# NAVAL POSTGRADUATE SCHOOL Monterey, California 



## DEVELOPMENT OF REGIONAL EXTREME MODEL ATMOSPHERES FOR AEROTHERMODYNAMIC CALCULATIONS (I)

by

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## ABSTRACT

A group of stations in the North American Arctic region have been analyzed for statistical determination of temperatures at mandatory pressure levels $p_{\text {. }}$. For each station the temperature at a key level (called the forcing-1evel temperature) peculiar to the station has been forced in at the first step, and retained at each subsequent step in the development of the stepwise regression equations giving temperature at the mandatory levels. In general, eight-step prediction equations in terms of other temperatures in the vertical were found to give specification of $T\left(p_{j}\right)$, with percentage explained variance of close to 0.99. As a resuit of this definitive property, the bestestimate of the regional atmosphere which is conditionally dependent upon the existence of an extreme $1 \%$ probability of the forcing-level temperature is obtained with a high degree of confidence.

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## 1. Introduction

A preliminary set of model atmospheres giving both the warm and cold extremes at certain reference elevations with a $1 \mathscr{D}$ level of expectancy has been proposed as the Preliminary MIL-STD-210B by ETAC (Environmental Technical Applications Center of the USAF). In essence this model of cold and warm extreme temperatures is based upon listing the coldest (warmest)temperatures at each of the eight standard levels listed in Table 1. These extreme temperatures are based upon observations of the location of extreme temperature noted on global temperature maps at the level noted. The extremes are observed to occur at the geographically diverse locations listed in Table 1.

The values of the extremes in Table 1 were computed from the assumed existence of a normal distribution of temperature in both the coldest (January, N. Hemisphere) and warmest (July, N. Hemisphere) months. Thus these extremes have been computed based upon the expectancy (true for a normal distribution) of finding $1 \%$ of all temperature-departures from the sample mean at each location and level at 2.3267 times the standard deviation $\sigma$ at that level and location. In the process of making the estimation of temperature-extremes, no allowance has been made for the possibility of inter-level correlations which have been shown to exist at typical stations (Cole and Nee, 1965).

Because of the neglect of vertical consistency in assembling the tem-perature-extreme data for the Preliminary MIL-STD-210B climatic model, an unrealistic range of temperatures is required in design of equipment needed to operate in these atmospheres. As a result, it has been proposed that the geographical sites which are proposed in Table 1, as corresponding to world-wide temperature extremesat the given levels, be used to

TABLE 1. Locations of proposed extreme temperatures in the cold world-wide and warm world-wide cases (after ETAC).

provide "forcing temperatures"at the level. Based upon inter-level tem-perature-statistics at each station, the most likely temperatures at the eight additional levels (the 70 mb level is to be included) in the atmosphere are then to be estimated with optimal accuracy.

It is to be noted that Table 2 lists only six different stations in definition of the cold-extreme, and five in definition of the warm-extreme atmosphere. Hence, a real-time data analysis would require a detailed climaticalogical examination of 11 different stations for the purpose of establishing regional extremes.
2. A statistical model for regional extreme atmospheres.

As indicated in the study by Cole and Nee (1965), high values of inter-level simple correlations of temperature frequently exist. It was decided in this study to obtain an initial data file on magnetic tape for the two-a-day rawinsonde stations (all in the North American sector) identified by code symbols C1 and W1 in Table 1. These symbols are employed to suggest the cold and warm atmosphere feasibility study. The data was provided by the Environmental Data Service, National Climatic Center, through the financial assistance and kind cooperation of the Commander, Naval Weather Service。

The other stations in Table 1 not bearing the code symbols C1 or W1 have also been provided through the data sources just listed. However, these secondary data records are not available in the convenient summary form, nor in such abundant population samples as those stations which were coded Cl or W1 so that more care is required in the data-handling of these secondary stations. This second set of data-stations is the subject of ongoing work which will appear as a later report in this series.

For the stations marked code symbols C1 and W1, a stepwise multiple regression technique was employed. This program, known as BIMED 02R, is available in the Program Library of the W. R. Church Computer Center of the Naval Postgraduate School. The temperature data was arranged in a decreasing pressure sequence but with standard pressure-spacings as follows (Table 2):

TABLE 2.

|  | $\mathrm{J}=1$ | $\mathrm{~J}=2$ | $\mathrm{~J}=3$ | $\mathrm{~J}=4$ | $\mathrm{~J}=5$ | $\mathrm{~J}=6$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
|  | 1000 mb | 950 | 900 | 850 | 800 | 750 |  |
| $\mathrm{~J}=7$ | 700 | 650 | 600 | 550 | 500 | 450 | $\mathrm{~J}=12$ |
| $\mathrm{~J}=13$ | 400 | 350 | 300 | 250 | 200 | 175 | $\mathrm{~J}=18$ |
| $\mathrm{~J}=19$ | 150 | 125 | 100 | 80 | 70 |  | $\mathrm{~J}=23$ |

Actually, each C1 and W1 radiosonde was listed with both a height and temperature at each of these levels. Over the four year data sample (1967-70), these stations provided the following population samples:
C-1 Stations
Hall Beach $(700 \mathrm{mb}) 243$ samples
Resolute $\quad(500 \mathrm{mb}) 237$ samples
Thule ..... $(300 \mathrm{mb}) 213$ samples
Thule .... $(200 \mathrm{mb}) 213$ samples
W-1 Stations
Alert $\ldots \ldots(200 \mathrm{mb}) 244$ samples
Alert $\ldots \ldots(150 \mathrm{mb}) 244$ samples
Thule $\ldots \ldots(150 \mathrm{mb}) 236$ samples

The program BIMED 02R permits for each station, the specification of a different dependent variable or predictand in each subproblem. Also, the number of independent variables to be tested for admission to regression equation can be limited. In the problems considered here an upper limit of eight independent variables was found to be given optimal specification of each of the dependent variables. Another convenient feature of the BIMED 02R program is that the forced entry into the regression of
the temperature value being tested as an extreme (the so-called "forcing temperature") is allowed, even though its contribution to the explained variance may become small. For example, in the case of Resolute, which corresponds to a winter extreme at 500 mb , the multiple regression equations at the various "mandatory levels" were set up in the forms:

$$
\begin{equation*}
T_{M}=A+B T_{11}+C X_{M} \tag{1}
\end{equation*}
$$

Here $T_{M}$ is the 8 -level predictand chosen alternately as

$$
\mathrm{T}_{\mathrm{M}}=\mathrm{T}_{1}, \mathrm{~T}_{4}, \mathrm{~T}_{7}, \mathrm{~T}_{15}, \mathrm{~T}_{17}, \mathrm{~T}_{19}, \mathrm{~T}_{21}, \mathrm{~T}_{23}
$$

where the subscript $M$ is the temperature identifier associated with the pressure level indicated in Table 2. In summary, the predictands desired are
$\mathrm{T}_{1}$, the temperature at 1000 mb
$\mathrm{T}_{4}$, the temperature at 850 mb
$\mathrm{T}_{7}$, the temperature at 700 mb
$\mathrm{T}_{11}$, the temperature at 500 mb
$\mathrm{T}_{15}$, the temperature at 300 mb
$\mathrm{T}_{17}$, the temperature at 200 mb
$\mathrm{T}_{19}$, the temperature at 150 mb
$\mathrm{T}_{21}$, the temperature at 100 mb
$\mathrm{T}_{23}$, the temperature at 70 mb

The form $X_{M}$ represents here a linear combination of up to 7 independent variables, apart from the forced temperature $\mathrm{T}_{11}$ in the case of Resolute, NWT. The variable $X_{M}$ varies from case to case, as the predictand is changed from $T_{1}$ to $T_{4}$, to $T_{7}$, etc. $X_{M}$ usually involves temperature predictors which are physically close to the level being specified as the dependent variable $T_{M}$. For example, at Resolute, $T_{4}[=T(850)]$ was best-fit by the following expression:

$$
\begin{aligned}
& T_{4}=.3745-.0849 T_{11}+1.0 \mathrm{X}_{4} \\
& \mathrm{X}_{4}=.3614 \mathrm{~T}_{3}+.8577 \mathrm{~T}_{5}-.2892 \mathrm{~T}_{6}+.0819 \mathrm{~T}_{7} \\
&+.0902 \mathrm{~T}_{9}+.0598 \mathrm{~T}_{14}-.0486 \mathrm{~T}_{17}
\end{aligned}
$$

with a multiple regression coefficient $R=$.9861. In general, somewhat similar results applied at each of the other mandatory pressure levels $1000,700,500,300,200,150,100,70 \mathrm{mb}$ except that the multiple $R$ expressing $T_{1}[=T(1000)]$ and $T_{15}[=T(300)]$ in terms of $T_{11}$ and $X_{M}$ were somewhat lower than the remaining correlations in the set. Similar statements applied to all stations at both extreme-times January and July.

The function $X_{M}$ is called in this study a "structure-function" in that to a large degree it accounts for most of the variation of the $T_{M}$ being predicted, even though the part played by the forcing variable is by no means negligible.

A least squares fit of form Eq. (1) may be shown to produce residuals (computed against the observed values) which are distributed normally relative to the regression-value, together with a variance given by the standard error of estimate $\sigma_{E}=\sigma \sqrt{1-R^{2}}$. In the set of dependent variable regressions tested here, most of the multiple regression coefficients had values well in excess of .99 (with the exception of the two levels already noted, 1000 mb and 300 mb ), so that a near functional relationship existed between the predictor and the predictand set. The specification statistics for the different stations in the North American sector of Table 1 are listed in Section 3.
3. Regression-specification of mandatory-level temperatures using the forcing-level temperature.

The stepwise regression procedure of BIMED 02R was employed with the appropriate forcing-level temperature required at the first step of the regression and retained in the selection of the following seven predictors. These seven predictors were allowed to enter the stepwise regression in accordance with the programmed requirement that at each step $k$, the variable added explained a maximum of the unexplained variance remaining after ( $k-1$ ) selections ( $k=2, \ldots, \ldots, 8$ ).

Tables $3 a, 4 a, \ldots, 9 a$ show the results of the stepwise regression applied to the specification of the temperatures $\mathrm{T}_{1}, \mathrm{~T}_{4}, \mathrm{~T}_{7}, \mathrm{~T}_{11}, \mathrm{~T}_{15}, \mathrm{~T}_{17}, \mathrm{~T}_{19}$, $\mathrm{T}_{21}, \mathrm{~T}_{23}$ at mandatory levels by an equation of form (1), using an eightpredictor equation for each $T_{M}$ just listed. Table 3a lists for the January full data-sample at Hall Beach the mean, standard deviation, multiple correlation coefficient and standard error $\sigma_{E}$ at each level other than at forcing level. Table 3 b lists analogous results for a nominal $10 \%$ of the January data sample corresponding to the $10 \%$ cold extreme sample.

Table 4a and 4b list similar results for Resolute in January with $\mathrm{T}_{11}[=\mathrm{T}(500)]$ as the forced variable. Tables 5 and 6 list analogous results for Thule, using $\mathrm{T}_{15}$ and $\mathrm{T}_{17}$ as forcing variables, respectively. It is to be noted that the fractional explained variance $R^{2}$ of the predictand in each case, does not fall of appreciably, in general, as one proceeds along any row of these tables from part (a) to part (b).

Tables 7, 8, 9 similarly list the July-warm statistics, using respectively the full data-samples at Alert [with forcing temperature $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(200)$ ], Alert $\left[T_{J}=T(150)\right]$ and Thule $\left[T_{J}=T(100)\right]$ in the left half of the tables. The prediction results corresponding to the nominal $10 \%$ of the warmest July forcing-1evel temperatures appear on the right half of these tables.

The selection of nominal $10 \%$ extreme cases was made by separating out of the full sample all cases where the forcing temperature $\mathrm{T}_{\mathrm{J}}$ lay in the ranges

$$
\begin{aligned}
& \mathrm{T}_{J}<\bar{T}_{J}-1.2817 \sigma_{J} \text { for cold extremes } \\
& \mathrm{T}_{J}>\bar{T}_{J}+1.2817 \sigma_{J} \text { for warm extremes }
\end{aligned}
$$

The criteria used in establishing these $90 \%$ nominal extremes was that the full data samples were normally distributed, a valid first approximation. The actual number of extreme data cases is listed at the top of Tables $3 b$, ...., 9b, for each case under consideration.

In order to determine the multiple correlation coefficients and standard errors for the right-side $10 \%$ extreme samples of Tables $3 b, \ldots, 9 b$, a regression equation of identical form to that developed for the full data-base predictands was generated for testing the $10 \%$ extreme-case data samples. The test-predictors for the extreme-data were then considered to be the forcing variable $T_{J}$ and the structure-variable $X$ of form determined for the full-sample cases. The high values of the multiple correlations thus determined for the January extreme-data cases serves to verify the concept that the predictands corresponding to the extreme-data may be anticipated through a joint knowledge of the forcing variable $\mathrm{T}_{\mathrm{J}}$ and of the structure function. The somewhat smaller correlation coefficients for the July data is due to the much smaller values of variance in summer, particularly in the lower stratosphere.

A listing of the nominal extremes of the forcing temperature for other probability levels of interest, e.g., the $5 \%$, the $1 \%$ was also made assuming the existence of a Gaussian distribution of temperature, and appears in Table 10. A separation of the $10 \%$ nominal extremes of $T_{J}$ was then made wherever possible into the classes suggested in Table 10 , that is beyond the . 01 probability level; in the probability range .05 to .01 and in the
range . 10 to .05. The results of these probability stratifications on the predictands at $\mathrm{T}_{1}, \mathrm{~T}_{4}, \mathrm{~T}_{7}, \mathrm{~T}_{11}, \mathrm{~T}_{15}, \mathrm{~T}_{17}, \mathrm{~T}_{21}, \mathrm{~T}_{23}$ are discussed in Section 4. These additional critical extremes are given by

$$
\begin{align*}
& \mathrm{T}_{\mathrm{J}}(.05)=\overline{\mathrm{T}}_{\mathrm{J}} \pm 1.640 \sigma_{\mathrm{J}}  \tag{2}\\
& \mathrm{~T}_{\mathrm{J}}(.01)=\overline{\mathrm{T}}_{\mathrm{J}} \pm 2.3267 \sigma_{\mathrm{J}} \tag{3}
\end{align*}
$$

Equations 2 and 3 tend to give excessive estimates of the critical extremes when the observed $\sigma_{J}$ is too large to be representative of a normal distribution. This happens primarily in the winter season at low stratospheric levels, e.g., Thule at 300 mb . At this locale there tends to be bimodal alternations between warm and cold stratospheric regimes and an unrepresentatively large $\sigma_{J}$ results from this sample. The ETAC estimate (Table 1) of the $1 \%$ extreme temperature at Thule at 300 mb in winter also seems to reflect this same difficulty.

The following temperature classes were then considered as input data at the C1 and W1 station forcing levels, respectively

Cold classes:
(a) $\mathrm{T}_{J}<\mathrm{T}_{.01}$;
(b) $\mathrm{T} .01<\mathrm{T}_{\mathrm{J}}<\mathrm{T} .05$;
(c) $\mathrm{T}_{.05}<\mathrm{T}_{\mathrm{J}}<\mathrm{T} .10$

Warm classes:
(a) $\mathrm{T}>\mathrm{T} .01$;
(b) $\mathrm{T} .01^{>} \mathrm{T}>\mathrm{T} .05$;
(c) $\mathrm{T}_{.05}>\mathrm{T}_{\mathrm{J}}>\mathrm{T}_{.10}$ In most cases studied, reasonable sample-sizes existed in each class (a), (b), (c). However, in several cases, the extremes did not reach the $1 \%$ nominal-value predicted from an assumed Gaussian distribution.
TABLE 3. Regression statistics at mandatory pressure levels at Hall Beach using $T_{T}=T(700)$ as the forcing-level temperature. Part (a) refers to the full-data January sample; part (b) refers to the nominal $10 \%$ cold extreme sample of $T(700)$.

| Level | $\underset{\mathrm{O}_{\mathrm{C}}}{\text { Mean }^{2}}$ | $\underset{\mathrm{C}}{\mathrm{Std}_{\mathrm{C}}} .$ | (a) $N=$ <br> Multiple <br> Correl. <br> Coeff. | ```cases Std. Error of Estimate ( }\mp@subsup{}{}{\circ}\textrm{C``` | Level | $\underset{\mathrm{C}}{\mathrm{Mean}^{2}}$ | $\underset{\mathrm{C}}{\text { Std. }_{\text {St. }}^{\text {dev }}}$ | (b) iv = <br> Multiple <br> Correl. <br> Coeff. | 6 cases <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | -26.11 | 7.602 | . 8992 | 3.383 | 1000 mb | -35.04 | 3.893 | . 7238 | 2.801 |
| 850 | -22.63 | 7.238 | . 9937 | . 824 | 850 | -35.60 | 3.295 | . 9757 | . 753 |
| 700 | -27.35 | 6.467 |  |  | 700 | -39.68 | 3.186 | - |  |
| 500 | -39.93 | 4.994 | . 9952 | . 496 | 500 | -45.98 | 2.847 | . 9750 | . 659 |
| 300 | -55.09 | 5.578 | . 9773 | 1.201 | 300 | -45.45 | 4.965 | . 9878 | 1.094 |
| 200 | -53.53 | 6.071 | . 9897 | . 883 | 200 | -45.88 | 5.279 | . 9878 | . 858 |
| 150 | -53.71 | 6.517 | . 9920 | . 836 | 150 | -47.40 | 6.796 | . 9867 | 1.151 |
| 100 | -55.49 | 7.689 | . 9943 | . 835 | 100 | -51.11 | 7.847 | . 9941 | . 887 |
| 70 | -57.38 | 8.942 | . 9946 | . 944 | 70 | -54.41 | 8.936 | . 9936 | 1.048 |

TABLE 4. Regression statistics at mandatory pressure levels at Resolute using $T_{\mathrm{J}}=\mathrm{T}(500)$ as the forcing-level temperature. Part (a) refers to the full-data January sample; part (b) refers to the nominal $10 \%$ cold extreme sample of $T(500)$.

| Level | $\underset{\mathrm{C}}{\mathrm{Mean}}$ | ${\underset{\mathrm{C}}{\mathrm{Std}} . \mathrm{dev} .}^{(2)}$ | (a) $\mathrm{N}=2$ <br> Multiple <br> Correl. <br> Coeff. | cases <br> Std. Error of Estimate ( ${ }^{\circ} \mathrm{C}$ ) | Level | $\underset{\mathrm{C}}{\mathrm{Mean}}$ |  | (b) $\mathrm{N}=2$ <br> Multiple <br> Correl. <br> Coeff. | cases <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | -29.10 | 6.349 | . 8522 | 3.379 | 1000 mb | -35.52 | 4.273 | . 7260 | 3.069 |
| 850 | -24.56 | 5.631 | . 9861 | . 953 | 850 | -31.18 | 3.457 | . 9925 | . 440 |
| 700 | -28.87 | 4.856 | . 9920 | . 625 | 700 | -35.71 | 3.175 | -.9941 | . 360 |
| 500 | -41.92 | 4.043 |  |  | 500 | -48.68 | 1.573 |  |  |
| 300 | -55.85 | 5.225 | . 9778 | 1.115 | 300 | -57.26 | 5.916 | . 9879 | . 958 |
| 200 | -53.89 | 6.121 | . 9883 | . 950 | 200 | -56.04 | 6.043 | . 9924 | . 777 |
| 150 | -54.56 | 7.036 | . 9946 | . 740 | 150 | -58.68 | 5.517 | . 9962 | . 499 |
| 100 | -56.48 | 8.510 | . 9965 | . 728 | 100 | -61.95 | 6.982 | . 9958 | . 665 |
| 70 | -58.34 | 7.578 | . 9962 | . 856 | 70 | -64.48 | 7.541 | . 9958 | . 725 |

TABLE 5. Regression statistics at mandatory pressure levels at thule using $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(300)$ as forcing-level January temperature. Part (a) refers to the fulldata sample; part (b) refers to the nominal $10 \%$ cold extreme sample of $T(300)$.

| Level | $\underset{\mathrm{C}}{\text { Mean }}$ | $\underset{\mathrm{C}}{\mathrm{Std} . \mathrm{dev}} .$ | (a) $\mathrm{N}=$ Multiple Correl. Coeff. | cases <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) | Level | $\underset{\text { C }}{\substack{\text { Mean }}}$ | $\underset{\mathrm{C}}{\mathrm{Std} . \operatorname{dev}} .$ | (b) $N=13$ cases <br> Multiple Std. Error of Correl. Estimate ( ${ }^{\circ} \mathrm{C}$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | -20.50 | 7.097 | . 9336 | 2.593 | 1000 mb | -21.41 | 5.931 | . 8765 | 3.127 |
| 850 | -21.91 | 7.622 | . 9924 | . 954 | 850 | -22.55 | 6.079 | . 9971 | . 505 |
| 700 | -27.04 | 6.187 | . 9937 | . 707 | 700 | -28.68 | 6.095 | -9991 | . 267 |
| 500 | -40.28 | 4.758 | . 9941 | . 528 | 500 | -42.41 | 3.887 | . 9958 | . 390 |
| 300 | -55.58 | 5.049 |  |  | -300 | -63.03 | 0.880 |  |  |
| 200 | -54.86 | 6.791 | . 9863 | 1.143 | 200 | -61.88 | 2.375 | . 9871 | . 417 |
| 150 | -55.22 | 7.216 | . 9907 | . 999 | 150 | -63.02 | 2.809 | . 9913 | . 405 |
| 100 | -57.10 | 9.084 | . 9968 | . 744 | 100 | -67.51 | 2.493 | . 9576 | . 787 |
| 70 | -59.12 | 10.804 | . 9979 | . 720 | 70 | -70.57 | 3.197 | . 9740 | . 793 |

TABLE 6. Regression statistics at mandatory pressure levels at Thule using $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(200)$ as forcing-level temperature. Part (a) refers to the full-data January sample; part (b) refers to the nominal $10 \%$ cold extreme sample of $T(200)$.

| Level | $\underset{\mathrm{C}}{\mathrm{Mean}}$ | $\underset{C}{\text { Std }} \cdot \underset{\mathrm{C}}{\mathrm{dev}} .$ | (a) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | samples <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) | Level | $\underset{\mathrm{C}}{\text { Mean }}$ | $\underset{\mathrm{C}}{\mathrm{Std} . \mathrm{dev}^{2}}$ | (b) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | $\begin{aligned} & \text { 3 samples } \\ & \text { Std. Error of } \\ & \text { Estimate ( }{ }^{\circ} \mathrm{C} \text { ) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | -20.50 | 7.097 | . 9336 | 2.592 | 1000 mb | -12.83 | 7.471 | . 9298 | 2.928 |
| 850 | -21.91 | 7.622 | . 9924 | . 958 | 850 | -13.18 | 6.948 | . 9933 | . 856 |
| 700 | -27.04 | 6.187 | . 9938 | . 703 | 700 | -20.51 | 5.885 | :9981 | . 383 |
| 500 | -40.28 | 4.758 | . 9941 | . 528 | 500 | -35.74 | 5.012 | . 9955 | . 491 |
| 300 | -55.58 | 5.049 | . 9739 | 1.168 | 300 | -59.37 | 2.170 | . 9486 | . 731 |
| 200 | -54.86 | 6.791 |  |  | 200 | -67.93 | 3.750 |  |  |
| 150 | -55.22 | 7.216 | . 9908 | . 995 | 150 | -66.57 | 4.252 | . 9460 | 1.467 |
| 100 | -57.10 | 9.084 | . 9968 | . 745 | 100 | -70.32 | 3.135 | . 9812 | . 644 |
| 70 | -59.12 | 10.804 | . 9979 | . 719 | 70 | -74.05 | 3.804 | . 9949 | . 410 |

TABLE 7. Regression statistics at mandatory pressure levels at Alert using $T_{J}=T(200)$ as forcing-level temperature. Part (a) refers to the full-data July sample; part (b) refers to the nominal $10 \%$ warm extreme of $\mathrm{T}(200)$.

| Leve1 | $\underset{\mathrm{C}}{\mathrm{Mean}}$ | $\text { Std. }_{\mathrm{C}}^{\mathrm{C}} \mathrm{dev} .$ | (a) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | samples <br> Std. Error of Estimate ( ${ }^{\circ}$ C) | Leve1 | $\mathrm{Mean}_{\mathrm{C}}$ | $\underset{\mathrm{C}}{\mathrm{Std}_{\mathrm{C}} . \operatorname{dev}}$ | (b) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | samples <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | 3.26 | 3.673 | . 8418 | 2.016 | 1000 mb | 1.30 | 1.317 | . 8924 | . 633 |
| 850 | - 0.91 | 3.203 | . 9764 | . 703 | 850 | - 4.40 | 2.363 | . 9831 | . 460 |
| 700 | $\therefore 9.02$ | 3.396 | . 9797 | . 693 | 700 | -13.43 | 2.296 | . 9479 | . 779 |
| 500 | -23.97 | 3.211 | . 9916 | . 422 | 500 | -28.11 | 2.435 | . 9930 | . 305 |
| 300 | -47.59 | 2.545 | . 6836 | 1.889 | 300 | -44.93 | 3.775 | . 6857 | 2.925 |
| 200 | -42.19 | 2.789 |  |  | 200 | -38.13 | 0.644 |  |  |
| 150 | -42.04 | 1.779 | . 9308 | . 6613 | 150 | -39.58 | 0.714 | . 6876 | . 552 |
| 100 | -41.87 | 1.263 | . 8921 | . 580 | 100 | -40.58 | 0.867 | . 7086 | . 651 |
| 70 | -40.87 | 1.104 | . 9175 | . 447 | 70 | -39.64 | 1.526 | . 9916 | . 210 |

TABLE 8. Regression statistics at mandatory pressure levels at Alert using $T=T(150)$ as the forcing-level temperature. Part (a) refers to the full-data july sample; part (b) refers to the nominal $10 \%$ warm extreme sample of $\mathrm{T}(150)$.

| Level | $\underset{\mathrm{C}}{\text { Mean }}$ | $\underset{\mathrm{C}}{\text { Std }} .$ | (a) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | samples <br> Std. Error of Estimate ( ${ }^{\circ} \mathrm{C}$ ) | Leve1 | $\underset{\mathrm{C}}{\mathrm{Mean}}$ | $\text { Std. }_{\mathrm{C}}^{\mathrm{C}} .$ | (b) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | 2 samples <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | 3.26 | 3.673 | . 8399 | 2.027 | 1000 mb | 2.19 | 2.434 | . 7853 | 1.585 |
| 850 | - 0.91 | 3.203 | . 9763 | . 704 | 850 | - 3.54 | 2.320 | . 9796 | . 490 |
| 700 | - 9.02 | 3.396 | . 9796 | . 694 | 700 | -12.25 | 2.644 | -9440 | . 917 |
| 500 | -23.97 | 3.211 | . 9916 | . 423 | 500 | -26.43 | 2.209 | . 9762 | . 504 |
| 300 | -47.59 | 2.545 | . 6851 | 1.885 | . 300 | -47.16 | 3.210 | . 5905 | 2.772 |
| 200 | -42.19 | 2.789 | . 9529 | . 860 | 200 | -38.93 | 0.939 | . 8574 | . 508 |
| 150 | -42.04 | 1.779 |  |  | 150 | -39.24 | 0.537 |  |  |
| 100 | -41.87 | 1.263 | . 8921 | . 581 | 100 | -40.27 | 0.498 | . 3888 | . 482 |
| 70 | -40.87 | 1.104 | . 9172 | . 447 | 70 | -39.59 | 1.325 | . 9840 | . 248 |

TABLE 9. Regression statistics at mandatory pressure levels at Thule using $T_{T}=T(100)$ as the forcing-level temperature. Part (a) refers to the full-data July sample; part (b) refers to the nominal $10 \%$ warm extreme sample of $T(100)$.

| Level | $\begin{gathered} \text { Mean } \\ \text { C } \end{gathered}$ | Std.dev. <br> C | (a) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | 6 samples <br> Std. Error of Estimate ( ${ }^{\circ} \mathrm{C}$ ) | Level | $\underset{\mathrm{C}}{\substack{\text { Mean }}}$ | $\underset{\mathrm{C}}{\text { Std. }}$ | (b) $\mathrm{N}=$ <br> Multiple <br> Correl. <br> Coeff. | samples <br> Std. Error of <br> Estimate ( ${ }^{\circ} \mathrm{C}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 mb | 3.70 | 3.199 | . 7686 | 2.082 | 1000 mb | 2.19 | 1.889 | . 7675 | 1.289 |
| 850 | - 0.73 | 3.033 | . 9736 | . 705 | 850 | - 3.43 | 2.551 | . 9830 | . 498 |
| 700 | - 7.76 | 3.147 | . 9704 | . 774 | 700 | -11.13 | 3.555 | .9805 | . 744 |
| 500 | -23.05 | 3.097 | . 9862 | . 522 | 500 | -26.48 | 3.409 | . 9902 | . 508 |
| 300 | -47.54 | 2.399 | . 7794 | 1.530 | 300 | -46.58 | 2.853 | . 7898 | 1.863 |
| 200 | -44.17 | 3.778 | . 9294 | 1.419 | 200 | -39.84 | 1.590 | . 8789 | . 808 |
| 150 | -43.57 | 2.770 | . 6735 | 2.083 | 150 | -40.62 | 1.035 | . 8113 | . 644 |
| 100 | -43.20 | 1.429 |  |  | 100 | -40.95 | 0.427 |  |  |
| 70 | -43.08 | 1.237 | . 9507 | . 390 | 70 | -40.62 | 0.626 | . 8721 | . 326 |

TABLE 10. Nominal temperature-extremes at the indicated stations and pressure levels at the $10 \%, 5 \%$ and $1 \%$ extreme values according to a Gaussian distribution.

| Station | T. 10 | T. 05 | $\left.{ }^{\mathrm{T} .01}{ }^{( }{ }^{\circ} \mathrm{C}\right)$ | Forcing Level $\mathrm{T}_{\mathrm{J}}$ | $90 \%$ extreme sample size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hall Beach | -35.6 | -37.9 | -42.3 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(700)$ | 26 |
| Resolute | -47.1 | -48.5 | -51.3 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(500)$ | 25 |
| Thule, Wi | -62.0 | -63.8 | -67.3 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(300)$ | 13 |
| Thule, Wi | -63.5 | -66.0 | -70.6 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(200)$ | 18 |
| Thule, Su | -41.3 | -40.8 | -39.8 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(100)$ | 18 |
| Alert, Su | -39.7 | -39.1 | -37.9 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(150)$ | 22 |
| Alert, Su | -38.6 | -37.6 | -35.7 | $\mathrm{T}_{\mathrm{J}}=\mathrm{T}(200)$ | 18 |

4. Temperature-estimates at mandatory levels corresponding to $1 \%$ extremes of forcing-level temperatures.
a. Winter extreme atmospheres

Corresponding to the four winter extreme cases, the multiple regression methods which led to Tables $3,4,5,6$ also lead to the results of Tables 11, 12, 13, 14. In the latter tables, the nominal $10 \%$ extreme sample is decomposed into the subsample extreme-classes (a), (b), (c). These classes were defined in Section 3, but their definition is also implicitly given in the top row of each table (e.g., Table 11: (a) T.01; (b) T. 01 T.05; (c) T.05-T.10. Corresponding to the stratified data-sample at each forcing level there exist also conditional data-sets at each of the mandatory levels above and below the forcing level. In virtually all cases, the mandatory-level data sets in classes (a), (b), (c) exhibit well-defined trends between the class-mean temperatures at all levels. This discovery afforded credibility to the listed extreme values of the class (a) mandatorylevel results of Tables 11, 12, 14. Note that in each stratification (a), (b), (c), the same sample size exists at each level within each class regardless of the level under consideration.

As noted in Section 3, no nominal 1\% data class was realized in Table 13; however, the class (b) sample $T_{J}=T(300)$ comprised a de facto extreme 1\% data-sample by actual count. In the comparison of $\bar{T}_{b}$ with $\bar{T}_{c}$ at mandatory levels over Thule (Table 13), there existed consistent inter-class temperature differences, again at all levels. In all cases, the reason for these consistent inter-class temperature deviations is that the mandatory level temperatures have been found to be strongly correlated with the forcing-1 evels temperature through the multiple regression

| Level$\text { ( } n h \text { ) }$ | ```Temperature-means ( }\mp@subsup{}{}{\circ}\textrm{C in classes``` |  |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) relative to class-mean |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (a) T .01 | ${ }_{\text {T }}^{\text {(b) }}$ ( $01^{-\mathrm{T}} .05$ | (c) $\mathrm{T} .05^{-\mathrm{T}} .10$ | $\sigma_{a}$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | -35.24 | -35.22 | -34.62 | 2.588 | 4.047 | 3.876 |
| 850 | -39.52 | -35.71 | -32.34 | 1.421 | 2.475 | 1. 943 |
| 700 * | -44.96 | -39.55 | -36.60 | 1.746 | 1.175 | 0.799 |
| 500 | -44.04 | -45.90 | -47.32 | 0.948 | 3.363 | 1.233 |
| 300 | -41.02 | -45.95 | -47.41 | 0.937 | 5.472 | 3.315 |
| 200 | -42.96 | -46.22 | -47.16 | 0.924 | 6.064 | 4.327 |
| 150 | -45.20 | -47.32 | -48.91 | 0.767 | 7.804 | 6.220 |
| 100 | -48.64 | -51.40 | -52.25 | 1.284 | 8.999 | 6.849 |
| 70 | -50.28 | -55.47 | -55.42 | 2.047 | 10.185 | 6.527 |
| Number <br> in <br> class | 5 | 13 | 8 |  |  |  |

TABLE 11. Temperature means at mandatory levels at Hall Beach (winter), corresponding to temperature extremes of $T_{f}(700)$ at the (a) $1 \%$ class, (b) 1-5\% class and (c) 5-10\% class of probability. The standard deviations $\sigma_{a}, \sigma_{b}, \sigma_{c}$ of the observed temperatures within each class are shown in the ${ }^{a}$ right half of the table.


FIG. 1. The heavy solid line shows the mean regression-determined vertical temperature sounding over Hall Beach, NWT, corresponding to the class (a) set of $1 \%$ cold extreme occurrences of $T_{J}(700)$ of Table 11. The thin solid line depicts the January mean Hall Beach vertical sounding (1967-70). The dashed curves the world-wide Preliminary MIL-STD 210 extreme atmospheres.

| $\begin{aligned} & \text { Level } \\ & \text { (mb) } \end{aligned}$ | ```Temperature-means ( }\mp@subsup{}{}{\circ}\textrm{C in classes``` |  |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) relative to class-mean |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { (a) } \\ & { }^{T} .01 \end{aligned}$ | $\mathrm{T}^{\text {T }} \mathrm{O} 1^{\text {(b) }} \mathrm{T} .05$ | $\begin{gathered} \text { (c) } \\ \mathrm{T} .05^{-\mathrm{T}} \cdot 10 \end{gathered}$ | $\sigma_{a}$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | -39.00 | -38.77 | -33.31 | 0.748 | 2.125 | 3.718 |
| 850 | -35.50 | -31.57 | -30.14 | 0.698 | 1.463 | 3.038 |
| 700 | -40.37 | -36.43 | -34.45 | 0.742 | 1.308 | 2.274 |
| 500 * | -51.73 | -49.70 | -47.59 | 0.173 | 0.782 | 0.342 |
| 300 | -54.03 | -56.20 | -60.48 | 1.225 | 2.027 | 4.070 |
| 200 | -55.20 | -55.24 | -59.15 | 0.648 | 2.280 | 3.561 |
| 150 | -59.37 | -58.09 | -61.12 | 0.881 | 2.489 | 3.098 |
| 100 | -65.70 | -61.00 | -64.59 | 0.860 | 3.745 | 3.248 |
| 70 | -70.77 | -62.50 | -65.88 | 0.480 | 5.748 | 4.064 |
| Number <br> in <br> class | 3 | 7 | 15 |  |  |  |

TABLE 12. Temperature means at mandatory levels at Resolute (winter), corresponding to temperature extremes of $T_{J}(500)$ at the (a) 1\% class, (b) 1-5\% class and (c) 5-10\% class of probability. The standard deviations $\sigma_{a}, \sigma_{b}, \sigma_{c}$ of the observed temperatures within each class are shown $a$ in the right half of the table.


FIG. 2. The heavy solid line shows the mean regression-determined vertical temperature sounding over Resolute, NWT, corresponding to the class (a) set of $1 \%$ cold extreme occurrences of $T_{J}(500)$ of Table 12. The thin solid line depicts the January mean Resolute sounding (1967-70). The dashed curves are identical to those of Figure 1.
relationships sumarized in Tables 3 through 9. These relationships also apply across classes (a), (b), (c), and give rise to the intex-class differences found in Tables 11,..., 17.

The listings in Tables $11,12,13,14$ of the class (a) set of temperatures [or the (b)-class if class (a) is non-existent] at mandatory levels makes possible the presentation of the results in graphical form. Thus the expected (cold) temperature data corresponding to the $1 \%$ forcing temperature appropriate to Tables $11,12,13$ and 14 have been reproduced as Figs. 1, 2, 3, 4. Here the heavy solid line depicts the cold or $1 \%$ "extreme"atmosphere in each case, whereas the thin solid line depicts the January (1967-70) mean sounding in each case. Note that the temperature deviations between "mean" and "extreme" in Figs. 1, 2, 3, 4 are indicative also of the class (a) to (c) deviations at the mandatory levels of Tables 11, 12, 13, 14.
b. Warm extreme atmospheric cases.

The procedure in these cases consists of an analysis of the nominal $10 \%$ extreme-warm July cases at Alert $\left(\left[T_{J}=T(200)\right],\left[T_{J}=T(150)\right]\right)$ and at Thule $\left[T_{J}=T(100)\right]$ after these extremes have been subdivided into the nominal probability classes (a), (b), (c), similar to the procedure of Section 4(a).

In two of the three cases, the class (a) nominal probability $\mathrm{T}_{\mathrm{y}}<$ T. 01 did not occur (see Tables 15 and 17 ), but an actual case count reveals that in either case there were 2 to 4 data-samples in class (b). These samples were taken as representative of the actual $1 \%$ extreme warm situation at the respective forcing levels, and also at the regressiondependent mandatory levels.

| $\begin{aligned} & \text { Level } \\ & (\mathrm{mb}) \end{aligned}$ | ```Temperature-means ( }\mp@subsup{}{}{\circ}\textrm{C in classes``` |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) relative to class-mean |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}_{.} \stackrel{(\mathrm{b})}{ } .01^{-\mathrm{T}} .05$ | $\mathrm{T} .0 \mathrm{c}^{(\mathrm{c})} \mathrm{T} .10$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | -25.20 | -20.27 | 2.371 | 4.942 |
| 850 | -24.83 | -21.87 | 1.637 | 3.736 |
| 700 | -29.73 | -28.36 | 5.287 | 3.858 |
| 500 | -42.90 | -42.26 | 2.032 | 3.822 |
| $300{ }^{*}$ | -64.47 | -62.60 | . 173 | . 341 |
| 200 | -62.83 | -61.60 | . 526 | 2.294 |
| 150 | -63.73 | -62.80 | . 911 | 2.889 |
| 100 | -67.93 | -67.38 | 1.496 | 2.581 |
| 70 | -71.50 | -70.29 | 2.921 | 3.026 |
| Number in class | 3 | 10 |  |  |

TABLE 13. Temperature means at mandatory levels at Thule, Greenland (winter), corresponding to temperature extremes of $T_{J}(300)$ at the (b) 1-5\% class, and (c) 5-10\% class of probability. The population of class (a) was zero. The standard deviations $\sigma_{b}$ and $\sigma_{c}$ of observed temperatures in the (b) and (c) classes are shown in the right side of the table.


FIG. 3. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the $1 \%$ cold extreme set of forcing temperatures $T_{J}(300)$ of Table 13. The thin solid line depicts the January mean Thule sounding (1967-70). The dashed curves are identical to those of Figure 1.

| $\begin{aligned} & \text { Level } \\ & \text { (mb) } \end{aligned}$ | $\begin{gathered} \text { Temperature-means }\left({ }^{\circ} \mathrm{C}\right) \\ \text { in classes } \end{gathered}$ |  |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) relative to class-mean |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (a) T .01 | ${ }^{T} .01^{(b)}$ | ${ }^{\text {T }} \mathrm{C}$ ( $05^{-\mathrm{T}} .10$ | $\sigma_{a}$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | -9.87 | -12.17 | -14.51 | 4.302 | 3.849 | 8.440 |
| 850 | -5.30 | -15.31 | -17.09 | 2.269 | 3.179 | 5.024 |
| 700 | -12.67 | -19.73 | -23.68 | 0.806 | 1.608 | 4.870 |
| 500 | -29.97 | -34.99 | -38.21 | 1.304 | 2.732 | 3.773 |
| 300 | -57.03 | -59.19 | -60.40 | 0.695 | 1.041 | 2.115 |
| 200 * | -74.70 | -68.61 | -64.80 | . 084 | 1.607 | . 480 |
| 150 | -73.57 | -66.81 | -63.74 | 0.173 | 2.496 | 2.216 |
| 100 | -73.43 | -71.30 | -68.29 | 0.733 | 2.413 | 2.339 |
| 70 | -77.50 | -75.46 | -71.52 | 1.145 | 3.156 | 2.767 |
| Number <br> in <br> class | 3 | 7 | 8 |  |  |  |

TABLE 14. Temperature means at mandatory levels at Thule (winter), corresponding to temperature extremes of $T_{f}(200)$ at the (a) $1 \%$ class, (b) $1-5 \%$ class and (c) 5-10\% class of probability. The standard deviations $\sigma_{a}, \sigma_{b}$, $\sigma_{c}$ of the observed temperatures within each class are shown in the right ${ }^{\text {b }}$ nalf of the table.


FIG. 4. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the $1 \%$ cold extreme occurrences of $T_{T}(200)$ of Table 14 . The thin solid line depicts the January mean Thule sounding (1967-70). The dashed lines are identical to those of Figure 1.

| $\begin{aligned} & \text { Level } \\ & \text { (mb) } \end{aligned}$ | $\begin{gathered} \text { Temperature-means }\left({ }^{\circ} \mathrm{C}\right) \\ \text { in classes } \end{gathered}$ |  | Std. deviations ( ${ }^{\circ}$ C) relative to class-mean |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}^{(\mathrm{b})} .01^{-\mathrm{T}} .05$ | (c) ${ }^{\text {T }} .05^{-\mathrm{T}} .10$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | 0.05 | 1.48 | . 350 | 1.147 |
| 850 | -6.65 | -4.07 | . 650 | 2.086 |
| 700 | -16.15 | -13.09 | . 250 | 1.953 |
| 500 | -31.30 | -27.79 | 2.000 | 4.086 |
| 300 | -40.05 | -45.54 | 2.250 | 3.443 |
| $200 *$ | -36.70 | -38.31 | . 600 | . 330 |
| 150 | -38.90 | -39.67 | 1.000 | . 575 |
| 100 | -40.95 | -40.47 | . 350 | . 860 |
| 70 | -40.25 | -39.56 | 1.150 | 1.498 |
| Number in <br> class | 2 | 16 |  |  |

TABLE 15. Temperature means at mandatory levels at Alert (summer), corresponding to temperature extremes of $T_{5}(200)$ at the (b) 1-5\% class and (c) 5$10 \%$ class of probability. The population of class (a) was zero. The standard deviations $\sigma_{b}, \sigma_{c}$ of the observed temperatures within classes $\underline{b}$ and $c$ are shown on the right side of the table.


FIG. 5. The heavy solid line shows the mean regression-determined vertical temperature sounding over Alert, NWT, corresponding to the $1 \%$ warm extreme occurrences of $T_{J}(200)$ of Table 15 . The thin solid Ine depicts the July mean Alert sounding (1967-70). The dashed curves are identical to those of Figure 1.

| $\begin{aligned} & \text { Level } \\ & \text { (mb) } \end{aligned}$ | Temperature-means ( ${ }^{\circ} \mathrm{C}$ ) in classes |  |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) relative to class-mean |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { (a) } \\ & \text { T. } 01 \end{aligned}$ | $\mathrm{T}^{\text {(b) }} 0 \mathrm{l}^{-\mathrm{T}} .05$ | $\begin{gathered} \text { (c) } \\ \mathrm{T} .05^{-\mathrm{T}} .10 \end{gathered}$ | $\sigma_{\mathrm{a}}$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | 0.00 | 2.68 | 2.29 | . 030 | 1.384 | 2.734 |
| 850 | -4.65 | -3.93 | -3.21 | 2.650 | 2.318 | 2.067 |
| 700 | -13.75 | -12.48 | -11.94 | 2.650 | 2.467 | 2.689 |
| 500 | -26.25 | -26.42 | -26.48 | 3.050 | 1.117 | 2.317 |
| 300 | -43.50 | -49.23 | -46.78 | 1.200 | 1.168 | 3.177 |
| 200 | -38.10 | -38.83 | -39.09 | . 800 | . 619 | . 909 |
| 150 * | -38.20 | -38.80 | -39.57 | . 300 | . 224 | . 226 |
| 100 | -40.60 | -40.33 | -40.33 | . 000 | . 500 | . 469 |
| 70 | -39.45 | -39.93 | -39.43 | . 349 | . 472 | 1.471 |
| Number in class | 2 | 6 | 14 |  |  |  |

TABLE 16. Temperature means at mandatory levels at Alert (summer), corresponding to temperature extremes of $T_{f}(150)$ at the (a) 1\% class, (b) 1-5\% class and (c) 5-10\% class of probability. The standard deviations $\sigma_{a}, \sigma_{b}$, $\sigma$ of the observed temperatures within each class are shown in the right side of the table.


FIG. 6. The heavy solid line shows the mean regression-determined vertical temperature sounding over Alert, NWT, corresponding to the $1 \%$ warm extreme occurrences of $T_{J}(150)$ of Table 16 . The thin solid line depicts the July mean Alert sounding (1967-70). The dashed curves are identical to those of Figure 1.

| $\begin{aligned} & \text { Level } \\ & \text { (mb) } \end{aligned}$ | $\begin{gathered} \text { Temperature-means }\left({ }^{\circ} \mathrm{C}\right) \\ \text { in classes } \end{gathered}$ |  | Std. deviations ( ${ }^{\circ} \mathrm{C}$ ) <br> relative to class-mean |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}_{\text {( } \mathrm{b})} .01^{-\mathrm{T}} .05$ | T. ${ }^{(c)}$ | $\sigma_{b}$ | $\sigma_{c}$ |
| 1000 | 0.90 | 2.56 | . 100 | 2.821 |
| 850 | -2.42 | -3.72 | 3.544 | 1.943 |
| 700 | -10.10 | -11.43 | 3.412 | 3.283 |
| 500 | $-25.80$ | -26.69 | 4.513 | 2.884 |
| 300 | -45.25 | -46.81 | 2.296 | 2.772 |
| 200 | -39.80 | -39.85 | 1.404 | 1.211 |
| 150 | -39.80 | -40.86 | 1.931 | . 875 |
| $100^{*}$ | -40.20 | -41.16 | . 122 | . 105 |
| 70 | -39.92 | -40.82 | 1.803 | . 699 |
| Number in class | 4 | 14 |  |  |

TABLE 17. Temperature means at mandatory levels at Thule (summer), corresponding to temperature extremes of $T_{5}(100)$ at the (b) $1-5 \%$ class and (c) $5-10 \%$ class of probability. The population of class (a) was zero. The standard deviations $\sigma_{b}, \sigma_{c}$ of the observed temperatures within classes $\underline{b}$ and $\subseteq$ are shown on the right side of the table.


FIG. 7. The heavy solid line shows the mean regression-determined vertical temperature sounding over Thule, Greenland, corresponding to the $1 \%$ warm extreme occurrences of $T_{J}(100)$ of Table 17. The thin solid line depicts the July mean Thule sounding (1967-70). The dashed curves are identical to those of Figure 1.

Table 15 shows that class (b) warm atmosphere is warmer than the class (c) case in the layer 400 mb to 125 mb . Above 125 mb Fig. 5 shows that the deviation between the mean case and the listed extreme at any level is small. For example, the standard deviation $\sigma_{\mathrm{T}}$ at Alert at the 100 mb level is $1.26^{\circ} \mathrm{C}$ (and is smaller still at 70 mb ). Below 400 mb , the temperature of the class (b) "extreme" atmosphere is consistently colder than that of the class (b) atmosphere. All of these features are also to be observed in Fig. 15 , which gives a comparison of the $1 \%$ warm extreme atmosphere and the July mean (1967-70) over Alert.

A similar set of conclusions applies to the regression-dependent class (a) atmospheres over Alert based upon $T_{J}=T(150)$ as forcing variable. Here the results included in Table 16 indicate that the 500 mb temperature is a level of small interdiurnal variability, and is in fact the location of a crossover between the $T$ (extreme) and $T$ (mean) curves. Below 500 mb , the inferred temperature structure corresponding to a forcing temperature at the $1 \%$ warm extreme at $p=150 \mathrm{mb}$ shows an atmosphere slightly colder than the July mean.

The sumner extreme atmosphere at Thule, based upon a forcing-level warm extreme in $T_{J}=T(100)$ is listed in Table 17 and also graphically depicted in Fig. 7. The results summarized in colum (b) of Table 17 show that the extrese warm case at 100 mb is associated with mandatory levels which are consistently warmer at all levels (other than the surface) than in the class (c) atmosphere.
5. Recommendations

For the best estimate of the forced-level extreme atmospheres considered here, one merely reads off the temperature of the left-hand extreme
class of Tables $11, \ldots, 17$. To be more realistic, one should allow a tem-perature-range of one standard-deviation $\pm \sigma_{a}$ about any class (a) extreme atmosphere. However, if a single temperature-estimate must be used at each level, the mean $T_{a}$-profile as a function of $P$ is to be recommended. If class (a) is lacking, then the $T_{b}$-profile should be used.

The work presented here does not include certain other levels listed in the preliminary MIL STD 210-B atmosphere. The four years of climatological data for the years 1967-70 for these additional stations (see Table 2) is still undergoing data analysis. The results for OJMJAKON, Siberia, have been analyzed statistically and have been found to determine regressiongenerated mandatory-level temperatures much like those derived for the North American stations. It is expected that a study of the additional stations 1 isted in Table 2 will yield fruitful results.

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## APPENDIX

It is not feasible due to the limited sample sizes to employ a ttest on the difference of means for the $1 \%$ class relative to the $1-5 \%$ class of Tables $11, \ldots, 17$. However, a well-defined test of significance of class-means is afforded by the t-test applied to the difference of sample-means obtained by the results of the predictions of the means of the $10 \%$ extreme relative to the full sample predictions (e.g., the means contained in Tables 3, 4,...,9).

The t-test on the predictions of the full-sample and the $10 \%$ extreme involves at every mandatory level and station a test for significance of the pooled t-statistic after prediction,

$$
\begin{equation*}
t_{P}(N+n-11)=\left(\bar{T}_{f u 11}-\bar{T}^{\prime} \cdot 10\right) /\left[\frac{(N-9) \sigma_{E}^{2}+(n-2) \sigma_{E, \cdot 10}^{2}}{(N+n-11)}\right]^{1 / 2}\left[\frac{1}{N}+\frac{1}{n}\right]^{1 / 2} \tag{A-1}
\end{equation*}
$$

Here the $\bar{T}$ 's are the sample temperature means at the mandatory levels based upon the identical prediction equations. These means are 1isted in columns 2 and 7 of Tables $3,4, \ldots, 9, \sigma_{E}$ is the standard error of estimate listed in column 5, and $\sigma_{E, .10}$ is that of colurn associated with the prediction of the extreme sample. The number of degrees of freedom associated with these $\sigma_{E}-$ values are $N-9$ and $n-2$ respectively, where

$$
\begin{aligned}
& \mathrm{N}=\text { full-sample size } \\
& \mathrm{n}=\text { nominal } 10 \% \text { extreme sample size }
\end{aligned}
$$

The pooled $t_{p}$-parameter is then tested using $(v+n-11)$ degrees of freedom for $t_{P}$.

At the $70-\mathrm{mb}$ level in Table 3, the $\mathrm{t}_{\mathrm{p}}$-statistic based upon the listed difference of the means is $t_{p}=3.113$. Using the number of degrees of freedom $N+n-11=258$, the sample $t_{P}$-statistic is significant at a level of $99.9 \%$ probability.

In a similar manner all sets of differences of full-sample and "extreme" sample means obtained by the prediction method of this study prove to be significant at leve1s in excess of $99 \%$ confidence. This may be verified by applying all the statistics of Tab1es 3, 4, $\ldots, 9(a, b)$ to the test-statistic $t_{P}$ of Eq. (A-1).

In this Appendix, t-test procedures for considering definitive tests of significant differences of means between full- and extremesample statistics at mandatory levels have been set forth. In defining the extreme-sample, the $10 \%$ probability extreme at the forcing leve1 has been considered as the basis of the sample, including the resulting regression-statistics. Beyond the $10 \%$ probability level, reduced sample sizes preclude the usual t-test procedures. However, the stratifications based upon extreme sub-samples (a), (b), (c) of Tables $11,12, \ldots, 17$ give rise to regression-generated $\bar{T}_{a}$ (mean) which might well have been extrapolated from the $10 \%$ subsample results of Tab1es $3,4, \ldots, 9$, respectively.

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3. ABSTRACT

A group of stations in the North American Arctic region have been analyzed for statistical determination of temperatures at mandatory pressure levels $p_{j}$. For each station the temperature at a key level (called the forcing-level temperature) peculiar to the station has been forced in at the first step, and retained at each subsequent step in the development of the stepwise regression equations giving temperature at the mandatory levels. In general, eight-step prediction equations in terms of other temperatures in the vertical were found to give specification of $T\left(p_{j}\right)$, with percentage explained variance of close to 0.99 . As a result of this definitive property, the best-estimate of the regional atmosphere which is conditionally dependent upon the existence of an extreme $1 \%$ probability of the forcing level temperature is obtained with a high degree of confidence.
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Extreme temperatures

Mandatory pressure levels
Stepwise regression technique
Temperature predictors

Forcing-level temperature
Verification of predictand temperature
Regional key stations

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