A

TREATISE

ON THE

Stability of Retaining Walls.

ILLUSTRATED BY

ENGRAVINGS AND DIAGRAMS.

BY

JOHN MURRAY,

CIVIL ENGINEER, M. INST. C.E.

FIRST PART.

LONDON:

PUBLISHED BY JOHN WEALE, 59, HOLBORN.

1855.

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ADDRESS.

Having on many occasions been required to make drawings of Retaining Walls for works brought under my notice as an Engineer, in Canals, Railways, Rivers, and Docks, I have, like others, only had experience to guide me. On referring to the meagre publications in the English language, I found their contents, for the most part, based on theory, with little or nothing practical on the subject. The French authors, who are more numerous, have entered minutely into the former, but have overlooked much appertaining to the latter: the theorems at last, are so complicated, that few engineers attempt to unravel them, and consequently no reliance is placed upon the results.

I have thus been induced to put some observations and notes into the form of a Treatise on the Stability of Retaining Walls. In commencing with simple, and arriving at more complex forms, I have made it my aim to give the principal theories in the easiest manner possible; and to draw conclusions from them applicable to practice. It will be seen how much the one is at variance with the other. If some of these theories be adopted, what a waste of material would be improperly applied: if practice only be followed, how much money would be spent, on the notion of obtaining strength to the structure, by placing the material where it could not be of the slightest use. In this, as in most instances connected with Physics, the one must be combined with the other. I have deemed it advisable therefore, after investigating the proper method of ascertaining the resistance of Retaining Walls, to give profiles of those constructed by the most eminent engineers of this country, and of the continent, with calculations on the thrust to which each would be exposed, and their resisting power.

Professional duties have unfortunately interrupted my work; and, for the present, I have only had leisure to complete the First Part of the subject. The remainder, connected with Counterforts and Casemated Revetments, as well as examples of walls constructed by English and foreign engineers, must, therefore, form a Second Part; trusting to send it in a short time to the press, and that my labours so far will be of some use to Practical Science.

JOHN MURRAY.

11, Great Queen Street, Westminster,

June, 1855.

THE

STABILITY OF RETAINING WALLS.

CHAPTER I.

ON THE CENTRE OF GRAVITY OF WALLS.

PREVIOUSLY to entering on the subject of the Stability of Retaining Walls, it is necessary to understand in what manner the pressure is applied to them, and how they are able to resist it. A knowledge must therefore first be acquired of the centre of gravity of the pressing power; and then, further, how the position of that point affects differently formed walls.

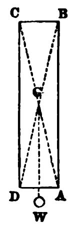
If a power be applied to a pivot or fulcrum of a given body, and so support it without disturbing the remaining parts, the point acted upon is in the direction of the centre of gravity.

The stability of the body depends upon the position in relation to the base of the line of direction of the centre of gravity. If that line fall within it, the body will stand firm; if on its edge, it is in a state of instability; and if it fall without the base, the body will overturn.

Its stability also depends upon the height of the centre of gravity. If that point be near the base it will remain firm: but if the base be small in reference to the height, causing the centre of gravity to be elevated, a slight force will overthrow it.

To find the centre of gravity of a Parallelogram.

Draw diagonals crossing each other, the intersecting point is the centre of gravity.



If we consider the superficial area of the plane to express weight; it can be collected in any point in the line of direction passing through its centre of gravity. Thus in the parallelogram ABCD, the height AB and the base AD may be called a and b respectively; its superficies will then be ab; which can be collected in a weight, W; hanging from a perpendicular line drawn from the centre of gravity G.

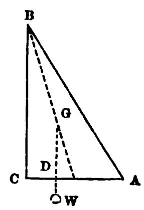
To find the centre of gravity of a Triangle.

From two of the angles draw lines bisecting their opposite sides. The point of intersection is the centre of gravity.

Let a be the length of the line drawn from the vertex which bisects the base.

d the distance of the vertex from the centre of gravity.

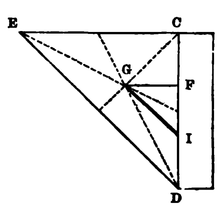
Then $d = \frac{2a}{3}$



If b represent the base AC of a right angled triangle ABC. Then a perpendicular let fall from the centre of gravity G on the base, trisects it in the point D.

Calling a the height CB, and b the base AC,

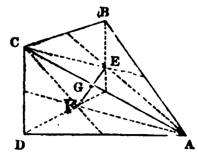
Then $\frac{ab}{2}$ is the value of W, in which we suppose may be collected the weight, or what is the same thing, the superficial area of the triangle ABC.



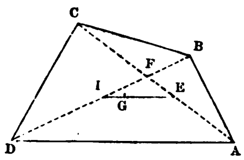
If earth, take a slope of 45°, then CE = CD.

The centre of gravity G, of the triangle DCE, is situated at $\frac{2}{3}$ of the perpendicular DC from the point D. If GF be drawn parallel to EC, then DF $= \frac{2}{3}$ DC. And if GI be drawn parallel to ED then DI $= \frac{1}{3}$ DC.

To find the centre of gravity of a Trapezium.



Divide it into two triangles by the diagonal AC, and find the centre of gravity, E and F, of these triangles, join EF; then the common centre G, or EG, $= \frac{ADC \times EF}{ABCD}$



Let ABCD be a quadrilateral figure. Draw the two diagonals. Bisect one of them, DB in I. Make AE equal to CF, the point of intersection of the diagonals. Then join the points I and E, and the centre of gravity is found at $\frac{1}{3}$ of IE in G.*

To find the centre of gravity of any other plane figure.†

Divide it into several triangles, and find the centre of gravity of each; then connect them together, and find their common centre, connect this and the centre of a third, and find the common centre of them, and so on; always connecting the last found common centre to another till the whole be included; and the last is the centre of gravity of the figure.

* Laisné Aide Mémoire du Génie.

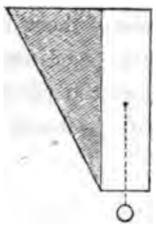
† Gregory's Mathematics for Practical Men, p. 212.

FORMULÆ FOR FINDING THE CENTRES OF GRAVITY OF DIFFERENTLY FORMED WALLS; ALL OF THEM BEING OF THE SAME HEIGHT, AND OF THE SAME SECTIONAL AREA.

The following symbols are employed:—

h = 30 feet the height common to all the walls.
b = the thickness at top of each.
c = the slope or batter in parts of the height.
x = the arm of the lever between the point of rotation and the perpendicular let fall from the centre of gravity.

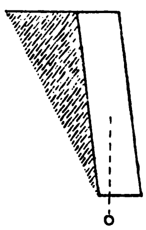
1. *The vertical wall with parallel sides.*



Calling b = the thickness at top 7·5 feet

$$\text{Then} \quad x = \frac{b}{2} = \frac{7 \cdot 5}{2} = 3 \cdot 75 \text{ feet.}$$

2. *The leaning wall with parallel sides, of $\frac{1}{8}$ of the height.*



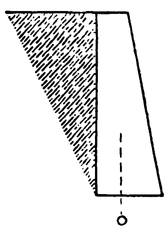
Let b = the thickness at top......... 7·5 feet.
c = the slope...................... 8

$$\text{Then } x = \frac{h + bc}{2c}$$

Substituting the true value for these symbols we have

$$x = \frac{30 + (7 \cdot 5 \times 8)}{2 \times 8} = 5 \cdot 625 \text{ feet.}$$

3. *The wall with an exterior slope of $\frac{1}{5}$ of the height.*

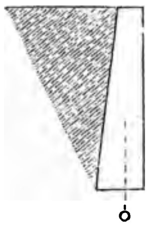


b = the thickness at top .... 4·5 feet.
c = the slope 5

$$\text{Then } x = \frac{2\,hb + 3\,b^2c}{3\,h + 6\,bc} + \frac{2h}{3c}$$

$$x = \frac{(2 \times 30 \times 4 \cdot 5) + (3 \times 4 \cdot 5^2 \times 5)}{3 \times 30 + (6 \times 4 \cdot 5 \times 5)} + \frac{2 \times 30}{3 \times 5} = 6 \cdot 55 \text{ feet.}$$

4. *The wall with a perpendicular exterior side, and a sloping side next the earth of $\frac{1}{8}$ of the height.*



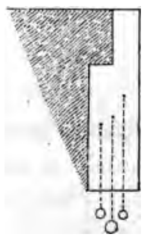
Let b = the thickness at top .... 5·625 feet.

c = the slope 8

$$x = \frac{hb + 3b^2c}{3h + 6bc} + \frac{h}{3c}$$

$$= \frac{(30 \times 5{\cdot}625) + (3 \times 5{\cdot}625^2 \times 8)}{3 \times 30 + (6 \times 5{\cdot}625 \times 8)} + \frac{30}{3 \times 8} = 8{\cdot}4375 \text{ ft.}$$

5. *Wall with perpendicular sides, but with an offset next the earth.*



Let b = the thickness at top .... 6·00 feet.

d = the breadth of the offset...... 2·25

e = the height above the base... 20·00

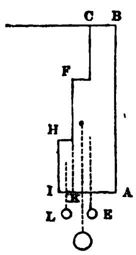
the other symbols, as before.

$$\text{Then } x = b + \frac{d}{2} - \frac{\left(\frac{b}{2} - \left(\frac{d}{2} + b\right)\right) \times hb}{ed + hb}$$

Substituting for this equation, its true value, we have

$$x = 6 + \frac{2{\cdot}25}{2} - \frac{\left(\frac{6}{2} - \left(\frac{2{\cdot}25}{2} + 6\right)\right) \times (30 \times 6)}{(20 \times 2{\cdot}25) + (30 \times 6)} = 3{\cdot}825 \text{ feet.}$$

6. *Wall with two offsets next the earth.*



		Feet.
Let b =	the thickness at top.................	6·00
	the breadth of each offset	1·50
	the height of the upper above the base................................	20·00
	ditto of the lower	10·00

The arm of lever of that part of the section ABCFK must first be found, as in the last case.

Let it be AE = 3·5357 feet.

$$\text{Then AK} + \frac{\text{KI}}{2} = \text{AL}$$

$$\text{And } \frac{(\text{AL} - \text{AE}) \times \text{ABCFK}}{\text{ABCFHI}} - \text{AL} = x \text{ the arm of the lever of the wall.}$$

Example.

$$\text{AL} = 6 + 1{\cdot}5 + 0{\cdot}75 = 8{\cdot}25 \text{ feet.}$$

$$\text{Then } x = \frac{(8{\cdot}25 - 3{\cdot}5357) \times 210}{225} - 8{\cdot}25 = 3{\cdot}85 \text{ feet.}$$

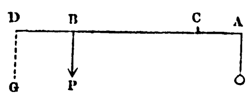
CHAPTER II.

ON THE LEVERAGE OF WALLS.

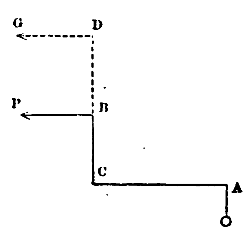
IT is requisite before proceeding further with the subject to consider the effect of levers, which in a great measure determine the thickness of walls when in equilibrium with the pressure which they have to sustain. Through want of attention to a few simple rules many persons are led into error.

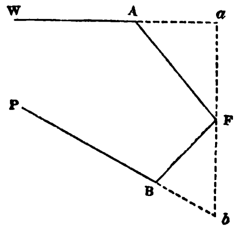
Two weights or forces, acting perpendicularly upon a straight lever, will balance each other, when the distances from the fulcrum are reciprocally proportional.

Let AB be a lever, with its fulcrum at C, and at A, its extremity, a weight be suspended, a power P may be applied at the point B, in equilibrium with it. If we alter this power to the extremity D of the lever CD, then in order to keep up the equilibrium we must multiply the weight or power applied at B by the length of the arm CB, and divide the product by the whole length, CD; the quotient will be the weight or power G capable of being applied at the extremity D.



If this lever be bent, making an angle ACB, and the power P be applied to the extremity B of the arm CB, where it acts according to a direction BP, and it be wished to transfer it to the extremity D of the lengthened lever, we must multiply the force of that power by the arm CB, and divide the product by the arm CD; when the quotient will give the force of the power G.





In bent levers the length of the arms are always considered as perpendiculars drawn from the fulcrum to the lines of direction in which the weight and the power acts:—

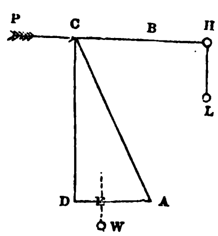
That is to say, that the virtual lengths of the arms are the perpendiculars F*a* and F*b*, to the directions WA and PB in which the weight and power acts. An equilibrium will ensue, (the arms being considered void of weight) when

$$W \times Fa = P \times Fb.$$

Two homogeneous planes, each at the extremity of the arm of a lever are said to be in equilibrium when the superficial areas of these planes are in the reciprocal ratio of the length of the lever.

As the planes of which we are about to speak represent profiles of earth or masonry, it is requisite to pay attention not only to their superficial area, but also to their specific gravity. For example, if we have a lever whose point of support is in the middle, and that a plane of 10 square feet of masonry be suspended at the extremity of one arm, this plane cannot be in equilibrium with another of 10 square feet of earth; because a cubic foot of masonry weighs more than a cubic foot of earth, nearly in the proportion of 3 to 2. To make the above lever in equilibrium with masonry, it would be necessary therefore to take $\frac{2}{3}$ of the power.

There being given a triangular profile of a wall ACD, it is required to know what the thickness should be at its base AD, with a power P acting upon it from P to C, and consequently tending to overturn it round the point A.



To resist this power we have to apply the weight W, equivalent to the superficial area of the triangle.

The sides AC, AE, of the angle CAE, form a bent lever, the fulcrum of which is at A, so that the power P, applied at the extremity C of the arm AC, pushes it in a horizontal and consequently oblique direction to the arm of lever. The weight W at the extremity E of the other arm AE, terminated by the line of direction drawn from the centre of gravity G of the triangle is applied downwards.

In place of the power P pushing from P to C, let it draw from C to H, in a direction parallel with the horizon, we may then suppose that the weight L is equivalent to that power, and capable of raising the perpendicular AB towards the line BH; the length of the arm of oblique lever AC, to the power, will in that case be reduced to the line AB.

If the arm of lever AB be called .... h
The base AD .... a
The other arm AE will be .... $\frac{2\,a}{3}$
And the weight W .... $\frac{a\,h}{2}$

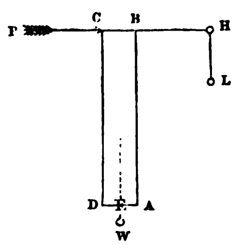
$$\text{Therefore, } P : \frac{ah}{2} :: \frac{2\,a}{3} : h.$$

$$\text{Hence } \frac{2\,a^2\,h}{6} = hP.$$

$$\text{Then as } \frac{a^2}{3} = P. \qquad a = \sqrt{3\ P.}$$

Therefore by tripling the power P, or the weight L, and extracting the square root of this product we obtain a, or AD.

There being given the height AB of a vertical wall, with the power P pushing it at the summit, in a direction PC; it is required to find the thickness BC, so that its weight may be in equilibrium with the effort of the power.



As in overturning the wall it is the same thing for the power P to push from P to C as to draw from B to H; let there be a weight L, attached to the extremity of the cord BH, equivalent to the force of the power; let also the whole superficies of the parallellogram be collected in the weight W, suspended from the centre of gravity G.

It is necessary to consider the lines AB and AE, which form the right angle BAE, as the arm of a bent lever, whose fulcrum is at A, the weight W at the extremity E of the little arm AE, and the power in the direction of the cord BH, which is attached to the extremity B of the great arm AB.

If we call h the arm AB
P the value of the power or the weight L,
a the base AD required.

Then ha = the superficies of the parallellogram; or, what is the same thing, the value of the weight W.

In order that the power and the weight may be in equilibrium they must be in the reciprocal ratio of their arms of lever.

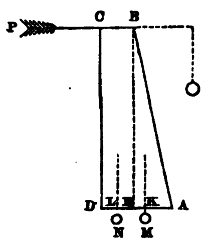
$$\text{Therefore } P : ha :: \frac{a}{h} : h.$$

$$\text{Which gives } hP = \frac{ha^2}{2}.$$

$$\text{Then } 2P = a^2. \quad \text{And } a = \sqrt{2P}.$$

Example. Supposing the power P, or the weight L, to be equivalent to a plane of 18 square feet, it would be necessary to double it making 36, the square root of which is 6, being the thickness AD required.

To find the thickness necessary to be given to the top of a wall, vertical at the back, but with an exterior slope; in equilibrium with the power tending to overturn it.



In sloping walls the base of the slope and consequently the triangle ABE is always given.

Let BE $= h$
AE $= d$
BC $= b$
Then $bh =$ the value of the weight N.
And $\frac{hd}{2} =$ the value of the weight M.

The weight N is suspended at the extremity L of the arm of lever AL; and the weight M at the extremity K of the arm AK.

Belidor* enters into the various steps of the problem, which we consider it unnecessary to follow, but to state the result of his investigations, viz:—

$$b = \sqrt{2P + \frac{d^2}{3}} - d.$$

Example.—Supposing that the power P be expressed by 52·5 square feet, double this value = 105, add to this quantity the third of the square of the line AE. If we call that line 6 feet, its square = 36, the third is 12, which being added to 105 gives 117, the square root of which is 10·816 feet, being the thickness of the base AD, consequently b or BC = 4·816 feet.

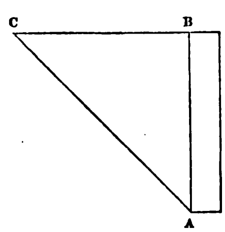
* La Science des Ingenieurs, liv. i., p. 19.

CHAPTER III.

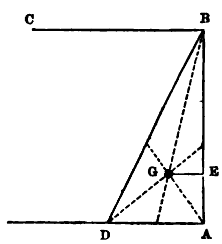
ON THE PRESSURE AGAINST VERTICAL WALLS.

Before entering on the form and thickness of walls with the least quantity of material necessary to resist the pressure behind them; the nature of that pressure, and the manner of its action must be considered. Several authors have written on the form of the prism of earth which tends to detach itself from the mass behind the wall. Many of them may be called mere displays of analytical transformations; some are extremely absurd, while others are of a different character; shewing much care bestowed on experiments, and great ingenuity in investigating them.

If newly made earth be not supported by a wall, the particles will loosen from each other by the action of the weather and tumble down, so as to make a slope, which plane is called the natural slope of the earth.



The angle BAC is different for each kind of soil. It is nothing for rock, and for other substances like it in consistency. It increases as the constituent parts of the earths have more tenuity, divisibility, or less adherence and friction. The limit of this increase is fluidity; that is when the line AC becomes horizontal, or when the angle BAC coincides with 90 degrees.



The pressure of water against the vertical plane increases from B towards A, in arithmetical progression. If we take half the depth of the water and multiply it by the superficial area of the plane, we obtain $\frac{AB^2}{2}$ as the expression of the pressure; being the area of the right angled isoceles triangle ABD. The centre of gravity of this triangle is the centre of pressure, and consequently found at the points G and E, at two-thirds of the depth.

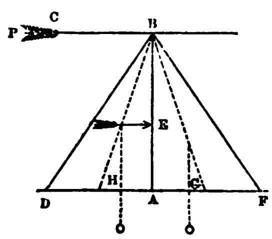
$$\text{Then } \frac{AB^2}{2} \times \frac{AB}{3} = \frac{AB^3}{6}$$

If the depth AB be 30 feet and the length unity.

$$\text{Then } \frac{AB^3}{6} = 4500 \text{ cubic feet.}$$

The weight of a cubic foot of water being 62·5 lbs.

4500 × 62·5 = 281,250 lbs., the pressure against the plane AB applied at the point E, tending to overturn it round the point A.*



If we reverse the position of the triangle and turn it outwards, then we may reduce the area, to obtain the same strength; in consequence of the increased length of the lever FG over AH; the triangle ADB having a tendency to overturn round the point A, and the triangle AFB round the point F.

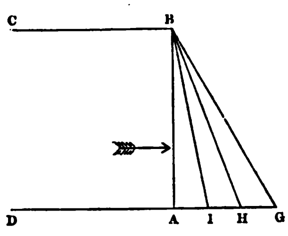
The pressure at E being 281,250lbs., and AB being 30 feet in height.

Then $\frac{281,250}{30 \times 62 \cdot 5} = 150$ cub. ft. on each lineal foot of the plane AB, at its extremity B, tending to overturn it round the point A.

Calling P this oversetting force.
y the length of the base AF.

$$\text{Then } y = \sqrt{3\,P} = \sqrt{3 \times 150} = 21{\cdot}213 \text{ feet.}$$

If the triangle ABG be composed of earth weighing 95lbs. per cnbic foot.



Then $\frac{281,250}{30 \times 95} = 98{\cdot}6842$ cubic feet on each lineal foot of the plane AB, at its extremity B, tending to overturn it rouud the point A.

Hence $\sqrt{3 \times 98{\cdot}6842} = 17{\cdot}206$ feet for the length of AG.

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* According to Prony:—Calling h the depth of the water.
w the density or weight per cubic foot.

Then the energy of the horizontal pressure of the water to overturn AB round the point A

$$= \left(\frac{1}{6}\,w\right) h^3 = \frac{62{\cdot}5}{6} \times (30)^3 = 281{,}250\text{lbs.}$$

If the triangle be of brickwork, weighing 105lbs. to the cubic foot.

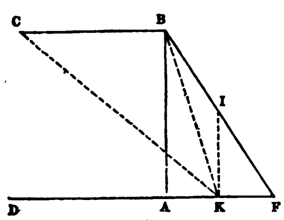
$$\text{Then } \frac{281,250}{30 \times 105} = 89{\cdot}2857.$$

$$\text{And } \sqrt{3 \times 89.2857} = 16{\cdot}366 \text{ feet for the length of AH.}$$

If the triangle be formed of rubble stone, weighing 138lbs. per cubic foot.

$$\text{Then } \frac{281,250}{30 \times 138} = 67{\cdot}9348.$$

$$\text{And } \sqrt{3 \times 67{\cdot}8348} = 14.276 \text{ feet for the length of AI.}$$



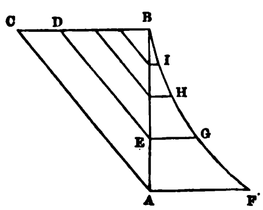
According to Ware* If the opposing side AB be vertical, the other side should be an inclined plane, the angle of inclination varying as the slope which the fluid or earth naturally assumes.

When the pressure is a fluid, the angle of inclination of the wall, or thrust AFB is 45°.

If the natural slope of the earth KC be the angle DKC, then the slope of the wall will be KB bisecting the angle IKC.

Then if KC make with the horizon an angle of 45°, the angle AKB will be 67½°. Consequently AK is the tangent of the angle of 22½°.

M. D'Antony, an Italian Engineer, in his researches on the pressure of earth,* takes it at an angle of 45°. He considers the wall, in each of its parts, to be acted upon by triangular prisms sliding on the hypothenuse; and, as the triangles are alike, he concludes that their effect is proportional to the squares of the heights of the wall pressed. For example, if AC mark the line of repose, the effect of the prism represented by this profile, is to that of BEB as AB^2 is to EB^2.



Then $AB^2 : EB^2 :: AF : EG$.
Consequently the curve FGHIB is parabolic.

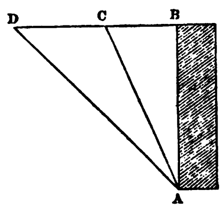
This is agreeable to the opinion of Dr. Young,† who says, that for resisting a force which tends to overset the wall, the form in which the weight gives the greatest strength is that of a conoid, or a solid, of which the outline is a parabola, concave towards the axis.

* Tracts on Vaults, pt. ii, p. 60. † Mayniel Traité de la Poussée des Terres, p. 50.
† Nat. Philos., vol. i. p. 159.

CHAPTER IV.

ON THE SLOPE OF EARTH, AND THE WEIGHT OF DIFFERENT SUBSTANCES EMPLOYED IN BUILDING WALLS.

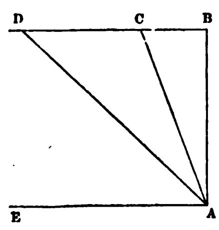
M. COULOMB was the first to consider the question of the slope of earth with all the physical circumstances attending it. The result obtained from his analysis is, that the line of separation or rupture bisects the angle between the natural slope of the earth and the back of the wall.



Let AD be the slope which loose earth, of itself, naturally assumes, then the line AC which determines the triangle of earth CAB that exerts the greatest horizontal stress against the vertical wall bisects the angle DAB.*

This bisected line is then the plane of maximum thrust.

The experiments made by Lieutenant Hope,† at Woolwich, with layers of coloured sand behind a glass, do not exactly agree with the result of M. Coulomb's theory; but they are sufficiently near for all practical purposes. The following is the mean of his experiments.



EAD	Natural slope of sand, mean of ten observations.	= 34° 0
DAC	Between plane of natural slope and plane of separation, or rupture	= 32° 0
CAB	Between plane of separation and the vertical.	= 24° 0

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* Hutton's Mathematics, Equa. 17, vol. i. p. 403.

† Professional Papers. Royal Engineers, vol. vii., On Retaining Walls.

TABLE I.

Shewing the natural slope of different Soils.

	Authority.	Slope.	Angle with the horizon.	Angle with the vertical.
Sand	Huber .. Bernand	......	From 30° to ... 33° rarely 35°	
Fine dry sand	Gadroy ...	5 hor. to 3 perp.	31° 0	59° 0
Very fine river sand....	Delanges ..	......	33° 0	57° 0
Dry yellow sand	Hope	......	34° 0	56° 0
Fine sand very dry, or pulverised freestone	Rondelet	Rather more than 3 hor. to 2 perp.	34° 30	55° 30
Lightest kind of sand ..	Barlow ...	5 hor. to 4 perp.	39° 0	51° 0
	Average of Sand		33° 41	56° 19
Gravel	Hope	4 hor. to 3 perp.	37° 0	53° 0
Loose shingle, perfectly dry	Pasley	5 hor. to 7 perp.	39° 0	51° 0
Ordinary earth, very dry and pulverised	Rondelet ..	......	46° 50	43° 10
Same earth, slightly humid	Rondelet ..	......	54° 0	36° 0
Very compact soil......	Barlow ...	5 hor. to 7 perp.	55° 0	35° 0
	Average of different soils, when not saturated with water;		44° 15	45° 45
	which for all practical purposes may be called		45° 0	45° 0

TABLE II.

Giving the weights of different kinds of Earth, &c.

	Authority.	Weight of a cubic foot.	Average weight of a cubic foot.	
		lbs.	lbs.	
Vegetable earth	Mayniel ...	69	83·50	General average weight, 95·33 which in practice may be called 95lbs.
Ditto do.	Murray ...	83		
Ditto do.	Rondelet ..	88		
Earth (free and open)	Ditto ..	94		
Sand	Mayniel ...	84	100·64	
Dry yellow sand, lightly thrown together	Hope	91		
Ditto, when shaken together	Ditto	98½		
Sand from the sea-shore, dry ...	Murray ...	90		
Ditto, when thoroughly wet ...	Ditto	116		
Sandy earth........................	Rondelet ..	106		
Pure sand	Ditto	119		
Dry clay	Mayniel ...	77	103·20	
Argillaceous earth	Rondelet ..	100		
Clay in clods	Murray ...	95		
Ditto, (probably more compact)	Rondelet ..	119		
Ditto, (probably still more so)	Tredgold...	125		
Vegetable earth, mixed with gravel, between the size of a hazel nut and a pigeon's egg	Mayniel ...	91	94·00	
Do., mixed with larger gravel, between the size of a walnut and a pigeon's egg............	Ditto	97		
Gravel.............................	Hope	95½	109·00	
Thames gravel	Murray ...	112		
Gravel	Moseley ...	120		
Rubbish, old materials, or the debris of rocks.................	Mayniel ...	110		
Clay, with gravel	Moseley ...	160		
Soapy earth, when dry	Ditto	64		
Ditto, when in a state of fluidity	Ditto	133		

TABLE III.

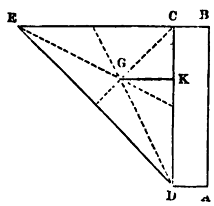
Giving the weights of different kinds of Building Materials.

	Authority.	Weight of a cubic foot.	Average weight of a cubic foot.
		lbs.	lbs.
Brickwork (London) 24 cubic feet = a ton...	Moseley	94	105
Ditto (Country) 22 cubic feet = a ton...	Ditto	102	
Ditto (new) ...... a rod = 16 tons......	Ditto	117	
Ordinary brickwork	Mayniel	98	
Ditto ditto	Murray	100	
Ditto ditto	Tredgold ...	117	
Wall with brick facing and rough stone hearting	Mayniel	122	
Rubble Work	Moseley	140	138
Masonry in rough stone	Mayniel	135	
Ditto ditto	Tredgold ...	140	
Ditto in flints or rounded pebbles ...	Mayniel	148	
Freestone (hewn) 16 cubic feet = a ton...	Murray	140	156
Stonework (hewn) 14 cubic feet = a ton...	Moseley	160	
Masonry in cut stone, 13¼ cubic feet = a ton...	Mayniel	169	
Ditto in granite, 13 cubic feet = a ton...	Murray	172	

CHAPTER V.

ON THE PRESSURE OF EARTH AGAINST VERTICAL WALLS.

A mass of earth when unsupported tends to slide down an inclined plane, until it take a natural slope, greater or less according to its consistency. If the earth be kept stationary by a vertical wall, then the sliding motion is prevented, and changed into a movement of rotation.



The triangle of earth DCE between the inclined plane DE and the back of the wall DC, represents a wedge, of which its own gravity is the acting power. It is supported by the inclined plane of earth and by the wall; consequently it may be supposed to have a tendency to turn round the point D. The centre of gravity of this mass is G, and the horizontal line GK has been imagined to be the thrust against the wall. In that case DK would be the arm of the lever of this force, and $= \frac{2}{3}$ DC.

If the angle CDE be 45°, then CE = CD, and consequently the area

$$CDE = \frac{CD^2}{2}$$

Then $\frac{CD^2}{2} \times \frac{2}{3} CD = \frac{1}{3} CD^3 =$ the momentum of the pressure of the earth; that is the oversetting force of earth supported by vertical walls of equal thickness, is as the cube of the height.

Supposing the height AB to be 30 feet, and the weight of a cubic foot of the earth to be 95lbs.,

$$\text{Then the resistance} = \frac{(30)^3}{3} \times 95 = 855{,}000\text{lbs.}$$

The counter resistance of the wall should be its superficial area multiplied by the length of the lever, measured from the oversetting point A along the line AD to the perpendicular let fall upon it from the centre of gravity of the wall.

In this case, $AB \times AD \times \frac{AD}{2}$ = the counter resistance.

Assuming the weight of a cubic foot of the wall to be 138lbs.

$$\text{Then } \frac{855{,}000}{138} = 6195{\cdot}66$$

$$\text{And } \frac{AD^2}{2} = \frac{6195{\cdot}66}{30} = 206{\cdot}52$$

Consequently $AD = 20{\cdot}323$ feet.

This thickness of wall being far greater than requisite in practice, leads to the conclusion that there must be some radical defect in the above theory.

In consequence of this discrepancy, the earth has been supposed not to act in a line with the horizon, but in a direction parallel with the inclined plane. The line of acting power would then pass from the centre of gravity of the earth to the back of the wall, and meet it at a lower point than before, reducing thereby the length of the arm of the lever to $\frac{1}{3}$ of DC.

$$\text{Then } \frac{DC^2}{2} \times \frac{1}{3} DC = \frac{1}{6} DC^3 = \text{the oversetting force of the earth,}$$

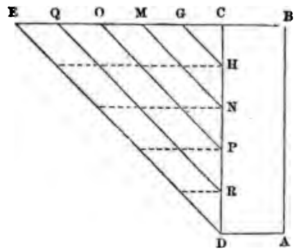
$$= \frac{(30)^3}{6} \times 95 = 427500\text{lbs.}$$

$$\text{And } \frac{427500}{138} = 3097{\cdot}83$$

$$\text{Consequently } \frac{AD^2}{2} = \frac{3097{\cdot}83}{30} = 103{\cdot}26$$

And $AD = 14{\cdot}37$ feet.

This thickness also being much greater than practice teaches us to be necessary, shews there must still be some egregious error in the calculation.



Belidor* considered the triangular prism of earth acting against a wall, vertical at the back, to be comprised within an angle of 45°, like DCE. As the earth acts with more or less force according to its distance from the summit C, he deemed it necessary to collect the whole pressure into that point. He therefore divided the height of the wall CD into a number of equal parts CH, HN, NP, PR, RD, and through each point of division he drew parallels to DE. He considered there was then pressing against the wall, and tending to overturn it round the point A, the little triangle CGH, and a number of trapeziums, GHNM, MNPO, &c., whose area is expressed by the differences of the squares of the terms of an arithmetic progression, 1, 3, 5, 7, and 9.

Calling z the effort of the triangle CGH, that of the trapeziums would be expressed by $3z$, $5z$, $7z$, and $9z$. He supposes the action of the triangle CGH to be collected at the point C, and those of the trapeziums at the points H, N, P, R.

The effect of the triangle would therefore be expressed by... $5z$
That of the first trapezium by .... $3 \times 4 = 12z$
the second do. by .... $5 \times 3 = 15z$
the third do. by...................... $7 \times 2 = 14v$
the fourth do. by .... $9 \times 1 = 9z$

The sum of all these effects would be expressed by ...... $55z$

As each power can be transferred to the extremity C of the arm DC, he divides the product of all the effects by that length of lever to obtain the expression of the total force DCE in C.

$$\frac{55z}{5} = 11z$$

Calling d the depth CH, then the area $= \frac{\text{CH}}{2} \times \text{CG} = \frac{d^2}{2}$

Belidor considers, that on account of its tenacity, a triangle sliding on its hypothenuse should be taken only at one-half of the mass if acted on like a spherical body on an inclined plane.

* La Science des Ingenieurs, liv. I, p. 30. Paris 1729.

If so, it would be expressed by

$$\frac{d2}{4} \times HC = z = \text{the effort of the little triangle CHG.}$$

$$\text{Then } \frac{d^2}{4} \times DC \times 11 = \text{effort of the whole area DCE.}$$

Example.

If DC = 30 feet be divided into five equal parts CH, HN, &c.

Then CH = 6 and the area $CGH = \frac{6^2}{2}$

But on account of tenacity this should be taken at

$\frac{6^2}{4} = 9 \times 30$ (arm of lever) $= 270$ effort of triangle CGH.

Then $270 \times 11 = 2970 \times 95$ (lbs. weight of a cubic foot of earth,) $= 282150$ lbs. momentum of the earths' pressure.

Otherwise we may take—

The effort of the triangle	CGH	= 9 × 30 lever	=	270
that of the 1st trapezium	GHMN	= 3 × 9 × 24 do.	=	648
„ „ 2nd „ „	MNPO	= 5 × 9 × 19 do.	=	810
„ „ 3rd „ „	OPRQ	= 7 × 9 × 12 do.	=	756
„ „ 4th „ „	QRDE	= 9 × 9 × 6 do.	=	486
				2,970 as before.

When the wall and earth are in equilibrio, calling b the thickness of the wall and the weight of a cubic foot 138lbs.

Then the momentum of the wall =

$$138 \times 30 \times \frac{b^2}{2} = 4140 \times \frac{b^2}{2} = 282150$$

$$\text{Conseqnently } \frac{b^2}{2} = \frac{282150}{4140} = 68{\cdot}152$$

And $b = 11{\cdot}674$ feet thickness of the wall.

The theory of Belidor is founded on an infinity of arbitrary suppositions, and it is especially defective from supposing that the total action of the earth is exerted at the summit of the wall. It is more natural to suppose, and experience proves, that the resultant passes through the centre of gravity of the falling mass. This resultant can only have two directions; one parallel to the inclined plane, the other horizontal, which, when the wall is vertical, determines the point of application at $\frac{1}{3}$ or $\frac{2}{3}$ of the height.

Belidor* gives a rule for calculating the thickness of the chamber walls of Sluices when the sides are vertical.

Calling k = the height of the water.

h = the height of the wall measured from the same base.

b = the thickness of the wall.

Then his general formula for finding the thickness to sustain water when the equilibrium of earth on the one part, and the resistance of the wall on the other, are alike.

$$b = \sqrt{\frac{7k^3}{36 \times h}}$$

Supposing the water and wall to be of the same height of 30 feet.

$$b = \sqrt{\frac{7 \times (30)^3}{36 \times 30}} = 13{\cdot}23 \text{ feet.}$$

But to make the resistance of the wall greater by one-half than the pressure of the water, he gives us a second formula quite safe for practice.

$$b = \sqrt{\frac{3}{2} \times \frac{7 \times k^3}{36 \times h}} = 16{\cdot}201 \text{ feet.}$$

Muller,† with others, has given a variety of tables for the thickness of Retaining Walls. This author supposes the action, or thrust, of the earth to be diminished $\frac{1}{3}$ by the resistance to motion along the inclined plane, or natural slope of the earth. The general result of his calculations for vertical parallel walls is, that the thickness is found by taking 0·385 of the height.

Then $30 \times 0{\cdot}385 = 11{\cdot}55$ feet.

Now this thickness of wall forms nearly an equilibrium to the thrust, and consequently is the smallest possible. It very far exceeds what is found sufficient in practice, consequently some important element must have been lost sight of in the calculation.

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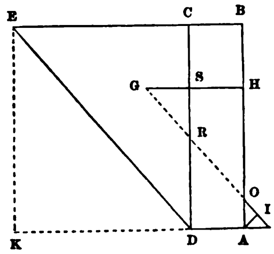
* La Science des Ingenieurs, liv. i, chap. iv, tom. III. See also Ibid, liv. iv, chap. tom. I.

† Treatise on the Practical Part of Fortification. Third edition. London, 1774.

Rondelet has adopted the ideas of Belidor, but in place of parallel slices or sections of equal thickness, he takes the product of the whole triangular prism, acting on a line drawn from its centre of gravity, parallel with the inclined plane. This is much less complicated than the method adopted by Belidor, independently of some of that author's suppositions, which cannot be correct.



If the earth take a slope of 45° as EDC, then G is the centre of gravity of the sliding mass; and the small triangle, GRS, has its sides proportional and equal to one-third of those of the large triangle EDC. We also know that GR is an oblique power acting at the extremity of the lever IA. The length of this lever is found by the triangles GRS, GHO, and AIO being alike, and having their sides proportional.

Calling P = the effort of the pressure.
h = the height of the wall.
b = CB its required thickness.

The surface of the section of earth EDC, which causes the pressure, is expressed by

$$h \times \frac{h}{2} = \frac{h^2}{2}.$$

Taking $\frac{3}{4}$ of this surface to be equal to the specific gravity of the masonry of the wall which sustains it, we have

$$\frac{h^2}{2} \times \frac{3}{4} = \frac{3h^2}{8}.$$

This mass acting on an inclined plane of 45°, its effort will be to its weight as the height KE of the plane is to its length DE; that is to say, as the side of the square is to its diagonal, or what is nearly the same thing as 70 is to 99.

We then have the expression of the effort of the pressure of the earth

$$P = \frac{3h^2}{8} \times \frac{70}{99} = \frac{210\,h^2}{792}.$$

Then, $\frac{210\,h^2}{792} \times$ IA = the power tending to overturn the wall at the point A.

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* L'Art de Batir, tom. IV., p. 198. 9th edition, Paris, 1842.

We must therefore find the length of IA, which depends upon b, the required thickness of wall. Rondelet conceives this ought to be made equal to twice the amount of pressure of the earth which it has to sustain. The formula which he gives is—

$$b = \sqrt{\frac{h^2}{8} + \left(\frac{3\,h}{16} \times \frac{3\,h}{16}\right)} - \frac{3\,h}{16}.$$

Knowing now the value of b, we have the pressure $= \frac{hb^2}{2}$.

The effort of the pressure is nearly equal to the resistance of a perpendicular wall, the height of which is five feet more than the actual height. Thus to obtain the pressure of earth against a wall 30 feet high, it is necessary to seek the resistance of it when 35 feet in height, and substitute that height for the value of h.

EXAMPLE.

To find the pressure of earth against a wall 30 feet high it is necessary to assume 35 feet for the height. Substituting that height in lieu of h, we have

$$b = \sqrt{\frac{35 \times 35}{8} + \left(\frac{35 \times 3}{16} \times \frac{35 \times 3}{16}\right)} - \frac{35 \times 3}{16} = 7{\cdot}497$$

Then, $\frac{hb^2}{2}$ expresses an effort equal to the pressure

$$= \frac{35 \times 7{\cdot}497 \times 7{\cdot}497}{2} = 1483{\cdot}59.$$

And 1483·59 × 95 lbs. weight of a cubic foot of the earth
= 140,936 lbs. the momentum of the earth.

Rondelet concludes his remarks on retaining walls by some easy rules, requiring only a knowledge of the first principles of arithmetic and geometry.

1. If 45° be the natural slope of the earth retained by the wall, the hypothenuse of the right-angled triangle behind the wall is the diagonal of a square, the side of which is the height of the retained earth. It is well known that the ratio of the side of a square is, to its diagonal, very nearly as 70 : 99.

2. Take then one-sixth of the hypothenuse for the thickness of the wall, and if it be wished to make the wall stronger, take one-fifth of

the hypothenuse, which will give a resistance nearly double the pressure.

3. If the wall be vertical at the back with an exterior slope or batter of one-sixth of the height the thickness at top may be one-ninth of the diagonal.

If the batter be one-eighth of the height, the thickness at top may be one-eighth of the diagonal.

4. If to a perpendicular wall, with parallel sides, counterforts be added of the same thickness as the wall, at distances of 18 feet from centre to centre; one-tenth of the diagonal need be the thickness of of the wall.

5. If the wall to which counterforts be added have an exterior slope or batter, the thickness of the wall at top may be found by determining the least thickness for 10 feet in height, in order to have a certain solidity, independently of what is necessary to sustain the pressure of the earth.

Calling that thickness 2 feet, for heights above 10 feet, add to each foot a quantity which ought to be as much greater as the slope will be less the following, viz:—

5 French lines for a slope of one-fifth.
6 „ for a slope of one-sixth.
7 „ for a slope of one-eighth.

Give to the counterforts the same thickness as the wall, and make their projection twice the thickness.

The French line is = 0·0888 English inch.

Example.

Suppose the wall to be 30 feet in height, with a slope of $\frac{1}{8}$:

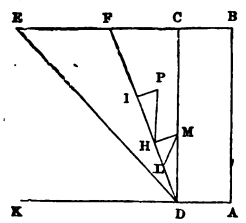
Then to.............................	2·000
Add 30 × 7 lines	1·554
Thickness of wall at top...... =	3·554

Counterforts { Breadth = 3·554; Length = 7.108 }

A formula is obtained from the experiments and theories of Rondelet which is remarkable for its extreme simplicity.*

Calling h = the height of the wall
b = its thickness
Then $b = h \times 0{\cdot}213$

In the celebrated memoir of Coulomb published in 1773,* the profile of a triangular prism of earth is considered as a solid right angled triangle, with one side vertical, and its hypothenuse touching an inclined plane, on which the triangle tends to slide, acted upon by its weight: it is retained there by a horizontal force, by its adhesion, and by its friction, which acts along that hypothenuse. This force is determined by the laws of statics, for the case of equilibrium.



Let ABCD be a vertical wall, and behind it is a mass of earth, DCEK, of which the triangular prism DCF would detach itself if it were not retained by the wall. This may be considered as a force M applied to the wall, the direction of which is perpendicular to DC.

This triangular prism would slide upon DF, by virtue of its weight P; but it is retained by the force M by its adhesion, and by its friction on DF. These forces can be resolved each into two others, of which one represents the normal pressure destroyed by the resistance FD; and the other acts parallel to DF. We have then for the components of P, the forces represented by PI, IH; and for those of M, the lines ML, LH. In consequence the prism can have motion only by virtue of IH; but LH acting in a contrary sense from D to F, opposes itself to IH, and is moreover assisted by cohesion and by friction. The equation of equilibrium will then be

$$IH = LH + \text{cohesion} + \text{friction}.$$

$$\text{Let } DC = h;\ CF = l;\ DF = \sqrt{h^2 + l^2}$$
$$PH = P;\ MH = M;$$

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* De Laistre. Science de l'Ingenieur, tom. II, p. 167.

* Memoires de Savans Etrangers, 1773.

We have $DF : CF :: PH : PI = \dfrac{Pl}{\sqrt{(h^2 + l^2)}}$

$DF : PH :: DC : HI = \dfrac{Ph}{\sqrt{(h^2 + l^2)}}$

$DF : DC :: MH : ML = \dfrac{Mk}{\sqrt{(h^2 + l^2)}}$

$DF : CF :: MH : LH = \dfrac{Mb}{\sqrt{(h^2 + l^2)}}$

If the density of the earth be expressed by y

then $P = \dfrac{yhl}{2}$

If c represent the cohesion, or unity of surface, having effect on the line DF; the total cohesion will be expressed by $c\sqrt{(h^2 + l^2)}$

Friction being proportional to the pressure, $\frac{1}{f}$ will express the constant ratio; but the normal pressure on $DF = ML + PI = \dfrac{Mh + Pl}{\sqrt{(h^2 + l^2)}}$ we have $\dfrac{MH + Pl}{f\sqrt{(h^2 + l^2)}}$ for the expression of the friction.

On these data the equation of equilibrium will be

$$\frac{Ph}{\sqrt{(h^2 + l^2)}} = \frac{Ml}{\sqrt{(h + l^2)}} + \frac{Mh + Pl}{f\sqrt{(h^2 + l^2)}}\sqrt{(h^2 + l^2)}$$

and consequently

$$M = \frac{fyh^2l - yhl^2 - 2cf(h^2 + l^2)}{2(fl + k)} = \frac{\frac{yhl}{2}\left(h - \frac{l}{f} - c(h^2 + l^2)\right.}{l + \frac{h}{f}}$$

Coulomb then proceeds to determine, by the theory of *maximis* and *minimis*, the triangle which exerts the greatest pressure. According to this theory we have

$$dM = \frac{2fyh^2(fl + h)dl - 4lyh(fl + h(dl - 8lcf(fl + h) -}{4(fl + h)^2}$$
$$\frac{-f^2yh^2ldl + 2fykl^2dl + 4cf^2(h^2 + l^2)dl.}{4(fl + h)^2}$$

from whence

$$\frac{dM}{dl} = o = l^2(-2fyh - 4f^2c) - l(4yh^2 + 8fhc) + h^2)2fyh + 4fc^2$$

Consequently the distance of the line of rupture from the back of the wall

$$= l = h\sqrt{1 + \frac{1}{f^2}} = \frac{h}{f} \quad *$$

Substituting this value of l in the expression of M,

$$= \frac{\frac{y h l}{2}\left(h - \frac{l}{f}\right) - c\left(h^2 + l^2\right)}{l + \frac{h}{f}}$$

$$\text{We have } M = h^2 \left[\frac{-\frac{y}{f} - \frac{y}{f^3} + \frac{y}{2}\sqrt{1 + \frac{1}{f^2}} + \frac{y}{f^2}\sqrt{1 + \frac{1}{f^2}}}{\sqrt{\left(1 + \frac{1^2}{f}\right)}}\right] -$$

$$- c\, h \left[\frac{\frac{2}{f^2} + 2 + \frac{2}{f}\sqrt{1 + \frac{1}{f^2}}}{\left(\sqrt{1} + \frac{1^2}{f}\right)}\right]$$

We now obtain an expression of the horizontal force necessary to retain the triangular prism of earth on its hypothenuse by making

$$A = \frac{-\frac{y}{f} - \frac{y}{f^3} + \frac{y}{2}\sqrt{1 + \frac{1}{f^2}} + \frac{y}{f^2}\sqrt{1 + \frac{1}{f^2}}}{\sqrt{\left(1 + \frac{1}{f^2}\right)}}$$

$$B = \frac{\frac{2}{f^2} + 2 + \frac{2}{f}\sqrt{1 + \frac{1^2}{f}}}{\sqrt{\left(1 + \frac{1}{f^2}\right)}}$$

And $M = A h^2 - B c h.$

Such is the theory of Coulomb, which determines the greatest pressure. It is, unfortunately, too analytical for practice; but as Mayniel, in the case of vegetable earths, makes by his experiments f, in the foregoing formula, equal

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* Supposing AB $= h = 30$ feet, and BF $= l$.

Then $l = h\sqrt{1 + \frac{1}{f^2}} - \frac{h}{f} = 30 \times 1{\cdot}118 - \frac{30}{2} = 33{\cdot}54 - 15 = 18{\cdot}54$ feet.

According to Mayniel $l = 0{\cdot}618\, h$. Then $30 \times 0{\cdot}618 = 18{\cdot}54$ feet.

to the number 2, we may therefore substitute it for f; and supposing the cohesion to be o; we thus calculate the following:—

Let the height AB = 30 feet
y = 95 lbs. the weight of a cubic foot of earth:

$$\text{Then } A = \frac{-\frac{95}{2} - \frac{95}{2^3} + \frac{95}{2}\sqrt{1 + \frac{1}{2^2}} + \frac{95}{2^2}\sqrt{1 + \frac{1}{2^2}}}{\sqrt{1 + \frac{1}{2^2}}}$$

$$= \frac{-4{\cdot}75 - 11{\cdot}875 + 4{\cdot}75 \times 1{\cdot}118 + 23{\cdot}75 \times 1{\cdot}118}{1{\cdot}118}$$

$$= \frac{59{\cdot}375 + 53{\cdot}104 + 26{\cdot}553}{1{\cdot}118} = \frac{20{\cdot}283}{1{\cdot}118} = 18{\cdot}142 \text{ feet.}$$

$$\text{And } B = \frac{\frac{2}{2^2} + 2 + \frac{2}{2}\sqrt{1 + \frac{1}{2^2}}}{\sqrt{\left(1 + \frac{1}{2^2}\right)}}$$

$$= \frac{\frac{2}{4} + 2 + 1 \times 1118}{1{\cdot}118} = \frac{3{\cdot}5 \times 1{\cdot}118}{1{\cdot}118} = 3{\cdot}5$$

Now omitting cohesion $M = A h^2 - B h$
$= (18{\cdot}142 \times 30^2) - (3{\cdot}5 \times 30) = 16222{\cdot}8$

And taking this to act at the extremity of an arm of the lever equal to $\frac{1}{3}$ of the height of the wall = 10 feet.

$16222{\cdot}8 \times 10 = 162228$ the momentum of the earth.

Let b be the breadth of the wall, and w = 138lbs. the weight of a cubic foot of the wall when formed of rubble stone.

Then $wh \times \frac{b^2}{2} = 138 \times 30 \times \frac{b^2}{2} = 4140 \times \frac{b^2}{2}$ = the resistance of the wall.

Consequently if both be in equilibrio

Then $\frac{b^2}{2} = \frac{162228}{4140} = 39{\cdot}185.$ $\qquad 4140 \times \frac{b^2}{2} = 162{,}228$

And b = 8·852 feet thickness of the wall.

If the wall be composed of brickwork, weighing 105lbs. then if both be in equilibrio, b = 10·149 feet, thickness of the wall.

M. Gauthey, a celebrated French engineer, published a paper on the subject of Retaining Walls in the Memoirs of the Academy of Dijon, in 1784. He criticises the theories of Couplet and others communicated to the Academy of Paris, as well as the profiles of Belidor and Vauban. The theory promulgated is founded, like that of many others, on the supposition that the prism of greatest pressure slides on an inclined plane, forming, with the horizon, an angle of 45°

Calling h = the height of the wall
b = its thickness
w = weight of a cubic foot of the masonry
y = do. of do. of the earth.

The result drawn from his enquiries is, that the weight of the triangle DCE is expressed by $\frac{y\,h^2}{2}$; and that the horizontal thrust against the vertical wall $= \frac{y\,h^2}{6}$ which he supposes to act at the extremity of an arm of lever equal to $\frac{1}{3}$ of the height of the wall.

The momentum of pressure will be therefore $= \frac{y\,h^2}{6} \times \frac{h}{3} = \frac{y\,h^2}{18}$

The resistance of the wall will be $= w\,h\,b \times = \frac{b}{2} = \frac{w\,h\,b^2}{2}$

From which he draws the equation of equilibrium $\frac{y\,h^3}{18} = \frac{w\,h\,b^3}{2}$

Example.

Let the wall be 30 feet high, the weight of a cubic foot be 138 lbs; and the weight of a cubic foot of the earth 95 lbs.

$$\text{Then } \frac{95 \times (30)^3}{18} = 142500 = \text{the momentum of the earth,}$$

$$\text{And } 138 \times 30 \times \frac{b^2}{2} = 4140 \times \frac{6^2}{2} = \text{the resistance of the wall.}$$

$$\text{Consequently if both be in equilibrio } 4140 \times \frac{b^2}{2} = 142500$$

$$\text{Then } \frac{b^2}{2} = \frac{142500}{4140} = 34{\cdot}42$$

And $b = 8{\cdot}296$ feet the thickness of the wall.

Hence we deduce the following formula when both sides are vertical,

$$b = \sqrt{\frac{y\,k^2}{9\,w}}$$

If the wall of rubble stone, weighing 138lbs. to the cubic foot.

$$\text{Then } b = \sqrt{\frac{95 \times (30)^2}{9 \times 138}} = 8{\cdot}296 \text{ feet.}$$

If the wall be composed of brickwork, weighing 105lbs. to the cubic foot,

$$\text{Then } b = \sqrt{\frac{95 \times (30)^2}{9 \times 105}} = 9{\cdot}512 \text{ feet.}$$

M. de Prony, in 1790,† treats of the pressure of earth, based on the theory of Coulomb, but considerably simplified by determining the co-efficient of friction, as a function of the natural slope of the earth; and by taking into consideration its cohesion or union of the constituent parts.

He has thus analyzed the subject with much care and exactness; and arrived at a theorem which gives the prism of the greatest pressure a very simple expression. It is further explained in his work on the Pressure of Earth.†

M. Gauthey, in his Treatise on the Construction of Bridges, edited by M. Navier, in 1809, has not thought fit, on account of the progress which Coulomb and Prony had made in the theory of the pressure of earth, to make allusion to the theory which he had formerly given. We are therefore led to conclude, from his observations on Prony's theorem, which is the only one noticed in the work, that he considered it more accurate than his own. We shall, therefore, extract what he says respecting the pressure of earth.‡

The different particles of a mass of earth are united by cohesion, and when they slide one on the other, friction opposes itself to their motion. We shall suppose here, conformably with experience, that the resistence produced by cohesion is proportional to the surface of rupture, and that friction is proportional to the normal pressure.

The pressure against a retaining wall is the prism of earth which would fall down if no wall was built. This prism is separated from the mass of earth by a surface which experience has shewn to be a plane. In all earth newly re-

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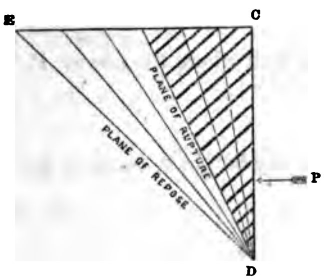
* Nouvelle Architecture Hydraulic. Paris, 1790. Art. 596, *et seq.*

† Recherches sur la Poussée des Terres. Paris, 1802.

‡ Gauthey, Traité de la Constrnction des Ponts. Paris, 1809. P. 372, *et seq.*

moved, the inclination of this plane is determined, for each kind of earth, by the value of the cohesion and friction. When the earth has little cohesion, the inclination of the plane is solely due to friction; but when the cohesion is taken into consideration, the analysis demonstrates that the angle formed by the plane of slope and the vertical depends on the height of the earth.

In this hypothesis the prism has no tendency to slide; for the inclination of the plane is such that cohesion and friction alone retain the earth in equilibrium. But the prism does not form a solid mass; consequently we may conceive a series of planes, less inclined than that of the slope (denoting the plane of repose) originating from the lowest point. One of these planes will have such a position, that the separating prism will have need of a greater opposing force to its sliding motion than any other.



Let us call P = the horizontal power which sustains the prism of earth.

a = EDC the angle which an inclined plane drawn through the lowest point of the prism makes with the vertical.

c = the force of cohesion or unity of surface.

f = the ratio of the normal pressure to friction.

i = the angle, of which the tangent is $\frac{1}{f}$.

h = DC the height of the earth.

y = the weight of a cubic foot.

The weight of the prism of earth resting on the plane, which makes, with the vertical, the angle a, is $\frac{1}{2} y h^2$ tang. a

The weight of this prism is in equilibrium with the force P, having regard to cohesion and friction, when

$$P\ (\text{sin.}\ a \times f \cos.\ a) = \tfrac{1}{2}\, y\, h^2\ \text{tang.}\ a\,(\cos.\ a - f\ \text{sin.}\ a) - \frac{h\,c}{\cos.\ a}$$

Retaining walls are in general built against newly removed earth, for which the value of cohesion is but then little. We may therefore suppress it: hence

$$P = \tfrac{1}{2} y\, h^2\, \frac{\text{tang.}\ a\ (1 - f\ \text{tang.}\ a)}{\text{tang.}\ a + f}$$

The *maximum* of this expression is found by varying the angle a. Corresponding with this,

$$\text{Tang.}\ a = f \pm \sqrt{(f^2 + 1)}$$

Substituting for f its value $\frac{1}{\text{tang. } i}$

$$\text{Tang. } a = \frac{-1 \pm \sqrt{\text{tang.}^2 a + 1}}{\text{tang. } i}$$

Finally tang. a = tang. $\frac{1}{2}i$.

When the cohesion of earth is nothing, the angle i is that which the plane of the slope makes with the vertical. Thus we have arrived at the same result as M. de Prony, and we have proved that in this hypothesis the prism of greatest pressure is given by the inclined plane which cuts the angle into two equal parts.

The value of the tang. a will then become,

$$P = \tfrac{1}{2} y h^2 \text{ tang.}^2 \tfrac{1}{2} i$$

Making tang. $\frac{1}{2}i = t$,

$$P = \tfrac{1}{2} y h^2 t^2.$$

From this equation is deduced the point of the wall where the power P, can be supposed to be applied.

The sum of the horizontal pressures will be for any height z, equal to $\frac{1}{2} y z^2 t^2$, of which the differential is $y z t^2 dz$.

This elementary pressure acting at the extremity of an arm of lever, whose length is $h - z$, its momentum will be represented by

$$y z t^2 (h - z) dz.$$

Integrating this expression. Since $z = 0$ even to $z = h$ we find for the sum of the moment,

$$\tfrac{1}{6} y h^3 t^2$$

$$\text{Consequently } \frac{\frac{1}{6} y h^3 t^2}{\frac{1}{2} y h^2 t^2} = \tfrac{1}{3} h$$

Hence the point of application of the force resulting from the pressure of the earth, or the length of the arm of lever is situated at $\frac{1}{3}$ of the height of the wall.

If we suppose the value of the friction to diminish, the angle of the slope will increase and approach continually to a right angle.

The force P, which augments at the same time as the angle of the slope, will approach also from its limit, of which we shall have the value by supposing $t = 1$.

This value is $P = \frac{1}{2} y h^2$, which is that deduced for the the theory of fluids.

Such is the result of Prony's formula as given by Gauthey in his work on Bridges.

EXAMPLE,

Let h = 30 feet height of the wall.
b = Its required thickness.
w = 138lbs. weight of a cubic foot of the wall.
y = 95lbs. ditto ditto earth.
t = tang. of half the angle which the slope of earth makes with the vertical, $= 45° \div 2 = 22° 30'$
Then the log. of $22° 30' = 9{\cdot}617224$
The nat. num. of which $= 0{\cdot}4142$.
P = The horizontal power which sustains the prism of earth.

Then $P = \frac{1}{2} y h^2 t^2$.

And $\frac{1}{6} y h^3 t^2 = \dfrac{95 \times (30)^3 \times (0{\cdot}4142)^2}{6}$

$= 73{,}359$ the momentum of the earth.

Now $138 \times 30 \times \dfrac{b^2}{2} = 4{,}140 \times \dfrac{b^2}{2}$ the resistance of the wall.

Consequently if both be in equilibrium,

$$4{,}140 \times \frac{b^2}{2} = 73{,}359$$

$$\text{Then } \frac{b^2}{2} = \frac{73{,}359}{4{,}140} = 17{\cdot}719$$

And $b = 5{\cdot}953$ feet thickness of the wall.

Hence we have this formula, when both sides are vertical,

$$b = h\sqrt{\frac{y\,t^2}{3\,w}} = h\,t\sqrt{\frac{y}{3\,w}}$$

$$= 30 \times 0{\cdot}4142\sqrt{\frac{95}{3 \times 138}} = 30 \times 0{\cdot}1984 = 5{\cdot}953 \text{ feet,}$$

for the thickness of the wall.

M. de Prony arrives at the same theorem,* without having neglected cohesion, as M. Gauthey has done to simplify the calculation. But then the angle i is not the angle of the natural slope of earth; but the inclination only of the plane on which the particles of earth maintain themselves in equilibrium by friction only.

If the cohesion be nothing $a = i$.

If it be very great $a = \frac{1}{2} i$.

The angle a is therefore always comprised between i and $\frac{1}{2} i$.

We have seen that friction is equal to the pressure, according to the theory of the inclined plane, $f = \frac{1}{\text{tan. } i.}$

Therefore if $i = 45°$

Then the tan. $45° = 10{,}000$, the natural number of which is 1.

$$\text{Consequently } f = \frac{1}{1} = 1.$$

M. Navier† takes notice of the observation of M. Mayniel,‡ respecting Prony's adoption of $\frac{1}{\text{tan. } i}$ for f, in the formula of Coulomb, i being the angle formed by the natural slope of the earth and the vertical. He says "This transformation of the constant f, is simply a result of the first principles of mechanics, and M. de Prony, by so doing, has only given this equation a more simple form." He concludes by saying "That there is no theory more general and more convenient for application, than that which has been given by M. de Prony."

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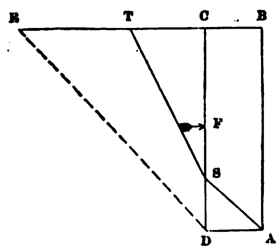
* Recherches sur la Poussée des Terres, p. 6, *et seq.*

† In Gauthey's Traité de la Construction des Ponts, tom. i., p. 385.

‡ Traité de la Poussée des Terres, p. 125, 126.

M. Navier* views the subject by supposing the vertical wall ABCD as being liable to be broken in the inclined direction AS, by the pressure which the earth makes against CS, like a force applied at F perpendicular to the wall. The rupture in this case, may occur either when the upper part, ABCS, slides on the plane of rupture, or when that upper part is overturned round the point A. The sliding rupture can be prevented by construction, and in general can only occur when the wall wants the necessary thickness to resist rupture by overturning. We shall therefore only consider this last case.



Calling h = the vertical height AB.
b = the thickness BC.
w = the weight of a cubic foot of the wall.
y = ditto ditto of the earth.
t = tangent of half the angle CDE which the vertical makes with the plane of repose DE.
z = the undetermined height SC.

Then $b = \sqrt{\dfrac{y\, z^2\, t^2\, (3h - 2z)}{w\,(h + 2z)}}$

The value of z which makes this expression a maximum, is

$$z = \tfrac{1}{2}\sqrt{c\,h}$$

And the least thickness of the wall is

$$b = h \times t\sqrt{\left(\frac{9\,y}{(12 + 8\sqrt{3})\,w}\right)} = 0{\cdot}51\,h\,t\sqrt{\frac{y}{w}}$$

As in the foregoing examples

$$b = 0{\cdot}51 \times 30 \times 0{\cdot}4142 \times 0{\cdot}83 = 5{\cdot}259 \text{ feet.}$$

M. Mayniel, in 1808, published a work* on the pressure of earth and on revetments. The following are the chief results at which he arrived.

1. In a vertical retaining wall, the direction of the resultant of the earth placed behind it, when unbeaten, strikes the back of the wall at one-third above the natural soil.

* Leçons sur l'Application de la Mecanique.

* Traité de la Poussée des Terres et des Murs de Revetments. 4to, Paris, 1808.

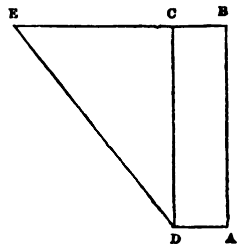
2. The ratio of the friction to the pressure (*f* in the formulae) is

For vegetable earths..................	0·5	that is, friction is one-half of the pressure.
Earth, mixed with small gravel	0·45	
Sand................................	0·4	
Rubbish, or the debris of rocks	1·0	

3. The loss of the effect by cohesion.

In vegetable earths, laid in courses with care............................	$\frac{5}{6}$	that is, cohesion diminishes the thrust full two-thirds.
But for more solidity this loss is called................................	$\frac{2}{3}$	

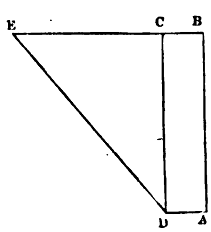
4. The *line of rupture* is supposed to slope from the interior angle D at the bottom of the wall, to a point E level with the top.



If h be the height of the wall AB.

The line of rupture CE is found thus:—

Nature of the material behind the wall.	Co-efficients.	Angle with the vertical.	Angle with the horizon.
Vegetable earth	0·618 h	31·43	58·17
Do. mixed with large gravel......	0·618 h	31·43	58·17
Do. mixed with small gravel.....	0·646 h	32·52	56·8
Sand	0·677 h	34·6	55·54
Rubbish, old materials, or the debris of rocks	0·414 h	22·29	67·36



Let DCE be a prism of vegetable earth, pressing against the back of a vertical wall, ABCD.

Calling AB $= h$
CE $= h \times 0{\cdot}618$

Then the area of the prism $= \frac{h^2}{2} \times 0{\cdot}618$

Calling y the density of the prism DCE.

$$\frac{y\,h^2}{2} \times 0{\cdot}618 = \text{its weight.}$$

Q = the force with which it acts against the wall.

$$\text{Then Q} = \frac{\frac{y\,h^2}{2} \times 0{\cdot}618 - \frac{y\,h^2}{2} f\,(0{\cdot}618)^2}{f + 0{\cdot}618}$$

$$= \frac{y\,h^2 \times \frac{0{\cdot}618 - (0{\cdot}5 \times (0{\cdot}618)^2)}{2}}{0{\cdot}5 + 0{\cdot}618} = y\,h^2 \times 0{\cdot}191$$

This force will act at one-third of the height above AD.

And its energy, or the effect of the action will be $\frac{y\,h^3}{3} \times 0{\cdot}191$.

If the prism of vegetable earth DCE be laid in courses, supposing each cubic foot weighs 69lbs. will have an expression for the effect of its action against the wall of

$$h^3\,\frac{69 \times 0{\cdot}191}{3} = h^3 \times 4{\cdot}393.$$

If the vegetable earth be laid with care, then, on account of cohesion, the effect of its action will be,

$$h^3\,\frac{4{\cdot}393}{3} = h^3 \times 1{\cdot}464.$$

If the backing material be dry clay, weighing 77lbs. per cubic foot, the decimal fraction 0·191 must be multiplied by it in place of 69lbs., and its effect will be expressed by,

$$h^3\,\frac{77 \times 0{\cdot}191}{3} = h^3 \times 4{\cdot}902.$$

If the clay be laid with care, this expression may be reduced one-third,

$$\text{Hence } h^3 \times \frac{4{\cdot}902}{3} = h^3 \times 1{\cdot}634.$$

If gravel be used to back the wall, weighing on an average 93½lbs. per cub. ft.

$$\text{Then Q} = \frac{y\,h^2 \times \frac{0{\cdot}632 - (0{\cdot}5 \times (0{\cdot}632)^2)}{2}}{0{\cdot}5 + 0{\cdot}632}$$

$$= \frac{y\,h^2 \times \frac{0{\cdot}632 - 0{\cdot}1997}{2}}{1{\cdot}132} = h^3 \times 5{\cdot}9528.$$

$$\text{And its energy} = h^3 \times \frac{93{\cdot}5 \times 0{\cdot}191}{3} = h^3 \times 5{\cdot}9528.$$

When laid with care this expression may be reduced one-third,

$$\text{Hence } h^3 \times \frac{5{\cdot}9528}{3} = h^3 \times 1{\cdot}9843.$$

If the backing be rubbish, old materials, or the debris of rocks, weighing 109lbs. per cubic foot,

$$\text{Then Q} = \frac{y\,h^2 \times \dfrac{0{\cdot}414 - (1{\cdot}0 \times (0{\cdot}414)^2)}{2}}{1{\cdot}0 + 414} = y\,h^2 \times 0{\cdot}0858$$

$$\text{And its energy} = h^3 \times \frac{109 \times 0{\cdot}0858}{3} = h^3 \times 3{\cdot}117.$$

If sand be used, weighing 84lbs. per cubic foot,

$$\text{Then Q} = \frac{y\,h^2 \times \dfrac{0{\cdot}677\ (0{\cdot}4 \times (0{\cdot}677)^2)}{2}}{0{\cdot}4 + 0{\cdot}677} = y\,h^2 \times 0{\cdot}229.$$

$$\text{And its energy} = h^3 \times \frac{84 \times 0{\cdot}229}{3} = h^3 \times 6{\cdot}412.$$

Soapy earths saturated with water, can scarcely preserve a slope with the horizon of 18°. When these are employed as a backing to the wall, they must be treated like a fluid; for all friction and cohesion must be destroyed.

$$\text{Then Q} = \frac{y\,h^2}{2}$$

$$\text{And its energy} = \frac{y\,h^3}{6}\ y\,h^3 \times 0{\cdot}166$$

As however it is impossible that they can become perfectly fluid, their weight may be calculated in a mean state,

		lbs.
When dry	=	64
When in a state of fluidity	=	133
		98½ average weight per cubic foot.

Then the expression for the energy, or the effect of the action against the wall will be,

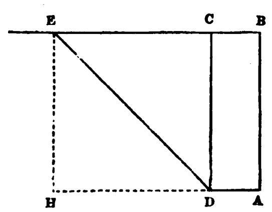
$$h^3 \times 98{\cdot}5 \times 0{\cdot}166 = h^3 \times 16{\cdot}351.$$

Tabular statement of the results of Mayniel's formulae, reduced to English measure, h being the height of the wall.

Nature of the material behind the earth.	Weight before compression.	Energy or effect of the action against a vertical wall.	If the wall be 30 feet in height.
	lbs.		lbs.
Vegetable earth, before compression ..	69	$h^3 \times 4{\cdot}393$	118,611
If spread in layers of 6 inches, this effect may be reduced one-third on account of cohesion; consequently	69	$h^3 \times 1{\cdot}464$	39,528
Dry clay, reduced one-third on account of cohesion..............................	77	$h^3 \times 1{\cdot}634$	44,118
Vegetable earth mixed with gravel, from the size of a hazel nut to a pigeon's egg	93·5	$h^3 \times 5{\cdot}9528$	160,726
But as this is susceptible of much cohesion, its effect may be reduced one-third	93·5	$h^3 \times 1{\cdot}9843$	53,576
Rubbish, old materials, or the debris of rocks.................................	109	$h^3 \times 3{\cdot}117$	84,159
Sand	84	$h^3 \times 6{\cdot}412$	173,124
Soapy earths, mean state	98·5	$h^3 \times 16{\cdot}351$	441,477

Thickness of Walls.

M. Mayniel next considers the thickness of walls necessary to resist the pressure of the retained earth.



Let ABCD be a vertical wall behind which is lodged a mass of vegetable earth, DCEH. It is required to find BC the thickness of the wall, on the supposition of it being overturned round the point A.

$$\text{Let AB} = h$$
$$\text{BC} = b$$

It is clear that the wall may be made to resist the action of the earth by virtue of its inertia. If w be the specific gravity of a cubic foot of masonry, the mass of the wall will be represented by whb, and as it is supposed to turn round the point A the length of the lever acting at that point will be expressed by $\frac{AD}{2} = \frac{b}{2}$

So that the effect of the inertia of the wall will be $\frac{w\,h\,b^2}{2}$

By reference to the last abstract, it will be seen that the effect of the action of vegetable earth laid in courses is expressed by $h^3 \times 1{\cdot}464$

It is necessary then that $\frac{w\,h\,b^2}{2} = h^3 \times 1{\cdot}464.$

Consequently $b = h \sqrt{\frac{1}{w}(2{\cdot}928)}$

If the wall be constructed of brickwork, weighing 105lbs. per cubic foot.

$$b = h \sqrt{\frac{2{\cdot}928}{105}} = h \times 0{\cdot}167.$$

Hence if the wall be 30 feet in height, its thickness ought to be 5·01 feet, to resist the pressure of vegetable earth laid in courses.

In a similar manner the thickness of other kinds of walls have been calculated, when backed with different sorts of materials, as delineated in the following table:—

*Retaining Walls, when both sides are Vertical.**

The required thickness is found by multiplying each co-effcient by the given height of the wall.

Nature of the material behind the wall.	Weight of a cubic foot.	Co-efficients for a wall of		
		Brick, 105lbs. per cub. ft.	Rubble stone. 138lbs. per cub. ft.	Hewn Freestone, 170lbs. per cub. ft.
	lbs.			
Vegetable earth, carefully laid, course by course	69	0·167	0·145	0·131
Clay, well rammed	77	0·176	0·154	0·140
Earth mixed with gravel	93½	0·194	0·169	0·152
Rubbish, or the debris of rocks...	109	0·243	0·211	0·211
Sand	84	0·349	0·303	0·274
Soapy earth, when not saturated with water } 64lbs. Do. when approaching to fluidity............ } 133lbs.	98½	0·558	0·446	0·438

* Mayniel, Traité de la Poussée des Terres, p.p. 254—265.

Calculations on the foregoing data of Mayniel.

To find a mean co-efficient for a wall, weighing on an average 138lbs. per cubic foot:—

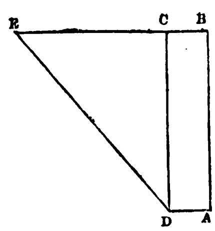
The line of rupture = 0·618 h for vegetable earths
„ „ 0·618 h ditto, mixed with large gravel
„ „ 0·646 h ditto, with small gravel
„ „ 0·677 h sand.

Average = 0·640 h

	Assumed by Mayniel, per cub. ft.	Taken in the following calculations, per cub. ft.
	lbs.	lbs.
The weight of vegetable earth........................	69	83.5
Do. clay	77	103·2
Do. gravel	93·5	94·0
Do. sand	84	100·64
Average	81	95

Friction is called by Mayniel = 0·5 for vegetable earth.
0·45 for earth mixed with gravel.
0·4 for sand.

Average......... = 0·45



Calling AB = h
CB = b
CE = h × 0·64.

Then the area of the prism = $0.64 \frac{h^2}{2}$

Calling the density of the prism DCE $= y$

Its weight $= 0{\cdot}64 \; \frac{y h^2}{2}$

The force with which it acts $=$ Q.

$$\text{Then } Q = \frac{\frac{y h^2}{2} \times 0{\cdot}64 - \frac{y h^2}{2} f \, (0{\cdot}64)^2}{f + 0{\cdot}64}$$

$$= y h^2 \times \frac{\frac{0{\cdot}64 - (0{\cdot}45 \times 0{\cdot}4096)}{2}}{0{\cdot}45 + 0.64} = y h^2 \times 0{\cdot}2549.$$

This force will act at one-third of the height above AD.

The energy, therefore or the effect of its action will be,

$$= \frac{y h^3}{3} \times 0{\cdot}2549.$$

Then a prism of earth of average density, each cubic foot weighing 95lbs., will have an expression for the effect of its action against the wall of

$$h^3 \times \frac{95 \times 0{\cdot}2549}{3} = h^3 \times 8{\cdot}0718.$$

But if the earth be laid with care, then on account of cohesion, the effect of its action may be reduced to,

$$h^3 = \frac{8{\cdot}0718}{3} = h^3 \times 2.6906.$$

It is necessary then that,

$$\frac{w\,h\,b^2}{2} = h^3 \times 2{\cdot}6906.$$

Consequently $b = h \sqrt{\frac{1}{w} \left(5{\cdot}3812\right)}$

If the average weight of the wall be 138lbs. per cubic foot,

$$b = h \sqrt{\frac{5{\cdot}3812}{138}} = h \times 0{\cdot}1974.$$

Then with a wall of 30 feet in height,

$30 \times 0{\cdot}1974 = 5{\cdot}922$ feet for its thickness.

The Theory of Woltman.

If h represent the height of the wall,
y the weight of a cubic foot of the earth,
a the angle of the plane of repose of the earth with the horizon.

Then the pressure which the earth exerts against a vertical wall

$$= h^2 y \times \tfrac{1}{2}\left(\frac{1 - \sin. a}{1 + \sin. a}\right)$$

Admitting $a = 0$, we have for the case of water and other fluids acting against the wall,

$$\frac{h^2 y}{2}.$$

The moment of the pressure, or the force capable of oversetting a vertical wall, is the product of the above formula by one-third of the height,

$$= \tfrac{1}{6}\, h^3 y \left(\frac{1 - \sin. a}{1 + \sin. a}\right)$$

If the angle T of the natural slope of the earth (the plane of repose) had been taken with the vertical, this last formula would be

$$= \tfrac{1}{6}\, h^3 y \tan.^2 \left(\frac{T}{2}\right)$$

Example.

Let $a = T = 45°$. Its sine $= 9{\cdot}849485$. Nat. num. of which $= 0{\cdot}707$.
$h = 30$ feet.
$y = 95$lbs. weight of a cubic foot.

Then $\tfrac{1}{6} h^3 y \left(\frac{1 - \sin. a}{1 + \sin. a}\right) = \tfrac{1}{6} \times 27{,}000 \times 95 \left(\frac{1 - 0{\cdot}707}{1 + 0{\cdot}707}\right)$
$= 4{,}500 \times 95 \times 0.1716 = 73{,}359$lbs.

Or, $\tfrac{1}{6}\, h^3 y \tan.^2 \left(\frac{45°}{2}\right) = \tfrac{1}{6} \times 27{,}000 \times 95 \times (0{\cdot}4142)^2 = 73{,}359$lbs.

Several of the French Engineers* have latterly employed a method of calculating the effects of pressure against rectangular vertical walls, different from that of M. de Prony.

To simplify the results, let us call

h = the height of the wall.
b = its thickness.
w = weight of a cubic foot of the wall.
y = ditto ditto of the earth.
R = the resistance $= \sqrt{\left(\frac{y}{3\,w}\right)}$.

1. When the backing acts like a fluid

$$b = h \times R$$

which agrees with the method adopted by Prony.

2. When the earth is newly removed and preserves a slope

$$b = h \text{ sin. } T \times R$$

T being the angle of the slope of the earth with the vertical.

Prony's formula for this case was,

$$b = h \times t \times R$$

t being the tangent of half the angle which the prism of earth makes with the vertical.

With the same example as before:—

Then $b = 30 \times \text{sin. } 45° \times \sqrt{\left(\frac{y}{3\,w}\right)}$

Sin. of 45° = 9·849485. Nat. numb. of which = 0·707.

Then $b = 30 \times 0{\cdot}707 \times \sqrt{\left(\frac{95}{3 \times 138}\right)}$

$= 30 \times 0{\cdot}707 \times 0{\cdot}479 = 30 \times 0{\cdot}339 = 10{\cdot}17$ feet.

The Engineers of the Water Staat (of Holland) in the construction of their sluice and quay walls are said† to use the following formula:—

Calling h = the height of the wall
Then the upper thickness $= 0{\cdot}3\,h$
The lower ditto $= 0{\cdot}42\,h$
The counterforts are square $= 0{\cdot}3\,h$

They are placed at central intervals of from $2{\cdot}5\,h$ to $3{\cdot}3\,h$.

* Delaistre. Science de l'Ingenieur, tom. II. p. 175.
† Merkes on Revetments, p. 99.

Summary of the foregoing Theories.

Calling h = 30 feet The height of the wall.

b .... Its required thickness.

y = 95 lbs. The weight of a cubic foot of the earth.

w = 138 lbs. Ditto do. of a rubble-stone wall.

z = 105 lbs. Ditto do. of a brick wall.

t .... Tangent of half the angle which the plane of repose makes with the vertical $= \frac{45}{2} = 22° \; 30'$

Logarithm of tangent of 22° 30′ .... = 9·617224

Its natural number .... = 0·4142

The square of which $(0{\cdot}4142)^2$ .... = 0·1716

	Thickness.	
	Rubblestone.	Brickwork.
	Feet.	Feet.
1. Navier's theory of minimum strength, $b = ht\sqrt{\left(\frac{9\,y}{(12 + 8\sqrt{3})\,w}\right)}$ $= 0{\cdot}51\,ht\sqrt{\frac{y}{w}} = 30 \times 0{\cdot}1753$	5·259	6·027
2. Average formula from Mayniel, $b = h \times 0{\cdot}1974$	5·922	6·791
3. Prony's formula, $b = h\sqrt{\left(\frac{y\,t^2}{3\,w}\right)} = ht\sqrt{\frac{y}{3\,w}} = 30 \times 0{\cdot}1984$	5·953	6·824
4. Rondelet's simple formula, (from the *Science de l'Ingenieur,*) $b = h \times 0{\cdot}213$	6·390	
5. Rondelet supposes earth to take a slope of 45°, then one-sixth of the hypothenuse is the thickness of the wall. $b = \frac{\sqrt{2\,h^2}}{6} = \frac{\sqrt{2 \times 30^2}}{6} = \frac{42{\cdot}426}{6}$	7·071	7·315
6. For greater strength, Rondelet takes one-fifth of the hypothenuse. $b = \frac{\sqrt{2\,h^2}}{5} = \frac{\sqrt{2 \times 30^2}}{5} = \frac{42{\cdot}426}{5}$	8·485	
Carry forward	39·080	26·955

	Thickness.	
	Rubble stone.	Brickwork.
	Feet.	Feet.
Brought forward..........	39·080	26·955
7. Gauthey's formula, (Memoirs of the Academy of Dijon). $b = \sqrt{\frac{y\,h^2}{9\,w}} = \sqrt{\frac{95 \times (30)^2}{9 \times 138}}$	8·296	9·512
8. Coulomb's theory coupled with the experiments of Mayniel..	8·825	10·149
	56·201	46·616
Then in the case of a rubble stone wall, $\frac{56·201}{8 \times 30} = 0·2342$ for an average co-efficient. Consequently $b = h \times 0·2342$	7.026	
If the wall be composed of brickwork, Then $\frac{46·616}{6 \times 30} = 0·25898$ Hence $b = h \times 0·259$..............................		7.769

Combining these results :

$$\frac{7·026 + 7.792}{2} = 7.3975 \div 30 = 0·24658$$

for an average co-efficient, which in practice, may be called 0·25. In other words, the thickness of the wall is one-fourth of the height.

This thickness is however, only in equilibrium with the pressure of the earth ; and therefore the smallest possible.

It is generally reckoned that the resistance of the wall ought to be made greater than the pressure of the earth which it has to sustain.

1. The momentum of the prism of earth supposing it to take naturally the slope of 45° is found, according to the theory of Coulomb, in the bisection of that angle. Consequently the line of separation or rupture is distant 12·426 feet from the back of the wall at the level of 30 feet above the base.

Hence the pressure of the prism is

$$30 \times \frac{12{\cdot}426}{2} \times \frac{30}{3} \times 95\text{lbs.} = 177{,}070\text{lbs.}$$

Necessary thickness of the wall. Feet.

Then $w \times h \times \frac{b^2}{2} = 138 \times 30 \times \frac{b^2}{2} = 177{,}070$lbs.

Consequently b .... 9·247

2. The investigations of Mayniel on the pressure of vegetable earths, gravel, and sand, give for the effect of their combined action against a vertical wall .... lbs. 217,940

But if the same be laid with care, then on account of cohesion, this may be reduced to 72,646

2)290,586

Mean state........................ 145,293

If $w \times h \times \frac{b^2}{2} = 138 \times 30 \times \frac{b^2}{2} = 145{,}292$lbs.

Then b .... 8·378

3. The formula of the French Engineers:—

$$b = h \sin. \mathrm{T} \sqrt{\frac{y}{3\,w}}$$

T being the angle of the slope of the earth with the vertical.

$$b = 30 \times \sin. 45^\circ \times \sqrt{\frac{95}{3 \times 138}}$$

$= 30 \times 0{\cdot}337$ .... 10·170

4. The formula of the Engineers of the Dutch Water Staat:—

$b = 0{\cdot}36\ h$ .... 10·800

38·595

Then $\frac{38{\cdot}595}{4} = 9{\cdot}64875$ average thickness.

And $\frac{9{\cdot}64875}{30} = 0{\cdot}3216$ for the average co-efficient.

Hence $b = h \times 0{\cdot}3216$ .... 9·648

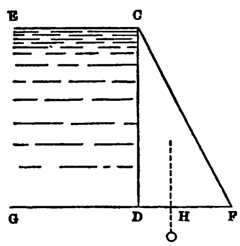
	Necessary thickness of the wall.
	Feet.
For the Case of Fluids.	
Prony's formula is also that of the French Engineers and of Woltman.	
$b = h \times \sqrt{\frac{y}{3w}} = 30 \times 0{\cdot}3855$	11·655
For Soapy earth saturated with water.	
Mayniel's formula gives,	
$b = h \times 0{\cdot}49$	14·700

The inference on the whole is, that if we make the thickness of the wall one-third of the height, it will be ample for the average kind of earth pressing against it; but if care be not taken to place the same properly, and there be likelihood of it being saturated with water, acting with the full force of a semi-fluid, we must of course increase its thickness. Whether this should be made nearly one-half the height, as recommended by Mayniel, is however questionable. There is no doubt that it may be very much avoided by having the backing executed, for a sufficient breadth, with proper care. By so doing a large reduction in the cost of the masonry may be advantageously applied in depositing the backing earth in layers, properly levelled, beaten, and carried up with the building of the wall. In practice this is sometimes done in a slovenly manner, without due superintendence, and is often the cause of failure in the structure.

CHAPTER VI.

ON THE TRANSFORMATION OF THE PROFILES OF WALLS.

In a former part of the work, it was ascertained* that the triangle of brickwork FCD is equal to resist the pressure of water contained in ECDG, and we wish now to transform the triangular profile into a parallelogram of brickwork of equal resistance, both being of the same height and unity of length.



When CD = 30 feet, DF has been found = 16·366 feet.

Now FCD, for each lineal foot of the wall,

$$= \frac{DC \times DF}{2} = \frac{30 \times 16{\cdot}366}{2} = 245{\cdot}5 \text{ cubic feet.}$$

The arm of lever FH of the triangle FCD,

$$= \frac{FD}{3} \times 2 = \frac{16{\cdot}366}{3} \times 2 = 10{\cdot}91 \text{ feet.}$$

Then the resistance of the triangle,

$$= 245{\cdot}5 \times 10{\cdot}91 = 2678{\cdot}4 \text{ cubic feet.}$$

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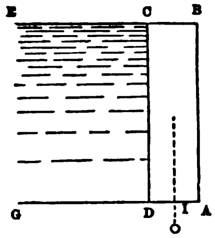
* Vide page 12.



Calling the thickness AD $= b$

Its arm of lever $= \frac{b}{2}$

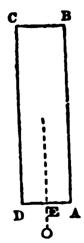
The height AB $= a$

Then $a \times b \times \frac{b}{2} = a \times \frac{b^2}{2} = 2678{\cdot}4$ the resistance.

And $b = \sqrt{\frac{2 \times 2678{\cdot}4}{30}} = 13{\cdot}362$, the thickness required.

A wall therefore having the section of a right angled triangle, with the hypothenuse for the exterior slope, is of equal resistance with a rectangular wall of the same height, although composed of three-fifths less masonry.

Vertical walls with parallel sides, may be transformed into others of the same height with an external slope, the base of which is given.*

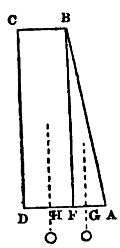


Let DC $= h$ the common height of both walls.

s sectional area of the given wall ABCD.

AE $= x$ distance between the point of rotation, and the perpendicular let fall upon the base AD, from the centre of gravity of the given wall.

Calling AF $= \frac{DC}{5} = e$ base of triangle representing the required slope.



z sectional area of this triangle.

AG $= d$ distance between the point of rotation and the perpendicular let fall upon AD, from the centre of gravity of the triangle.

BC $= a$ thickness at top of the wall required.

$$\text{Then } a = \sqrt{\left(\frac{2\,(s x - z d)}{h} + e^2\right)} - e.$$

* Laisné Aide Memoire, p 62.

1[mo]. Let us ascertain the strength per lineal foot, of the vertical wall with parallel sides.

Calling DC = 30 feet; BC = 7·5 feet.

Then AE $= x = \frac{7 \cdot 5}{2} = 3 \cdot 57$ feet

And $s = 225$

Consequently the resistance of the wall

$= 225 \times 3 \cdot 75 = 843 \cdot 75$ cubic feet.

2[do]. Let us next find the dimensions and strength of the wall, with an external slope, and vertical next the earth.

1. To find BC the breadth at top.

Substitute the real values for the foregoing symbols, and make AF = one-fifth of the height DC.

$$\text{Then BC} = a = \sqrt{\frac{2\,(225 \times 3 \cdot 75) - (90 \times 4)}{30} + 36} - 6.$$

$$= \sqrt{\frac{967 \cdot 5}{30} + 36} - 6 = 8 \cdot 2613 - 6 = 2 \cdot 2613 \text{ feet.*}$$

2. To find AH the length of the arm of the lever.

Calling c the slope or batter of the wall in parts of the height.

$$\text{AH} = \frac{2ha + 3a^2c}{3h + 6ac} + \frac{2h}{3c}$$

$$= \frac{(2 \times 30 \times 2 \cdot 2613) + (3 \times (2 \cdot 2613)^2 \times 5)}{(3 \times 30) + (6 \times 2 \cdot 2613 \times 5)} + \left(\frac{2 \times 30}{3 \times 5}\right)$$

$$= \frac{212 \cdot 3805}{157 \cdot 839} + 4 = 5 \cdot 3456.$$

3[tio].* Let us now ascertain the strength of the wall with an external slope and vertical next the earth.

$$\text{AD} = \text{BC} + \text{AF} = 2 \cdot 2613 + 6 = 8 \cdot 2613$$

---

* The following formula gives a similar result.

Let BC = b the thickness of the given vertical parallel wall,

And c the slope or batter of the wall in parts of the height.

$$\text{Then } a = \sqrt{b^2 + \frac{h^2}{3c^2}} - \frac{h}{c}$$

$$= \sqrt{7 \cdot 5 + \frac{900}{3 \times (5)^2}} - \frac{30}{5} = \sqrt{56 \cdot 25 + 12} - 6$$

$$= 8 \cdot 2613 - 6 = 2 \cdot 2613 \text{ feet.}$$

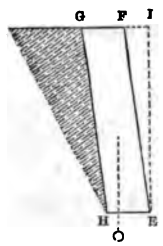
$$\text{Then } 30 \times \frac{8{\cdot}2613 + 2{\cdot}2613}{2} = 157{\cdot}836$$

And the resistance of the wall

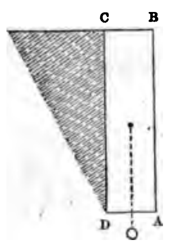
$$= 157{\cdot}839 \times 5{\cdot}3456 = 843{\cdot}75$$

as previously calculated in the case of the vertical wall with parallel sides.

General Sir Charles Pasley, in his Course of Military Instruction,* a work long since out of print, has given us the following formulæ for the transformation of profiles of retaining walls.



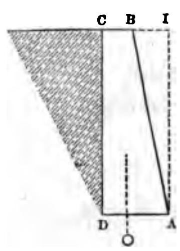
1. To find the thickness of a wall EFGH, leaning against the earth, one-fifth of the height of 30 feet; equal in strength to the vertical wall ABCD, with parallel sides of the same height and 7·5 feet thick.



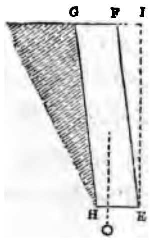
		Feet.
Calling the height common to both walls	$h =$	30·00
the thickness CB of the given wall	$b =$	7·50
ditto FG of the proposed wall	a	
the slope or batter IF in parts of the height	$c =$	5·00

$$\text{Then } a = \sqrt{\left(b^2 + \frac{h^2}{4c^2}\right)} - \frac{h}{2c}$$

$$= \sqrt{56{\cdot}25 + \frac{900}{100}} - 3 = 5{\cdot}077 \text{ feet.}$$



2. When the given wall ABCD has an external slope IB of one-fifth of the height, but vertical next the earth, and it be wished to find the thickness of the wall EFGH, having parallel sides with a similar batter or slope IF; but leaning against the earth, both being of the same height of 30 feet.



Employing the symbols as before, viz:—

$$h = 30;\ b = 2{\cdot}2613;\ c = 5.$$

$$\text{Then } a = \sqrt{\left(b^2 + \frac{2hb}{c} + \frac{2h^2}{3c^2}\right) + \frac{h^2}{4c^2}} - \frac{h}{2c}$$

$$= \sqrt{5{\cdot}1135 + 27{\cdot}1356 + 24 + 9} - 3$$

$$= \sqrt{65{\cdot}2491} - 3 = 5{\cdot}077 \text{ feet.}$$

All profiles of walls, with interior vertical sides, having the same height and the same stability, but the outer faces inclined at least one-sixth from the vertical, have the same thickness measured at one-ninth of their common height above the base.

Hence we have the following simple rule:—

To transform one profile into another of a different slope, turn the exterior side of the given profile round an horizontal axis at one-ninth above the base, until the side has taken the required inclination.

(*Vide* PLATE I.)

Various sections of retaining walls are given in Plate I. They shew at one view their different dimensions; each being of the same height, and of the same stability, with varying exterior slopes. The least profile and the greatest resisting power is the triangle No. 1, vertical next the earth, with an inclined plane for its exterior face. In practice however, we would never construct such a prism, reduced to zero at the summit; and therefore, naturally prefer No. 2, with an exterior slope of one-fifth of the height. This profile, we must bear in mind, is only in equilibrium with the earth, averaging 95lbs. to the cubic foot; frequently we meet with it much heavier, and subjected to the additional power of water. The great exterior slope of one-fourth and one-fifth of the perpendicular height, so generally adopted by the old Engineers in their fortifications, became in time, much injured by exposure to the weather, from the rain water falling on the surface, lodging in the joints of the brick-work, and percolating into the interior work. The Engineers of a later date have therefore, greatly diminished the slope, and some writers have even proposed to run into a contrary extreme, and abandon it altogether.

CHAPTER VII.

ON DIFFERENT FORMS OF WALLS CAPABLE OF RESISTING THE PRESSURE OF EARTH.

(*Vide* PLATE II.)

FOR all practical purposes vegetable earths, sands, and gravels, not saturated with water, may be considered generally to take a slope of 45° with the horizon; and the surface is called the *Line of Repose.* It has been proved by Coulomb that the *Line of Rupture* bisects the angle formed between it and the vertical. If AL be the line of Repose, AM becomes that of Rupture; consequently the prism of earth ABM is a falling mass, and requires to be kept in its position by a wall built in front of AB; whilst the prism of earth AML is a sliding mass, and is retained by the prism ABM and the wall in front of AB. If either change their position by pressure, forcing them outwards, then the sliding prism AML begins to exercise its pressure also; and when combined with that of the falling prism ABM, will tend to overturn the wall in front of AB.

Now the centre of gravity of AML is G; and the line of direction of its sliding action is GH parallel to AL. The centre of gravity of ABM is E; and a perpendicular let fall from it, will intersect in D the line GH. Hence, so long as this intersection takes place, the prism AML is kept in its position by the prism ABM. A change in ABM to A*bm* by the pressure outwards of the wall AB, supposed to turn on a pivot or hinge round the point A, disturbs the centre of gravity. But, in this instance, although removed from E to *e*, which gives the falling mass an unstable position, yet the prism AML if transferred to A*m*L is partially maintained, and the effect of retention is not wholly destroyed until the centre of gravity *e* be forced beyond the vertical AB. If the falling mass A*bm* be wholly removed, the prism A*m*L has then the power to slide in a direction drawn from its centre of gravity *g*, in a line parallel with the plane of repose AL.

Assuming that the prism of earth AML can be retained in a stationary position by the prism ABM; then EF of the last-mentioned prism, is the direction of the pressure parallel to AM, and AF is the length of the arm of lever = one-third of the total height AB.

If AB be 30 feet in height,

Then BM = $0{\cdot}4142 \times 30 = 12{\cdot}426$ feet.

And the area of ABM = $\dfrac{30 \times 12{\cdot}426}{2} = 186{\cdot}39$ square feet.

Now AF = $\dfrac{30}{3} = 10$

Then $186{\cdot}39 \times 10 = 1863{\cdot}9$.

And calling the weight of a cubic foot of earth 95lbs. Then the pressure of the earth against each lineal foot of the plane AB,

$$= 1863{\cdot}9 \times 95 = 177{,}070 \text{ lbs.}$$

Hence the power of overturning it round the point A, on each lineal foot of the plane AB, at its extremity B

$$= \frac{177{,}070}{30 \times 95} = 62{\cdot}13 \text{ cubic feet.}$$

Consequently the length of the base AO of a triangle of earth placed outside of AB, capable of supporting the prism of earth ABM

$$= \sqrt{3 \times 62{\cdot}13} = 13{\cdot}652 \text{ feet.}$$

If the triangle outside of AB be composed of masonry weighing 138lbs. per cubic foot, then,

$$\frac{177{,}070}{30 \times 138} = 42{\cdot}7702 \text{ cubic feet.}$$

And the length of the base AP of the triangle of masonry placed outside of AB, capable of supporting the prism of earth ABM*

$$= \sqrt{3 \times 42{\cdot}7702} = 11{\cdot}327 \text{ feet.}$$

It must be remembered that the triangle of masonry ABP is only *in equilibrium* with the pressure of the earth composing the prism ABM. In practice it is necessary to make the wall of greater resistance, in consequence of inferior materials or bad workmanship being employed in its construction. There is also the possibility of water percolating through unknown crevices and saturating

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* See Page 12.

the retained earth, which may cause the Line of Repose to be much less than the angle of 45° with the horizon. If such be the case the Line of Rupture is considerably altered, and hence the falling prism ABM is increased in its area. How much must be left to the judgment of the engineer, who, from a proper examination of the soils which he has to encounter, conjoined with long practical experience, generally exercises a sound opinion on the subject. But sometimes he is deceived, especially with soapy clays, saturated with water, and consequently in a most treacherous condition. The Line of Repose is therefore of great consequence in our calculations, for the necessary strength of Retaining Walls; and must require due consideration before proceeding with the design.

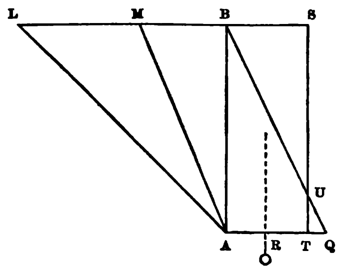
We are of opinion, that if proper care be taken to cut down the earth for the reception of the wall, and back it up again in layers as the wall is built, sufficient allowance may be made, by calculating the falling prism equivalent to 25 per cent. more than its theoretical area. But taking into consideration the chance of encountering treacherous ground, let this be doubled, making 50 per cent. more than the area of the prism ABN.

As that is comprised in the angle of 22° 30′ with the vertical, this addition of 50 per cent. will increase the angle to about 31° 51′ 9″. Taking this as the Line of Rupture, then twice the angle, viz., 63° 42′ 18″, leaves 26° 17′ 42″ as the angle with the horizon for the Line of Repose, being something less than two to one; a slope so gentle that with it, almost every soil may with safety be considered stationary, unless it approach semi-fluidity.

Again, we must take into consideration the possibility of the materials employed in the construction of the wall being of indifferent quality, coupled with inferior workmanship. On this account if 25 per cent. be added to the area of the falling prism ABM, it may be ample. Combining therefore these contingencies, if we make the strength of the masonry equivalent to 75 per cent. more than the pressure of the falling prism ABM we arrive at a standard value for ordinary practice.

If, however the materials be of very inferior quality, or no care be taken to back the work properly, then a larger per centage must be taken, which consequently requires the wall to be increased in thickness.

Assuming 75 per cent. to be sufficient, then $177{,}070 \times \frac{3}{4} = 132{,}803$.



And $\frac{177{,}070+132{,}803}{30 \times 138} = \frac{309{,}873}{4{,}140} = 74{\cdot}8485$.

Hence the length of the base AQ,

$$= \sqrt{3 \times 74{\cdot}8485} = 14{\cdot}985 \text{ feet.}$$

$$ABQ = \frac{AB \times AQ}{2} = \frac{30 \times 14{\cdot}985}{2} = 224{\cdot}775 \text{ square feet.}$$

The leverage QR of this triangle,

$$= \frac{AQ}{3} \times 2 = \frac{2}{3} \times 14{\cdot}985 = 9{\cdot}99 \text{ feet.}$$

Then the resistance of the triangle of masonry, ABQ,

$$= 224{\cdot}775 \times 9{\cdot}99 = 2{,}246 \text{ square feet.}$$

Hence, each lineal foot of the wall, taking a cubic foot at 138lbs.,

$$= 2{,}246 \times 138 = 309{,}948\text{lbs.}$$

Some authors transform this triangle into a parallelogram ABST, of the same resistance as the triangle ABQ in the following manner:—

Calling the thickness AT $= b$

Its arm of lever............ $= \frac{b}{2}$

The height AB............ $= h$

Then the resistance....... $= h\,\frac{b^2}{2} = 2{,}246$

And the thickness AT required,

$$= b = \sqrt{2\,\frac{2{,}246}{30}} = 12{\cdot}236 \text{ feet.}$$

If the triangle ABQ be of sufficient strength, then this thickness of 12·236 feet from the bottom to the top is absurd, and the upper part BSU, of the parallel-sided wall is useless, and a complete waste of material.

(*Vide* PLATE III.)

This will be still further evident by subdividing the triangle of pressure ABM into others *b*Bc, *d*Be, &c., and transforming the external triangle ABQ, into another figure of equal strength, having a series of offsets placed externally. Assuming the weight of a cubic foot of the earth to be 95lbs., and that of the masonry at 138lbs, the dimensions of these offsets have been ascertained by the following calculations:—

As regards the Pressure of the Earth.	Areas of the subdivided triangles.
	Square Feet.
B*c* = 5 × 0·4142 = 2·071 × 5 = 10·555 ÷ 2 =	5·1775
B*e* = 10 × 0·4142 = 4·142 × 10 = 41·420 ÷ 2 =	20·7100
B*g* = 15 × 0·4142 = 6·213 × 15 = 93·195 ÷ 2 =	46·5975
B*i* = 20 × 0·4142 = 8·284 × 20 = 165·680 ÷ 2 =	82·8400
B*l* = 25 × 0·4142 = 10·355 × 25 = 258·875 ÷ 2 =	129·4375
BM = 30 × 0·4142 = 12·426 × 30 = 372·780 ÷ 2 =	186·3900

We proceed further with the Pressure of the Earth as follows:—

Areas of the subdivided triangles.	Leverage of each triangle.	Weight of a cubic foot.	Actual pressure of each triangle.
Sq. Ft.	Ft.	lbs.	lbs.
5·1775	× $\frac{5}{3}$	× 95	= 820
20·7100	× $\frac{10}{3}$	× 95	= 6,558
46·5975	× $\frac{15}{3}$	× 95	= 22,134
82·8400	× $\frac{20}{3}$	× 95	= 52,750
129·4375	× $\frac{25}{3}$	× 95	= 102,471
186·3900	× $\frac{30}{3}$	× 95	= 177,071

Having arrived at this stage, we suppose that the triangles of pressure require to be increased 75 per cent. for the reasons previously given. Then

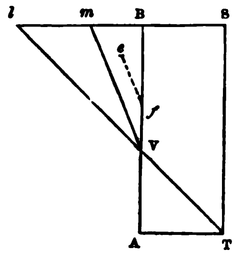
Pressure of the Earth.				Resistance of the Wall.	
	lbs.	lbs.	lbs.		Feet.
$bBc =$	$(820 +$	$615) =$	1,435	$bm = \sqrt{2 \frac{1,435}{5 \times 138}} =$	2·039
$dBe =$	$(6,558 +$	$4,919) =$	11.477	$dn = \sqrt{2 \frac{11,477}{10 \times 138}} =$	4·078
$fBg =$	$(22,134 +$	$16,661) =$	38,795	$fo = \sqrt{2 \frac{38,795}{15 \times 138}} =$	6·119
$hBi =$	$(52,750 +$	$39,562) =$	92,313	$hp = \sqrt{2 \frac{92,313}{20 \times 138}} =$	8·158
$kBl =$	$(102,471 +$	$76,854) =$	179,325	$kq = \sqrt{2 \frac{179,325}{25 \times 138}} =$	10·196
$ABM =$	$(177,071 +$	$132,804) =$	309,875	$AT = \sqrt{2 \frac{309,875}{30 \times 138}} =$	12·236

We have therefore the wall AB*mnopq*T, of 214·13 superficial feet in area, above the line of the horizon, equivalent to resist the pressure of the prisms *b*B*c*, *d*B*e*, &c., of the subdivided triangle ABM, with 75 per cent. more for contingencies.

As Retaining Walls are seldom built with offsets externally, we may reverse the Profile, and place them inwardly with advantage. By so doing we obtain additional strength to the wall, from the gravity of the backing earth, which then rests upon the offsets; and which circumstance allows the masonry of the walls to be reduced in thickness accordingly.

In truth, both the triangle of masonry ABQ, and the wall with the external offsets, AB*mnopq*T, are much stronger than necessary, for the calculations are based upon the erroneous supposition of the wall turning round at the point A, instead of the points T and Q. Both may however be forced outwardly, in a partial manner, along the line AQ, before being overturned; but such can only occur when the wall is too weak in itself, or improperly constructed: and as this may be wholly avoided where the foundation proves good, we shall consider only the case of overturning at the points T and Q.

It is admitted that ordinary soils remain stationary when cut to a slope of 45° with the horizon. If this be so with earthwork, it must likewise stand good with the masonry of the wall itself, the buttresses (if there be any), in front, or the counterforts behind, embedded as they usually are in the earth.



Hence, if from an assumed point of rotation T, a line TV*l* be drawn at the angle of 45° with the horizon, and cutting the back of the wall AB in the point V. Then all below that line must remain stationary; and the effects above it of resistance on the part of the wall, and pressure on that of the earth have only to be considered.

By this assumption we err on the safe side; for it is probable that brickwork and masonry, being of greater consistency than earths, and such like substances, remain stationary at a much less angle than that of 45°. But without taking this into consideration, if we proceed by bisecting the angle VB*l* by the line V*m*, then the falling mass comprises the prism VB*m*, the centre of gravity of which is *e*; the line *ef* parallel to V*m*, is the direction of the thrust; and its length of lever is V*f*, equal to one-third of the height VB.

If we subdivide the falling prism VB*m* into a series of minor triangles, and calculate the pressure of each separately, then the figure, with internal offsets, may be found by the following calculations:—

(*Vide* PLATE IV.)

Let it be required to find the proper thickness of a wall 30 feet in height, perpendicular in front, with offsets behind, capable of resisting the ordinary pressure of the earth, and 75 per cent. more for contingencies.

Divide AB into any number of equal spaces that may be considered desirable for the height of the offsets at the back of the wall, which are here assumed to be six in number :—

$$\text{Then BN, NP, \&c.} = \frac{AB}{6} = \frac{30}{6} = 5 \text{ feet.}$$

Through N, P, R, T, and V, draw parallel lines to AL, and let us assume the wall to be 18 inches thick at the level of the coping. Then the first cuts the back of the wall in the point *a*. Through it draw *a*O parallel to AM.

The leverage of the triangle a C O $= \frac{5 - 1{\cdot}5}{3} = \frac{3{\cdot}5}{3} = 1{\cdot}1667$ feet.

	lbs.
The pressing force of a C O $= 95 \times \frac{(5 - 1{\cdot}5) \times 0{\cdot}4142 \times (5 - 1{\cdot}5)}{2} \times 1{\cdot}1667 =$	281
To which add 75 per cent. for contingencies.................. =	210
Pressure .... =	491
The resistance of the wall NBCa $= 138 \times \frac{5 + 3{\cdot}5}{2} \times 1{\cdot}5 \times 0{\cdot}75$ .... =	660

Consequently this portion of the wall is stronger than necessary, arising from too great a thickness having been given to it under the coping.

The next division of the wall is that comprised within PBcb.

Draw through b a line parallel to AM.

The leverage of the triangle bCQ is

$$= \frac{10 - 1{\cdot}5}{3} = \frac{8{\cdot}5}{3} = 2{\cdot}833 \text{ feet.}$$

	lbs.
The pressing force of bCQ $= 95 \frac{(10 - 1{\cdot}5) \times 0{\cdot}4142 \times (10 - 1.5)}{2} \times 2{\cdot}833 =$	4,027
To which add 75 per cent. .... =	3,020
Pressure .... =	7,047
The resistance of the wall PBCb $= 138 \frac{10 + 8{\cdot}5}{2} \times 1{\cdot}5 \times 0,75$ .... =	1,436
Consequently the wall is much too weak, and requires an offset at the back; which we may assume to project 18 inches,	
Then to the above add for the masonry of the offset $138 \times \frac{5 + 3{\cdot}5}{2} \times 1{\cdot}5 \times 2{\cdot}25$ .... =	1,979
Add for the earth resting upon it $95 \times (5 - 1{\cdot}5) \times 1{\cdot}5 \times 2{\cdot}25$ .... =	1,122
Resistance .... =	4,537

The wall therefore, is now of sufficient strength, to resist the pressing triangle, as it is reduced in area to dOg by the above projection of 18 inches,

Making its leverage

$$= \frac{10 - 3}{3} = \frac{7}{3} = 2{\cdot}332$$

	lbs.
And the pressing force $= 95 \times \frac{(10 - 3) \times 0.4142 \times (10 - 3)}{2} \times 2{\cdot}333 =$	2,218
To which add 75 per cent. .... =	1,663
Pressure .... =	3,881

The next division of the wall is comprised within RBC*ace*.

Draw through *e*, a line parallel to AM.

The leverage of the triangle *e*OS

$$= \frac{15 - 3}{3} = \frac{12}{3} = 4 \text{ feet.}$$

	lbs.
The pressing force of *e*OS $= 95 \; \frac{(15 - 3) \times 0.4142 \times (15 - 3)}{3} \times 4$ ..... =	11,143
To which add 75 per cent. .... =	8,356
Pressure .... =	19,499
The resistance of the wall PBC*acd*, ... Brought forward	4,537
Add the portion RP*de* = 138 × 5 × 3 × 1·5...... =	3,105
Resistance .... =	7,642

The wall therefore, requires additional strength by another offset. If this project 18 inches, we proceed thus—

The above resistance .... Brought forward	7,642
Add for the masonry of the offset $= 138 \; \frac{5 \times 3{\cdot}5}{2} \times 1{\cdot}5 \times 3{\cdot}75$ .... =	3,299
And for the earth resting upon it 95 × (15 − 3) × 1·5 × 3·75 .... =	3,740
Resistance .... =	14,681

The pressing force is now reduced to *ghs*	lbs.
$= 95 \frac{(15 - 4{\cdot}5) \times 0{\cdot}4142 \times (15 \times 4{\cdot}5)}{2} \times \frac{10{\cdot}5}{3} =$	7,692
To which add 75 per cent. .... =	5,768
Pressure .... =	13,460

Proceeding in this manner, by projections of 18 inches at the back of the wall, the figure at last is found to assume that of ABC*acdfgklmnop*. Its resistance is ascertained in the following manner :—

Masonry, for each lineal foot, of the wall and its offsets,

Heights. Feet.		*Leverages.* Feet.		Feet.		lbs.
29·25	×	0·75	=	31·9375		
24·25	×	2·25	=	54·5625		
19·25	×	3·75	=	72·1875		
14·25	×	5·25	=	74·8125		
9·25	×	6·75	=	62·4375		
4·25	×	8.25	=	35·0625		
				331·0000 × 1·5 (Feet) × 138 (lbs.)	=	62,307

Earth, for each lineal foot, resting upon the several offsets,

3·5	×	2·25	=	7·875		
7·0	×	3·75	=	26·250		
10·5	×	5·25	=	55·125		
14·0	×	6·75	=	94·500		
17·6	×	8·25	=	144·375		
				328·125 × 1·5 × 95	=	46,758
				Total Resistance		115,275

As regards the Pressure of the Earth,

The leverage of the triangle *p*UW

$$= \frac{30 - 9}{3} = \frac{21}{3} = 7$$

The pressing force of *p*UW	
$= 95 \times \frac{(30 - 9) \times 0{\cdot}4142 \times (30 - 9)}{2} \times 7 \ldots =$	60,735
To which add 75 per cent. .... =	45,551
Total pressure............ =	106,286

Hence this form of wall, with a sectional area of 191·25 square feet above the line of the horizon, is adequate to resist the pressure of the prism of earth, *p*UW, with 75 per cent. additional for contingencies.

The thickness at top for Quays, and such like Walls, is in a great measure regulated by the width of the coping. A sufficient strength is necessarily given to the wall immediately beneath it for the purpose of withstanding the shock of a vessel. Practically this is made from three to five feet. In the following example we shall take three feet for the thickness at the level of the coping.

Considerable strength is given to all retaining walls by making the face have a batter or slope. This varies from one-fifth to one-tenth of the total perpendicular height. In modern practice, however, it is deemed preferable not to do so, more particularly in the upper part of quay walls, where the sides of the ships rub against and injure them. In narrow entrances, the batter of the upper part of the wall has been also the cause of inconvenience. In locks, the side walls, at the level of the coping, are usually made wider than the distance between the hollow quoins, to make allowance for the batter; but sometimes, for want of proper attention, they are not placed sufficiently apart. In consequence, they are more contracted at the low water line than they ought to be, and some difficulty is then experienced in passing the vessels through the entrance or lock.

(*Vide* Plate V.)

Assuming therefore, to meet these difficulties, that the upper part of the wall BP, be made with a perpendicular face for one-third of the total height, and the lower part with a batter to gain strength. Then through P draw a line parallel to AL, cutting the vertical back of the wall in the point *a*, from which draw *a*O, parallel to AM.

It is unnecessary to calculate how much stronger that part of the wall comprised within PBC*a* is, than the pressure of the prism of earth *a*CO, as it will be taken into consideration in the next lower division of the wall, bearing in mind that it is here made thicker than requisite, for the reasons previously given.

Divide PA into four, or any number of equal parts P*r*, *rt*, *tv*, *v*A; and draw lines through them, parallel with the horizon. Make the slope or batter *r*R, *t*T, of the middle part of the wall, between P and T, equal to one-tenth of the perpendicular height, and through R and T draw lines parallel to AL. The first cuts the back of the wall C*a* produced in *b*, and through it draw *b*Q, parallel to AM. We find by calculation that the wall requires to be strengthened by an offset. Assuming this to be 12 inches, we proceed to find the resistance per lineal foot of the wall RPBC*acd*, in the following manner:—

1. The area of the battering portion comprised within RPw,

$$= \frac{5 \times 0{\cdot}5}{2} - \frac{0{\cdot}5 \times 0{\cdot}5}{2} = 1{\cdot}125$$

Its leverage $= \frac{2}{3} \times 0{\cdot}5 = 0{\cdot}333$

	Cub. ft.		
Then the resistance $= 1{\cdot}125 \times 0{\cdot}333$	0·375		
2. The area of the portion comprised within wBCb, $= \frac{14{\cdot}5 + 11{\cdot}5}{2} \times 3 = 39$; Its leverage $= \frac{3}{2} + 0{\cdot}5 = 2$; Then 39 × 2 .... =	78·000		
3. The area of *bacd* $= \frac{4{\cdot}5 + 3{\cdot}5}{2} \times 1 = 4$; Its leverage $= \frac{1}{2} + 3{\cdot}5 = 4$; Then 4 × 4................................... =	16·000	lbs.	lbs.
	94·375	× 138=	13,024
4. The area of the earth resting on the offset *ac*, which is considered to form part of the wall, $= (10 - 3) \times 1 = 7$; Its leverage $= \frac{1}{2} + 3{\cdot}5 = 4$; Then 7 × 4 × 95lbs. .... =			2,660
Resistance .... =			15,684

Pressure of the Earth per lineal foot.

The leverage of the triangle, *de*Q, $= \frac{15 - 4{\cdot}5}{3} = \frac{10{\cdot}5}{3} = 3{\cdot}5$

	lbs.
The pressing force $= 95\,\frac{(15 - 4{\cdot}5) \times 0.4142 \times (15 - 4{\cdot}5)}{2} \times 3{\cdot}5 =$	7,592
To which add 75 per cent. for contingencies .... =	5,694
Pressure .... =	13,286

The offset therefore might have had a trifle less projection, as this part of the wall, at the point R, is stronger than necessary.

The next portion is that comprised within TPBC*acdgh*; assuming another offset *dg* to be required, with a projection also of 12 inches. Draw T*h* parallel to AL, cutting the line *cd* produced in *f*, and the back of the wall in *h*. Then *hi*S is the prism of the pressing earth.

Resistance of the Wall per lineal foot.

	Cub. ft.		lbs.
1. That portion above R*d*, with the earth resting on the offset *ac*, ... Brought forward =			15,684
2. The battering portion TR*x* = RP*w* as above calculated .... =	0·3750		
3. The area of the portion comprised within *x*R*df*, $= 4·5 \times 4·5 = 20·25$ Its leverage $= \frac{4·5}{2} + 0·5 = 2·75$ Then $20·25 \times 2·75$ .... =	55·6875		
4. The area of *fdgh* = *bacd*, = 4·0 Its leverage $\frac{1}{2} + 4·5 + 0·5 = 5·5$ Then $4 \times 5·5$ .... =	22·0000		
	78·0625	× 138 lbs. =	10,773
5. The area of the earth, resting on the offset *dg*, $= 10·5 \times 1 = 10·5$ Its leverage $\frac{1}{2} + 4·5 + 0·5 = 5·5$ Then $10·5 \times 5·5 \times$ 95lbs. .... =			5,486
Resistance.............. =			31,943

Pressure of the Earth per lineal foot.

The leverage of the triangle *hi*S, $= \frac{15 - 1}{3} = \frac{14}{3} = 4·666$	
The pressing force $= 95 \frac{(15 - 1) \times 0·4142 \times (15 - 1)}{2} \times 4·666 =$	17,996
To which add 75 per cent. .... =	13,497
Pressure .... =	31,493

So far therefore the profile of the wall, at the point T, is equivalent in strength to the pressure of the earth with 75 per cent. more for contingencies.

Strength being gained by the wall having a battering face, we may advantageously increase it; and as the outline of ships is usually such that they cannot injure the lower part of the wall, we may assume the batter, below T, to be one-fifth of the perpendicular height, making $vV = 2$ feet, and $AX = 3$ feet.

Let us now calculate the strength of the wall comprised within VBC*acdghlm*, assuming another offset *hl* to be necessary, of 12 inches projection. Through V draw a line parallel to AL, cutting *gh* produced in *k*, and the back of the wall in *m*. Then *mn*V is the prism of the pressing earth.

Resistance of the Wall per lineal foot.

	Cub. ft.		lbs.
1. That part above T*h*, with the earth resting on the offsets, *ac* and *dg* .... Brought forward			31,943
2. The area of the battering portion VT*y*,			
$= \frac{5 \times 1}{2} - \frac{1 \times 1}{2} = 2$			
Its leverage $\frac{2}{3} \times 1 = 0·666$			
Then the resistance $= 2 \times 0·666$ ...... =	1·3333		
3. The area of the part *y*T*hk*,			
$= 5 - 1 \times 6 = 24$			
Its leverage $= \frac{6}{2} + 1 = 4$			
Then the resistance $= 24 \times 4$ .... =	96·0000		
4. The area of *khlm*,			
$= \frac{(5-1) + (4-1)}{2} \times 1 = 3·5$			
Its leverage $\frac{1}{2} + 6 + 1 = 7·5$			
Then $3·5 \times 7·5$ .... =	26·2500	lbs.	
	123·5833	× 138 =	17,054
5. The area of the earth resting on the offset *hl*,			
$= (15 - 1) \times 1 = 14$			
Its leverage $\frac{1}{2} + 7 = 7·5$			
Then $14 \times 7·5 \times 95$lbs .... =			9,975
Resistance .... =			58·972

Pressure of the earth per lineal foot.

The leverage of the triangle mnU,

$$= \frac{20 - 3}{3} = \frac{17}{3} = 5{\cdot}666$$

The pressing force,

		lbs.
$= 95 \dfrac{17 \times 0{\cdot}4142 \times 17}{2} \times 5{\cdot}666$	=	32,220
To which add 75 per cent...	=	24,165
Pressure	=	56,385

The wall, at the point V, is now stronger than necessary from having assumed too much projection for the offset.

Let us however proceed with the last portion comprised within XTPBC*acdg hlmpq*, assuming another offset, *mp*, to be requisite with a projection of 12 inches.

Through X draw a line parallel to AL cutting *lm*, produced in *o*, and the back of the wall in *q*. Then qYZ is the prism of the pressing earth.

Resistance of the Wall per lineal foot.

	Cub. ft.		lbs.
1. That part above Vn with the earth resting on the offsets, *ac*, *dg*, and *km*................ Brought forward			58,972
2. The battering portion, XVz = VTy as above calculated	1·3333		
3. The area of the portion comprised within zVmo = $(5 - 1) \times 8 = 32$			
Its leverage $\frac{8}{2} + 1 = 5$			
Then the resistance = 32×5...........	160·0000		
4. The area of $ompq$ = $khlm$ = $3{\cdot}5$			
Its leverage = $\frac{1}{2} + 8 + 1 = 9{\cdot}5$			
Then the resistance = $3{\cdot}5 \times 9{\cdot}5$... =	33·2500	lbs.	
	194·5833	× 138 =	26,852
Carried forward			85,824

	lbs.
Brought forward	85,824
5. The area of the earth resting on the offset, *mp*,	
$= (20 - 3) \times 1 = 17$	
Its leverage $= \frac{1}{2} + 8 + 1 = 9{\cdot}5$	
Then $17 \times 9{\cdot}5 \times 95$lbs. .... =	15,343
Total Resistance...... =	101,167
Pressure of the earth per lineal foot.	
The leverage of the triangle qYZ	
$= \frac{20}{3} = 6{\cdot}667$	
The pressing force	
$= 95 \frac{20 \times 0{\cdot}4142 \times 20}{2} \times 6{\cdot}666$ .... =	52,465
To which add 75 per cent. .... =	39,348
Total Pressure	91,813

(*Vide* PLATE VI.)

The external part of the wall below P may be curved, instead of having straight lines as above calculated; thus giving the profile a more elegant outline, and at the same time allowing the courses of masonry to radiate to the centre, from the foundation course upwards.

Having ascertained from the preceding calculation, that the wall at R could admit of a slight reduction in thickness, while at the point T it would still remain of adequate strength, we assume P and T to be given. If it be required to find the radius of a curve, in a line through P, parallel with the horizon, so as to make PB a tangent, we adopt the following rule:—

Square the half chord, to which add the square of the versed sine, and divide the product by twice the versed sine, will give the radius required.

Calling W the centre of the curve,

$$\text{Then PW} = \text{TW} = \frac{(\text{P}t)^2 + (\text{T}t)^2}{2\,(\text{P}t)} = \frac{10^2 + 1^2}{2 \times 1} = \frac{101}{2} = 50{\cdot}5 \text{ feet.}$$

Conversely when the half chord, and the radius of the curve are given, to know the versed sine.

Find the square root of the difference of the squares of the radius and of half the chord; the difference between it and the radius is the versed sine.

The versed sine $Vv = aP$.

Hence $\sqrt{(VW)^2 - (Va)^2} = aW = \sqrt{(50{\cdot}5)^2 - (15)^2} = 48{\cdot}2208$.

And $PW - aW = aP = 50{\cdot}5 - 48{\cdot}2208 = 2{\cdot}2792$ feet.

The versed sine $AX = Pb$.

Then $\sqrt{XW^2 + Xb^2} = bW = \sqrt{50{\cdot}5^2 - 20^2} = 46{\cdot}3707$.

And $PW - bW = bP = 50{\cdot}5 - 46{\cdot}3707 = 4{\cdot}1293$ feet.

Graphically we may adopt the following method:—

Join PX, and bisect it at *c;* make *c*T at right angles to $PX = \frac{AX}{4}$

Join PT, TX; bisect them in *d* and *f*, and let *de*, and *f*V at right angles to PT and TX respectively $= \frac{cT}{4}$

Join P*d*, *d*T, TV, VX, and so on *ad infinitum.*

Then P, *d*, T, V, and X are points in the curve.

In consequence of *v*V and AX being greater than in the figure delineated on Plate V, the projections of the offsets *hl* and *mp* may be reduced, and thus diminish the sectional area of the wall.

Assuming that all above the line T*h* remains as heretofore, which is near enough for every practical purpose, we proceed to ascertain the altered condition of the wall below T*h*.

Resistance of the wall per lineal foot.

	Cub. ft.		lbs.
1. The portion comprised within TPBC*h*, with the earth resting on the offsets ....Brought forward			31,943
2. Taking TV as a straight line, the area of the battering part comprised within VT*r*, $=\frac{5\times1{\cdot}2792}{2}-\frac{1{\cdot}2792\times1{\cdot}2792}{2}=2{\cdot}3798$			
Its leverage $= \frac{2}{3} \times 1{\cdot}2792 = 0{\cdot}8528$			
Then the resistance $= 2{\cdot}3798 \times 0{\cdot}8528$............ =	2·0295		
Carried forward	2·0295	...	31,943

	Cub. ft.		lbs.
Brought forward...... =	2·0295		31,943
3. The area of the portion comprised within $rThk$,			
$= (5 - 1·2792) \times 6 = 22·3248$			
Its leverage $= \frac{6}{2} + 1·2792 = 4·2792$			
Then $22·3248 \times 4·2792$ .... =	95·5323		
4. Assuming the projection of the offset hl to be 10 inches,			
Then $kh = 5 - 1·2792 = 3·7208$			
$ml = 3·7208 - 0·8333 = 2·8875$			
The area of $khlm$,			
$= \frac{3·7208 + 2·8875}{2} \times 0·833 = 2·7524$			
Its leverage $= \frac{0·8333}{2} + 6 + 1·2792 = 7·6959$			
Then $2·7524 \times 7·6959$.................. =	21·1823	lbs.	
	118·7441	× 138 =	16,387
5. The area of the earth resting on the offset hl			
$= (15 - 0·8333) \times 0·8333 = 11·8051$			
Its leverage $= \frac{0·8333}{2} + 7·2792 = 7·6959$			
Then $11·8051 \times 7·6959 \times 95$lbs. .... =			8,630
Resistance .... =			56,960

Pressure of the earth per lineal foot.

The leverage of the triangle mnU,

$$= \frac{20 - 2·8333}{3} = \frac{17·1667}{3} = 5·7222$$

Its area $= \frac{17·1667 \times 0·4142 \times 17·1667}{2} = 61·0311$

	lbs.
And the pressing force $= 61·0311 \times 5·7222 \times 95$lbs......................... =	33,177
To which add 75 per cent. .... =	24,883
Pressure .... =	57,360

The resistance at V is therefore nearly equivalent to the pressure.

The whole *Resistance of the Wall, per lineal foot*, including the lower part XV*mqq*, now remains to be ascertained.

	Cub. ft.		lbs.
1. The portion above V*m*, with the offsets of Earth...Brought forward			56,960
2. Assuming VX to be a straight line, the area of the battering part comprised within XV*s* $= \frac{5 \times (4{\cdot}1293 - 2{\cdot}2792)}{2} - \frac{1{\cdot}8501 \times 1{\cdot}8501}{2}$ $= 2{\cdot}9138$			
Its leverage $= \frac{2}{3} \times 1{\cdot}8501 = 1{\cdot}2334$			
Then the resistance $= 2{\cdot}9138 \times 1.2334$........... =	3·5939		
3. The area of the part comprised within *s*V*mo* $= (5 - 1{\cdot}8501) \times 8{\cdot}1125 = 25{\cdot}5536$			
Its leverage $= \frac{8{\cdot}1125}{2} + 1{\cdot}8501 = 5{\cdot}9064$			
Then $25{\cdot}5536 \times 5{\cdot}9064$ .... =	150·9298		
4. Assuming the projection of the offset *mp* to be 4 inches,			
Then $om = 5 - 1{\cdot}8501 = 3{\cdot}1499$			
$qh = 3{\cdot}1499 - 0{\cdot}4167 = 2{\cdot}7332$			
The area of *ompq* $= \frac{3{\cdot}1499 + 2{\cdot}7332}{2} \times 0{\cdot}4167 = 1{\cdot}2257$			
Its leverage $= \frac{0{\cdot}4167}{2} + 8{\cdot}1125 + 1{\cdot}8501 = 10{\cdot}171$			
Then $1{\cdot}2257 \times 10{\cdot}171$ .... =	12·4666	lbs.	
	166·9903	× 138 =	23,045
5. The area of the earth resting on the offset *mp*, $= (25 - 8{\cdot}1125) \times 0{\cdot}4167 = 7{\cdot}037.$			
Its leverage $= \frac{0{\cdot}4167}{2} + 8{\cdot}1125 + 1{\cdot}8501 = 10{\cdot}171$			
Then $7{\cdot}037 \times 10{\cdot}171 \times 95$lbs................................. =			6,799
Total Resistance ...... =			86,804

Pressure of the Earth per lineal foot.

The leverage of the triangle qYZ.

$$= \frac{30 - 10{\cdot}3793}{3} = \frac{19{\cdot}6207}{3} = 6{\cdot}5402$$

Its area

$$= \frac{19{\cdot}6207 \times 0{\cdot}4142 \times 19{\cdot}6207}{2} = 79{\cdot}7277$$

		lbs.
And the pressing force		
= 79·7277 × 6·5402 × 95lbs.	=	49,537
To which add 75 per cent.	=	37,153
Total Pressure		86,690

The profiles of the walls, delineated on the plates V. and VI., have each a sectional area above the horizon AX of 186·5 and 181·23 square feet respectively. We have proved by these calculations that each is capable of resisting something more than the ordinary pressure of the earth and other contingencies. We have also seen that a wall having a curved face, and perpendicular behind with offsets, has the least profile yet investigated; and that the lowest offsets may be less than those in the middle where the wall is subjected to the greatest pressure. It would thus appear that the back of the wall might be in a curvilineal form, for if a line be drawn through q parallel with the horizon, the centre of such a curve would be found to touch q and B, and nearly intersect the outer points of the offsets p, l, g, and C. The upper part of the wall CB, remaining three feet in thickness, as before, would intersect the curve, while the earth resting on the rough projecting courses of the back of the wall would add stability equivalent to a series of different ordinates between CD and Yq.

We have to say a few words on the part below Xq. As it must be immoveable, if properly constructed, there can be no necessity for further offsets at the back of the wall; and consequently it may be made perpendicular, like the line pq produced. If the foundation be good, oversailing courses may be used from the bottom upwards, parallel to TX; and if the ground be very sound, the commencement of these courses may approach even nearer than this to the point X. Other means of saving materials may be adopted which we shall refer to hereafter, when we enter into the manner how counterforts add strength to the profile. It must be evident, that by placing the foundation course 12 or 18 inches below X, and radiating it in part, or throughout the section, to the centre of the curved wall, we may obtain a profile capable of resisting the pressure of the earth based on the principles of SCIENCE, aided by the observations of PRACTICE.

Before concluding this part of the subject, a caution may be given, that the foregoing calculations on Quay Walls are founded on the supposition that the backing earth only acts against them. Whereas in practice they are often loaded with heavy goods, either ready for shipment, or discharged from vessels, waiting to be conveyed elsewhere. Mercantile sheds, and even warehouses are sometimes planted upon the quays and bearing upon the ground behind them. In fortifications and other military works, the backing earth has to support the weight of the rampart; as well as to sustain the action of military warfare. In all such cases of additional load an allowance must be made to the thickness of the wall, and this is advantageously done by placing Counterforts at certain intervals along the back of the wall; or by introducing Casemated Revetments, for those of the ordinary form.

END OF FIRST PART.

Plates

TO

THE FIRST PART.

PROFILES

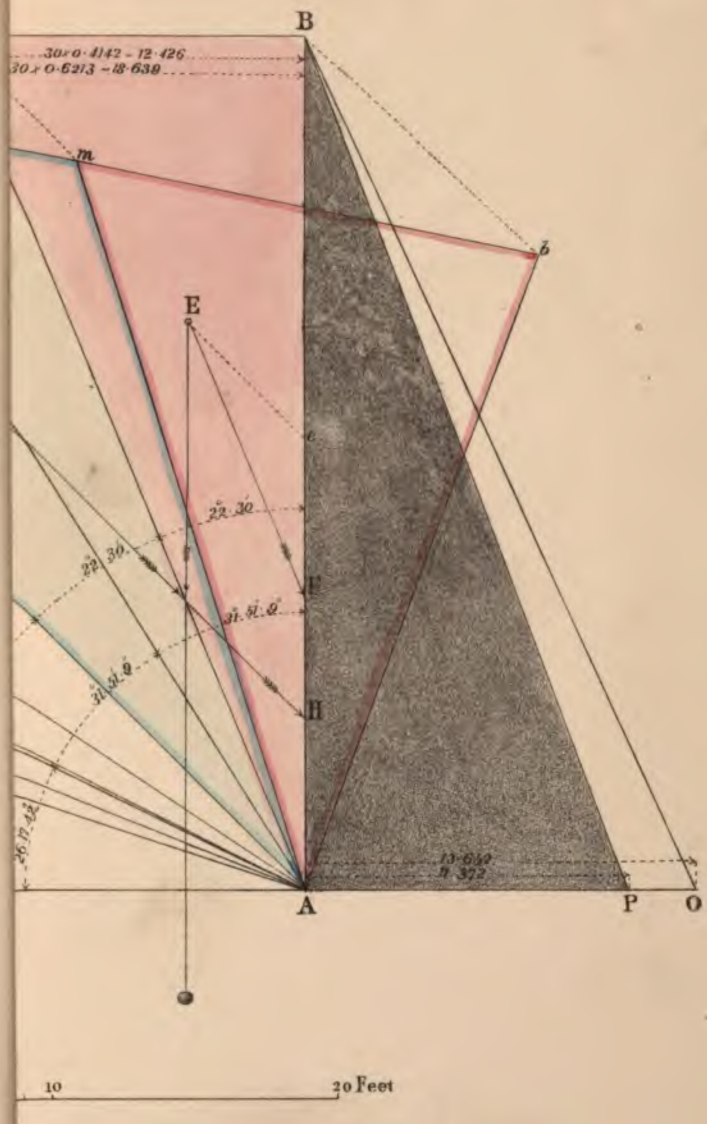
All

Plate I.

N	Nº6	Nº7
		
8 .	7 . 54917	7 . 50
3 .	6 . 04917	7 . 50
$\frac{30}{6}$ =	$\frac{30}{20}$ = 1.50	0 . 00
5 .	4 . 13653	3 . 75
166 .	203 . 9751	225 . 00
843 .	843 . 75	843 . 75
1 : 1 ·	1 : 1 · 4804	1 : 1 · 6329

J Aresti, Imp. 61, Greek St

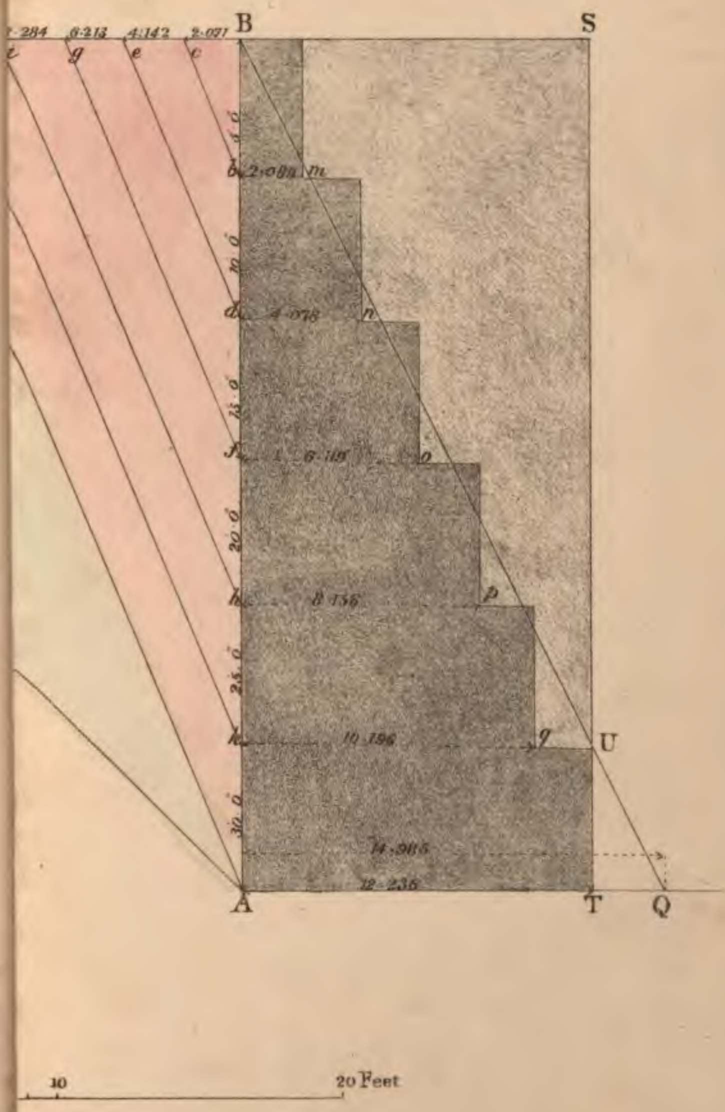
ND SLOPE OF EARTH.



Imp. J. Aresti, 61, Greek St.

Plate III.

EXTERNAL OFFSETS.



*

LL WITH INTERNAL OFFSETS.

PROFILES Plate I.

All

N	Nº6	Nº7
		
8 .	7 . 54917	7 . 50
3 .	6 . 04917	7 . 50
$\frac{30}{6}$ =	$\frac{30}{20}$ = 1.50	0 . 00
5 .	4 . 13653	3 . 75
166 .	208 . 9751	225 . 00
843 .	843 . 75	843 . 75
1:1·	1:1 · 4804	1:1 · 6329

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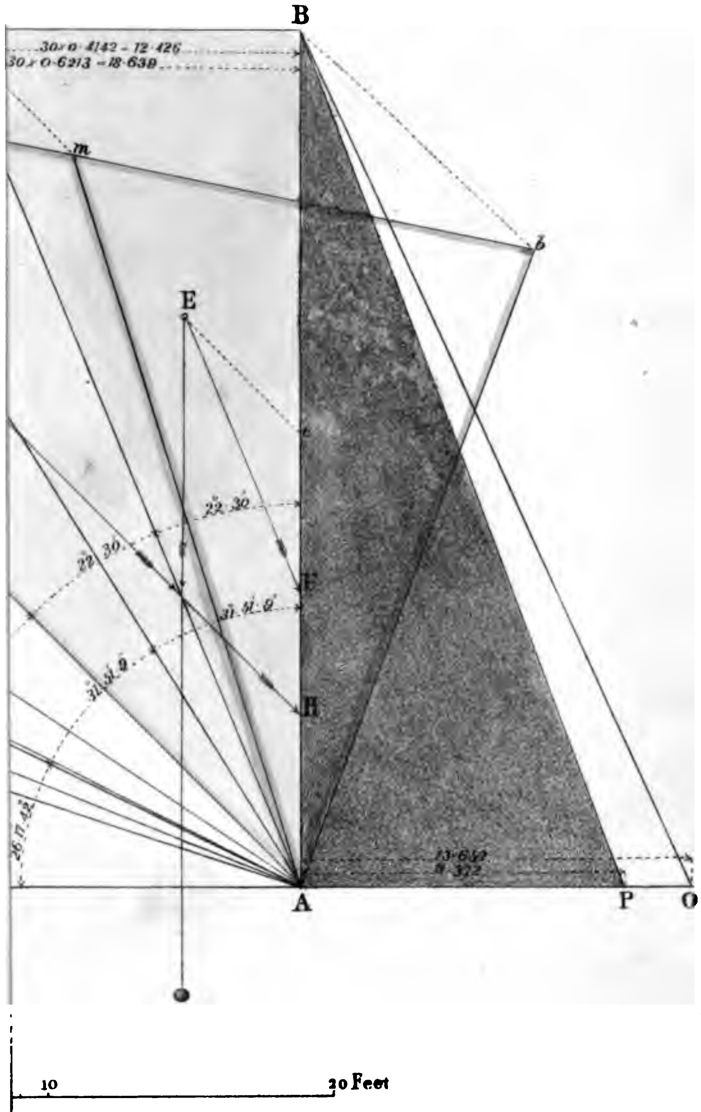
Plate II

ND SLOPE OF EARTH.



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