 for 66 days, at 6 per cent. at 365 days to the year?

7.00
6 *days* $=\frac{1}{10}=$.70

7.70 *bank interest.*
Deduct 5 *days* $=\frac{1}{73}=$.105

Ans. $7.59 *int. for* 66*ds. at* 365*ds. to the year.*

From the foregoing examples, we see with what *facility* we may cast interest by the application of the No. 100, the *simplicity, ease and certainty* of which, *is manifest in these solutions, where many figures and tedious calculations are dispensed with. Currency is calculated on the same principles, by reducing the lower denominations to a decimal of the higher.*

NOTE.—Let the student form tables similar to table 2nd, on this principle.

CASE VI.

When interest is to be cast on an account current:

RULE.—*Multiply the several balances by the number of days they have been due, add the products into one sum, which divide by*
7200 *for* 5 *per cent.*
or 6000 *for* 6 *per cent.*
or 5142.857 *at* 7 *per cent. the quotient will be the interest required.*

EXAMPLE.

George Johnson in account current with John Glover.

				days.
1803. July	14, to merchandize	600	× 15 =	9000
	29, to do.	300		
		900	× 20 =	18000
Aug.	18, by cash	450		
		450	× 12 =	5400
	30, to goods	250		
		700	× 15 =	10500
Sep.	14, by cash	600		
		100	× 20 =	2000
Oct.	4, by cash	100		
			6,000)	44,900

Interest on the several balances $7.48 *Ans.*

When several notes are to be discounted, the discount may be found by one operation.

EXAMPLE.

Discounted James Young's note to run	30 days for	500 =	15000
George Clark's	25	700 =	17500
John Caldwell's	45	300 =	13500
William Steele's	60	500 =	30000
Wm. Armstrong's	40	200 =	8000
at 5 per cent. - - - - - - -			84000

then 84000÷7200=$11.66 *Ans.*

New-York, July 20th, 1802.

35. Borrowed on account $370. Aug. 1, paid $250; Aug. 21, borrowed $640; 29, borrowed $340; September 10, paid $1000; September 20, paid off in full: What was the last payment, interest being charged at 6 per cent.? *Ans.* $104.52.

36. Discounted the following notes: James Thomson's for $700, to run 20 days; William Howard's $800 at 25 days; Charles Jones' $300 at 20 days; Henry Jackson's $700 at 16 days; and George Bond's $750 at 12 days: How much am I to receive, deducting interest at 5 per cent. per annum?

Ans. $3291.09.

New-York, October 1st, 1802.

37. Thomas Par, Dr. to merchandize $350

Oct. 12,	to	do.	250	
Nov. 1,	by	cash		$500
15,	to	merchandize	600	
Dec. 10,	by	cash		500
30,	by	do.		200

Required the interest due on the several balances at 6 per cent.? *Ans.* $6.51.

CASE VII.

Given—the rate, time and amount, to find the principal:

$$\text{Theorem } \frac{a}{r\ t+1} = p.$$

EXAMPLE.

38. What principal will amount to $5000 in 5 years at 5 per cent.?

Ratio .05
Time 5 *years.*

.25
more 1.

1.25) 5000 (4000 *dlls. Ans.*

NOTE.—When days are the given time, reduce them to the decimal of a year.

39. What principal lent out for 45 days at 6 per cent. will amount to $1007.50? *Ans.* $1000.

40. How many dollars must I lend at 5 per cent. for 72 days, that the amount may be $454.50? *Ans.* $450.

CASE VIII.

Given—amount, principal and time, to find the ratio, or rate per cent.:

$$\text{Theorem } \frac{a-p}{tp} = r$$

EXAMPLE.

41. What rate of interest must be charged on $1000, that it may amount to $1007.50 in 45 days?

45 *days*=.125 *year* × 1000=125.000

a 1007.50
p 1000.

7.50÷125.000=.06 *ratio, or* 6 *per cent. Ans.*

42. At what rate must I lend $2000, that the amount may be $2010 in 36 days? *Ans.* 05 *ratio, or* 5 *per cent.*

CASE IX.

Given—amount, principal and rate, to find the time:

$$\text{Theorem } \frac{a-p}{rp} = t$$

EXAMPLE.

43. In what time will $1000 amount to $1007.50, at 6 per cent.?

```
                 1007.50
                 1000
                 -------
1000×.06=60.00) 7.50 (.125 year, or 45 days.
```

44. In what time will $700 amount to $710.50, at 6 per cent.?
Ans. 90 *days.*

45. In what time will $4000 amount to $5000, at 5 per cent.?
Ans. 5 *years.*

TABLE IV.

RATIOS *at the following rates:*

Rate.	*Ratio.*	*Rate.*	*Ratio.*	*Rate.*	*Ratio.*
$\frac{1}{8}$ = *pr. ct.*	.00125	1 *pr. ct.*	.01	5 *pr. ct.*	.05
$\frac{2}{8}$ or $\frac{1}{4}$	.0025	2	.02	6	.06
$\frac{4}{8}$ = $\frac{1}{2}$	.005	3	.03	7	.07
$\frac{6}{8}$ = $\frac{3}{4}$	.0075	4	.04	8	.08

Annuities or Pensions in arrear.

ANNUITIES are payable the day they become due; if they are withheld beyond that day, they are said to be in arrear.

RULE.—*Find the interest of the Annuity for one year, then for two, three, &c. up to the given time, less one, because there is no interest due on the last payment, it being only due. Multiply the Annuity by the time, add the product and several interests together for the amount, &c.*

EXAMPLE.

What is the amount of an annuity of $500 forborn 5 years, interest computed at 6 per cent.?

```
as 100 : 6 :: 500 : 30 interest for 1 year.
                    60=         2 do.
                    90=         3 do.
                   120=         4 do.
     and 500×5=2500 amount annuity.
                  ----
                  2800 amount, Answer.
```

But if interest be allowed on half yearly, or quarterly payments, it will be still more in favour of the receiver.

EXAMPLE.

What would the foregoing annuity have amounted to, if it had been payable at quarterly payments?

NOTE.—When the number of payments or terms are many, as they form an arithmetical progression, I would recommend that rule, as the shortest for the solution *.

5 × 4 = 20 *payments.*

1.875 *first term.*
20—1 = × 19

35.625 *last term.*

37.50 *sum of first and last.*
× 9½ *half no. of terms.*

356.25 *sum of interest.*
500 × 5 = 2500.

Ans. 2856.25 *amount.*

4) 500

125 *quarterly.*
.06

4) 7.50 *int. for* 1 *year.*

1.87½ *for* 1 *quarter, and first term.*

From this it appears evident, that half-yearly payments are more advantageous to the receiver than yearly, and quarterly than either, by the interest of the respective sums, for the anticipated time.

If $600 payable quarterly, be forborn 6 years, what will be due at the end of the term at 5 per cent.? *Ans.* $4117.50 *amt.*

An annuity payable half yearly is in arrear 7 years: What is the amount due at 5 per cent.? *Ans.* 4882.50.

An annuity of $1000 payable half-yearly, remains unpaid 6: What is the amount due at 6 per cent.? *Ans.* $6990.

A house-rent of $460 payable quarterly, being disputed at law, the tenant withheld the rent 5½ years: What had he to pay at the end of the term, if he was charged 4 per cent.?

Ans. $4882.50.

* See Problem III. Arithmetical Progression.

Rebate or Discount.

DISCOUNT is *an allowance* made for the payment of money *before it becomes due*, and is less than *interest* by the *interest of the interest* for the given time.

Therefore, the *present worth of any sum*, must be such, that if put out at interest at the *given rate*, *the amount should equal the given sum* on the day it becomes *due*.

CASE I.

Given—the sum to be discounted, rate per cent. and time, to find the discount.

RULE.—As 100 more, *the rate per cent.* is to the *rate per cent.* so is the *given sum*, to the *discount*.

EXAMPLE.

What is the discount of $916.90 due 1 year hence at 6 per cent.?

```
      100
        6
      ---
as 106 : 6 :: 916.90
                   6
              ------
        106) 5501.40 ($51.90 Ans.
```

CASE II.

Given—the sum to be discounted, rate per cent. and time, to find the present worth.

RULE.—As 100 more, the rate per cent. is to 100, so is the given sum to the *present worth*.

EXAMPLE.

Required the present worth of $916.90 due one year hence, discount allowed at 6 per cent.?

as 106 : 100 :: 916.90 : to $865 *Ans.*

Where the time is *long*, the calculation of DISCOUNT *may be much abridged* by the following *proportion*, viz.

As 1 more, the product of the ratio and time, is to 1, *so is the given sum* to its present worth. See table of ratios, simple interest, case 9.

EXAMPLE.

What is the present worth of $5750 due 3 years hence at 5 per cent. discount?

```
      .05 ratio.
        3 time.
      ---
      .15
add   1.
      ---
as 1.15 : 1 :: 5750 : to $5000 Ans.
              575
```

Therefore it is evident, if we divide any sum, *by the amount of 1 for the given time*, the quotient will be the *present worth.*

From this axiom, the following tables were calculated, which will stand for *universal divisors* for all sums to be discounted at their *rates and time.*

Calculation of the following table:

per cent.

as 100 > 6 < 1
365 > 6 < 1 day comes .00016438355 *int. of* 1 *for* 1 *day*
to which add 1.

1.00016438355 *amt. of* 1 *for* 1 *day*
add .00016438355

1.0003287671 *amt. of* 1 *for* 2 *days*
add .00016438355

1.00049315065 *amt. of* 1 *for* 3 *days*

Use.

1st. *To find the present worth of any sum:*

Rule.—Divide the sum to be discounted by the number opposite the days, months or years to be discounted for: *the quotient will be the present worth.*

EXAMPLE.

What is the present worth of $2431.56 for 80 days at 6 per cent.?
Tab. No. at 6 *per cent.*
For 80 *days*, 1.01315) 2431.56 (2400 *Ans.*

2nd. The *amount* of any sum at simple interest, may be found by multiplying the principal *by the amount of one*, *for the given time.*

EXAMPLE.

Required the amount of $600 for 60 days at 6 per cent. per annum? 600 × 1.00986 Tab. No. for 60 days = $605.91 6/10 *Ans.*

3rd. The *interest* of any sum at simple interest may be found by subtracting 1 from the Tab. No. and multiplying as before.

EXAMPLE.

Required the interest on $600 for 60 days at 6 per cent.?
1.00986 — 1 = .00986 × 600 = $5.91|600.

TABLE IV,

Showing the amount of 1 *dollar or* 1 *pound, from* 1 *day to* 16 *years, computed at* 365 *days to the year.*

Days.

	6 *per cent.*	7 *per cent.*		6 *per cent.*	7 *per cent.*
1	1.00016438355	1.00019178	20	1.003287	1.003835
2	1.000329	1.000383	21	1.003453	1.004026
3	1.000493	1.000575	22	1.003616	1.004218
4	1.000657	1.000767	23	1.003780	1.004411
5	1.000822	1.000958	24	1.003944	1.004602
6	1.000986	1.00115.	25	1.004107	1.004794
7	1.001151	1.001342	26	1.004274	1.004986
8	1.001315	1.001534	27	1.004438	1.005178
9	1.001479	1.001726	28	1.004602	1.005368
10	1.001643	1.001917	29	1.004766	1.005660
11	1.001808	1.002109	30	1.004931	1.005952
12	1.001972	1.002301	31	1.005095	1.006044
13	1.002137	1.002493	32	1.005260	1.006136
14	1.002301	1.002684	33	1.005424	1.006327
15	1.002465	1.002976	40	1.006574	1.007670
16	1.002630	1.003068	50	1.008214	1.009588
17	1.002794	1.00326	60	1.009862	1.011904
18	1.002959	1.003452	90	1.014793	1.017256
19	1.003123	1.003643			

Months.

	6 *per cent.*	7 *per cent.*		6 *per cent.*	7 *per cent.*
1	1.005	1.0058'3	7	1.035	1.04083'
2	1.01	1.0116	8	1.04	1.04666'6
3	1.015	1.0175	9	1.045	1.0525
4	1.02	1.02'3	10	1.05	1.05833'3
5	1.025	1.02916'6	11	1.055	1.06416'
6	1.03	1.035	12	1.06	1.07

Years.

	6 *per cent.*	7 *per cent.*		6 *per cent.*	7 *per cent.*
1	1.06	1.07	9	1.54	1.63
2	1.12	1.14	10	1.6	1.7
3	1.18	1.21	11	1.66	1.77
4	1.24	1.28	12	1.72	1.84
5	1.3	1.35	13	1.78	1.91
6	1.36	1.42	14	1.84	1.98
7	1.42	1.49	15	1.9	2.05
8	1.48	1.56	16	1.96	2.12

To collect a divisor for days, months, and years:

Rule.—Take the decimal parts for the *different denominations of the given time*—the sum more one will be the *divisor.*

EXAMPLE.

Required the present worth of $500, due 10 years, 4 months, 14 days hence, discount at 6 per cent. per annum.

.6 - - *decimal part by the table for* 10 *years.*
.02 - - *do.* - - - - - - - - - 4 *months.*
.0023 - *do.* - - - - - - - - - 14 *days.*
add 1.

1.6223 *amount of* 1. *for* 10 *years,* 4 *months, and* 14 *days,* and a divisor for any sum at that rate and time.

Then 1.6223) 500,000 (308.20. *present worth, Answer.*
7140 *remainder.*

To prove Discount, cast interest on the present worth, for the given time and rate—the *amount* should equal the given *sum* to be discounted.

308.20 *present worth as above.*
6 *per cent.*

18.492 *interest for* 1 *year.*
10 *years.*

184.92 *interest for* 10 *years.*
3 *months* $=\frac{1}{4}=$ 4.623 *do. for* 3 *months.*
1 *do.* $=\frac{1}{3}=$ 1.541 *do.* 1 *do.*
10 *do.* $=\frac{1}{3}=$ .513 *do.* 10 *days.*
3 *do.* $=\frac{1}{10}=$ .154 *do.* 3 *do.*
1 *do.* $=\frac{1}{3}=$ .051 *do.* 1 *do.*
308.20 *present worth.*

$500. *proof.*

8. What is the present worth of $4000, due 75 days hence, discount at 6 per cent. per annum? *Ans.* $3951.29.

9. What ready money must be paid for a note of $4500, due 60 days hence, discount at 6 per cent.? *Ans.* $4456.06.

10. What discount must I deduct from a note of $500, due 45 days hence, at 7 per cent.? *Ans.* $4.28.

11. What is the present worth of a bond for $8000, due 4 years, 6 months, 20 days hence, at 6 per cent.? *Ans.* $6282.95.

New-York, 1*st Dec.* 1802.

12. Discounted the three following notes, viz.
George Barnwall's, for $500 due 30th January;
Wm. Constable's, - - 720 - 15th do.
John Glover's, - - - 800 - 30th instant:

What sum must be deducted for discount, at 6 per cent.? *Ans.* $13.96.

The difference of Discount and Interest.

Interest is almost universally used by merchants for the calculation of discount. For a short time, and small sum, the error is not worth correcting; but when the time is considerable, and the sum large, the error becomes greater than we might at first apprehend. This can be better illustrated by example than argument:

EXAMPLE.

Required the difference of the discount and interest of $1000 for 1 month, 2 months, 4 months, 8 months, 1 year, 5 years, and 10 years, at 6 per cent. per annum?

	Interest.	*Discount.*		
1 *month*	$5	$4.97	*difference*	$0.03
2	10	9.90		.10
4	20	19.78		.22
8	40	38.46		1.54
1 *year*	60	56.61		3.39
5	300	230.77		69.23
10	600	375.		225.

Thus the seller of a bill or note of $1000, will lose by interest in one month only about 3 cents, for 4 months 22 cents, for 8 months $1.54, for a year $3.39, and for 10 years $225.

Present worth of Annuities.

The present worth of Annuities is found by dividing *each payment* by the *amount* of 1 for the *given time*, *the sum* of the different terms found, is the *present worth*.

At first view this appears *absurd;* because, if interest be calculated on the *present worth*, the amount will be more than *the sum of the annuity;* but if interest be counted on the *present worth of each payment*, for its *respective time*, the amount will be equal to the *annuity for that time*.

EXAMPLE.

What is the present worth of an annuity of $1000 to continue 6 years, discount at 6 per cent. per annum?

$1000÷	1.06=943.40	*present worth of*	*1st yr's.*	*annuity.*
	1.12=892.85	- - - -	*2nd do.*	*do.*
	1.18=847.46	- - - -	*3rd do.*	*do.*
	1.24=806.45	- - - -	*4th do.*	*do.*
	1.3 =769.23	- - - -	*5th do.*	*do.*
	1.36=735.29	- - - -	*6th do.*	*do.*

$4994.68 *present worth, Ans.*

Some imagine this method of purchasing annuities unjust; that they should be computed at compound interest. Leaving the buyer and seller to settle this matter, we shall give a few examples for the learner's sake, and proceed to factorage and exchange.

2. What is the present worth of an annuity of \$500, to continue 5 years, at 5 per cent. per annum? *Ans.* \$2179.12.

3. What is the present worth of \$1200 yearly rent for 8 years, at 6 per cent. per annum? *Ans.* \$

4. What is the present worth of \$600, annuity to continue 7 years, and not to commence till the expiration of 4 years, discount allowed at 7 per cent. per annum? *Ans.* \$2844.06.

5. What is the present worth of £50, annuity to continue 6 years, at 5 per cent. per annum? *Ans.* £256 13 7.

6. What is £80 annuity worth in ready money, to continue 5 years, at 6 per cent. per annum? *Ans.* £340 15 1.

7. What is the present worth of £100 annuity to continue 5 years, at 6 per cent. per annum? *Ans.* £425 18 9½.

Equation of Payments.

When several debts are payable at *different times*, and the Dr. and Cr. mutually agree that all the sums should be paid *at once*, and at *such a time* that neither party should sustain *loss* thereby; this is called *equating* the time *of payment*, for which this is the

RULE.—*Multiply each particular payment by its time, add the products, and divide the sum by the whole debt*, the quotient *is* equated time *for the payment of the* whole.

EXAMPLE.

1. A owes B \$600; whereof 200 is payable at 3 months, 150 at 4 months, the remainder at 6 months; but they agree that the entire should be paid at once: Required the equated time:

$$200 \times 3 = 600$$
$$150 \times 4 = 600$$
$$250 \times 6 = 1500$$

600) 2700 (4½ *months, Answer.*

Though the above method is in general use, *yet it is not strictly true*, for we must suppose that *in paying a debt before it is due*, the *discount, not interest, should be lost.* But the error is so *trifling*, and the true method so *prolix*, that this rule *will never go out of use.*

2. A bought goods of B, to the amount of \$460, to be paid as follows: 260 in 5 months, the remainder in 7 months; but they agree to make one payment of the whole: What is the equated time? *Ans.* *5 months* $26\frac{2}{23}$ *days.*

3. C owes D a certain sum, payable as follows: $\frac{1}{2}$ in 3 mo. $\frac{1}{3}$ in 4 months, and $\frac{1}{6}$ in 9 months; but they agree to make one payment of the entire: Required the equated time:

Ans. 4 *months* 10 *days.*

4. A debt is to be discharged thus: $\frac{1}{4}$ immediately, $\frac{1}{4}$ at 4 months, $\frac{1}{4}$ at 5 months, and the rest at 6 months: At what time may the whole be paid? *Ans.* 5 *months.*

5. E is indebted to F £240, which by agreement, is to be paid 5 months hence; but E is willing to pay down £40, provided he has a longer time for the payment of the remainder, which is agreed to: The time of payment is required: *Ans.* 6 *months.*

Factorage or Commission, Brokerage, &c.

A Premium estimated at so much per cent. for the transaction of business by a factor, is called *factorage, commission* or *brokerage.*

RULE.—Multiply the amount sales, &c. by the rate per cent. divide by 100, the quotient will be *the amount, commission, brokerage, &c.*

Otherwise,

Multiply the amount sales by the ratio, (see Tab. 3, simple interest page 117) the product will be the commission, &c.

EXAMPLE.

1. What is the commission on an account sales of $6754.50, at $3\frac{1}{2}$ per cent. *Ans.* $236.40.

2. What is the brokerage on $654.40, at $\frac{1}{4}$ per cent.?

Ans. $1.63..

3. What brokerage must be paid on $800.50, at $\frac{3}{4}$ per cent.?

Ans. $6.00.+

4. Sold goods to amount of $345.60: What is my commission on the sales at $3\frac{3}{4}$ per cent.? *Ans.* $12.96.

6. A factor owes a merchant $1000 neat proceeds, and has orders to lay it out on merchandize, and hold 2 per cent. for his commission and charges: How much must the factor lay out on these terms? *Ans.* $980.40.

7. A factor owed his employer $456.50 neat proceeds; the merchant thinking it more advantageous to have merchandize for the amount, than to draw, ordered the factor to buy and ship on his account. The factor's invoice is $443.20, the remainder he holds for commission and charges: What rate did he charge?

Ans. 3 *per cent.*

Insurance.

A Writ, or Policy of Insurance, is a security passed from the *insurer* to the *insured*, whereby the *insurers* or *underwriters*, in consideration of a certain premium, are bound to make good to the *assured all such property mentioned therein, or an equivalent thereto, against all hazards, perils, losses, &c. or against some particular event.*

This Instrument, when once effected, becomes the *property of the assured*, and is executed by the insurer alone, subscribing his name, from which he is called an *underwriter*.

The Premium is rated at so much per cent. on the property insured, and rises or falls, according to the *risk* or *hazard run*.

In cases of total loss, the underwriter makes good to the assured the amt. insured, deducting 2 per cent.: this is called *Discount*.

Abandonment is that right which the assured has, in case of partial loss, to *relinquish* his interests in the property insured to the underwriter, and claim the *amount insured*, as if for a *total loss*.

Salvage is an *allowance* or *premium* made by the *owners of ship and cargo*, for saving the ship or cargo, or both, from the dangers of the sea, pirates, or enemies; and, till such premium is made, *the person saving the property is justified in retaining it.*

General average is a *contribution proportionably borne by all concerned in ship and cargo*, where a partial loss is incurred for the preservation of the rest, though it is ultimately made good by the underwriters, in proportion to the sums they have underwritten.

Partial loss signifies the *damage* a ship or goods may have sustained, in her voyage from some of the perils insured against.

Average loss must be sustained by the owners of ship and cargo, or freight, in proportion to their respective adventures at hazard. *The average loss must amount to 5 per cent.*

CASE I.

The calculation of insurance is simply the same as simple interest for years.

EXAMPLE.

What is the insurance on $1000, from New-York to Havanna, at 7 per cent.?

as 100—7 *per cent.* :: 1000
10

Ans. $70

But adventurers generally cover the adventure, that is, they take out a policy for such amount as will be equal to the sum at hazard, together with the insurance.

RULE.—*As* 100 *less the premium is to* 100, *so is the given invoice to the amount that covers it.*

EXAMPLE.

What amount must I take out a policy for to cover an invoice of $7440, at 7 per cent.?

$$\begin{array}{l} 100 \\ \underline{\ \ 7} \\ \text{as } 93 : 100 :: 7440 : \text{to } \$8000 \textit{ Answer.} \end{array}$$

In giving theorems in the following cases,

Let x=100.
p=premium.
a=amount to be insured.
s=sum to be covered.
r=rebate or discount.

3. A merchant adventures $6640.83: For what amount must he take out a policy to cover the same, at 15 per cent. premium, and 2 per cent. discount, in case of loss? *Ans.* $8001.

4. What shall the amount of a policy be to cover an adventure of $4173.60 premium, 4 per cent. discount, 2 per cent. in case of loss? *Ans.* $4440.

5. Shipped goods for Jamaica, as per invoice, $987: What amount will cover the same, at 7 per cent. premium, and 2 per cent. in case of loss? *Ans.* $1084.61

CASE II.

When the amount of the policy and premium are given to find the sum covered.

Theorem. As $x : x-p :: a : s$.

$$\textit{Or thus, } \frac{\overline{x-p} \times a}{x} = s.$$

EXAMPLE.

6. A merchant insuring an adventure, at 7 per cent. took out a policy for $8000: Required the sum at hazard.

$$\begin{array}{l} 100 \\ \underline{\ \ 7} \\ \text{as } 100 : 93 :: 8000 : \text{to } \$7440 \textit{ Answer.} \end{array}$$

For questions 7, 8 *and* 9, *transpose and prove questions* 3, 4 *and* 5 *in this manner.*

CASE III.

When the amount and sum covered are given to find the premium and rebate.

Theorem. As $a : s :: x : x - p + r$.

Otherwise, $x - \frac{sx}{a} = p + r$.

EXAMPLE.

A merchant insuring $4173.60, took out a policy for $4440: The premium and rebate are required.

As 4440 : 4173.60 :: 100 : 94,
and 100—94=6 *premium and discount.*

Then 6—2=4 *per cent. premium, and* 2 *per cent. discount.*

For questions 11, 12 *and* 13, *transpose and prove questions* 2, 3 *and* 5 *in the same manner.*

CASE IV.

When the adventure is continued from one port to another at the same, or different risks, to find the amount that will cover the adventure all round.

RULE.—*Subtract each premium from* 100, *and multiply the several remainders for your first number—then raise* 100 *to that power denoted by the number of risks for the second, and make the sum adventured the third of a statement in the Rule of Three: the fourth number found, from this proportion, will be the amount of the policy to be insured all round.*

EXAMPLE.

14. A merchant insures an adventure of $4000 to New-Orleans, at 7 per cent.; thence to Kingston, Jamaica, at 5 per cent.; thence to New-York, at 9 per cent.: For how much must he take out a policy to cover the adventure round?

First, 100 100 100
7 5 9
——— ——— ———
93 × 95 × 91=803985 *first number.*
and 100^3=1000000 *second do.*

Then as 803985 : 1000000 :: 4000 : *to* $4975.21 *Answer.*

15. A merchant in New-York insures an adventure to London, say $23.085, at 10 per cent.; thence to Oporto at 5; thence to Philadelphia, where the voyage ceases, at 14 per cent.: For how much must he take a policy to cover this adventure?

Ans. $30.000.

16. A in New-York insures \$10119.33 to Philadelphia, at $2\frac{1}{2}$ per cent.; thence to New Orleans at 7; thence to Havanna, at 4, and from Havanna home, at 5 per cent.: What amount will cover this adventure round with 2 per cent. in case of loss?

Ans. \$12500.

CASE V.

When the adventure is at different risks round, and a premium required equal to a mean, or tantamount to the several rates per cent.:

RULE.—*Find the amount to be insured by the last case, subtract the sum adventured from the amount, the remainder is the entire of the discount and premium.*

Then say, as amount is to the difference, so is 100 to the rate per cent. from which subtract the discount, the remainder is the premium you have to pay the underwriters, &c.

$$\text{Theorem. } a - s \times \frac{x}{a} = p + r \quad \text{and } pr - r = p$$

EXAMPLE.

17. Amount of 16th question is \$12500
Sum adventured 10119.33 *insu. and discount.*
then as 12500 : 2380.67 :: 100 : *to* 19.045 *prem. and rebate.*
subtract rebate 2.

17.045 *rate per cent.*

and tantamount to all the risks.

Then from the amount 12500
take 12500 *at* 17.035 *per cent.* 2140.665
12500 *at* 2 *per cent.* 250.
2380.665

Rems. sum covered \$10119.33

For questions 18 and 19, transpose and prove questions 15 and 16.

CASE VI.

To find the equal premium paid on a policy taken out to cover an adventure to two or more places.

$$\text{Theorem } x - \frac{\sqrt[3]{s \times x^3}}{a} = p$$

RULE.—*Multiply the adventure by* 100 *as often as there are risks, and divide this product by the policy.* Second.—*Extract that root of the quotient denoted by the number of risks; this root subtracted from* 100 *will give the equal rate.*

EXAMPLE.

A in New-York, adventures \$2187 to London: from thence to Jamaica; from Jamaica home; to cover which, he took out a policy for \$3000: What was the rate per cent. premium?

$$100-\frac{\sqrt[3]{2187\times 100\times 100\times 100}}{3000}=10 \text{ per cent. Ans.}$$

21. A merchant in Boston adventures \$1038.23, to New-York; thence to New-Orleans; and thence to Jamaica; to cover which, he took out a policy for 1250 dollars: Required the premium per cent.: *Ans.* 6 *per cent.*

By this case is estimated the premium between the insurer and the insured. When the adventure is to be continued from one port to another at equal risks, or to all the ports at one risk, and that the voyage ceases in any port short of the ship's first destination.

If half the voyage be performed, it is estimated at two equal risks; if one third, at three; if one fourth, at four; and from this to find what rate per cent. premium must be paid to the insurer on the part accomplished, use the foregoing rule.

EXAMPLE.

A in Boston ships an adventure of \$800 to New-York; thence to Charleston; thence to Havanna, insurance at 20 per cent. round; but the voyage ceasing in New-York: What must the underwriters receive per cent. estimating one-third of the voyage perfected?

First. $100-20=80$

then as $80 : 100 :: 8000 : $ *to* $10{,}000$

and $8000\times 100,^3=8{,}000{,}000{,}000,\div 10{,}000=800000$

then $\sqrt[3]{800000}=92.83+$ *and* $100-92.83=7.16$ *per cent. Ans.*

Exchange.

Exchange, considered as a rule in *arithmetic*, is chiefly comprehended in this Problem, viz. *how to reduce the money of one country into that of another.* It may likewise be defined a fixing of the *actual* and *momentary* value of *money.*

Gold and *silver*, as metals, have their *intrinsic value;* but, as they are capable of becoming the *sign* or *medium* by which *merchandize* may be estimated, they may receive an additional value; for were gold and silver no more than merchandize, their value would be less fluctuating.

As *money*, the legislature can fix a value on gold and silver in some cases, and in others they cannot: they can fix a proportion between silver as *metal*, and silver as *money;* between the several *metals* made use of to *pass as money*, establishing the weight and standard of each denomination, and assigning to it that *ideal value by which it is made current.*

On the reverse, if we compare the *money* of one country with *that* of another, it receives a new value, which is fixed by the *current course of commerce, and the general opinion of merchants*, but never by the laws of any particular nation; because it is liable to incessant variations, and depends on the *accidental circumstances of trade*, the *money transactions* between nations, and the state of their *public credit.*

The *mutability of the course of exchange* is not *real*, but *relative:* For instance, when Boston has greater occasion for funds in Norfolk, than Norfolk has in Boston, though the specie of both places be the same, as to currency and standard, yet, when there is not a *fund of credit at* Norfolk equal to the debit, the *price of bills*, not of *money*, will rise of course at Boston.

To set this in a more particular point of view, let us consider that, if our purchases in *foreign countries* balance their purchases in *ours*, there will be enough of bills in the *one* to settle accounts with the *other*, and the *exchange* will be at *par;* but if a nation supplies us with more than it purchases, there will be a *balance against us*, to discharge which the *money*, or *bills of exchange* on *that nation*, rises *above par*, and puts ours *below* it, which constitutes the *course of exchange:*

From which we may naturally infer,

1st. That the course of exchange between two nations is a *herald*, that *publicly proclaims the state of commerce* between them, and which of the *two* is indebted to the *other.*

2nd. The nation which is indebted has the *disadvantage*, both in *commerce* and *money negociations.*

3rd. That the balance of trade *naturally imports specie*, and renders *money at home* more valuable *abroad.* On the reverse, when the balance is against a *nation*, their specie is *exported*, and becomes thereby of *less value.*

A Bill of Exchange *is a written order delivered in one place for value received there, (according to the rate of exchange agreed on) for the like value to be paid to the holder of the bill in the place on which it is drawn.*

Par, *in exchange, is the supposed equality between the monies of one country and that of another—that is, when the money received for a bill of exchange is equal in value to the money paid for it, then the exchange is at par.*

The course of Exchange, *being variable, may be defined the fluctuation of par, because it depends on the particular demands* *of merchants for bills on the place to which the balance is due: and likewise, that the money paid for a bill of exchange is esteemed equivalent to the sum expressed on the face of the bill:*—Consequently, bills of exchange are a kind of merchandize that rise and fall in price, in proportion to the demand for them.

USANCE *is a particular time specified for the payment of a bill of exchange after acceptance, being in some places* 30 *days, in others* 60 *or* 90.

Domestic Exchange.

THE *reciprocal exchanges* of the U. S. are for *dollars* and *cents*; yet, as many affect to keep their accounts *(both foreign and domestic)* in currency, we give the following *proportionate scale*, which serves to change the *currency* of any one *state* into that of *all the others*.

The value of the Federal Dollar, estimated in the currency of the different states, is as follows:

States of N. England, Virginia, Kentucky, and Tennessee.	*at* 6*s*.
New-York, North-Carolina.	8*s*.
South-Carolina, Georgia.	4*s*. 8*d*.
New-Jersey, Pennsylvania, Delaware, Maryland.	7*s*. 6*d*.

From the foregoing proportion we establish the following Theorems:

To change the currency of the states of
New-England,
Virginia,
Kentucky, and
Tennessee,
Into the currency of

New-York, North-Carolina.	Theorem. *Add* $\frac{1}{3}$, *or* $\times 4 \div 3$.
New-Jersey, Pennsylvania, Delaware, Maryland.	Theorem. *Add* $\frac{1}{4}$, *or* $\times 5 \div 4$.
South-Carolina, Georgia.	*Subtract* $\frac{2}{9}$, *or* $\times 7 \div 9$.

To change the currency of the states of
New-York and
North-Carolina,
Into the currency of the states of

New-England, Virginia, Kentucky, and Tennessee.	Theor. *Subtract* $\frac{1}{4}$, or $\times 3 \div 4$.
New-Jersey, Pennsylvania, Delaware, Maryland.	*Subtract* $\frac{1}{16}$, or $\times 15 \div 16$.
South-Carolina, Georgia.	*Subtract* $\frac{5}{12}$, or $\times 7 \div 12$.

To change the currency of the states of
New-Jersey, Pennsylvania,
Delaware, Maryland,
Into the currency of the states of

New-England, Virginia, Kentucky,. Tennessee.	Theor. *Subtract* $\frac{1}{5}$, or $\times 4 \div 5$
New-York, North-Carolina.	*Add* $\frac{1}{15}$ or $\times 16 \div 15$
South-Carolina, Georgia.	*Multiply by* 28, *divide by* 45.

To change the currency of the states of
South-Carolina and Georgia,
Into the currency of the states of

New-England, Virginia, Kentucky, Tennessee.	*Add* $\frac{2}{7}$ or $\times 9 \div 7$
New-York, North-Carolina.	*Add* $\frac{5}{7}$, or $\times 12 \div 7$
New-Jersey, Pennsylvania, Delaware, Maryland.	$\times 45 \div 28$

Or to the currency of South-Carolina or Georgia
add $\frac{1}{2}$ *the sum,*
more $\frac{1}{7}$ *of that half,*
more $\frac{1}{2}$ *of that seventh part,*
the sum is the answer required.

EXAMPLES.

1. Given—£112 10*s.* Virginia and states of New-England currency, to be changed into that of

New-York and North-Carolina.	*Ans.* £150.
Pennsylvania, Maryland, Delaware, &c.	*Ans.* £140 12 6.
South-Carolina, Georgia.	*Ans.* £87 10.

2. Given—£150, New-York and North-Carolina currency, to be changed into that of

Pennsylvania, Delaware, Maryland.	*Ans.* £140 12 6.
New-Hampshire, Virginia, &c.	*Ans.* £112 10.
South-Carolina, Georgia.	*Ans.* £87 10.

3. Given—£140 12*s.* 6*d.* Pennsylvania, Delaware, and Maryland currency, to be changed into that of

New-Hampshire, Virginia, Kentucky, &c.	*Ans.* £112 10.
New-York, North-Carolina.	*Ans.* £150.
South-Carolina, Georgia.	*Ans.* £87 10.

4. Given—£87 10*s.* South-Carolina and Georgia currency, to be changed into that of

New-Hampshire, Virginia, Kentucky, &c.	*Ans.* £112 10.
New-York, North-Carolina.	*Ans.* £150.
Pennsylvania, Delaware, Maryland.	*Ans.* £140 12 6.

Exchange with Great-Britain.

The Bank of England was incorporated by charter in the reign of William and Mary, and is under the direction of a governor, deputy-governor, and 24 directors, who are chosen annually. By their *charter* they are not *to trade*, nor suffer any person in trust for them, *to trade* in any *merchandize*, but they may buy or sell *bills of exchange*, *bullion*, *foreign gold and silver coins*, *&c.*

Bank Stock is *personal*, not *real estate*: therefore, no contract, either in word or writing, can be good in law, for buying or selling bank stock, unless it be *registered* in the books of the bank in the space of seven, and the stock *transferred* in fourteen days after the contract.

It is felony to counterfeit the seal of the Bank, or any sealed bank bill or note.

This company is said to have upwards of twelve millions sterling of paper in circulation!

In Great-Britain they keep their accounts in currency, or pounds, shillings and pence.

4 farthings	*make*	*1 penny.*
12 pence		*1 shilling.*
20 shillings		*1 pound, or £, which is imaginary.*

The real coins of Great-Britain are

Guineas of (gold)	21	*shillings.*
Half-guineas	10*s.*	6*d.*
Quarter-guineas	5	3
Crowns (silver)	5	
Half-crowns	2	6

Shillings of 12 pence, sixpences, and pence.

The standard for the *gold coin* of *Great-Britain* is 22 parts of *fine gold*, and 2 parts of *alloy* (copper). From a pound Troy of this mixture, they coin 44½ guineas; therefore, the guinea should weigh 5 dwts. 9$\frac{39}{89}$ grs.; but allowance being necessary for wear in circulation, an act was passed by government, establishing the current wt. at 5 dwts. 8 grs. and the half-guinea at 2 dwts. 16 grs.

The standard for silver coin, is 11 oz. 2 dwts. fine silver, and 18 dwts. of alloy; a pound of this mixture is coined into 62 shillings. The crown of 5 shillings should weigh about 19 dwts. 8½ grs. and the shilling 3 dwts. 20$\frac{2}{10}$ grs. nearly.*

---

* Note.—The par between the U. States and Great-Britain, does not seem to be calculated or establised on the intrinsic value of the English coin. To prove which, let us make the following remarks:

The par between the United States and Great-Britain as established by public opinion, is found thus:

	s. 6*d.* *Eng.*		$		*s.* *Eng.*		$
as	4 6	:	1	::	20	:	$4\frac{4}{9}$

and

	£ *Eng.*		$				$	
as	1	:	$4\frac{4}{9}$	::	100	:	$444\frac{4}{9}$ or £177 15 $6\frac{2}{3}$	N. York and N. Carolina currency.

And to find the par for the other states, take the foregoing proportion,

viz. £177 15*s.* $6\frac{2}{3} - \frac{1}{16} =$ £166 13*s.* 4*d.* — Pennsylvania and Maryland currency.

£177 15*s.* $6\frac{2}{3}$*d.* $- \frac{1}{4} =$ £133 6*s.* 8*d.* — Virginia and States of N. Eng.

£177 15*s.* $6\frac{2}{3}$*d.* $\times 7 \div 12 = 103$ 14 $0\frac{8}{9}$ — South-Carolina and Georgia.

1st. If a British guinea of 5 dwt. 8 grs. be current in England at 21 shillings, and in the United States at $88\frac{8}{10}$ cents the dwt. we shall find the value to be 4 dlls. 73 cents 6 mills.

2nd. The weight of the British imaginary £ is ascertained by this proportion: as 21*s.* : $5\frac{1}{3}$ *dwts.* :: 20*s* : $5\frac{5}{63}$ *dwts.* and its value in federal money, at 88.8 cents, is 451 cts. $0\frac{10}{21}$ mills.

Or thus:

as 21*s.* : $473\frac{6}{10}$ *cts.* :: 20 : 4 *dols.* 51 *cents*, $0\frac{10}{21}$ *mills*, as before; but 4 dollars 51 cents is near enough for calculation.

From these considerations it would appear that the following would be a par for the United States with England:

	£ st'g.		$.		£ st'g.		$.	
as	1	:	4.51	::	100	:	451 or £$180\frac{2}{5}$	New-York and N. Carolina currency.

And to find the par for the other states:

$180\frac{2}{5} - \frac{1}{16} = 169\frac{1}{8}$ — Pennsylvania and Maryland currency.

$180\frac{2}{5} - \frac{1}{4} = 135\frac{3}{10}$ — Virginia and States of New-England.

$180\frac{2}{5} \times 7 \div 12 = 105\frac{7}{10}$ — Georgia and South-Carolina currency.

For if a dollar be current in England, 4*s.* 6*d.* the £ sterling

will contain $4\frac{4}{9}$ dollars, value in the United States	$$4.44\frac{4}{9}$
the guinea will contain $4\frac{6}{9}$, value in do.	$4.66\frac{6}{9}$
the standard guinea of 5 dwts. 8 grs. is valued at	$4.73\frac{6}{10}$
by which it appears that a guinea is better by	$06\frac{14}{15}$

than its value in dollars in the exchange, &c.

The foregoing calculations are given under this idea, that as a guinea of 5 dwts. 6 grs. which is current in the U. S. and estimated as a par for $466\frac{6}{10}$ cents, would not pass as coin in Great-Britain, but be sold as bullion.

Short rules for changing British sterling into federal, and the currencies of the several states at par, and the contrary.

Sterling into Federal.

RULE.—Multiply the sterling by 40, divide by 9.

£100 *sterling.*
40

9) 4000

\$444.44$\frac{4}{9}$ *Ans.*

Federal into Sterling.

RULE.—Multiply the federal by 9, divide by 40.

\$444.44$\frac{4}{9}$
9

40) 4000.00

£100. *sterl. Ans.*

British guineas into federal.

RULE.—Multiply by 42, and divide by 9.

100 *guineas.*
42

9) 4200

\$466.66$\frac{6}{9}$ *Ans.*

Federal into British guineas.

RULE.—Multiply by 9, and divide by 42.

\$466.66$\frac{6}{9}$
9

42) 4200.

100 *guin. Ans.*

Sterling into N. York and North-Carolina currency.

RULE.—Multiply the sterling by 2, subtract $\frac{1}{9}$, the remainder is the currency.

£100 *sterling.*
2

200
= 22 4 5$\frac{1}{3}$

Ans. £177 15 6$\frac{2}{3}$ *currency.*

Or thus,

To the sterling add $\frac{1}{3}$ of itself, to that sum add $\frac{1}{3}$ of itself, the last amount will be the currency.

£100 *sterling.*
$\frac{1}{3}$= 33 6 8

133 6 8
$\frac{1}{3}$= 44 8 10$\frac{2}{3}$

Ans. £177 15 6$\frac{2}{3}$ *as before.*

New-York and North-Carolina to sterling.

RULE.—Divide the currency by 2, add $\frac{1}{8}$ of the quotient to itself, the sum will be the sterling.

2) £177 15 6$\frac{2}{3}$ *currency.*

88 17 9$\frac{1}{3}$
$\frac{1}{8}$ = 11 2 2$\frac{2}{3}$

£100 *sterling.*

Or thus,

From the currency subtract $\frac{1}{4}$, from the remainder subtract $\frac{1}{4}$ of itself, the last remainder will be the sterling.

177 15 6$\frac{2}{3}$ *currency.*
$\frac{1}{4}$= 44 8 10$\frac{2}{3}$

133 6 8
$\frac{1}{4}$= 33 6 8

£100 *sterling, as before.*

Sterling to Virginia and states of New-England currency.

Rule.—To the sterling add $\frac{1}{3}$ of itself.

100 *sterling.*
$\frac{1}{3}$=33 6 8

£133 6 8 *currency.*

Virginia and states of New-England currency to sterling.

Rule.—From the currency subtract $\frac{1}{4}$.

133 6 8 *currency.*
$\frac{1}{4}$=33 6 8

£100 0 0 *sterling.*

Sterling to Pennsylvania, Delaware, Maryland, and New-Jersey currency.

Rule.—Multiply the sterling by 5, divide the product by 3, the quotient is the answer.

£100 *sterling.*
5

3) 500

£166 13 4 *currency, Ans.*

Pennsylvania, Delaware, Maryland, and New-Jersey currency to sterling.

Rule.—Multiply the currency by 3, divide the product by 5 for the sterling.

£166 13 4 *currency.*
3

5) 500

£100 *sterl. Ans.*

Sterling to Georgia and South-Carolina currency.

Rule.—To the sterling add $\frac{1}{27}$ part, the sum is the currency.

£100 0 0 *sterling.*
$\frac{1}{27}$= 3 14 0$\frac{2}{3}$

Ans. £103 14 0$\frac{2}{3}$ *currency.*

Georgia and South-Carolina currency to sterling.

Rule.—From the currency subtract $\frac{1}{28}$ part, the remainder is sterling.

£103 14 0$\frac{2}{3}$ *currency.*
$\frac{1}{28}$= 3 14 0$\frac{2}{3}$

Ans. £100 0 0 *sterling.*

EXAMPLE.

1. A in New-York owes B in London £654 18 8 currency: How much has B debit in his books, exchange at par?

£654 18 8
$\frac{1}{4}$=163 14 8

491 4 0
$\frac{1}{4}$=122 16 0

£368 8 0 *Answer.*

2. A merchant in Philadelphia draws on London for £464 18*s*. 8*d*. sterling: How much must he credit his correspondent for, exchange at par? *Ans*. £774 17 9½.

3. A merchant in Norfolk remits a bill of £478 16 2, Virginia currency, to London: How much will his correspondent credit him for in sterling? *Ans*. £359 2 1½.

As accounts are generally kept in federal money, and the calculation of exchange more simple in that specie than any other, we shall use the federal, unless in a few instances, for the sake of those who do not perfectly understand decimals.

TABLE OF EXCHANGE,

*For buying and selling bills of exchange above and below par.**

Rate		Rate		Rate	
¼	1.0025	4	1.04	7¾	1.0775
½	1.005	4¼	1.0425	8	1.08
¾	1.0075	4½	1.045	8¼	1.0825
1	1.01	4¾	1.0475	8½	1.085
1¼	1.0125	5	1.05	8¾	1.0875
1½	1.015	5¼	1.0525	9	1.09
1¾	1.0175	5½	1.055	9¼	1.0925
2	1.02	5¾	1.0575	9½	1.095
2¼	1.0225	6	1.06	9¾	1.0975
2½	1.025	6¼	1.0625	10	1.1
2¾	1.0275	6½	1.065	10¼	1.1025
3	1.03	6¾	1.0675	10½	1.105
3¼	1.0325	7	1.07	10¾	1.1075
3½	1.035	7¼	1.0725	11	1.11
3¾	1.0375	7½	1.075	11¼	1.1125

USE OF THIS TABLE.

When the exchange is above par:

Multiply the amount of the bill by the number opposite the rate per cent. the product is the value of the bill.

When the exchange is below par:

Divide the amount of the bill by the number opposite the rate per cent. the quotient will be the value of the bill.

These tables are not given with an intent to exempt the student from calculation, but are designed for the use of those whose business requires certainty and dispatch.

---

* Note.—It is the practice with some merchants, in buying and selling bills of exchange below par, to deduct the rate per cent. from 100, and calculate the value of the bill on this proportion:

Thus, the value of a bill of $1000, at 5 per cent. below par, is only $950. This appears absurd from these principles:

TABLE II.

Showing the decimals of a £ currency or sterling.

$\frac{1}{4}$ *penny, is*	£0.00104	4 *shillings are*	£0.2
$\frac{1}{2}$ - -	.00208	5 - -	.25
$\frac{3}{4}$ - -	.00312	6 - -	.3
1 - -	.00416′	7 - -	.35
2 - -	.0083′	8 - -	.4
3 - -	.0125	9 - -	.45
4 - -	.0166′	10 - -	.5
5 - -	.0208	11 - -	.55
6 - -	.025	12 - -	.6
7 - -	.02916′	13 - -	.65
8 - -	.0333′	14 - -	.7
9 - -	.0375	15 - -	.75
10 - -	.0416′	16 - -	.8
11 - -	.04583′	17 - -	.85
1 *shilling*	.05	18 - -	.9
2 - -	.1	19 - -	.95
3 - -	.15		

Use.

Required the decimal of a £ equal to £0 17*s.* 4$\frac{1}{2}$*d.*

Decimal of 17*s.* = .85
of 4*d.* .01666
of $\frac{1}{2}$*d.* .00208

.86874 *Ans.*

1st. Because the rise or fall of exchange is no more than the fluctuation of the par, or agio, by which the value of foreign money is estimated.

2nd. The seller has a right to discount on the money he does not receive, as in common rebate; otherwise the buyer has no right to deduction on the money he does not pay.

3rd. The worth or value of a bill of exchange below par, calculated at the rate per cent. advance, should be equal to the sum expressed on the face of the bill, which cannot be so in this case, for \$950 × 1.05 = 997.50, being an error of \$2.50.

The true value of the above sum should be found by this proportion:

As 1.05 : 1 :: 1000 :: *to* \$952.38 *value nearly.*

And 952.38 × 1.05 = 999.99 + *or* \$1000.

This is the method pursued by all commercial towns in Europe, when they calculate by rates per cent.

But the general usage of merchants is the criterion by which all mercantile matters must be regulated: the author, therefore, hopes that this digression will be treated with candour, even by those who pursue the methods of calculation hinted at above.

Thus the decimal of any number of shillings and pence may be had by inspection.

EXAMPLE,

In using the two foregoing tables:

A in New-York owes B in Glasgow £654 15*s.* 6*d.* to pay which, he buys a bill at $2\frac{1}{2}$ per cent. below par: How much must he pay in federal money?

£654 15 6=*by the 2nd table* 654.775 *sterling.*

```
                    40
               ─────────
            9) 26191.000
               ─────────
            $2910.11 1/9 value in fed. money at par.
```

Then to find the value of the bill at $2\frac{1}{2}$ per cent. below par:

```
Tab. No. for 2½ per }                 $
cent. by tab. 1st.  } 1.025) 2910.111' (2839.13½ nearly, Ans.
                             2050
                             ────
                              8601
                              8200
                              ────
                               4011
                               3075
                               ────
                                9361
                                9225
                                ────
                                 1361
                                 1025
                                 ────
                                  3361
                                  3075
                                  ────
                                  2,86
                                 ───── =⅕ nearly.
                                 10,25
```

5. A in New-York owes B in London £1000 sterling; to pay which, he buys a bill at 4 per cent. below par: How many dollars will he have to pay for the same? *Ans.* $4273.50

6. B in Philadelphia orders C in Glasgow to ship on his account to the amount of £1000 sterling, with orders to draw on D of London with $2\frac{1}{2}$ per cent. commission: For how much must the Philadelphia merchant credit D, if the exchange be at par?

Ans. £1708 6 8 *currency, or* $5000 *federal.*

7. E in London owes F in Philadelphia £630 sterling: How many dollars must F receive for his draft at $2\frac{1}{2}$ per cent. below par? *Ans.* \$2731.70$\frac{10}{41}$

8. Prove the above bill. *Ans.* £

9. A factor in New-York owes his merchant in London, neat proceeds amounting to £1500 sterling; to pay which, he buys a bill at 4 per cent. above par: How many dollars must he pay for the same? *Ans.* \$6933.33$\frac{1}{3}$

10. Prove the above bill. *Ans.* £ *sterling.*

11. B in Philadelphia draws in favour of D in London, (with an intention of closing their accounts) for £1708 6*s.* 8*d.* currency: What sum had D Dr. in his books? *Ans.* £1025 *sterl.*

Concerning the gain and loss by the rise and fall of exchange.

From the foregoing examples it is evident, that a merchant may turn both rise and fall of exchange to his advantage, by buying and remitting when it is low; thus establishing a fund for which he may draw to advantage when the exchange is high.

For instance:

A in New-York remits a bill of \$1450.25 to London, which he bought at $4\frac{1}{2}$ per cent. below par; but on the rise of exchange, he drew for the same at $3\frac{1}{2}$ per cent. above par: What was his gain?

1450.25 × 1.035 =	\$1501	*sold for.*
1450.25 ÷ 1.045 =	1387.80	*bought for.*
	113.20	*gain, Ans.*

There is likewise a gain in importing gold from Great-Britain instead of dollars.

For instance:

C in London owes B in Philadelphia 1000 guineas; whether is it better for B to have dollars remitted him at 4*s.* 6*d.* sterling, or standard guineas of 5 dwts. 8 grs. in return?

$\frac{1000 \times 42}{9} = 4666.66\frac{2}{3}$ *value of the guineas, if dollars are imported; but it appears that the intrinsic value of the guinea of 5 dwts. 8 grs. is 473.6 cts.—Therefore,*

1000 × 473.6 = 4736 *the value of the guineas.*
4666.66$\frac{2}{3}$ *value in dollars.*

the 1000*g. are better by* 69.33$\frac{1}{3}$ *than dollars to import, &c.*

Great-Britain *exchanges with*

Switzerland, Nuremburgh, Leipsic, Dresden, Osnaburgh, Brunswick, Cologn, Liege, Strasburgh, Cracow, Denmark, Norway, Riga, Revel, and Narva, on the rix dollar; the par being 4*s.* 6*d.* or 54*d.* sterling.

With Ireland, *at* 8⅓ *per cent. advance.*

France,	*for the ecu of exchange, at*	29.365*d.*	*sterling.*
Holland, Flanders, Brabant,	*for* 36 *schil.* 7 *groot, at*	£1 0*s.* 0*d.*	*do.*
Hamburgh, Antwerp,	*for* 35 *sch.* 6⅔ *groot, at*	£1 0*s.* 0*d.*	*do.*
Spain,	*for the piastre of exch. or* 8 *rials, at*	43*d.*	*do.*
Portugal,	*on the milrea, at*	67½*d.*	*do.*
Venice,	*on the ducat of* 20 *sols d'or, at*	50¼*d.*	*do.*
Genoa and Leghorn,	*on the pezzo, at*	54*d.*	*do.*
Rome,	*on the crown, at*	73½*d.*	*do.*
Naples,	*on the ducat, at*	40½*d.*	*do.*
Florence,	*on the crown, at*	64½*d.*	*do.*
Bologna,	*on the dollar, at*	51*d.*	*do.*
Sicily,	*on the crown, at*	60*d.*	*do.*
Vienna,	*on the rix dollar, at*	56*d.*	*do.*
Bremen,	*on the rix dollar, at*	42*d.*	*do.*
Breslaw,	*on the rix dollar, at*	39*d.*	*do.*
Berlin,	*on the rix dollar, at*	48*d.*	*do.*
Frankfort on the main,	*on the florin, at*	36*d.*	*do.*
Stetin,	*on the mark, at*	18*d.*	*do.*
Embden,	*on the rix dollar, at*	42*d.*	*do.*
Bolsena,	*on the rix dollar, at*	44*d.*	*do.*
Dantzick,	*on* 13⅓ *florins, at*	£1	*do.*
Stockholm, Upsal, Thorn,	*on the rix dollar, at*	54*d.*	*do.*
Petersburgh,	*on the ruble, at*	53*d.*	*do.*

More or less, according to the course of exchange.

Exchange with Ireland.

They keep their accounts in Ireland under the same denominations as in England; they have scarcely any other than *English coin* in circulation.

The par between England and Ireland is 8⅓ *per cent. advance—that is,* £100 *English is equal to* £108 6 8 *Irish currency; but the course of exchange is from* 5 *to* 12 *per cent.*

The national bank was established in the year 1783, under the management of a governor, deputy-governor, and fifteen directors, chosen annually from among the subscribers, with this restriction, that five new directors at least be chosen every year. &c.

Bills on Ireland are usually drawn for sterling money; therefore the most necessary rule is, how to change *Irish to English*, and the contrary.

To change Irish currency into English at par.

RULE.—From the Irish subtract $\frac{1}{13}$ part, the remainder will be English.

EXAMPLE.

£108 6 8	*Irish.*
$\frac{1}{13}$ = 8 6 8	
£100 0 0	*sterl.*

To change English into Irish currency at par.

RULE.—To the English add $\frac{1}{12}$ part, the sum will be the Irish.

EXAMPLE.

£100 0 0	
$\frac{1}{12}$ = 8 6 8	
£108 6 8	*Irish.*

GENERAL RULES.—1st. Reduce the Irish currency to halfpence, and divide by 117, the quotient will be dollars.—Multiply the dollars by 117, the product will be halfpence Irish.

2nd. Or multiply the Irish by 160, and divide by 39, the quotient will be dollars—or multiply dollars by 39, and divide by 160, the quotient will be the Irish currency.

EXAMPLE.

By rule 2nd. 1. How many dollars are equal to £244 14 6 Irish currency, if the par of a dollar be 4*s.* 10½*d.* Irish?

```
£244 14 6=244.725×160
          -----------
          14683500
          244725
          ----------
      39) 39156.000 ($1004 Answer.
          39
          --
          ..156
            156
```

2. Prove the above. *Ans.*

RULE 3.—Multiply the Irish by 4.102564, the product will be dollars:

Or, divide the dollars by 4.102564, the quotient will be Irish currency.

EXAMPLE.

3. In $1004 how much Irish currency?

4.102564) 1004.000000 (£244.725, or £244 14 6 *Ans.*

4. How many dollars are equal to £700 Irish, exchange at par? *Ans.* \$2871.79$\frac{2}{10}$.

5. In \$6850 how much Irish currency at par?
Ans. £1669 13 9.

6. Required a multiplier and divisor at par, between the U. States and Ireland, a dollar passing at 4*s.* 10½*d.* Irish?
As 4*s.* 10½*d.* : 1 :: 20*s.* *Irish to* 4.102564 *Ans.*

These methods may be taken as proofs. For buying and selling, examples sufficient have been given in the exchange with G. Britain.

Exchange with France.

To the man of *science*, as well as the *merchant*, a particular account of the *weights* and *measures* of France must be agreeable: This idea alone must be the excuse, if any is necessary, for giving the particulars in this place.

For several centuries past *reason*, *good faith*, and all the principles of *social order*, have exclaimed against the *diversity* of *weights* and *measures* in France. A general and uniform scale, though often demanded, and as often *promised by government*, was never adopted till the late *revolution*, when a system of *weights* and *measures* was established on principles so *simple*, *perfect* and *advantageous*, that, were *books*, *records*, *monuments* and *measures* lost, posterity might recover them with ease.

This scale is established on the measure of a *degree of the meridian*, equally distant from the *north pole* and the *equator*, which is found to be 57027 toises—this, multiplied by 90 degrees, gives 5132430 toises, or 30794580 feet from the pole to the equator, *and is the assumed unity expressed in ancient measure, from the iron toise of the philosophical academy, and the origin from which all their measures are derived.*

This being established, they chose the number 10 as the simplest and most perfect number for a divisor, and after *seven successive divisions produced the number* 3.0794580, *or* 3 *feet* 11½ *lines nearly, and this portable measure was adopted to take place of the foot*, the *toise*, and the *ell*, and called a *metre*.

Proceeding from this measure, as the *common unity* in a tenfold progression, they assumed the following names:

		French feet.
Metre,	*equal to*	3.079458
Decametre,		30.79458
Hectometre,		307.9458
Kilometre,		3079.458
Myriametre,		30794.58

The metre is subdivided as follows;

		French feet.	
Viz.	*Decimetre, equal to*	.3079458 *or,*	3 *inches* 8 *lines* 4 *points.*
	Centimetre,	.03079458	4 *lines* 5 *points.*
	Millimetre	.003079458	$5\frac{1}{3}$ *points.*

From the foregoing, they have deduced the following:

ITINERARY, OR LONG-MEASURE.

		Toises.
	1 *decametre, equal to*	5.
10 *decametre*	1 *hectometre,*	51.
10 *hectom*	1 *kilometre*	513.
10 *kilom.*	1 *myriametre,*	5132.

AGRARIAN MEASURE.

1 *square decametre, or* 100 *square metres, make* 1 *are.*
10,000 *square metres,* 100 *sq. ares,* 1 *hectare.*

These are substituted in place of the perch and acre (arpent) of France.

CUBIC MEASURES.

1 *cubic metre is called a stere; the* 10*th part of the stere is called a decistere, and is used for carpentry, &c.*

A vessel of a cubical form, whose side is 1 *decimetre, or a cylindrical vessel of the same capacity, or solid content, is called a libre; it contains about* 2 *lb. of water, or* 25 *oz. of wheat, and is assumed as the element, or standard for measures of capacity.*

10 *decilitre make* 1 *litre.*
10 *litre* 1 *decalitre.*
10 *decalitre* 1 *hectolitre.*
10 *hectolitre are equal in capacity to* 1 *cubic metre.*

WEIGHT.

The original weight is taken to be equal to the quantity of distilled water contained in a cubic vessel, the side of which, is equal to the one hundreth part of a metre. This water weighed in vacuo at the temperature of melting ice, is equal to 18.841 grains; to this weight is given the denomination of *gramme,* and from the *gramme,* all the superior and inferior weights are deduced by multiplication and division.

	1 *milligramme*	.001
10 *milligramme are*	1 *centigramme*	.01
10 *centigramme*	1 *decigramme*	.1
10 *decigramme*	1 *gramme*	1.
10 *gramme*	1 *decagramme*	10.
10 *decagramme*	1 *hectogramme*	100.
10 *hectogramme*	1 *kilogramme*	1000.
10 *kilogramme*	1 *myriagramme*	10000.

We see, therefore, that the *gramme* is an unit placed between two series of like numbers, the one increasing in a tenfold proportion to the *myriagramme*, by which *merchandize* will be ascertained; the other decreasing in the same proportion to the *milligramme*, or $\frac{1}{1000}$th part of a *gramme;* and is, therefore, adapted to *medicinal* and *monetary* weights and mixtures.

The foregoing compared with English measures.

The *metre is equal in length to*	39.383 *inches*	English.
litre is equal to	61.083 *cubic inches*	
gramme is equal to	22.966 *grs. Troy*	
kilogramme is equal to 3 *lb.*	$4\frac{1}{2}$ *oz. avoirdupoise*	
hectare is equal to	11968 *square yards,*	
which is rather less than	$2\frac{1}{2}$ *acres*	

From the above proportions, it is easy to find the content of any invoice in known names required.

EXAMPLES.

1. What is the content of 1000 litres in wine gallons?

61.083 × 1000=61083 *cubic inches—this divided by* 231, *the cubic inches in a wine gallon, gives* 264.43 *gallons, the answer required.*

2. In 100 metres, how many yards?

1 *metre is* 39.383 *inches.*
100

36) 3938.3

Ans. $309\frac{1}{3}$ *yds nearly.*

The French money of account is *francs* and *centimes.* The real monies are connected with the general system of weights, &c. and are as follow:

Copper is coined into pieces of

1 *centime, weight*	1 *gramme.*
5 *centimes,*	5 *do.*
1 *decime,*	10 *do.*
2 *decimes,*	20 *do.*

Silver is coined into pieces of

1 *franc, weight*	5 *grammes and*
5 *francs*	25 *grammes.*

Gold is coined into pieces of

10 *grammes.*

The *franc* does not differ from the ancient livre *tournoise* in *value;* it is, therefore, estimated by merchants as a par with the United States for 19 cents federal *money.*

NOTE.—These denominations are not universally used in France yet; some using the old measures and new money in accounts, others the old money and new measures.

To change francs into dollars.

RULE.—Multiply the francs and centimes by the rate of exchange—divide by 100, the quotient will be dollars.

EXAMPLE.

A merchant in Bourdeaux owes a merchant in New-York 3000 fran. 50 centimes: What is the value in federal dollars, at 19 cents per franc?

Ans. $570.09½ *cents.*

```
        Fr.
     3000.50
          19
    --------
     2700450
     300050
    --------
1,00) $570,09,50
```

To change federal dollars into francs.

RULE.—Reduce the dollars to cents, and divide by the rate for a franc; if a remainder occurs, continue the division to two decimal places for centimes.

EXAMPLE.

2. A factor in Bourdeaux owes a merchant in New-York neat proceeds of a consignment, say $570.09½ cts.: How many francs must he draw for, if the exchange is 19 cents per franc?

Ans. 3000 *fr.* 50 *centimes.*

```
                    Fr.
19) $570.095 (3000.50 Ans.
     57
    ----
     ..0095
         95
```

3. A in New-York draws on B of Paris for 5000 francs, at 15 cents per franc: How much does he receive? *Ans.* $750.

4. C in Philadelphia buys a bill of exchange on Bourdeaux for 5000 francs, at 18½ cents per franc: How many dollars does he pay for the same? *Ans.* $925.

The ancient monies of France were divided as follow:

12 *deniers make*	1 *sol.*
20 *sols*	1 *livre, valued at about* 19 *cents.*
3 *livres*	1 *cr. of exch. intrinsically worth* 29.365*d. sterl.*
6 *livres*	1 *crown d'argent.*
4 *crowns d'arg.*	1 *louis d'or, or guinea.*

It would appear that the franc and livre are both rated too high in the United States; for, by the arrette of June 1726, the French mark of 7 oz. 18 dwts. 18.24 grs. English, was to be coined into 8⅓ crowns of 6 livres each, intrinsically worth 9.788*d.* sterling per livre, or franc; this, valued in federal money, is only about 18⅓ cents, and any thing paid more or less than this, must be a gain or loss in exchange.

5. F in New-York sells his draft on Bourdeaux for 5000 francs, and receives $925: What was the course of exchange?

Ans. 0.18\frac{1}{2}$ *per franc.*

6. A in New-York owes B in Paris $800: How many francs will A be debited for, if the exchange be at 18 cents per franc?

Ans. fr. 4444.44$\frac{4}{9}$.

7. If A in New-York buys a draft on Brest of 4444 fr. 44$\frac{4}{9}$ centimes for $800: What was the course of exchange?

Ans. $0.18

8. What is the amount of 4444 francs 44$\frac{4}{9}$ centimes in federal money, at 18 cents per franc? *Ans.* 800.

New-York, 24th June, 1803.

Exchange for 8059 fr. 60 cent.

At usance, pay this first of exchange (second and third of the same tenor and date unpaid) *to Pier Lamotte or order, eight thousand fifty-nine francs sixty centimes, value received, and charge the same to account, with or without advice.*

Charles Wilkes.

Messrs. Loftes & Co.
merchants, Paris.

What is the value of the above bill at 20 cents per franc?

Ans. 1611.92

Paris, 16th April 1802.

Exchange for 750\frac{40}{100}$.

Twenty days sight, pay the second of exchange (first and third of same tenor and date unpaid) *to George L'Estrange or order, seven hundred fifty dollars forty cents, value received, and charge the same to account as per advice.*

Pier Dumont.

Jean J. Montaigne,
merch't New-York.

What did Dumont receive for his draft, at 20 cents per franc?

Ans. 3752 *francs.*

Exchange with Spain.

The monies of Spain are of two sorts, viz. the one called *velon*, the other *plate money*. The *velon* is to the *plate money* as 34 to 64. Dealers and commissioners of excise keep their accounts in *rials and mervadies velon,*

viz. 2 mervadies velon			make	1 quartile.
4 do.		do.		1 quarto.
8$\frac{1}{2}$ quartos,	or 34	mer. vn.		1 rial velon.
15 rial vn.	or 510	do.		1 peso, or current dollar.
20 do.	or 680	do.		1 hard dollar.
16 quartos,	or 64	do.		1 rial plate.

But bankers, remitters, and some merchants, keep their accounts in old plate, or money of exchange,

viz. 34 mervadies plate	make 1 rial plate,	value in the U.S.	$0.10
8 rials plate	1 piastre of exchange,		.80
10 do.	1 dollar,		1.
11 do.	1 ducat of exchange,		1.10
32 do.	1 doubloon of exchange,		3.20

In the following table the first column shows the assay to be worse than American or British gold and silver—*second,* the absolute weight—*third,* the standard weight—*fourth,* its value in federal money.

SILVER.	*dwt.*	*dwt. gr.*	*dwt. gr.*	*value of*
Peso, or piastre, of 8 rials, now reduced to 10,	*w.* 1	17 12	17 10.2	$1
Pillar, piece of 8 rials, now reduced to 10,	*stand*	17 9	17 9	1 *nearly.*
Mexico piece, - - - - -	1	17 10	17 8.14	99¾ *cts. current at* 1 *dollar.*
New-Seville piece of 8, - -	1½	14 0	13 21.15	79⅚ *cts. current at* 80 *cents.*
Pistrine, of 2 rials plate, -	-	- -	- - -	20 *cents.*
GOLD.				
Old Spanish doubloon, - -	½	17 8	17 5.12	*These coins are current in the U. S. at* 87½*cts the dwt. nearly.*
half do. or double pistole,	½	8 16	8 14.16	
Pistole, - - - - - - - -	½	4 8	4 7.8	
New-Seville double pistole,	3	8 16¾		
pistole, - - - - - -	3	4 8⅜		

To reduce rials velon to rials of exchange:

RULE.—Multiply the rials velon by 8½, the quartos in a rial velon, adding one for every 4 mervadies—the product divide by 16, the quotient will be the rials of exchange.

EXAMPLE.

1. Reduce 4564 rials 24 mer. vn. to rials of exchange or plate.

```
      rials. m. v.
      4564  24
         8½    quartos in a rial velon.
      ------
      36518    product of 8, to which 6 is added for 24 mer. vn.
       2282
      ------
  16) 38800 (2425 rials of exch. Ans.
```

2. In 12142 rials 12 mer. vn. how many dollars plate?
Ans. 645 *dollars*, 0 *rials*, 20 *mer. vn.*

3. In 10000 rials velon, how many rials of exchange?
Ans. 5312 *rials*, 16 *mer. vn.*

To reduce rials of exchange to rials velon:

Rule.—Multiply the rials of exchange by 16, the quartos in a rial of exchange; add one for every two mervadies—divide this product by $8\frac{1}{2}$, the quotient will be the rials velon.

EXAMPLE.

4. Reduce 6450 rials 20 mer. plate to rials velon.

```
           6450.20
             16
           -------
8½=8.5)  103210 product by 16, to which 10 is added for 20 mer.
          85
          --            (12142 rials 12 mer. Ans.
          182
          170
          ---
           121
            85
           ---
            360
            340
            ---
             200
             170
             ---
              30 rem.
              34 mer.
             ----
             1020 (12
             1020
             ====
```

5. Reduce 2425 rials exchange to rials velon.
Ans. 4564 *rials* 24 *mer. vn.*

6. In 10 rials of plate, how many rials velon?
Ans. 18 *rials* 28 *mer. vn.*

To reduce Spanish to Federal:

Rule.—Reduce the odd rials and mervadies to the decimal of a piastre; multiply the given Spanish by .80—the amount will be dollars, and the decimal of a dollar; of which two places will be sufficient to retain for cents.

EXAMPLE.

7. A in Valencia is indebted to B in Baltimore 875 piastres, 6 rials, 25½ mervadies plate: What number of dollars is A debited in B's books. *Ans.* $700.67½.

$$875.6.25\tfrac{1}{2} = 875.84375$$
$$\times .80$$
$$700.67,500$$

TABLE

Of mervadies and rials expressed in the decimal of a piastre.

Mer.		*Mer.*		*Mer.*	
1	.00367	16	.05881	31	.11395
2	.00735	17	.06249	32	.11763
3	.01102	18	.06616	33	.12131
4	.0147	19	.06984		
5	.01838	20	.07352	*Rials*	
6	.02205	21	.07719	1	.125
7	.02573	22	.08087	2	.25
8	.0294	23	.08455	3	.375
9	.03308	24	.08822	4	.5
10	.03676	25	.0919	5	.625
11	.04043	26	.09557	6	.75
12	.04411	27	.09925	7	.875
13	.04778	28	.10293		
14	.05146	29	.1066		
15	.05514	30	.11028		

USE.

Find the value of 6 rials, 17 mervadies, in the decimal of a piastre.

6 *rials* = .75
17 *merv.* = .06249

.81249

To reduce Federal to Spanish:

RULE.—Divide the federal by .80—the quotient will be Spanish piastres, and the decimal of a piastre: The decimal should be continued to 4 or 5 places.

EXAMPLE.

New-York, 14*th March*, 1803.

1. Exchange for $6403.75.

At twenty days sight, pay this second of exchange (first and third of the same tenor and date unpaid) *to Benjamin Page, or order, six thousand four hundred three dollars seventy-five cents, value received, which charge to account as per advice.*

Charles Wilkes.

Don Juan De Langara, }
merch't, Barcelona. }

How much does Don Juan debit the drawer in Spanish piastres?

.8|0) 6403.75

8004.6875
8 *rials in* 1 *piastre.*

5.5000
34 *merv.* 1 *rial.*

17.0000

Ans. 8004 *p.* 5 *r.* 17 *mer.*

9. A merchant in New-York draws on Madrid for $700.67½: How many piastres will be paid for the same?

Ans. 875 *p.* 6 *r.* 25½ *mer.*

10. What is the amount of 675 pias. 6 r. 16 merv. plate, at 82 cents per piastre? *Ans.* $554.16'6'2'5

11. Prove the above. *Ans. piast.*

12. Sold a bill of exchange for 756 piast. 4 r. 17 merv. plate: What is the amount, at 75 cents per piastre? *Ans.* $567.42,'2

13. Prove the above bill. *Ans. piast.*

When bills are bought or sold at rates per cent.

Barcelona, 4*th may*, 1803.

14. Exchange for 640 p. 4 r. 24 merv.

Sixty days sight, pay this third of exchange, *(first and second of the same tenor and date unpaid)* to Don Alonzo Viola, or order, six hundred forty piastres four rials twenty-four mervadies plate, and charge the same to account as per advice.

Charles De Alvarez.

Messrs. Lynch and Stoughton, }
merchants, New-York. }

What is the amount in federal money paid for this bill?

Ans. $512.47.

15. A in New-York sells his draft on Cadiz, and receives 666.06\frac{8}{10}$, at 4 per cent. above par: What is the amount in piastres? *Ans.* 800 *p.* 4 *r.* 17 *mer.*

16. Drew on Don Juan De Modena for 1456 p. 4 r. 16 mer. plate, at 5 per cent. above par: What is the value in federal money? *Ans.* $1223.51 *nearly.*

17. Sold a draft on Don Juan De Modena, at 5 per cent. above par, for which I received $1223.51: I want to know how much he is to debit me for in piastres.

Ans. 1456 *p.* 4 *r.* 16 *mer.*

18. Remitted a bill for $1004 to Cadiz, when the exchange was at 3 per cent. below par; soon after, I drew for the same, when the exchange was $5\frac{1}{2}$ per cent. above par: I want to know how much was gained by this transaction? *Ans.* $84.47.

19. A merchant in Bilboa is indebted to a merchant in New-York 8004 p. 5 r. 17 merv. plate: How much federal money does the New-York merchant receive for his draft at $\frac{1}{2}$ per cent. above par? *Ans.* 6435.76\frac{8}{10}$

The following rules and examples are given for those who do not wish to use decimals. The pupil may be directed to solve the foregoing questions in this manner.

To change Federal to Spanish without using decimals.

Rule.—Divide the federal reduced to cents by 80 for piastres.
Divide the remainder by 10 for rials.
Multiply the remainder by $3\frac{2}{5}$ for merv.

EXAMPLE.

In $6478.33, how many piastres?

```
80) 6478.33      (8097 piastres.
    -------
      10) 73 rems.
        ----(7 rials.
           3 rems.
           3 2/5
          -----
          10 1/5 merv.      Ans. 8097 p. 7 r. 10 1/5 m.
```

To change Spanish to Federal.

Rule.—Multiply the piastres by 80, add 10 cents for each rial, and $2\frac{1}{2}$ for every $8\frac{1}{2}$ mervadies; divide the sum by 100.

EXAMPLE.

Change 8097 p. 7 r. $10\frac{1}{5}$ mer. plate to federal money.

```
          8097  7  10 1/5
            80
        ------
        647760
            70 =7 rials.
         2 1/2=8 1/2 mervadies.
           1/2=1 1/2   do.
     ----------
1.00) $6478.33  Ans.
```

Exchange with Portugal.

The Portuguese keep their accounts in milreas and reas; 1000 reas make one milrea, valued in the United States at 1 dollar 25 cents.

The milrea is an imaginary piece of account. The real monies of Portugal are as follow:

NOTE.—In estimating invoices for the calculation of American duties, it is ordered that the milrea shall be valued at $1.24 cents; but from the intrinsic value of the joannes, the milrea seems to be justly valued at 67½ pence sterling, or 125 cents federal money exactly.

SILVER.

Crusado of 400 *reas, not stamped, value in the U. S.*		$0.50
Crusado of 480 *reas, stamped in* 1643 - - - -		.60
12 *vintin piece, or half crusado of* 240 *reas* - - -		.30
5 *vintin piece,* - - - - - -	*of* 100 *do.* - - - -	.12½
2½ *vintin piece,* - - - - - -	*of* 50 *do.* - - - -	.06¼

GOLD.

Double Joannes, or piece of	25*m*.600*r*. *value in the U. S.*	$32.
Single do. - - - -	12.800 - - - - - - -	16.
Half do. - - - -	6.400 - - - - - - -	8.
Quarter do. - - - -	3.200 - - - - - - -	4.
Eighth do. - - - -	1.600 - - - - - - -	2.
Testoon, or $\frac{1}{16}$*th* - - -	.800 - - - - - - -	1.
Moydore - - - -	4.800 - - - - - - -	6.
Half-moydore - - - -	2.400 - - - - - - -	3.
Quarter do. - - - -	1.200 - - - - - - -	1.50

To change milreas and reas into dollars and cents.

RULE.—To the milreas and reas add one fourth part, the sum will be the dollars and cents.

EXAMPLE.

1. In 3867 m. 400 r. how many dollars?
Ans. $4834.25

```
     3867.400
¼ =   966.850
     --------
    $4834.25
```

To change dollars and cents into milreas and reas.

RULE.—From the dollars and cents subtract one fifth part, the remainder will be the milreas and reas.

EXAMPLE.

2. In 809.59\frac{3}{10}$, how many milreas?
Ans. 647 *m.* 675 *reas.*

$809.593
$\frac{1}{5}$=161.918

mil. 647.675 *Ans.*

NOTE.—The *third figure that occurs in the reas may be omitted* when changed into cents, and when *dollars* are changed into *reas* and *milreas*, if there be only two or one place in the reas, add a cypher, or cyphers, till the decimal consists of three places for reas.

3. In 10808.79\frac{1}{2}$, how many milreas? *Ans.* 8647 *m.* 036 *r.*

4. In 700 m. 320 r. how many dollars? $875.40.

5. In 0 m. 640 r. how many dollars? $0.80.

6. In 8647 m. 036 r. how many dolls.? 10808.79\frac{1}{2}$.

7. Reduce $875.40 to milreas, 700 *m.* 320 *r.*

8. What is the amount of a draft for 1000 mil. 500 r. at 120 cents per milrea? *Ans.* $1200.60

9. Prove the above. *Ans. mil.*

10. What must be paid for a draft of 900 mil. 280 r. at $1.24 per milrea? *Ans.* $1116.34,7'2

11. Prove the above. *Ans. mil.*

New-York, 16*th May*, 1803.

12. Exchange for 6000 m. 400 r.

Sixty days after sight, pay this first of exchange (*second and third of the same tenor and date unpaid*) to Joannes De Silva or order, six thousand milreas four hundred reas, value received, and charge the same to account as per advice.

Alfred Prince.

Messrs. Monteiro De Silva & Co.
merchants, Lisbon.

What is the value in dollars, at 5 per cent. above par?
Ans. 7875.52\frac{1}{2}$

13. Sold my draft on Alphonzo De Gama, at 3 per cent. under par, for which I received $6552: How many milreas was the bill drawn for? *Ans.* 5398 *m.* 848 *r.*

14. Drew on Messrs. Monteiro De Silva & Co. for 647 m. 800 reas: How many dollars must I receive for the same, at 1 per cent. above par? *Ans.* 817.84\frac{1}{4}$

15. Sold my draft on Monteiro De Silva & Co. at 1 per cent. above par, for which I received 817.84\frac{1}{4}$: How many milreas were paid for the same? *Ans.* 647 *m.* 800 *r.*

Exchange with Holland,

The Batavian Republic, or Seven United Provinces.

THERE are two banks in Holland—one in Amsterdam, the other in Rotterdam; that of Amsterdam (the most considerable in Europe) was established on the 31st of January, 1609, by the authority of the states general, and under the direction of the burgomasters of the city, who having constituted themselves perpetual cashiers of the merchants of Amsterdam, are security for the bank.

By the establishment of this bank, it is ordained, that bills of exchange and wholesale goods, shall only be paid in bank, unless the sum be under 300 guilders; in which case, it cannot be entered in the bank under a duty of 6 stivers, except by the East or West-India company.

No director can use the freedom of abstracting money from the bank, even for a single day, on pain of death.

The strictness and fidelity with which this bank is kept, is productive of so much security and dispatch, that a bank payment is reckoned from 3 to 5 per cent. better than cash, beside a premium allowed the bank for every deposit.

The bank of Amsterdam is supposed to contain the greatest treasure in the world of gold and jewels: The specie kept here, is computed at 36 millions sterling.

In Holland they keep their accounts in guilders, stivers and pennings.

2 groot, or 16 *pennings, make*	1 *stiver.*
20 *stivers*	1 *guilder or florin.*
6 *guilders*	1 *£, or pound Flemish.*

Or in pounds, schillings and groot:

8 *pennings make*	1 *groot.*
12 *groot*	1 *schilling.*
20 *schillings*	1 *£ Flem.*

The par of a Dutch guilder, florin, or piece of 20 stivers, is calculated by Sir Thomas Bond (a late writer) at 21.86 pence sterling, which in federal money, is about 40½ cents nearly.

This seems something above its intrinsic value, for the true par may be easily estimated from Sir Isaac Newton's table of the assay weight and value of foreign coin, published in London by order of the privy council 1740, viz.

Standard weight 6 dwts. 17.05 grs.; assay 2 dwts. worse than British or American silver. Value in London at 20.08 pence sterling, and in the United States about $0.37.

The U. S. exchange with Holland, at 40 cents the guilder, more or less; but from the above comparison, it appears reasonable, the par should be fixed at 37 cents.

RULE.—Multiply the guilders by the rate of exchange, and divide by 100, the quotient is the answer in dollars.—If stivers and pennings occur, take practical parts, or otherwise reduce them to the decimal of a guilder.

EXAMPLE.

1. A merchant in New-York sold his bill on Amsterdam for 1000 guilders, 15 stivers, 8 pennings, at 40 cents the guilder: What was the amount in federal money?

Ans. 400 *dollars* 31 *cts.*

Decimally — 1000.15.8

= 1000.775
40

1,00) 400.31000

Otherwise,

1000.15.8
40.

40000
10 = $\frac{1}{2}$ = 20
5 = $\frac{1}{2}$ = 10
8*pen.* = $\frac{1}{10}$ = 1

1,00) 400.31

2. A in Amsterdam is indebted to B in New-York 3400 guilders: How much will they amount to, at 35 cents the guilder?

Ans. $1190.

3. C in Philadelphia owes D in Amsterdam $2000: How many guilders will it amount to, at 33$\frac{1}{3}$ cents the guilder?

Ans. 6000 *guild.*

4. Prove the above questions.

5. E in New-York draws on F in Rotterdam for 4000 guilders, at 36$\frac{1}{2}$ cents the guilder: What is the amount in dollars?

Ans. $1460.

6. Prove the above. *Ans.*

OBSERVATION. By the foregoing examples, a merchant may see his advantage in remitting when it is low, and drawing when it is high; for it is evident I can have a credit for more guilders when the exchange is at 32, than when it is at 36 cents. On the contrary, I will receive more dollars for a bill when the exchange is at 36, than when it is at 32 cents the guilder. The merchant who has money to spare, will remit when the exchange is low, in order to have a fund for which he may draw to advantage when the exchange rises.

Holland exchanges with France upon the French crown for 54 grotes, more or less; Hamburgh upon the dol. for 32 Flemish schillings; Spain, upon the ducat for 375 merv. for 97 grotes; with Portugal, upon the crusado of 400 reas for 44 grotes; Genoa, upon the piastre of 5 liv. banco for 92 grotes; Venice, upon the ducat of 24 gross banco for 88 grotes; Leghorn, upon the piastre of 20 sols d'or for 86 grotes; England, upon the £ sterling for 36*s*. 6*d*. Flemish, more or less, according to the rate of exchange.

Exchange with the Austrian Netherlands.

ANTWERP, or ANVERS, is the principal place of *exchange* in *Austrian Netherlands*, and was once the metropolis of the *Seventeen Provinces;* but its commerce was ruined by the Dutch, who closed the navigation of the Scheldt soon after they threw off the Spanish yoke. This was the more cruel, as the people of Antwerp had been their allies and fellow-sufferers in the cause of liberty. The change of politics in France effected a happy change for the people of Antwerp; for their commercial privileges are again restored, and Antwerp bids fair to rival the most flourishing commercial cities on the continent.

Antwerp is pleasantly situated on the Scheldt, which is 20 feet deep at low water, and rising 20 feet more at flood, allows vessels of the greatest burthen to come up to the quays and unload.

Their money calculations and exchange are the same as in Holland; therefore, no other examples are necessary.

Exchange with Hamburgh.

Hamburgh is one of the most flourishing commercial cities in Europe: it has the sovereignty of a small district about 10 miles in circuit; and though the king of Denmark affects certain privileges within its walls, yet it may be considered as *a well regulated independent commonwealth.*

It is well situated for trade on the Elbe, which forms two spacious harbours. The stately public and private edifices, the number of canals and bridges, give the town an air of grandeur not to be surpassed by many towns in Europe.

The funds of the bank are inferior to those of Amsterdam; yet the integrity with which it has been conducted, has gained it a very high reputation.

The inspection of this bank comes not under the cognizance of the senate, but as the corporation and citizens are their sureties, they choose their directors by a majority of votes.

Though the capital of this bank be very considerable, it is uncertain the amount, as the clerks and cashiers are bound to secrecy by oath; therefore, no seizure can be made on any deposit or bank account, as no one knows how another stands with the bank.

They keep their accounts in rix dollars, sous, and denier lubs.

Viz.	12 denier lubs make	1 sous lub, value in the U. S.	$0.0208125
	16 sous lubs	1 mark banco (or mark lub)	.33⅓
	3 m. banco	1 rix dollar	1.
	2½ rix dols.	1 £ gross	2.50

Or in rix dollars, schillings and denier gross.

6 denier lubs make	1 denier gross. - -	.0104
2 denier gross	1 sous lub. - - -	.0208
6 sous lub	1 schilling. - - -	.125
8 schilling	1 rix dollar. - - -	1.
2½ r. dols.	1 £ gross. - - -	2.50

The money of exchange, generally called bank or banco, on account of its convenience and undoubted security in trade, is esteemed better than the current from 3 to 6 per cent. which excess is called agio. The current monies are said to have been so much adulterated of late that the agio has risen to 15 per cent. and upwards, but all bills of exchange are paid in bank.

To change banco to currency:

RULE.—As 100 banco : 100 + agio :: given banco : currency.

To change currency to banco:

RULE.—As 100 + agio :: 100 :: given currency : banco.

The U. States exchange with Hamburgh, at 33⅓ cents per mark banco, 3 marks being equal to 1 dollar; but the course of exchange is from 27 to 35 cents per mark banco.

To reduce federal to marks banco:

RULE.—Reduce the federal to cents and divide by the rate of exchange for the marks banco.

To reduce marks banco to federal:

RULE.—Reduce the sous and deniers to the decimal of a mark, and then multiply by the rate of exchange for the cents; or, multiply the marks by the rate of exchange and take parts for the lower denominations.

EXAMPLE.

1. A bill of exchange on Hamburgh for 12546 marks 10 sous 8 deniers banco, is sold at 30 cents per mark: What is the amount in dollars?

By Practice.

	12546 m. 10 s. 8 d.	
	30	
s.	376380	
8. $=\frac{1}{2}=$	15	
2. $=\frac{1}{4}=$	$3\frac{3}{4}=\frac{12}{16}$	
.6d. $=\frac{1}{4}=$	$\frac{15}{16}$	
2. $=\frac{1}{3}=$	$\frac{5}{16}$	
	\$3764.00 *Ans.*	

By Decimals.

12546 m. 10 s. 8 d.

12546.6′
.30

\$3764.000 *Ans.*

2. A in New-York draws on Hamburgh for 11564 m. b. 8 s. 10 d. and receives at the rate of $33\frac{1}{3}$ cents per mark banco: What is the value in dollars? *Ans.* \$3854.85

3. C in New-York draws on D in Hamburgh for 12546 m. 10 s. 8 den. in favour of E. who pays him \$3764: What was the course of exchange? *Ans.* \$0.30 per m. b.

4. A bill of exchange on Hamburgh, is sold for \$3000.75: What is the amount in rix dollars at 35 cents per mark banco. *Ans.* 2857 rix d. 2 m. b. 9 s. $1\frac{5}{7}$ d.

5. What is the mark banco worth in Boston, if 8573 m. 9 s. $1\frac{5}{7}$ d. be sold for \$3000.75? *Ans.* \$0.35.

Hamburgh exchanges with

France, *upon the* crown *of* 60 *sols for* 27 schil. lubs	*More or less, according to the course of exchange.*
Spain, *upon the* ducat *of* 375 *merv. for* 93 gross	
Portugal, *upon the* crusado *of* 400 *reas for* 42 gross	
Venice, *upon the* ducat *of* 24 *gross for* 86 gross	
England, *upon the* £ sterling *for* 35*s.* $6\frac{1}{3}$ Flemish	

Exchange with Denmark, Norway, &c.

The exchanges with Denmark and Norway, are intrinsically the same with that of Hamburgh. Their monies being of the same value, divided in the same manner, and nearly of the same denominations:

Viz.—12 pennings	make	1 sch. lub, value in the U. S.	$0.0208125
16 schillings lub		1 mark,	.333′
3 marks		1 rix dollar,	1.
6¼ marks		1 ducat,	2.08\3′

The examples and questions in exchange with Hamburgh, are sufficient for this.

For exchange with Sweden and Russia, see the general tables.

Exchange with Genoa, Leghorn, Florence, and Corsica.

Saint George's Bank, in Genoa, is constituted of such parts of the public revenue as have been mortgaged by government during the exigencies of the commonwealth. It is become a kind of inferior senate, of powerful sway in the republic, and often breaks the uniformity of their aristocratic government. The president's office is during life.

Accounts are kept in the bank in pezzoes *, or piastres, soldi and denari.

12 denari	make	1 soldi, value in the U. S.	$0.05.
20 soldi		1 pezzo, piastre, or dollar,	1

This is the money of exchange; but, in general, accounts are kept in *lire*, *soldi*, and *denari*, divided as before. The lire money of Leghorn is equal in value only to ⅓ of the exchange; and the lire of Genoa is only ⅕ value of the exchange money. The par of a piastre, lire money, is only 20 cents.

12 denari	make	1 soldi, value in the U. S.	$0.01
20 soldi		1 lire,	20
4 do.		1 chevalet,	04
30 do.		1 testoon,	30
6 testoons		1 genoni,	1.80

To reduce lire money to money of exchange:

Rule.—Divide the lire money by 5, the quotient will be the money of exchange.

EXAMPLE.

In 794 pezzoes, 19 soldi, 6 denari lire, how much money of exchange?

	pez.	*s.*	*d.*	
5)	794	19	6	
	158	19	10⅘	*exchange.*

* *Pezzoes are frequently called dollars.*

To reduce money of exchange to lire money:

RULE.—Multiply the money of exchange by 5, the quotient will be the lire money.

EXAMPLE.

In 684 pez. 15 s. 9 d. exchange, how much lire money?

```
      684  15  9
                5
      ------------
pez. 3423  18  9  lire, Ans.
```

To reduce pezzoes, soldi and denari to federal money;

RULE.—Reduce the soldi and denari to the decimal of a pezzo, and the exchange money will answer to dollars and cents at par.

EXAMPLE.

In 648 pez. 7 s. 6 d. how much federal money?

```
6 d. =      .005
7 s. =      .37
    pez. 648.
         ----------
         $648.37½ Ans.
```

NOTE.—This currency, or money, being divided like pounds, shillings and pence, to find the decimal value of the lower denominations, use table 2nd, page 140.

Otherwise,

Take the pezzoes as so many dollars, and for the soldi and denari take parts of a dollar for the cents.

Foregoing example. pez. 648.

```
5 soldi   = 1/4    .25
2 s. 6 d. = 1/2    .12½
                 ----------
                 $648.37½ Ans.
```

To reduce dollars and cents to pezzoes, soldi and denari:

RULE.—Take the dollars as so many pezzoes—multiply the cents by the lower known names in the exchange money for soldi and denari, &c.

EXAMPLE.

Reduce $847.37½ to pez. &c. &c.

```
$847. 375
       20
    -----
    7.500
       12
    -----
    6.000 = 847 pez. 7 s. 6 d. Ans.
```

When the exchange is so many cents per pezzo, above or below par:

RULE.—Reduce the lower denominations to the decimal of a pezzo, and multiply by the rate of exchange.

EXAMPLE.

What is the value of 500 pez. 14 s. 6 d. in federal money, at 90 cents per pezzo? *Ans.* 450.65\frac{1}{4}$.

New-York, 15*th June*, 1803.

Exchange for 4000 pez. 10 s. 6 d.

At usance, pay this first of exchange (*second and third of the same tenor and date unpaid*) to Pier De Foscari, or order, four thousand pezzoes, 10 sols, 6 den. exchange, which charge to account as per advice. *George L'Estrange.*

To Messrs. Francisco & Co.
merchants, Leghorn.

What is the value of the above, at $1.04 per pezzo?

Ans. 4160.54\frac{6}{10}$.

Exchange with Venice.

THE BANK OF VENICE, or *Banco del Giro*, is a public depository of the merchants' money, as established by an edict of the republic, ordering the payment of *merchandize* by wholesale, and *bills of exchange* to be made in *bank*. All debtors and creditors are obliged to pay or receive their money or accounts in the bank by transfer, from one account to the other.

A ready money account has been opened for the accommodation of strange merchants, and to supply the necessities of a *real trade*: this, instead of diminishing the funds, has been found to augment them. The funds of the bank were fixed at 5,000,000 ducats.

Their monies are as follow:

5$\frac{1}{4}$ soldi make	1 gross,	value in the U. S.	$
24 gross	1 ducat,		

But merchants and bankers, for the sake of ease in calculation, keep their accounts in ducats, sols and deniers d'or.

12 deniers d'or make	1 sol,	value in the U. S.	$0.0465+
20 sols	1 ducat,		.9305

But 93 cents are near enough for calculation.

This money of exchange is imaginary, 100 ducats whereof make 120 ducats current—the difference is called agio.

The money of *Venice* is of three sorts, viz. *Bank*, *Banco current* and *Picoli*. The bank money is 20 per cent. better than the banco current, and banco current is 20 per cent. better than the picoli; but all bills of exchange are paid in bank.

The real monies are as follow:

SILVER.	assay. dwt.	absolute wt. dwt. gr.
The old ducat of Venice, - - - - - -	w. 23½	14.15
new do. with No. 124, or 124 sols, - -		18.2
crusado, or St. Mark, with No. 140, or 140 sols,		20.6
piece of 20 jewels, - - - - - - - -	b. 6	3.17.7′
GOLD.		dw. gr. m.
double ducat, - - - - - - - - - -	b. 1½	4.18.8
single do. - - - - - - - - -		2. 9.4

To reduce Venetian ducats, soldi and denari to federal money:

Rule.—Reduce the soldi and denari to the decimal of a ducat, then multiply the given sum by .93—the product will be dollars and cents.

Note.—*This money being divided as currency, the decimal parts for the lower denominations will be found in table 2nd, page* 140.

EXAMPLE.

1. In 3254 ducats, 16 s. 6 d. how many dollars, if 93 cents be a par for one ducat?

```
6 denari =     .025
16 soldi =     .8
           3254.
           ---------
           3254.825
                 93
           ---------
           9764475
          29293425
           ---------
         $3026.98,7'2'5' Ans.
```

To reduce federal money to Venetian ducats, &c.

Rule.—Divide the dollars and cents by .93, the quotient will be ducats and decimal of ducats—then find the value of the decimal in the known denominations.

EXAMPLE.

2. Bought a bill of exchange on Venice, amounting to $3464.50: What is the amount in Venician ducats of exchange, at 93 cents per ducat? *Ans.* 3725 *d.* 5 *s.* 4½.

```
.93) 3464.50 (3725.2688
                     20
                 ------
                 5.3760
                     12
                 ------
                 4.5120
```

3. Prove the above examples.

New-York, June 20th, 1803.

Exchange for 750 duc. 16 sol. 6 den. d'or.

At usance, pay this second of exchange *(first and third of the same tenor and date unpaid)* to Messrs. Borelli & Co. seven hundred fifty ducats, sixteen sols, six den. bank, which charge to account, with or without advice. *Jonathan Burrall.*

To Messrs. Davilla, Dane & Co. merchants, Venice.

What is the amount of this draft, at 90 cents per ducat?

Ans. $675.74\frac{1}{4}$.

Having thus minutely proceeded with the most particular commercial nations, and (I conceive) sufficiently illustrated the different monies and exchanges, I deem it useless to pursue calculation farther; *but, as exchange is a subject so extensive, its consideration so useful and advantageous to the young, as well as to the old merchant,* I shall here subjoin tables of the different monies in use with the several trading nations of the world, *the foregoing rules and applications being sufficient directions for the different exchanges that may be required.*

Tables of Exchange.

PIEDMONT, SAVOY AND SARDINIA.

TURIN, CHAMBERRY, CAGLIARI.

12 denari	make	1 soldi, value in the U. S.	$0.013'8
12 soldi		1 florin, - - - - - - -	.166
20 soldi		1 lire, - - - - - - -	.277
6 florins		1 scudi, - - - - - - -	1.
7 florins		1 ducatoon, - - - - -	1.166
13 lires		1 pistole, - - - - - -	3.611
16 lires		1 louis d'or, - - - - -	4.444

MILAN, PAVIA, MODENA, PARMA.

12 denari	make	1 soldi, - - - -	$.0.
20 soldi		1 lire, - - - -	.162
115 soldi		1 scudi cur. - - -	.931
117 soldi		1 scudi of exch. - -	.944
6 lires		1 philip, - - -	.972
22 lires		1 pistole, - - -	3.555
23 lires		1 Spanish pistole, -	3.61

ROME.

CIVITA-VECCHIA, ANCONA, &c.

5 quatrini	make	1 bayoc, - - - -	$0.
8 bayocs		1 julio, - - - - -	.111
10 bayocs		1 stampt julio, - -	.138
24 bayocs		1 testoon, - - -	.333
10 julios		1 current crown, - -	1.111
12 julios		1 stampt do. - -	1.333
18 julios		1 chequin, - - -	2.
31 julios		1 pistole, - - - -	3.444

BOLOGNA, RAVENNA, &c.

6 quatrini	make	1 bayoc, value in the U. S.	$0.
10 bayocs		1 julio, - - - - - -	.11
20 bayocs		1 lire, - - - - - -	.22
3 julios		1 testoon, - - - - -	.333
85 bayocs		1 scudi of exch. - - -	.944
105 bayocs		1 ducatoon, - - - -	1.166
100 bayocs		1 crown, - - - - -	1.11
31 julios		1 pistole, - - - -	3.444

NAPLES.

GAIETA, CAPUA.

3 quatrini	make	1 grain, value in the U. S.	$0.
10 grains		1 carlin, - - - - - -	.074
2 carlins or 20 grs.		1 tarin, - - - - - -	.148
2 tarins or 40 grs.		1 testoon, - - - -	.296
2½ tarins or 100 grs.		1 ducat of ex. - - -	.74
23 tarins		1 pistole, - - - -	3.407
25 tarins		1 Spanish pistole, - -	3.70

SICILY AND MALTA.

PALERMO, SYRACUSE, MESSINA and VALETTA.

6 pichili	make	1 grain, value in the U. S.	$0.
10 grains		1 carlin, - - - - - -	.029
20 grains		1 tarin, - - - - - -	.058
6 tarins		1 florins of exch. - -	.35
13 tarins		1 ducat of exch. - -	.73
60 carlins		1 ounce, - - - - -	1.71
2 ounces		1 pistole, - - - - -	3.41

Germany.

AUSTRIA, SWABIA, BOHEMIA, SILESIA.

VIENNA, BLENHEIM, PRAGUE, BRESLAW.

HUNGARY, FRANCONIA, &c.

PRESBURGH, FRANKFORT, NUREMBERG, &c.

2 fennings make	1 dreyer, value in the U. S.	$0.
4 fennings	1 cruitzer, - - - - -	
4 cruitzers	1 batzen, - - - - - -	.034
15 batzen	1 gould, - - - - - -	.517
90 cruitzer	1 rix dol. - - - - - -	.777
2 florins	1 hard dol. - - - - - -	1.
60 batzen	1 ducat, - - - - - -	2.074

COLOGNE.

MENTZ, LIEGE, TRIERS, MUNICH, MUNSTER, PADERBORN, &c.

3 dutes make	1 cruitzer, value in the U. S.	$0.
2 cruitzers	1 alb, - - - - -	
8 dutes	1 stiver, - - - - -	.013
3 stivers	1 palpert, - - - - -	.039
4 palperts	1 copstuck, - - - -	.155
40 stivers	1 guilder, - - - -	.517
2 guilders	1 hard dol. - - - -	1.034
4 guilders	1 ducat, - - - - -	2.074

BANDENBURG AND POMERANIA.

BERLIN, POTSDAM, and STETIN.

18 deniers make	1 grosh, value in the U. S. at	$0.
20 groshen	1 mark, - - - - -	.173
30 groshen	1 florin, - - - - -	.259
90 groshen	1 rix dol. - - - - -	.777
108 groshen	1 albertus, - - - -	.926
240 groshen	1 ducat, - - - - -	2.074

SAXONY, HOLSTEIN AND HANOVER.

DRESDEN, LEIPSIC, &c. WISMAR, KEIL, &c. LUNENBURG.

2 hellers make	1 fenning, value in the U. S.	$
6 hellers	1 dreyer, - - - - -	
16 hellers	1 marien, - - - - -	
12 fennings	1 grosh, - - - - -	.032
16 groshen	1 gould, - - - - -	.517
24 groshen	1 rix dol. - - - - -	.777
32 groshen	1 hard dol. - - - -	1.034
4 goulds	1 ducat, - - - - -	2.074

Switzerland.

BASIL AND ST. GALL.

ZURICH, ZUG, &c. APPENSAL, &c.

4 fennings make	1 cruitzer, value in the U. S.	-	$0.0092
12 fennings	1 sol,	-	.027
20 sols	1 livre,	-	.555
60 cruitzers	1 gould,	-	.555
107 cruitzers	1 rix dol.	-	1.

BERN.

LUCERNE, NEUFCHATTEL, &c.

4 deniers make	1 cruitzer, value in the U. S. at	-	$0.0074
3 cruitzers	1 sol,	-	.022
20 sols	1 livre,	-	.444
75 cruitzers	1 gulden,	-	.555
135 cruitzers	1 crown,	-	1.

GENEVA.

BONNE, &c.

12 deniers make	1 sol, value in the U. S. at	-	$0.0138
20 sols current	1 livre,	-	.277
6 sols	1 florin,	-	.083
10½ florins	1 patacoon,	-	.875
15¾ florins	1 croisade,	-	1.312
24 florins	1 ducat,	-	2.

SWEDEN AND LAPLAND.

STOCKHOLM, UPSAL and TORNEA.

8 runstics make	1 copper marc, value in the U. S.	-	$0.027
3 cop. marks	1 silver mark	-	.081
4 cop. marks	1 copper dolar,	-	.111
3 cop. dols.	1 silver dol.	-	.333
3 silver dols.	1 rix dol.	-	1.
2 rix dols.	1 ducat,	-	2.

POLAND AND PRUSSIA.

CRACOW, WARSAW, and THORN, DANTZIC, KONINGSBERG, &c.

3 shelon make	1 grosh,	-	$0.
30 groshen	1 florin,	-	.266′
90 groshen	1 rix dol.	-	.80
8 florins	1 ducat,	-	2.133′
5 rix dols.	1 Frederick d'or,	-	4.00

RUSSIA AND MUSCOVY.

PETERSBURG, ARCHANGEL, and MOSCOW.

2 denuscas	make	1 copec, - - -	$0.01
50 copecs		1 poltin, - - -	.50
100 copecs		1 rouble, - - -	1.
2 roubles		1 xervonitz, - - -	2.

TURKEY.

GREECE, CANDIA, CYPRUS, &c.

4 mangar	make	1 asper, value in the U. S.	$0.011
5 asper		1 bestic, - - -	.055
2 bestic		1 ostic, - - -	.111
20 aspers		1 solota, - - -	.222
4 solota		1 piastre, - - -	.888
5 solota		1 caragrouche, - -	1.111
2 caragrouche		1 xeriff, - - -	2.222

Africa.

MOROCCO.

SANTACRUZ, MEQUINEZ, FEZ, &c.

24 fluce	make	1 blanquil, value in the U. S.	$0.037
3 blanq.		1 ounce, - - - - - - - -	.148
7 blanq.		1 octavo, - - - - - - - -	.259
2 octavo		1 quarto, - - - - - - - -	.518
2 quarto		1 media, - - - - - - - -	1.037
27 blanq.		1 dol. - - - - - - - -	1.
54 blanq.		1 xequin, - - - - - - - -	2.
100 blanq.		1 pistole, - - - - - - - -	3.72

BARBARY.

ALGIERS, TUNIS, TRIPOLI.

		1 asper, value in the U. S.	$0.0125
10 asper	make	1 rial, - - - - - - - -	.125
2 rials		1 double, - - - - - - - -	.25
4 doubles		1 dol. - - - - - - - -	1.
24 medins		1 chequin, - - - - - - -	.75
32 medins		1 dol. - - - - - - - -	1.
180 aspers		1 zequin, - - - - - - -	1.96
15 doubles		1 pistole, - - - - - - -	3.72

EGYPT.

CAIRO, ALEXANDRIA, &c.

	1 asper, value in the U. S.	$0.0104
3 aspers make	1 medin, - - - - - - -	0.0212
24 medins	1 Italian ducato, - - - - -	.75
80 aspers	1 piastre, - - - - - - - -	.888
32 medins	1 dol. - - - - - - - - -	1.
96 aspers	1 crown, - - - - - - - -	1.111
192 asp. or 2 cr.	1 sultanin, - - - - - - -	2.222
72 medins	1 pargo dol. - - - - - - -	2.333

Asia.

PERSIA.

ISPAHAN, ORMUS, and GOMBROON.

10 coz make	1 shahee, value in the U. S.	$0.074
4 shahees	1 abashee, - - - - - - -	.296
5 abashees	1 or, - - - - - - - - -	1.48
12 do.	1 bovelo, - - - - - - -	3.555
50 abashees	1 tomond, - - - - - - -	14.81

ARABIA

MEDINA, MECCA, and MOCHA.

7 caret	1 comashee, value in the U. S.	$0.016
18 comashee	1 abyss, - - - - - - -	.30
60 do.	1 piastre, or dol. - - - - -	1.
100 do.	1 sequin, - - - - - -	1.666
9 sequin	1 tomond, - - - - - -	15,

MOGOL, EAST-INDIES.

SURAT, CAMBRAT, GUZZURAT.

4 pieces make	1 fanam, value in the U. S.	$0.034
4 fanam	1 ana, - - - - -	.138
4 anas	1 rupee, - - - - -	.555
2 rupees	1 crown, - - - - -	1.11
14 anas	1 pagoda, - - - - -	1.94
4 pagoda	1 gold rupee - - - -	7.77

MALABAR.

BOMBAY, DUBAL, &c.

5 rez make	1 pice, value in the U. S.	$0.00647
20 pice	1 quarter, - - - -	.129
4 quarters	1 rupee, - - - -	.518
14 do.	1 pagoda, - - - -	1.814
60 quarters	1 gold rupee, - - -	7.77

GOA, VISAPOUR, &c.

20 rez	1 vintin, - - - -	$0.0231
42 vintin	1 tangu, - - - -	.972
4 tangu	1 paru, - - - -	3.888
2 paru	1 gold rupee, - - -	7.77

COROMANDEL.

MADRAS, PONDICHERRY, &c.

4 cash make	1 pice, value in the U. S.	$0.0069
8 pices	1 fanam, - - - -	.055
10 fanam	1 rupee, - - - -	.555
2 rupees	1 crown, - - - -	1.11
35 fanam	1 pagoda, - - - -	1.944
4 pagoda	1 gold rupee, - - -	7.77

BENGAL.

CALICUT, FORT WILLIAM, &c.

4 pice make	1 fanam, value in the U. S.	$0.0115
3 fanam	1 ana, - - - -	.034
16 ana	1 rupee, - - - -	.555
2 rupee	1 crown, - - - -	1.11
56 ana	1 pagoda, - - - -	1.94

SIAM.

PEGU, MALACCO, CAMBODIA, SUMATRA, JAVA, BORNEO, &c.

800 cori make	1 fettee, value in the U. S.	$0.0011
125 fettee	1 sataleer, - - -	.138
250 fettee	1 sooco, - - -	.277
500 fettee	1 tutal, - - -	.555
900 fettee	1 dollar, - - -	1.
1000 fettee	1 crown or rial, - -	1.11

CHINA.

PEKIN, CANTON, &c.

10 caxa make	1 candareen, value in the U. S.	$0.014
10 candareen	1 mace, - - - - -	.143
35 candareen	1 rupee, - - - - -	.50
2 rupee	1 dol. - - - - -	1.
10 mace	1 tale, - - - - -	1.43

JAPAN.

	1 piti, value in the U. S.		$0.0037
20 pitis make	1 mace,	- - -	.074
15 maces	1 oz. silver,	- - -	1.076
20 do.	1 tale,	- - - -	1.481
30 do.	1 ingot,	- - -	2.22
13 oz. silver	1 oz. gold,	- - -	14.
2 oz. gold,	1 japanese,	- - -	28.
10 japanese	1 cattee,	- - -	280.

West-Indies.

ENGLISH.

JAMAICA, BARBADOES, &c.

4 farthings make	1 penny, value in the U. S.	
12 pence	1 shilling, - - -	
20 shillings	1 pound-currency, - -	
7½ pence	1 bit, - - - -	
75 pence	1 dollar, - - - -	
84 pence	1 crown, - - - -	
24 shillings	1 pistole, - - -	
30 shillings	1 guinea, - - -	

FRENCH.

ST. DOMINGO, MARTINICO, &c.

15 sols make	1 scalin, value in the U. S.	$
20 sols	1 livre, - - - -	
7 livres	1 dol. - - - -	
8 do.	1 ecu, - - - -	
26 do.	1 pistole - - - -	
32 do.	1 louis d'or - - - -	

In Nova-Scotia, Canada, Florida, Cayenne, &c. where English, French and Spanish monies circulate, the currency alters according to the plenty and scarcity of specie.

For the currencies of the Spanish, Portuguese, Dutch, Danish, &c. in the West-Indies, see their respective exchanges.

For the information of the readers of history, the monies of the ancients shall be explained here.

JEWISH MONIES.

	1 gerah, value in the U. S.	$0.025'3'
10 gerah make	1 bekah, - - - - -	.253'
2 bekah	1 shekel, - - - - -	.506'
50 shekel	1 mina, - - - -	25.333
60 mina	1 talent silver, - -	1520.
1 sextula gold	- - - - - -	2.704'
3 sextula	1 siculus, - - - -	8.111'
3000 siculus	1 talent gold, - -	24333.333'

GRECIAN MONIES.

	1 chalcus, value in the U. S.	$0.006
2 chalcus	1 dichalcus, - - -	.011
2 hemibolum	1 obolus, - - -	.0239
2 obolus	1 diabolum, - - -	.047
2 diabolum	1 tebrabolum, - -	.094
1½ tetrabolum	1 drachma, - - -	.143
100 drachma	1 mina, - - -	143.50
60 mina	1 talent, - - -	861.

ROMAN MONIES.

	1 teruncius, - - -	.0035
2 teruncius	1 semilibella, - - -	.0071
2 semilib.	1 libella, - - -	.0142
2½ libella	1 sestertius - - -	.0357
2 sestertius	1 victoriatus - - -	.0715
2 victoriatus	1 demarius - - -	.143
1000 sestertius	1 sestertium, - - -	35.80
100 sestertia	1 decies, - - -	35800.

Arbitrations of Exchange.

It is of the utmost consequence to the merchant, who has foreign concerns, to be well acquainted with the mode of arbitrating the exchanges between two or more places; to have a knowledge of their weights and measures and the proportion they bear to each other. By this means he may make his gains certain, his knowledge as a merchant respected; and may likewise acquire valuable correspondents abroad that otherwise might never hear of his name.

To the rule of proportion belongs the solution of all questions in *arbitration;* but as continual statements are not only tedious but liable to error, let the questions be solved by the following rule, called *conjoined proportion.*

First.—*Distinguish the members of the arbitration into antecedents and consequents, placing the antecedents on the left and the consequents on the right.*

Second.—*The first consequent must be of the same name with the second antecedent, which order must be observed through the equation, and the last consequent must be of the same name with the first antecedent.*

Third.—*If any of the terms in the equation have a fraction annexed, multiply the whole numbers by the denominator, adding the numerator, and set the said denominator on the opposite side below; however, this may be dispensed with by taking parts for the fraction, &c.*

Fourth.—*Multiply the antecedents for a divisor, and the consequents for a dividend, if the place of the antecedent be blank; or multiply the consequents for a divisor, and the antecedents for a dividend, if the place of the consequent be blank.*

EXAMPLE.

1. Amsterdam owes New-York 6000 guilders: Whether is it better to draw at 37 cents the guilder, or have the money remitted by the following route, viz.—To Paris, at 54 d. flemish, for 3 francs; thence to Genoa, at 5 francs per piastre, thence to London, at 50d. sterling per piastre; and thence to New-York, at par.

18	*d. flem.*	5′4′=3′ *fr.*		
	fr.	5′=1 *piastre*		
	piast.	1=5′0′. *d ster.* 10		
9	*d. ster.*	5′4=′1 *dol.*	*dividend,*	
	dol.	?=2′4′0000·*d flem.*	4000,0	($2469.13
and		18×9=162 *divisor) by the way of London.*		

$2469.13

then 6000×37 *cents,*=2220

$249.13 *gain by remitting.*

This rule for abridging the antecedents and consequents, is founded on the 19th proposition of the 5th book of Euclid, which says, "If a whole magnitude, be to a whole, as a magnitude taken "from the first, is to a magnitude taken from the other, the re- "mainder shall be to the remainder, as the whole to the whole."

☞ See the application of this, case 4th of contractions in the rule of three direct, page 54.

The foregoing question is given here by proportion, to show how much unnecessary work is dispensed with.

	d. flem.	*fr.*	*d. flem.*	*fr.*
First.—As	54 :	3 ::	24000 :	13333.3′,
	fr.	*piast.*	*fr.*	*piast.*
Second.—As	5 :	1 ::	13333.3′, :	2666.6′
	piast.	*d. ster.*	*piast.*	*d. ster.*
Third.—As	1. :	50 ::	2666.6 :	133333.3′
	d. ster.	*dol.*	*d. ster.*	
Fourth.—As	54 :	1 ::	133333.3′ :	to $2469.13 as before.

Let the pupil work this solution in full.

2. C in London owes D in Philadelphia £400 sterling; D orders the money to be remitted on his account to Hamburgh, at 52d. sterling per rix dollar; thence to Lyons, at 7 rix dollars for 13 crowns; thence to Venice, at 12 crowns for 7 ducats; thence to Malaga, at 7 ducats for 6 Spanish ducats of exchange; and thence to Philadelphia, at 110 cents per ducat: What will be his gain or loss if the exchange between Philadelphia and London be at par? *Ans.* $107.94. *gain.*

3. E of Leghorn is indebted to F of New-York 800 piastres, exchange between Leghorn and New-York 99 cents per piastre; F considers it better to have the money remitted, and E remits as follows: To Venice, at 94 piastres for 100 ducats; thence to Cadiz, at 320 mervadies per ducat; thence to Lisbon, at 630 reas for 272 mervadies; to Amsterdam, at 50d. flemish for 400 reas; to Paris, at 56d. flemish for 3 francs; thence to London, at 31½d. sterling per 3 francs; and from London to New-York, at 54d. sterling per dollar: What is the arbitrated price of a piastre between New-York and Leghorn, and what does F gain by the transaction? *Ans. arbitrated price,* $1.02+ *gain,* $24.

4. G of Venice is indebted to H of Philadelphia 300 ducats, exchange on Venice at 90 cents per ducat: H considers it better to have the money remitted, and G remits as follows: To Barcelona, at 320 mervadies per ducat; thence to Oporto, at 420 mervadies for 1 milrea; thence to Nantz, at 140 reas per franc; thence to London, at 30d. sterling for 3 francs; and thence to Philadelphia, at par: What is the arbitrated price of a ducat, between Venice and Philadelphia, and what does H gain or lose by the transaction? *Ans. arbitr. p. of a d.* $1.0078, *gain,* $32.34

Concerning the gain and loss by exchange.

Suppose K in London owes L in New-York $1000, the exchange between New-York and London, at 2 per cent. above par, and the exchange between London and New-York, at 52d. sterling per dollar: Whether is it better for L to draw, or K to remit? *Ans. better remitted, by* $18.46

Foreign Weights and Measures compared with American and British.

1. If 6 lb. of a. be equal to 5 lb. of b. and 3 lb. b. equal 4 lb. of c. and 8 lb. of c. equal 9 lb. of d. and 5 lb. of d. equal 6 lb. of e. how many lb. of e. are equal to 40 lb. of a.?

$$\begin{array}{rl} & a\ 6' = 5'\ b \\ & b\ 3' = 4'\ c \\ 2 : & c\ 8' = 9'\ d : 3 \\ & d\ 5' = 6'\ e \\ & e\ ? = 40a \\ 2) & 120 \\ & 60\ lb.\ Ans. \end{array}$$

2. If 10 lb. at London be equal to 9 lb. at Amsterdam, and 45 lb. at Amsterdam equal 49 lb. at Bruges, and 98 lb. at Bruges equal 116 lb. at Dantzic—how many lb. at Dantzic are equal to 112 lb. at London? *Ans.* 129.97 lb.

3. If 109 lb. at London equal 100 lb. at Amsterdam, and 60 lb. at Amsterdam equal 90 lb. at Genoa, and 100 lb. at Genoa equal 70 lb. at Leipsic, and 105 lb. at Leipsic equal 105 lb. at Leghorn, and 100 lb. at Leghorn equal 109 lb. at Seville, and 63 lb. at Seville equal 100 at Milan—how many lb. at Milan will equal 63 lb. at London? *Ans.* 105 lb.

4. What will one lb. of pepper cost, if 3 lb. of cloves cost as much as 6 lb. of pepper, and $2\frac{1}{2}$ lb. cinnamon cost as much as 4 lb. of cloves, and 3 lb. cinnamon cost 8 shillings? *Ans.* 10*d.*

5. If 7 aunes of Geneva make 9 yards of London, and 36 yds. of London equal 49 aunes of Holland, and 7 aunes of Holland 9 braces of Milan, and 3 braces of Milan equal 2 vares of Arragon, and 5 vares of Arragon 2 canes of Montpelier, 9 canes of Montpelier 10 canes of Thoulouse, and 4 canes of Thoulouse 9 aunes of Troyes—how many aunes of Troyes will be equal to 100 aunes of Geneva? *Ans.* 150 *aunes of Troyes.*

6. Suppose a merchant in Hamburgh hath orders to procure 81 yards of English cloth, so that 7 ells of Hamburgh may be procured for £3 sterling: now if 7 camos of Barcelona make 9 yards in London, and 7 ells of Holland make 4 camos in Barcelona, and 1 ell of Holland makes $1\frac{1}{5}$ ell of Hamburgh—how much will the said cloth amount to, at Hamburgh, exchange being at 33*s.* gross per £ sterling. *Ans.* 701 *m.* 10*s.* $7\frac{1}{5}$ *den.*

TABLE,

Showing how many pounds and decimals of pounds avoirdupoise are equal to 100 lb. in the following places. Comparative tables of weights and measures, for the most part, give 100 lb. of London, Amsterdam, or Paris, as the standard to which all the rest are made equal; but it will be found to facilitate calculation, by making 100 lb. foreign the standard, and show how many lb. avoirdupoise are equal thereto.

Foreign.	American or English. equals
HOLLAND.	
100lb. at Amsterdam, Dort,	109.lb.
Leyden,	102.8
Rotterdam,	109.
BRABANT.	
Bergen Opzome,	112.3
Antwerp,	103.8

Foreign.	American or English. equals
BRABANT.	
100 lb. at Arschot,	lb. 103.8
Bois le Duc,	103.8
Lovain,	103.8
DENMARK.	
Copenhagen,	101.46
Hamburgh,	106.8
Lubec,	103.8

Weights of Muscovy.

		lb.	oz.	
1 poede is equal to		8	11.42	avoirdupoise.
10 do.	1 berkewitz,	87	3.24	

Weights of Turkey. Great weight.

		lb.	oz.	
1 occo is equal to		3	12.8	avoirdupoise.
6 do.	1 batman,	32	12.8	
44 do.	1 quintal,	167	3.2	

Small weight.

1 occo is equal to		lb. 0	15.	avoirdupoise.
6 do.		1 batman, 5	10.	
1 rottoli of Egypt,	144 drams,	1	6.4	
1 do. (large) Aleppo,	720 do.	7		
1 do. (small) do.	624 do.	6	1.22	
1 do. Seyda,	600 do.	5	13.28	

Measures of foreign Nations compared with Inches, &c.

FRANCE.

Foreign.		*Inches.*
Metre		39.383
Aune of	Abbeville, Bayone, Bourdeaux, Caen, Calais, Elberuf, Havre de Grace, Nantz, Paris, St. Quintins, Rochelle, Rouen, Sedan	42.
Aune of	Cambray	28.8
Aune of	Lyons	44.5

BRABANT.

Foreign.		*Inches.*
Aune of	Abo, Bergen Opzome, Bois le Duc, Breda, Brussels	27.4
Aune of	Louvain, Maestreicht, Malines, Narva, Strasburg	27.4
Aune of	Antwerp	27.6

HOLLAND.

Foreign.		*Inches.*
Aune of	Amsterdam, Campvier, Hague, Haerlem, Leyden, Rotterdam, Flushing, Middleburg	27.
Aune of	Gueldres	26.1

FLANDERS.

Foreign.		*Inches.*
Aune of	Arras	27.7
Aune of	Bruges, St. Omers, Ostend	26.3
Aune of	Lisle	28.8

FLANDERS.

	Foreign.		*Inches.*
Aune of	Douay, Ypres		28.
Aune of	Dunkirk, Ghent		26.6
Aune of	Namur		26.1
Aune of	Tournay		24.
Aune of	Valenciennes		26.

SPAIN.

	Foreign.		*Inches.*
Barras, cont'g 2⅓ palmas	Almeria		27.4
Barras, cont'g 2⅓ palmas	St. Andero, Bilboa, St. Sebastian		33.3
Barras, cont'g 2⅓ palmas	Cadiz	Silk	32.4
Barras, cont'g 2⅓ palmas	Cadiz	Cloth	33.8
Barras, cont'g 2⅓ palmas	Carthagena, Saley		32.4
Barras, cont'g 2⅓ palmas	Corunna, Madrid		33.1
Aune of Cadiz			27.4
Camas of	Alicant		37.6
Camas of	Barcelona		63.
Camas of	Saragossa		71.
Camas of	Tortosa		66.
Camas of	Valencia		37.6

Spanish Islands.

	Foreign.	*Inches.*
Camas of 8 palmas, or 4 brazzes.	Minorca	63.
Camas of 8 palmas, or 4 brazzes.	Majorca	37.8

PORTUGAL.

Foreign.	*Inches.*
Barras of Lisbon	44.3
Cavido of do.	27.

SWITZERLAND.

	Foreign.		*Inches.*
Ell	Basil, Berne		22.7
Ell	St. Gall	woollen	24.1
Ell	St. Gall	linen	31.3

ITALY.

	Foreign.		*Inches.*
Aune of Geneva			44.5
Ell	Ravena		26.5
Ell	Trent		21.6
Camas of	Capua, Gaieta, Messina, Naples, St. Remo		88.2

ITALY.

	Foreign.		*Inches.*
Camas of	Salerno, Savona		88.2
Camas of	Rome, Civita		81.4
Camas of	Florence		93.6
Camas of	Genoa	woollen	88.3
Camas of	Genoa	linen	98.1
Camas of	Palermo, Syracuse	Sicily	88.2
Camas of	Malta		88.2
Brazzas	Bergama, Bologna		26.5
Brazzas	Rome, Farara		36.
Brazzas	Florence, Luca		23.4
Brazzas	Genoa		22.8
Brazzas	Parma, Mantua, Modena, Placencia		26.8
Brazzas	Milan	silk	20.9
Brazzas	Milan	cloth	26.5
Brazzas	Venice	silk	22.5
Brazzas	Venice	cloth	26.5

GERMANY.

	Foreign.		*Inches.*
Ell of	Aix-la-chapelle, Dusseldorf, Mentz		21.6
Ell of	Augsburg	linen	35.
Ell of	Augsburg	woollen	31.7
Ell of	Bonn, Coblentz, Cologn, Manheim, Philipsb'g, Triers		22.
Ell of	Nurenburg		27.
Ell of	Osnaburg		44.7
Aune of Friburgh			30.6

SAXONY.

	Foreign.	*Inches.*
Ell of	Bremen, Leipsic, Naurenburg	23.

Measure	Foreign.	*Inches.*
	SAXONY.	
Ell of	Dresden, Wesmar	21.2
	SILESIA.	
Ell of	Breslaw	21.6
	AUSTRIA.	
Ell of	Vienna	22.
Ell of	Inspruck, Bolsano	21.6
	BAVARIA.	
Ell of	Munich	21.2
Ell of	Ratisbon, Saltsburg	21.6
	BOHEMIA.	
Ell	Prague	21.6
	BRANDENURG.	
Ell	Potsdam	23.4
	LOW COUNTRIES.	
Aune	Nimeguen	26.1
Aune	Liege	24.1
Aune	Ruremond	27.
	HAINAULT.	
Aune	Mons	25.2
	FRANCONIA.	
Ell	Frankfort	22.3
	HANOVER.	
Ell	Zell	21.6
	DENMARK.	
Ell	Copenhagen	24.3
Ell	Kiel	22.
Ell	Altona, Lubec	22.4
	SWEDEN.	
Ell	Abo, Christianople	23.4
	NORWAY.	
Ell	Bergen, Christigna, Drontheim	24.3
	POLAND.	
Ell	Warsaw, Cracow	21.6
Ell	Thorn	23.
Ell	Dantzic	24.
	PRUSSIA.	
Ell	Berlin	23.4
Ell	Stetin	21.6

Measure	Foreign.	*Inches.*
	POMERANIA.	
Ell	Embden, Paderbon	21.2
Ell	Munster	21.6
	RUSSIA.	
Arsheen of 16 Vesh.	Archangel, Narva, Petersburg, Moscow, Mitaw	25.6
	GREECE.	
Pico	Athens, Lacedemon	18.
	LESSER ASIA.	
Pico	Ephesus, Smyrna	24.
	ROMANIA.	
Pico	Adrianople	27.
Pico	Galiopili	18.
	PALESTINE.	
Pico	Jerusalem, Acre	24.
	SYRIA.	
Pico	Aleppo	24.
Pico	Damascus	27.
Pico	Scanderoon	24.
	ARABIA.	
Pico	Bassora	24.
Pico	Medina, Mecca, Suez	27.
Cavido	Mocka, Goa	27.
	AFRICA.	
Pico	Egypt, Alexandria, Sayde	24.
Pico	Tunis & Tripoli — Cloth	27.
Pico	Tunis & Tripoli — Linen	24.
Pico	Tunis & Tripoli — Silk	18.
Pico	Oran	27.
Pico	Algiers	27.
Pico	Barca — Cloth	24.
Pico	Barca — Silk	18.

Profit and Loss.

PROFIT AND LOSS instructs us how to estimate the value of goods from cost and charges, &c. and the gain that may be acquired on their sales.

CASE. I.

To estimate imports and exports.

RULE.—To the prime cost add the inferior charges, the sum is what the article stands you in.

EXAMPLE.

1. Imported from London 50 reams of paper, which stand me per invoice in $4 per ream; paid duty here 50 cents per ream, freight $10.37½,i nsurance 7½ per cent. porterage, lighterage, &c. $3.45½: How must I sell it per ream, to clear 20 per cent.?

50 reams, at 4 dolls	=200.
do. at 50 cents duty	25.
freight,	10.37½
insurance on 200 dlls. at 7½	15.
charges	3.45½
total cost	253.83
to gain 20 per cent. add ⅕	50.76⅗
	50) 304.59⅗

may be sold for $6.09 per ream nearly.

New-York, November 1st, 1802.

2. Shipped on board the Draper, for Dublin, 100 hogsheads of Flax-seed, at $8.75 cents per hhd.; paid porterage 20 cents per hhd. storage 21 cents, insurance on the whole at 7 per cent.: How much do they stand me in on board? *Ans.* $980.12.

3. Imported from Dublin 40 pieces linen cloth, which stand me in (as per invoice) £3.15 Irish per piece; paid duty here 15 per cent. freight $14.65, porterage 75 cents, insurance 5 per cent. on the invoice: How may I sell this linen per piece, and gain 25 per cent.? *Ans.* $23.55 *per piece.*

New-York, 21st November, 1801.

4. Shipped on board the Joseph for Dublin, and consigned to Joseph Wilson, to sell for M. A. 200 barrels superfine flour, at $9.50 per barrel; paid storage 25 cents per bbl. porterage $20, insurance on the whole, at 7½ per cent.: What is the whole cost on board? *Ans.* $2117.75.

5. Imported from Lisbon per the Good Intent, 30 casks port wine, containing 3500 gallons, at 300 reas per gallon; paid freight $9.50 per cask, insurance 5 per cent. duty 6 cents per gallon, porterage $30: How may I sell it per gallon, to clear 50 per ct.?

Ans. 0.81\frac{5}{7}$ *nearly.*

6. Imported from Amsterdam 10 casks of Geneva, containing 1180 gallons, at 2$\frac{1}{4}$ guilders* per gallon; paid here, freight $120, porters $7.50, insurance on the invoice 10 per cent. gauging $\frac{1}{2}$ cent per gallon on 1169 galls. the rest having leaked: How may I sell it per gallon, to clear 30 per cent.?

Ans. $1.35 *nearly.*

7. If I buy merchandize to amt. £20, and sell them for £22 10*s.* what do I gain per cent.? *Ans.* 12$\frac{1}{2}$.

8. Bought 1000 bushels of wheat, at 10*s.* per bushel; paid charges £10, and sold it again at 11*s.* 3*d.* per bushel: What is the gain per cent.? *Ans.* 10$\frac{5}{17}$.

9. When I sell a yard of cloth for 11*s.* 6*d.* I gain 15 per cent.; but if sold at 12*s.* what should I gain? *Ans.* 20 *per cent.*

CASE II.

When goods are bought and sold on time.

EXAMPLE.

10. Bought goods to am't £500 ready money, and sold them for £584, at 8 months credit: What is gained per ct. per ann.?

584 − 500 = 84 *gain in 8 months.*

And to find the rate per cent.

5 *as* £5'0'0' } 84 { 1'0'0'
2 *mo.* 8' } 3 { 1'2' − 3
—
10)

£25.2 *Ans.* or £25$\frac{1}{5}$ *per cent.*

11. Bought goods for £18 ready money, sold them again for £35 with 4 months credit: What is gained per cent. per ann.?

Ans. 116$\frac{2}{3}$ *per cent.*

12. Bought 300 lb. coffee, at 4*s.* 2*d.* ready money, sold at 5*s.* per pound payable in 8 months: What was gained on the entire, allowing discount at 6 per cent. per annum; and what per cent. per annum was gained?

Ans. £9 12 3$\frac{9}{13}$ *entire gain, and* 30 *per cent.*

13. Bought a hogshead of brandy, at $1.10, by accident 9 gals. leaked: How may the remainder be sold at 6 months credit, and sustain no loss, interest at 6 per cent. per annum?

Ans. $1.32 *per gallon.*

---

* Guilder = 37 cents.

14. Bought 100 yards of cloth, at 14s. per yard, which I want to sell at 25 per cent. profit; and if I sell upon time, to have 5 per cent. per annum for forbearance, how may I sell it per yard at 6 months, and make both these gains? *Ans.* 17s. $11\frac{1}{4}$d.

15. Bought raisins at 50s. per cwt. payable in 9 months; sold them for 52s. payable in 15 months: What is the profit per ct. per annum, at that rate?

15 − 9 = 6 mo. and 52 − 50 = 2 gain.

then as 50 : 2 :: 100 ; 6 : 2 :: 12 *Ans.* 8 *per cent.*

16. Bought linen cloth at 5s. 3d. per yard, payable in 6 months; sold the same at 5s. $10\frac{1}{2}$d. payable in 10 months: How much per cent. per annum is gained by the transaction?

Ans. $35\frac{5}{7}$.

17. If 1 cwt. of merchandize be bought for 56s. payable in 9 months, and sold at $6\frac{1}{4}$d. per lb. payable in 12 months: What is the gain per cent. per annum? *Ans.* $16\frac{2}{3}$.

18. Bought merchandize, at $9\frac{1}{4}$d. payable in 3 months; sold it at $11\frac{1}{3}$d. payable in 7 months: How much per cent. per ann. was gained at that rate? *Ans.* $60\frac{10}{37}$.

CASE III.

Foreign sales estimated.

19. Shipped for Rotterdam 500 salt ox hides, wt. neat 400 cwt. 2 qrs. 14 lb. at $5 per cwt.; paid duty, charges, &c. $$60.87\frac{1}{2}$; the hides weighed in Rotterdam 39370 lb. and my correspondent sells them at 15 guilders for 100 lb.; he deducts for freight, charges, &c. 650 g. $17\frac{4}{5}$ stiv. and for his commission 2 per cent. the neat proceeds he remits to London, at 2s. sterling per guild. for which I draw at 2 per cent above par: What do I gain by the adventure?

400 cwt. 2 qrs. 14 lb. at 5 dols. per					$2003.12\frac{1}{2}$
Duty, &c.					$60.87\frac{1}{2}$
				first cost	$2064.

sales	*g.*		*lb.*		*g. stiv.*
as 100 :	15	::	3937	: *comes*	5905 . 10

charges.

commission at 2 per cent.	$118.2\frac{1}{5}$	
freight, &c.	$650.17\frac{4}{5}$	
		769.
		5136.10 *nt. pro.*

to London, at 2s. per = 20546 *sixpences* ÷ 9 = $$2282.88\frac{8}{9}$ —

this at 2 per cent. above par = $$2328.54\frac{1}{2}$ *the value of my draft.*

first cost 2064.

$$264.54\frac{1}{2}$ *gain, Ans.*

20. Shipped to Bourdeaux 200 barrels flour, at 5 dollars per barrel, 600 cwt. 2 qrs. butter, at \$5.50 per cwt.; pays duty, porterage, &c. \$200.25; my correspondent sells the flour at 32 liv. per barrel, and the butter at 20 liv. the 100 lb. (wt. there 67850 lb.) deducts for freight, charges, &c. 1500 liv. and 2 per cent. for his commission; the neat proceeds he remits to London, at 16*d.* sterling per livre, for which I draw at par: Do I gain or loose by the adventure? *Ans.* \$851.25 *gain.*

21. Shipped for Lisbon 500 barrels wheat, at \$6 per barrel, 110 cwt. 2 qrs. 14 lb. butter, at \$7 per cwt. 400 tanned ox hides wt. 106 cwt. 3 qrs. at $12\frac{1}{2}$ cents per lb.; paid duty, charges, &c. \$150, insurance on the entire at 10 per cent. By my factor's bill of sales, the wheat measured 5000 alquires, at 600 reas per alq.; the butter weighed 387 arobes and 6 lb. at $2\frac{1}{2}$ milreas the arobe; the leather weighed 373 arobe 20 lb. at 200 reas per lb. (arobe=32 lb.) he charges for freight, duty, charges, &c. 832 m. $193\frac{3}{4}$ reas, and 3 per cent. commission. For the neat proceeds, drew at 5 per cent. above par: What is the gain or loss on the adventure? *Ans.* \$1044.10 *gain.*

22. A merchant in Dublin ships to Ostend 10 ton 17 cwt. 3 qrs. tallow, at £30 10*s.* 9*d.* per ton; pays duty, &c. £25 10*s.* and premium of insurance £55 10*s.* The tallow weighs at Ostend 24388 lb. and sells at 18 guilders the 100 lb.; the factor pays freight, duty and other charges 395 guilders, 10 stivers, 4 pen. and reckons for his commission 2 per cent.; the neat proceeds he remits to London, at 33*s.* 9*d.* Flemish per £ sterling, and the Irish merchant draws it from thence, at 4 per cent. advance: Does he gain or lose by the adventure?

Ans. he loses £32 15 $1\frac{866429}{1350000}$.

Partnership, or Company.

The intent of this rule is principally to divide the gain or loss that may arise in partnership (when the stocks are unequal) proportionably among the partners.

CASE I.

The gain or loss, with the several sums at hazard, given to find the proportion each partner is to have.

Rule.—Add the several stocks into one sum—then it will be, as the whole amount is to the entire gain or loss, so is each particular share to the proportion of gain or loss.

EXAMPLE.

1. Two mer. join stocks; A put in \$1000; B put in \$1500; they gain \$700, which they agree to withdraw from the common stock: What is the proportion each is to receive?	1000 1500 —— *as* 2500 : 700 :: 1000 *comes* \$280 *as* 250 : 700 :: 1500 *to* \$420

2. A and B have gained in trade £182; A put in £300, and B £400: I demand each man's share of the profit.

Ans. A £78, B £104.

3. Two persons are to share £100, of which B is to have $\frac{1}{2}$, and C $\frac{1}{4}$: I demand what is each man's particular share.

Ans. B $66\frac{2}{3}$, C $33\frac{1}{3}$.

4. A merch't being deceased, 'tis found he owes to A £500, to B £900, though he left but £1100 behind him: I demand how much each is to have in proportion to his debt?

Ans. A £$392\frac{6}{7}$, B £$707\frac{1}{7}$.

5. Three merc'ts load a ship with corn; A puts in 200, B 300, and C 400 barrels; at sea the master is obliged, by storm, to heave 180 barrels overboard: Tell me how many barrels each merchant ought to lose. *Ans.* A 40, B 60, C 80.

6. Two cities, Cork and Dublin, distant, by computation, say 100 m.; A sets out from Dublin towards Cork, at the same time B sets out from Cork towards Dublin; A goes 12 miles, B goes 8 miles a day: I demand how many miles each has travelled when they meet? *Ans.* A 60 *m.* B 40 *m.*

7. Three persons made a company; A put in 150, B 260, and they gained £65 sterling, whereof C's share came to £19: I would know A and B's part of the gain, and how much C put in.

Answer. A £16 16 $7\frac{1}{41}$, B 29 3 $11\frac{40}{41}$, 169 6 $11\frac{11}{23}$ *put in by* C.

8. Three persons join stocks; A and B put in certain sums; C put in £1090; they gain £110, whereof A took £35, and B £29: Tell me how much A and B put in, and C's share of the gain.

Answer. A *put in* £829 6 $11\frac{11}{23}$, B *put in* 687 3 $5\frac{17}{23}$, C *gains* 46

CASE II.

In estimating bankruptcies, considerable labour will arise, calculating the dividends by the foregoing methods: to obviate which, follow this

RULE.—Find the proportion of £1 or \$1, according to the rule—this proportion will be a multiplier for the several creditors.

EXAMPLE.

9. A merc't failing in trade, delivers up his accounts as follows: cash, $375; wares, $720; good book-debts, $3654.50. He owes as follows: to George Loftus, $1000; Henry Jackson, $760; James Thomson, $3600; Geo. Adams, $250; John Jones, $375; and James Wells, $800: What part must each creditor receive of the estate?

Drs. cash,	375	Crs.	Loftus,	1000
wares,	720		Jackson,	760
good debts,	3654.50		Thomson,	3600
			Adams,	250
	4749.50		Jones,	375
			Wells,	800
				$6785

Then as 6785 : 4749.50 :: 1 to .70 cents per dollar.

Then 1000 × .70 =	700.00	Loftus'	Dividend.
760 × .70 =	532.	Jackson's	
3600 × .70 =	2520.	Thomson's	
250 × .70 =	175.	Adams'	
375 × .70 =	262.50	Jones'	
800 × .70 =	550.	Wells'	
	4749.50		

Let the pupil work this example after the usual method.

CASE III.

When stocks are considered in respect of time:

RULE.—Multiply each man's stock by its time, and the products add together; then say, as the whole stock and time is to the whole loss or gain, so is each man's particular stock and time, to each man's particular loss or gain.

EXAMPLE.

10. Two merchants made a company; A put in £100 for 4 months, and B £136 for three months, and they gain £50: I demand each man's share of the profit.

100 × 4 = 400
136 × 3 = 408

808

as 808 : 50 :: 400 to £24 $15\frac{5}{101}$ } *Answer.*
Second, as 808 : 50 :: 408 to £25 $4\frac{96}{101}$ }

11. Three persons hired a piece of land for £12 10 6; A put in 20 sheep for 5 days, B put in 16 for 7 days, and C put in 25 for 4 days: I demand how much each must pay.

Ans. A £4 0 $3\frac{6}{13}$, B £4 9 $11\frac{1}{13}$, C £4 0 3 $\frac{6}{13}$.

12. Three merc'ts put in a stock; A puts in the 1st of January £120 till the 23rd of March; B puts in the 10th of February £176 till the 12th of April; C put in the 2nd of February £295 till the 25th of April, and they gain £800: I want to know each man's part of the gain, allowing 28 days to February.

Answer. { A £175 16 $11\frac{20502}{44766}$.
B 191 17 $2\frac{16784}{44766}$.
C 432 5 $10\frac{7500}{44766}$.

13. Four persons hired a coach to go 24 miles for 19 *s.* but being come within 9 miles of the place, there came two persons who desired to come into the coach, on condition of paying proportionably: Tell me how much each person is to pay.

Ans. 4 persons are to pay 16*s. and* 2 *persons* 3*s.*

14. Four persons hired a boat to go 50 miles for 40 shillings; now, when they were gone 20 miles, they met two persons who desired to come into the boat, with condition to pay proportionably: How much is each party to pay?

Ans. 30*s.* $9\frac{3}{13}$*d. for* 4 *persons, and* 9*s.* $2\frac{10}{13}$*d. for* 2 *persons.*

Alligation

Is that rule whereby we resolve questions concerning the mixing of several simples or commodities into one compound quantity.

Alligation is either Medial or Alternate.

Alligation Medial is, when having the several quantities and rates of divers simples proposed, we discover the rate of a mixture compounded of these simples.

Rule.—*Find, accord'g to the given rates, the value of each quantity, and then taking the sum of these quantities, and the sum of their value, say,*

as the sum of values : sum of q'ties :: q'ty proposed : Ans.

EXAMPLE.

1. A vintner mixed $31\frac{1}{2}$ glls. malaga sack worth 7*s.* 6*d.* per gallon, 18 gallons canary, at 6*s.* 9*d.* $13\frac{1}{2}$ gallons sherry wine at 5*s.* and 27 gallons white wine at 4*s.* 3*d.* per gallon: What is one gal. of this mixture worth?

galls.		*s.*	*d.*	*s.*	*d.*
$31\frac{1}{2}$	*at*	7	6	236	3
18		6	9	121	6
$13\frac{1}{2}$		5		67	6
27		4	3	114	9

90 *galls.*) : 540 :: 1 *gal.*

Ans. 6*s. per gal.*

2. There are melted and mixed together, two sorts of silver; 4 ounces at 5 shillings, and 8 ounces at 4 shillings per oz.: What is the value of 1 oz. of this mixture? *Ans.* 4*s.* 4*d. per oz.*

3. A goldsmith melts 8 lb. $5\frac{1}{2}$ oz. of gold bullion, 14 carats* fine, with 12 lb. $8\frac{1}{2}$ oz. of 18 carats fine: How many carats fine is this mixture? *Ans.* $16\frac{51}{127}$ *carats fine.*

Alligation Alternate,

Is, when we have the several ingredients to be mixed, and the mean rate of the mixture given to find such quantities of the simples as, being mixed together, will bear the common rate.

RULE.—*The rates being all reduced to one denomination, set down in a column under each other, and the mean rate which the mixture is to bear to the left hand of these.*

Connect, or link the rates, so that every one less may be linked to one greater than the mean rate. Take the difference between the rate and the several simples, and write it over against all the simples with which that one whose difference it is is linked; then the sums of these differences standing against every simple rate, are such quantities of the several simples against which they stand, as answer the question.

EXAMPLE.

1. A merchant would mix wines at 14, 15, 19, and 22 shillings per gallon, so that the mixture should stand him in 18 shillings the gallon: What quantity of each sort must he take?

18	14	4 at	14	per gallon
	15	1	15	
	19	3	19	
	22	4	22	

Or 18	14	1 at	14	per gallon
	15	4	15	
	19	4	19	
	22	3	22	

Or 18	14	4+1=5 at	14	per gallon.
	15	1+4=5	15	
	19	3+4=7	19	
	22	4+3=7	22	

Proved by alligation medial.

4 at	14	comes	56
1	15		15
3	19		57
4	22		88

As 12 gallons cost 216 : 1 comes 18 shillings, proof.

* NOTE.—With refiners, a carat is not a fixed quantity, but the 24th part of any quantity: thus we suppose an ounce of gold divided into 24 parts, and an ounce of copper divided into 24 parts, if 22 parts of gold, and 2 parts of alloy, be added, it is still an ounce, but only 22 carats fine, &c.

NOTE.—*Besides the different answers produced by the different methods of linking the simple prices, questions in Alligation (being of that kind algebraists term unlimited problems) have an infinite variety of other answers; for any other numbers in proportion to those found by this rule (as above) will answer the question.*

2. How much rye at 3s. the bushel, barley at 4s. and oats at 2s. will make a mixture worth 2s. 6d. per bushel?

Ans. $\frac{1}{2}$ *at* 4s. $\frac{1}{2}$ *at* 3s. *and* 2 *at* 2s.

3. A vintner would make a mixture of malaga worth 7s. 6d. per gallon, with canary at 6s. 9d. sherry at 5s. and white wine at 4s. 3d. per gallon: What quantity of each must be taken that the mixture may be sold for 6s. per gallon?

Ans. 12 *gallons malaga*, 21 *canary*, 18 *sherry*, 9 *white wine.*

4. A Goldsmith has gold of 17, 18, 22 and 24 carats fine: How much must he take of each sort, to make it 21 carats fine?

Ans. 3 *at* 17, 1 *at* 18, 3 *at* 22, 4 *at* 24 *carats fine.*

Alligation Partial.

The particular rates of the ingredients proposed to be mixed, the mean rate of the whole mixture, and any one of the quantities to be mixed, given, to find how much of every one of the other ingredients are requisite to compose the mixtures.

RULE.—*Set down all the particular rates, and find their difference; then say, As the difference standing against the price of which the quantity is given, is to the said given quantity, so is each other difference to the quantity required.*

EXAMPLE.

1. How much water must be mixed with 63 gallons of brandy of 5s. 5d. the gallon, to reduce it to 4s. 6d. per gallon?

$$4s.\ 6d. = 54 < \begin{matrix} 0 & \text{—}11 \\ 65 & \text{—}54 \end{matrix}$$

gals. *gals.*

then as 54 : 63 :: 11 : 12$\frac{5}{6}$ *water, Ans.*

2. How much brass of 14d. per lb. pewter of 10$\frac{1}{2}$d. must I mix with 50 lb. copper worth 16d. the pound, that the whole mixture may stand me in 12d. per lb.? *Ans.* 50 *lb. at* 14d. & 200 *at* 10$\frac{1}{2}$d.

3. With 60 gallons of brandy, at 6s. per gallon, I mixed brandy at 5s. 4d. per gallon, and some water; then I found it stood me in 3s. 6d. per gallon: I demand how much water and brandy I took? *Ans.* 74$\frac{6}{7}$ *water,* 60 *gallons brandy at* 5s. 4d.

4. How much malaga of 7s. 5d. sherry at 5s. 2d. white wine at 4s. 2d. the gallon, must be mixed with 20 gallons of canary at 6s. 8d. the gallon, so that the mixture may stand in 6s. the gallon? *Ans.* 34 *at* 4s. 2d. 16 *at* 5s. 2d. *and* 44 *at* 7s. 5d. *per gal.*

5. How much alloy, and how much gold of 21, and 23 carats fine, must be put to 30 oz. of 20 carats fine, to bring it to 18 carats fine? *Ans.* $16\frac{2}{3}$ *oz. alloy,* 18 *oz. at* 21, *and* 18 *oz. at* 23 *ca. fine.*

O *stands for alloy, or water.*

Alligation Total.

The particular rates of all the ingredients proposed to be mixed, the sum of all their quantities, with the mean rate of that sum being given, to find the particular quantities of the mixture.

RULE.—*Set down all the particular rates with the mean rate as before, and find their differences; add all the differences into one sum; then say, as the sum of all the differences, is to the sum of all the quantities given, so is each particular difference to its particular quantity required.*

EXAMPLE.

1. A goldsmith has two sorts of silver bullion; the one of 10 oz. the other of 5 oz. fine, and has a mind to mix a pound of it so that it may be 8 oz. fine: How much of each sort must he take?

$$8\begin{cases}10 & 3\\ 6 & 2\end{cases}$$

$$\overline{5}$$

$$\textit{then as } \overset{oz.}{5} : 12 :: \begin{cases}3 : to\ 7\frac{1}{5}\\ 2 ; to\ 4\frac{4}{5}\end{cases}\overset{oz.}{} \begin{matrix}at\ 10\ oz.\\ at\ 5\ oz.\end{matrix}$$

2. A vintner has 3 sorts of wine, viz. of 24, 22 and 18*d.* per gallon, and has a mind to mix 60 gallons, so that he may sell it at 20*d.* per gallon: How much of each sort must he take?

Ans. 12 *gal. at* 24*d.* 12 *at* 22*d. and* 36 *at* 18*d.*

3. A goldsmith has 3 sorts of silver, viz. of 11, 8 and 5 oz. fine, and has a mind to make a piece of work that shall weigh 10 lb. of 9 oz. fine: How much of each must he take?

Ans. $5\frac{1}{3}$ *at* 11 *oz. fine,* $2\frac{2}{3}$ *at* 8 *oz. and* $2\frac{2}{3}$ *at* 5 *oz. fine.*

4. A cask of 58 gallons, is filled with liquors at 7, 8, and 10*d.* per gallon; then it stands in $9\frac{1}{4}d.$ the gallon: I would know how many gallons of each sort was taken?

Answer, $\begin{cases}8\frac{7}{10} & at\ 7d\ per\ gallon.\\ 8\frac{7}{10} & 8\\ 40\frac{6}{10} & 10\end{cases}$

5. A druggist has simples of 18, 15, 12, 9, 8 and 5*d.* per lb.: How much of each sort must he take to make up 318 lb. so that they may stand him in 11*d.* per lb.?

Answer, $\begin{cases}82\frac{22}{23} & at\ 18d.\ per\ lb.\\ 43\frac{11}{23} & 15\\ 27\frac{15}{23} & 12\\ 13\frac{19}{23} & 9\\ 55\frac{7}{23} & 8\\ 96\frac{18}{23} & 5\end{cases}$

Single Position,

Or the Rule of False, is so called because by supposed numbers taken at adventure and worked with according to the nature of the question, the true number sought is discovered.

EXAMPLE.

1. There is a cistern with water which has three cocks; when the first cock is opened all the water runs out in an hour; when the second is opened it runs out in two hours; and when the third is opened it runs out in three; but if all the cocks are left open, how long will it be in emptying?

first, $\frac{1}{1}=\frac{6}{6}$

second, $\frac{1}{2}=\frac{3}{6}$

third, $\frac{1}{3}=\frac{2}{6}$

hour. *minutes.*

Ans. as $\frac{11}{6} : \frac{1}{1} :: 6 : 32\frac{8}{11}$.

2. Suppose I set at interest £240, and that in 5 years I receive for principal and interest £300: I demand the rate per cent. *Ans. 5 per cent.*

3. A gamester loses at 4 turns of dice 160 shillings, and trebled each turn the sum he put in: How much did he play for the first and last times? 4*s. the first time,* 108 *the last. Ans.*

4. Peter drinks a cask of beer, quantity 16 gallons, in 6 days; and John, when he goes about it, can do it in 4 days; now if they should both drink together, how long would it stand them? *Ans. 2 days* $9\frac{3}{5}$ *hours.*

5. Suppose I set at interest, a certain sum of money at the rate of 8 per cent. simple interest, and at the end of 10 years I receive both principal and interest, £600: How much was the principal lent out at first? *Ans.* £333 6 8.

6. A master mason can finish a piece of work in $2\frac{1}{2}$ days, his journeyman can do it in $3\frac{3}{4}$ days, and his apprentice can do it in $4\frac{1}{2}$ days: In what time would they finish the work if they all worked together? *Ans.* $1\frac{1}{8}$ *days.*

7. There are 3 mills, the first grinds 3 bushels in 2 hours, the second 5 bushels in 3 hours, the third 7 bushels in 5 hours: In how many hours will they grind 36 bushels? *Ans.* $7\frac{121}{137}$ *hours.*

8. Four merchants, A, B, C and D, have gained £140, which they divide in this manner; that $\frac{1}{3}$, the share of A, is equal severally to $\frac{1}{2}$ the share of B, $\frac{1}{4}$ of C, and $\frac{1}{5}$ of D; the question is, what is each man's share of the gain?

Ans.
30 *A's part.*
20 *B's part.*
40 *C's part.*
50 *D's part.*

Double Position.

Make choice of any number and work with it according to the nature of the question; if your error be more, mark it thus (+); choose another number, and work as before; if it be less, mark it thus (—); multiply the supposed numbers and their errors crosswise, that is, each error into the opposite supposed number; add their products if your errors are more and less, and divide by the sum of the errors—the quotient will be the answer; otherwise, divide the difference of the products by the difference of the error; if both be marked (+) or (—) the quotient will be the answer.

EXAMPLE.

1. A asked B to lend him the 100 crowns he had in his pocket; B answered he had not so much money, but said, if he had ½ as much more, with ¼, ⅛ and ⅙ of the numbers less three, there would be 46; well then, says A, you have only twenty-four crowns.

	suppose	*suppose*	
	12	16	
	6	8	
	3	4	
	1½	2	
	2	2⅔	
	24½	32⅔	
	3	3	
	21½	29⅔	
should be	46	46	*should be*
(—)	24½	16⅓	(—)
	X		
	12	16	
	196	392	
		196	

— 24½
— 16⅓
8⅙ *divisor.*

8⅙) 196 (24 *crowns. Ans.*

2. A man on horseback sets out from a certain town; a coach sets out from another; at a certain distance they meet, the coachman asks the man on horseback, how many miles he had rode: he answered, if he had rode 1½ times as far as he had done, he would have rode 36 miles; well then, says the coachman, I have rode ½ as far and ¼ as far as you: How many miles are the towns asunder? *Ans.* 25⅕ *miles.*

3. A master hires a journeyman, on this condition, that he shall have 12 pence a day for every day he works, but for every day he does not work, he is to pay his master 6 pence per day for his diet. On the 30th day they settle, when neither receives

or pays money: How many days did the journeyman work, and how many was he idle? *Ans. He worked* 10, *and was idle* 20.

4. A departs from Philadelphia at the same time that B departs from New-York, the distance between Philadelphia and New-York, say 100 miles; when they meet, the miles A had travelled being multiplied by $8\frac{3}{4}$, and the miles B had travelled being multiplied by $5\frac{1}{4}$, the difference of their products will be $75\frac{3}{4}$: How much farther has one travelled than the other?

Ans. A *travelled* $33\frac{18}{36}$ *miles more than* B.

5. There are 2 numbers; when you add them, they make 30, the $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ of the one, are equal to $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$ of the other: What are they? *Ans.* 18 *and* 12.

6. If from a piece of cloth, I cut $\frac{2}{3}+11$ yards, then there is left $\frac{1}{2}$ less 17 yards: How many yards were in the piece?

Ans. 36 *yards.*

7. A gentleman has two horses, Chesnut and Swift; and a saddle worth 50 pounds, which set on the back of Chesnut, makes his value double that of Swift; but the saddle set on the back of Swift, makes his value 3 times that of Chesnut: What was the value of each horse? *Ans. Chesnut* £30, *Swift* 40.

Arithmetical Magazine, &c.

PART IV.

Extractions, Progressions, &c.

First.—*Of Involution, or raising of powers.*

If any number be multiplied into itself any number of times, the several products are called *powers of that number*, and are distinguished by *first, second, third, &c. powers;* and the number multiplied by, is called the *root* of each power particularly.

Thus 4 *first power, or root.*
4
—
16 *square, or second power of* 4 *and* 4, *the sq. root of* 16.
4
—
64 *cube, or third power of* 4 *and* 4, *the cube root of* 64.
4
—
256 *biquad. or 4th power of* 4 *and* 4, *the biquad. root of* 256.

5th power, is called sursolid; 6th, square cube; 7th, second sursolid; 8th, square biquadrate; 9th, cube cubed; 10th, sq. sursolid.

The number denoting the power, is called the *index*, or *exponent*, because it shows how often the *root*, or *first power* is to be involved, *thus* $4,^5 = 1064$, *the 5th power of 4 or 4 involved 5 times and 5, the exponent of that power.*

RULE.—*Multiply the given number, root, or power, as often as the exponent contains units less 1 (because the given root or power equals 1st power) the last product will be the power required.*

Thus, $3,^4 = 81$, the 4th power of 3 and 3, the biquadrate root of 81.

Second.—*Evolution, or extraction of roots:*

To find the root of any assigned power is called *evolution*, or *extraction*. The given power whose root is to be extracted, is distinguished by this sign $\sqrt{}$, as $\sqrt[2]{36}$ shows that the square root of 36 is required (the exponent of the square root may be omitted) $\sqrt[3]{216}$, the cube root of 216 is required, &c.

If the power be expressed by several connected numbers, a vinculum should be drawn from the top of the sign over the entire connexion,

thus, $\sqrt[3]{354-3}$ cube root of 354 less 3 is required.

$\sqrt[4]{864+27}$ biquadrate root of 864 more 27 is required.

Square Root.

RULE.—Distinguish the given power into *periods of two figures each*, beginning at unit's place; if there be decimals, the *periods must be pointed off both ways from the separating point.*

Find the *greatest square* in the left hand period, and place the root in the quotient, subtract the square from the period, and to the remainder bring down the next period for a *dividend.*

Double the quotient for a *divisor*, and try how often it will be contained, *putting the trial figure for unit's place in the divisor*, and when found nearest, *put it in the quotient.*

Subtract as before, bring down the next period for a dividend, *double the quotient for a divisor*, and proceed in the trial as directed above.

If a remainder occurs after the last period is brought down, add cyphers and proceed as before, *the quotient thence arising will be a decimal.*

Surd numbers are those whose roots can never be exactly found; but by the addition of cyphers, we may approximate to the truth to a sufficient degree of exactitude.

EXAMPLE.

Required the side of a square whose superficial content is 572199960721 square feet.

	57'21'99'96'07'21'	
7×7=	49 ' ' ' ' '	(756439 *feet*, *Ans.*
145	821	
	725	
1506	9699	
	9036	
15124	66396	
	60496	
151283	590007	
	453849	
1512869	13615821	
	13615821	

Extract the Square Root of	25097636	*Ans.* 4806
	151321 - - -	389
	14122564 - - -	3758
	2985984 - - -	1728
	15437041 - - -	3929
	2990667969 - -	54687

To extract the square root of a fraction:

RULE.—Extract the root of the numerator for a numerator, and the root of the denominator for a denominator.

Thus we find the square root of $\frac{9}{16}$ to be $\frac{3}{4}$—but the numerators and denominators of fractions being generally surds, it is better to reduce the fraction to a decimal, and extract the root of the decimal for the answer.—Thus, $\frac{9}{16}$=.5625, and $\sqrt{5625}$=.75 the answer=$\frac{3}{4}$.

What is the square root of	$\frac{25}{64}$	*Ans.* .625
	$\frac{5}{8}$ - - - - -	.25
	$\frac{7}{8}$ - - - - -	.2958+
	$\frac{1}{8}$ - - - - -	.1118+
	64832.5465	254.622+

Use.

CASE I.

To find a mean proportional between two given numbers:

RULE.—*Multiply the given numbers together, and extract the root of the product for the answer.*

EXAMPLE.

What is the mean proportional between 5 and 20?

$\sqrt{5 \times 20} = 10$ *Ans.*

What is the mean proportional between 4 and 9? *Ans.* 6.

What is the mean proportional between 16 and 36? *Ans.* 24.

CASE II.

Having the content of a superfices or a given figure to find the side of square equal thereto, &c.

RULE.—*Extract the root of the given content for the side of a square equal thereto.*

EXAMPLE.

4. In a square plantation are 23716 trees: How many rows are there, and how many trees in a row? *Ans.* 154.

5. The area of a circle is 4276.5 feet: Required the side of a square equal thereto. *Ans.* 65.395.

6. An eliptical fishpond contains 9 acres, 2 roods, 15 perches: Required the side of a square one to contain the same quantity? *Ans.* 274.2535 *yards.*

7. If the content of a given circle be 160 yards, what is the side of a square equal thereto? *Ans.* 12.649.

CASE III.

The area of a circle given to find the diameter:

RULE.—*As* 355 : 452 :: *area* : *square diameter.*

Otherwise,

Divide the area by the common number .7854, *the quotient will be the square of the diameter.*

8. A horse in the midst of a meadow, suppose, made fast to a stake by a rope from his nose, how long must the cord be, that feeding around, the horse may graze neat just an acre of ground? *Ans.* 39.25 *yard*

9. How many yards must be added to the rope to let the horse graze two acres? *Ans.* 16.25.

CASE IV.

Any two sides of a right angled triangle given to find the other.

1st. Having the base and perpendicular given to find the hypothenuse, or longest side:

RULE.—*Square the given sides—add their squares, and extract the square root of their sum for the hypothenuse, or side required.*

2nd. Having the hypothenuse, and one of the other sides given to find the other:

RULE.—*Square the given sides as before, and extract the square root of their difference for the side required.*

EXAMPLE.

A castle 45 yards high, is surrounded by a fosse 60 yards broad: What length must a ladder be to reach from the outside of the fosse to the top of the wall? *Ans.* 75 *yards.*

Suppose a light-house built on the top of a rock, the distance between the place of observation, and that part of the rock level with the eye, and directly under the building is given 310, the distance from the top of the rock to the place of observation 423, and from the top of the building 425 fathom; the height of the lighthouse is required? *Ans. height of the rock* 287.8 *fathom.*
of the house 2.9315+ *do.*

A ladder 40 feet long, reaches a window 33 feet from the ground on one side of the street; but if raised, and laid against the opposite house without moving the foot, it will reach a window 21 feet from the ground; required the breadth of the street. *Ans.* 57.08 *feet.*

Let the foregoing questions be transposed by the pupil for the application and elucidation of the rule.

Cube Root.

A Cube is a figure whose length, breadth and thickness are equal: Or,

It is that power of a number which was involved three times, thus, $4^3=64$ the Cube and 4 the Cube Root of 64, &c.

To extract the Cube Root:

RULE 1.—Point off the given power into periods of three figures each from unit's place.

2. Write the next greatest Cube under the left hand period, and note the Root for the first figure of the quotient or required Root.

3. Subtract, and to the remainder annex the next period, this call your Resolvend.

To find a Divisor:

RULE.—Multiply the square of the quotient by 300 for a Divisor, see how often it is contained in the Resolvend—this figure must find the three following numbers:

First—The product of the Divisor.

Second—The product of its square and the foregoing part of the quotient multiplied by 30.

Third—Its Cube.—The sum of these three numbers is the Subtrahend, which subtract from the Resolvend; to the remainder, annex the next period and proceed as before.

If it be not an exact Cube, annex periods of cyphers to the remainder; proceed in all respects as before; the part of the Root thence resulting will be a decimal.

Required the Cube Root of 9302348

```
                         9'302'348'      (210.312 Ans.
                     2³=8
                         ----
                         1302=1st Resolvend.
              divisor.   ----
      2,²×300=1200=1200        (1
      1,²×2×  30=    60
          cube of 1=  1
                         ----
                         1261=subtrahend.
                         ----
                          41,348,000=2d. and 3d. resolv.
            divisor.      ----------
    21,²×300=132300,00=39690000        (0,3
    .3,²×210×   30=       56700
                3,³=         27
                          --------
                          39746727   subtrahend.
                          --------
                           1601273,000=4th resolvend.
            divisor.       ----------
  2103,²×300=1326782700=1326782700   (1
     1,²×2103×   30=        63090
                1,³=            1
                           ----------
                           1326845791   subtrahend.
                           ----------
                           274427209,000=5th resolvend.
              divisor.     -------------
21031,²×300=132690888300=265381776600   (2
      2,²×21031×30=           2523720
                2,³=                8
                           ------------
                           265384300328=5th subt'nd.
                           ------------
                              9042908672 remainder.
```

ELUCIDATION.

After deducting the Cube 8 and annexing the next period, we find the Resolvend 1302, then to find a Divisor, take the quotient figure 2×2=4×300=1200 Divisor, which being found once I place under the Resolvend;

Then 1 × 1 = 1 × 2 (the foregoing part of the quotient) = 2 × 30 = 60, which I place in order under the Resolvend, and, lastly, the Cube of 1. The sum of these three numbers, viz, 1261, I take from the Resolvend 1302, and there remains 41, to which I annex the next period, viz, 348 = 41348, second Resolvend; then I find a Divisor as before, viz, 21 × 21 × 300 = 132300 Divisor, but I find it is not contained in the Resolvend, therefore I put 0 in the Quotient, annex a period of cyphers to the Resolvend and two cyphers to the Divisor, (because 210 × 210 × 300 = 13230000 Divisor) and find it contained 3 times—the number thence resulting is a decimal.

Required the Cube Root of	41278242816	*Ans.* 3456
	5859375	180.28
	164566592	548.
	705919947264	8904.
	673373097125	8765.
	41421736	346.
	146363183	527.

To extract the Cube Root of a Fraction:

Reduce the fraction to a decimal, and proceed as if it was a whole number.

Required the Cube Root of $\frac{27}{64}$, *Ans.* .75
$\frac{1}{2}$, - - .793+
$\frac{1}{4}$, - - .63+

The root of some of the higher powers is attainable without much labour; of others the task is arduous without a knowledge of Algebra. As algebraic elucidation is useless to a student in arithmetic, we shall dispense with those parts which depend on it for solution.

To extract the root of the bequadrate or fourth power, extract the square root; then extract the square root of that root, for the biquadrate root required.

Square Cube, or sixth power:

Extract the square root of the given power, then extract the cube root of the root found, for root required.

Square Biquadrate, or eighth power:

Extract the square r. = 4th power = extract the square root of that root for the second power or square; then extract the square root of the last root found for the root required.

Cube Cubed, or ninth power:

Extract the cube root of the given power, then extract the cube root of the last found root for the root required.

By applying to the problematical part of geometrical progression, this will be clearly understood, and the pupil may be directed to higher extractions if the teacher thinks necessary.

The Use of the Cube Root.

CASE I.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

Rule.—Extract the cube root of the solid content of the given body, which root will be the side of the cube required.

EXAMPLE.

1. There is a stone of a cubic form, which contains 21952 solid feet: What is the superficial content of one of its sides?

Ans. 28 *feet.*

CASE II.

Having the dimension of any solid body, to find the dimensions of another similar solid, that shall be any number of times greater or less than the solid given.

Rule.—Multiply the cube of each side by the difference between the solid given and that required, if greater (or divide by the difference if less) than the solid given; then extract the cube root of each product or quotient, which will give the dimensions of the solid required.

2. Suppose the length of a ship's keel to be 125 feet, the breadth of the midship-beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship of the same form, that will carry three times the burthen.

Ans. { *Length of the keel,* 180. 28 *feet.*
Breadth of the beam, 36. 05
Depth of the hold, 21. 63 }

3. Again, I demand the dimensions of another ship of the same form, that shall be only half the burthen of that whose dimensions are given as above.

Ans. { *Length of the keel,* 99. 21 *feet.*
Breadth of the beam, 19. 84
Depth of the hold, 11. 9 }

CASE III.

Having the dimension and capacity of a solid, to find the dimensions of a similar solid of a different capacity.

Rule.—Like solids are in triplicate proportion to their homologous sides, therefore it will be, as the cube of a dimension : is to its given weight :: so is the cube : of any like dimension to the weight sought.

EXAMPLES.

4. If a ship of 300 tons burthen be 75 feet long in the keel, I demand the burthen of another ship, whose keel is 100 feet long. *Ans.* 711. 111 *tons.*

5. Suppose a ball of 4 inches diameter weighs 18 lb. I demand the diameter of another that weighs 114 lb.

Ans. 7. 4 *inches.*

CASE IV.

To find two mean proportionals between two given numbers:

RULE.—Divide the greater extreme by the less, and the cube root of the quotient, multiplied by the less extreme, gives the lesser mean; multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

EXAMPLES.

6. What are the two mean proportionals between 7 and 189?

Ans. 21 *and* 63.

7. Find two mean proportionals between 4 and 256.

Ans. 16 *and* 64.

Arithmetical Progression.

ANY rank or series of numbers increasing or decreasing, by a common difference, is said to be in arithmetical progression.

Any series increasing by a common difference, is called an ascending progression:

As 1, 3, 5, 7, 9, 11, &c.

And any number decreasing by a common difference, is called a descending progression:

As 11, 9, 7, 5, 3, 1.

The numbers which form an arithmetical progression, are called the terms of the progression; and the number whereby one term exceeds, or is deficient of another, is called the common difference.

The first and last terms of a progression, are called the extremes; and the other terms, the means.

From the foregoing progressions, it is easy to conceive, that an increasing arithmetical progression may be continued to infinity; but a decreasing progression cannot be continued farther than till the last term becomes less than the common difference.

In an arithmetical progression these five particulars are to be noted:

1st. The first term.
2nd. The last term.
3rd. The common difference.
4th. The number of terms.
5th. The sum of the series.

From any three of which being given the rest may be found.

Numbers in arithmetical progression have sundry peculiar properties, some of which are as follows:

PROPOSITION 1. In any increasing series, if the first term be added to the product of the common difference, multiplied by the number of terms less 1, the sum will be the last term.

EXAMPLE.

Given 2 the first term, 12 the number of terms, and the common difference, to find the last term.

$12-1=11\times2+2=24$, *the last term.*

In any decreasing series, if the said product be subtracted from the first, the remainder will be the last term.

$24-11\times2=2$, *last term.*

PROPOSITION 2. If odd numbers compose an arithmetical progression, twice the mean or middle numbers will equal the sum of the extremes:

Thus, 1, 3, 5, 7, 9, 11, 13.
Now $7\times2=14$, *and* $1+13=14$.

But if even numbers compose an arithmetical progression, the sum of the means is equal to the sum of the extremes.

3, 6, 9, 12, 15, 18, 21, 24.
Now $12+15=27$, *and* $3+24=27$, *or* $6+21=27$.

Therefore, a progression whose terms are odd, the mean term being doubled, is equal to the sum of any two terms equally distant therefrom; and if the terms be even, the sum of the means are equal to the sum of any two terms equally distant therefrom.

Hence, if any two numbers be added, and their sum halved, that half is an arithmetical mean between the said numbers.

PROBLEM I. Having the first term, common difference, and number of terms, to find the last term:

RULE.—*Multiply the number of terms less 1 by the common difference—to that product add the first term, the sum is the last term.*

EXAMPLE.

1. A man bought 40 yds. of cloth, at 1*s*. for the first, 3*s*. for the second, &c.: How many shillings did he pay for the 40th?

$40-1=39\times2$, *common dif.* $+1$, *first term* $=79s$. *Ans.*

NOTE.—If the first term equals the common difference, the number of terms multiplied by the common difference will produoe the last.

PROBLEM II. Having the first term, the last term, and number of terms, to find the sum of the series.

RULE.—*Add the first and last terms together—multiply half the sum by the number of terms, or the whole sum by half the number of terms, the product will be the sum of the series.*

EXAMPLE.

2. Bought 30 yds. of ribband; for the first I gave a penny, for the last 30*d.*: How much did the 30 yards cost?

$1+30=31\times 15$, *half the terms*$=465$ *pence*, £1 18 9 *Ans.*

PROBLEM III. Having the first term, the common difference, and number of terms, to find the sum of the series.

RULE.—*Find the last term by problem* 1*st*, *and the sum of the series by problem 2nd.*

EXAMPLE.

3. Bought 35 yards of silk, at 1*s.* for the first, 3 for the second, 5 for the third, &c.: How much is paid for the silk, and how much lost, if the silk be valued at 24*s.* per yard?

$35-1\times 2+1=69$, *last term.*
And $69\times 17\frac{1}{2}$, *half number of terms* $=1207\frac{1}{2}$*s.* $=$ £60 7 6
And 35 *yards, at* 24*s.* - - - - - - - - - 42

Ans. loss £18 7 6

4. How many strokes doth a regular clock strike in a natural day, or 24 hours? *Ans.* 156.

5. A man buys 17 yards of Kersey; for the first yard he gave 2 shillings, for the last 10*s.* the price of each yard increasing in an arithmetical progression: How much did the whole amount to? *Ans.* £5 2*s.*

6. How many strokes doth the clocks of Venice (which go on to be 24 o'clock) strike in the compass of a natural day? *Ans.* 300.

7. The length of my garden is 94 feet; now if eggs be laid along the pavement 1 foot asunder, and be fetch'd up singly to a basket, removed 1 foot from the last, how much ground must he traverse that does it?

Ans. 1 *mile*, 5 *fur.* 21 *poles*, $3\frac{1}{2}$ *feet English.*

8. A merchant hires a clerk by covenant for 14 years, to give him £5 the first year, and raise his salary 40*s.* a year during the term: The question is, to discover how much he paid him one year with another on an average? *Ans.* £18.

9. Supposing a press-gang having a warrant to press for 30 days, press the first day 300 men, and every succeeding day 10 more than the former: How many men will they raise in 30 days? *Ans.* 13350.

Annuities in Arrear.

By this rule, interest on annuities in arrear may be easily calculated; for the several interests form an arithmetical progression, of which the last interest due may be taken as the first term and common difference; and the number of years, half-years, or quarters less one, the number of terms; because there is no interest due upon the last annuity, it being only due.

EXAMPLE.

1. If the payment of an annuity of £70 be forborne 5 years, what will be due at the end of the term, simple interest being computed at 5 per cent.?

£70 at 5 per cent. comes	£3	10	0	*interest and first term.*
5 years — 1 =			4	
	14	0	0	*last term.*
	17	10	0	*sum of first and last.*
			2	*half the number of terms.*
	35	0	0	*int. or sum of the series.*
£70 for 5 years =	350			
Amount due,	£385	0	0	*Answer.*

But if interest be allowed on half-yearly, or quarterly payments, the advantage will be still more in favour of the receiver.

EXAMPLE.

2. If the above annuity was payable half-yearly, what was the amount? *Ans.* £398 7 6.

3. If interest be allowed on quarterly payments, what would the above annuity have amounted to? *Ans.* £391 11 3.

4. If the payment of a pension be omitted for 7 years, what will it amount to in that time, supposing the annuity £56, at 6 per cent. simple interest? *Ans.* £462 11 $2\frac{2}{5}$.

The foregoing propositions and problems being of real use in arithmetic, I have given them separate. The following problems are here subjoined, for the use of those who wish to investigate the peculiar properties of Progression.

Problem IV. Given the first term, the last term, and number of terms, to find the common difference.

Rule.—*Divide the difference of the extremes by the number of terms less 1—the quotient will be the common difference.*

EXAMPLE.

Given 3, the first, 39 the last, and 19 the number of terms: Required the common difference.

39 − 3 = 36, and 19 − 1 = 18, then 36 ÷ 18 = 2 common dif.

PROBLEM V. Given the first term, the last term, and common difference, to find the number of terms.

RULE.—*Divide the difference of the extremes by the common difference—the quotient + 1, will be the number of terms required.*

EXAMPLE.

Given the first, 7 the last, 51 common, excess 4: Required the number of terms.

51 − 7 = 44 ÷ 4 = 11 + 1 = 12, *number of terms, Ans.*

PROBLEM VI. Given the extremes and common difference, to find the sum of the series.

RULE.—*Multiply the sum of the extremes by their difference, increased by the common difference, and the product divided by twice the common difference will be the sum of the series.*

EXAMPLE.

Given 3 and 39, extremes, 2 common difference: Required the sum of the series.

39 + 3 = 42, *and* 39 − 3 = 36, *then* 42 × 36 + 2 = 1596,
and 1596 ÷ 2 × 2 = 399, *sum, Ans.*

PROBLEM VII. Given the extremes and sum of the series, to find the common difference.

RULE.—*Divide the product of the sum, and difference of the extremes by the difference of twice the sum of the series and the sum of the extremes—the quotient will be the common difference.*

EXAMPLE.

Given 3 and 39, extremes, and 399 the sum: Required the common difference.

39 + 3 = 42 *sum, and* 39 − 3 = 36 *difference,*
then 42 × 36 = 1512 ÷ (399 × 2 − 42) = 756 = 2 *com. dif. Ans.*

PROBLEM VIII. Given the extremes and sum of the series, to find the number of terms.

RULE.—*Divide twice the sum of the series by the sum of the extremes—the quotient will be the number of terms.*

EXAMPLE.

Given 7 and 51, extremes, 348 sum: Required the number of terms.

348 × 2 = 696 ÷ 7 + 51 = 12, *number of terms, Ans.*

Problem IX. Given first term, common difference, and sum of the series, to find the number of terms.

Rule.—*To the square of the difference of twice the first term, and the common difference, add the product of the sum, and the common difference multiplied by 8, and extract the square root of the sum; from the root take twice the first term less the common difference; divide the remainder by twice the common difference—the quotient will be the number of terms.*

EXAMPLE.

Given 3 the first term, 2 common difference, 399 sum of the series, to find the terms.

$3\times2=6-2=4$, *and* $4^2=16$, *square of difference; then sum* 399×2 *com. dif.* $\times8=6384$, *and* $6384+16$, *sq. of dif.* $=6400$; *then* $\sqrt{6400}=80$, *root; then* $80-4$, *twice first term less common dif.* $=76\div4=19$, *number of terms, Ans.*

Problem X. Given the first term, the common difference, and sum of the series, to find the last term.

Rule.—*To the square of the difference of twice the first term and common difference, add the product of the sum and common difference multiplied by 8, and extract the square root of their sum, from which root take the common difference, the remainder divided by 2 will give the last term.*

Given 3 the first term, common difference 2, sum of the series 399, to find the last term.

$3\times2=6-2=4$ *and* $4^2=16$ & $399\times2\times8=6384+16$ *the sq.* 6400 *then* $\sqrt{6400}=80$ *and* $80-2\div2=39$ *the last term.*

Problem XI. Given the first term, number of terms, and sum of the series, to find the common difference.

Rule.—*From the sum subtract the product of the extremes, twice the remainder divided by the square of the terms less the number of terms, will give the common difference.*

Given 399 sum, first term 3, number of terms 19, to find the common difference.

$399-3\times19=342$ *and* $342\times2=684\div19^2-19=2$ *the com. diff.*

Problem XII. Given the first term, number of terms, and sum of the series, to find the last term.

Rule.—*Divide twice the sum by the number of terms; from the quotient subtract the first term, the remainder will be the last.*

Given 3 the first term, 19 number of terms, 399 sum: Required to find the last term.

$399\times2=798\div19=42$ *and* $42-3=39$ *number of terms.*

Problem XIII. Given the common difference, last term, and sum of the series, to find the first.

Rule.—*From the square of twice the last term more the common difference, take 8 times the product of the sum and common difference; the square root of the remainder, either added to, or subtracted from, the common difference* (as the case may require) *and divide by 2, will give the first term.*

EXAMPLE.

Given 39 the last term, 399 sum of the series, 2 common difference, to find the first term.

$39\times2+=80$ *and* $80^2=6400$, *then* $399\times2\times8=6384$
and $6400-6384=16$, *and* $\sqrt{16}=4$ *the square root of* 16
then $4+2=6\div2=3$ *the first term*, &c.

Problem. XIV. Given the common difference, last term, and sum of the series, to find the number of terms.

Rule.—*From the square of twice the last term more the common difference, subtract 8 times the product of the sum and common difference, the square root of the remainder added, or subtracted from twice the last term more the common difference, as the case may require, and the sum, or remainder, divided by twice the common difference, will give the number of terms.*

EXAMPLE.

Given 2 common difference, 39 last term, sum 399, to find the number of terms.

last $39\times2+2=80$ *and* $80^2=6400$ *then* $399\times2\times8=6384$

and $\sqrt{6400-6384}=4$, *then* $39\times2+2=80$ *and* $80-4=76$
then $76\div4=$ *twice the common difference* $=19$ *the number of terms.*

Problem XV. Given the last term, number of terms, and sum of the series, to find the first term.

Rule.—*Divide twice the sum, by the number of terms—from the quotient, subtract the last, the remainder will be the first term.*

EXAMPLE.

Given last term 48, number of terms 10, sum of the series 255: Required the first term.

$255\times2\div10=51-48$ last term $=3$ first term. *Ans.*

Problem XVI. Given the last term, number of terms, and sum of the series, to find the common difference.

Rule.—*From the product of the last term, and number of terms, subtract the sum of the series—twice the remainder, divided by the square of the number of terms, less the number of terms, will be the common difference.*

EXAMPLE.

Given 65 the last term, 16 number of terms, 560 sum of the series, to find the common difference.

$-65\times16=1040-560=480\times2=960$

and $960\div16^2-16=4$ *common difference. Ans.*

Problem XVII. Given the last term, number of terms, and common difference, to find the first term.

Rule.—*Multiply the number of terms less 1 by the common difference, and subtract the product from the last term, the remainder will be the first term.*

EXAMPLE.

Given 65 the last term, 16 number of terms, 4 common difference, to find the first term.

$16-1=15\times4=60$ and $65-60=5$ *the first term. Ans.*

Problem XVIII. Given the last term, number of terms, and common difference, to find the sum of the series.

Rule.—*Multiply the number of terms less 1, by half the common difference, subtract this product from the last term, the remainder multiplied by the number of terms will be the sum of the series.*

EXAMPLE.

Given 65 the last, 16 the number of terms, and 4 the common difference, to find the sum of the series.

$16-1=15\times2=$ *half common difference* $=30$

then $65-30=35$ *and* $35\times16=560$ *sum of the series.*

Problem XIX. Given the common difference, number of terms, and sum of the series, to find the last term.

Rule.—*Divide the sum of the series by the number of terms, to this quotient add the product of the number of terms less 1, multiplied by half the common difference, the sum will be the last term.*

EXAMPLE.

Given 4 common difference, 16 number of terms, and 560 sum of the series, to find the last term.

$560\div16=35$, *and* $16-1=15\times2$ *half common difference* $=30$ *then* $30+35=65$ *the last term.*

Problem XX. Given the common difference, number of terms, and sum of the series, to find the first term.

Rule.—*Divide the sum of the series by the number of terms, from this quotient subtract the product of the number of terms less 1, multiplied by half the common difference, the remainder will be the first.*

EXAMPLE.

Given 4 common difference, 16 number of terms, 560 sum of the series, to find the first term.

$560\div16=35$ *and* $16-1=15\times2$ *half the common difference* $=30$ *then* $35-30=5$ *first term required.*

Geometrical Progression.

When any rank or series of numbers increase by a common multiplier, or decrease by a common divisor, they are said to be in Geometrical Progression, or proportion continued; and the common multiplier or divisor is termed the ratio of the progression.

The numbers which compose the series, are called the terms of the progression, and

The first and last numbers are called the extremes.

Thus 2, 4, 8, 16, 32, 64, 128, are a geometrical series, where 2 is the ratio, and 2 and 128 the extremes.

If over a Geometrical Progression proceeding from unity, we place 0 over the first, 1 over the second term, &c. and so proceed orderly according to the natural progression of numbers, thus

0	1	2	3	4	5	6	7	8	indices,
1,	2,	4,	8,	16,	32,	64,	128,	256,	powers

The indices, or numbers 1, 2, 3, &c. will express what power of the ratio the term is over which it stands; they are, therefore, called indices, or exponents of the powers.

If the 1st term given be equal to the ratio, then every succeeding term is the same power of the ratio, as the order of its place for one must be placed over the first term.

1	2	3	4	5	6	7	indices.
2,	4,	8,	16,	32,	64,	128,	powers.

But in all other cases, whether the first term be one or more, the indices must begin with a cypher.

Proposition 1. In any geometrical progression beginning with unity, there will be this coherence or relation between the powers and their indices, viz.

The sum of the indices of any two powers, will be the index to that power expressed by the product of the said two powers; and if the index of any power be doubled, its double will be the index of the square of said term, and doubly distant from the first term.

0	1	2	3	4	5	6	7
1,	2,	4,	8,	16,	32,	64,	128,

now 3+3=6 sum of indices, or 3+4=7 sum of indices,
and 8×8=64 power and 8×16=128 power.

Proposition 2. In any Geometrical Progression not proceeding from unity, the square of any term; or the product of any two terms divided by the first term, gives that term of the progression expressed by the sum of the indices.

0	1	2	3	4	5	6	7	8
3,	6,	12,	24,	48,	96,	192,	384,	768,

now $4+4=8$ and $\frac{48^2}{3}=768$ the 9th term whose index is 8

and $3+4=7$ then $\frac{24\times48}{3}=384$ the 8th term, index 7

PROBLEM I. To find any assigned term of a geometrical progression proceeding from unity without producing all the terms.

RULE.—*Continue the progression to the 6th term whose index is* 5, *square this 6th term and it produces that whose index is* 10, *this term squared gives that power whose index is* 20, *and from these terms we may find any others, whose indices are even tens, for* $20+10=30$ *and* $30+20=50$, *&c. if odd numbers are found in the series, multiply the last term found by that term whose index denotes the distance of the assigned term, or by the product of any two terms, the sum of whose indices denotes the said distance.*

EXAMPLE.

A man bought 43 yards of velvet, at 1 shilling for the first, 2 for the second, 4 for the third, &c.: How many shillings should he pay for the 43d yard, suppose the merchant bestows him the rest?

0	1	2	3	4	5
1,	2,	4,	8,	16,	32

```
                      32
                      32
                    ----
                    1024 power whose index is 10.
                    1024
                 -------
                 1048576   index 20.
                 1048576
           -------------
           1099511627776   index 40, and 41 term.
                      4 × whose index is 2.
           -------------
           4398046511104 shillings Ans. 43 term.
         or £219902325555 4s.!!
```

PROBLEM II. To find any assigned term of a geometrical progression not proceeding from unity, without producing all the terms.

RULE.—*Proceed directly as in the last problem, only observe that you divide the product, or square of any power or powers by the first term.*

EXAMPLE.

A man bargained for 26 horses, was to give 2 cents for the first, 6 for the second, 18 for the third, &c. in triple proportion geometrical: What was the price of the last horse.

0	1	2	3	4	5
2,	6,	18,	54,	162,	486

486

first term 2) 236196

118098 *index* 10.
118098

2) 13947137604

6973568802 *index* 20.
multiplied by 486 *index* 5.

2) 3389154437772

1694577218886 *index* 25, *and* 26*th term.*
=$16945772188.86 *Ans.*

PROBLEM III. To find the sum of any Geometrical Progression.

RULE.—*Find the last term by problem first or second, then subtract the least term from the greatest, divide the remainder by the ratio of the progression less* 1, *and to the quotient add the greatest, or last term, their sum will be the sum of the series.*

EXAMPLE.

Bought 7 yards broadcloth, at $1 for the first yard, 3 for the second, 9 for the third, &c.: What is the price of the 7 yards?

1, 3, 9, 27, 81, 243, 729 *greatest.*
1 *least.*

ratio 3−1=2) 728

364 *quotient.*
729 *greatest.*

$1093 *sum, Ans.*

4. A man bought a horse and was to give a farthing for the first nail, 2 for the second, 4 for the third, &c. in geometrical progression: The number of nails was to be 7 in each shoe, viz. 28 nails in all: What must be paid for the horse?

Ans. £279620 5 3¾.

5. A merchant sold 15 yards of sattin; the first for 1*s.* the second for 2*s.* the third for 4*s.* the fourth for 8*s.* I demand the price of the 15 yards? *Ans.* £1638 7*s.*

6. A draper sold 20 yards of superfine cloth; the first yard for 3*d.* the second for 9*d.* the third for 27*d.* &c. in triple proportion geometrical: I demand the price of the cloth.

Ans. £21792402 10.

7. A goldsmith sold 1 lb. of gold at a farthing for the first oz. a penny for the second, 4*d.* for the third, &c. in quadruple proportion geometrical: I demand what he sold the whole for; also, how much he gained by the sale thereof, supposing he gave for it £4 per ounce?

Ans. { *He sold it for* £5825 8 $5\frac{1}{4}$. *and gained* 5777 8 $5\frac{1}{4}$. }

8. A cunning servant agreed with a master (unskilled in numbers) to serve him 11 years, without any other reward for his service but the produce of 1 wheat-corn for the first year; and that product to be sowed the second year, and so on from year to year, until the end of the time: What is his wages, allowing the increase to be but in a tenfold proportion; that 7680 wheat-corns make a pint, and is sold at 50 cents per bushel? - *Ans.* $101725.25.

9. A thresher worked 20 days at a farmer's, and received for the first day's work 4 barley-corns, for the second 12 barley-corns, for the third 36 barley-corns, and so on in geometrical proportion: I demand what the 20 days labour came to, supposing the pint to contain 7680 corns, and the whole quantity to be sold at 2*s.* 6*d.* per bushel? *Ans.* £1773 7 6, *rejecting remainders.*

10. A merchant sold 30 yards of fine velvet trimmed with gold very curiously, at 2 pins for the first yard, 6 pins for the second, 18 pins for the third, &c. in triple proportion geometrical: I demand how much the velvet produced when the pins were afterwards sold at an hundred for a farthing: also, whether the said merchant gained or lost by the sale thereof, and how much, supposing the said velvet to have been bought at £50 per yard?

Ans. { *The velvet produced* £2144699292 13 $0\frac{1}{2}$. *The merchant gained* 2144697792 13 $0\frac{1}{2}$. }

PROBLEM. IV. Given the extremes and number of terms to find the sum of the series.

RULE.—*Divide the greatest by the least; extract that root of the quotient denoted by the number of terms less* 1, *for the ratio.—Second, subtract the least from the greatest, divide the difference by the ratio less* 1, *add the greatest term to the quotient for the sum of the series.*

EXAMPLE.

Given the extremes 3 and 768, number of terms 9, to find the sum of the series.

$768 \div 3 = 256$ *and* $\sqrt[8]{256} = 2$ *ratio.*

then $768 - 3 = 765 \div$ *ratio* $2 - 1 = 765 + 768$ *greatest* $= 1533$ *sum.*

PROBLEM V. Given the greatest term, number of terms, and ratio, to find the sum of the series.

RULE.—*Raise the ratio to that power denoted by the number of terms less 1, divide the last term thereby, the quotient is the first term; then find the sum by the last problem.*

EXAMPLE.

Given, 768 the greatest, 9 the number of terms, and ratio 2, to find the sum of the series.

$768 \div 2^8 = 3$ *the first term*—*then* $768 - 3 = 765 \div$ *ratio* $2 - 1 = 765$ *and* $765 + 768 = 1533$, *sum of the series, Ans.*

PROBLEM VI. Given the first term, ratio, and sum of the series to find the last term.

RULE.—*Multiply the sum by the ratio less 1, the product more the first term, divided by the ratio, will give the last term.*

EXAMPLE.

Given the first term 4, ratio 4, sum of the series 87380, to find the last term.

$87380 \times 4 - 1 = 262140 + 4 \div 4 = 65536$ *last term.*

PROBLEM. VII. Given the ratio, number of terms, and sum of the series, to find the greatest term.

RULE.—*From that power of the ratio equal the number of terms, subtract that power of the ratio denoted by the number of terms less 1, divide the remainder by the greatest power of the ratio less unity, the quotient multiplied by the sum of the series gives the greatest term.*

EXAMPLE.

Given ratio 4, number of terms 4, and sum 340, to find the last term.

$4,^4 = 256$ *and* $256 - 1 = 255$
$4,^3 = 64$

diff. $192 \div 255 = \frac{192}{255} \times \frac{340}{1} = \frac{65280}{255} = 256$ *last term.*

PROBLEM VIII. Given the ratio and extremes to find the number of terms.

RULE.—*Divide the greatest by the least, raise the ratio to a power equal to the quotient, the exponent of that power more 1 will be the number of terms.*

EXAMPLE.

Given ratio 2, extremes 3 and 768, to find the number of terms.

$768 \div 3 = 256$ *and* $2^8 = 256$ *consequently* $8 + 1 = 9$ *number of terms.*

PROBLEM IX. Given the first term, ratio and sum of the series, to find the number of terms.

RULE.—*Multiply the sum of the series by the ratio less* 1, *and add the first term; this divided by the first term, will give that power of the ratio denoted by the number of terms, &c.*

EXAMPLE.

Given first term 3, ratio 2, and sum of the series 1533, to find the number of terms.

$1533 \times 2 - 1 = 1533 + 3$ *first term* $= 1536 \div 3 = 512$

and $2^9 = 512$ *the 9th power of ratio and number of terms.*

PROBLEM X. Given the extremes and sum of the series to find the ratio.

RULE.—*From the sum of the series subtract the least term, divide the remainder by the sum of the series less the greatest term, the quotient will be the ratio.*

EXAMPLE.

Given 4 and 65536 extremes, 87380 sum, to find the ratio.

$87380 - 4 = 87376$ *and* $87380 - 65536 = 21844.$

then $87376 \div 21844 = 4$ *ratio.*

PROBLEM XI. Given the extremes and number of terms to find the ratio.

RULE.—*Divide the greatest by the least, extract that root of the quotient denoted by the power of the progression less* 1 *for the ratio.*

EXAMPLE.

Given first term 3, last 768, no. of terms 9, to find the ratio.

$768 \div 3 = 256$ *and* $\sqrt[8]{256} = 2$ *ratio, Ans.*

PROBLEM XII. Given the ratio, greatest term, and number of terms, to find the least term.

RULE.—*Divide the greatest term by that power of the ratio denoted by the number of terms less* 1, *the quotient will be the least term.*

EXAMPLE.

Given 2 the ratio, 9 number of terms, and 768 the greatest, to find the least term.

$9 - 1 = 8$ *then* $768 \div 2,^8 = 3$ *the first term, Ans.*

PROBLEM XIII. Given the ratio, number of terms, and sum of the series, to find the first term.

RULE.—*Divide the ratio less* 1 *by that power of the ratio denoted by the number of terms less* 1, *the quotient multiplied by the sum of the series gives the first term.*

EXAMPLE.

Given 4 ratio, 4 number of terms, and 340 sum, to find the first term.

$4-1=3\div4^4-1=\frac{3}{255}\times\frac{340}{1}=\frac{1020}{255}=4$ *the first term, Ans.*

PROBLEM XIV & XV. The last term and sum of the series given to find the first term and ratio.

RULE.—*Divide the sum of the series by the difference of the greatest term and sum of the series, the quotient will be the ratio, and remainder the first term.*

EXAMPLE.

Given 65536 the greatest term, 87380 sum of the series, to find the ratio and first term.

```
        87380
        65536
        -----
        21844) 87380 (4 ratio.
               87376
               -----
                   4 first term, Ans.
```

PROBLEM XVI. Given, the ratio, last term, and sum of the series, to find the least term.

RULE.—*Subtract the last term from the sum of the series, multiply the remainder by the ratio; this product taken from the sum of the series will leave the first term.*

EXAMPLE.

Given the ratio 4, greatest term 65536, sum of the series 87380, to find the first term.

$87380-65536=21844\times4=87376$ *and* $87380-87376=4$ *the first term.*

Compound Interest.

If any sum be lent out at interest, and the interest be not paid at the end of the term, it is added to the sum lent, and thus arises a new principal, on which interest will be calculated for the ensuing term; and if it runs on a third or fourth, the said principal and interests arising form a new principal.

Thus, £500 lent out at 8 per cent. and the interest forborne 3 years, will form these several principals:

As 100 : 108 :: 500 : 540	*new principal first year.*	
100 : 108 :: 540 : 583.4	*do.*	*2nd.*
100 : 108 :: 583.4 : 629.17.1$\frac{11}{23}$		*3rd.*

Otherwise,

$500 \times 1.08=540$ *1st year.*

$540 \times 1.08=583.2$ *2nd do.*

$583.2\times 1.08=629.856$ *3rd do.*

The several amounts £540, £583.2, £629.856, being produced from £500, by the continual multiplication thereof by 1.08. The principal and several amounts are in a geometrical progression, whose ratio is the amount of £1 for one year, viz. £1.08, and the number of years indices to the progression.

Likewise, if the amount of £1, for any number of years, be multiplied by any given principal, the product will be the amount of the principal for that time, from which we draw the following

RULE.—*Find the amount of £1 or $1, for the given time (which is to find a term in a geometrical progression, proceeding from unity, whose index is the number of years given) and multiply that amount by the given principal for the amount.*

EXAMPLE.

How much will $320 amount to in 10 years, at 5 per cent. compound interest?

```
0    1      2        3
1.  1.05.  1.1025.  1.157625
                      52011   second term reversed.
                   ---------
                   1.276281   fifth term.
                   1.826721   do. reversed.
                   ---------
                   1.628893   tenth term.
Multiplied by           320   principal.
                 ----------
                 421.245760   am't, or $421.24½, Ans.
```

At 5 per cent. the continual multiplier is 1.05, or $1\frac{1}{20}$; wherefore $1.\times 1.05=1.05$, for second term, and $1.05\times 1.05=1.1025$, for the third; but this may be done by adding $\frac{1}{20}$ of itself to every new term, thus producing the next as follows:

```
          1.
1/20 =    .05
        ------
          1.05      amount of 1. first year.
1/20 =    .0525
        ------
          1.1025    do. second do.
1/20 =    .055125
        --------
          1.157625  do. third do.
```

But 1.06, the amount of £1 or $1, at 6 per cent. = 1.05 + .01, or $1+\frac{1}{20}+\frac{1}{5}$ of $\frac{1}{20}$: Therefore,

	1.06 *first year.*
$\frac{1}{20}$ =	.053
$\frac{1}{5}$ =	.0106
	1.1236 *second.*
$\frac{1}{20}$ =	.05618
$\frac{1}{5}$ =	.011236
	1.191016 *third.*

Or thus,

1.06 *first year.*
1.06
636
1060
1.1236 *second.*
.601
11236
67416
1.191016 *third.*

Likewise 1.07, the amount of £1 or $1, at 7 per cent. = 1.05 + .02, or $1+\frac{1}{20}+\frac{2}{100}$, or $1+\frac{1}{20}+\frac{2}{5}$ of $\frac{1}{20}$: Therefore,

	1.07 *first year.*
$\frac{1}{20}$ =	.0535
$\frac{1}{5}$ =	.0107
$\frac{1}{5}$ =	.0107
	1.1449 *second.*
$\frac{1}{20}$ =	.057245
$\frac{1}{5}$ =	.011449
$\frac{1}{5}$ =	.011449
	1.225043 *third.*

Or thus,

1.07 *first year.*
1.07
749
1070
1.1449 *second.*
107
80143
114490
1.225043 *third.*

Thus may tables be calculated, by which the amount of any sum at compound interest may be found, by multiplying by the tabular number answering to the number of years.

TABLE I.

Showing the amount of £1 or $1, for 21 years, at the following rates.

Years.	*5 per cent.*	*6 per cent.*	*7 per cent.*
1	1.05	1.06	1.07
2	1.1025	1.1236	1.1449
3	1.157625	1.191016	1.225043
4	1.215506	1.262477	1.310796
5	1.276281	1.338225	1.402552
6	1.340096	1.418519	1.50073
7	1.4071	1.503630	1.605781
8	1.477455	1.593848	1.718185
9	1.551328	1.689479	1.838458
10	1.628895	1.790848	1.96715
11	1.710339	1.898298	2.10485
12	1.795856	2.012196	2.252189
13	1.855649	2.132928	2.509842
14	1.979932	2.260904	2.57853
15	2.078928	2.396558	2.759027
16	2.182874	2.540352	2.952158
17	2.292018	2.692773	3.158809
18	2.406619	2.854339	3.379925
19	2.526950	3.025599	3.616519
20	2.653298	3.207135	3.869675
21	2.785962	3.399564	4.140552

The index of 1, the first term, is 0, of 1.05, is 1, &c.; the number of years thus standing for indices to the several amounts. Therefore, to find the tabular number for any term more than is expressed in the table, add any two indices which will make the index required, and multiply the opposite numbers, the product will be the term sought, which multiply by the principal; the product will be the amount.

1. What sum will $450 amount to in 3 years, at 5 per cent. per annum, compound interest? *Ans.* $520.93.

2. What will £256 10, amount to in 7 years, at 6 per cent. per annum, compound interest? *Ans.* £385 13 7½.

3. What will £136 13 6, amount to in 20 years, at 6 per cent. per annum, compound interest? *Ans.* £438 13 1½.

4. What will £500 amount to in 4 years, at 4½ per cent. per annum, compound interest? *Ans.* £596 5 2.

CASE II.

When the rate, time and amount, are given to find the principal.

RULE.—*Divide the amount by the amount of 1, for the given time and rate; the quotient will be the principal.*

EXAMPLE.

The amount of a sum at 7 per cent. compound interest, for 14 years, is $12892.65: Required the principal lent.

12892.65÷2.57853, *tabular number for* 14 *years*, = $5000. *Ans.*

CASE III.

The principal, rate and amount, given to find the time.

RULE.—*Divide the amount by the principal, the quotient will be the tabular number, or amount of* 1, *opposite which, find the number of years, &c.; otherwise, divide the amount of* 1, *continually by the ratio, till nothing remains; the number of divisions will be equal to the number of years.*

Lent out $5000, at 7 per cent. compound interest; on settlement, received $12892.65: Required the time the money was at interest.

12892.65÷5000=2.57853, tabular number for 7 per cent. which, on inspecting the table, I find opposite 14 years. *Ans.*

CASE IV.

The principal, time and amount, given to find the rate per cent.

RULE.—*Divide the amount by the principal, for the tabular number, then extract that root, denoted by the number of years, for the amount of* 1 *in one year; this last, less unity, will be the ratio.*

Lent $5000, which in 8 years amounts to $7969.24: Required the rate per cent.

7969.24÷5000=1.593848, tabular number for 8 years, and $\sqrt[8]{1.593848}$=1.06, amount of 1 for one year, and 1.06−1=.06 ratio. *Ans.*

Annuities, or Pensions in Arrear, at Compound Interest.

RULE 1.—Find the amount of the given yearly sum, for the number of years less 1, which will be the last term of a geometrical progression, of which the given sum is the first; then find the sum of that progression and it is the amount of the annuity required.

2. Otherwise, find the amount of 1, for the given years less 1; then find the sum of that progression, whose first term is 1, and last term the said amount; and multiply the sum by the given annuity for the amount required.

EXAMPLE.

An annuity of $320, being 11 years in arrear; required the amount due, compound interest computed at 5 per cent. on every payment.

By Rule 1.

In table 1, the amount of 1,
for 10 years, is
1.628895
multiply 320 *annuity*

521.246400 *last term*
320. 1*st term.*

1.05–1=.05)201.246400 *dif.*

4024.928
521.2464 *greatest.*
$4546.174'4'. *Ans.*

By Rule 2.

1.628895 *tab, no. for* 10 *yrs.*
1. *first term.*

,05) .628895

12.5779
1.628895 *greatest.*

14.206795 *sum.*
320 *annuity.*

$4546.174'4'. *Ans.*

TABLE II.

Showing the amount of £1 or $1, annuity forborne 21 years, at the following rates.

Years.	*5 per cent.*	*6 per cent.*	*7 per cent.*
1	1.	1.	1.
2	2.05	2.06	2.07
3	3.1525	3.1836	3.2149
4	4.310125	4.374616	4.439943
5	5.525631	5.637093	5.570739
6	6.801913	6.975318	7.153291
7	8.142008	8.393837	8.654021
8	9.549108	9.897467	10.259802
9	11.026564	11.491316	11.977987
10	12.577892	13.180794	13.816445
11	14.206787	14.971643	15.783595
12	15.917126	16.869940	17.888345
13	17.712982	18.182137	20.140534
14	19.598631	21.015065	22.550376
15	21.578563	23.275969	25.128906
16	23.657491	25.672527	27.887933
17	25.840366	28.212879	30.840091
18	28.132384	30.905651	33.998900
19	30.539003	33.759992	37.378825
20	33.065954	36.785590	40.995344
21	35.719251	39.992727	44.865019

This Table is constructed from Table 1, of compound interest.

Thus—to 1. the amount of 1 for 1 year's annuity,
add 1.07, the amount of 1 at compound int.—tab. 1.

2.07,= amount of 1 annuity at 2 years end,
add 1.1449, 2d term of table 1.

3.2149,=amount of 1 annuity at 3 years end,
add 1.225043, 3d term of table 1.

4.439943,=amount of 1 annuity at 4 years end, at 7 per cent.

USE.

Multiply the annuity by the amount of 1, for the given time; the product will be the amount and answer required.

An annuity of $20 per annum, is forborne 7 years: How much is due at the end of that term, allowing the annuitant 6 per cent. per annum, compound interest?

Tabular number for 6 years, in table 1,=1.418519

	20	*annuity.*
	28.370380	*greatest term.*
	−20.	*least term.*
1.06−1=.06)	8.37038	*difference.*
	139.50	
	28.37	*greatest.*
	$167.87	*sum. Ans.*

Or thus, by Table 2.

Amount of 1 for 7 years, 8.3938
Multiply by the annuity, 20

$167.87 *Ans.*

The foregoing may be explained by the following calculation.

20.	*due first year.*
×1.06	
21.20	
+20.	
41.20	*due second year.*
×1.06	
43.672	
+20.	
63.672	*third year.*
×106	
67.49232	
+20	
87.49232	*fourth year.*
×1.06	
92.741859	
+20	
112.741859	*fifth year.*
×1.06	
119.506370	
+20.	
139.50637	*sixth year.*
×1.06	
147.876772	
+20	
167.876772	*seventh year, or $167.87, as before.*

3. If £30 yearly rent be forborne 9 years, what will it amount to in that time, at 6 per cent. per annum, compound interest?

Ans. £344 14 9¼.

4. Suppose a person who has an annuity of £20 per annum, suffers it to be in arrear 15 years, what has he to receive at the end of that term, compound interest being computed at 6 per cent.? *Ans.* £465 10 4¾.

5. An annuity of $80 per annum is forborne 17 years: How many dollars has the annuitant to receive on settlement, compound interest computed at 7 per cent. per annum?

Ans. $2467.20.

CASE II.

The amount, rate and time given, to find the annuity:

RULE.—*Divide the amount by the amount of one for the rate and time* (see tab. 2nd) *the quotient is the annuity.*

EXAMPLE.

Amount of an annuity for 17 years, at 7 per cent. is $2467.20: Required the annuity.

2467.20÷30.84 *tab. no. for* 17 *yrs. tab.* II.=80 *dolls. Ans.*

CASE III.

Annuity, rate, and amount given to find the time:

RULE.—*Multiply the amount by one,+the rate, and add the annuity; from this sum subtract the amount; divide the remainder by the annuity, the quotient will be the amount of* 1 *at the given rate, which in table first stands opposite the number of years required—or divide this amount continually by the ratio till nothing remains, the number of divisions will give the number of years, &c.*

EXAMPLE.

Amount of an annuity is $2467.20, rate 7 per cent. annuity $80: Required the time.

2467.20×1.07=2639.904+80=2719.904

and 2719.904—2467.20=252.704÷80=3.1588 *tabular number for* 17 *years, at* 7 *per cent. in table first.*

Rebate or Discount at Compound Interest.

TABLE III.

Showing the present worth rebate of £1 or $1, for 21 years, as follows:

Years.	*5 per ct.*	*6 per ct.*	*7 per ct.*
1	.952381	.943396	.934579
2	.907030	.889996	.873438
3	.863838	.839619	.816297
4	.822703	.792093	.762894
5	.783526	.747258	.712985
6	.746215	.70496	.666341
7	.710681	.665057	.622748
8	.676839	.627412	.582007
9	.644609	.591898	.543931
10	.613913	.558394	.508346
11	.584679	.526787	.475089
12	.556837	.496969	.444008
13	.530321	.468839	.414960
14	.505068	.442301	.387813
15	.481017	.417265	.362442
16	.458111	.393647	.338730
17	.436296	.371364	.316570
18	.415520	.350353	.295859
19	.395734	.330513	.276504
20	.376889	.311804	.258415
21	.358942	.294155	.241509

Construction of the above Table.

Let £1 or $1 be divided by 1 more the ratio,* the quotient will be the value of 1 at the year's end, and the first term; then this quotient being divided by the above divisor, gives its value at the end of the second year and second term.

Thus $1 \div 1+.07 = .934579$, value of £1 or $1, due 1 y. hence.

And $.934579 \div 1.07 = .873438$, value of 1, due 2 years hence.

USE.—Multiply the sum to be discounted by the number answering to the number of years and rate per cent.—the product is the present worth. *Otherwise,*

Find the amount of 1 for the given time and rate per cent.; divide the sum to be rebated thereby—the quotient will be the present value.

EXAMPLES.

1. Suppose £521 4 11¼ were to fall due 10 years hence, what is the present worth of the same, discount being allowed at 5 per cent. per annum, compound interest? *Ans.* £320.

2. What is the present worth of £629 17 1½, due 3 years hence, at 8 per cent. per ann. compound interest? *Ans.* £500.

3. A legacy of £520 18 7½ is left payable in 4 years; but the executor is willing to pay it at the end of one year, on being allowed discount at 5 per cent. per annum, compound interest: This being agreed to, what had he to pay? *Ans.* £450.

CASE II.

Rate per cent. debt, and present worth, given to find the time.

Rule.—Divide the debt by the present worth—the quotient will be the amount of 1 for the time required, (which seek in table 1st) or divide this amount of 1 continually by the ratio—the number of divisions will be the time.

EXAMPLE.

4. Given £521 4 11½, debt; present worth, £320; rate per cent. 5, to find the time.

521.24687÷320=1.62889, tab. number, or amount of £1, found opposite 10 years in table 1st.

CASE III.

Time, debt and present worth, given to find the ratio:

Rule.—Having found the tab. number, or amount of 1 for the given time, seek the number in table 1st, and at the head of the column you will find the ratio:

Otherwise,

Extract that root of the number found, denoted by the number of years—the root will be the ratio.

EXAMPLE.

Last question. 1.62889, tab. number for 10 years, at 5 per ct.

Or thus, $\sqrt[10]{1.62889}=.05$, *Ans.*

Present worth of Annuities.

TABLE IV.

Showing the present worth rebate of £1 or $1, annuity to continue 21 years.

Years.	5 *per ct.*	6 *per ct.*	7 *per ct.*
1	0.952381	0.943396	0.934579
2	1.85941	1.833392	1.808017
3	2.723248	2.673012	2.624314
4	3.54595	3.465105	3.387208
5	4.329477	4.212363	4.100193
6	5.075692	4.917324	4.766534
7	5.786373	5.582381	5.389282
8	6.463212	6.209792	5.971289
9	7.107821	6.801691	6.51522
10	7.721734	7.360086	7.023566
11	8.306414	7.886673	7.498655
12	8.863251	8.383843	7.942663
13	9.393572	8.852682	8.357623
14	9.89864	9.294983	8.745436
15	10.379658	9.712248	9.107878
16	10.837769	10.105894	9.446608
17	11.274065	10.477258	9.763178
18	11.689586	10.827602	10.059037
19	12.08532	11.158115	10.335541
20	12.462209	11.46992	10.593956
21	12.821152	11.764075	10.835465

Construction of the foregoing Table.

This table is constructed from table 3d, thus, to the amount of 1 as due one year hence, viz.

.934579 *being first term at 7 per cent.*
Add .873438 *second term of table* 3d.

1.808017=*present value of* 1. *annuity for* 2 *years.*
Add .816297 *third number of table* 3d.

2.624314=*present value of* 1. *annuity for* 3 *years, rebate at 7 per cent. per annum.*

USE.

Multiply one year's annuity by the tabular number opposite the number of years the annuity has to continue, and under the rate per cent. the product will be the present value of the annuity, &c.: Otherwise,

Find the present worth of the first and last year's annuity, which are the greatest and least terms of a geometrical progression; then find the sum of that progression, which is the present worth of the annuity.

EXAMPLES.

1. What is £30 yearly rent worth, in ready money, to continue 7 years, at 6 per cent. per annum, compound interest?
Ans. £167 9 5.

2. An annuity of £20, to continue 7 years, is to be sold at 5 per cent. compound interest: What is it worth in ready money?
Ans. £115 14 6.

3. A rent of £365, paid yearly, is to be sold for 12 years, at 5 per cent. per annum, compound interest: What is the present worth in ready money? *Ans.* £3235 1 9.

CASE II.

Present worth, rate per cent. and time, given to find the annuity.

Rule.—*Divide the present worth, by the present worth of* 1, *for the given time and rate; the quotient will be the annuity.*

EXAMPLE.

4. Present worth, £3235 1 9, time 12 years, and rate per cent. 5: Required the annuity the above would purchase.

3235.087248÷8.863251 *present worth of* 1, *at* 5 *per cent. Table 4th*=365 *annuity. Ans.*

CASE III AND IV.

Annuity, present worth and ratio, given to find the time.

Rule.—*Divide the present worth by the annuity, the quotient will be the present worth of* 1; *which seek for in table* 4, *under the* *ratio, opposite to which you find the time: Otherwise, divide the present worth of 1 continually by the ratio, till nothing remains—the number of divisions will be equal to the time.*

EXAMPLE.

5. Annuity £365, is sold for £3235 1 9, at 5 per cent.: Required the time of its continuance.

3235.087248÷365=8.863252, *present worth of* 1. *which is found under* 5 *per cent. table* 4, *answering to* 12 *years. Ans.*

Annuities in Reversion.

When an annuity or yearly rent does not commence till after the expiration of some time, or when an annuity belongs to several, each of whom shall enjoy it in succession; then the annuity or rent is said to be in reversion.

RULE.—*Find the present worth of the annuity, as if commencing immediately. Second—find what ready money ought to be paid for that sum, rebate at compound interest, being allowed for the term of years, till the commencement of the annuity or lease, &c.*

EXAMPLE.

1. Required the present value of $100 yearly rent, to continue 14 years, and not to commence till 6 years hence; discount computed at 7 per cent. per annum, compound interest.

100÷1.07 *amount of* 1 *for* 1 *year*,=93.457, *first term or value at* 1 *year's end.*

100÷1.07,^{14}=(2.5785=1 *for* 14 *y.*)=38.781, *last term.*

1.07−1=.07) 54.676, *difference.*

781.085
93.457, *first term.*

$874.54, *present worth of the annuity or rent, as if commencing immediately; but as it does not commence till after* 6 *years, this present worth of the annuity must be discounted as a sum due* 6 *years hence: Therefore,*

874.54÷1.07,^{6}=(1.50073)=$582.74, *the real value of the annuity or rent. Ans.*

2. Required the present worth of an annuity of £75, which is not to commence till 10 years hence, and then to continue 7 years after that time, at 6 per cent. compound interest.

Ans. £233 15 9.

3. An annuity of £24, to begin 7 years hence, and to continue 21 years, is to be sold at 6 per cent. compound interest: What is it worth to the purchaser? *Ans.* £187 15 5.

CASE II.

Present worth, ratio, time and reversion, given to find annuity.

RULE 1st.—*Divide the present worth by the present worth of* 1, *for the time given, (according to table* 3*); this gives the value of the reversion as commencing immediately.*

2nd. *Divide this present worth by the present worth of* 1 *annuity for the number of years, the annuity is in reversion, (by table* 4*); this quotient will be the annuity.*

EXAMPLE.

A, purchasing the reversion of an annuity, to commence 2 years hence, and to continue 4 years, pays £185.035876, being allowed compound interest at 6 per cent.: Required the annuity.

185.035876÷.89, *present worth of* 1 *for* 2 *years*,=207.906, *present worth of the annuity, as commencing immediately—and*

207.906÷3.4651, *present worth of* 1 *annuity for* 4 *years*,=£60. *Answer.*

Present worth of Annuities, or Leases for ever.

CASE I.

To find for how many years purchase an annuity, or rent for ever, may be bought at any given rate per cent.; and then to find the present worth.

RULE.—*Divide* 100 *by the rate per cent. the quotient will be the number of years required; this number multiplied by the annuity, gives the present worth.*

EXAMPLE.

What is the present worth of $500, annuity for ever; allowing the purchaser 8 per cent. for his money?

100÷8=12.5=12½ *years purchase.*

then 12.5×500=$6250. *present worth. Ans.*

What is the present worth of an estate, whose yearly rent is $3000, at 5 per cent.? *Ans.* $60000.

CASE II.

When the annuity and rate are given to find the present worth.

RULE.—*Divide the annuity by the ratio, the quotient is the present worth.*

EXAMPLE.

1. Prove 1st example of case first.

500.÷*ratio* .08=$6250. *Ans.*

2. Prove example second. *Ans.*

CASE III.

When the present worth and rate are given to find the annuity.

RULE.—*Multiply the present worth by the ratio, the product will be the annuity.*

EXAMPLE.

An estate is sold at 8 per cent. it amounts to \$6250: Required the yearly rent. 6250.×.08=\$500. *Ans.*

CASE IV.

Present worth, and yearly rent, given to find the ratio.

RULE.—*Divide the annuity by the present worth, the quotient will be the ratio.*

EXAMPLE.

An annuity of \$500, is sold for \$6250: Required the rate per cent.

500.00÷6250=.08 *ratio or* 8 *per cent.* *Ans.*

CASE V.

The number of years purchase, given to find the rate per cent.

RULE.—*Divide* 100 *by the given years, the quotient will be the rate per cent.*

EXAMPLE.

An annuity is bought at $12\frac{1}{2}$ years purchase: Required the rate per cent. 100÷12.5=8 *per cent.* *Ans.*

CASE VI.

Annuities for ever in reversion.

Having the annuity, rate, and time of reversoin, given to find the present worth:

RULE.—*Find the present worth by case* 1*st* or 2*d*, (*as if commencing immediately*); *divide this present worth by that power of the ratio more* 1. *denoted by the number of years the reversion continues; this quotient is the present worth.*

EXAMPLE.

What sum must be paid for the reversion of an annuity or rent of \$500, to commence after 6 years, at 7 per cent. compound interest?

500÷.07=7142.85, *present worth as commencing immediately; and* 7142.85÷$1.07,^6$(=1.5)=\$4761.90, *present worth.* *Ans.*

An annuity of \$300. for ever, 6 years in reversion, is sold at 5 per cent.: Required the present worth. *Ans.* \$4477.60.

CASE VII.

Present worth, rate per cent. and time of reversion, given to find the annuity or yearly rent.

RULE.—*Find that power of the ratio more* 1. (*see table* 1*st, compound interest*) *denoted by the number of years in reversion; multiply this power by the ratio, the product multiplied by the present worth, gives the annuity.*

EXAMPLE.

An estate six years in reversion, at 7 per cent. amounts to \$4761.90: Required the yearly rent.

$1.07,^6$=1.5 *amount for* 6 *years*, × *ratio* .07=.105 *and* 4761.90×.105=\$500. *Ans.*

An annuity 6 years in reversion, being sold at 5 per cent. amounts to \$4477.61: Required the annuity. *Ans.* \$300.

Duodecimals;

OR,

Multiplication of Feet and Inches.

THEIR use in arithmetic being valuable to accountants, in finding the content of packages and the calculation of freight, a few examples shall be given to render calculation easy.

PROPOSITION 1. Feet multiplied by feet are feet.
2. Inches multiplied by feet are inches.
3. Feet multiplied by inches are inches.
4. Inches multiplied by inches are seconds.
5. Seconds multiplied by inches are thirds, &c.

RULE.—*Place feet and inches under feet and inches; multiply all the denominations of the upper line by the feet of the lower, placing the surplus of each denomination under its proper head; then multiply by inches, placing the surplus of each result one place to the right: Proceed with the lower denominations in the same manner, still removing one denomination to the right—add up all the results for the content of the figure required.*

12 inches make	1 foot.
12 seconds	1 inch.
12 thirds	1 second, &c.
144 inches	1 square foot.
1728 cubic inches	1 solid or cubic foot.

EXAMPLE.

Required the superficial content of a figure whose length is 7 feet 5 inches 9″, breadth 3 feet 5 inches 3″.

ft.	*in.*	″.	‴.	⁗.	
7.	5.	9			
3.	5.	3			
22.	5.	3			*amount of* 3 *feet.*
3.	1.	4.	9		=*do. of* 5 *inches.*
	1.	10.	5.	3.	*do. of* 3″.
25.	8.	6.	2.	3.	*Ans.*

Answers.

2. Multiply 10 *ft.* 4 *in.* 5″ by 7 *ft.* 8 *in.* 6″.—79 *ft.* 11 *in.* 0″ 6‴ 6⁗
3. Multiply 311 *ft.* 4 *in.* 7″ by 36 *f.* 7 *i.* 5″. 11402. 2. 4. 11. 11.
4. Multiply 321 *f.* 7 *i.* 3″ by 9 *f.* 3 *i.* 6″ 2988. 2. 10. 4. 6.
5. What is the price of a marble slab 5 feet 7 inches long, and 1 foot 10 inches broad, at 6 shillings per foot? *Ans.* £3 1 5.

In the calculation of Freight 40 cubic feet make 1 Ton.

Length, breadth, and depth, given to find the solid content.

RULE.—*Multiply the length, breadth, and thickness, continually together—the product will be the solid content in feet, &c.*

If lower denominations arise, it is better to reduce them to the decimal of the higher, in calculating the price.

EXAMPLE.

Required the solid content and freight of a bale of dry goods, 4 feet 2 inches long, 2 feet 6 inches broad, by 1 foot 8 inches deep, at $30 per ton.

4 *ft.* 2 *in.* × 2 *ft.* 6 *in.* = 10 *ft.* 5 *in. superfice.*
10 *ft.* 5 *in.* × 1 *ft.* 8 *in.* = 17 *ft.* 4 *in.* 4″ *solid contents.*
And 17 *ft.* 4 *in.* 4″ = 17.361 *ft.*
Then as 40 *ft.* : 30 :: 17.361 : *to* $13.02. *Ans.*

Required the solid content and freight of the following bales of dry goods.—viz.

	Long.		Broad.		Deep.		Solid content.			
	ft.	*in.*	*ft.*	*in.*	*ft.*	*in.*	*ft.*	*in.*	″	‴
No. 1.	4	6	2	4	1	7.				
2.	5	2	2	2	2	4.				
3.	3	6	3	0	1	6.				
4.	4	0	2	3	1	4.				
At $20 *per ton.*						*Solid content,*	70	5	11	4.

Ans. $35.25, *freight.*

Required the freight of a bale of goods 8 *ft.* 4 *in.* long, 7 *ft.* 2 *in.* broad, 3 *ft.* 6 *in.* deep, at $10. *per ton.* *Ans.* 32.25.

Arithmetical Magazine, &c.

PART V.

A short Introduction to Mechanics,

IN WHICH THE SEVERAL POWERS ARE EXEMPLIFIED IN THE CALCULATION OF SUNDRY PRACTICAL QUESTIONS.

THE theory of mechanics is justly considered an indispensable part of polite education; it is a science of great utility and pleasing speculation; by it we are taught how to overcome the inactivity and gravity of matter, and compel it to act subserviently to our wishes.

By a combination of the several powers we construct machines, by which arts, sciences, and manufactures are improved.

In this short system is given calculations and explanations of the several powers sufficiently for common uses. The compass of such a volume as this must be the only excuse for its brevity; for the chief design of this essay is to give a spur to genius, and create a taste where lucid talents lie concealed, that require only the breath of novelty to blow the latent spark, that may yet illumine our American hemisphere.

Definitions.

DENSITY is the proportion or quantity of matter in any body, to the quantity of matter in any other body, of the same dimensions.

FORCE is a power exerted on a body to move it: if it act instantaneously, it is called *precision* or *impulse*; if constantly, it is called *accelerative force*.

VELOCITY is a property of motion, by which a body passes over a certain space in a certain time, and is greater or less, as it passes over a greater or less space in a certain time.

QUANTITY OF MOTION is the motion a body has considered, both in regard to its velocity and quantity of matter.

Vis inertiæ is the innate force of matter, by which it resists any change, striving to preserve its present state of rest or motion.

Gravity is that force wherewith a body endeavours to fall downwards: it is called *absolute gravity* in empty space, and *relative gravity* when immersed in a fluid.

Centre of gravity is a certain point of a body, upon which the body, when suspended, will rest in any position.

Centre of motion is a fixed point, about which a body moves; and the *axis of motion* is a fixed line it moves about.

The alteration of motion, by any external force, is always proportionable to that force, and in direction of the right line in which that force acts.

The Foundation of Mechanics.

If we consider bodies in motion, and compare them together, we may do it either with respect to the quantities of matter they contain, or the velocity with which they are moved; for the heavier any body is, the greater is the power required to move it, or stop its motion; and the swifter it moves, the greater is its force: so that the whole force of a moving body is the result of its quantity of matter, multiplied by its velocity; and when the products of the respective quantities of matter in two or more bodies are equal, their forces will be so too.

EXAMPLE.

A body, wt. 40 lb. moves with a velocity of 40 feet in one second.
Another, wt. 8 lb. moves with a velocity of 200 ft. in do.
Another, wt. 4 lb. moves with a velocity of 400 ft. in do.
Required the proportion of their forces.

$$\left.\begin{array}{r}40\times 40=1600\\ 8\times 200=1600\\ 4\times 400=1600\end{array}\right\}$$ *Answer. The forces of these bodies are equal.*

Upon this easy principle the whole of mechanics depends; for it holds universally true, that when two bodies are suspended on any machine, so as to act contrary to each other, if the machine be put in motion, and the perpendicular ascent of the one multiplied into its weight, be equal to the perpendicular descent of the other multiplied into its weight, these bodies, however unequal, will balance each other in all positions; for, as the whole ascent of the one is performed in the same time with the whole descent of the other, their respective velocities must be directly as the spaces they move through; and the excess of weight in the one, is compensated by the excess of velocity in the other. Upon this principle, it is easy to compute the power of any machine; for it is only finding how much swifter the power moves than the weight.

Velocities acquired by heavy bodies falling.

PROPOSITION. The motion of a body falling freely by the attraction of gravitation, is uniformly accelerated; for, a new impression being made every instant by the action of gravity, and the effect of the former impulse remaining, the velocity of the falling body must continually increase.

The velocity acquired by a falling body, in the first second of time, is found to be about 16.083, and this velocity is accelerated in proportion to the square of the times; but 16.1 feet is near enough for calculation.

Therefore, to find what space any body has fallen through in any given time:

RULE.—*As sq.* 1 *sec.* : 16.1 *feet* :: *sq. given time* : *feet required.*

EXAMPLE.

Required the space fallen through by a body which has been in motion 11 seconds.

sec. *feet.* *sec.* *feet.*
As $1,^2$ :: 16.1 :: $11,^2$: 1948.1 *Ans.*

Otherwise,
Multiply 16.1 by as many of the odd numbers, beginning from unity, as there are seconds in the given time; the sum total will be the space fallen through.

16.1 ×	1 = 16.1	passed over in 1 sec.
	3 = 48.3	- - - - - 2
	5 = 80.5	- - - - - 3
	7 = 112.7	- - - - - 4
	9 = 144.9	- - - - - 5
	11 = 177.1	- - - - - 6
	13 = 209.3	- - - - - 7
	15 = 241.5	- - - - - 8
	17 = 273.7	- - - - - 9
	19 = 305.9	- - - - - 10
	21 = 338.1	- - - - - 11

Space fallen through in 11 *seconds,* 1948.1 *as before.*

Therefore, to find the acquired velocity of a falling body, in any given second of time, *take the seconds as indices, and the odd numbers as the terms of an arithmetical progression—the product of any given term, and* 16.1, *will be the acquired velocity for that time; twice the exponent, or index,* —1, *will be the term required.*

A stone, falling from a precipice, was $19\frac{1}{2}$ seconds in descent: What space did it fall through? *Ans.* 6122.025 *feet.*

To find the time of ascent or descent:

RULE.—As 16.1 feet, : $1,^2$ sec. :: distance fallen : square of time in seconds; then $\sqrt[2]{}$=time.

EXAMPLE.

In what time would a bullet pass through 400 feet descent?

Ans. 5 *sec. nearly.*

The same force which accelerates a falling body, acting in an opposite direction on a body thrown upwards, must retard it; and since the action of gravitation is uniform, in whatever time it generates any velocity in a falling body, it must, in the same time, destroy the same velocity in a rising body: therefore a body will descend from any height in an equal space of time, and with equal force to that with which it was projected to that height.

The force with which a body descends to the earth, is as its quantity of matter multiplied into its acquired velocity, in the last second of its descent.

EXAMPLE.

Suppose a bullet, 7 lb. wt. be projected from a piece of ordnance perpendicular to the horizon, and that the time of ascent and descent be 1′ 20″: Required the height to which the ball was thrown, and the force with which it descends to the earth.

First. $1.' 20.'' \div 2 = 40''$ *time of ascent or descent.*

Then as $1,^2 : 16.1 :: 40,^2 : 25760$ *feet high; and as it descends so far in* 40″, *the ratio of descent for the last second is* $(79 \times 16.1) = 1271.9$ *feet: this, multiplied by the quantity of matter, viz.* 7 *lb. gives* 8903.3 *lb. the force or momentum with which it strikes the ground.*

Mechanical Powers.

OF THE LEVER.

The lever is a bar movable about a fixed point, which is called its *fulcrum* or *prop*. In theory, it is considered as an inflexible line, without weight.

The lever is of three orders.

In the first, the fulcrum or prop is between the weight and the power.

In the second, the weight is between the prop and the power; and,

In the third, the power is between the weight and the prop.

FIRST ORDER.

PROPOSITION 1. A power and weight, acting on the arms of a lever, will balance each other, when the distance of the power from the prop is to the distance of the prop from the weight, as the weight is to the power reciprocally.

Let the lever, *a*, *b*, *c*, be supported by the prop *b*; if the wt. *d*, multiplied into *a b*, be equal to the power *c*, multiplied into *b c*, the weight and power will balance each other: Or, *a b* will be to *b c*, as the weight is to the power reciprocally.

EXAMPLE.

What weight will a man be able to raise, who presses with a force of $1\frac{1}{2}$ cwt. on the arm of an equipoised handspike, 100 inches long, which rests on a fulcrum, $7\frac{1}{2}$ inches from the other end?

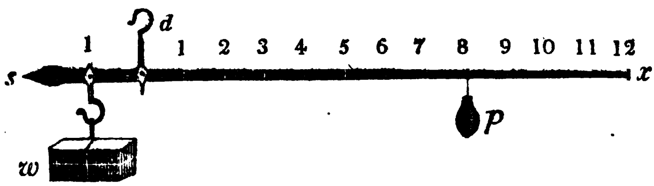
$100-7\frac{1}{2}=92\frac{1}{2}$ *longer arm.*

Then reciprocally.

	in.		cwt.		in.		cwt.	
As	$92\frac{1}{2}$	:	$1\frac{1}{2}$	::	$7\frac{1}{2}$	:	$18\frac{1}{2}$	*ans.*

PROPOSITION 2. The statera, or steelyard, is of this order. By this machine the different weights of bodies are ascertained, by one single weight, which acts as the power.

Let the bar, *s x*, be divided into any number of equal parts, and exactly poised or balanced at *d*; let the power, *p*, equal to 1 lb. be applied on any part of the longer arm, *d x*, it shall counterpoise as many lb. suspended at *w*, as are equal to the number of parts counted from *d* to *p*.



For if *p* equals 1 lb. and placed at the first division in the arm, *s x*, it would balance 1 lb. at *w*; but being placed at 8, it counterpoises 8 lb. at *w*.

What weight, applied 70 inches from the centre of motion, will equipoise a hhd. of tobacco, wt. $9\frac{1}{2}$ cwt. freely suspended from a steelyard, at 2 inches from the said centre? *Ans.* $30\frac{2}{5}$ *lb.*

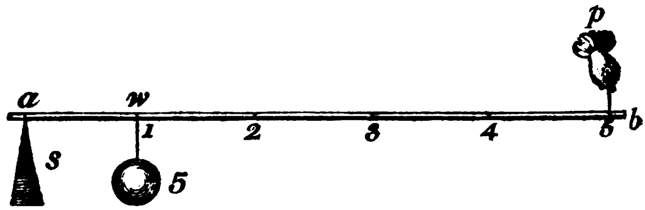
The shorter arm of a balance is 27 inches, the longer arm 36 inches: How many lb. suspended on the longer end, will equipoise 20 lb. on the other? *Ans.* 15 *lb.*

The beam of a balance measures 63 inches, and 20 lb. on one end equipoises 15 lb. on the other: Required the length of each arm. *Ans.* 27 *and* 36 *inches.*

SECOND ORDER OF LEVERS.

A lever of the second order has the weight between the prop and the power.

In this, as well as the former, the advantage gained is, as the distance of the power from the prop, is to the distance of the weight from the prop; for the respective velocities of the power and weight are in that proportion.



PROPOSITION 3. Thus, if *a b* be a lever on which the weight, *w*, 5 lb. hangs, at the distance of 1 inch from the prop, *s* and *p*, equal to 1 lb. 5 inches from the prop, the power will just support the weight; and a small addition to the power will raise the weight 1 inch for every 5 inches the power ascends.

This lever explains why two men, carrying a load on a pole between them, bear unequal shares of the weight, in the inverse proportion of their distance from it; for the nearer either is to the weight, the greater part he bears; and if he goes directly under it, he bears the entire.

This may be applied to *two horses of unequal strength,* (in drawing weighty drafts, or ploughing) *by dividing the beam they pull, so that the point of attraction may be as much nearer to the stronger horse than the weaker, as the strength of the one exceeds that of the other.*

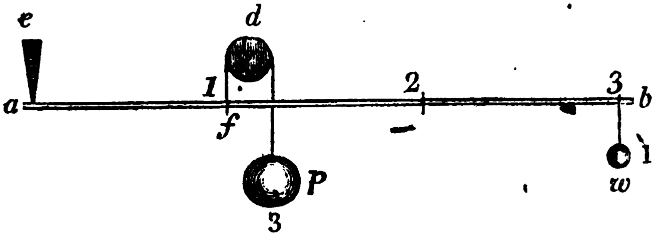
If a lever be 100 inches long, what weight, lying $7\frac{1}{2}$ inches from the end, may be moved with a power of 168 lb. lifting at the other end?	$100-7\frac{1}{2}=92\frac{1}{2}$ *Then reciprocally.* *in. lb. in. lb.* *As* $92\frac{1}{2}$ :: 168 :: $7\frac{1}{2}$: *to* 2072 *ans.*

Oars, rudders of ships, cutting-knives fixed at one end, doors moving on hinges, &c. &c. belong to this order.

THIRD ORDER.

In this lever, we suppose the power and the weight to change places, so that the power may be between the weight and the prop. That there may be a balance between the power and the weight, the intensity of the power must exceed the intensity of the weight, just as much as the distance of the weight from the prop exceeds the power.

It is evident, that the force of the power is increased by the two first kinds of lever, being adapted to produce slow motion by a swift one: but this order serves to produce a swift motion of the weight, by a slow motion of the power. This order may be applied to the muscles of animals, particularly a man's arm.



PROPOSITION 4. Let *e* be the prop of the lever; *a*, *b* and *w*, a weight of 1 lb. placed three times as far from the prop as the power, *p*, which acts at *f*, by the cord going over the fixed pulley, *d*. In this case, the power must be equal to 3 lb. in order to support the weight of 1 lb.

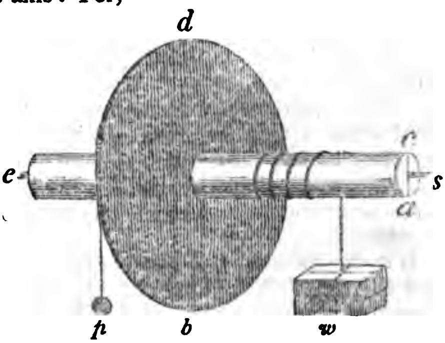
A weight of $1\frac{1}{2}$ lb. placed on a man's shoulder, is no more than its absolute weight: What difference will he feel when the wt. is applied at his elbow, being 12 inches, and in the palm of his hand, being 28 inches from his shoulder; and how much must his muscles draw to support it at right angles, that is extended right out? *Ans.* 18 *lb. at the elbow*, 42 *lb. at the palm*, *dif.* 24.

The second mechanical power is the wheel and axis.

In this machine, the power is applied to the circumference of the wheel, and the weight to the axis, by means of a cord winding round it.

PROPOSITION. An equilibrium is produced in this machine when the weight is to the power, as the diameter of the wheel is to the diameter of the axis: For,

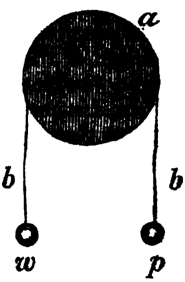
Let *d b* be the diameter of the wheel; *c a*, the diameter of the axis; *w*, the wt.; and *p*, the power.—When the wheel has performed one revolution, by means of the power, *p*, the wt. *w*, is raised through a space equal to the circumference of the axis; therefore, the velocity of the power exceeds the velocity of the weight, as the circumference of the wheel exceeds that of the axis.



Or thus,

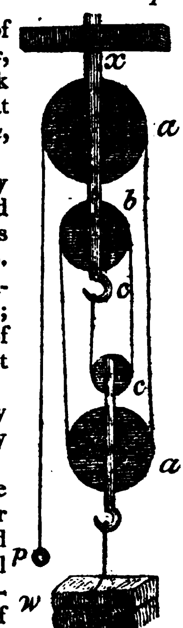
It is plain, that the wheel and axis is a lever of the *first order*, of which the centre of motion is the line, *e s*, in the centre of the axis; the weight, *w*, is applied at the distance of half *c a* from the centre of motion. If, therefore, the radius, or semi-diameter of the wheel, exceed the semi-diameter of the axis, as the weight does the power, the momenta will be equal, and the power and weight will balance each other. To the wheel and axis we may refer cranes, mills, the capstan, winch, &c.

The third mechanical power is a pulley, or system of pullies.



A single pulley that only moves on its axis, may serve to change the direction of the weight or power, *but can give no mechanical advantage*, being only as the beam of a balance whose arms are of equal length and weight.

Thus, *a* is a single pully supporting the equal weights *w* and *p*: The cord *b b*, to which they are appended, will be equally drawn, and the pulley *a* sustain both the weights, or be drawn with a force equal to twice *p*.

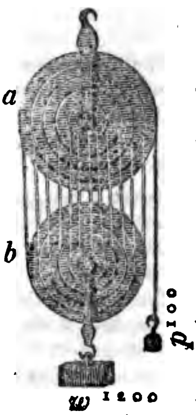


A combination, or system of pullies, whereof two, or more, as *a* and *b*, run in a fixed block *x*, and two others, *c* and *d*, in the movable block which raises the weight, *w*, by pulling the cord at *p*, which goes successively round the pullies, *a*, *d*, *b*, *c*, and is fastened to the fixed block at *s*.

The purchase of this machine is known by considering that the cord is equally stretched throughout, by applying two such weights as will counterpoise each other. Thus, if *p* be 4 lb. it will counterpoise *w*, 16, because *p* is sustained by a single cord, and *w* by 4 folds of the same; therefore a man who pulls at *p* with a force of 100 lb. will poise a weight of 400 lb. or raise it by a small additional force.

A system of pullies can be applied in many cases where other engines cannot; particularly in raising great weights.

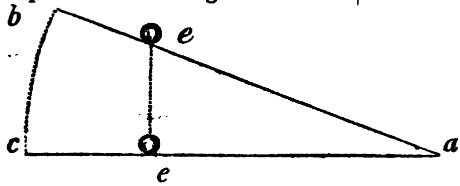
If in the solid block *a*, grooves be cut, whose radi are 1, 3, 5, 7, &c. and in the block *b* other groves be cut, whose radi are 2, 4, 6, 8, &c. and a cord be placed round the grooves, they will answer the purpose of as many distinct pullies—every point in each, moving with the velocity of the cord, in contact with it the whole friction will be removed to the two centres of motion in the blocks *a* and *b*, which is a great advantage over common pullies; therefore, a man pulling at *p* with a force of 100 lb. will poise a weight of 1200 at *w*, because *p* is sustained by one fold of the cord and *w* by 12 folds: *P* will equipoise 1200 (for 12 × 100 = 1200) and a small additional force will raise *w* higher.



THE INCLINED PLANE.

An inclined plane makes an acute or an obtuse angle with the plane of the horizon.

The force with which a body descends down an inclined plane, is to that with which it would descend freely as the elevation of the plane is to its length.



Let *a b*, be an inclined plane, *c b*, its elevation, and *a c*, a plane parallel to the horizon, the body *e* will remain at rest on any part of the horizontal plane, or a small impulse will roll it in any direction. But let the body *e* be placed on the inclined plane *a b*, the force with which it will descend along the plane, will be to its absolute gravity, as the elevation of the plane to its length reciprocally.

EXAMPLE.

Given an inclined plane 15, elevation 3, weight 40 lb: Required the force requisite to sustain it on any part of the plane.

	Elevation.		*lb.*		*Length.*
as	3′	:	40	::	15′
	—				—
	5)		————		5
			8 lb.		*Ans.*

A man who exerts a force of 100 lb. in rolling a body up an inclined plane of the same proportions, will sustain 5 times as much, and a small additional force will roll the body up the plane.

The motion of a body descending down an inclined plane, is uniformly accelerated; therefore whatever was demonstrated concerning the gravitation of bodies, is equally applicable to their descent down inclined planes; the motion in both cases being the effect of the same power: *Therefore,*

The velocity acquired in any given time, by a body descending down an inclined plane, is to the velocity acquired by a body falling freely in the same time, as the elevation of the plane is to the length; *and*

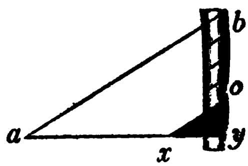
The time in which a body moves down an inclined plane, is to that in which it would fall perpendicularly from the same height, as the length of the plane is to its elevation.

THE SCREW.

The screw is a cylinder which has either a prominent part or a hollow line passing round it in a spiral form, so inserted in one of the opposite kind that it may be raised or depressed at pleasure, with the weight upon its upper part or suspended beneath its lower surface.

In the screw, the equilibrium will be produced when the power is to the weight as the distance between the contiguous threads, in a direction parallel to the axis of the screw, is to the circumference of the circle described by the power in one revolution.

The screw is not a simple machine, because it requires a lever or winch to work it, and then it becomes a compound machine of very great force.



Cut a piece of paper to the figure of the inclined plane, *a b y*, and fold it round a pencil, it will then convey a perfect idea of the screw. While the screw performs one revolution, the weight may be supposed to be raised up one of the spirals or an inclined plane, whose height is equal to the distance between two contiguous spirals, and whose base, *y x*, the circumference of the cylinder and length; *x o*, is the spiral line.

THE WEDGE.

The wedge is composed of two inclined planes, whose bases are joined: Its properties are so well known that it is superfluous to say more than that it is a power of very great force and use.

When the resisting forces, and the powers which act on the wedge, are in equilibrio, the weight will be to the power, as the height of the wedge to a line drawn from the middle of the base to one side, and parallel to the direction in which the resisting force acts on that side.

SCHOLUM.

In theory of the mechanical powers, we suppose all planes and bodies perfectly smooth; levers to have no weight; cords to be perfectly pliable, and the parts of machines, to have no friction; but it is found, in practice, that few compound machines but require a third part more to work them when loaded, than what is sufficient to constitute an equilibrium between the weight and the power.

The inequality of the surface, on which any body moves, is called friction, which prevents the accurate agreement of many experiments in mechanics with theory.

Friction is an uniformly retarding force in hard bodies, not subject to alteration by velocity, except when the body is covered with a woollen cloth; in that case, the friction ceases a little with the velocity.

Friction increases in a less ratio than the weight of the body; being different in different bodies.

The smallest surface has the least friction.

Wheel carriages are used to avoid friction as much as possible: A wheel turns round on its axis, because the several points of its circumference are retarded in succession by attrition, while the opposite points move freely. Large wheels meet with less resistance than smaller from external obstacles, and from the friction of the axle, and are more easily drawn, having their axles level with the horses; but in uneven roads small wheels are used, that in ascents the action of the horse may be nearly parallel with the plane of the ascent, and therefore may have the greatest effect.

Vibration of Pendulums.

A pendulum is a heavy body hanging by a cord or wire, and movable with it upon a centre.

The vibrations of a pendulum are produced by the force of gravitation.

The time of ascent and descent of a pendulum, is equal to the time in which a body falling freely, would descend through eight times the length of the pendulum.

The times in which pendulums of different lengths perform their vibrations, are as the square roots of their lengths.

The lengths of pendulums, are to each other, reciprocally, as the square of their vibrations in the same space of time.

A pendulum, whose length is 39.2 inches from the point of suspension to the centre of oscillation, will make 60 vibrations in a minute. *This is called standard length.*

EXAMPLE.

What difference is there in the length of two pendulums, the one vibrating $\frac{1}{2}$ seconds, or 120 times in a minute, the other vibrating double seconds, or 30 times in a minute?

$120,^2 = 14400$ square vibrations.
$30,^2 = 900$ do.
$60,^2 = 3600$ do. do. in 1 hour.

Then it will be reciprocally,

	vib.		inches.			inches.	
as	3600	:	39.2	::	14400 :	9.8	*vib. half seconds.*
					900 :	156.8	*vib. double seconds.*

Answer.

If the length of a pendulum be given, and the number of vibrations in a minute required.

RULE.—*As given length : standard length :: square of 60 vib. : square of the number required.*

EXAMPLE.

Required the number of vibrations in a minute, by a pendulum 156.8 inches long.

$60^2 = 3600$ *in.*   *in.*   *sq. vib.*   *sq. vib.*

Then as 156.8. : 39.2 :: 3600 : 900

and $\sqrt{900} = 30$ *vibrations, Ans.*

What difference is there between the number of vibrations made by a pendulum 6 inches long and another of 12 inches, in an hour's time? *Ans.* 2695.08.

What difference is there in the length of two pendulums, the first swinging 30 and the second 100 times in an hour?

Ans. 42806 *feet* 4.8 *inches.*

Required the length of the pendulums that will swing once in a third, once in a minute, once in an hour, and once in a day.

Ans.
1. *A pendulum that vibrates thirds* .0108 *inches.*
2. *Once in a minute,* 2 *miles* 1 *fur.* 32 *poles* 4 *yards.*
3. *Once in an hour,* 8018 *miles* 1 *f.* 18 *p.* 1 *y.*
4. *Once in a day,* .4618472 *miles* 5 *f.* 32 *p.* 4 *y.*

English (2–4)

Miscellaneous Exercise.

1. The globe of the earth, under the line, is 360 degrees in circumference, each degree $69\frac{1}{2}$ miles; and this body being turned on its own axis in 23 hours 56 minutes, at what rate an hour are the inhabitants of Bencoolen, situate under the line, carried about from west to east by this rotation? *Ans.* $1045\frac{1}{3}\frac{4}{5}\frac{1}{9}$ *miles.*

2. Three merchants, A, B and C, trade in company; at making up accounts, it appears that A and B together gained £13 10; B and C together £12 12; and A and C together £11 16 6: What did they severally gain?

Ans. A £6 7 3, B £7 2 9, and C £5 9 3.

3. Some others advance in trade, viz. W, X and Y raised £350 10; W, X and Z £344 10; X, Y and Z made up a stock of £400, and W, Y and Z contribute £378 4: in the conclusion, they parted with their joint property for 450 guineas: What did they gain or lose by their adventure?

Ans. £18 11 4 *sterling loss.*

4. There is a mast or pole, $\frac{1}{9}$ of its length stands in the ground, 12 feet of it in the water, and $\frac{5}{6}$ of its length in the air, or above water: What is the whole length? *Ans.* 216 *feet.*

5. What quantity of water must I add to a pipe of mountain wine, value £33, to reduce the first cost to 4*s.* 6*d.* per gallon?

Ans. $20\frac{2}{3}$ *gallons.*

6. There are 2 pieces of linen; the one is 9 yards shorter than the other, and cost £3 $1\frac{1}{5}$s.; the other piece, at the same price, cost £3 12s.: I demand how many yards are in both pieces, and the price of 1 yard.

Ans. longest 60, *shortest* 51 *yards, at* $1\frac{1}{5}$ *shillings per yd.*

7. A piece of satin cost a certain sum, and being sold for £3 10s. there is lost $\frac{2}{5}$ in a shilling: What was the first cost?

Ans. £5 16 8.

8. With 13 gallons Canary, at 6s. 8*d.* the gallon, I mingled 20 gallons white wine, at 5s. and 10 gallons cyder at, 3s. the gallon: How am I to sell a quart of this mixture, so as to clear 10 per cent.? *Ans.* $16\frac{2}{4}\frac{7}{3}$*d. per quart.*

9. A is dispatched on a commission from Washington to Boston, distant by computation, say 350 miles, and his route is settled 22 miles a day; 4 days after, B is sent after him with fresh orders and is to travel 32 miles a day: Whereabout on the road will B overtake A? *Ans.* $68\frac{2}{5}$ *miles on this side Boston.*

10. If 6 lb. pepper be worth 13 lb. ginger, and 19 lb. ginger be worth $4\frac{1}{4}$ lb. cloves, and 10 lb. cloves be equal to 63 lb. of sugar, at 5*d.* per lb. what is the value of 1 cwt. of pepper?

Ans. £7 19 3.

11. If 30 men can perform a piece of work in 11 days, how many men must be added to finish another 4 times as big in $\frac{1}{5}$ of the time? *Ans.* 600 *men.*

12. A may-pole, 50 feet 11 inches high, at a certain time of day casts a shadow 98 feet 6 inches long: I would hereby find the breadth of a river that is running 20 feet 6 inches from the foot of a steeple, 300 feet 8 inches high, when the extremity of the shadow of the steeple reaches 30 feet 9 inches beyond the stream. *Ans.* 530 *feet* 5 *inches, nearly.*

13. A tradesman increased his estate annually $\frac{1}{3}$, abating £100, which he usually spent in his family; and at the end of $3\frac{1}{4}$ years found that his neat estate amounted to £3154 11 8: What had he at out-setting? *Ans.* £1411 12 $9\frac{3}{4}$.

14. A can do a piece of work in 10 days, B alone in 13 days: Set them both about it together, in what time will it be finished? *Ans.* $5\frac{1}{2}\frac{5}{3}$ *days.*

15. A cistern is supplied with water by one pipe, of such bigness that if the cock A, at the end of the pipe, be set open, the cistern will be filled in $\frac{1}{2}$ an hour; but at the bottom of the cistern are two other cocks, B and C, whose capacities are such that, by the cock B, set open alone, (all the rest being stopt) the cistern, supposed to be full, will be emptied in $1\frac{1}{7}$ hours, and by C alone in $2\frac{1}{3}$ hours. Now, because more water will be infused by the cock A, than can be expelled by the cocks B and C in the same time, the question is, in what time the cistern will be filled, if all the three cocks are set open at once? *Ans.* $1\frac{9}{61}$ *h. nearly.*

16. There is an island 36 miles in compass: Now if, at the same time, and from the same place, two footmen, A and B, set forward to travel round said island, and follow each other in such a manner that A travels every day 9 miles, and B 7 miles: In what time will they meet again—also, how many miles, and how often round the island has each travelled? *Ans.* $4\frac{1}{2}$ *times for A,* $3\frac{1}{2}$ *times round for B,* 18 *days before they come together.*

17. If £100, put forth at interest at a certain rate, will, at the end of 2 years, be augmented to £$112\frac{36}{100}$, compound interest being computed: What principal and interest was due at the first year's end? *Ans.* £106.

18. If £100, put forth at interest at a certain rate, will, at the end of 3 years, be augmented to £115.7625, compound interest, what principal and interest will be due at the first year's end? *Ans.* £105.

19. An accountant told a gentleman who had constantly 8 persons at his table, that he would gladly make a ninth, and was willing to give 200 guineas for his board so long as he could place the same company at dinner, differently from any one day before; this being accepted, what did his entertainment cost him per year? *Ans.* $50\frac{25200}{36288}$*d.*

20. A has kerseys at £4 5 0 a piece, ready money; in barter he charges them at £5 6 0, and half of that required down; B has flax at 3*d.* per lb.: How ought he to rate it in truck, not to be hurt by the extortion of A? *Ans.* $4\frac{31}{32}$*d. or 5d. nearly.*

21. Put out £384 at interest, and in $8\frac{1}{4}$ years there were £542 8*s.* found to be due: What rate of interest could then be implied? *Ans.* 5 *per cent.*

22. M of Amsterdam, orders N of London, to remit O of Paris, at 54*d.* sterling per crown, and to draw on P of Antwerp, for the value at $33\frac{1}{2}$ shillings Flemish per pound sterling; but as soon as N received this commission, the exchange was on Paris at $54\frac{1}{2}$*d.* per crown: At what rate of exchange ought N to draw on P, to execute his orders, and be no loser? *Ans.* 33*s.* $2\frac{34}{109}$ *Flem.*

23. Three persons enter joint trade together, to which A contributes £210, B £312; they clear £140, whereof £37 10*s.* belongs of right to C; C's stock, and the several gains of the other two, are required?

Ans.

C's stock,	£190	19	6
A's gain,	41	4	$8\frac{288}{522}$
B's gain,	61	5	$3\frac{234}{522}$

24. A, B and C, will trench a field in 12 days; B, C and D, in 14; C, D and A, will do it in 15, and D, A and B, in 18 days: In what time will it be done by all of them together, and by each of them singly?

Ans. 10.83095 *days by them all together.*

A	47.848 *days.*	*singly.*
B	38.969 *do.*	
C	27.194 *do.*	
D	111.176 *do.*	

25. A, B and C, make a company; A put in share of the stock for 5 months, and laid claim to $\frac{1}{5}$ of the profit: B put in his for 8 months, C advanced £400 for 7 months, and required on the balance, $\frac{2}{3}$ of the gain; the stocks of the other two adventurers are sought. *Ans.* { £168, *A's stock.* 70, *B's do.* }

26. If £120$\frac{1}{5}$ is to be distributed among 3 persons, A, B and C, in such sort that as often as A takes 5, B shall take 4, and as often as B takes 3, C shall take 2: What is each man's share?

Ans. A £51$\frac{60}{105}$, *B* £41$\frac{27}{105}$, *C* £27$\frac{53}{105}$.

27. What sum of money, at 3$\frac{1}{2}$ per cent. per annum, will clear £38 10 in a year and quarter's time? *Ans.* £880.

28. X, Y and Z, working together, will complete a stair-case in 12 days; Z can do it alone in 24 days, X in 34: In what time can Y get it done himself? *Ans.* 81$\frac{3}{5}$ *days.*

29. Two merchants enter into partnership, and each put in 12 pieces of cloth; but those of A cost £48 more than those of B: The cloth being sold, they find they have gained £273$\frac{1}{5}$, of which B has for his share £122$\frac{2}{5}$: The question is, to know at how much a piece each of their cloths was valued.

Ans. A's at £21, *B's at* £17 *per piece.*

30 A person dying, left his wife with child, and by his will ordered, that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she had a daughter, he appointed the mother $\frac{2}{3}$, and the daughter $\frac{1}{3}$; but it happened she was delivered of both a son and daughter, by which she lost in equity £2000 more than if it had been only a girl: What would have been her dowry if she had only a son?

Ans. £1750.

31. In a distress at sea, they threw out 17 hhds. sugar, worth £34 per hhd. the worth of which came up to but $\frac{4}{7}$ of the indigo they threw overboard; besides which they threw out 13 iron guns, worth £18 10 a piece; the worth of all these came to $\frac{4}{7}$ of $\frac{9}{13}$ of the ship and cargo: What value came into port?

Ans. £4337 15 6$\frac{2}{3}$.

32. Three persons purchase a West-India ship, toward the payment whereof A advanced $\frac{1}{8}$, B $\frac{1}{7}$, and C £140: How much paid A and B, and how much of the vessel had C?

Ans. A paid £267 $\frac{3}{11}$, *B* £305$\frac{5}{11}$; *C's part* $\frac{11}{56}$.

33. A and B join stocks and purchase brandies: A's stock was £19 19 8 more than B's; now by selling their brandy at 55 shillings per anker, A cleared £74 11, and B £52 10: The quantity of brandy is required, and the gain upon the anker.

Ans. 88 *ankers.* *Gain,* £1 8 10$\frac{1}{11}$.

34. A lets B have a hhd. of sugar, of 18 cwt. worth 31*s.* for 42*s.* the cwt. $\frac{1}{3}$ of which he is to pay in cash; B has paper which cost 14*s.* the ream, which he gives A at 15*s* and 6*d.*: Which gained most by the bargain? *Ans. A gains* £7 9 3 *more than B.*

35. A, for a 9 months adventure, received £20, B for one of 7, received 23 guineas and 1s. and 9d. over; and C, for lying out of his contribution 5 months, had a title to £32 Irish ; the total of their adventures multiplied into their respective times, is 640 : What were their particular stocks ?

A £18 3 6. B £30 13 5. C £52 6 10¾.

36. A had 15 pipes of Malaga wine, which he parted with to B, at 4½ per cent. profit, who sold them to C for £38 11 6 advantage; C made them over to D for £500 16 8, and cleared thereby 6½ per cent.: What did this wine cost A per gallon ?

Ans. £0 4 4½ *per gallon.*

37. A, B and C, make a company, and put in together, £3860; A's money was in 3 months, B's 5, and C's 7 months; they gained £234, which was so divided that ½ of A's gain was equal to ⅓ of B's, and ⅓ of B's to ¼ of C's : What did each gain and put in? *Ans. A gained* £52, *B* £78, *C* £104; *A's stock* 1400, *B's* 1260, *C's* 1200.

38. If 3 dozen gloves be equal in value to two pieces of ribbon, 3 pieces of ribbon to 7 doz. points, 6 doz. points to 2 yards Flanders lace, and 3 yards of Flanders lace to 81 shillings, how many dozen pair of gloves may be bought for 28 shillings ?

Ans. 2 *dozen.*

39. A, with intention to clear 30 guineas on a bargain with B, rates hops at 16 pence the lb. that stood him in 10d. : B, apprised of that, set down malt, which cost him 10s. 10d. per barrel, at an adequate price : How much malt did they contract for ?

Ans. 420 *bushels.*

40. A, in order to put off to B 720 ells of damaged Holland, worth 5s. per ell, at 6s 8d. proposes, in case he has half the value in money, to give B a discount of 10 per cent. ; the rest A is to take out in saffron, which B, apprised of the whole management, rates, in justice, at 30s. per lb. : Pray what was it really worth in ready money, and what quantity of saffron was he to deliver on the change. *Ans.* 72 lb. 20s. *per* lb.

41. Laid out on a lot of muslin, £488 11 ; on examination of which, two parts in seven proved damaged, so that I could make but 5s. and 6d. per yard of the same, and by so doing lost £49 6 : At what rate per ell am I to part with the undamaged muslin to make up said loss ? *Ans.* 12s 3d.$\frac{12}{160}$ *per ell.*

42. A young hare starts 5 rods before a greyhound and is not perceived by him till she is up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog, on view, makes after her at the rate of 20 : How long will the course hold, and what ground will be run, beginning with the outsetting of the dog?

Ans. 1702¼ *feet and* 58$\frac{1}{32}$ *seconds.*

43. A reservoir of water hath 2 cocks to supply it, by the first it may be filled in 44 minutes, by the second just in an hour; and it hath a discharging cock by which it may, when full, be

emptied in $\frac{1}{2}$ an hour: Now suppose these three cocks, by accident, should all of them be left open, and the water should chance to come in; what time would this cistern be in filling?

Ans. $2\frac{3}{4}$ *hours.*

44. There are 100 stones, which lie 1 yard from the other; and ther is a person employed to gather them up, one by one, and bring them to a basket that stands one yard from the first stone: The question is, to know how many miles he will go before the last stone is brought into the basket?

Ans. 5 *miles*, 5 *furlongs*, 220 *yds.*

45. What ought a man to give down in ready money, for the reversion of £1000 a year, to continue 20 years on a lease that cannot commence till 10 years hence, allowing the purchaser compound interest at 6 per cent. per ann.? *Ans.* £8570 19 10.

46. A minor of 14, had an annuity left him of £70 a year, the proceeds of which, by will, was to be put out both principal and interest yearly, as it fell due at 5 per cent. until he should arrive at 21 years of age; the utmost improvement being thus made on this part of his fortune, what had he to receive?

Ans. £569 18 $9\frac{1}{2}$.

47. Value the lease of a house in tolerable repair the rent £54 17 a year, the ground rent 7 guineas *, 3 years of it only to come, the rent payable every 6 months, discount per compound interest on this kind of purchase at 10 per cent.

Ans. £120 10 11+.

48. A lease for 7 years is agreed for at £250, fine on the old rent £44 a year; but the purchaser being desirous to reduce the rent to £20 per annum, and pay a proper fine, computing, as before, after the rate of £10 a year: To what must the fine be advanced? *Ans.* £366 16 10.

49. Suppose I would add 5 years to a running lease of 15 years yet to come, the improved rent being £186 7 6 per annum, what ought I to pay down for the favour, discount being allowed at 4 per cent. per annum, compound interest? *Ans.* £460 14 $1\frac{1}{2}$.

50. A person dying, left a piece of land on a 20 years lease, for £31 10 tax free; the profits of this he bequeathed to the poor of the parish where he was born, for the first 4 years after his decease; the proceeds of the next 6 years he left to the poor of the parish where he lived, and the residue, or last 10 years, he left to his niece: Now this young woman having money, and being willing to come into immediate possession of her uncle's land, comes to a compromise with the parishes on a discount of 10 per cent.: What did it cost her? *Ans.* £193 11 0.

51. A and B are on opposite sides of a wood 134 fathoms about; they begin to go round it both the same way at the same time—A goes 11 fathom in 2 minutes and B 17 fathom in 3; the question is, how many times will they surround this wood before the

* Guinea £1 1s.

nimbler overtakes the slower. *Ans. A* 16$\frac{1}{2}$ *times, B* 17 *times round before they meet.*

52. In what time will the morning gun be heard in Newark, allowing the distance, between York and Newark, to be 9 miles?

Ans. 41$\frac{37}{115}$ *seconds.*

NOTE—*Sound, (if not interrupted) is found, by experiment, to move about* 1150 *feet in a second of time.*

53. If I see the flash of a piece of ordnance fired by a vessel at sea, and hear the report a minute and three seconds afterwards, how far is she off, reckoning the passage of sound as before?

Ans. 13 *miles* 5 *fur.* 30$\frac{1}{11}$ *poles.*

54. Divide £450 between A, B, C, D, E and F, and give B twice as much as A, + 5, C three times as much as A, + 1, D as much as B and C together, + 4, E as much as D and C, + 5, and to F, as much as A, B, C, D and E, + 6. Query the amount of each? *Ans.* A £10, B 25, C 31, D 60, E 96, and F 228.

55. A gentleman a chaise did buy,
A horse and harness too;
Which cost the sum of three score pound—
Upon my word 'tis true.
The harness came to half the horse,
The horse to half the chaise,
But if thou canst their values find,
Take them and go thy ways.

Ans. chaise £34 5 8$\frac{4}{7}$, *horse* £17 2 10$\frac{2}{7}$, *harness* £8 11 5$\frac{1}{7}$.

56. When first the marriage knot was ty'd,
Betwixt my wife and me;
My age did hers as far exceed,
As three times three does three;
But after ten and half ten years,
We man and wife had been,
Our ages then appear'd to be,
As eight is to sixteen.
Now Tyro, skilled in numbers, say,
What were our ages on our wedding day?

Ans. her age was 15, *his* 45.

57. A vintner has a cask of wine, quantity 500 gallons; of which he drew 50 gallons, and then filled it up with water, and did so 5 times: I demand how much wine and water there was left in the cask? *Ans.* 295$\frac{49}{200}$ *wine,* 204$\frac{151}{200}$ *water.*

58. A vintner has a cask of wine containing 500 gallons, of which he drew 50 gallons and filled it again with water; he drew the next time 40, the next time 30, the next time 20, and the next time 10 gallons, still filling it with water as he drew: I demand how much wine and how much water is in the cask?

Ans. 366$\frac{3804}{31250}$ *wine,* 133$\frac{27446}{31250}$ *water.*

59. Four merchants gain £98 in trade; B takes twice as much as A, +£2, C thrice as much as B, +£4, and D as much as B and C together, +£6: How much is each man's part?

Ans. A £4, B 10, C 34, D 50.

60. Says A to B, give me 5 shillings of your shillings, and I will have 3 times as many shillings as you have left; no says B, it is more reasonable that you give me 5 shillings of your shillings, for then our money will be equal. *Ans.* A 25, B 15.

61. One being asked the time of the day, answered "the time passed from noon, is equal to $\frac{2}{13}$ of the time remaining till midnight: What o'clock was it? *Ans.* 1 *hour* 36 *minutes* P. M.

62. If from a piece of cloth I cut $\frac{9}{16}$ — 12 yards, then there is left $\frac{3}{8}$ + 20: I demand how many yards were in the whole piece? *Ans.* 128 *yards.*

63. If a mother and two daughters can spin 3 lb. of flax in one day, and that the mother can do it in $2\frac{1}{2}$ days, and the eldest daughter in $2\frac{1}{4}$ days, in what time would the youngest do it alone? *Ans.* $6\frac{3}{7}$ *days.*

64. Lent £75, at compound interest, which, at the end of 4 years, amounted to £98.2897: I demand what interest was due at the end of the first year? *Ans.* £5 5*s.*

65. A young lady was left a legacy of £1000, which her guardian (to improve as much as possible) lent out at compound interest. At the end of 8 years the young lady, being of age, takes the management of her fortune into her own hands, when her guardian pays her £1593.848: I demand what interest was due at the end of the first year? *Ans.* £60.

66. A vintner had a hhd. of sherry wine, containing 63 galls. of which he drew 10 gallons, and filled it with malaga; he drew the next time 12, the next 20, and the next time 30 gallons, still filling it with malaga as he drew: I demand the quantity of sherry and malaga remaining in the cask, if, after this, he draws out one half without filling it?

Ans. $7\frac{111633}{166698}$ *sherry*, $23\frac{138414}{166698}$ *malaga.*

67. How many pears can I have for 11*s.* 3*d.* if 70 almonds are worth 50 chesnuts, and 48 chesnuts 1 pomegranate, and 28 lemmons 18 pomegranates, and 25 pears 10 lemmons, and 108 almonds cost $2\frac{1}{4}$*d.*? *Ans.* 375 *pears.*

68. A merchant loads 2 vessels; in A 150 hhds. and in B 240 hhds. of wine; the ships coming by a certain pass, A pays for toll 1 hhd. and receives 12 shillings back; B pays also 1 hhd. and 36 shillings more: I demand at how much each hhd. was valued. *Ans.* £4 12*s.*

69. A puts in £100 for 14 months, draws £112 gain;
B puts in —— for 12 months, - - - 128
C puts in £100 for ——, - - - - - 116
Required B's stock and C's time.

Ans. *B's stock* £$117\frac{5}{7}$, *C's time* $18\frac{2}{3}$ *months.*

70. Divide £1000 between A, B, C, D and E, and give B twice as much as A, (−£10) C as much as A and B together, (+£10) D as much as B and C together, (−£40) and E as much as A, B, C and D, (+£20): Query each man's share.

Ans. A £50, *B* 90, *C* 150, *D* 200, *E* 510.

71. One at a country fair had a mind to a string of 20 fine horses; but, not caring to take them at 20 guineas a head, the jockey consented, that if he thought good to take them at a single farthing for the first, and doubling it only to the 19th, he would give the 20th into the bargain: This being agreed to, what did they sell for a piece? *Ans.* £27 6 $1\frac{4}{10}$.

72. A minor, 12 years of age, was left an estate of £150 per annum; his guardian was left, by his father's will, £40 per ann. for his board, education, and other contingent charges, and was to put out the surplus at interest for his benefit, at 5 per cent. simple interest: Now, supposing no loss of principal or interest, what sum was his guardian to pay him when he was of age?

Ans. £1188.

73. A set out from London for Lincoln, at the very same time that B set out from Lincoln for London: at 8 hours end they meet on the road, and it then appeared that A had travelled $2\frac{1}{2}$ miles an hour more than B: At what rate an hour did each travel, if the distance between these towns be 100 miles?

Ans. A $7\frac{1}{2}$, *B 5 miles an hour.*

74. A leaves Exeter at 10 o'clock in the morning, for London, and goes at the rate of 2 miles an hour, without intermission; B sets out from London for Exeter at 6 the same evening, and rides 3 miles an hour constantly: The question is, whereabouts on the road will they meet, if the distance of the two cities be 130 miles? *Ans.* $61\frac{1}{5}$ *miles from Exeter.*

75. A and B, in partnership, equally divide the gain; A's money was £84 12 6, for 19 months, and B's for no more than 7 months: The adventure of the latter is sought?

Ans. £229 13 $11\frac{1}{7}$.

76. Suppose I would exchange £527 17 6 English, for dollars, at 4*s.* 6*d.* ducats at 5*s.* 8*d.* and crowns at 6*s.* 1*d.* per piece, and would have 2 dollars for 1 ducat, and 3 dollars for 2 crowns, how many of each sort must I have?

Ans. 309 *crowns,* 927 *dollars,* $463\frac{1}{2}$ *ducats.*

77. A gay young fellow had £18200 left him by an old uncle, to whose memory he expended 3 per cent. of his whole fortune, in a sumptuous funeral and monument; 9 per cent. of the remainder he made over to his cousins, forgotten by the old man; with $\frac{2}{7}$ of the remainder he bought a fine seat, and with $\frac{1}{8}$ of the residue a stud of horses: he squandered £550 in gaming; and after having lived at the rate of £2000 a year, for 19 months, and ruined his health, he died: What was there left for his sister, who was heir-at-law? *Ans.* £6324 0 11.

FINIS,