ELEMENTARY ARITHMETIC

PART THREE

CHARLES SCRIBNER'S SONS

MOREY'S ARITHMETICS

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PART THREE

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PREFACE

THIS Elementary Arithmetic, the outcome of many years of observation and actual teaching experience, is based on the idea that number is essentially abstract, and that the prime object in the first years of school is to teach number as number. To secure accuracy and facility, a large amount of drill work in the fundamental processes is provided, and is so arranged as to furnish thorough and frequent review of all subjects previously studied.

The application of number to the affairs of everyday life has not been neglected. An abundance of oral and written problems within the limits of the comprehension of pupils furnishes material for concrete work.

Technical explanations of processes, necessarily confusing to immature minds, are purposely omitted. A minimum of theory and a maximum of practice are generally conceded to be the wisest method of teaching the principles of arithmetic to young pupils.

Experience proves that an elementary arithmetic should be simple, progressive, and teachable, and in a direct and practical way aim to develop arithmetical power. It is

the earnest hope of the author that the present book will be found to fulfill these requirements.

The author wishes to acknowledge his indebtedness to all who have assisted in the preparation of the manuscript, and especially to Mr. Myron T. Pritchard, Master of the Everett School, Boston, Massachusetts, for wise counsel and criticism.

C. W. M.

CONTENTS

PART III

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NOTATION AND NUMERATION

1. How many units make 1 ten? How many tens make 1 hundred? How many hundreds make 1 thousand?

2. The middle 3 in the number 333 represents how many times as many units as the right-hand 3?

3. The left-hand 3 represents how many times as many units as the right-hand 3.

Each figure in a number has a value determined by its place in the number.

4. Compare the value of the 2's in 22; 202; 220; 2200; 2020; 2002.

5. Using 4's and 0's write a number in which one 4 represents one hundred times as many as the other 4.

Separate into groups, and read:

6. 8067	**11.** 20387	**16.** 480465	**21.** 1378543
7. 9350	**12.** 58706	**17.** 896302	**22.** 5490876
8. 7006	**13.** 93042	**18.** 107069	**23.** 9040732
9. 8360	**14.** 10087	**19.** 316400	**24.** 27438564
10. 6040	**15.** 80649	**20.** 300602	**25.** 764312857

26. When we separate numbers into groups of three figures each, what is the right-hand group called? The next group to the left? The next group?

Write in figures:

1. Three thousand forty.
2. Seventeen thousand nine hundred twenty-six.
3. Sixty thousand six hundred six.
4. One hundred thirty-nine thousand.
5. One hundred thousand thirty-nine.
6. Three hundred four thousand one hundred ten.
7. Eight hundred twenty thousand twenty-four.
8. One million two hundred twelve thousand.
9. Three million forty-six thousand seventeen.
10. Two hundred sixty-seven million eight hundred four thousand seventy-six.

ROMAN NOTATION AND NUMERATION

Letters used	I	V	X	L	C	D	M
Values	1	5	10	50	100	500	1000

By combining these letters we can express any number by following these rules:

I. When a letter is followed by the same letter or by one of less value, add the values of the letters. Thus, XX = 20; XIII = 13.

II. When a letter is followed by one of greater value, subtract the letter of less value from the letter of greater value. Thus, IX = 9; XL = 40.

Read:

1. XIX XXXVII LXV CIV DC

Write in Roman notation:

2. 8 14 25 43 52 66 78 81 99

DRILL IN FUNDAMENTAL PROCESSES

NOTE. Each exercise should begin with a short, rapid oral drill in the fundamental processes. This daily drill should be continued until accuracy and facility render such work unnecessary.

ADDITION *Oral*

Add 2 to each number:

3	1	5	2	7	0	4	8	6	9

Add 4; 6; 8; 1; 3; 9; 5; 7.

Addition is the process of uniting two or more numbers into one number.

The *sum* or *amount* is the result of addition.

DRILL TABLE

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
1.	35	28	36	52	61	44	70	86	91	60
2.	20	12	78	37	53	62	45	71	87	92
3.	93	21	13	79	38	54	63	46	72	88
4.	10	30	22	14	80	39	55	64	47	73
5.	74	94	31	23	15	81	48	56	65	29
6.	98	59	40	32	24	16	82	95	57	66
7.	89	75	67	41	33	25	17	83	49	58
8.	99	68	76	50	42	34	26	18	84	97
9.	69	96	90	77	51	43	11	27	19	85

Add 2 to each number; add 3; 4; 5; 6; 7; 8; 9.

Add 20 to each number; add 30; 40; 50; 60; 70; 80; 90.

Give the sum of each number and any number of two figures. Thus, 35 + 78. This means 35 + 70 + 8. Think 35, 105, 113. Say 113.

Find the sum of each column. Of each row.

ORAL PROBLEMS

1. Miriam used her weekly allowance as follows: 7 cents for candy, 2 cents for a pencil, 6 cents for flower seeds, 5 cents for a soda, and 5 cents for the school savings bank. How much was her weekly allowance?

2. At the playground 15 boys enter the potato race, 12 the three-legged race, and 9 the running race. How many boys in the three races?

3. How much did it cost Sarah to go to the picnic, if she spent 20 cents for car fares, 5 cents for lemonade, 15 cents for a steamer ride, and 10 cents on the merry-go-round?

4. Mr. Kennedy buys Harold a knife for 25 cents, Frank a box of crayons for 20 cents, and Alice a doll for 50 cents. How much does he pay for all?

5. Mrs. Hovey canned 16 jars of blueberries, 9 jars of raspberries, 11 jars of strawberries, and 8 jars of cherries. How many jars in all?

6. We sold from our garden 6 bushels of pears, 2 bushels of plums, 13 bushels of apples, and 3 bushels of grapes. How many bushels of fruit did we sell?

7. John bought a hat for 3 dollars, a coat for 12 dollars, a pair of shoes for 4 dollars, and collars and cuffs for 1 dollar. How much did he pay for all?

8. A farmer brings us a dozen ears of corn for 12 cents, two boxes of blueberries for 25 cents, and a dozen eggs for 40 cents. How much do all cost?

9. Fred entered the primary school when he was 6 years old. He spent 3 years in the primary school, 5

years in the grammar school, 4 years in the high school. How old was he when he graduated from the high school?

10. At the settlement house there are 13 girls in the dressmaking class, 17 in the millinery class, and as many in the cooking class as in both the other classes. How many in the cooking class? How many in the three classes?

SUBTRACTION

Oral

Take 4 from:

10	13	16	19	11	14	17	12	15	18

Take 3; 6; 9; 1; 5; 8; 2; 7.

Subtraction is the process of taking one number from another, or of finding the difference between two numbers.

The *minuend* is the number from which something is taken.

The *subtrahend* is the number taken from the minuend.

The *remainder* or *difference* is the result of subtraction.

Take 2 from each number in the table on page 227. Take 3; 4; 5; 6; 7; 8; 9.

From 100 take each of the numbers in the table. Thus, $100 - 57 = 100 - 50 - 7$. Think 100, 50, 43. Say 43.

Give differences between any number of two figures and the numbers in the table.

ORAL PROBLEMS

1. Six pupils out of a class of 40 were not promoted. How many were promoted?

2. Frank earned 25 cents on Monday and 9 cents less on Tuesday. How much did he earn on Tuesday?

3. Out of a flock of 37 chickens, a hawk caught 3 and 8 died. How many were left?

4. Joe sells 43 papers and Sam 15 less. How many does Sam sell?

5. A party of 45 people started to climb Mt. Adams; 19 went half way up. How many reached the top?

6. In a box of 3 dozen eggs 9 were broken. How many were good?

7. There were 30 men and 50 women in a hospital. How many patients were there after 40 were discharged as cured?

8. What is the change from a 50-cent piece given in payment for oranges for 18 cents, tomatoes for 8 cents, and lettuce for 5 cents?

9. A party of 50 children went on a picnic down the river; 18 of them went on the boat, and the rest on the cars. How many went on the cars?

10. I gave a two-dollar bill to pay for a 75-cent cap. What was my change?

MULTIPLICATION *Oral*

Multiply by 2:

3	7	5	0	9	2	4	6	8

Multiply by 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

Multiplication is the process of combining several *equal* numbers into one number.

The *multiplicand* is one of the equal numbers. This is the number to be multiplied.

The *multiplier* is the number by which we multiply. It shows how many times the multiplicand is to be taken.

The *product* is the result of multiplication.

Multiply by 4 the numbers in the table on page 227. Thus, 68 multiplied by 4: $68 = 60 + 8$. $4 \times 60 = 240$; $4 \times 8 = 32$; $240 + 32 = 272$.

Multiply by 2; 3; 5; 6; 7; 8; 9.

ORAL PROBLEMS

1. If a steamer makes a 2-mile trip 6 times every day, how many miles does it run in a week?

2. If 2 pears are sold for 5 cents, what will 20 cost?

3. What will Ella's vacation of 3 weeks cost her, if she pays 8 dollars a week for board and 4 dollars each week for laundry and other expenses?

4. What will 24 oranges and 12 lemons cost at 25 cents a dozen?

5. How many children in the march if there are 4 lines and 15 children in each line?

6. What must I pay for 5 melons at 6 cents apiece and 2 boxes of berries at 12 cents a box?

7. Mr. Hubbard brings vegetables to the city twice a week. He lives 7 miles away. How many miles does he travel each week?

8. What will $\frac{1}{2}$ dozen bananas and 4 apples cost at 3 cents apiece?

9. How much will 5 packages of cereal cost at 15 cents a package?

10. Grace sends 8 Christmas cards. If she pays 5 cents for each card, 1 cent for each envelope, and puts a 2-cent stamp on each envelope, how much does she pay for all?

DIVISION *Oral*

Division is the process of finding how many times one number is contained in another number, or of finding one of the equal parts of a number.

The *dividend* is the number to be divided.

The *divisor* is the number by which we divide.

The *quotient* is the result of division.

Divide by 2 the numbers in the table on page 227; divide by 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

ORAL PROBLEMS

1. I have 84 pounds of salt. How many 7-pound packages can I make from it?

2. How many feet long is a steel rod that measures 108 inches?

3. How many berries at 12 cents a box must Ralph sell to earn a football worth $1.20?

4. Mrs. Miller sold the grocer 2 dozen eggs at 30 cents a dozen and took her pay in sugar at 6 cents a pound. How many pounds did she receive?

5. John had 50 cents. He lost 8 cents, and spent the rest for firecrackers at 6 cents a bunch. How many bunches did he buy?

6. Mr. Fisher earns 2 dollars a day. How long will it take him to earn 72 dollars?

7. How many calls does a district nurse average a week if she makes 160 in 4 weeks?

8. Carrie pledged $1 to the children's aid society. How long will it take her to pay it, if she earns 15 cents every week and her mother gives her 5 cents every week?

9. Lucy spends 10 days of her vacation at the seashore, 14 days in the country, and 4 days at home. How many weeks is her vacation?

10. Eight girls have a sale of fancy articles. They pay $2 for advertising and $3 for other expenses. They take in $61. What is each girl's share of the profits?

UNITED STATES MONEY *Oral and Written*

1. Read: $4.00; $6.00; $2.40; $1.08; $0.27; $0.20; $0.05.

2. How many cents make one dollar? How many cents in $2.00? $3.00? $2.50? $1.67? $1.07?

3. How many dollars in 500 cents? 600 cents? 800 ¢? 1000 ¢?

4. Write as dollars and cents: 125 cents; 260 cents; 308 ¢; 203 ¢.

5. Write with the dollar sign: 25 cents; 60 cents; 4 ¢; 1 ¢.

Remember in addition and subtraction to place the decimal points one under another. Why?

6. Add: $8.04, $3.17, $2.80, $7.05, $9.62.

7. Add: $0.08, $0.56, $0.47, $0.40, $0.83, $0.05.

8. Add: $3.00. $3.30, $3.03, $0.30, $0.33, $0.03.

10. Grace
cents for each
2-cent stamp
for all?

Division is
number is co
of the equal
The *divide*
The *divisor*
The *quotie*

Divide by 2
5; 6; 7; 8; 9

1. I ha
packages c
2. How
108 inches
3. Hov
sell to earn
4. Mr
cents a d
pound.
5. Jo
rest for
bunches
6. M
it take h

	7.	8.	9.	10.
$0.74	$678	$0.87	68	96
.08	7	37.56	706	8453
.76	8	.17	9083	473,584
.09	803	.08	67,384	6708
.58	49	9.04	307	403
.29	28	.28	26,308	27
.90	7	57.01	49	8

nd the difference, and test your work:

1. $34.65 – $6.80
2. 7623 – 930
3. $12,500 – $6700
4. $58.34 – $20.70
5. 8542 – 3719
6. 32,706 – 10,834
7. $43.52 – $17.56
8. 3627 – 2864
9. 17,280 – 12,780
10. $27.90 – $18.25
11. 5625 – 4096
12. 35,060 – 12,087
13. $34.20 – $15.05
14. 8070 – 4308
15. 67,324 – 34,827
16. From 8000 take 8; 80; 800; 88; 880; 808; 888.

Minuend	Subtrahend	Remainder		Minuend	Subtrahend	Remainder
?	$6.95	$1.38	18.	722	266	?
$8.00	$3.69	?	20.	?	392	827
$5.23	?	$3.65	22.	648	?	209
?	$5.26	$0.79	24.	900	253	?
$4.60	$1.87	?	26.	?	539	278
$9.05	?	$3.88	28.	753	?	167

NOTE. There should be frequent dictation of numbers to be led and subtracted.

PROBLEMS *Written*

1. One lot of cloth contained 850 yards, another 1285 yards, and a third 1460 yards. How many yards in all?

2. An iceman cut 2250 tons of ice. How much had he left after selling 1780 tons?

3. A farmer raised 375 bushels of corn in one year, and in the next year 250 bushels more than in the first year. How many bushels did he raise in both years?

4. Mr. Morse bought a house for $2800, and another for $3650. He sold both for $7290. How much did he gain?

5. Mr. Cook paid $1096 for a house lot and on it built a house for $3265. He sold both at a gain of $475. How much did he receive?

6. Mr. Wright's bank account showed a deposit of $1296 on Monday morning. On Monday he deposited $582 and withdrew by check $653; on Tuesday he deposited $498 and withdrew $379; on Wednesday he deposited $889 and withdrew $1498; on Thursday he deposited $756. What were his total deposits? How much had he to his credit on Friday morning?

7. A butcher's charges against a family for one week were $1.37, $0.69, $2.08, $0.87, $1.75, and $0.98. What change ought he to give back if he is given a ten-dollar bill in payment?

8. Find the cost of a desk for $27.50, a chair for $9.75, a table for $12, a bookcase for $18.50, and a set of reference books for $67.80.

9. James bought a geography for $1.15, an arithmetic for $0.65, a grammar for $0.48, a block of paper for $0.08, and a pencil for $0.03. How much less than $3.00 did he pay for all?

DRILL IN MULTIPLICATION *Written*

$3 \times 4 \times 5 = ?$ $4 \times 5 \times 3 = ?$ $5 \times 3 \times 4 = ?$

The order in which numbers are multiplied together does not affect the product.

Multiply, selecting your multipliers so as to make your work as easy as possible:

1. $18 \times 50 \times 2$	2. $20 \times 24 \times 5$	3. $25 \times 45 \times 4$
4. $10 \times 36 \times 50$	5. $75 \times 26 \times 2$	6. $15 \times 19 \times 8$
7. $35 \times 15 \times 4$	8. $60 \times 57 \times 20$	9. $16 \times 32 \times 5$
10. 308×64	11. 876×75	12. 963×56
13. 3729×78	14. 5087×46	15. 7567×75
16. 436×208	17. 804×279	18. 225×306
19. 506×3468	20. 2387×207	21. 5682×256
22. 5468×357	23. 864×7678	24. 546×6807
25. 624×9034	26. 504×6327	27. 4657×406

28. At $16.75 each, what will 5 gas stoves cost?

29. What must be paid for 14 hammocks at $2.98 each?

30. A crate of berries contains 32 quart baskets. How many quarts in 5 lots of 12 crates each?

31. Mr. Howe bought 3 32-quart crates of strawberries at 12 cents a quart and sold them at 15 cents a quart. How much did he make?

32. Mr. Parker raised 17 bushels of pears. He sold 8 bushels at $1.05 a bushel, and the rest at $0.85 a bushel. How much did he receive for them?

33. After buying 6 head of cattle at $65 each, Mr. Turner had $27 left. How much money had he at first?

DRILL IN DIVISION *Written*

Divide, and test your work:

	A	*B*	*C*
1.	$801 ÷ 27	2765 ÷ 44	247,583 ÷ 64
2.	$765 ÷ 34	8327 ÷ 65	627,862 ÷ 75
3.	$896 ÷ 56	6754 ÷ 36	837,921 ÷ 29
4.	$27.95 ÷ 65	46,810 ÷ 84	247,583 ÷ 304
5.	$52.48 ÷ 82	67,632 ÷ 95	507,381 ÷ 409
6.	$34.08 ÷ 76	26,981 ÷ 43	729,843 ÷ 652
7.	$64.86 ÷ 138	48,366 ÷ 54	720,480 ÷ 432
8.	$133.92 ÷ 124	51,302 ÷ 208	837,641 ÷ 751
9.	$528.75 ÷ 225	64,730 ÷ 352	808,732 ÷ 364
10.	$739.44 ÷ 316	90,387 ÷ 525	976,068 ÷ 575

11. At 9 cents a yard Ella paid 45 cents for cloth. How many yards did she buy? (As many yards as 9 is contained times in 45.)

12. Esther paid 72 cents for 6 boxes of raisins. What was that a box? (One box cost ⅙ of 72 cents.)

13. At $6 a cord how many cords of wood can be bought for $912?

14. The grocer paid $702 for 54 barrels of sugar. What was the price per barrel?

15. A lot of land cost $6244. It was divided into 28 lots. What was each lot worth?

16. A stable keeper bought horses at $137 each. He paid $1096. How many did he buy?

17. Three lawn mowers were sold for $19.35. What was that apiece?

COMPARISON OF NUMBERS *Oral*

1. Compare 18 with 6. 18 is 3 times 6.

2. Compare 6 with 18. 6 is $\frac{1}{3}$ of 18.

Compare:

3. 10 with 2 4 with 20 30 with 6 5 with 40 27 with 9

4. 24 with 6 8 with 24 48 with 12 3 with 21 28 with 4

5. 56 with 7 9 with 36 54 with 9 7 with 63 32 with 8

NOTE. This exercise may be extended by comparing the second number in each couplet with the first.

6. A newsboy buys 5 papers for 3 cents. How many does he get for 15 cents?

HINT. Compare 15 cents with 3 cents.

7. Six boxes of raisins cost 75 cents. What will 2 boxes cost?

HINT. Compare 2 boxes with 6 boxes.

8. The grocer sells 4 pounds of sugar for a quarter. How many pounds does he sell for a dollar? For a dollar and a half?

9. Eight bars of soap weigh 36 pounds. What do 2 bars weigh?

10. Chester pays 25 cents for 8 oranges. Two dozen will cost ——.

11. Harriet buys 12 papers of needles for 20 cents. This is how many papers for 5 cents?

12. Mr. Perry pays $49.50 for 15 sheep. What will 5 more cost at the same rate?

13. For 2 cords of wood I paid $13.50. What will 10 cords cost?

DICTATION EXERCISES

1. $24 \div 3, \times 9, \div 12, \times 9, - 5, \div 7, + 20, \div 3 = ?$
2. $17 + 8, \times 2, + 4, \div 6, + 3, \times 7, + 6, \div 10, \times 5 = ?$
3. $56 \div 8, \times 4, + 2, \div 5, \times 7, + 3, \div 5, - 7, \times 5 = ?$
4. $42 \div 7, \times 9, + 6, \div 5, + 8, \div 2, - 7, \times 9, + 7 = ?$
5. $32 \div 4, + 7, \div 5, \times 8, + 4, \div 7, \times 16, - 4, \div 6 = ?$

NOTE. For securing concentration of attention, this form of oral drill is unexcelled if used daily for a few moments. Numbers must be dictated rapidly to make exercise effective.

MISCELLANEOUS PROBLEMS *Written*

1. The following represents the cash receipts of a coal firm for one week:

Kind	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Totals
Furnace	$420.87	$473.19	$296.89	$318.54	$387.53	$464.59	$
Stove	384.60	297.64	372.23	376.53	455.90	278.83	$
No. 1 Nut	297.83	308.07	424.86	565.49	387.37	588.10	$
No. 2 Nut	378.69	420.00	375.50	482.96	300.87	249.50	$
Soft	684.17	367.29	294.73	783.59	462.82	539.42	$
Totals	$	$	$	$	$	$	$

(*a*) Find the amount received each day.

(*b*) Find the amount received for the week.

(*c*) Find the total receipts for each kind of coal for the week.

(*d*) Find the total receipts for all kinds during the week.

2. If 15 books cost $12.75, what is the cost of 1?

3. At $12.75 each, what will 15 plows cost?

4. At $0.75 a bushel, what is the value of the corn raised on 26 acres, if each acre produces 37 bushels?

5. A bushel of corn in the ear weighs 70 pounds. How many bushels are there in a car of 15,750 pounds?

6. How many times at $2 a time must the blacksmith shoe the farmer's horse to pay for 5 bushels of potatoes at $0.50 a bushel and 2 barrels of apples at $1.75 a barrel?

7. Lime absorbs $2\frac{1}{4}$ times its weight in water. How many pounds of water will be required to slake 6 casks of lime of 240 pounds each?

8. What will 30 acres of land cost at the rate of 6 acres for $336?

9. Mr. Clark buys of Mr. Hodge 3 acres of land at $84 an acre. Mr. Hodge buys of Mr. Clark 18 tons of hay at $16 a ton. In order to settle the account how much money must be paid, and who must pay it?

10. A farmer had $440. With $192 he bought 24 sheep. With the rest he bought 4 cows. What did each sheep cost? Each cow?

11. A coal dealer paid $900 for coal at $ 5 a ton. He sold it at $6.50 a ton. How much did he gain?

12. A fruit dealer bought 36 baskets of peaches for $30.60. He sold 27 baskets at $0.95 each and the rest at $1.15 each. How much did he gain?

13. By selling 42 acres of timber land for $2148 a man gained $804. What did the land cost him an acre?

14. The railway fare to a place 18 miles away is 54 cents. How far away is a place the fare to which is 72 cents?

15. On a lot costing $896 there was built a house costing $4\frac{1}{2}$ times as much. What was the cost of the entire property?

16. What is the cost of 6 cases of straw hats, each case containing 12 dozen, and each hat costing 79 cents?

17. Mr. Adams bought an automobile for $975, paying $450 in cash, and agreeing to pay the rest at $75 a month. How long did it take him to pay for it?

18. A 36-pound tub of butter was bought for $9.90 and retailed at 32 cents a pound. Did the dealer lose or gain? How much?

19. A cask of 84 gallons of molasses cost $37.80. Seven gallons leaked out and the rest was sold at 48 cents a gallon. Did the grocer gain or lose? How much?

20. The pupils of the Adams school spent $10.65 for their school garden. They bought 9 dozen bulbs at 35 cents a dozen and 15 shrubs. How much did each shrub cost?

FACTORS *Oral*

When two or more numbers are multiplied together, the result is a *product.*

The numbers multiplied together are the *factors* of the product. Thus, 3 and 5 are the factors of their product, 15. 2, 3, and 5 are the factors of 30.

Any product is exactly divisible by any of its factors.

Find the missing factors:

1. —— × 9 = 54 9 × —— = 63 —— × 6 = 72
6 × —— = 42

2. —— × 5 = 30 7 × —— = 56 —— × 7 = 35
4 × —— = 32

3. —— × 7 = 63 3 × —— = 36 —— × 9 = 72
8 × —— = 96

4. —— × 12 = 84 6 × —— = 54 —— × 12 = 144
12 × —— = 132

The process of separating a number into its factors is *factoring.*

Separate into two factors:

5. 14 22 33 45 81 42 70 63 66 35

6. 56 64 21 32 72 84 54 96 55 108

7. Separate 24 into as many groups of two factors as you can. Thus, 2 × 12, 3 × 8, 4 × 6.

Name all the groups of two factors that make:

8. 16 28 20 40 50 80 72 90 84 42

9. 18 30 48 60 32 96 36 64 90 100

Separate each of these numbers into three factors:

10.	12	18	27	30	28	50	63	45	70	100
11.	32	40	66	48	20	72	54	60	56	144

Name the two equal factors of:

12.	4	9	25	49	81	64	144
13.	100	900	2500	4900	8100	3600	400

NOTE. Every number, of course, may be said to be made up of two factors consisting of itself and 1, but in giving the factors of a number the number itself and 1 are not generally included.

Numbers that can be separated into factors are *composite* numbers.

14. Name the composite numbers below 20.

Numbers that cannot be separated into factors are *prime* numbers.

15. Name the prime numbers below 20.

A *prime factor* is a prime number used as a factor.

16. What are the prime factors of 72?

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Dividing 72 by the prime number 2, we get 36; dividing 36 by 2, we get 18; dividing 18 by 2, we get 9; dividing 9 by the prime number 3, we get 3. All these divisors and the last quotient are prime numbers, and their product is 72. $2 \times 2 \times 2 \times 3 \times 3 = 72$. Therefore, the prime factors of 72 are 2, 2, 2, 3, and 3.

NOTE. The above example is inserted for illustration. The method given may be used if necessary, but pupils should be taught to find prime factors by inspection whenever possible. Thus, 72 may be thought of as 8×9; then 8 may be thought of as $2 \times 2 \times 2$, and 9 as 3×3.

Name the prime factors of:

17. 18 20 24 30 32 36 45 48 60 56

18. 84 50 66 80 90 64 81 63 54 100

GREATEST COMMON DIVISOR *Oral*

A number that will exactly divide a given number is an *exact divisor.*

1. Name a number that will exactly divide both 6 and 9; 8 and 12; 10 and 15; 12 and 18.

A number that will exactly divide two or more numbers is a *common divisor.*

2. Name the greatest number that will exactly divide 12 and 16; 18 and 24; 24 and 32; 30 and 40.

The greatest number that will exactly divide two or more numbers is their *greatest common divisor* (*g. c. d.*).

The greatest common divisor of two or more numbers is often called their greatest common factor.

Name the greatest common divisor of:

3. 16, 20	4. 22, 33	5. 18, 27	6. 27, 36
7. 14, 35	8. 32, 40	9. 11, 15	10. 36, 48
11. 35, 42	12. 20, 35	13. 28, 42	14. 63, 72
15. 56, 63	16. 45, 54	17. 28, 49	18. 24, 32
19. 6, 9, 12	20. 8, 12, 20	21. 12, 15, 18	22. 10, 15, 25
23. 15, 18, 30	24. 18, 24, 30	25. 12, 15, 21	26. 21, 28, 35
27. 18, 27, 45	28. 22, 33, 55	29. 24, 32, 40	30. 24, 36, 60

LEAST COMMON MULTIPLE *Oral*

When two or more whole numbers are multiplied together, their product is a *multiple* of each of the numbers. Thus, 15 is a multiple of both 3 and 5.

Any multiple of a number is exactly divisible by the number.

1. Name all the factors whose product is 12. Thus,

$$2 \times 6, \quad 3 \times 4, \quad 2 \times 2 \times 3.$$

12 is a *common multiple* of 2, 3, 4, and 6, and is exactly divisible by each of them.

24, 36, 48, 60 are also common multiples of 2, 3, 4, and 6.

As 12 is the *least multiple* that contains 2, 3, 4, and 6, it is their *least common multiple* (*l. c. m*).

The least common multiple of two or more numbers is the least number that is exactly divisible by each of the numbers.

What is the least common multiple of 5 and 6? Of 4 and 6? Of 3 and 9?

Find the least common multiple of:

2. 4 and 8	3. 7 and 8	4. 6 and 8	5. 6 and 9
6. 8 and 9	7. 8 and 12	8. 4 and 10	9. 6 and 10
10. 9 and 12	11. 6 and 15	12. 5 and 15	13. 8 and 24
14. 2, 4, 8	15. 4, 8, 16	16. 2, 3, 4	17. 3, 4, 6
18. 2, 4, 5	19. 3, 6, 9	20. 4, 5, 10	21. 2, 5, 20
22. 3, 6, 5	23. 4, 9, 36	24. 4, 5, 6	25. 3, 4, 9
26. 4, 6, 8	27. 6, 9, 12	28. 4, 8, 12	29. 3, 4, 5

CANCELLATION *Oral and Written*

$60 \div 20 = 6 \times 10$ divided by 2×10.

What common factor is found in both dividend and divisor?

By taking out the common factor 10 from both dividend and divisor, is the quotient changed?

What is the quotient of $60 \div 20$? Of $6 \div 2$?

Dividing both dividend and divisor by the same number does not affect the quotient.

Tell what common factors may be taken out of, or canceled from, both dividend and divisor:

1. 12×3 divided by 5×3
2. 10×3 divided by 10×2
3. 10×8 divided by 3×8
4. 21×7 divided by 4×7
5. 11×5 divided by 11×3
6. 11×12 divided by 12×3

7. In the expression 12×10 divided by 8×3 what common factors will divide both dividend and divisor? What in 14×10 divided by 5×7?

The process of dividing both dividend and divisor by the same number, or of striking out factors common to both dividend and divisor, is *cancellation.*

8. Divide 16×35 by 4×7.

$$\frac{\overset{4}{\cancel{16}} \times \overset{5}{\cancel{35}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{7}}} = \frac{20}{1} = 20$$

Write the dividend above a line and the divisor below it. Divide the 16 in the dividend and the 4 in the divisor by the common factor 4, writing the quotient 4 over the 16, and the quotient 1 under the 4. In like manner divide both dividend and divisor by the common factor 7.

The factors remaining in the dividend are 4 and 5, and their product is 20. The factors remaining in the divisor are 1 and 1, and their product is 1. $\frac{20}{1} = 20$.

In practice we do not write the 1's. We always remember, however, that when a factor is canceled 1 is understood to take its place.

9. Divide 56×18 by 8×9.

10. What is the quotient of 42×10 divided by 7×5?

11. How many times is 4×3 contained in 6×8?

Find quotients:

12. $\frac{4 \times 12}{2 \times 6}$ 13. $\frac{6 \times 25}{3 \times 5}$ 14. $\frac{20 \times 30}{15 \times 10}$ 15. $\frac{18 \times 30}{6 \times 5}$

16. $\frac{27 \times 18}{9 \times 3}$ 17. $\frac{28 \times 35}{4 \times 7}$ 18. $\frac{15 \times 50}{5 \times 5 \times 5}$ 19. $\frac{60 \times 30}{5 \times 12}$

20. $(22 \times 18) \div (11 \times 6)$ 21. $(35 \times 42) \div (14 \times 7)$

22. $(35 \times 42) \div (49 \times 6)$ 23. $(63 \times 72) \div (24 \times 21)$

24. $(33 \times 48) \div (12 \times 22)$ 25. $(54 \times 54) \div (6 \times 18)$

26. $(60 \times 27) \div (18 \times 45)$ 27. $(35 \times 84) \div (49 \times 30)$

Divide:

28. $\frac{6 \times 10 \times 15}{25 \times 2 \times 2}$ 29. $\frac{12 \times 15 \times 24}{20 \times 4 \times 18}$ 30. $\frac{9 \times 8 \times 10}{30 \times 2 \times 3}$

31. $\frac{18 \times 30 \times 22}{33 \times 10 \times 9}$ 32. $\frac{50 \times 42 \times 20}{35 \times 25 \times 12}$ 33. $\frac{36 \times 45 \times 27}{18 \times 15 \times 9}$

34. $\frac{11 \times 30 \times 28}{15 \times 22 \times 7}$ 35. $\frac{60 \times 42 \times 54}{9 \times 20 \times 6}$ 36. $\frac{44 \times 56 \times 96}{48 \times 77 \times 16}$

FRACTIONS

A unit is a single thing; as 1 apple.

A fraction is one or more of the equal parts of a unit; as $\frac{3}{4}$ of an apple.

$\frac{3}{4}$ of an apple means that an apple has been divided into 4 equal parts and 3 of these parts taken.

1. In the expression $\frac{2}{3}$ of a yard, what figure shows the number of equal parts into which the unit is divided?

The figure below the line is the *denominator;* it denominates or names the number of parts into which the unit is divided; it is the *namer.*

2. In the expression $\frac{2}{3}$ of a yard, what figure shows the number of parts taken?

The figure above the line is the *numerator;* it numerates or tells the number of parts taken; it is the *numberer.*

The numerator and the denominator are the *terms* of the fraction.

3. Read these fractions and tell what the terms of each fraction show: $\frac{5}{8}$; $\frac{6}{7}$; $\frac{4}{5}$; $\frac{9}{10}$; $\frac{17}{20}$.

4. Write in figures and tell what each fraction means: five sixths; eight ninths; eleven twelfths; thirteen twenty-firsts.

5. Write an expression which will show that something has been divided into nine equal parts and four of those parts taken.

6. Explain $\frac{1}{5}$ of a mile; $\$\frac{1}{5}$; $\frac{3}{5}$ bu.; $\frac{3}{4}$ gal.

A unit may also be regarded as a group of things treated as a single thing. Thus, $\frac{3}{4}$ of a dozen oranges means that 12 oranges have been separated into 4 equal groups of 3 oranges each, and that 3 of these groups, or 9 oranges, have been taken.

In studying fractions remember:

First. The only difference between an integer, or whole number, and a fraction is that an integer is a whole thing, while a fraction is part of the whole thing.

Second. The denominator of a fraction simply tells with what kind of things we are dealing; that is, it simply gives a name to the fraction.

Third. The numerator simply tells the number of parts taken.

Fourth. A fraction must always be treated as if it were a whole number.

A *proper fraction* is a fraction whose numerator is less than its denominator; as $\frac{7}{8}$, $\frac{5}{7}$, $\frac{11}{12}$.

An *improper fraction* is a fraction whose numerator is equal to or greater than its denominator; as $\frac{5}{5}$, $\frac{8}{8}$, $\frac{16}{5}$, $\frac{7}{3}$.

7. Write a proper fraction whose denominator is 5; 8; 12; 10; 3.

8. Write a proper fraction whose numerator is 3; 7; 9; 4; 10.

9. Write an improper fraction whose numerator is 7; 6; 4; 3; 5.

10. Write an improper fraction whose denominator is 3; 6; 8; 9; 10.

A *mixed number* is a whole number and a fraction united; as $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{5}{9}$.

CHANGING THE FORM OF FRACTIONS

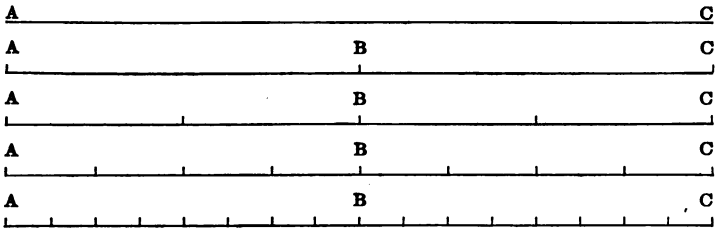
1. Cut from paper a strip 1 inch wide and 12 inches long. Place the ends together and fold into two equal parts. Show that $1 = \frac{2}{2}$.

2. Fold again and crease into four equal parts. Show that $1 = \frac{4}{4}$. Show that $\frac{1}{2} = \frac{2}{4}$.

3. Fold and crease into eight equal parts. 1 = how many eighths? $\frac{1}{2}$ = how many eighths? $\frac{1}{4}$ = how many eighths?

4. Fold another strip into two equal parts. Fold this double strip into three equal parts. 1 = how many sixths? 1 = how many thirds? $\frac{1}{2}$ = how many sixths? $\frac{1}{3}$ = how many sixths? $\frac{2}{3}$ = how many sixths?

TO THE TEACHER: Simple fractions and their equivalents may be shown in this or some other simple manner. The extent to which such work is carried must be determined by the needs of individual pupils. While objective presentation should be used freely, care should be taken not to make pupils dependent on its use. That which is at first a help may easily become a hindrance to progress.



If the line AC be divided into two equal parts, AB is $\frac{1}{2}$ of AC; if divided into four equal parts, AB is $\frac{2}{4}$; if divided into eight equal parts, AB is $\frac{4}{8}$; if divided into sixteen equal parts, AB is $\frac{8}{16}$; that is, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ of the line AC are of equal value, and represent the same thing — the line AB.

Notice, that in changing $\frac{1}{2}$ to $\frac{2}{4}$ we have twice as many parts in the line AC, and also twice as many parts in the line AB. In changing $\frac{1}{2}$ to $\frac{4}{8}$, we have four times as many parts in the line AC, and also four times as many parts in the line AB. In changing $\frac{1}{2}$ to $\frac{8}{16}$, we have eight times

as many parts in the line AC, and eight times as many parts in the line AB.

$$\frac{1\times 2}{2\times 2}=\frac{2}{4} \qquad \frac{1\times 4}{2\times 4}=\frac{4}{8} \qquad \frac{1\times 8}{2\times 8}=\frac{8}{16}$$

In changing $\frac{8}{16}$ to $\frac{4}{8}$, we have one half as many parts in in the line AC, and one half as many parts in the line AB. In changing $\frac{8}{16}$ to $\frac{2}{4}$, we have one fourth as many parts in the line AC, and one fourth as many parts in the line AB. In changing $\frac{8}{16}$ to $\frac{1}{2}$, we have one eighth as many parts in the line AC, and one eighth as many parts in the line AB.

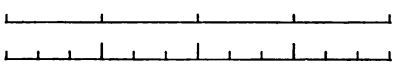
$$\frac{8\div 2}{16\div 2}=\frac{4}{8} \qquad \frac{8\div 4}{16\div 4}=\frac{2}{4} \qquad \frac{8\div 8}{16\div 8}=\frac{1}{2}$$

From this exercise we learn that

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

CHANGING TO HIGHER TERMS *Oral*

1. Change $\frac{3}{4}$ to twelfths.



$$\frac{3\times 3}{4\times 3}=\frac{9}{12}$$

The fraction $\frac{3}{4}$ shows that the unit has been separated into 4 equal parts and 3 of those parts taken. If we separate the unit into twelfths, or three times as many parts, we have three times as many parts for the numerator. $\frac{1}{4}=\frac{3}{12}$; $\frac{3}{4}=\frac{9}{12}$.

To change a fraction to higher terms, we multiply both terms of the fraction by that number which will give the required denominator.

2. Why must we multiply both terms of the fraction by the same number?

Change:

3. To fourths: $\frac{1}{2}$

4. To sixths: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

5. To eighths: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$

6. To ninths: $\frac{1}{3}$ $\frac{2}{3}$

7. To tenths: $\frac{1}{2}$ $\frac{1}{5}$ $\frac{2}{5}$

8. To twelfths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{5}{6}$

9. To fifteenths: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

10. To sixteenths: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{5}{8}$

11. To eighteenths: $\frac{1}{2}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{1}{9}$ $\frac{5}{9}$

12. To twentieths: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{7}{10}$

CHANGING TO LOWER TERMS *Oral and Written*

1. Change $\frac{6}{9}$ to thirds.

$\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

This means change $\frac{6}{9}$ to a fraction with 3 for its denominator.

In order to get 3 for a denominator, we divide 9 by 3. In order not to change the value of the fraction, we must also divide the numerator by 3.

NOTE. If necessary, let pupils show by folding paper or by diagram that $\frac{6}{9} = \frac{2}{3}$.

2. Change to 4ths: $\frac{2}{8}$ $\frac{3}{12}$ $\frac{9}{12}$ $\frac{4}{16}$ $\frac{12}{16}$ $\frac{5}{20}$

3. Change to 5ths: $\frac{2}{10}$ $\frac{4}{10}$ $\frac{8}{10}$ $\frac{4}{20}$ $\frac{12}{20}$ $\frac{6}{30}$

4. Change to 6ths: $\frac{2}{12}$ $\frac{10}{12}$ $\frac{3}{18}$ $\frac{15}{18}$ $\frac{4}{24}$ $\frac{5}{30}$

5. Change to 9ths: $\frac{2}{18}$ $\frac{4}{18}$ $\frac{8}{18}$ $\frac{10}{18}$ $\frac{3}{27}$ $\frac{4}{36}$

6. Change to 12ths: $\frac{2}{24}$ $\frac{10}{24}$ $\frac{14}{24}$ $\frac{8}{36}$ $\frac{4}{48}$ $\frac{5}{60}$

7. Change $\frac{15}{20}$ to its simplest form.

$\frac{15 \div 5}{20 \div 5} = \frac{3}{4}$

Since the factor 5 is common to both numerator and denominator, we can divide both terms by 5 without changing the value of the fraction. As the numerator and denominator of the fraction $\frac{3}{4}$ have no common factor, the fraction $\frac{15}{20}$ has been changed to its simplest form, or, as we say, to its lowest terms.

A fraction is in its lowest terms when its numerator and denominator have no common factor.

Change to lowest terms:

8. $\frac{6}{9}$ $\frac{8}{10}$ $\frac{10}{16}$ $\frac{6}{14}$ $\frac{12}{18}$ $\frac{15}{21}$ $\frac{14}{24}$ $\frac{20}{20}$

9. $\frac{8}{12}$ $\frac{6}{15}$ $\frac{6}{6}$ $\frac{12}{16}$ $\frac{8}{18}$ $\frac{6}{20}$ $\frac{14}{21}$ $\frac{6}{24}$

10. $\frac{6}{12}$ $\frac{8}{14}$ $\frac{9}{12}$ $\frac{12}{15}$ $\frac{8}{20}$ $\frac{6}{21}$ $\frac{2}{28}$ $\frac{15}{24}$

11. $\frac{5}{15}$ $\frac{12}{21}$ $\frac{12}{20}$ $\frac{18}{21}$ $\frac{9}{24}$ $\frac{16}{16}$ $\frac{14}{22}$ $\frac{18}{27}$

12. $\frac{10}{18}$ $\frac{9}{15}$ $\frac{10}{12}$ $\frac{16}{20}$ $\frac{14}{16}$ $\frac{9}{21}$ $\frac{10}{24}$ $\frac{19}{19}$

13. $\frac{30}{42}$

$\frac{30 \div 2}{42 \div 2} = \frac{15 \div 3}{21 \div 3} = \frac{5}{7}$

Or

$\frac{30 \div 6}{42 \div 6} = \frac{5}{7}$

Dividing both terms of $\frac{30}{42}$ by 2, we get $\frac{15}{21}$; dividing both terms of $\frac{15}{21}$ by 3, we get $\frac{5}{7}$.

We can change this fraction more quickly by dividing both terms by their greatest common factor, 6.

14. $\frac{18}{42}$ **15.** $\frac{49}{63}$ **16.** $\frac{48}{78}$ **17.** $\frac{48}{84}$ **18.** $\frac{75}{135}$

19. $\frac{30}{48}$ **20.** $\frac{42}{70}$ **21.** $\frac{60}{72}$ **22.** $\frac{84}{144}$ **23.** $\frac{60}{150}$

24. $\frac{24}{54}$ **25.** $\frac{56}{63}$ **26.** $\frac{35}{80}$ **27.** $\frac{90}{120}$ **28.** $\frac{96}{108}$

29. $\frac{42}{63}$ **30.** $\frac{42}{54}$ **31.** $\frac{63}{84}$ **32.** $\frac{48}{96}$ **33.** $\frac{105}{150}$

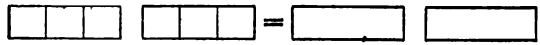
34. $\frac{30}{72}$ **35.** $\frac{72}{84}$ **36.** $\frac{70}{98}$ **37.** $\frac{90}{108}$ **38.** $\frac{125}{175}$

To change a fraction to its lowest terms, we cancel the factors common to both numerator and denominator; or we divide both terms by their greatest common factor.

NOTE. Do not now require pupils to give rules or technical explanations of process. The main thing at present is to see that pupils understand and apply the principles.

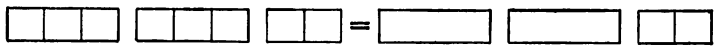
CHANGING IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS *Oral and Written*

1. Change $\frac{6}{3}$ to a whole number.



$\frac{6}{3} = 2$ Since there are 3 *thirds* ($\frac{1}{3}$) in 1 unit, in 6 *thirds* ($\frac{1}{3}$) there are as many units as there are 3's in 6; that is, 2 units.

2. Change $\frac{8}{3}$ to a mixed number.



$\frac{8}{3} = 2\frac{2}{3}$ Since there are 3 thirds in 1 unit, in 8 thirds there are as many units as there are 3's in 8; that is, 2 units and 2 thirds of a unit.

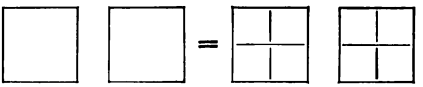
To change an improper fraction to a whole or mixed number, we divide the numerator by the denominator.

Change to whole or mixed numbers:

3. $\frac{24}{7}$	4. $\frac{36}{9}$	5. $\frac{15}{8}$	6. $\frac{42}{5}$	7. $\frac{56}{7}$
8. $\frac{27}{4}$	9. $\frac{28}{9}$	10. $\frac{24}{3}$	11. $\frac{48}{8}$	12. $\frac{20}{6}$
13. $\frac{43}{8}$	14. $\frac{36}{9}$	15. $\frac{40}{9}$	16. $\frac{25}{7}$	17. $\frac{30}{5}$
18. $\frac{15}{2}$ in.	19. $\frac{17}{4}$ ft.	20. $\frac{28}{4}$ yd.	21. $\frac{19}{2}$ ft.	22. $\frac{11}{2}$ qt.
23. $\frac{35}{4}$ gal	24. $\frac{35}{8}$ pk.	25. $\frac{44}{4}$ bu.	26. $\$\frac{5}{5}$	27. $\$\frac{20}{4}$

CHANGING WHOLE OR MIXED NUMBERS TO IMPROPER FRACTIONS *Oral and Written*

1. Change 2 to fourths.



$2 = \frac{2}{1}$

$\frac{2 \times 4}{1 \times 4} = \frac{8}{4}$

Since there are 4 *fourths* in one unit, in 2 units there are 2 times 4 *fourths*, or 8 *fourths*.

To change a whole number to an improper fraction, we multiply the whole number by the required denominator, and write the product over the required denominator.

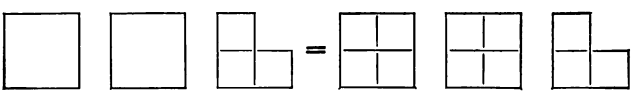
2. Change to halves: 1 2 3 4 5

3. Change to thirds: 1 2 4 6 9

4. How many fifths are there in 1? 3? 6? 7? 8?

5. Express as fractions with 8 for a denominator: 3 5 7 8 10

6. Change $2\frac{3}{4}$ to fourths.



$2\frac{3}{4} = \frac{11}{4}$

Two units equal 8 *fourths*; 8 *fourths* and 3 *fourths* are 11 *fourths*.

To change a mixed number to an improper fraction, we multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the sum over the denominator.

Write as improper fractions:

7. $4\frac{2}{5}$	8. $7\frac{3}{4}$	9. $5\frac{3}{8}$	10. $2\frac{5}{9}$
11. $4\frac{1}{6}$	12. $8\frac{2}{3}$	13. $3\frac{5}{7}$	14. $5\frac{2}{5}$
15. $2\frac{5}{6}$	16. $3\frac{1}{10}$	17. $3\frac{3}{9}$	18. $5\frac{7}{8}$
19. $4\frac{9}{10}$	20. $4\frac{2}{7}$	21. $6\frac{3}{5}$	22. $3\frac{1}{8}$
23. $4\frac{2}{3}$ ft.	24. $1\frac{1}{2}$ pt.	25. $3\frac{1}{2}$ qt.	26. $5\frac{3}{4}$ in.
27. $\$4\frac{2}{5}$	28. $7\frac{5}{8}$ pk.	29. $9\frac{3}{4}$ bu.	30. $5\frac{5}{8}$ mi.

REVIEW EXERCISE *Written*

1. Write a proper fraction using 5 and 3 for its terms.

2. Change the form of the fraction you have written without changing its value.

3. Change $\frac{2}{3}$ to ninths; $\frac{3}{4}$ to 12ths; $\frac{3}{8}$ to 16ths.

4. In the fraction $\frac{9}{12}$, what factor is common to both terms? To what simpler fraction is $\frac{9}{12}$ equal?

5. Take out the common factors in these fractions: $\frac{4}{20}$ $\frac{15}{18}$ $\frac{10}{16}$ $\frac{10}{24}$ $\frac{5}{30}$.

6. Take out all the common factors in $\frac{30}{48}$.

7. Change to lowest terms: $\frac{12}{16}$ $\frac{24}{30}$ $\frac{36}{60}$ $\frac{42}{63}$ $\frac{72}{84}$.

8. Write an improper fraction whose terms are 12 and 3. Change it to a whole number.

9. Write two improper fractions that can be changed to mixed numbers.

10. What is a mixed number?

11. Write five mixed numbers and change them to improper fractions.

12. Change 3 to halves; 6 to fifths; 5 to eighths; 4 to twelfths.

ADDITION OF FRACTIONS *Oral and Written*

1. Add $\frac{3}{7}$ and $\frac{2}{7}$.

$$\frac{3}{\text{apples}} + \frac{2}{\text{apples}} = \frac{5}{\text{apples}}$$

$$\frac{3}{\text{sevenths}} + \frac{2}{\text{sevenths}} = \frac{5}{\text{sevenths}}$$

$$\tfrac{3}{7} + \tfrac{2}{7} = \tfrac{5}{7}$$

The denominator names the fraction; it simply tells the kind of things with which we are dealing.

Add:

2. $\frac{1}{6} + \frac{1}{6} + \frac{3}{6}$ **3.** $\frac{1}{5} + \frac{1}{5} + \frac{2}{5}$ **4.** $\frac{3}{8} + \frac{2}{8} + \frac{1}{8}$

5. $\frac{2}{9} + \frac{3}{9} + \frac{2}{9}$ **6.** $\frac{1}{11} + \frac{4}{11} + \frac{5}{11}$ **7.** $\frac{5}{12} + \frac{1}{12} + \frac{1}{12}$

8. $\frac{3}{16} + \frac{1}{16} + \frac{5}{16}$ **9.** $\frac{3}{20} + \frac{7}{20} + \frac{9}{20}$ **10.** $\frac{4}{25} + \frac{3}{25} + \frac{11}{25}$

11. Add $\frac{5}{8}$ and $\frac{3}{4}$.

$\frac{5}{8} + \frac{3}{4} = ?$

$\frac{3}{4} = \frac{6}{8}$

$\frac{5}{8} + \frac{6}{8} = 1\frac{1}{8} = 1\frac{3}{8}$

$$\frac{5}{\text{quarts}} + \frac{3}{\text{pecks}} = ?$$

Since these quantities do not represent things of the same kind, they cannot be added. But, since 1 peck is equal to 8 quarts, 3 pecks may be expressed as 24 quarts. $\frac{5}{\text{quarts}} + \frac{24}{\text{quarts}} = \frac{29}{\text{quarts}}$.

Similarly, $\frac{5}{8} + \frac{3}{4}$. Since eighths and fourths represent unlike things, we cannot add them until we express them as like things; that is, as fractions having the same denominator, which we call a common denominator. The common denominator is 8. $\frac{3}{4} = \frac{6}{8}$.

12. Add $\frac{1}{4}$ and $\frac{1}{6}$.

$c.\,d. = 12$

$\frac{1}{4} = \frac{3}{12}$

$\frac{1}{6} = \frac{2}{12}$

$\frac{5}{12}$

We can express these fractions as 24ths, for 24 is a multiple of both 4 and 6. 12 is also a multiple of 4 and 6, and is the least multiple common to both. It simplifies our work to use the least common multiple of the denominators for the common denominator.

To add fractions, we express the fractions as equivalent fractions having a common denominator, and write the sum of the numerators over the common denominator.

NOTE. As much as possible of the work in fractions, both abstract and concrete, should be done orally.

Add, changing the fraction in the answer to its lowest terms:

13. $\frac{1}{2}+\frac{1}{6}$	14. $\frac{1}{2}+\frac{3}{8}$	15. $\frac{3}{8}+\frac{5}{6}$	16. $\frac{3}{4}+\frac{5}{8}$
17. $\frac{7}{10}+\frac{1}{2}$	18. $\frac{5}{12}+\frac{1}{2}$	19. $\frac{1}{2}+\frac{5}{16}$	20. $\frac{2}{3}+\frac{1}{12}$
21. $\frac{3}{4}+\frac{5}{12}$	22. $\frac{2}{15}+\frac{1}{3}$	23. $\frac{3}{5}+\frac{2}{15}$	24. $\frac{3}{4}+\frac{9}{16}$
25. $\frac{2}{5}+\frac{7}{10}$	26. $\frac{5}{6}+\frac{5}{12}$	27. $\frac{3}{8}+\frac{7}{16}$	28. $\frac{3}{4}+\frac{7}{12}$
29. $\frac{3}{10}+\frac{4}{5}$	30. $\frac{2}{5}+\frac{7}{15}$	31. $\frac{3}{4}+\frac{7}{16}$	32. $\frac{5}{6}+\frac{7}{12}$

Find sum of:

33. $\frac{1}{2}+\frac{1}{3}$	34. $\frac{1}{4}+\frac{1}{3}$	35. $\frac{1}{2}+\frac{3}{5}$	36. $\frac{2}{3}+\frac{2}{5}$
37. $\frac{3}{7}+\frac{1}{2}$	38. $\frac{1}{2}+\frac{2}{9}$	39. $\frac{2}{5}+\frac{1}{2}$	40. $\frac{2}{3}+\frac{3}{4}$
41. $\frac{1}{3}+\frac{4}{5}$	42. $\frac{1}{2}+\frac{5}{7}$	43. $\frac{2}{3}+\frac{4}{5}$	44. $\frac{2}{3}+\frac{1}{2}$
45. $\frac{4}{5}+\frac{1}{2}$	46. $\frac{2}{3}+\frac{3}{5}$	47. $\frac{4}{5}+\frac{1}{2}$	48. $\frac{5}{6}+\frac{1}{4}$
49. $\frac{3}{4}+\frac{5}{6}$	50. $\frac{1}{3}+\frac{3}{4}$	51. $\frac{1}{2}+\frac{1}{7}$	52. $\frac{1}{3}+\frac{1}{5}$

PROBLEMS

1. Mr. Smith has $\frac{1}{2}$ of an acre in one lot and $\frac{5}{8}$ of an acre in another lot. How many acres are there in both lots?

2. Miriam's spelling book cost $\frac{1}{5}$ of a dollar and her arithmetic $\frac{1}{2}$ of a dollar. What part of a dollar did both cost?

3. A cook used $\frac{7}{8}$ of a ton of coal in January and $\frac{3}{4}$ of a ton in February. How much did she use in both months?

4. Maggie bought $\frac{5}{6}$ of a yard of lace for an apron, and $\frac{2}{3}$ of a yard for a waist. How much lace did she buy?

5. A spelling lesson takes $\frac{1}{5}$ of an hour, and a reading lesson $\frac{1}{3}$ of an hour. What part of an hour is taken for both lessons?

To the Teacher: Many simple oral problems illustrating the principle under consideration should be given by the teacher. As far as possible, the problem material should be within the realm of the pupils' interest and experience. Local conditions will determine the character and content of problem work.

Pupils should be encouraged and required to make original problems based on their observation of the affairs of everyday life.

ADDITION *Written*

Find the sum of:

1. $\frac{1}{3} + \frac{3}{4} + \frac{1}{6}$

$c.\,d. = 12$

$\frac{1}{3} = \frac{4}{12}$

$\frac{3}{4} = \frac{9}{12}$

$\frac{1}{6} = \frac{2}{12}$

$\frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$

2. $\frac{5}{8} + \frac{1}{2} + \frac{3}{4}$

3. $\frac{1}{2} + \frac{1}{6} + \frac{3}{4}$

4. $\frac{3}{10} + \frac{2}{5} + \frac{1}{2}$

5. $\frac{2}{3} + \frac{5}{6} + \frac{5}{6}$

6. $\frac{5}{6} + \frac{3}{4} + \frac{7}{12}$

7. $\frac{1}{6} + \frac{1}{2} + \frac{1}{3}$

8. $\frac{2}{3} + \frac{1}{2} + \frac{5}{6}$

9. $\frac{7}{15} + \frac{2}{3} + \frac{3}{5}$

10. $\frac{3}{5} + \frac{3}{10} + \frac{1}{2}$

11. $\frac{1}{2} + \frac{4}{7} + \frac{3}{14}$

12. $\frac{1}{4} + \frac{2}{3} + \frac{1}{2}$

13. $\frac{1}{4} + \frac{5}{6} + \frac{2}{3}$

14. $\$\frac{1}{2} + \$\frac{3}{5} + \$\frac{1}{10}$

15. $\frac{3}{4}$ gal. + $\frac{5}{6}$ gal. + $\frac{1}{2}$ gal.

16. $\frac{1}{6}$ yr. + $\frac{2}{3}$ yr. + $\frac{7}{12}$ yr.

17. $\frac{1}{3}$ yd. + $\frac{3}{4}$ yd. + $\frac{5}{12}$ yd.

18. $\frac{1}{2}$ bu. + $\frac{3}{8}$ bu. + $\frac{1}{4}$ bu.

19. $\frac{2}{3}$ mi. + $\frac{3}{4}$ mi. + $\frac{1}{12}$ mi.

PROBLEMS

1. John spent $\frac{1}{6}$ of his money for candy, $\frac{1}{3}$ for a ball, and $\frac{1}{4}$ for a bat. What part of his money did he spend?

2. Mary earned $\frac{2}{5}$ of a dollar, $\frac{1}{2}$ of a dollar, and $\frac{3}{10}$ of a dollar. How much did she earn in all?

3. Mr. Wright has $\frac{1}{16}$ of an acre of corn, $\frac{1}{8}$ of an acre of potatoes, and $\frac{1}{4}$ of an acre of onions. What part of an acre is used for all?

4. A bag of flour cost $\frac{7}{10}$ of a dollar, a bushel of potatoes $\frac{1}{2}$ of a dollar, and a peck of apples $\frac{1}{5}$ of a dollar. How much did all cost?

5. Mrs. Whiting paid $\$\frac{1}{5}$ for oranges, $\$\frac{1}{5}$ for sugar, and $\$\frac{1}{2}$ for peaches. What part of a dollar did she pay for all?

ADDITION OF MIXED NUMBERS *Written*

1. Add $9\frac{2}{3}$ and $6\frac{3}{4}$

$c.d. = 12$

$9\frac{2}{3} = 9\frac{8}{12}$

$6\frac{3}{4} = 6\frac{9}{12}$

$16\frac{5}{12}$

Express both fractions as 12ths.
$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$. Write $\frac{5}{12}$ under the fractions and add 1 to the sum of the whole numbers.

Add:

2. $3\frac{3}{4}+4\frac{1}{2}$	3. $2\frac{1}{2}+7\frac{5}{6}$	4. $3\frac{5}{8}+4\frac{1}{2}$	5. $6\frac{11}{16}+7\frac{1}{2}$
6. $4\frac{2}{3}+2\frac{5}{6}$	7. $5\frac{1}{6}+8\frac{1}{2}$	8. $4\frac{5}{8}+9\frac{3}{4}$	9. $5\frac{9}{16}+3\frac{7}{8}$
10. $7\frac{1}{2}+3\frac{7}{10}$	11. $5\frac{3}{10}+3\frac{4}{5}$	12. $7\frac{11}{16}+2\frac{3}{4}$	13. $4\frac{2}{3}+3\frac{5}{9}$
14. $7\frac{5}{6}+8\frac{7}{12}$	15. $3\frac{3}{4}+5\frac{7}{12}$	16. $8\frac{5}{12}+6\frac{2}{3}$	17. $5\frac{1}{2}+8\frac{1}{12}$
18. $8\frac{2}{3}+8\frac{1}{2}$	19. $5\frac{1}{2}+2\frac{1}{5}$	20. $9\frac{3}{4}+5\frac{2}{3}$	21. $7\frac{2}{5}+6\frac{2}{3}$
22. $3\frac{3}{4}+7\frac{5}{6}$	23. $2\frac{2}{5}+1\frac{1}{2}$	24. $3\frac{2}{3}+7\frac{3}{4}$	25. $8\frac{2}{5}+8\frac{3}{10}$
26. $9\frac{7}{12}+6\frac{3}{4}$	27. $2\frac{1}{2}+2\frac{2}{5}$	28. $3\frac{7}{9}+4\frac{2}{3}$	29. $2\frac{4}{9}+3\frac{2}{3}$

NOTE. Special attention should be paid to the manner of arranging work on paper, as well as to accuracy and neatness. A slovenly paper is usually indicative of a careless and inaccurate mind.

Find the sum of:

30. $2\frac{1}{2} + 2\frac{1}{3} + 3\frac{3}{4}$

31. $4\frac{1}{5} + 2\frac{1}{2} + 3\frac{3}{10}$

32. $1\frac{2}{3} + 3\frac{1}{2} + 2\frac{1}{4}$

33. $5\frac{3}{4} + 5\frac{1}{2} + 4\frac{2}{3}$

34. $4\frac{1}{3} + 2\frac{1}{6} + 1\frac{3}{4}$

35. $2\frac{1}{4} + 3\frac{1}{3} + 4\frac{5}{12}$

36. $1\frac{1}{2} + 4\frac{2}{3} + 2\frac{5}{12}$

37. $2\frac{1}{10} + 3\frac{1}{5} + 4\frac{1}{2}$

38. $4\frac{1}{6} + 2\frac{2}{3} + 1\frac{1}{2}$

39. $2\frac{3}{4} + 5\frac{1}{2} + 6\frac{7}{16}$

PROBLEMS

1. A railroad train ran the first mile in 2 minutes, the second mile in $1\frac{7}{8}$ minutes, and the third mile in $1\frac{3}{4}$ minutes. How long did it take to run the three miles?

2. Susie is $8\frac{1}{2}$ years old, Ella is $10\frac{1}{4}$ years old, and Annie is $9\frac{1}{8}$ years old. What is the sum of their ages?

3. A clerk sold $1\frac{1}{8}$ yards of cloth, $2\frac{3}{4}$ yards, and $4\frac{1}{2}$ yards. How many yards did he sell?

4. A farmer sold a calf for $\$7\frac{1}{2}$, a pig for $\$5\frac{1}{2}$, and a sheep for $\$7\frac{3}{4}$. How much money did he receive?

5. It takes $5\frac{1}{2}$ yards of braid for Mary's skirt, $3\frac{1}{8}$ yards for her waist, and $4\frac{1}{8}$ yards for her jacket. How many yards does it take for the suit?

NOTE. Care should be taken not to proceed too rapidly in the study of fractions. It takes a long time and much patient labor to lay a secure foundation. A new process should not be taken up until pupils show by their mastery of the present work that they are prepared for advanced work.

SUBTRACTION OF FRACTIONS *Oral and Written*

1. $\frac{5}{\text{apples}} - \frac{2}{\text{apples}} = ?$ **2.** $\frac{5}{\text{sevenths}} - \frac{2}{\text{sevenths}} = ?$

3. $\frac{5}{7} - \frac{2}{7} = ?$ **4.** $\frac{3}{4} - \frac{1}{4} = ?$

5. $\frac{7}{8} - \frac{3}{8} = ?$ **6.** $\frac{8}{9} - \frac{2}{9} = ?$

7. Subtract $\frac{3}{4}$ from $\frac{7}{8}$.

c.d. = 8

$\frac{7}{8} = \frac{7}{8}$

$\frac{3}{4} = \frac{6}{8}$

$\frac{1}{8}$

Only like quantities can be subtracted. Change $\frac{3}{4}$ to 8ths. $\frac{3}{4} = \frac{6}{8}$. $\frac{7}{8} - \frac{6}{8} = \frac{1}{8}$.

To subtract one fraction from another, we express the fractions as equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.

Find the difference:

8. $\frac{1}{2} - \frac{1}{6}$	**9.** $\frac{1}{2} - \frac{1}{10}$	**10.** $\frac{1}{2} - \frac{1}{12}$	**11.** $\frac{2}{3} - \frac{1}{6}$
12. $\frac{2}{3} - \frac{4}{9}$	**13.** $\frac{2}{3} - \frac{1}{12}$	**14.** $\frac{3}{4} - \frac{5}{12}$	**15.** $\frac{5}{6} - \frac{1}{2}$
16. $\frac{5}{6} - \frac{5}{12}$	**17.** $\frac{7}{8} - \frac{7}{8}$	**18.** $\frac{8}{9} - \frac{2}{3}$	**19.** $\frac{7}{10} - \frac{1}{2}$
20. $\frac{9}{10} - \frac{2}{5}$	**21.** $\frac{2}{3} - \frac{2}{15}$	**22.** $\frac{2}{3} - \frac{2}{9}$	**23.** $\frac{1}{2} - \frac{5}{14}$
24. $\frac{7}{15} - \frac{1}{3}$	**25.** $\frac{2}{3} - \frac{5}{9}$	**26.** $\frac{11}{15} - \frac{1}{3}$	**27.** $\frac{7}{15} - \frac{1}{5}$

Find the difference:

28. $\frac{5}{6} - \frac{3}{4}$	**29.** $\frac{3}{4} - \frac{2}{3}$	**30.** $\frac{3}{4} - \frac{1}{6}$	**31.** $\frac{2}{3} - \frac{1}{2}$
32. $\frac{2}{3} - \frac{2}{5}$	**33.** $\frac{1}{2} - \frac{1}{7}$	**34.** $\frac{2}{3} - \frac{1}{4}$	**35.** $\frac{2}{3} - \frac{1}{6}$
36. $\frac{4}{5} - \frac{1}{2}$	**37.** $\frac{2}{5} - \frac{1}{3}$	**38.** $\frac{3}{4} - \frac{3}{10}$	**39.** $\frac{3}{4} - \frac{1}{6}$
40. $\frac{4}{5} - \frac{2}{3}$	**41.** $\frac{2}{3} - \frac{1}{6}$	**42.** $\frac{5}{7} - \frac{1}{2}$	**43.** $\frac{2}{3} - \frac{3}{5}$
44. $\frac{5}{6} - \frac{1}{4}$	**45.** $\frac{3}{4} - \frac{1}{3}$	**46.** $\frac{5}{6} - \frac{3}{4}$	**47.** $\frac{3}{5} - \frac{1}{3}$

48. From $\frac{3}{4}$ yr. take $\frac{1}{2}$ yr. **49.** Take \$$\frac{2}{5}$ from \$$\frac{7}{10}$.

50. From $\frac{3}{4}$ bu. take $\frac{3}{8}$ bu. **51.** Take $\frac{1}{2}$ acre from $\frac{3}{5}$ acre.

52. From $\frac{2}{3}$ hr. take $\frac{1}{2}$ hr. **53.** Take $\frac{2}{3}$ yd. from $\frac{3}{4}$ yd.

PROBLEMS

1. Mary has $\frac{7}{8}$ of a yard of ribbon. She gives $\frac{1}{4}$ of a yard to her sister. How much has she left?

2. A man buys $\frac{3}{4}$ of a ton of coal. After using $\frac{2}{3}$ of a ton, how much has he left?

3. James walks $\frac{1}{2}$ of a mile to school, and William walks $\frac{1}{6}$ of a mile. James walks how much farther than William?

4. A grocer buys eggs for $\frac{2}{5}$ of a dollar a dozen, and sells them for $\frac{1}{2}$ of a dollar. How much does he make?

5. Mr. Merrill had $\frac{5}{8}$ of a bushel of potatoes. $\frac{1}{4}$ of a bushel decayed. What part of a bushel was good?

SUBTRACTION OF MIXED NUMBERS *Written*

1. Subtract $6\frac{3}{4}$ from $9\frac{7}{12}$.

c.d. $= 12$

$9\frac{7}{12} = 9\frac{7}{12} = 8\frac{19}{12}$

$6\frac{3}{4} = 6\frac{9}{12} = 6\frac{9}{12}$

$2\frac{10}{12} = 2\frac{5}{6}$

Since $\frac{9}{12}$ cannot be taken from $\frac{7}{12}$, we take one of the 9 units in the minuend, change it to 12ths, $1 = \frac{12}{12}$, and add it to the $\frac{7}{12}$, making $\frac{19}{12}$. $9\frac{7}{12} = 8\frac{19}{12}$.

Find differences:

2. $3\frac{3}{4} - 2\frac{1}{4}$ **3.** $4\frac{1}{4} - 1\frac{3}{4}$ **4.** $3\frac{1}{6} - 1\frac{5}{6}$ **5.** $4\frac{2}{5} - 1\frac{4}{5}$

6. $2\frac{7}{8} - 1\frac{1}{2}$ **7.** $4\frac{3}{10} - 1\frac{2}{5}$ **8.** $4\frac{5}{9} - 1\frac{2}{3}$ **9.** $5\frac{1}{2} - 2\frac{3}{14}$

10. $8\frac{2}{3} - 5\frac{1}{6}$ **11.** $6\frac{1}{4} - 5\frac{3}{8}$ **12.** $4\frac{5}{12} - 2\frac{1}{4}$ **13.** $6\frac{2}{3} - 2\frac{5}{12}$

14. $6\frac{2}{15} - 2\frac{1}{5}$ **15.** $6\frac{1}{2} - 2\frac{1}{5}$ **16.** $12\frac{1}{2} - 5\frac{1}{7}$ **17.** $7\frac{1}{5} - 2\frac{1}{3}$

18. $10\frac{3}{4} - 2\frac{2}{3}$ **19.** $8\frac{1}{3} - 4\frac{1}{2}$ **20.** $7\frac{2}{3} - 2\frac{1}{2}$ **21.** $9\frac{2}{5} - 5\frac{1}{2}$

PROBLEMS

1. A farmer had $10\frac{1}{6}$ dozen eggs. He sold $8\frac{1}{2}$ dozen. How many dozen had he left?

2. Mrs. Street earns $\$10\frac{1}{2}$ a week and spends $\$8\frac{2}{5}$. How much does she save?

3. A fruit dealer bought 7 boxes of oranges. After selling $2\frac{2}{5}$ boxes, how many had he left?

4. Margaret bought $12\frac{2}{3}$ yards of ribbon. After using $4\frac{1}{4}$ yards, how much has she left?

5. Mr. Price planted $8\frac{2}{3}$ acres of corn and $5\frac{1}{5}$ acres of wheat. How many more acres of corn than wheat did he plant?

REVIEW EXERCISE *Oral and Written*

1. Write two fractions that can be added or subtracted without changing their form.

2. Write two fractions that cannot be added or subtracted without changing their form.

3. Change the form of these fractions without changing their value: $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{12}$

4. Find the sum of $\frac{3}{4} + \frac{7}{12}$; $\frac{1}{2} + \frac{2}{3}$; $\frac{2}{3} + \frac{5}{6}$; $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

5. Add: $\frac{3}{4}$ mile + $\frac{3}{8}$ mile + $\frac{1}{2}$ mile; $\frac{2}{3}$ yd. + $\frac{5}{6}$ yd. + $\frac{1}{2}$ yd.

6. Add: $2\frac{3}{4} + 5\frac{1}{2} + 6\frac{7}{8}$; $5\frac{3}{5} + 5\frac{1}{2} + 4\frac{3}{10} + 3$.

7. What is the difference between $\frac{5}{6}$ and $\frac{5}{12}$? $\frac{1}{2}$ and $\frac{1}{7}$? $\frac{5}{12}$ and $\frac{3}{4}$? $\frac{1}{2}$ and $\frac{7}{10}$?

8. Subtract $\frac{1}{2}$ hr. from $\frac{2}{3}$ hr.; $\frac{2}{3}$ acre from $\frac{3}{4}$ acre.

9. From 4 take $\frac{3}{4}$. Express one of the units as 4ths. Then $4 = 3\frac{4}{4}$. $3\frac{4}{4} - \frac{3}{4} = 3\frac{1}{4}$.

10. From $3\frac{7}{8}$ take $1\frac{1}{2}$. Take $5\frac{3}{8}$ from $6\frac{1}{4}$.

11. Which is larger, $\frac{4}{5}$ of a dollar or $\frac{7}{10}$ of a dollar How much larger?

12. What must be added to $\frac{3}{5}$ to make $\frac{7}{10}$?

13. What must be taken from $\frac{5}{6}$ to leave $\frac{1}{4}$?

14. From 5 take $\frac{2}{5}$. Take $\frac{7}{9}$ from 7.

15. From 6 take each of these fractions:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{7}{8}$ $\frac{2}{9}$ $\frac{7}{9}$

Read:

16. $\frac{3}{4}$ $\frac{7}{7}$ $\frac{5}{3}$ $\frac{5}{8}$ $\frac{9}{9}$ $\frac{9}{8}$ $\frac{7}{10}$ $\frac{5}{5}$ $\frac{9}{5}$ $\frac{13}{12}$

17. $\frac{8}{9}$ $\frac{11}{6}$ $\frac{8}{8}$ $\frac{3}{7}$ $\frac{5}{11}$ $\frac{3}{2}$ $\frac{17}{15}$ $\frac{18}{25}$ $\frac{13}{13}$ $\frac{27}{20}$

Select from fractions in 16 and 17:

18. All the proper fractions, and name the smallest fraction that must be added to each to make it an improper fraction.

19. All the improper fractions that are equal to 1.

20. All the improper fractions that are greater than 1, and name the fraction that must be taken from each to leave 1.

NOTE. As pupils progress, all processes should be reviewed frequently.

MULTIPLICATION OF FRACTIONS *Oral and Written*

1. 2 times $\frac{3}{\text{apples}} = ?$ 2. $2 \times \frac{3}{\text{fifths}} = ?$ 3. $2 \times \frac{3}{5} = \frac{6}{5} = 1\frac{1}{5}$

Find:

4. $3 \times \frac{5}{7}$	5. $4 \times \frac{2}{3}$	6. $3 \times \frac{3}{8}$	7. $4 \times \frac{3}{5}$
8. $5 \times \frac{3}{4}$	9. $6 \times \frac{3}{7}$	10. $9 \times \frac{2}{5}$	11. $5 \times \frac{5}{6}$
12. $7 \times \frac{5}{8}$	13. $8 \times \frac{5}{9}$	14. $9 \times \frac{3}{10}$	15. $5 \times \frac{7}{9}$
16. $7 \times \frac{4}{5}$	17. $4 \times \frac{4}{7}$	18. $5 \times \frac{3}{9}$	19. $9 \times \frac{7}{10}$
20. $4 \times \frac{9}{11}$	21. $6 \times \frac{4}{13}$	22. $5 \times \frac{7}{12}$	23. $8 \times \frac{2}{15}$
24. $10 \times \frac{2}{3}$	25. $15 \times \frac{7}{8}$	26. $12 \times \frac{6}{7}$	27. $17 \times \frac{3}{4}$
28. $12 \times \frac{3}{5}$	29. $11 \times \frac{8}{15}$	30. $15 \times \frac{4}{7}$	31. $21 \times \frac{4}{5}$
32. $20 \times \frac{5}{7}$	33. $17 \times \frac{5}{8}$	34. $14 \times \frac{4}{5}$	35. $13 \times \frac{7}{15}$
36. $14 \times \frac{3}{9}$	37. $18 \times \frac{4}{7}$	38. $14 \times \frac{5}{9}$	39. $22 \times \frac{7}{9}$
40. $13 \times \frac{8}{11}$	41. $23 \times \frac{3}{8}$	42. $19 \times \frac{5}{12}$	43. $24 \times \frac{9}{13}$

PROBLEMS

1. A family uses $\frac{3}{5}$ of a bushel of apples a month. How many bushels will they use in 6 months?

2. If a yard of silk costs $\frac{3}{4}$ of a dollar, what will 5 yards cost?

3. If a pound of butter costs $\frac{2}{5}$ of a dollar, what will 4 pounds cost?

4. A horse eats $\frac{3}{4}$ of a peck of oats a day. How many pecks will he eat in 9 days?

5. John walks $\frac{3}{8}$ of a mile to school. How many miles does he walk in 5 mornings?

6. Prescott's hens lay $\frac{2}{3}$ of a dozen eggs every day. How many dozen do they lay in a week?

7. Mabel bought 3 hair ribbons, each $\frac{5}{8}$ of a yard long. How many yards of ribbon did she buy?

8. At $\frac{4}{5}$ of a dollar a yard, what will 9 yards of poplin cost?

9. Henry paid $\frac{4}{5}$ of a dime for marbles. What would he pay for 5 times as many?

10. If 2 handkerchiefs cost $\frac{1}{3}$ of a dollar, what will a dozen cost?

MULTIPLYING A WHOLE NUMBER BY A FRACTION

Oral and Written

1. Find $\frac{1}{3}$ of 12; 24; 36; 48; 60.

2. Find $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{3}$, of the above numbers.

3. What is $\frac{3}{8}$ of 32?

4. What is $\frac{5}{12}$ of 72?

5. What is $\frac{5}{9}$ of 63?

6. What is $\frac{7}{8}$ of 56?

7. What is $\frac{3}{9}$ of 72?

8. What is $\frac{5}{6}$ of 54?

Finding a fractional part of a number is called multiplying by a fraction.

9. Find $\frac{2}{5}$ of 9.

$\frac{1}{5}$ of 9 is $\frac{9}{5}$; $\frac{2}{5}$ of 9 is 2 times $\frac{9}{5}$, or $\frac{18}{5}$, or $3\frac{3}{5}$.

10. $\frac{2}{3}$ of 4

11. $\frac{2}{5}$ of 8

12. $\frac{5}{6}$ of 5

13. $\frac{3}{4}$ of 5

14. $\frac{3}{7}$ of 8	15. $\frac{3}{9}$ of 4	16. $\frac{4}{5}$ of 9	17. $\frac{6}{7}$ of 6
18. $\frac{5}{9}$ of 8	19. $\frac{2}{3}$ of 7	20. $\frac{5}{8}$ of 3	21. $\frac{3}{8}$ of 9

The sign $\times$ in the expression $\frac{2}{3} \times 7$ is equivalent to the word "of."

Find products:

22. $\frac{5}{7} \times 8$	23. $\frac{4}{5} \times 8$	24. $\frac{3}{5} \times 6$	25. $\frac{5}{6} \times 7$
26. $\frac{2}{9} \times 10$	27. $\frac{2}{3} \times 10$	28. $\frac{3}{4} \times 11$	29. $\frac{2}{5} \times 4$
30. $\frac{2}{5} \times 14$	31. $\frac{5}{6} \times 11$	32. $\frac{3}{7} \times 9$	33. $\frac{3}{10} \times 7$
34. $\frac{3}{4} \times 17$	35. $\frac{2}{3} \times 16$	36. $\frac{5}{6} \times 22$	37. $\frac{2}{5} \times 18$
38. $\frac{3}{5} \times 24$	39. $\frac{3}{7} \times 25$	40. $\frac{4}{5} \times 16$	41. $\frac{5}{7} \times 12$
42. $\frac{3}{8} \times 27$	43. $\frac{6}{7} \times 18$	44. $\frac{5}{8} \times 15$	45. $\frac{4}{7} \times 10$
46. $\frac{7}{8} \times 15$	47. $\frac{2}{9} \times 40$	48. $\frac{4}{9} \times 32$	49. $\frac{5}{9} \times 28$

PROBLEMS

1. At 20 cents a dozen, what will $\frac{3}{4}$ of a dozen of bananas cost?

2. George picked 15 quarts of berries and sold $\frac{3}{5}$ of them. How many quarts did he sell?

3. Olive has an allowance of 10 cents a week. She saves $\frac{1}{10}$ of it. How much does she spend?

4. A butcher bought 3 pairs of chickens for 6 dollars. How much did he pay for each chicken?

5. Ethel is 18 years old and her sister is $\frac{5}{6}$ as old. How old is her sister?

6. There are 36 shade trees on a street. Seven ninths of them are maples. How many of other kinds?

7. When oranges are 60 cents a dozen, what will one cost? 4? $\frac{1}{4}$ of a dozen? $\frac{2}{3}$ of a dozen? $\frac{3}{4}$ of a dozen? $\frac{1}{2}$ of a dozen? $\frac{5}{6}$ of a dozen?

8. I have 32 raspberry bushes and $\frac{3}{8}$ as many currant bushes. How many currant bushes?

9. If $\frac{3}{7}$ of the days are stormy, how many pleasant days in 2 weeks? In 5 weeks?

10. George has 4 dollars. Henry has $\frac{4}{5}$ as much. How much has Henry?

MULTIPLYING A MIXED NUMBER BY A WHOLE NUMBER

Written

1. Multiply $2\frac{3}{5}$ by 8.

$2\frac{3}{5}$
$\underline{8}$
$4\frac{4}{5} = 8 \times \frac{3}{5}$
$\underline{16} = 8 \times 2$
$20\frac{4}{5} = 8 \times 2\frac{3}{5}$

This means 8 times 2 + 8 times $\frac{3}{5}$.
$8 \times \frac{3}{5} = \frac{24}{5} = 4\frac{4}{5}$
$8 \times 2 = 16$
$16 + 4\frac{4}{5} = 20\frac{4}{5}$

Multiply:

2. $2\frac{2}{3}$ by 4	**3.** $12\frac{1}{2}$ by 7	**4.** $16\frac{5}{8}$ by 7
5. $37\frac{1}{2}$ by 27	**6.** $3\frac{3}{4}$ by 5	**7.** $10\frac{1}{4}$ by 9
8. $17\frac{2}{9}$ by 9	**9.** $16\frac{2}{3}$ by 25	**10.** $4\frac{5}{6}$ by 7
11. $15\frac{5}{8}$ by 5	**12.** $20\frac{4}{9}$ by 8	**13.** $87\frac{1}{2}$ by 35
14. $5\frac{3}{8}$ by 8	**15.** $16\frac{5}{6}$ by 7	**16.** $18\frac{3}{10}$ by 9
17. $66\frac{2}{3}$ by 28	**18.** $3\frac{7}{8}$ by 7	**19.** $15\frac{5}{9}$ by 5
20. $20\frac{7}{12}$ by 5	**21.** $80\frac{2}{3}$ by 24	**22.** $83\frac{1}{3}$ by 15

PROBLEMS

1. What will 6 yards of muslin cost at $8\frac{1}{3}$ cents a yard?

2. Sugar is $5\frac{3}{4}$ cents a pound. How much must be paid for 4 pounds?

3. At $6\frac{1}{4}$ cents a pound, what will 9 pounds of meat cost?

4. Mary earns $8\frac{2}{5}$ dollars a week. How much will she earn in 4 weeks?

5. At $12\frac{1}{2}$ cents apiece, what will half a dozen collars cost?

MULTIPLYING A WHOLE NUMBER BY A MIXED NUMBER

Written

1. Multiply 12 by $7\frac{2}{5}$.

$$\begin{array}{l} 12 \\ \underline{7\frac{2}{5}} \\ 4\frac{4}{5} = \frac{2}{5} \text{ of } 12 \\ 84 = 7 \times 12 \\ \overline{88\frac{4}{5}} = 7\frac{2}{5} \times 12 \end{array}$$

This means 7 times 12 + $\frac{2}{5}$ of 12.
$\frac{2}{5}$ of $12 = \frac{24}{5} = 4\frac{4}{5}$
$7 \times 12 = 84$
$84 + 4\frac{4}{5} = 88\frac{4}{5}$

Multiply:

2. 8 by $2\frac{3}{8}$
3. 7 by $3\frac{1}{6}$
4. 9 by $5\frac{3}{4}$
5. 12 by $2\frac{4}{5}$
6. 5 by $3\frac{3}{8}$
7. 7 by $2\frac{5}{8}$
8. 11 by $4\frac{5}{6}$
9. 9 by $4\frac{7}{8}$
10. 8 by $4\frac{2}{9}$
11. 9 by $4\frac{3}{5}$
12. 10 by $3\frac{5}{7}$
13. 7 by $5\frac{3}{8}$
14. 12 by $8\frac{3}{4}$
15. 20 by $4\frac{3}{5}$
16. 18 by $3\frac{5}{6}$
17. 24 by $5\frac{5}{8}$
18. 17 by $2\frac{4}{5}$
19. 14 by $4\frac{3}{8}$
20. 16 by $4\frac{2}{3}$
21. 25 by $6\frac{3}{10}$
22. 36 by $5\frac{3}{5}$

MULTIPLYING A FRACTION BY A FRACTION

Oral and Written

$\frac{2}{3} \times \frac{3}{5}$ means $\frac{2}{3}$ of $\frac{3}{5}$.

1. $\frac{2}{3}$ of $\frac{3}{\text{apples}} = ?$
2. $\frac{1}{3}$ of $\frac{3}{\text{apples}} = \frac{1}{\text{apple}}$
3. $\frac{2}{3}$ of $\frac{3}{\text{apples}} = \frac{2}{\text{apples}}$
4. $\frac{2}{3}$ of $\frac{3}{5} = ?$
5. $\frac{1}{3}$ of $\frac{3}{5} = \frac{1}{5}$
6. $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$

Notice that $\frac{2}{3}$ of $\frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$

and $\frac{2 \times \cancel{3}}{\cancel{3} \times 5} = \frac{2}{5}$

To multiply a fraction by a fraction, we write the product of the numerators over the product of the denominators, canceling when possible.

This rule applies to all cases of multiplication of fractions, for every whole number may be written as a fraction with 1 for its denominator. Thus, $8 = \frac{8}{1}$; $\frac{2}{3}$ of 8 may be written $\frac{2}{3} \times \frac{8}{1}$; 8 times $\frac{2}{3}$ may be written $\frac{8}{1} \times \frac{2}{3}$.

Find:

7. $\frac{1}{4}$ of $\frac{4}{5}$	8. $\frac{1}{5}$ of $\frac{5}{6}$	9. $\frac{1}{8}$ of $\frac{6}{7}$	10. $\frac{2}{3}$ of $\frac{6}{7}$
11. $\frac{2}{3} \times \frac{3}{4}$	12. $\frac{3}{5} \times \frac{5}{6}$	13. $\frac{3}{7} \times \frac{7}{8}$	14. $\frac{3}{4} \times \frac{4}{5}$
15. $\frac{1}{2} \times \frac{1}{2}$	16. $\frac{2}{3} \times \frac{1}{4}$	17. $\frac{3}{5} \times \frac{7}{9}$	18. $\frac{9}{10} \times \frac{5}{8}$
19. $\frac{2}{3} \times \frac{7}{8}$	20. $\frac{3}{4} \times \frac{5}{6}$	21. $\frac{5}{7} \times \frac{1}{15}$	22. $\frac{8}{9} \times \frac{1}{2}$
23. $\frac{3}{4} \times \frac{1}{8}$	24. $\frac{2}{3} \times \frac{1}{2}$	25. $\frac{4}{5} \times \frac{1}{4}$	26. $\frac{7}{8} \times \frac{1}{2}$

Find the product of:

27. $\frac{1}{4} \times \frac{8}{15}$ 28. $\frac{1}{4} \times \frac{12}{17}$ 29. $\frac{2}{7} \times \frac{2}{7}$ 30. $\frac{5}{6} \times \frac{24}{25}$

31. $\frac{3}{10} \times \frac{5}{6}$ 32. $\frac{8}{15} \times \frac{3}{8}$ 33. $\frac{5}{8} \times \frac{3}{4}$ 34. $\frac{5}{21} \times \frac{14}{25}$

35. $\frac{5}{8} \times \frac{8}{9}$ 36. $\frac{3}{4} \times \frac{5}{7}$ 37. $\frac{6}{7} \times \frac{4}{5}$ 38. $\frac{11}{16} \times \frac{8}{33}$

39. $\frac{14}{15} \times \frac{5}{7}$ 40. $\frac{4}{9} \times \frac{7}{12}$ 41. $\frac{2}{3} \times \frac{4}{9}$ 42. $\frac{12}{25} \times \frac{15}{16}$

43. $\frac{5}{9} \times \frac{9}{20}$ 44. $\frac{4}{5} \times \frac{7}{8}$ 45. $\frac{2}{3} \times \frac{5}{6}$ 46. $\frac{5}{14} \times \frac{14}{35}$

PROBLEMS

1. What will $\frac{1}{2}$ of a yard of silk cost at $\frac{3}{5}$ of a dollar a yard?

2. If Alfred picks $\frac{1}{2}$ of a peck of cherries and sells $\frac{3}{4}$ of them, what part of a peck does he sell?

3. Blanche had $\frac{3}{4}$ of a pound of candy. She gave $\frac{1}{3}$ of it to Susie. How much did she give to Susie?

4. Mrs. Whiting bought $\frac{7}{8}$ of a yard of ruching and used $\frac{2}{3}$ of it. What part of a yard did she use?

5. What will $\frac{2}{3}$ of a yard of lace cost at $\frac{3}{5}$ of a dollar a yard?

6. It takes Frank $\frac{3}{4}$ of an hour to mow his lawn. Herbert can mow it in $\frac{2}{3}$ the time. How long does it take Herbert?

7. Mr. Kimball bought $\frac{4}{5}$ of a ton of oats. He had to throw away $\frac{1}{6}$ of the lot. What part of a ton did he lose?

8. Two thirds of $\frac{3}{7}$ of an acre of corn is sweet corn. How much sweet corn is there?

9. I have $\frac{4}{5}$ of a dollar. If I spend $\frac{3}{4}$ of it for a book, what part of a dollar does the book cost?

10. The schoolhouse is $\frac{5}{6}$ of a mile from my home. The church is $\frac{2}{3}$ as far. What part of a mile do I walk in going to church?

MULTIPLYING A MIXED NUMBER BY A MIXED NUMBER

Written

1. Find $1\frac{2}{3} \times 1\frac{2}{5}$. Change to improper fractions.

$$1\frac{2}{3} \times 1\frac{2}{5} = \frac{5}{3} \times \frac{7}{5} = \frac{7}{3} = 2\frac{1}{3}$$

Find products:

2. $2\frac{1}{3} \times 1\frac{1}{5}$	**3.** $3\frac{3}{4} \times 2\frac{2}{3}$	**4.** $2\frac{2}{5} \times 1\frac{3}{4}$	**5.** $1\frac{4}{5} \times 3\frac{1}{3}$
6. $\frac{3}{4} \times 3\frac{2}{5}$	**7.** $\frac{5}{6} \times 4\frac{1}{2}$	**8.** $\frac{7}{8} \times 2\frac{1}{3}$	**9.** $\frac{8}{9} \times 2\frac{5}{7}$
10. $1\frac{3}{5} \times 2\frac{1}{2}$	**11.** $3\frac{3}{4} \times 1\frac{5}{7}$	**12.** $5\frac{1}{3} \times 2\frac{1}{4}$	**13.** $2\frac{1}{7} \times 2\frac{1}{10}$
14. $2\frac{5}{6} \times \frac{3}{5}$	**15.** $4\frac{3}{8} \times \frac{8}{9}$	**16.** $3\frac{5}{6} \times \frac{7}{8}$	**17.** $5\frac{2}{3} \times \frac{5}{7}$
18. $1\frac{5}{6} \times 3\frac{3}{5}$	**19.** $2\frac{1}{12} \times 4\frac{4}{5}$	**20.** $4\frac{2}{7} \times 1\frac{5}{9}$	**21.** $2\frac{7}{8} \times 3\frac{3}{5}$

FINDING WHAT PART ONE NUMBER IS OF ANOTHER

Oral

1. What part of 4 dollars is 1 dollar? 2 dollars? 3 dollars?

2. Express as parts of a gallon: 2 quarts; 1 quart.

3. Express as parts of a dollar: 50 cents; 25 cents; 75 cents; 20 cents; 40 cents; 60 cents; 80 cents.

4. What part of a bushel is 1 peck? 3 pecks?

5. What part of 12 inches is 1 inch? 5 inches? 7 inches? 11 inches?

6. Express as parts of a foot: 6 inches; 3 inches; 9 inches; 4 inches; 8 inches; 2 inches; 10 inches.

Express as parts of an hour:

7. 30 minutes
8. 15 minutes; 45 minutes
9. 20 minutes; 40 minutes
10. 10 minutes; 50 minutes
11. 6 minutes; 18 minutes
12. 12 minutes; 48 minutes
13. 5 minutes; 35 minutes
14. 3 minutes; 9 minutes
15. 4 minutes; 16 minutes

What part of:

16. 24 is 8
17. 32 is 4
18. 40 is 8
19. 20 is 10
20. 12 is 9
21. 20 is 12
22. 25 is 10
23. 40 is 30
24. 84 is 7
25. 63 is 9
26. 54 is 6
27. 56 is 8
28. 84 is 21
29. 63 is 35
30. 54 is 36
31. 56 is 32

32. Elsie solves 8 of her 10 problems. What part does she solve?

33. Out of 20 words Jack misspelled 2. What part did he misspell?

34. James bought 24 newspapers. He sold 20. What part had he left?

FINDING THE WHOLE WHEN A PART IS GIVEN *Oral*

1. Four dollars is $\frac{1}{2}$ of my money. What is the whole of it?

2. Ned sold his rabbit for 30 cents. This was $\frac{3}{5}$ of what he paid. What did he pay for the rabbit?

SOLUTION. Since 30 cents is 3 fifths, 1 fifth is $\frac{1}{3}$ of 30 cents, or 10 cents; 5 fifths is 5 × 10 cents, or 50 cents.

3. 12 is $\frac{2}{3}$ of what number? 4. 9 is $\frac{3}{4}$ of what number?

5. 24 is $\frac{6}{7}$ of what number? 6. 15 is $\frac{5}{6}$ of what number?

7. 20 is $\frac{4}{5}$ of what number? 8. 28 is $\frac{4}{7}$ of what number?

9. Maggie paid 40 cents for a veil. This was $\frac{4}{5}$ of what she paid for a pin. How much did she pay for her pin?

10. In one pasture there are 10 cows. This is $\frac{2}{3}$ of the number in another pasture. How many in the second pasture?

11. A baseball team won 12 games. This was $\frac{3}{4}$ of the number played. How many games did it play?

12. 60 miles is $\frac{5}{6}$ of the distance between two cities. How far apart are the cities?

13. I have read 48 pages of a book. This is $\frac{4}{7}$ of the book. How many pages in the book?

Find the number of which:

14. 27 is $\frac{3}{5}$ 15. 60 is $\frac{5}{8}$ 16. 84 is $\frac{7}{12}$ 17. 35 is $\frac{7}{8}$

18. 96 is $\frac{2}{3}$ 19. 96 is $\frac{3}{4}$ 20. 96 is $\frac{8}{9}$ 21. 72 is $\frac{3}{4}$

22. A man spends $\frac{7}{9}$ of his yearly wages. He spends $630. How much does he earn?

DICTATION EXERCISES

1. $9 \times 8, \div 6, +4, \div 8, \times 5, \times 7, -4, \div 11, +9 = ?$
2. $8 + 4, \times 5, -6, \div 9, \times 7, +3, \div 9, +7, +8 = ?$
3. $63 \div 7, \times 3, +8, \div 7, +4, \times 6 -5, \div 7, +5 = ?$
4. $84 \div 12, \times 8, -2, \div 9, \times 7, -6, \div 4, \times 7, +9 = ?$
5. $48 \div 4, +3, -7, \times 4, +8, \div 8, +3, \times 7, +4 = ?$

DRILL EXERCISE

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
1.	6	12	2	16	4	24	18	14	10	22	8	20
2.	12	24	4	32	8	48	36	28	20	44	16	40
3.	24	48	8	64	16	96	72	56	40	88	32	80
4.	36	72	12	96	24	144	108	84	60	132	48	120
5.	9	18	3	24	6	36	27	21	15	33	12	30
6.	18	36	6	48	12	72	54	42	30	66	24	60
7.	27	54	9	72	18	108	81	63	45	99	36	90
8.	15	30	5	40	10	60	45	35	25	55	20	50
9.	30	60	10	80	20	120	90	70	50	110	40	100
10.	21	42	7	56	14	84	63	49	35	77	28	70
11.	33	66	11	88	22	132	99	77	55	121	44	110

Each number in

Row 1, or column *E*, is $\frac{2}{3}$ of what number?

Row 2, or column *K*, is $\frac{4}{5}$ of what number?

Row 3, or column *D*, is $\frac{8}{9}$ of what number?

Row 4, or column *F*, is $\frac{1}{2}$ of what number?

Row 5, or column *A*, is $\frac{3}{4}$ of what number?

Row 6, or column *B*, is $\frac{6}{7}$ of what number?

Row 7, or column *G*, is $\frac{9}{10}$ of what number?

Row 8, or column *I*, is $\frac{5}{6}$ of what number?

Row 9, or column *L*, is $\frac{10}{11}$ of what number?

Row 10, or column *H*, is $\frac{7}{8}$ of what number?

Row 11, or column *J*, is $\frac{11}{12}$ of what number?

Each number in column *C* is $\frac{1}{2}$ of what number? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

REVIEW EXERCISE *Written*

1. Using the numbers 7 and 8, write a proper fraction; an improper fraction.

2. Write an improper fraction that can be changed to a whole number. Change it.

3. Write a proper fraction that can be changed to lower terms. Change it.

4. Write an improper fraction that can be expressed as a mixed number. Write the mixed number.

5. Write five fractions that are each equal to $\frac{2}{3}$.

6. A man said he owned $\frac{20}{52}$ of a mill. What simpler fraction might he have used?

7. Write a fraction that will show what part of the days of a week you attend school.

8. Using any of the numbers from 1 to 10, write the largest proper fraction you can; the smallest proper fraction.

9. Using the numbers 3, 4, 4, 5, write two fractions that can be added without changing their form. Add them.

10. Using the same numbers, write two fractions that cannot be added without changing their form. Change their form and add them.

11. Write as fractions: 1; 5; 8.

12. From 8 take $\frac{1}{8}$; $1\frac{1}{8}$; $2\frac{7}{8}$; $2\frac{3}{4}$; $3\frac{1}{2}$.

13. Find 4 times $\frac{5}{7}$; $\frac{7}{8}$; $2\frac{1}{4}$; $2\frac{3}{5}$; $3\frac{2}{9}$.

14. Find $2\frac{1}{2}$ times 2; $\frac{3}{5}$; $1\frac{2}{5}$; $1\frac{1}{2}$; $2\frac{5}{8}$.

15. Find $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{6}{7}$; $\frac{1}{2}$; $1\frac{1}{2}$; $2\frac{3}{5}$.

16. Find $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{7}{9}$; $1\frac{1}{2}$; 2; $2\frac{2}{3}$.

17. What is $\frac{3}{4}$ of 12?

18. 12 is $\frac{3}{4}$ of what number?

19. John sold 18 doves. This was $\frac{3}{5}$ of the number he had at first. How many had he at first?

DIVISION *Oral and Written*

1. $\frac{4}{\text{apples}} \div \frac{2}{\text{apples}} = ?$

2. $\frac{4}{\text{fifths}} \div \frac{2}{\text{fifths}} = ?$

3. $\frac{4}{5} \div \frac{2}{5} = ?$

4. $\frac{5}{8} \div \frac{1}{8} = ?$

5. $\frac{5}{8} \div \frac{2}{8} = ?$

6. $\frac{5}{8} \div \frac{3}{8} = ?$

7. $\frac{4}{5} \div \frac{2}{3} = ?$ Change to like fractions.

$1\frac{2}{15} \div \frac{10}{15}$

$$\frac{12}{\text{fifteenths}} \div \frac{10}{\text{fifteenths}} = 12 \div 10 = 1\frac{2}{10} = 1\frac{1}{5}$$

8. Divide 4 by $\frac{2}{3}$.

$4 = \frac{4}{1}$, for any whole number may be expressed as a fraction with 1 for its denominator.

$\frac{4}{1} \div \frac{2}{3}$ Change to like fractions.

$1\frac{2}{3} \div \frac{2}{3} = 12 \div 2 = 6$

9. Divide $\frac{5}{6}$ by 2.

$\frac{5}{6} \div \frac{2}{1}$ Change to like fractions.

$\frac{5}{6} \div 1\frac{2}{6} = 5 \div 12 = \frac{5}{12}$

Any number may be divided by a fraction by changing both numbers to fractions having a common denominator,

and then dividing the numerator of the dividend by the numerator of the divisor.

By multiplying the dividend by the divisor inverted, we obtain the same results as in the process just described.

Thus,
$$\frac{4}{5} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\underset{1}{\cancel{2}}} = \frac{6}{5} = 1\tfrac{1}{5}$$

$$4 \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}} = 6$$

$$\frac{5}{6} \div 2 = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

To divide fractions, we change to like fractions and divide the numerator of the dividend by the numerator of the divisor; or, for convenience, we invert the divisor and multiply, canceling when possible.

Divide by changing to like fractions, or by inverting the divisor:

	A	*B*	*C*	*D*	*E*
10.	$4 \div \frac{1}{2}$	$\frac{1}{5} \div 2$	$9 \div \frac{3}{7}$	$\frac{2}{5} \div \frac{1}{2}$	$\frac{2}{3} \div \frac{1}{5}$
11.	$6 \div \frac{1}{3}$	$\frac{3}{4} \div 3$	$\frac{3}{4} \div 5$	$\frac{3}{4} \div \frac{2}{3}$	$\frac{4}{5} \div \frac{3}{4}$
12.	$3 \div \frac{1}{4}$	$\frac{7}{8} \div 7$	$12 \div \frac{4}{5}$	$\frac{3}{5} \div \frac{5}{8}$	$\frac{2}{3} \div \frac{3}{4}$
13.	$5 \div \frac{1}{6}$	$\frac{4}{5} \div 6$	$\frac{5}{6} \div 4$	$\frac{5}{8} \div \frac{5}{7}$	$\frac{6}{7} \div \frac{2}{3}$
14.	$2 \div \frac{1}{8}$	$\frac{5}{7} \div 5$	$\frac{3}{8} \div \frac{3}{4}$	$\frac{4}{9} \div \frac{5}{6}$	$\frac{3}{5} \div \frac{3}{4}$
15.	$8 \div \frac{3}{4}$	$\frac{8}{9} \div 3$	$\frac{3}{4} \div \frac{7}{8}$	$\frac{3}{5} \div \frac{6}{7}$	$\frac{3}{8} \div \frac{7}{8}$
16.	$5 \div \frac{2}{5}$	$\frac{4}{5} \div 2$	$\frac{2}{3} \div \frac{8}{9}$	$\frac{8}{9} \div \frac{2}{3}$	$\frac{4}{9} \div \frac{2}{3}$
17.	$10 \div \frac{5}{6}$	$\frac{3}{8} \div 5$	$\frac{4}{5} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{5}{6}$	$\frac{5}{6} \div \frac{3}{4}$

DIVIDING MIXED NUMBERS *Written*

1. Divide $9\frac{2}{7}$ by 4.

(1) $9\frac{2}{7} \div 4 = \frac{65}{7} \div 4 = \frac{65}{7} \times \frac{1}{4} = \frac{65}{28} = 2\frac{9}{28}$

(2) $4)\overline{9\frac{2}{7}}$ $\frac{1}{4}$ of 9 is 2, and 1 over

$2\frac{9}{28}$ $\frac{1}{4}$ of $1\frac{2}{7} = \frac{1}{4}$ of $\frac{9}{7} = \frac{9}{28}$

Give quotients:

2. $3\frac{1}{5} \div 2$	**3.** $16\frac{2}{3} \div 7$	**4.** $16\frac{2}{3} \div 4$	**5.** $14\frac{4}{5} \div 6$
6. $4\frac{7}{8} \div 3$	**7.** $8\frac{1}{3} \div 5$	**8.** $14\frac{2}{3} \div 5$	**9.** $12\frac{2}{5} \div 7$
10. $8\frac{2}{3} \div 4$	**11.** $8\frac{1}{2} \div 6$	**12.** $15\frac{5}{8} \div 4$	**13.** $24\frac{3}{4} \div 3$
14. $12\frac{1}{2} \div 3$	**15.** $12\frac{2}{5} \div 7$	**16.** $33\frac{1}{3} \div 8$	**17.** $14\frac{4}{5} \div 6$
18. $9\frac{5}{7} \div 5$	**19.** $7\frac{2}{3} \div 5$	**20.** $18\frac{3}{4} \div 2$	**21.** $33\frac{1}{3} \div 8$
22. $20\frac{2}{3} \div 6$	**23.** $15\frac{7}{9} \div 4$	**24.** $17\frac{7}{8} \div 7$	**25.** $25\frac{1}{2} \div 9$

26. Divide $3\frac{1}{2}$ by $1\frac{3}{4}$.

$$3\frac{1}{2} \div 1\frac{3}{4} = \frac{7}{2} \div \frac{7}{4} = \frac{7}{2} \times \frac{\overset{2}{\cancel{4}}}{7} = 2$$

Give quotients:

27. $2\frac{1}{4} \div 1\frac{1}{8}$	**28.** $\frac{7}{8} \div 1\frac{5}{8}$	**29.** $\frac{3}{4} \div 1\frac{1}{2}$	**30.** $1\frac{2}{3} \div \frac{5}{9}$
31. $1\frac{1}{2} \div 2\frac{1}{4}$	**32.** $1\frac{1}{4} \div \frac{2}{5}$	**33.** $3\frac{1}{8} \div \frac{5}{8}$	**34.** $\frac{7}{8} \div 2\frac{1}{3}$
35. $3\frac{1}{3} \div \frac{2}{5}$	**36.** $\frac{5}{6} \div 2\frac{1}{4}$	**37.** $\frac{2}{3} \div 3\frac{1}{3}$	**38.** $3\frac{3}{4} \div 1\frac{7}{8}$
39. $2\frac{2}{5} \div 5\frac{2}{5}$	**40.** $3\frac{3}{5} \div 2\frac{1}{4}$	**41.** $1\frac{2}{3} \div 3\frac{2}{3}$	**42.** $5\frac{2}{5} \div 2\frac{1}{4}$
43. $4\frac{1}{3} \div 1\frac{1}{2}$	**44.** $2\frac{3}{4} \div 1\frac{5}{6}$	**45.** $6\frac{1}{4} \div 1\frac{7}{8}$	**46.** $2\frac{2}{7} \div 1\frac{1}{8}$

PROBLEMS *Written*

1. If 1 pound of coffee costs $\frac{1}{4}$ of a dollar, how many pounds can be bought for $6\frac{1}{2}$ dollars?

2. Mrs. Martin paid $4\frac{2}{5}$ dollars for $5\frac{1}{2}$ yards of cloth. What was the price a yard?

3. Two boys walked 4 miles in $2\frac{3}{4}$ hours. How far did they walk in 1 hour?

4. Alice paid the photographer $1\frac{2}{5}$ dollars for finishing 20 pictures. What was the cost of each picture?

5. At $5\frac{1}{2}$ cents a pound, how many pounds of sugar can be bought for 55 cents?

6. If 9 yards of carpeting cost $12\frac{3}{5}$ dollars, what will 1 yard cost?

7. A boy picked 4 boxes of strawberries and sold them for 50 cents. How much did he receive a box?

8. A farmer's coal cost 33 dollars. He paid for it in apples worth $2\frac{3}{4}$ dollars a barrel. How many barrels did it take?

9. If a man earns $1\frac{4}{5}$ dollars a day, how long will it take him to earn 9 dollars?

10. Three barrels of flour cost $19\frac{1}{2}$ dollars. What was the price of a barrel?

11. A field containing $2\frac{5}{8}$ acres is cut up into 7 equal lots. What part of an acre is each lot?

12. Mrs. Jones sold some eggs for $\frac{2}{5}$ of a dollar a dozen. She received $1\frac{3}{5}$ dollars for them. How many dozen did she sell?

13. If $4\frac{1}{5}$ dollars will pay for 7 pounds of tea, what is the cost of a pound?

14. In 6 minutes a railroad train ran $4\frac{2}{7}$ miles. What was the rate per minute?

15. A woman received $11\frac{1}{4}$ dollars for 5 days' work. What did she receive a day?

16. At $\frac{2}{5}$ of a dollar a pound, how many pounds of chocolate can be bought for $2\frac{2}{5}$ dollars?

17. How many half-gallon bottles will be required to bottle $3\frac{1}{2}$ gallons of vinegar?

18. How many strips of paper $\frac{5}{8}$ of a yard wide will be needed to cover the side of a room 5 yards long?

19. Julia uses $\frac{1}{5}$ of a yard of cretonne to make a work-bag. How many bags can she make from 4 yards?

20. Mrs. Danforth divided $3\frac{1}{2}$ pounds of candy among 4 children. What part of a pound did she give to each child?

REVIEW EXERCISE *Oral and Written*

1. Name the largest of these quantities: $\frac{1}{3}$ of a dollar, $\frac{1}{4}$ of a dollar, $\frac{1}{2}$ of a dollar.

2. What is the product of $10 \times \frac{3}{5}$? $12 \times \frac{6}{7}$? $20 \times \frac{5}{12}$?

3. What does the expression $\frac{4}{5} \times 9$ mean?

4. Find the product of $\frac{4}{5} \times 8$; $\frac{3}{7} \times 9$; $\frac{3}{8} \times 2$.

5. Multiply $3\frac{3}{4}$ by 5; $5\frac{3}{8}$ by 8; $12\frac{1}{2}$ by 7.

6. Multiply 8 by $2\frac{2}{3}$; 12 by $4\frac{2}{3}$; 20 by $3\frac{3}{5}$.

7. What is $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{5}{9}$ of $\frac{9}{20}$? $\frac{2}{3}$ of $\frac{4}{5}$?

8. Find $2\frac{3}{5} \times 2\frac{1}{2}$; $\frac{5}{6} \times 4\frac{1}{2}$; $2\frac{2}{7} \times 2\frac{1}{10}$.

9. Divide $\frac{2}{3}$ by $\frac{5}{6}$; $\frac{8}{9}$ by $\frac{4}{5}$; $\frac{9}{10}$ by $\frac{5}{6}$.

10. Divide 9 by $\frac{3}{7}$; 12 by $\frac{4}{5}$; $\frac{5}{6}$ by 4; $\frac{7}{8}$ by 7.

11. Divide $3\frac{3}{5}$ by $2\frac{1}{4}$; $4\frac{1}{5}$ by $\frac{5}{7}$; $\frac{4}{7}$ by $3\frac{3}{7}$; $4\frac{3}{8}$ by 5.

12. What is the value of $\frac{5}{6} + \frac{3}{4}$? $\frac{5}{6} - \frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{4}$? $\frac{5}{6} \div \frac{3}{4}$?

MISCELLANEOUS PROBLEMS *Written*

1. What is the cost of $3\frac{3}{4}$ pounds of coffee at $\frac{2}{5}$ of a dollar a pound?

Omitting fractions, read "What will 3 pounds cost at 1 dollar a pound?" 3 times 1 dollar.

Similarly, $3\frac{3}{4}$ pounds will cost $3\frac{3}{4}$ times $\frac{2}{5}$ of a dollar.

$$3\tfrac{3}{4} \times \frac{2}{5} = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} = \frac{3}{2}, \text{ or } 1\tfrac{1}{2} \qquad \text{Answer, } \$1\tfrac{1}{2}.$$

Notice that in the mechanical work we treat the quantities as abstract numbers.

2. What must I pay for $2\frac{1}{2}$ tons of coal at $6\frac{4}{5}$ dollars a ton?

3. A bushel of oats weighs 32 pounds. What is the weight of a load of $20\frac{5}{8}$ bushels?

4. Mr. Farmer has 280 sheep. Mr. Harlow has $2\frac{1}{8}$ times as many. How many has Mr. Harlow?

5. How many quarts of pickles are there in 15 jars if each jar holds $1\frac{3}{4}$ quarts?

6. At 12 cents a pound, how much must be paid for 6 cheeses, each weighing $12\frac{5}{8}$ pounds?

7. If $15\frac{5}{8}$ yards of cloth cost $6\frac{1}{4}$ dollars, what is the cost of 1 yard?

Omitting fractions, read "If 15 yards cost 6 dollars, what will 1 yard cost?" $\$6 \div 15 =$ cost of 1 yard.

Similarly, $\$6\frac{1}{4} \div 15\frac{5}{8} =$ cost of 1 yard.

$$6\tfrac{1}{4} \div 15\tfrac{5}{8} = \frac{25}{4} \times \frac{8}{125} = \frac{2}{5}$$

Answer, $\frac{2}{5}$ of a dollar.

8. How many bushels of potatoes at $\frac{4}{5}$ of a dollar a bushel can be bought for 20 dollars?

9. In 6 days James earned $\$10\frac{1}{2}$. What were his daily wages?

10. For $5\frac{1}{2}$ days' work a gardener received $13\frac{3}{4}$ dollars. How much did he receive a day?

11. It takes $\frac{5}{8}$ of a yard of cloth to make an apron. How many aprons can be made from $7\frac{1}{2}$ yards of cloth?

12. If $\frac{7}{8}$ of a yard of cloth is used for an apron, how many yards must be bought to make 20 aprons?

13. How much cloth is used for an apron when 22 aprons are made from $8\frac{1}{4}$ yards?

14. A small park contains $5\frac{3}{5}$ acres. In the same city there is another park $8\frac{3}{4}$ times as large. What is the size of the larger park?

15. A clerk receives $\$60$ a month. He spends $\$20\frac{3}{4}$ for board, $\$7\frac{1}{2}$ for room rent, $\$5\frac{1}{4}$ for clothing, and $\$1\frac{3}{4}$ for car fares. How much does he save?

16. A carpenter agreed to do a piece of work at $\$3\frac{3}{4}$ a day. He worked $7\frac{1}{2}$ days. How much did he charge?

17. Oil is worth at the wells $37\frac{1}{2}$ cents a barrel. What are 1000 barrels worth?

18. Mr. Jenkins received $\$108\frac{1}{2}$ for his apples. He sold them at $\$1\frac{3}{4}$ a barrel. How many barrels did he sell?

19. From $6\frac{1}{2}$ acres of land there were cut $9\frac{3}{4}$ tons of hay. What was the yield of one acre?

20. A can contains $8\frac{1}{2}$ quarts of milk. How much is left after $1\frac{1}{2}$ quarts are sold to one customer and twice as much to another customer?

RELATION OF ONE NUMBER TO ANOTHER *Oral*

1. What is the relation of 8 to 2?

The relation of 8 to 2 is found by dividing 8 by 2. $8 \div 2 = 4$.

The relation of one number to another is called their *ratio.*

This principle is nothing new, as every division expresses a ratio, as, also, does every fraction.

Ratio is expressed by the sign : written between the two numbers or quantities. This sign is equivalent to the sign of division, and means that the first number is to be divided by the second. $5)\underline{3}$, $3 \div 5$, $\frac{3}{5}$, 3 : 5, and the ratio 3 to 5, all mean the same thing.

2. What is the ratio (1) of 12 to 3; (2) of 3 to 12?

(1) The ratio of 12 to $3 = \frac{12}{3} = 4$.

(2) The ratio of 3 to $12 = \frac{3}{12} = \frac{1}{4}$.

Find the ratio of:

3. 20 to 4	**4.** 27 to 9	**5.** 2 to 10	**6.** 3 to 15
7. 54 : 9	**8.** 56 : 7	**9.** 12 : 60	**10.** 5 : 40

11. 36 : 6 **12.** 35 : 5 **13.** 8 : 48 **14.** 7 : 63

15. 28 : 7 **16.** 84 : 7 **17.** 12 : 72 **18.** 8 : 32

What is the ratio of:

19. 56 days to 8 days **20.** 5 boys to 60 boys

21. 32 men to 4 men **22.** 8 barrels to 48 barrels

23. Mr. Rich is 40 years old. His son Harry is 8 years old. What is the ratio of the father's age to the son's age?

24. Harriet solved 9 out of 10 problems. What is the ratio of the number solved to the number given?

25. Jennie has 2 dolls. Maggie has 6. What is the ratio of Jennie's dolls to Maggie's dolls?

26. A shoe dealer sold 40 pairs of shoes in the afternoon and 20 pairs in the evening. What is the ratio of the afternoon sales to the evening sales?

The following table contains pairs of fractions whose sums, differences, and quotients have no denominators greater than 16. The exercises should be used frequently for a few moments at a time for quick oral work until pupils acquire accuracy and facility in the use of these simple fractions.

1. Add the fractions in each couplet.

2. Subtract the second fraction in each couplet from the first fraction.

3. Find the product of the fractions in each couplet.

4. Divide the first fraction in each couplet by the second fraction.

5. Compare the first fraction in each couplet with the second fraction.

6. Compare the second fraction in each couplet with the first fraction.

7. Make up simple problems based upon the fractions given in the table.

DRILL TABLE IN FRACTIONS

	A	*B*	*C*	*D*	*E*	*F*
1.	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{2}$			
2.	$\frac{1}{2}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{6}$	$\frac{2}{3}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{3}$	$\frac{5}{6}$ $\frac{2}{3}$
3.	$\frac{1}{2}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{2}$				
4.	$\frac{1}{4}$ $\frac{1}{8}$	$\frac{3}{4}$ $\frac{1}{8}$	$\frac{3}{8}$ $\frac{1}{4}$	$\frac{5}{8}$ $\frac{1}{4}$	$\frac{7}{8}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{3}{8}$
5.	$\frac{3}{4}$ $\frac{5}{8}$	$\frac{7}{8}$ $\frac{3}{4}$	$\frac{1}{2}$ $\frac{1}{8}$	$\frac{1}{2}$ $\frac{3}{8}$	$\frac{5}{8}$ $\frac{1}{2}$	$\frac{7}{8}$ $\frac{1}{2}$
6.	$\frac{1}{3}$ $\frac{1}{9}$	$\frac{1}{3}$ $\frac{2}{9}$	$\frac{4}{9}$ $\frac{1}{3}$	$\frac{5}{9}$ $\frac{1}{3}$	$\frac{7}{9}$ $\frac{1}{3}$	$\frac{8}{9}$ $\frac{1}{3}$
7.	$\frac{2}{3}$ $\frac{1}{9}$	$\frac{2}{3}$ $\frac{2}{9}$	$\frac{2}{3}$ $\frac{4}{9}$	$\frac{2}{3}$ $\frac{5}{9}$	$\frac{7}{9}$ $\frac{2}{3}$	$\frac{8}{9}$ $\frac{2}{3}$
8.	$\frac{1}{2}$ $\frac{1}{5}$	$\frac{1}{2}$ $\frac{2}{5}$	$\frac{3}{5}$ $\frac{1}{2}$	$\frac{4}{5}$ $\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{10}$	$\frac{1}{2}$ $\frac{3}{10}$
9.	$\frac{7}{10}$ $\frac{1}{2}$	$\frac{9}{10}$ $\frac{1}{2}$	$\frac{1}{5}$ $\frac{1}{10}$	$\frac{8}{10}$ $\frac{1}{5}$	$\frac{7}{10}$ $\frac{1}{5}$	$\frac{9}{10}$ $\frac{1}{5}$
10.	$\frac{2}{5}$ $\frac{1}{10}$	$\frac{2}{5}$ $\frac{3}{10}$	$\frac{7}{10}$ $\frac{2}{5}$	$\frac{9}{10}$ $\frac{2}{5}$	$\frac{3}{5}$ $\frac{1}{10}$	$\frac{3}{5}$ $\frac{3}{10}$
11.	$\frac{7}{10}$ $\frac{3}{5}$	$\frac{9}{10}$ $\frac{3}{5}$	$\frac{4}{5}$ $\frac{1}{10}$	$\frac{4}{5}$ $\frac{3}{10}$	$\frac{4}{5}$ $\frac{7}{10}$	$\frac{9}{10}$ $\frac{4}{5}$

12.	$\frac{1}{2}$ $\frac{1}{12}$	$\frac{1}{2}$ $\frac{5}{12}$	$\frac{7}{12}$ $\frac{1}{2}$	$\frac{11}{12}$ $\frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{3}$
13.	$\frac{7}{12}$ $\frac{1}{3}$	$\frac{11}{12}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{12}$	$\frac{2}{3}$ $\frac{5}{12}$	$\frac{2}{3}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{2}{3}$
14.	$\frac{1}{4}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{4}$	$\frac{7}{12}$ $\frac{1}{4}$	$\frac{11}{12}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{12}$	$\frac{3}{4}$ $\frac{5}{12}$
15.	$\frac{3}{4}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{3}{4}$	$\frac{1}{3}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{2}{3}$
16.	$\frac{1}{6}$ $\frac{1}{12}$	$\frac{5}{12}$ $\frac{1}{6}$	$\frac{7}{12}$ $\frac{1}{6}$	$\frac{11}{12}$ $\frac{1}{6}$	$\frac{5}{6}$ $\frac{1}{12}$	$\frac{5}{6}$ $\frac{5}{12}$
17.	$\frac{5}{6}$ $\frac{7}{12}$	$\frac{11}{12}$ $\frac{5}{6}$				

18.	$\frac{1}{2}$ $\frac{1}{14}$	$\frac{1}{2}$ $\frac{3}{14}$	$\frac{1}{2}$ $\frac{5}{14}$	$\frac{9}{14}$ $\frac{1}{2}$	$\frac{11}{14}$ $\frac{1}{2}$	$\frac{13}{14}$ $\frac{1}{2}$
19.	$\frac{1}{2}$ $\frac{1}{7}$	$\frac{1}{2}$ $\frac{2}{7}$	$\frac{1}{2}$ $\frac{3}{7}$	$\frac{4}{7}$ $\frac{1}{2}$	$\frac{5}{7}$ $\frac{1}{2}$	$\frac{6}{7}$ $\frac{1}{2}$

20.	$\frac{1}{3}$ $\frac{1}{5}$	$\frac{2}{5}$ $\frac{1}{3}$	$\frac{2}{5}$ $\frac{1}{3}$	$\frac{4}{5}$ $\frac{1}{3}$	$\frac{2}{3}$ $\frac{1}{5}$	$\frac{2}{3}$ $\frac{2}{5}$
21.	$\frac{2}{3}$ $\frac{3}{5}$	$\frac{4}{5}$ $\frac{2}{3}$				

22.	$\frac{1}{2}$ $\frac{1}{16}$	$\frac{1}{2}$ $\frac{3}{16}$	$\frac{1}{2}$ $\frac{5}{16}$	$\frac{1}{2}$ $\frac{7}{16}$	$\frac{9}{16}$ $\frac{1}{2}$	$\frac{11}{16}$ $\frac{1}{2}$
23.	$\frac{13}{16}$ $\frac{1}{2}$	$\frac{15}{16}$ $\frac{1}{2}$	$\frac{1}{4}$ $\frac{1}{16}$	$\frac{1}{4}$ $\frac{3}{16}$	$\frac{1}{4}$ $\frac{5}{16}$	$\frac{7}{16}$ $\frac{1}{4}$
24.	$\frac{9}{16}$ $\frac{1}{4}$	$\frac{11}{16}$ $\frac{1}{4}$	$\frac{13}{16}$ $\frac{1}{4}$	$\frac{15}{16}$ $\frac{1}{4}$	$\frac{3}{4}$ $\frac{1}{16}$	$\frac{3}{4}$ $\frac{3}{16}$
25.	$\frac{3}{4}$ $\frac{5}{16}$	$\frac{3}{4}$ $\frac{7}{16}$	$\frac{3}{4}$ $\frac{9}{16}$	$\frac{3}{4}$ $\frac{11}{16}$	$\frac{13}{16}$ $\frac{3}{4}$	$\frac{15}{16}$ $\frac{3}{4}$
26.	$\frac{1}{8}$ $\frac{1}{16}$	$\frac{3}{16}$ $\frac{1}{8}$	$\frac{5}{16}$ $\frac{1}{8}$	$\frac{7}{16}$ $\frac{1}{8}$	$\frac{9}{16}$ $\frac{1}{8}$	$\frac{11}{16}$ $\frac{1}{8}$
27.	$\frac{13}{16}$ $\frac{1}{8}$	$\frac{15}{16}$ $\frac{1}{8}$	$\frac{3}{8}$ $\frac{1}{16}$	$\frac{3}{8}$ $\frac{3}{16}$	$\frac{3}{8}$ $\frac{5}{16}$	$\frac{7}{16}$ $\frac{3}{8}$
28.	$\frac{9}{16}$ $\frac{3}{8}$	$\frac{11}{16}$ $\frac{3}{8}$	$\frac{13}{16}$ $\frac{3}{8}$	$\frac{15}{16}$ $\frac{7}{8}$	$\frac{5}{8}$ $\frac{1}{16}$	$\frac{5}{8}$ $\frac{3}{16}$
29.	$\frac{5}{8}$ $\frac{5}{16}$	$\frac{5}{8}$ $\frac{7}{16}$	$\frac{5}{8}$ $\frac{9}{16}$	$\frac{11}{16}$ $\frac{5}{8}$	$\frac{13}{16}$ $\frac{5}{8}$	$\frac{15}{16}$ $\frac{5}{8}$
30.	$\frac{7}{8}$ $\frac{1}{16}$	$\frac{7}{8}$ $\frac{3}{16}$	$\frac{7}{8}$ $\frac{5}{16}$	$\frac{7}{8}$ $\frac{7}{16}$	$\frac{7}{8}$ $\frac{9}{16}$	$\frac{7}{8}$ $\frac{11}{16}$
31.	$\frac{7}{8}$ $\frac{13}{16}$	$\frac{15}{16}$ $\frac{7}{8}$				

DRILL EXERCISE IN RAPID ADDITION AND SUBTRACTION

7	4	6	1	9	5	3	8	2	6	8	5
5											9
3											7
8											2
6											6
4											3
7											8
9											6
3											7
2											5
7											8
5	3	8	9	4	7	2	6	8	7	5	9

1. Beginning with any number in the margin and going in either direction, rapidly add the numbers until 100 or any given number is reached.

2. Beginning with 100 or any given number, rapidly subtract the successive numbers in the margin.

NOTE. The above exercise is valuable only when additions and subtractions are performed rapidly.

MISCELLANEOUS PROBLEMS *Written*

1. Two men do a piece of work for 84 dollars. One does $\frac{3}{7}$ of the work. How much ought each to receive?

2. John had $\frac{1}{2}\frac{5}{0}$ of a dollar. He gave $\frac{2}{5}$ of it to his sister. How much had he left?

3. If he had given $\frac{2}{5}$ of a dollar to his sister, how much would he have had left?

4. After spending $\frac{2}{3}$ of his money for a knife, Austin had 24 cents left. How much had he at first?

5. What is the cost of a yard of cloth when $\frac{1}{2}$ of a yard costs $\frac{1}{2}$ of a dollar?

6. What is the cost of $\frac{1}{2}$ of a yard of cloth at $\frac{1}{2}$ of a dollar a yard?

7. What is the cost of $1\frac{1}{2}$ yards of cloth at $1\frac{1}{2}$ dollars a yard?

8. Rope is sold for $2\frac{5}{8}$ cents a foot. How much will 176 feet cost?

9. A merchant paid $9\frac{3}{5}$ dollars for a dozen hats. He sold them at cost. How much did he receive for each hat?

10. A book which cost $\frac{9}{10}$ of a dollar was sold for $\frac{4}{5}$ of a dollar. What was the loss?

11. What are the daily wages of a man who earns $13\frac{1}{2}$ dollars in a week?

12. A telephone pole 30 feet long was set 6 feet in the ground. What part of the pole was in the ground?

13. A grocer had 72 gallons of molasses. He sold $\frac{1}{4}$ of it to one customer and $\frac{1}{8}$ of it to another. How many gallons had he left?

14. Mr. Brown has 100 dollars. If he pays the grocer $17\frac{1}{2}$ dollars, and buys 8 cords of wood at $5\frac{3}{4}$ dollars a cord, how much will he have left?

15. By the single package, raisins are 12 cents; by the dozen packages, $10\frac{3}{4}$ cents. What is saved by buying a dozen packages at a time?

16. What is the change from 2 ten-dollar bills given to pay for $2\frac{1}{2}$ tons of coal at $6\frac{3}{5}$ dollars a ton?

17. Two boys started from the same point and walked in opposite directions. One walked $3\frac{5}{8}$ miles and the other $2\frac{3}{4}$ miles. How far apart were they then?

18. What is the cost of $\frac{3}{4}$ of a pound of tea at $\frac{2}{5}$ of a dollar a pound?

19. A man earns $17\frac{1}{2}$ dollars a week and saves $\frac{3}{7}$ of it. How much can he save in 4 weeks?

20. Three fourths of a fish line is 36 feet. How long is the line?

21. Another line is $\frac{2}{3}$ of 36 feet. How long is this line?

22. What part of a dollar is 50 cents? 25 cents? 75 cents?

23. What part of a dollar is 20 cents? 40 cents? 60 cents? 80 cents?

24. What part of a dollar is 10 cents? 30 cents? 70 cents? 90 cents?

25. If land is worth 100 dollars an acre, what part of an acre can be bought with 50 dollars? 25 dollars? 75 dollars?

26. What part of a century (one hundred years) is 20 years? 40 years? 60 years? 80 years?

27. A bundle of 10 pencils is what part of 100 pencils? A bundle of 30? A bundle of 70? A bundle of 90?

28. How long will it take Joseph to save 21 dollars for a bicycle if he saves $1\frac{3}{4}$ dollars a week?

29. Every Saturday night Robert puts $\frac{3}{4}$ of a dollar in the savings bank. How much will he save in 20 weeks?

30. If you earn 2 dollars and save $\frac{3}{4}$ of it, how much do you save?

31. If you earn 2 dollars and save 1 cent out of every 10, how much do you save?

MEASURING DISTANCES *Oral and Written*

Distances have one dimension — length.

We measure short distances in inches, feet, and yards.

1. How many lines 1 inch long will make a line 1 foot long?

2. How many feet long is a yardstick?

3. How many inches make a yard?

We measure long distances in rods and miles.

4. $5\frac{1}{2}$ yards make a rod. How many feet in a rod?

5. A distance of 320 rods is a mile. How many feet in a mile?

6. Write the table of long or linear measure.

7. My desk is 54 inches long. Express its length in feet and inches. In feet.

8. One foot minus 3 inches is how many inches? What part of a foot?

9. One stick is $1\frac{1}{2}$ feet long and another 15 inches. Both together will reach how far?

10. A house lot is 4 rods long. What is its length in feet?

11. It is 32 rods around a running track in a playground. How many times will Henry and Forrest go around it in running a mile? $\frac{1}{2}$ of a mile?

12. If your steps are $1\frac{2}{3}$ feet long, how many will you take in walking a mile?

MEASURING SURFACES *Oral*

NOTE. An accurate conception of surface area is rare with young pupils. A little time spent now in developing this idea will prove of great help in subsequent work.

Surfaces have two dimensions — length and width.

We measure surfaces or areas in square inches, square feet, square yards, square rods, acres, and square miles.

A square inch is a square 1 inch long and 1 inch wide.

1. A square foot is a square —— long and —— wide.

2. A square 1 foot long is 12 inches long and 12 inches wide. It contains 12 times 12 square inches, or —— square inches.

3. A square yard is a square —— long and —— wide. It is equal to a square —— feet long and —— feet wide. Its area is —— square feet.

4. $30\frac{1}{4}$ square yards make a square rod. How many square feet make a square rod?

160 square rods make 1 acre.

5. Learn:

TABLE OF SQUARE OR SURFACE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	
	or	= 1 square rod (sq. rd.)
$272\frac{1}{4}$	square feet	
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)

The side on which a figure seems to stand is its *base*.

The height of a figure from the base is its *altitude*.

6. How many sides has a rectangle?

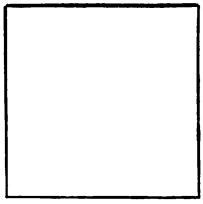
7. How many corners or angles has a rectangle? How do they compare in size?

A RECTANGLE

Each of the angles of a rectangle is a *right angle*.

A figure bounded by four straight lines and having four equal angles is a *rectangle*.

8. How many sides has a square? How do they compare in length?



A SQUARE

9. How many angles has a square? How do they compare in size?

A figure bounded by four equal straight lines and having four equal angles is a *square*.

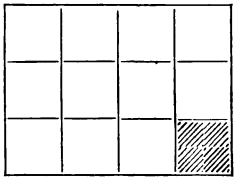
10. In what respects are squares and rectangles alike?

11. In what respect are squares and rectangles unlike?

12. Are all squares rectangles?

The number of square units in the surface of a figure is its *area*.

Let the figure at the right represent a rectangle 4 inches long and 3 inches wide.



The shaded part represents the unit of measurement — one square inch.

13. How many of these units are there in the lower row?

How many rows of these units are there?

Then in the whole figure there are 3 times 4 square inches. The area is 12 square inches.

Note that in finding areas we take these steps:

First. Determine the unit of measurement.

Second. Find the number of these units in one row.

Third. Multiply the number of units in one row by the number of rows.

Think first of the unit of measurement.

The area of a rectangle can always be found by multiplying together its length and its width, when both are expressed in the same unit of measurement (inches, feet, yards, etc.).

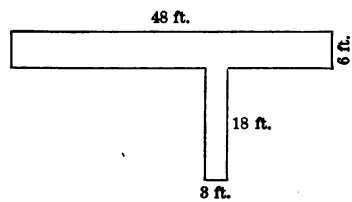
NOTE. Pupils should draw diagrams of rectangles and other plane figures in their problem work until they apprehend the principle involved. The extent to which diagrams are used must be determined by the needs of individuals, since some pupils acquire the powers of visualization and generalization earlier than others.

PROBLEMS *Oral and Written*

1. A post card is 5 inches by 3 inches. How many square inches of writing surface on one side?

2. What is the area of a walk 40 feet long and 3 feet wide?

3. This diagram shows the sidewalk in front of a house and the walk leading to the front door. Find the area of each walk. Express the dimensions of each walk in yards. How many square yards in both walks?



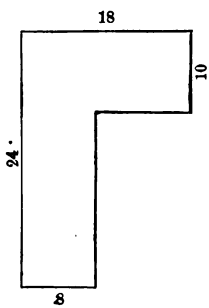
4. A room is 27 feet long and 18 feet wide. Express its dimensions in yards. How many square yards in the floor of the room?

5. A box cover is 15 inches long. Its width is $\frac{2}{3}$ the length. What is the area of the top of the cover?

6. A book is $6\frac{1}{2}$ inches long and 4 inches wide. How much space does it cover on the table?

7. A picture 13 inches by 10 inches is surrounded by a frame 1 inch wide. What are the dimensions of the frame? How much space does the framed picture cover on the wall?

8. The perimeter of a square table is 12 feet. What is the length of one side? What is the area of the top of the table?



9. This diagram represents a field whose dimensions are given in rods. Divide into rectangles and find the area.

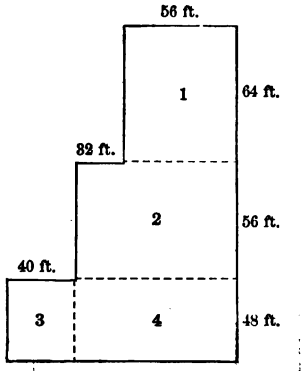
10. Give the dimensions and perimeters of all the different rectangles you can make with 36-inch squares, using all the squares each time. With 48-inch squares. With 60-inch squares.

Find areas and perimeters of these rectangles:

	Length	Width		Length	Width
11.	18 in.	12 in.	**12.**	17 ft.	6 ft.
13.	15 ft.	$8\frac{2}{3}$ ft.	**14.**	14 yd.	16 yd.
15.	$8\frac{3}{4}$ ft.	$2\frac{1}{2}$ ft.	**16.**	25 ft.	5 yd.
17.	$4\frac{1}{2}$ in.	$4\frac{1}{2}$ in.	**18.**	4 yd.	$3\frac{3}{4}$ ft.
19.	30 in.	$2\frac{1}{2}$ ft.	**20.**	$16\frac{1}{2}$ in.	$10\frac{2}{3}$ in.

21. The following diagram represents a plot of ground which was cut up into house lots as indicated by the

dotted lines. Lot 1 was sold for 15¢ a square foot; lot 2 for 12¢ a square foot; lot 3 for 18¢ a square foot; and lot 4 for 20¢ a square foot. Find the selling price of each lot.



22. How many yards of tape will it take to bind a rug $2\frac{1}{2}$ yards long and 1 yard wide? How much space will the rug cover?

Give the areas and the perimeters of:

23. A 4-inch square

24. A 9-yard square

25. A 5-inch square

26. A 10-inch square

27. A 6-foot square

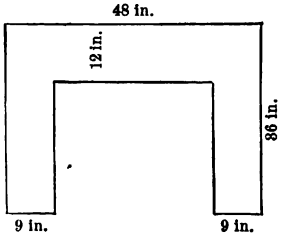
28. An 11-foot square

29. A 7-yard square

30. A 12-yard square

31. An 8-yard square

32. A 20-rod square



33. Find the area of this figure by dividing it into rectangles. Find its perimeter.

34. At 12 cents a square foot, what is the cost of a lot of land 75 feet by 40 feet?

35. A house lot is 75 feet by 48 feet.

(1) Express its dimensions in yards.

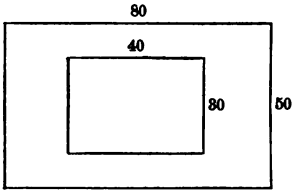
(2) Express the area in square yards. In square feet.

(3) Express the perimeter in feet. In yards.

36. What is the length in yards of a tablecloth that covers 54 square feet if it is 2 yards wide?

37. How many square feet of sod will it take to make a lawn 18 yards long and 9 yards wide?

38. How many strips of turf 4 feet long and 1 foot wide must be used to cover a space 28 feet by 15 feet?



39. A house lot is 50 feet on the street side and has a depth of 80 feet. At 15¢ a foot, what will it cost to fence it? How many square feet in the lot? What is it worth at 18¢ a square foot?

40. A house 30 feet by 40 feet stands in the center of the lot. How far from the street is the front of the house? How far from the sides of the lot does the house stand? How many square feet does the house cover? What part of the lot does it cover?

41. A lot of land is 160 rods long and 1 rod wide. Express its area in square rods. What other name is given to this area?

42. A lot contains 1 acre of land. It is 40 rods long. How wide is it?

43. A farmer has a field containing 2000 square rods. How many acres in the field?

44. It takes 80 rods of fence to inclose a square field. How many acres in the field?

45. What is the area in acres of a square park $\frac{1}{2}$ of a mile on each side?

46. The distance around a square field is $\frac{1}{2}$ of a mile. How many acres in the field?

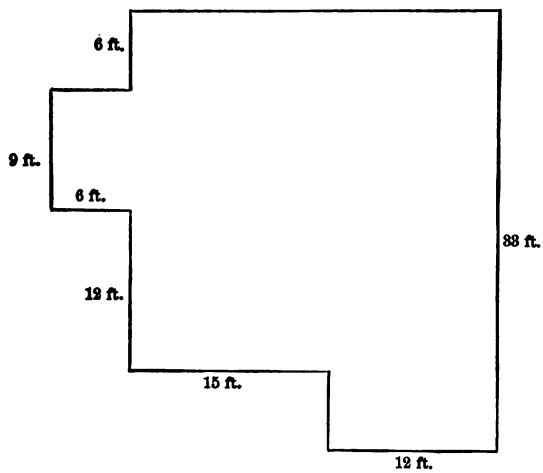
47. How many acres in a lot $\frac{1}{2}$ of a mile long and $\frac{1}{4}$ of a mile wide?

48. At \$45 an acre, what is a field 40 rods by 20 rods worth?

49. A lot of land 9 rods by 6 rods was sold for \$1188. What was the price per square rod? Per acre?

50. At 50 cents a square yard, what will it cost to lay a sidewalk 60 feet by 6 feet?

51. The following diagram represents the ground plan of a house. Find its perimeter. Find its area.



Express:

52. 2880 square inches as square feet.

53. 2880 square feet as square yards.

54. 2880 square feet as square inches.

55. 2880 square yards as square feet.

56. 2880 square rods as acres.

57. 2880 acres as square rods.

DRAWING TO SCALE *Oral and Written*

1. Draw a line 4 inches long. Divide it into four equal parts. If 1 inch represents 1 foot, how many feet does the line represent? $\frac{1}{2}$ of the line? $\frac{3}{4}$ of the line?

2. If 1 inch represents 2 feet, how many feet does the line stand for? $\frac{1}{2}$ of the line? $\frac{3}{4}$ of the line? A line twice as long?

3. On a map a street is represented by a line 12 inches long. If 1 inch represents 1 rod, how long is the street?

4. Letting 1 inch stand for 5 feet, draw a line that will represent 15 feet. How many inches long is your line?

This is drawing to a scale. The scale you have just used is 1 inch to 5 feet.

Scales on which plans, maps, or diagrams are made are usually indicated in this way: $1'' = 5'$, the sign $''$ meaning inches and the sign $'$ feet. Scale $1'' = 5'$ means that 1 inch represents 5 feet.

5. If on a map a line 1 inch long represents the distance from New York to Philadelphia — 90 miles — what is the scale?

6. From New York to Albany is 140 miles. On the scale $1'' = 14$ miles, how long a line will represent the distance between these two cities?

7. On a scale of 1 inch to 4 feet draw a line that will represent 12 feet.

8. On a scale of 1 inch to 3 feet, how many feet does a line 9 inches long represent?

9. Draw a 4-foot square on a scale of 1 inch to 2 feet.

10. On the scale $1'' = 3'$, what length of lines must you draw to represent a square 1 yard long? A rectangle 12 feet by 9 feet?

11. What is the scale when 3 inches stands for 18 rods?

12. On a map a street 60 rods long is represented by a line 10 inches long. What is the scale?

13. My desk is 5 feet long and 3 feet wide. Draw a picture or diagram of its top, letting 1 inch represent 1 foot.

(1) How many inches long is your diagram?

(2) How many inches wide?

(3) What is the perimeter of the diagram? How many feet does it represent?

(4) What is the area of the diagram? How many square feet does it represent?

14. A flower bed is 60 inches by 40 inches. Draw a plan of it on a scale of 1 inch to 10 inches.

15. Another flower bed is 6 yards square. Draw a plan on a scale of 1 inch to 2 yards.

16. On the scale $1'' = 4'$ draw the diagram of a blackboard 4 feet wide and 24 feet long.

17. What are the dimensions of a room represented by a diagram 8 inches long and 5 inches wide if the scale is 1 inch to 2 feet? What is the floor area?

18. On a builder's plan, drawn to scale 1 foot = 10 feet, a house is represented by a rectangle 4 feet by 3 feet. What are the dimensions of the house? Its area?

19. A dining room is $16'$ by $12'$. Draw diagram to scale $1'' = 4'$.

20. The dining room table is $8'$ by $4'$. Draw a diagram of it in the diagram of the room.

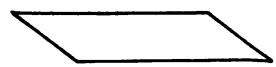
21. Letting 1 inch stand for 20 inches, draw the diagram of a window 60 inches high and 40 inches wide.

22. On a scale of 1 inch to 15 inches draw the diagram of a window sash having 4 panes of glass, each 30 inches by 15 inches.

PARALLELOGRAMS *Oral and Written*

Lines that run in the same direction are *parallel lines.*

A four-sided figure whose opposite sides are parallel is a *parallelogram.*



A PARALLELOGRAM

1. If the shaded part of figure 1 is cut off and placed in the position indicated by the dotted lines, what kind of a figure will you have? See figure 2.

FIG. 1

FIG. 2

2. How does the base of the parallelogram compare with the base of the rectangle?

3. How does the altitude of the parallelogram compare with the altitude of the rectangle?

4. Compare the areas of the parallelogram and the rectangle.

5. Draw on paper a parallelogram 3 inches long and 2 inches wide. Cut it out. Cut the parallelogram into two pieces and arrange them to make a rectangle. Compare bases, altitudes, and areas of the parallelogram and rectangle.

6. Draw other parallelograms. Cut, and arrange the parts until you see that a parallelogram is equal to a rectangle having the same base and the same altitude as the parallelogram.

7. How can you find the area of a parallelogram?

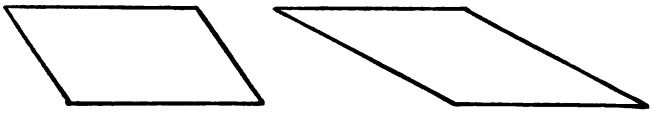
To find the area of a parallelogram, we find the product of its base and its altitude.

8. Draw a rectangle $3\frac{1}{2}$ inches long and 2 inches high. Write the area in the rectangle.

9. Draw a parallelogram whose base is $3\frac{1}{2}$ inches and whose altitude is 2 inches. Write the area in the parallelogram.

10. How do the areas of the two figures you have just drawn compare?

11. Compare the bases and the altitudes of these parallelograms:



12. Find and compare their areas.

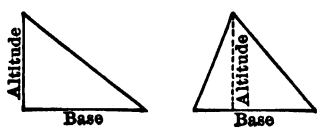
Find areas of parallelograms of these dimensions:

	Base	Altitude		Base	Altitude
13.	12 inches	8 inches	**14.**	18 inches	5 inches
15.	9 feet	10 feet	**16.**	8 yards	9 yards
17.	12 inches	$8\frac{1}{2}$ inches	**18.**	16 feet	$5\frac{3}{8}$ feet
19.	$10\frac{2}{3}$ yards	6 yards	**20.**	$16\frac{1}{2}$ feet	12 feet
21.	4 yards	8 feet	**22.**	18 inches	3 feet

NOTE. Measurement of plane figures made from or drawn on cardboard will prove helpful and interesting. A variety of these figures should be prepared by the teacher, numbered consecutively, and a record of their dimensions and areas kept to facilitate the checking of pupils' work.

Finding measurements and areas of plane figures from the actual figures and from diagrams drawn on the blackboard should precede finding of areas from data given by the teacher.

TRIANGLES *Oral and Written*



A three-sided figure is a *triangle.*

The height of a triangle is its *altitude.*

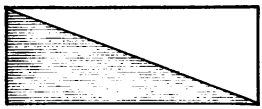


FIG. 1

FIG. 2

1. What kind of a figure is the shaded part of figure 1?

2. Compare the base of the triangle with the base of the rectangle.

3. Compare the altitude of the triangle with the altitude of the rectangle.

4. What part of the area of the rectangle is the area of the triangle?

5. How, then, can the area of a triangle be found?

6. In like manner compare the shaded part of figure 2 with the whole parallelogram.

7. Draw on paper a rectangle 4 inches by 3 inches. Cut it into two parts as in figure 1. Compare areas.

8. Draw on paper other parallelograms. Cut each into two parts along the diagonal. Compare areas.

9. Draw on paper a triangle whose base is 4 inches and whose altitude is 2 inches. Cut it out. Cut another triangle exactly like this. Arrange the two triangles so as to form a parallelogram. What are the dimensions of the parallelogram? Compare the base of the parallelogram and the base of the triangles. Compare the altitude of the parallelogram and the altitude of the triangles. What is the area of the parallelogram? The area of each triangle is what part of the area of the parallelogram? What is the area of each triangle?

The area of a triangle is equal to one half the product of its base and its altitude.

To find the area of a triangle, we find one half the product of its base and its altitude.

Note that the dimensions must be expressed in like units.

NOTE. Work like the above should be continued until pupils grasp the principle involved. Different pupils should draw, cut, and compare parallelograms and triangles of different dimensions. Care should be taken that most of the parallelograms and triangles are not rectangles and right-angled triangles.

Give the areas of these triangles:

	ALTITUDE	BASE		ALTITUDE	BASE
10.	10 inches	12 inches	**11.**	15 inches	18 inches
12.	9 inches	3 inches	**13.**	11 inches	7 inches
14.	25 feet	18 feet	**15.**	17 feet	12 feet
16.	13 rods	8 rods	**17.**	7 yards	9 yards

18. What are the base and altitude of the largest triangle you can cut from a piece of paper 4 inches square?

19. What are the dimensions of the largest triangle you can cut from a piece of paper 5 inches by 3 inches?

20. What are the base and altitude of the largest triangle you can draw on a sheet of your arithmetic paper? How does the area of this triangle compare with the area of the sheet on which it is drawn?

21. In the corner of a room is a triangular shelf. The two sides that touch the wall are each 10 inches in length. What is the area of the shelf? On the shelf stands a box

4 inches long and $2\frac{1}{2}$ inches wide. How many square inches of the shelf does it cover?

22. Three roads form the sides of a triangular lot. The base of the lot is 22 feet and the altitude is 18 feet. How many square feet in the lot?

23. A field 32 rods long and 20 rods wide is separated into two equal triangular parts by a path running between two opposite corners. What are the base and the altitude of each part? How many acres in each part?

24. At 15 cents a square foot, what is the value of a three-sided lot of land whose base is 64 feet and whose altitude is 40 feet?

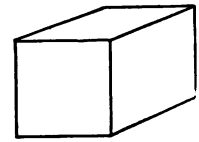
25. How many square yards are there in a triangular lot whose base is 18 yards and whose altitude is one half the length of the base?

26. The height of a triangle is 24 inches. The base is $\frac{3}{4}$ as long. What is the area of the triangle?

27. A triangular flower bed is 36 inches on each side. How many feet of wire netting will inclose it?

MEASURING VOLUMES *Oral and Written*

A number of 1-inch cubes should be used in teaching this subject.



1. How many sides or faces has a cube?

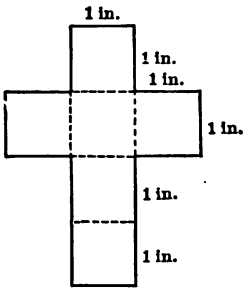
2. How do the sides compare in shape?

3. How do the sides compare in size?

A solid bounded by six equal sides or faces is a *cube*.

A solid has three dimensions—length, breadth, and thickness.

4. Draw on cardboard a figure like this. Cut it out and fold on dotted lines. Paste, sew, or pin the edges together. You have made a cube 1 inch long, 1 inch wide, and 1 inch high. This is called an inch cube or a cubic inch. How many sides or faces has it? How do they compare in size? What is the shape of each face? What is the area of each face? What is the area of all the faces?



5. Could you have told the area of the surface of the cube from the diagram?

6. With the inch cubes build a solid like figure 1, 3 inches long, 2 inches wide, and 1 inch thick. This is a rectangular solid or rectangular prism.

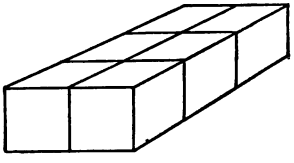


FIG. 1

How many cubic inches are there in 1 row? In both rows?

We say its contents or volume is 6 cubic inches.

The number of cubic units in a solid is its *volume*.

7. With the inch cubes build a solid like figure 2, 3 inches long, 2 inches wide, and 3 inches high.

How many cubic inches in 1 row of the bottom layer? Then in the bottom layer there are 2 times 3 cubic inches, or 6 cubic inches.

How many layers are there? Then in the whole solid there are 3 times 6 cubic inches, or 18 cubic inches.

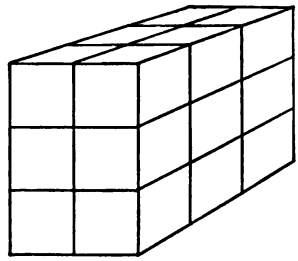


FIG. 2

Note that in finding volumes we take these four steps:

First. Determine the unit of measurement.

Second. Find the number of these units in one row of the lower layer.

Third. Multiply the number of units in one row by the number of rows.

Fourth. Multiply the number of units in one layer by the number of layers.

Think first of the unit of measurement.

The volume of a solid can always be found by multiplying together its length, its width, and its height, when all are expressed in the same unit of measurement (inches, feet, yards, etc.).

NOTE. Practice in computing volumes of blocks, boxes, and so forth, from measurements made by pupils, should precede the solution of problems from data given by the teacher.

Give the volumes of these rectangular prisms:

8. 2 in. by 4 in. by 5 in.

9. 3 in. by 4 in. by 2 in.

10. 4 in. by 5 in. by 3 in.

11. 6 in. by 5 in. by 2 in.

12. 5 in. by 8 in. by 2 in.

13. 3 in. by 8 in. by 2 in.

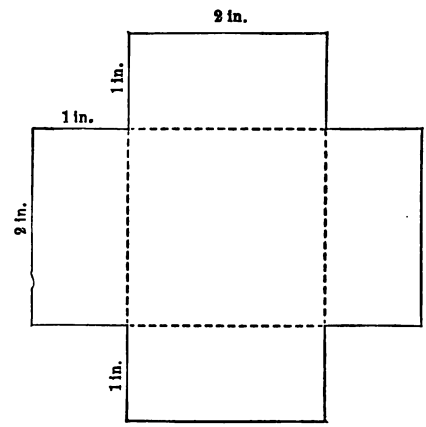
14. 3 in. by 5 in. by 4 in.

15. 6 in. by 2 in. by 8 in.

16. 10 in. by 3 in. by 6 in.

17. 12 in. by 5 in. by 4 in.

18. With the help of this diagram construct a box that will hold 4 cubic inches.



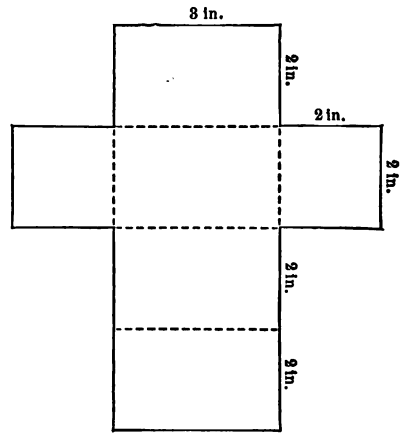
19. How high will you have to make the sides of a box of the same base to hold twice as much? Make one.

20. With the diagram below as an aid, construct a rectangular prism 3 inches by 2 inches by 2 inches.

21. How many sides has this rectangular prism?

22. Of what shape are the sides? Are all the sides equal?

23. What is the area of the two ends? Of the four sides? What is the total area of the six sides?



24. Could you determine the surface area from the pattern?

25. How many 1-inch prisms could you put into the prism you have just made?

26. A cube 1 foot long, 1 foot wide, and 1 foot high is a cubic foot.

27. Express its dimensions in inches.

28. A cubic foot contains $12 \times 12 \times 12$ cubic inches, or —— cubic inches.

29. Describe a cubic yard.

30. Express its dimensions in feet.

31. What is its volume in cubic feet?

32. Write the table of cubic measure.

33. Make a pattern of a 2-inch cube. Cut it out and fold it into a cube.

How long is this cube? How wide? How high?

What is the area of one of its faces? Of all its faces?

What is its volume? How many 1-inch cubes will it take to make a 2-inch cube?

34. What is the volume of a 3-inch cube?

35. How many inch cubes can you put into a box 4 inches on each edge?

36. How many cubic inches are occupied by a book 6 inches long, $3\frac{1}{2}$ inches wide, and 1 inch thick?

37. The inside measurements of a box are 5 inches, 3 inches, $1\frac{1}{2}$ inches. What is its capacity?

38. A drawer in a desk is 8 inches by 5 inches by $1\frac{3}{4}$ inches. What is its capacity?

39. A coal bin is 10 feet by 6 feet by 4 feet. How many cubic feet of coal will it hold when even full?

Find the volumes of these rectangular prisms:

	Length	Width	Height		Length	Width	Height
40.	10 ft.	6 ft.	5 ft.	41.	9 in.	7 in.	4 in.
42.	15 ft.	12 ft.	9 ft.	43.	18 in.	15 in.	1 ft.
44.	14 ft.	4 ft.	$\frac{3}{4}$ ft.	45.	20 in.	$8\frac{1}{2}$ in.	5 in.
46.	$18\frac{3}{4}$ ft.	16 ft.	$4\frac{1}{2}$ ft.	47.	27 in.	$4\frac{2}{3}$ in.	$\frac{1}{2}$ ft.
48.	$3\frac{5}{8}$ in.	3 in.	$1\frac{1}{3}$ ft.	49.	$1\frac{1}{4}$ ft.	1 ft.	18 in.

50. Find the surface areas of the prisms in examples 8 to 17 on page 311.

WOOD MEASURE

Wood is usually sold by the cord.

1. A pile of wood 8 ft. by 4 ft. by 4 ft. is a cord. How many cubic feet in a cord?

2. $\frac{1}{8}$ of a cord is a cord foot. How many cubic feet in a cord foot?

3. Learn:

16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet = 1 cord (cd.)
128 cubic feet = 1 cord

4. How many cords of wood in a pile 8 feet long, 4 feet wide, and 8 feet high? How many in a pile 16 feet by 4 feet by 8 feet?

5. A wagon body 4 feet wide and 12 feet long has on it a pile of wood 6 feet high. How many cords?

6. By the roadside near a farmer's house I saw a pile of wood 4 feet wide, 6 feet high, and 18 feet long. How many cords in the pile?

7. A leather firm bought from this farmer a pile of hemlock bark 4 ft. × 4 ft. × 16 ft. How many cords?

8. Express 1 cord, 16 cord feet as cords.

9. How many cubic feet in three quarters of a cord?

10. How many cords in 1728 cubic feet of bark?

DECIMALS *Oral and Written*

Dimes, cents, and mills are decimal parts of a dollar.

Dimes are written in the first place at the right of the decimal point as *tenths* of a dollar; cents are written in the second place at the right as *hundredths* of a dollar; mills in the third place at the right as *thousandths* of a dollar.

A dime, or a tenth of a dollar, is written $.1.

A cent, or a hundredth of a dollar, is written $.01.

A mill, or a thousandth of a dollar, is written $.001.

$0.87 may be read 87 hundredths of a dollar.

$0.875 may be read 875 thousandths of a dollar.

Read as parts of a dollar:

1. $0.6; $0.05; $0.003; $0.802; $0.025.

2. What do the 0's show in the numbers you have just read?

3. Write decimally $\frac{5}{10}$ of a dollar; $\frac{50}{100}$ of a dollar; $\frac{87}{100}$ of a dollar; $\frac{125}{1000}$ of a dollar; $\frac{25}{1000}$ of a dollar; $\frac{5}{1000}$ of a dollar.

Write decimally:

4. 7 hundredths of a dollar.

5. 70 hundredths of a dollar.

6. 75 hundredths of a dollar.

7. 75 thousandths of a dollar.

8. 225 thousandths of a dollar.

9. 5 thousandths of a dollar.

10. How many places are used to express tenths of a dollar? Hundredths of a dollar? Thousandths of a dollar?

We can express other things besides dimes, cents, and mills as tenths, hundredths, and thousandths. Thus, .25 yd. This means 25 hundredths of a yard.

11. Read: .5 bu.; .75 A.; .287 mi.; .08 rd.

2.25 yd. means 2 whole yards and 25 hundredths of a yard. It is read two and twenty-five hundredths yards.

Whenever we read a number made up of a whole number and a decimal, we always use the word *and* to mark the decimal point.

12. Read: 2.5 ft.; 3.275 mi.; 4.08 sq. rd.; 7.006 A.

13. Write decimally:

$\frac{7}{10}$; $\frac{8}{100}$; $\frac{47}{100}$; $\frac{84}{1000}$; $\frac{6}{1000}$;
$\frac{75}{100}$ bu.; $\frac{5}{10}$ in.; $\frac{22}{1000}$ A.; $\frac{205}{1000}$ mi.

14. Write decimally:

$5\frac{3}{10}$; $2\frac{4}{100}$; $3\frac{15}{100}$; $6\frac{2}{1000}$;
$3\frac{7}{10}$ yd.; $7\frac{25}{100}$ in.; $5\frac{8}{100}$ sq. yd.; $8\frac{4}{1000}$ mi.

Read:

15.	.8	.96	.07	.519	.806	.087	.005
16.	.3	.03	.33	.303	.033	.003	.333
17.	4.7	3.64	5.07	7.602	8.319	9.054	2.008

Write in figures:

18. Seven tenths.
19. Five hundredths.
20. Nine thousandths.
21. Seventeen thousandths.
22. Sixty-eight hundredths.
23. One hundred two thousandths.
24. Three hundred eighty-seven thousandths.
25. Four and nineteen hundredths.
26. Thirty-two and four hundred seven thousandths.
27. Sixteen and six thousandths.

CHANGING DECIMALS TO COMMON FRACTIONS

1. Write as common fractions: .1; .01; .001.

$$.1 = \tfrac{1}{10} \quad .01 = \tfrac{1}{100} \quad .001 = \tfrac{1}{1000}$$

Write these decimals as common fractions:

2.	.2	.4	.6	.8	.3	.5	.7	.9
3.	.12	.07	.67	.05	.83	.07	.56	.03
4.	.125	.402	.019	.009	.047	.004	.103	.005

5. Write .6 as a common fraction and change to its simplest form:

$$.6 = \frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

Express these decimals as common fractions in their simplest form:

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
6.	.2	.4	.6	.8	.02	.04	.06	.08
7.	.25	.50	.75	.66	.32	.56	.24	.48
8.	.15	.45	.65	.35	.85	.64	.84	.76

Express as common fractions in their lowest terms:

.5 .50 .500

How do .5, .50, and .500 compare in value?

Ciphers annexed to a decimal do not change its value. Why?

CHANGING COMMON FRACTIONS TO DECIMALS

1. Write as tenths of a dollar: $\frac{1}{2}$ of a dollar; $\frac{1}{5}$ of a dollar; $\frac{2}{5}$ of a dollar; $\frac{3}{5}$ of a dollar; $\frac{4}{5}$ of a dollar.

2. Write as hundredths of a dollar: $\frac{1}{2}$ of a dollar; $\frac{1}{4}$ of a dollar; $\frac{3}{4}$ of a dollar; $\frac{1}{5}$ of a dollar; $\frac{2}{5}$ of a dollar; $\frac{3}{5}$ of a dollar; $\frac{4}{5}$ of a dollar.

3. Write as hundredths of a dollar: $\frac{1}{20}$ of a dollar; $\frac{7}{20}$ of a dollar; $\frac{1}{25}$ of a dollar; $\frac{12}{25}$ of a dollar; $\frac{1}{50}$ of a dollar; $\frac{9}{50}$ of a dollar.

4. Express decimally, first as tenths, then as hundredths: $\frac{1}{2}$; $\frac{1}{5}$; $\frac{3}{5}$; $\frac{2}{5}$; $\frac{4}{5}$.

5. Express decimally as hundredths: $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{9}{10}$; $\frac{1}{20}$; $\frac{11}{20}$; $\frac{1}{25}$; $\frac{6}{25}$; $\frac{9}{25}$; $\frac{1}{50}$; $\frac{7}{50}$; $\frac{27}{50}$.

Write as whole numbers and decimals:

6. $2\frac{1}{2}$; $2\frac{1}{4}$; $5\frac{3}{4}$; $2\frac{1}{5}$; $1\frac{3}{5}$; $7\frac{3}{10}$.

7. $5\frac{1}{20}$; $3\frac{7}{20}$; $6\frac{1}{25}$; $2\frac{4}{25}$; $7\frac{1}{50}$; $5\frac{17}{50}$.

8. Express as the decimal of a foot: 6 inches; 3 inches; 9 inches.

9. What decimal part of an hour is 3 minutes?

3 minutes $= \frac{3}{60} = \frac{1}{20} = \frac{5}{100} = .05$ of an hour.

Express as decimals of an hour:

10. 30 minutes; 15 minutes; 45 minutes.

11. 12 minutes; 24 minutes; 36 minutes; 48 minutes.

12. 6 minutes; 18 minutes; 42 minutes; 54 minutes.

13. 9 minutes; 21 minutes; 33 minutes; 57 minutes.

DECIMALS: ADDITION *Oral and Written*

Add:

1. \$0.60	2. 6 dimes	3. 6 tenths	4. .6
.20	2 dimes	2 tenths	.2
5. \$0.05	6. 5 cents	7. 5 hundredths	8. .05
.04	4 cents	4 hundredths	.04
9. \$0.375	10. 375 thousandths		11. .375
.233	233 thousandths		.233

In adding decimals, why must tenths come under tenths, hundredths under hundredths, and so on?

Add by rows and by columns:

	A	*B*	*C*	*D*	*E*
12.	6.73	+ 18.4	+ 8.5	+ 60	+ 4.003
13.	5.8	+ 7.29	+ 7.06	+ 6	+ .87
14.	.97	+ 3.07	+ 4.12	+ .6	+ .008
15.	.008	+ 15.007	+ 10.01	+ .06	+ 5.17
16.	70.49	+ 3.9	+ 9.004	+ .006	+ 4.09

Write in columns and add:

17. .5, .27, .08, .762, .007.
18. .007, .64, .303, .09, .3.
19. .606, .04, .005, .008, .7, .3.
20. .302, .08, .009, .54, .16, .016.
21. .97, .087, .07, .05, .09, .008.
22. .07, .017, .009, .108, .05, .012.
23. 4.37, 2.05, 9.007, .03, 4.1.
24. 8.007, .37, 6.09, 4.304, .006.
25. 5.5, .004, 3.018, 6.704, .076.
26. 4.85, 3.001, 5.07, .008, .02.

DECIMALS: SUBTRACTION *Oral and Written*

Subtract:

1. \$0.63	2. \$0.80	3. \$0.08	4. \$0.40
.44	.60	.05	.37
5. \$1.00	6. \$0.625	7. \$0.600	8. \$0.008
.05	.375	.045	.005
9. 1.000	10. 1.000	11. .087	12. .308
.025	.004	.009	.088
13. .402	14. .072	15. .067	16. .6
.391	.006	.059	.27

In example 16 think .6 as hundredths.

17. .8	18. .57	19. .7	20. .69	21. .1
.34	.3	.07	.6	.05
22. .563	23. .427	24. .8	25. .5	26. .3
.5	.42	.425	.463	.292

27. From .4 take .4; .04; .004.

28. From 8 take .8; .08; .008.

29. From one take one tenth; one hundredth; one thousandth.

30. From one tenth take one tenth; one hundredth; one thousandth.

31. From one hundredth take one hundredth; one thousandth.

32. From ten take one tenth.

33. From one hundred take one hundredth.

34. From one thousand take one thousandth.

35. From 13.7 take 6.08.

36. Take .017 from 6.6.

37. From 1.672 take 1.005.

38. Take .305 from 1.055.

39. From 27.06 take 14.3.

40. Take 14.07 from 70.04.

41. From 3.002 take .998.

42. Take 7.006 from 10.04.

43. From 643.7 take .691.

44. Take 1.125 from 11.325.

PROBLEMS *Oral and Written*

1. Edward walked .3 of a mile and rode .5 of a mile. How far did he go?

2. A stick of braid contained 4 yards. The dressmaker used .5 of a yard. How much was left?

3. In making candy, Emma used .25 of a pound of chocolate and .75 of a pound of sugar. What was the weight of both?

4. If you spend .6 of your money, how much will you have left?

5. Charles bought a necktie for .25 of a dollar and a collar for .15 of a dollar. What part of a dollar did he pay for both?

6. My pencil was 7 inches long. How long was it after I had used 1.75 inches?

7. The grocer sold .5 of a bushel of potatoes to one customer and .625 of a bushel to another. How many bushels did he sell?

8. Mr. Hollis has three pastures, one of 4.75 acres, one of 25.5 acres, and one of 8.42 acres. What is the area of the three?

9. William raised a bushel of strawberries. He sold .125 of a bushel to Mrs. Waite, .25 of a bushel to Mrs. Long, and the rest to the grocer. What part of a bushel did he sell to the grocer?

10. The three sides of a triangle are 12.4 ft., 18.65 ft., and 15.75 ft. What is the perimeter?

11. The perimeter of a triangle is 8.5 ft. Two sides are respectively 2.25 ft. and 3.8 ft. What is the length of the third side?

12. A playground contains 7.32 acres. In it is a pond covering 1.67 acres. What is the area not covered by the pond?

13. A tub of maple sugar weighs 34.625 pounds. The tub itself weighs 3.875 pounds. What is the weight of the sugar?

14. The weights of five tubs of butter were 30.125 lb., 28.5 lb., 29.875 lb., 30.25 lb., and 27 lb. What was the total weight?

DECIMALS: MULTIPLICATION *Oral and Written*

1. 3 times 3 apples = ——
2. 3 times 3 tenths = ——
3. $3 \times \frac{3}{10} = \frac{9}{10} = .9$
4. $3 \times .3 = .9$

5. Multiply .3 by .3.

$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100} = .09$

$.3 \times .3 = .09$

Express both decimals as common fractions.

Multiplying $\frac{3}{10}$ by $\frac{3}{10}$, we get $\frac{9}{100}$, which, written decimally, is .09.

In multiplying .3 by .3 it is clear that, since the denominators are 10 and 10, the denominator of the product must be 10×10, or 100. A decimal expressing hundredths occupies two decimal places, which is the sum of the decimal places in the multiplicand (.3) and the multiplier (.3).

6. Multiply .03 by .3.

$\frac{3}{100} \times \frac{3}{10} = \frac{9}{1000} = .009$

$$\begin{array}{r} .03 \\ .3 \\ \hline .009 \end{array}$$

The product of the denominators is 1000. A decimal expressing thousandths occupies three decimal places. This is the sum of the decimal places in the multiplicand (.03) and the multiplier (.3).

To multiply decimals, we multiply as in whole numbers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.

Note that the "pointing off" is the multiplying together of the denominators.

7. How many decimal places are there in the product when we multiply units and tenths? $3 \times .2$.

8. How many when we multiply units and hundredths? $3 \times .02$.

9. How many when we multiply units and thousandths? $3 \times .002$.

10. How many when we multiply tenths and tenths? $.3 \times .2$.

11. How many when we multiply tenths and hundredths? $.3 \times .02$.

Multiply, orally, by 2 each number in the table:

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
12.	1	4	7	2	6	9	3	5	8
13.	.1	.4	.7	.2	.6	.9	.3	.5	.8
14.	.01	.04	.07	.02	.06	.09	.03	.05	.08

15. Use 1, 3, 5, 7, 9, 2, 4, 6, 8, 10, 11, 12 as multipliers.

16. Use .1, .3, .5, .7, .9, .2, .4, .6, .8, 1.1, 1.2 as multipliers.

17. Multiply each number in the first two rows by .01, .02, .03, .04, .05, .06, .07, .08, .09.

18. Victor is 7.5 years old and Hubert is twice as old. How old is Hubert?

19. Sarah has 50 cents. Marion has .5 as much. How many cents has Marion?

20. There are 80 trees in an orchard. .3 of them are pear trees. How many pear trees?

21. What is the area of a square .5 of a yard long? What is its perimeter?

22. The three sides of a triangle are each 2.4 feet long. What is the total length of the sides?

23. How many square rods in a rectangle .7 of a rod long and .6 of rod wide? What is the perimeter?

Multiply:

24. .76	**25.** 2.07	**26.** 3.4	**27.** 6.25	**28.** 3.07
42	63	8.7	1.4	8.9
29. 39	**30.** .045	**31.** 27.3	**32.** 78.5	**33.** 4.55
.07	52	4.4	.08	6.6
34. .096	**35.** .308	**36.** 408	**37.** 78.5	**38.** .875
75	47	.027	1.07	64
39. 25.04	**40.** 500.5	**41.** 648	**42.** 39.3	**43.** 720
56	3.17	.035	2.06	.225
44. 30.05	**45.** 520.7	**46.** 28.76	**47.** 1728	**48.** 17.28
.48	4.05	1.15	.375	37.5

49. The multiplicand is .643; the multiplier is 867; what is the product?

50. Multiply sixty and six tenths by ten and one tenth.

51. $6.06 \times 5.5 \times 2.002 = ?$

MULTIPLYING BY 10, 100, 1000

$$.222 \times 10 = 2.22$$
$$.222 \times 100 = 22.2$$
$$.222 \times 1000 = 222.$$

1. In multiplying .222 by 10, the decimal point was moved how many places to the right? How many places to the right was it moved in multiplying by 100? How many places to the right was it moved in multiplying by 1000?

2. Change the decimal point in 1.234 so that you will have a number 10 times as great. So that you will have a number 100 times as great. So that you will have a number 1000 times as great.

Write numbers 10 times as great as these:

3. .284 3.75 42.6 .008 .06 .3

4. 3.706 .903 4.62 .05 .4 .007

5. Write numbers 100 times as great; 1000 times as great.

6. What is the weight of 10 chickens if each weighs 3.75 pounds?

7. What is the total length of 100 boards, each 6.25 feet long?

8. How many yards of cloth in 1000 pieces, each of which contains 27.5 yards?

9. Frederick's cap cost \$0.25; his shoes cost 10 times as much, and his suit 100 times as much. How much did the shoes cost? The suit? How much did all cost?

MULTIPLYING BY .1, .01, .001

$$222 \times .1 = 22.2$$
$$222 \times .01 = 2.22$$
$$222 \times .001 = .222$$

1. In multiplying 222 by .1 the decimal point was moved how many places to the left? How many in multiplying by .01? How many in multiplying by .001?

2. Change the decimal point in 3456 so that you will have a number .1 as great. So that you will have a number .01 as great. So that you will have a number .001 as great.

Multiply by .1:

3.	$525	$37.50	$2.45	$0.70	$0.66	$0.04
4.	236	42.5	3.17	.5	.75	.03

Multiply by .01:

5.	4325	372.5	30.4	100.1	.2	1.1

Multiply by .001:

6.	46,800	1000	144	36	8	1

7. Howard had 80 marbles. He lost .1 of them. How many did he lose?

8. Mr. Wilson paid $1000 for an automobile and .01 as much for a license to run it. How much did he pay for the license?

9. Out of 125,000 yards of cloth .001 was found imperfect. How many yards were poor?

PROBLEMS *Written*

1. There are 16.5 feet in a rod. What is the length in feet of a fence 7 rods long?

2. How many feet in 320 rods or 1 mile?

3. What is the area of a square 15.4 yards long?

4. Henry has 8 rows of peas. He gathers 2.75 bushels from 1 row. How many bushels will he probably get from the other rows?

5. Mr. Moulton mowed 2.6 acres of grass in a day. How many acres will he mow in 3.5 days?

6. A square lot is 32.07 rods on a side. How many rods of wall will inclose it?

7. A cubic foot of water weighs 62.5 pounds. What weight of water will a tank 2 feet square and 3 feet high hold?

8. Ice weighs .92 as much as water. What is the weight of a cubic foot of ice?

9. How far will a railroad train run in 2.4 hours if the rate is 40.75 miles an hour?

10. A cow gives 3.2 gallons of milk a day. How many pounds is this if a gallon weighs 8.625 pounds?

11. Mr. Slater's house-lot contains .65 of an acre. His pasture is 10 times as large, and his garden is .1 as large. What is the size of the pasture? Of the garden?

12. Our school paid 75 dollars for trees for the school grounds, .1 as much for flowering shrubs, and .01 as much for seeds for the vegetable garden. How much was paid for shrubs? For seeds? How much was paid for all?

DECIMALS: DIVISION *Oral and Written*

Divide:

1. 2)8 dollars
2. 2)$8.00
3. 2)$1.68
4. 2)$0.68
5. 2)$0.60
6. 2)6 tenths
7. 2).6
8. 2)64 hundredths
9. 2).64
10. 2)648 thousandths
11. 2).648
12. 2).608
13. 2).812

Note that in dividing a decimal by a whole number the decimal point in the quotient comes directly under the decimal point in the dividend. The first step in division is to write the decimal point in the dividend.

Divide, and test your work:

	A	*B*	*C*	*D*	*E*
14.	3)3.696	3)36.96	3)3.696	3).603	3).906
15.	4)3.08	4)177.2	4)1.984	4)2.24	4)4.08
16.	5)14.5	5)2.045	5)4.05	5)3.55	5)2.65
17.	6)2.76	6).72	6)8.4	6).84	6).726
18.	7)8.05	7)85.4	7).924	7)285.6	7)35.7
19.	8)1.28	8)34.4	8)4.32	8).808	8)11.52
20.	9)12.78	9)7.2	9)63.36	9)54.72	9)9.009

21. Divide .36 by 9.

9).36 There being no tenths in the quotient, we write a 0 in
.04 the tenths' place.

22. Divide .008 by 4.

4).008 Why do we write two 0's in the quotient in this divi-
.002 sion?

23. Divide .2 by 5. .2 may be written .20.

Divide, and test your work:

	A	*B*	*C*	*D*	*E*
24.	3).018	2).08	4).036	7).056	5).005
25.	6).072	7).049	4).028	5).14	2).01
26.	8).04	5).065	9).729	6).426	8).056
27.	7).084	6).006	8).12	7).28	9).198

28. Divide 12.88 by 28.

.47
28)13.16
11 2
196
196

In long division be careful to place the decimal point in the quotient directly over the decimal point in the dividend.

Divide, and test your work:

	A	*B*	*C*	*D*
29.	22.68 by 27	34.68 by 34	9.90 by 45	1.44 by 1
30.	17.92 by 32	5.184 by 24	51.84 by 72	15.75 by 1
31.	17.28 by 36	172.8 by 24	1.728 by 48	34.56 by 1
32.	345.6 by 16	3.456 by 32	34.56 by 54	.3456 by 2
33.	35.68 by 16	775.2 by 19	7.752 by 38	77.52 by 5

Find the quotient of :

34.	$4.536 \div 42$	$26.20 \div 35$	$5.12 \div 64$	$21.28 \div 76$
35.	$46.72 \div 73$	$4.672 \div 146$	$.522 \div 29$	$5.04 \div 84$
36.	$74.16 \div 72$	$68.4 \div 90$	$374.48 \div 62$	$17.385 \div 57$
37.	$416.56 \div 82$	$6.916 \div 28$	$38.52 \div 36$	$1297.8 \div 63$
38.	$.552 \div 92$	$3.12 \div 39$	$44.8 \div 56$	$816.08 \div 202$

PROBLEMS *Written*

1. A coal dealer sent out 5.25 tons of coal in 3 equal loads. What was the weight of each load?

2. Maggie used .5 of a yard of cloth in making 2 dresses for her dolls. How much was used for each dress?

3. A merchant sold 8 pairs of shoes for $13.20. How much was this a pair?

4. What is the side of a square whose perimeter is 36.24 square rods?

5. Richmond rode his bicycle 17.4 miles on Tuesday and $\frac{1}{2}$ as far on Wednesday. How far on Wednesday?

6. It took 15 fence rails to build a fence 118.5 feet long. What was the length of each rail?

7. If 57.75 tons of hay were cut from 7 fields, what was the average cut from each field?

8. My gas bills for six months were $1.89, $2.16, $2.43, $1.80, $2.70, $2.52. What was the average cost of the gas a month?

9. At the rate of 17 miles an hour, how long will it take to go to a place 40.8 miles away?

10. In 6 days a range burned 2.4 thousand cubic feet of gas. What part of a thousand cubic feet was this a day?

BILLS AND RECEIPTS

WASHINGTON, July 1, 1910.

Mr. CHARLES R. WATSON

Bought of CROSBY & MARSH

Mar. 3	3 pr. Shoes	@	$2.15	$6	45	
Apr. 7	3 pr. Slippers	@	.83	2	49	
June 1	2 pr. Rubbers	@	.69	1	38	
						$10 32

Received payment
July 15, 1910
CROSBY & MARSH
By Goodwin.

When were the above purchases made?

By whom were the goods bought?

From whom were they bought?

What was bought?

What did each kind cost?

What did all cost?

When was the bill paid?

What shows that the bill has been paid?

Was the money paid directly to the owners of the store or to one of their clerks? How do you know?

Who is the creditor in the above bill? Why?

Who is the debtor? Why?

Mr. Ames sells his black horse to Mr. Baker. Who is the debtor?

Mr. Childs buys a house from Mr. Burns. Who is the debtor?

1. Complete the following bill :

BUFFALO, July 29, 1910.

MRS. HENRY P. DUNCAN

Bought of ARTHUR P. DAVIS

2 lb. Figs	@	$0.20	$		
3½ lb. Raisins	@	.14			
5¾ lb. Mixed Nuts	@	.16			
4 lb. Candy	@	.35			
					$
Received payment ARTHUR P. DAVIS.					

When purchases are made at one time, the date is written in the heading only.

Make out bills for the following school supplies. Buyer, the city in which you live. Seller, yourself.

2.

120 reams of paper @ 35¢
12 boxes of pens @ 32¢
25 dozen pencils @ 18¢

3.

50 arithmetics @ 65¢
75 arithmetics @ 42¢
20 number cards @ 3½¢

4.

68 grammars @ 54¢
38 geographies @ 95¢
18 geographies @ 75¢

5.

4 wall maps @ $3.75
100 spelling books @ 18¢
35 readers @ 25¢

6. Make out the bill for 10 grammars, 12 number cards, and 20 spelling books at the prices given above.

7. Mr. George R. Hamilton used 14,000 pounds of ice during the year 1909. Make out his bill at $3 a ton.

8. Mr. Alfred Smith buys $6\frac{1}{2}$ tons of coal at $6.50 a ton and 2 tons at $6.75 a ton. Make out his bill.

9. The pupils in the Jackson school bought the following seeds for their school garden: 8 10-cent packets of nasturtiums, 6 5-cent packets of poppies, and 5 5-cent packets of asters. Make out the bill.

10. Make out your bill for cutting your neighbor's lawn three times: on July 10 you work 6 hours, on July 24 you work $6\frac{1}{2}$ hours, and on Aug. 7, $5\frac{1}{2}$ hours. You receive 20 cents per hour.

11. Imagine that you sell to a hotel 4 barrels of potatoes at $3.35 per barrel, 2 bushels of peas at $1.75 per bushel, 2 boxes of lettuce at 65 cents each, and $1\frac{1}{2}$ bushels of beans at $1.12 per bushel. Make out the bill.

12. Robert put electric bells in his house. He paid $0.75 for one bell and $0.60 for the other. It took $1\frac{1}{2}$ pounds of wire at 20 cents a pound. He used a 6-cent paper of tacks, and 2 buttons at 12 cents each. Make out the bill, using your own name as seller.

13. Make out the bill for three articles purchased by your mother at the grocer's.

14. Make out your milk bill for the month of April.

15. Make out the bill for three kinds of fruit you see every day in the stores.

16. Make out other bills for goods purchased at different stores, using the prices given in the daily paper.