Hayes

Diagnostic study in arithmetic.

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DIAGNOSTIC STUDY

IN

ARITHMETIC

BY

MARGARET HAYES

Rural School Supervisor, Craven County

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INTRODUCTION

The materials in this bulletin were prepared by Miss Margaret Hayes, rural supervisor in Craven County. The purpose of the investigation was to determine as far as possible the various causes for retardation in the fundamental processes of arithmetic among the public school children of that county and to suggest remedial measures.

Upon the recommendation of Mr. L. C. Brogden, Director of the Division of Elementary Instruction, and of Miss Hattie Parrott of the same division this bulletin is printed in the belief that it will prove valuable to superintendents, principals and supervisors in their efforts to improve class room instruction in arithmetic in the public schools of the State.

A. T. Allen

State Superintendent of Public Instruction.

9-20-28—1M

A DIAGNOSTIC STUDY IN ARITHMETIC

The need for this study arose when the school children of Craven County, North Carolina, showed persistent retardation in arithmetic fundamentals. This condition did not yield to continued efforts on the part of teachers and supervisor. Therefore a systematic investigation was made with the purpose of securing data about this situation, studying the bearing on the problem of these data, and arriving at some helpful conclusions.

Statement of The Problem

The problem, briefly stated, is this: What causes these pupils to be unable to do satisfactory work in arithmetic, and how can this condition be improved?

APPROACHES

There are four approaches to the problem: (1) A statistical treatment of the scores made by pupils in grades 3-8, inclusive, on Woody Fundamentals. (2) A practical analysis of the papers just referred to. This analysis included (a) a tabulation of examples missed in each grade and calculation of percents to find what types of exercises are generally missed, and (2) an analysis of the types of errors. (3) Individual diagnostic tests in whole numbers and fractions to find out mental habits that retard the work and make it inaccurate. The Buswell-John test was used, and supplemented by skillful questioning. (4) Close observation of the children over a period of two years.

It will readily be seen that each of these methods of approach has certain defects, inherent in its nature. However, the strong points of one make up for the deficiencies of another. Each method gives information that could not be secured in any other way; but it is felt that the most significant information was that gained by the individual tests and the close observation of the pupils themselves. These revealed the mental habits of the pupils.

Statistical Treatment of Data From Survey Tests

Scores available for investigation were secured by administering the Woody Series B Form 1 test in fundamentals to 745 pupils in the Craven County schools, distributed through the grades as follows: 195, 151, 160, 144 and 95 pupils in the fourth, fifth, sixth, seventh and eighth grades, respectively. Tests were given at the beginning of the school year 1926-1927.

Data are given in the frequency table below:

(1) Grade 4		(2) Grade 5		(3) Grade 6		(4) Grade 7		(5) Grade 8	
Score	*Frequency*	*Score*	*Frequency*	*Score*	*Frequency*	*Score*	*Frequency*	*Score*	*Frequency*
42	1	50	1	56	2	63	3	68	1
41	0	48	3	54	4	62	3	67	1
40	1	46	2	52	6	61	3	66	5
39	2	44	5	50	12	60	3	65	6
38	2	42	11	48	13	59	4	64	3
37	4	40	12	46	17	58	7	63	4
36	5	38	21	44	23	57	12	62	6
35	5	36	25	42	20	56	13	61	7
34	3	34	20	40	21	55	3	60	6
33	8	32	14	38	13	54	9	59	6
32	10	30	9	36	8	53	6	58	6
31	7	28	9	34	9	52	8	57	6
30	12	26	5	32	5	51	6	56	6
29	6	24	3	30	1	50	3	55	7
28	8	22	4	28	2	49	6	54	4
27	11	20	4	26	2	48	5	53	3
26	19	18	1	24	1	47	7	52	6
25	11	16	0	22	0	46	5	51	4
24	8	14	0	20	0	45	4	50	1
23	7	12	1	18	1	44	5	49	0
22	10	10	1	--	--	43	4	48	0
21	7	--	--	--	--	42	3	47	3
20	12	--	--	--	--	41	7	46	1
19	4	--	--	--	--	40	3	45	1
18	7	--	--	--	--	39	1	44	3
17	3	--	--	--	--	38	2	--	--
16	2	--	--	--	--	37	4	--	--
15	6	--	--	--	--	36	1	--	--
14	4	--	--	--	--	35	3	--	--
13	3	--	--	--	--	34	1	--	--
12	2	--	--	--	--	--	--	--	--
11	2	--	--	--	--	--	--	--	--
10	1	--	--	--	--	--	--	--	--
9	0	--	--	--	--	--	--	--	--
8	2	--	--	--	--	--	--	--	--

As a means to interpreting the data the following measures were computed: Arithmetic means and medians; absolute and semi-interquartile ranges; mean and standard deviations; and total, partial and multiple correlations of the fundamental scores with reading and reasoning scores for the same pupils obtained at the same time. When it was considered to be of value errors of the measures were computed.

The arithmetic means of grades 4, 5, 6, 7, and 8 are 25.98, 35.48, 43.28, 50.98, and 58.50, respectively, with P. E. of .33579, .37289, .34795,

.409197, and .39427, respectively. Mediums are: 26.34, 36.36, 43.74, 50.62 and 58.42.

Absolute ranges for grades 4, 5, 6, 7 and 8 are 34, 40, 38, 29, and 24 respectively; semi-interquartile ranges: 5.58, 6.98, 6.47, 5.41, and 3.88 with probable errors of .26415, .29334, .27371, .32231, and .31015, respectively. Standard deviations: 6.92, 6.80, 6.50, 7.28 and 5.67, with probable errors of .23741, .26365, .24363, .289319, and .27876, respectively; 10-90 percentile ranges are: 19.18, 15.98, 15.07, 18.45, and 13.92, respectively.

Where reading is represented by 1, reasoning 2, and fundamentals 3, total correlations for the grades all taken together are as follows:

$$r_{12} = .636; r_{23} = .822; \text{and } r_{13} = .715.$$

(Scatter diagram was used with Karl Pearson's formula.)

Using the same notation, partial correlations were found as follows:

$$r_{12.3} = .121; r_{13.2} = .437; r_{23.1} = .687.$$

Values found for multiple correlations are:

$$R_{1.23} = .72; R_{2.13} = .83; R_{3.12} = .86.$$

It will be seen that fourth, fifth, sixth, seventh and eighth grade students are retarded 5, 7, 6, 7, and 3 months respectively, using grade norms. The lesser retardation of grade 8 is probably due to the fact that no scores were available for the repeaters in this grade. Pupils in the rural schools of this county are doing work in fundamentals over half a year before the national standard. Also, if a large number of means for these grades in the rural schools of North Carolina were computed, it is reasonably certain that the true means will lie within the Craven County means $\pm$ 2 P. E., as follows:

25.30 to 26.66, grade ability 3.3 to 3.4.
34.74 to 36.22, grade ability 4.1 to 4.2.
42.58 to 43.98, grade ability 5.1 to 5.2.
50.16 to 51.80, grade ability 6.1 to 6.2.
57.72 to 59.28, grade ability 7.4 to 7.7.

The absolute range though not very significant, is wide in all cases, widest in grade 5 and narrowest in grade 8. The latter is probably due to the same reason given for relatively small retardation. Semi-interquartile ranges show a greater clustering of the scores of the middle 50 per cent in grade 8, with grade 7 next and the greatest scattering in grade 5. 10-90 percentile ranges give a closer clustering in grade 4. Probably there are more pupils with low I. Q's, in grade 5, but the general preparation for normal pupils may be poor in grade 4. The most important measure of variability, the standard deviation, narrows the picture of the distribution by showing that 68.26 per cent of the cases fall between mean $\pm$ sigma, or in these cases:

19.06 to 32.90, grade ability 3.1 to 3.8.
28.68 to 46.28, grade ability 3.5 to 5.6.
36.78 to 49.78, grade ability 4.2 to 6.1.
43.70 to 58.26, grade ability 5.3 to 7.5.
52.83 to 64.17, grade ability 6.4 to 8.4.

This shows a rather wide range for normal pupils within the grade.

The relationships as indicated by the total correlations are very close for reasoning with fundamentals and reading with fundamentals, and close for reading with reasoning. Evidently the subjects are all three highly interrelated and the values found suggested that fundamentals play the most important part in the trio. Since the number of cases (745) makes this a fair sample there is probably a high degree of relationship between these subjects in the rural schools of North Carolina as a whole. The partial correlations give some insight into the nature of the relationships shown by the total correlations. With fundamentals held constant, the correlation between reading and reasoning reduces from .636 to .121, therefore the correlation between reading and reasoning is dependent largely upon the skills in fundamentals. With reading held constant the correlation reduces from .822 to .687. Evidently some, but not many of the difficulties in problem solving and working exercises in fundamentals are due to inability to read the problems. With reasoning held constant the correlation between reading and fundamentals reduces from .715 to .437. This is an indication that the relation between fundamentals and problem solving is very much stronger than that between fundamentals and other subjects, both when these other factors are present, and when they are eliminated. This is further borne out by the fact that the multiple correlations between reading and the combined effects of the other two is relatively lower than the correlation of the combined effects of reading and fundamentals on reasoning, and also, lower than the correlation of the combined effects of reading and reasoning on fundamentals. All these can be taken as indications only, but they lead to the belief that a knowledge of fundamentals is the first and most important essential in the arithmetic situation and the reading and reasoning are both of great importance in this connection.

Practical Analysis of Pupils' Papers

In the study of the actual exercises the procedure was different. The point of view here was a seeking for *cause*, while the statistical treatment was designed to bring out the meaning of the data.

a. Tabulation to find types of examples missed

The practical treatment of the results consisted of the following steps: (1) a determination by the course of study of the examples on the test sheet that pupils completing each grade might be expected to work; (2) a tabulation of the number of pupils missing these examples; (3) a conversion of these into percentages; (4) an analysis of examples missed by as many as ten per cent of the pupils, with a view to determining types of errors, difficulties and interesting peculiarities incident to teaching and learning fundamentals; (5) a summary of conclusions drawn from the study outlined above.

b. Analysis of examples missed by types of errors

Following this plan a careful tabulation of errors counting numbers of pupils making each type of error rather than actual number of errors made, the information that follows was obtained. Since in many cases of unfinished examples some types of errors connote others, the errors given in all cases are a minimum number.

In general pupils entering grade 4 from grade 3 may be expected to work 32 examples on the test paper, distributed as follows: subtraction, examples 1-17; multiplication 1-18; addition 1-16; division 1-19. Pupils in grade 3 have failed to fix examples of the type 13, 14, and 17, in subtraction, 8, 9, 11, 13, 16 and 18 in multiplication. 7, 10, 13, 14, 16 in addition, and 1, 2, 7, 8, 11, 14, 15, 17, 19 in division.

Several interesting facts were observed. In multiplication by zero pupils so often say "one times" the number. Often the minuend was taken for the subtrahend when the latter was larger, as 16 — 9 = 13. The answer for an example of equation form was often reversed, as ¼ of 128 = 23. A number times zero is often given as one times the number. When there is a zero in the multiplier and a carried number is to be added to the multiplier times it, pupils often put down the zero only.

Types of errors most generally made in the beginning of grade 4 in order of importance as shown by the tabulation of errors are as follows:

In subtraction, errors are due to ignorance of the more difficult subtraction facts, ignorance of the type of exercises with two figures in minuend and subtrahend, incorrect borrowing and incorrect handling of the zero in subtraction.

Errors in multiplication are due to insufficient knowledge of the more difficult multiplication facts, incorrect handling of zero in multiplier and multiplicand, and incorrect carrying where multiplicand is larger.

In division, errors are due to ignorance of the more difficult division facts, zero difficulties in division, trouble with remainder, unfamiliar form (equation) and inability to handle exercises with large dividends.

In addition, errors are due to ignorance of the more difficult additive facts, incorrect carrying, errors in higher decade addition, unfamiliar form and ignorance of type of exercise with two figures or more in addends.

For pupils entering grade 5 from grade 4 it was found that in general they may be expected to work 41 examples on the test sheet, distributed as follows: 1-19 in subtraction, 1-18 in multiplication, 1-16, 22, 28 in addition, and 1-23 in division. Pupils in grade 4 have failed to fix examples of the types 14, 17, 19, in subtraction; 9, 11, 12, 13, 16, 18, 26 in multiplication; 14, 16, 22, 23, in addition and 11, 14, 15, 17, 19, 23, 28 in division.

Several interesting points were observed. Pupils sometimes put down whole number in the product instead of carrying this number. A very prevalent error is giving the answer in long division with zero omitted as 2 9/29 for 20 9/29. Zero is sometimes put in quotient where it does not belong as 7032 for 732. Where the number is carried and there is a zero in the multiplier pupils often put zero in product, instead of the number carried. Remainders are often placed over dividend instead of divisor. In addition carried number is sometimes drawn down as 529 for 79. Often a zero in multiplier causes wrong placing of products. Errors in the more difficult combinations sometimes connote errors in carrying. Where there are errors in estimating quotient these would usually be accompanied by errors in handling remainders but since the example is unfinished it is impossible to tell. In the same way

errors in bridging nearly always connote errors in carrying. Example 23 in division contains a zero difficulty but since it was tried by so few the errors could not be tabulated. Therefore the errors tabulated in each case are the minimum number.

Types of errors made by pupils completing grade 4 and entering grade 5 are:

In subtraction, errors are due to ignorance of the more difficult subtraction combinations, reversing process when subtrahend digit is larger than corresponding digit in minuend, those due to zero difficulties, borrowing, unfamiliar equation forms, confused process and careless errors (copying, etc.)

In multiplication, types of errors made are those due to ignorance of more difficult multiplicative facts, carrying zero difficulties, unfamiliar equation form, confusion of process (harmful transfer) incorrect placing of partial products, and careless errors (such as placing decimal where they do not belong, copying, etc.)

In addition, types of errors, are those due to ignorance of the more difficult addition combinations, inability to handle higher decade addition, zero difficulties, carrying, unfamiliar equation form, confused process, and careless errors.

For pupils entering grade 6 from grade 5 it was found that in general pupils may be expected to work 48 examples distributed as follows: 1-20 in subtraction, 1-18 in multiplication, 1-23 in addition, and 1-27 in division. Pupils in grade 5 have failed to fix examples of the type 1, 7, 19, 20 in subtraction, 8-18, 24, 26 in multiplication, 20-23 in addition and 17, 19, 23, 27 in division.

Several interesting facts were observed. Remainders were often put over the dividend instead of the divisor. Errors in estimating quotients usually connote errors in bringing down terms of dividend. Subtraction in long division is often wrong because figures are not put in the right places to be subtracted.

Types of errors in subtraction made by pupils entering grade 6 are those due to ignorance of more difficult subtraction combinations, incorrect borrowing, zero difficulties, confused processes, careless errors, failure to reduce to lowest terms, putting denominator under integer, unfamiliar equation form, drawing down fractions, not multiplying by numerator or dividing by denominator, and subtracting the denominator from the multiplicand.

Types of errors in multiplication are: more difficult combinations, carrying, zero, partial products, confused errors in computation, failure to reduce to lowest terms, error in reducing to whole or mixed number, putting denominator under integer, equation, drawing down fractions, not trying, not multiplying by numerator or dividing by denominator, and subtracting denominator from multiplicand.

Types of errors in addition are those due to ignorance of more difficult addition combinations, incorrect carrying, zero difficulty, inability to handle higher decade addition, confused process, careless errors, failure to reduce to lowest terms, unfamiliar equation form, failure to place decimal, and multiplying denominator.

Types of errors in division are those due to ignorance of more difficult combinations, inability to estimate quotients, zero difficulties, incorrect bringing down, confused process, careless errors, failure to reduce to lowest terms, unfamiliar equation form, inability to handle remainders, incorrect placing of decimal and adding in numerator.

For pupils entering grade 7 from grade 6 it was found that in general pupils may be expected to work 55 examples on the test, distributed as follows: 1-25 in subtraction, 1-29 multiplication, 1-36 in addition, and 1-27 in division. Pupils in grade 6 have failed to fix examples of the type 19, 20, 24, 25 in subtraction, 12-29 in multiplication, 16, 20-36 in addition, and 17, 19, 23, 27 in division.

Types of errors in subtraction made by pupils entering grade 7 are those due to ignorance of the more difficult combinations, incorrect borrowing, zero difficulties, drawing down fractions, integer treated as a fraction, not getting to common denominator and confused process.

Types of errors in multiplication are those due to insufficient knowledge of more difficult combinations, incorrect carrying, zero difficulties, incorrect handling of partial products, bringing down fractions, incorrect placing of decimals, careless, confused process and failure to reduce to lowest terms.

Errors in addition are those due to ignorance of more difficult addition combinations, incorrect carrying, inability to do higher decade addition, adding denominator, unfamiliar equation form, multiplying denominator, not getting common denominator, inability to handle denominate numbers, incorrect placing of decimal and not reducing to lowest terms.

Errors in division are those due to inability to estimate quotients, zero difficulties, incorrect bringing down, unfamiliar equation form, inability to handle remainders, incorrect placing of decimal, adding numerator, careless and harmful transfer.

For pupils entering grade 8 from grade 7 it was found that in general pupils may be expected to work 58 examples distributed as follows: 1-27 in subtraction, 1-33 in multiplication, 1-36 in addition and 1-30 in division. Pupils in grade 7 have failed to fix the following types of examples: 25, 27 in subtraction; 18, 26, 27, 29, 33 in multiplication 20, 21, 22, 24, 30, 33, 36 in addition; and 19, 23, 27, 28, 30 in division. In many cases fractions were not tried.

Types of errors in subtraction made by pupils entering grade 8 from grade 7 are those due to incorrect borrowing, drawing down fractions, integers treated as fractions, not getting common denominator, harmful transfer and inability to handle denominate numbers.

Types of errors in multiplication are those due to insufficient knowledge of more difficult combinations, incorrect carrying, incorrect handling of partial products, drawing down fractions, incorrect placing of decimal, failure to reduce to lowest terms, harmful transfer, careless, failure to multiply by numerator or divide by denominator, and multiplying the denominator.

Types of errors in addition are those due to incorrect carrying, incorrect borrowing, inability to do higher decade addition, failure with denominate numbers, incorrect placing of decimal, not reducing to lowest terms, and not getting a common denominator.

Types of errors in division are those due to inability to estimate quo-

tients, zero difficulties, incorrect bringing down terms of dividend, inability to handle remainders, subtraction in long division, harmful transfer, wrong inversion and incorrect placing of the decimal.

Several conclusions may be briefly shown from the above study (a) there has been insufficient drill on the more difficult combinations in all four fundamentals; (b) some hard things such as handling the zero, long division, carrying and borrowing have been inadequately taught and insufficient drill has been furnished; (c) drills given have not always fitted children's needs.

Individual Diagnostic Tests to Fnd Out Mental Habits That Slow Up Work and Make It Inaccurate

The third approach to the problem is the use of the individual diagnostic tests to determine the mental habits that slow up the pupil's work and make it inaccurate.

a. A test admirably suited for individual testing is the Buswell-John. The pupils' sheet contains exercises involving all the known difficulties in the four fundamentals. The teachers' sheet contains the same exercises with a list of the bad habits usually observed in each of the four fundamental operations.

b. As the pupils work aloud the examiner carefully observes and questions him and is able to discover his bad habits.

PLAN OF WORK, RESULTS AND INTERPRETATION OF RESULTS

In this case the supervisor selected three children, one slow, one average, and one superior, from each of the grades 3-7 in six schools of widely varying types in the county. This made a group of representative grades, each grade comprising eighteen children: six superior, six average, and six slow pupils. A careful testing of these ninety pupils gave information as to the number, and kinds of bad habits prevalent in each group in each grade, and also showed types of examples missed by each group. These were arranged in order of frequency as shown as follows:

For Grade 4

1. *Addition*: Errors in combinations, counting added carried number last, forgot to add carried number, repeated work after partly done, carried wrong number, wrote number to be carried, irregular procedure in column, used wrong fundamental operation, and dropped back one or more tens.
2. *Subtraction*: Errors in combinations, did not allow for having borrowed, counting, subtracted minuend from subtrahend, put zero in front of answer, (as follows, 06 for 6), failed to borrow (gave zero as answer), added instead of subtracting, and ignored a digit.
3. *Multiplication*: Errors in combinations, errors due to zero in multiplier, errors in addition, errors in single zero combinations (zero as multiplier), omitted digit in multiplier, error in adding the carried number, carried a wrong number, forgot to carry, counted to carry, wrote rows of zeros, errors in position of partial product, errors in writing product, used wrong process (added), wrote carried number, confused products when multiplier had two or more digits, and errors in carrying zero.

4. *Division*: Errors in division combinations, errors in subtraction, errors in multiplication, found quotient by trial multiplication, omitted digit in dividend, used remainder larger than divisor, omitted final remainder, omitted zero resulted from another digit, not reducing remainder to lowest terms, neglected to use remainder within problem, used long division for short division, and used too large a product.

Examples missed by pupils, in order of frequency are: (See Buswell-John Test Sheet).

For superior group of pupils:
Addition: 21, 19, 5, 18, 20, 22, 23.
Subtraction: 17.
Multiplication: 16, 17, 18, 14, 15.
Division: 16, 17, 15, 11, 12, 13, 14, 8, 9.

For average group of pupils:
Addition: 19, 20, 21, 22, 8.
Subtraction: 15, 18, 8, 9, 10, 11, 12, 13, 14, 16.
Multiplication: 16, 17, 18, 12, 9, 6, 13, 15, 14.
Division: 16, 17, 10, 12, 14, 15, 13, 8, 9.

For slow pupils:
Addition: 21, 22, 19, 18, 4, 6, 9, 14, 17, 10, 12, 15, 20, 23.
Subtraction: 11, 18, 14, 17, 15, 4, 5, 12, 13, 16.
Multiplication: 13, 14, 15, 16, 17, 11, 12, 9, 18, 5, 2, 10.
Division: 13, 14, 15, 16, 17, 10, 5, 11, 12, 2, 3, 6, 8, 9.

For Grade 5

1. *Addition*: Errors in combinations, added carried number last, counting, forgot to add carried number, repeated work after partly done, irregular procedure in column, carried wrong number, and wrote the number to be carried.
2. *Subtraction*: Did not allow for having borrowed, errors in combinations, counting, deducted two from minuend after borrowing, and subtracted minuend from subtrahend.
3. *Multiplication*: Errors in combinations, carried a wrong number, error in adding carried number, errors due to zero in multiplier, omitted digit in multiplier, error in single zero combinations (zero as multiplier) and wrote carried number.
4. *Division*: Errors in division combinations, found quotient by trial multiplication, omitted zero resulting from another digit, errors in subtraction, used remainder larger than divisor, used long division form for short division, omitted final remainder, failed to reduce to lowest terms (remainder), omitted digit in dividend, and repeated part of multiplication table.

Examples missed by pupils in order of frequency are:

For superior pupils:

Addition: 21, 22, 18, 23, 14, 16.
Multiplication: 14, 17, 22, 13, 15, 16, 18.
Division: 17, 16, 18, 21, 12, 15, 20.

For average group of pupils:

Addition: 21, 16, 18, 19, 22, 12, 15, 20, 23.
Subtraction: 18, 21, 22, 13, 8, 15, 17, 19, 20.
Multiplication: 13, 15, 17, 18, 19, 20, 22, 12, 16, 21, 9, 14.
Division: 19, 16, 17, 18, 21, 13, 14, 15, 11, 20, 12, 10.

For slow group of pupils:

Addition: 21, 22, 19, 23, 8, 9, 13, 17, 18, 20.
Subtraction: 17, 18, 21, 22, 10, 19, 20, 13, 15.
Multiplication: 17, 19, 21, 22, 12, 13, 15, 16, 20, 6, 18, 1, 2, 3, 5, 11, 7.
Division: 12, 15, 16, 17, 18, 19, 20, 21, 8, 11, 13, 14, 10, 5, 2, 6, 9.

For Grade 6

1. *Addition*: Errors in combinations, counting, added carried number last, forgot to add carried number, repeated work after partly done, wrote number to be carried, irregular product in column, and added carried number irregularly.
2. *Subtraction*: Errors in combination, did not allow for having borrowed, counting said example backwards, deducted 2 from the minuend after borrowing, and put zero in front of answer as 06 for 6.
3. *Multiplication*: Errors in combinations, errors due to zero in multiplication, counted to carry, error in single zero combinations (zero as multiplier), omit digit in multiplier, errors in addition, wrote rows of zeros, carried a wrong number, and wrote carried numbers.
4. *Division*: Errors in subtraction, found quotient by trial multiplication, used long division for short division, omitted zero resulting from another digit, errors in combinations, errors in multiplication, omitted digit in dividend, not reducing remainder to lowest terms, counted in subtracting, used remainder larger than divisor, omitted final remainder, and neglected to use remainder within problem.

Examples missed by pupils in order of frequency are:

For superior group of pupils:

Subtraction: 17, 18, 20, 26.
Multiplication: 14, 16, 19, 20, 7, 13, 18.
Division: 21, 16, 17, 18, 10, 13.

For average group of pupils:

Subtraction: 19, 21, 16, 22, 14, 15, 18.
Addition: 21, 19, 22, 18, 23, 3, 8, 17, 20.
Multiplication: 19, 21, 16, 22, 14, 15, 18.
Division: 20, 21, 15, 16, 17, 12, 13, 18, 10, 11, 13, 19.

For slow group of pupils:

Addition: 21, 19, 23, 8, 17, 4, 6, 10, 18, 20, 22.
Subtraction: 19, 20, 22, 15, 17, 13, 21, 10, 18.
Multiplication: 16, 22, 14, 17, 18, 21, 15, 19, 20, 6, 12.
Division: 16, 17, 18, 20, 21, 8, 11, 12, 13, 19, 1, 9, 10, 14.

For Grade 7

1. *Addition*: Errors in combinations, added carried number last, counting, repeated work after partly done, and wrote number to be carried.
2. *Subtraction*: Errors in combinations, did not allow for having borrowed, and counting.
3. *Multiplication*: Errors in combinations, errors in single zero combinations (zero as multiplier). errors in addition, counted to carry, error in adding the carried number, wrote rows of zeros, wrote carried number, errors due to zero in multiplier, forgot to carry and omitted digit in multiplier.
4. *Division*: Found quotient by trial multiplication, errors in multiplication, errors in subtraction, used long division combinations, omitted digit in dividend, omitted zero resulting from another digit, counted in subtracting, did not reduce remainder to lowest terms, used remainder larger than divisor, and used digit in dividend twice.

Examples missed by pupils in order of frequency are:

For superior group of pupils:

Addition: 21, 19, 22.
Subtraction: 18.
Multiplication: 21, 22.
Division: 20, 13, 17, 21, 15, 18.

For average group of pupils:

Addition: 21, 8, 18, 19, 22.
Subtraction: 18, 19, 22, 14, 15, 17.
Multiplication: 14, 21, 15, 16, 18, 19, 20.
Division: 17, 21, 15, 16, 18, 19, 20.

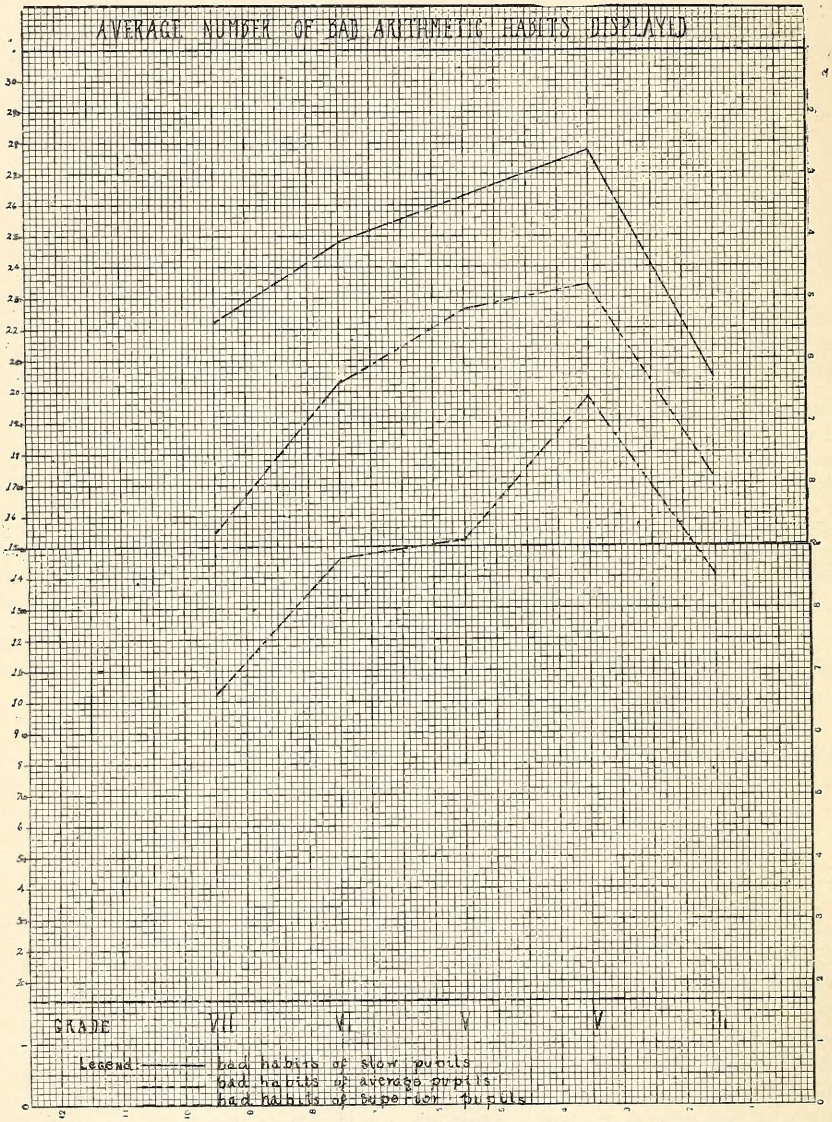
For slow group of pupils:

Addition: 21, 18, 19, 23, 8, 10, 15, 20, 22.
Subtraction: 18, 21, 22, 15, 16, 17, 19, 20.
Multiplication: 22, 13, 14, 19, 21, 15, 16, 18, 17.
Division: 21, 17, 19, 15, 16, 18, 10, 13, 14, 11, 8, 12, 20.

It was observed that slow pupils had far more bad habits than average pupils, and that avérage pupils had more bad habits than superior pupils. Also superior pupils worked more exercises than average pupils, and average pupils surpassed slow pupils in this respect. Another significant point was that the schools varied among themselves in achievement and number of bad habits displayed. In general pupils in schools with small teacher load, more highly trained teachers, and superior teaching morale showed relatively greater achievement and fewer bad habits of work. Pupils in one school, whose teachers are nearly all of the older, more conservative type, living in the community, showed a larger number of bad habits of work. This may, or may not be significant. Small schools displayed a relatively large number of bad habits of work.

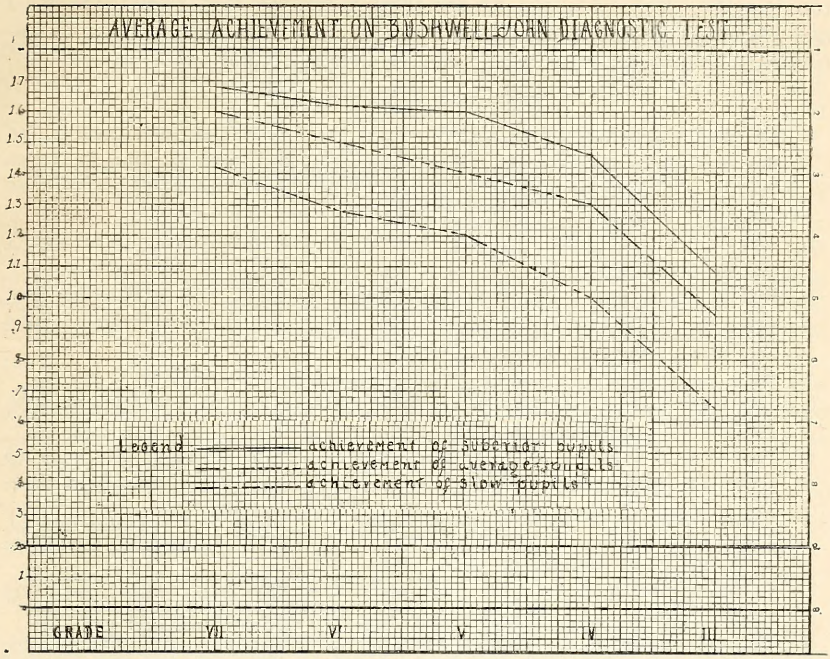
Another interesting point is that far more habits, not listed on the Buswell-John test, were discovered. These are: (1) Not reducing re-

mainder to lowest terms (d.36); (2) putting a zero in front of the answer in subtraction (s. 25); (3) not placing remainder over divisor (d. 37); and (4) writing borrowed number (s. 26).



The accompanying graphs illustrate the comparisons given above as to bad habits displayed and achievement.

These are significant in that they shed light on variations in instruction in different types of schools in the county and on variations in habits and achievement of groups within the school.



SUMMARY OF DATA AND BEARING ON PROBLEM

From the foregoing interpretation of data we may draw the following conclusions in regard to the problem:

1. The chief cause of retardation in arithmetic is the formation of bad habits of work that make the work slow and inaccurate. These habits are due partly to negligence and ignorant procedure on the part of the teacher, and partly to habits formed independently by pupils. These habits are usually not evident except to the trained observer.

2. Another cause is the inadequate teaching of certain difficult material and inadequate drill on newly learned material, especially the combinations.

3. Another cause is failure to classify pupils properly.

4. A fourth cause is ignorance on the part of the teacher of the exact difficulties in her class, which has prevented a close adjustment of the instruction to the needs of the pupils.

USE OF DATA IN SUGGESTING REMEDIAL MEASURES

Having completed the first two steps in the study (i.e. (1) finding by a study of the data what conditions exist; and (2) investigating the causes of such conditions), the third step is to plan definite remedial action. Here follows a few particular suggestions for the use of the information gained. First, and most important of all, definite remedial measures should be worked out to counteract each bad habit discovered in the pupils. These should be constantly revised and added to. The

importance of the co-operation of the child in this work cannot be stressed too heavily. This is a list of remedial measures that have been used and found valuable, though it is thought best that the teacher be encouraged to use her own originality in devising remedial measures.

Remedial Suggestions

1. Often let an individual pupil, usually a slow one, work aloud at board, pupils and teacher noting good and bad habits. Discuss these habits with pupils, let them see why they hinder the work and arouse a desire for self-improvement. This is very important.
2. Develop self-control attitude in pupils and have each pupil work on his own difficulties.
3. When teaching a new process be sure to point out good and poor methods of procedure, giving advantages and disadvantages. Give careful explanation of new process.
4. Give drill only when material is thoroughly understood.
5. Use any device you can think of to arouse pupils' interest: games, posters, competition, projects, etc.
6. In reteaching types of examples imperfectly understood, be sure to point out repeatedly what causes difficulty, as for instance the prevalent habit of leaving out a zero in the quotient.
7. Slow pupils have more bad habits of work than average or bright ones, so work especially with these pupils.
8. Co-operative work at the blackboard and checking helps. Let one pupil perform one step, another one the next and so on.
9. Present examples in such order that only one new difficulty is presented at a time.
10. Pupils' names and combinations causing them difficulty may be placed on the board for reference and study. Encourage pupils to work on these before and after school and at study period.
11. Each pupil might keep record of his errors in a little book.
12. To remedy irregular column procedure examples might be written in words.
13. Using concrete numbers instead of abstract ones often helps when a pupil has zero difficulties. (For instance, marbles, apples, etc.)
14. Occasionally going through examples slowly in correct form with whole class helps to form correct habits.
15. Commend good form in work whenever found.
16. Observe what bad habits your pupils have and try to work out something to combat each bad habit. Use any method you can think of.
17. The Scott-Foresman work books are excellent help, both for diagnosis and drill. Good drill material can be made by the teacher.
18. Try to prevent careless errors by arousing pupils' pride.
19. Keep a combination posted until it is mastered (can be given without hesitation).
20. Remember that the time to forestall bad habits is when the material is first presented. The pupils will make up for themselves poor and time-wasting methods that we must discover and get rid of. Demonstrate the advantage of a better method.

21. Gaining pupils' co-operation will make it easier to discover bad habits and easier to get rid of them.

NOTE: Ask the teacher to write down any method she finds effective. At the close of the school term ask her to report what specific things she did for the bad habits found during the term. Some most excellent suggestions for remedial work are given by G. T. Buswell in his "Diagnostic Studies in Arithmetic"—University of Chicago.

SUGGESTIONS FOR FURTHER STUDY AND EXPERIMENTATION

At the beginning of the year teachers should be furnished with list of errors similar to ones given in first part of this paper—arranged in order of prevalence. The teacher should use this list as a guide in her review work at the beginning of the term since it shows what the pupils have failed to get in the preceding grade. These same lists will indicate (from last year's failures) what should be especially stressed this year.

All teachers of grade 3-8 inclusive should lay special stress on the more difficult combinations in all four fundamentals. Special practice cards would be of service here if they were supplemented by extra drill on combinations missed by a particular class.

Remedial drills should be modeled on the types of exercises that the study showed that pupils had failed to fix. Diagnostic drills should be made accordingly.

Teachers should demand absolute accuracy, in view of the large number of careless errors found.

Also the teacher should always be on the lookout for individual difficulties and correct these at once. It is felt that what will bring about a great deal of benefit in this particular situation is the finding of the needs of the children and fitting instruction and drills closely to them.

In this diagnostic study, the teachers then undertook, with the aid of the supervisor, to put the remedial suggestions into effect. Frequent checks were made by the teachers, and at the close of the year another form of the survey test (Woody Fundamentals) was used to measure progress. It was found that the average retardation in this subject was 5.4 month instead of 8.0 as previously. It was felt by supervisor and teachers that the right method was being employed and should be continued the next year and carried beyond the field of integers. The pupils' difficulties in fractions indicated that a similar diagnosis of fractional difficulties would be most profitable. The next step after that would logically be an analysis of problem difficulties and study of the technique of problem solving. At present a diagnostic test in fractions is not on the market, but a satisfactory one could be made by the teacher.

TESTS FOR SURVEY AND DIAGNOSTIC PURPOSES

A word here would not be out of place concerning the advantages and disadvantages of the tests used in the study. The survey test used had the obvious advantage of being a practical testing medium for a large number of pupils and easily scored. The resulting scores were suited to a rather extensive statistical treatment. Progress was easily measured with this test. Its limitation is that it gives a vague picture of the pupil's difficulties. The Buswell-John diagnostic test, on the other hand, is impractical for a large number of pupils, since it must be given in-

dividually. Its advantage lies in the fact that the pupil works aloud and the examiner observes the way his mind works when encountering the exercise. One test supplements the other. Briefly, the survey test shows *what* the pupils can do; the diagnostic test shows *how* he does it.

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