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GENERAL

PREFACE.

THIS volume, which is issued as Part II. of an Elementary Text-Book of Physics, deals with the elements of *Sound.*

An effort has been made to give, as simply and as clearly as possible, a fairly complete exposition of the fundamental facts and principles of the subject. The experiments described in the text are intended to illustrate and develop the theory, but in most cases the descriptions are given with sufficient experimental detail to be of service in the laboratory.

This Part is complete in itself, but a knowledge of some of the elementary principles of Dynamics dealt with in Part I. is necessarily assumed.

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CONTENTS.

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SOUND.

CHAPTER I.

SIMPLE HARMONIC VIBRATION.

1. **Simple Harmonic Motion.**—Let P (Fig. 1) be any point on the circumference of the circle APB, and AB *any* diameter of the circle. From P draw P*p* perpendicular to the diameter

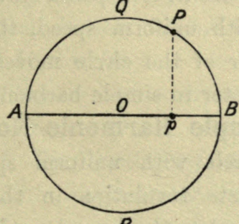

Fig. 1.

AB, and meeting the diameter at the point *p*. The point *p* is the *projection* of the point P on the diameter AB. For different positions of the point P on the circumference of the circle, the point *p* will have different positions on the diameter AB, for the point *p* will in all positions be the foot of the perpendicular from P on to the diameter.

Now, imagine the point P to move round the circumference of the circle with uniform speed, and consider the corresponding motion of the point p along the diameter AB. It will be seen that as P moves round and round the circle the point p moves backwards and forwards along the diameter AB, making a complete backward and forward movement for each complete revolution made by P. The point P makes a complete revolution from any starting point on the circumference of the circle every time it passes through the starting point; the point p, therefore, makes a complete backward and forward movement from any starting point on the diameter every time it passes through the starting point in the same direction as it had at the instant of starting. Thus, when P moves round the circle from B through Q, A, and R back to B, the point p moves along the diameter AB from B, through O, to A, and back through O to B. Or, as P moves from Q through A, R, and B back to Q, the point p moves from O to A, back through O to B, and then back to O again.

The point p is said to move with **simple harmonic motion** along the line AB. That is, if a point moves round the circumference of a circle with uniform speed, the projection of this point on any diameter of the circle moves backwards and forwards along the diameter in simple harmonic motion.

2. **Period of Simple Harmonic Motion.**—The point P moves round the circle with uniform speed, and, therefore, describes each complete revolution in the same time. The point p makes a complete backward and forward movement for each complete revolution made by P, and must, therefore, describe each complete movement in a definite constant period of time equal to the time occupied by P in making one complete revolution. This period of time is known as the **period** of the motion. In the case of a point moving in simple harmonic motion, the **period** of the motion may, therefore, be defined as the time occupied by the point in making one complete backward and forward movement. The motion of a point in simple harmonic motion is evidently a repetition, period after period, of

complete backward and forward movements, each described in a constant period of time. For this reason the motion of the point is said to be **periodic**. Simple harmonic motion is the simplest type of periodic motion.

3. **Amplitude of Simple Harmonic Motion.** — The motion of the point *p*, as described above, is defined as simple harmonic motion. The line AB is the path of the motion, and O is the middle point or centre of the path of motion. The distance OA or OB is the greatest distance the point travels from O during its motion; this distance is known as the **amplitude** of the motion.

In the case of a point moving in simple harmonic motion the **amplitude** of the motion may, therefore, be defined as the greatest distance the point moves from the centre of its path during the motion.

The distance of a point in simple harmonic motion from the centre of its path is sometimes called the **displacement** of the point. The term displacement, used in this way, implies that the centre of the path is the normal position of the moving point when at rest. If this term is used the amplitude of the motion may be defined as the maximum displacement of the point during the motion.

4. **Variation of Displacement during Simple Harmonic Motion.**—The displacement of a point moving in simple harmonic motion varies, during a complete period, in a manner specially characteristic of the motion.

If the point *p* (Fig. 2) moving in simple harmonic motion along AB be supposed to start a complete backward and forward movement from the centre of its path, and to move from O towards A, the successive positions of the point at intervals of one-sixteenth of a period from the start are indicated by the points 0, 1, 2, 3, . . . 16 on the line AB, and the corresponding successive displacements of the point are, therefore, represented by the distances 01, 02, 03, . . . measured from O towards A or B. These distances represent, therefore, successive values of the displacement Op at intervals of one-sixteenth of a period, and indicate the manner in which this displacement varies during a complete period. The points 0, 1, 2, 3, . . . 16 on the line AB are easily obtained in the following manner:—The point P moves round the circle with uniform speed; its successive positions at intervals of one-sixteenth of the period of motion from the start are, therefore, obtained by dividing the circumference of the circle into sixteen equal parts. As the point p is supposed to start from 0, and to move towards A, the point P must start from the point marked 0 on the

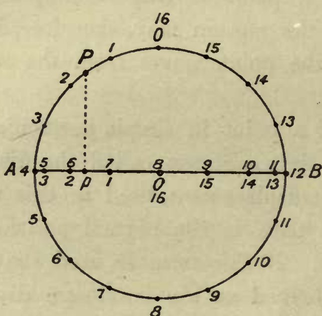

Fig. 2.

circumference of the circle in the same direction, and the division of the circumference into sixteen equal parts must, therefore, begin at this point, giving the points 0, 1, 2, 3 . . . 16 shown on the circumference. The corresponding successive positions of the point p during its motion along AB are merely the projections of these points on the line AB.

5. **The Displacement Curve for Simple Harmonic Motion.**—The variation of the displacement of a point in simple harmonic motion during a period starting from the instant the point passes through the centre of its path is conveniently exhibited by means of a curve. The motion of the point p, as

above described, is represented by the curve shown in Fig. 3. In this figure distance along OX represents time, and the scale of representation is such that the distance OX represents the period of the motion. The distances 01, 12, 23, . . . being each one-sixteenth of OX, evidently represent one-sixteenth of the period. The ordinates of the curve at the points 1, 2, 3, &c., represent successively the displacements of the point at intervals of one, two, three, &c., sixteenths of a period from the start, as shown in Fig. 2.

The curve OBX is, therefore, drawn so that its ordinate at any point represents the displacement of the point p at a particular instant. Thus, if the point A is so placed that OA is

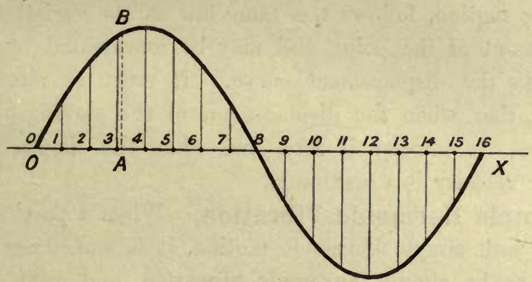

Fig. 3.

one-fifth of OX, then AB, the ordinate of the curve at A, represents the displacement of the point p at an instant one-fifth of a period after the point passes to the left through the centre of its path of motion.

A curve drawn in this way is usually called a **displacement curve.** It shows how the displacement of a point moving in simple harmonic motion varies from instant to instant during a complete period. The **form** of the curve is characteristic of the motion, for whatever may be the period or amplitude of the motion the form of the curve is always the same.

6. **Velocity of a Point in Simple Harmonic Motion.**— The velocity of a point in simple harmonic motion evidently

varies from point to point in its path. From Fig. 2 it can be seen that the point p moves over the unequal distances 01, 12, 23, 34, &c., in successive equal intervals of time. Its velocity, therefore, changes from point to point in its path, and by comparing the distances passed over in successive equal intervals of time, it is evident that it decreases as the point travels away from the centre of the path, and increases as it travels towards the centre. At the extreme points of the path, the points 4 and 12 in Fig. 2, the point is for an instant at rest. As it passes through the centre of its path, in either direction, its velocity is a maximum.

The variation of the velocity of a point moving in simple harmonic motion, follows the same law as the variation of the displacement of the point, and may be represented by a curve similar to the displacement curve. It must be remembered, however, that when the displacement of the moving point is a maximum, its velocity is zero, and when the displacement is zero, the velocity is a maximum.

7. **Simple Harmonic Vibration.**—When a particle moves with simple harmonic motion, it is sometimes said to be in **simple harmonic vibration,** and each complete backward and forward movement of the particle is called a **complete vibration.** A heavy particle, such as a shot or small bullet, suspended, as shown at O in Fig. 4, by a fine elastic thread, may be readily set in simple harmonic vibration. When displaced vertically and released, it moves or **vibrates** up and down along the vertical path AB in simple harmonic motion. If the period of the motion, or the period of vibration, be determined by noting the time of a number of vibrations, and then calculating the time of one from the data obtained, it will be found that the period is constant as long as the motion continues.

Fig. 4.

The **amplitude** of the motion will be observed to decrease as the motion continues, until the particle finally comes to rest, but the period of the motion remains constant throughout.

The displacement of the particle from the centre of its path (which, in this case, is the normal position of rest for the particle) varies also, in the manner described in Art. 5, for simple harmonic motion. If the particle moved sufficiently slowly for its position and displacement to be observed at intervals of one-eighth or one-sixteenth of a period, and if the observed displacements were plotted to give a curve as shown in Fig. 3, it would be found that the curve would have the form characteristic of simple harmonic motion.

The motion of the bob of a simple pendulum is a familiar example of simple harmonic vibration. Thus, if a small bullet or brass ball be suspended by a fine thread, about a metre long, and set in vibration as a pendulum, with swings *of small amplitude*, it will be found that the motion of the bob approximates very closely to simple harmonic vibration.

The bob O, as shown in Fig. 5, moves backwards and forwards along a small arc AB. If the amplitude of vibration is small, the arc is very short, and may be considered, without serious error, as a straight line. The **path** of vibration of the bob is, therefore, practically a straight line.

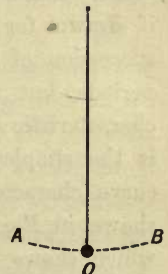

Fig. 5.

If the **period** of vibration is determined by direct observation, it will be found to be constant as long as the pendulum continues to swing. The amplitude of vibration decreases as the motion dies away, but the period of the motion is constant throughout.

The **displacement** of the bob from the centre of its path (the position of the bob when at rest) varies also in the manner required for simple harmonic vibration, and the displacement curve, if drawn from experimental data, would be found to have the form characteristic of simple harmonic motion.

From what has been said above, it will be plain that a particle in simple harmonic vibration fulfils three main conditions.

(1) The path of motion is a straight line.

(2) The period of motion is constant and independent of the amplitude. This condition is characteristic of every type of periodic motion, and is not sufficient by itself to specify simple harmonic motion. It is sometimes referred to as the **isochronism** of the motion.

(3) The displacement of the particle varies from instant to instant, in the manner described in Art. 5, and the displacement curve for the motion of the particle has the form characteristic of simple harmonic motion.

This last condition is specially characteristic of simple harmonic vibration. The law of variation of the displacement, or the form of the displacement curve, is the criterion which serves to distinguish the different types of periodic motion from each other. For any type of periodic motion the displacement curve, if drawn for a number of successive periods, consists of a succession of exactly similar curve lengths for the successive periods, but the form of the curve length for a period is characteristic of the type of motion. Simple harmonic motion is the simplest type of periodic motion, and the displacement curve characteristic of it is represented by the simple curve shown in Fig. 3. This curve is similar in form to the curve which shows the variation of the sine of an angle with the magnitude of the angle, and is, therefore, sometimes called the **sine curve**.

Experiment 1.—Set up a simple pendulum so that its bob moves in front of a wall or vertical screen. By means of a movable mark on the wall or screen, take different points in the path of motion of the bob as the starting point for determining complete vibrations. Note that, wherever the starting point be taken, the bob describes successive complete vibrations in the intervals between its successive passages, *in the same direction*, through the starting point. Determine the time occupied by 5, 10, 15, and 20 complete vibrations, as given by three or four different starting points, and calculate from each observation the average period of vibration of the pendulum. It will be found that, within the limits of experimental error, the average value of the period is the same for all the observations taken. This indicates that the period of vibration is constant.

Experiment 2.—Set up a simple pendulum with a heavy bob and a length of at least 4 metres. Arrange that the bob moves in front of a plainly marked scale, placed parallel to the path of the bob. The scale should be divided into inches, and the divisions numbered from a zero at the centre outwards in both directions. This scale should be placed so that when the bob is at rest, a fine pointer attached to the bob is immediately in front of the zero mark on the scale. With a scale marked and arranged in this way, the position of the bob at any instant during its vibration can be read off on the scale, and the displacement of the bob at that instant can therefore be determined. Now adjust a metronome to give 120 ticks per minute, or a clock to tick half-seconds. Then, when the pendulum is in steady motion, read off, on the scale, the position of the bob at each successive tick of the metronome or clock. From the data thus obtained the displacement curve for the motion of the bob can be plotted. It will be found that the form of the curve is that characteristic of simple harmonic motion.

8. **Frequency of Vibration.**—It has been explained that in the case of simple harmonic vibration, the period of the motion is constant. That is, the time occupied by one complete vibration is constant. It follows from this that a particle in simple harmonic vibration makes a constant number of complete vibrations in a second. **The number of complete vibrations per second** made by a particle in simple harmonic vibration is called the **frequency** of the vibration, or, simply, the **vibration frequency.**

The frequency and the period of any simple harmonic vibration are evidently related, so that one is the reciprocal of the other. Thus, if a particle makes n complete vibrations per second—that is, if its frequency is n vibrations per second—then the time of one complete vibration or the period of the motion is evidently $1/n$ second. Similarly, if t seconds be the period of the motion, then the vibration frequency is $1/t$ vibrations per second.

Numerical Examples.—If the period of vibration of a particle is ·05 second ($\frac{1}{20}$ second), then the vibration frequency for the particle is 20 vibrations per second.

If the period of vibration is ·003 second ($\frac{3}{1000}$ second), the vibration frequency is $\frac{1000}{3}$ or 333$\frac{1}{3}$ vibrations per second.

If a particle takes 84 seconds to make 50 complete vibrations, the vibration period is 1·68 seconds.

9. **Vibration of Extended* Bodies.**—Under certain conditions an extended body, such as a solid rod or strip, a string or wire, a metal plate, a membrane, a bell, or a column of gas or liquid in a pipe, may be set in vibration. Each particle in the body vibrates in a manner determined by its position in the body, and by the mode of vibration established; and the concerted vibration of the system of particles which make up the body constitutes what is called the vibration of the body.

Thus a thin strip of wood or steel, such as a flat ruler or steel scale, may be set in vibration by fixing one end in a vice, as shown in Fig. 6, and plucking the free end aside. When the free end is pulled aside from its position of rest to the point A, and then let go, the strip flies back from the dotted position shown at A to the dotted position shown at B, and then vibrates backwards and forwards with gradually decreasing amplitude between two extreme positions—corresponding to those shown at A and B in the figure.

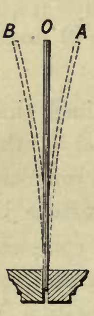

Fig. 6.

In this case the path of vibration of each particle is evidently a short arc joining the two extreme positions of the strip at the point where the particle is placed. The amplitude of vibration is greatest for the particle at the free end of the strip, and decreases to zero at the fixed end, but the period of

* The term *extended* body is used to distinguish it from a particle or body of negligible small dimensions. An "extended" body is a body which extends to appreciable dimensions and consists of a very large number of particles.

vibration is the same for each particle, and the strip, as a whole, has a definite constant period of vibration.

A string or wire stretched between two fixed points may be set in vibration by plucking it aside, or striking it lightly at any point; a metal plate, when properly mounted, may be set in vibration by striking it or bowing its edges; a stretched membrane vibrates when struck lightly with a suitable hammer; a bell vibrates when struck by the tongue; and a column of air in a pipe may be set in vibration by blowing obliquely across the open end of the pipe.

In the case of the vibration of extended bodies, the particles of the body very rarely vibrate in simple harmonic vibration. The motion is always periodic, but the type of motion is usually a compound or complex type of periodic motion, and not the simple harmonic motion described above. In the case of the vibration of the thin strip dealt with above, however, the motion of the particles approximates very closely to simple harmonic motion.

10. **Longitudinal and Transverse Vibration.**—In the case of extended bodies, such as a rod or string, of which the dimension usually called the **length** is obvious and well marked, it is necessary to distinguish between two possible modes of vibration, known as longitudinal and transverse vibration. If the paths of vibration of the particles of the body in vibration are parallel to the length of the body, the vibration is said to be **longitudinal**, but if the paths of vibration are at right angles to the length of the body, the vibration is **transverse.**

Thus, if a rod, or strip of wood or metal, is fixed in a vice, as in Fig. 6, and set in vibration by plucking the free end aside laterally every particle in the rod vibrates in a path at right angles to the length of the rod, and the vibration is said to be transverse vibration. But if the rod is grasped lightly about the middle with a glove or rubber lightly sprinkled with powdered rosin, and the hand drawn smartly along the rod towards the free end, the rod can be set in longitudinal vibration. When excited to vibration in this way, every particle in the rod vibrates in a path parallel to the length of the rod.

The vibration of a stretched string, when plucked aside at any point in its length, is also an instance of transverse vibration. Each point on the string vibrates in a path at right angles to its length. A stretched string or wire may, however, be set in longitudinal vibration by rubbing it lengthways with a suitable rubber.

CHAPTER II.

PRODUCTION OF SOUND.

11. **Sound may be Produced by Vibration.**—It will be explained later that a body in vibration is not always a source of sound, and that all sounds are not produced by a body in vibration. Thus, the bob of a simple pendulum in vibration is not a source of sound, and the report of a gun is not produced by a vibrating body as its source.

With certain limitations, however, a vibrating body is a source of sound, and it will be found that practically all musical sounds have a body in vibration as their source.

When a tightly stretched violin string is set in transverse vibration by plucking it or bowing it a sound is heard, and it can be seen from the blurred appearance of the string that it is in rapid vibratory motion. As the vibration dies away the sound becomes fainter and fainter and ultimately ceases. If the vibration is suddenly stopped by touching the string the sound at once ceases.

If a glass bell-jar is set in vibration by bowing the rim a sound is at once produced, and the quivering motion of the rim which accompanies the vibration of the jar can be clearly seen. Even when the frequency of the motion is too high or the amplitude too small for the motion of the rim to be visible, it can be detected by bringing a pith ball suspended by a thread near the side of the jar, or by touching it with the point of a pencil held lightly in the hand. It will always be found that if a sound is heard the jar is in vibration. Also, as in the case of the string, as the vibration dies away the sound becomes fainter and fainter, and ultimately ceases when the vibration ceases. If the vibration is suddenly stopped by touching the jar the sound at once ceases to be heard.

If a tuning fork is struck or bowed so as to set the prongs in transverse vibration a sound is at once produced, and ceases only when the vibration dies away or is suddenly stopped. The vibration of the fork is readily detected by touching the tongue or teeth with one of the prongs, or by sprinkling sand on the face of one of the prongs held in a horizontal position. It can be made evident to the eye in an interesting way by attaching a short and fairly stiff bristle or wire, B, to the end of one prong of the fork P, as shown in Fig. 7, and then, when the fork is in vibration, drawing the end of the bristle lightly along the smoked surface of a piece of glass. A wavy line, such as that shown in the figure, is traced by the bristle point on the smooth smoked surface. The best way of smoking the glass surface is to hold it in the smoky part of the flame of a piece of burning camphor. The vibration of a tuning fork is also conveniently exhibited to the eye by attaching a small, light plane mirror with wax to the flat outer surface of one of the prongs of the fork. The image of a bright point seen in this mirror is a point when the fork is at rest, but becomes a short vertical line when the fork is in vibration. If the fork is moved quickly sideways when the eye is fixed on this vertical line, the line assumes the wavy form shown in Fig. 7.

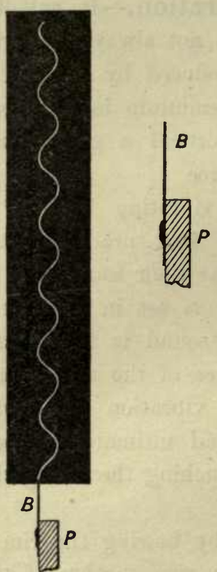

Fig. 7.

12. Sound Produced by the Vibration of a Rod or Strip.—If a short and fairly stiff steel rod be fixed in a vice, as shown in Fig. 6, and set in transverse vibration by striking it lightly with a small mallet, a sound similar to that emitted by a tuning fork is produced. A tuning fork, which consists essentially of a steel rod bent at the middle into a narrow U-shaped fork, is an example of the application of the transverse vibration of a rod for the production of sound. A common form of the instrument is shown in Fig. 8.

Similarly, a thin strip of sheet metal may, if short enough, be set in transverse vibration so as to produce a sound. The tongues of the **reeds** used in some musical instruments, such as the harmonium, are short rectangular strips or tongues of thin metal fixed on a metal plate so as to cover, or partly cover, rectangular openings in the plate. The vibration of these tongues is not the source of sound in the instrument in which

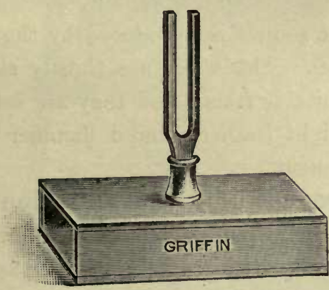

Fig. 8.

they are used, but they control by their vibratory motion the passage of an air blast through the rectangular openings in the plate on which they are fixed, and the rapid succession of air puffs thus obtained through these openings is the real source of sound.

A thin strip of metal resting horizontally on two supports, placed at points about one-third of the length of the strip from each end, may be set in transverse vibration, and become a source of sound, by striking it about the middle point with a light wooden hammer. In the musical toy called the **harmonicon** the sounds are produced in this way.

If a glass rod or tube is clamped in a horizontal position at the middle point, and excited to longitudinal vibration by rubbing it lengthways with a pad of cotton wool dipped in alcohol, it emits a sound of very high pitch produced by the longitudinal vibration of the rod. The longitudinal vibration of rods has no application in musical instruments.

13. **Sound Produced by the Transverse Vibration of a String or Wire.**—A suitable string or wire tightly stretched between two points emits a musical sound when set in transverse vibration by plucking, bowing, or striking it lightly at any point.

In all stringed instruments of the violin and banjo type the sounds are produced by the transverse vibration of the strings of the instrument.

In the piano the sounds are produced by the transverse vibration of steel wires. The wires are tightly stretched between wrest-pins on a suitable frame, and they are set in vibration by the stroke of a light leather-covered hammer actuated by the keys of the instrument.

14. **Sound produced by the Vibration of a Plate or Membrane.**—A metal plate, mounted as shown in Fig. 9, may be set in vibration by striking it lightly at particular points, or by bowing it across an edge; when in vibration it emits a clear, musical sound. The vibration of the plate is easily made apparent to the eye by sprinkling sand on it. If the plate, when at rest, is lightly sprinkled with sand, and then set in vibration, the sand is at once disturbed by the motion of the plate, and collects along certain nodal lines in a pattern characteristic of the mode of vibration of the plate.

The circular metal gongs, in common use as dinner gongs, are examples of the use of metal plates as a source of sound. They are usually set in vibration by striking them with a soft, leather-covered hammer.

A bell is practically a plate bent into the usual bell form. In form it bears something the same relation to a plate as a tuning fork bears to a rod. When struck by the tongue, the bell is set in vigorous vibration, and produces a well-defined and familiar sound.

A membrane tightly stretched across a circular frame, as in the case of a drum or tambourine, may be set in vibration, and made to act as a source of sound by striking it with a soft hammer or with the hand.

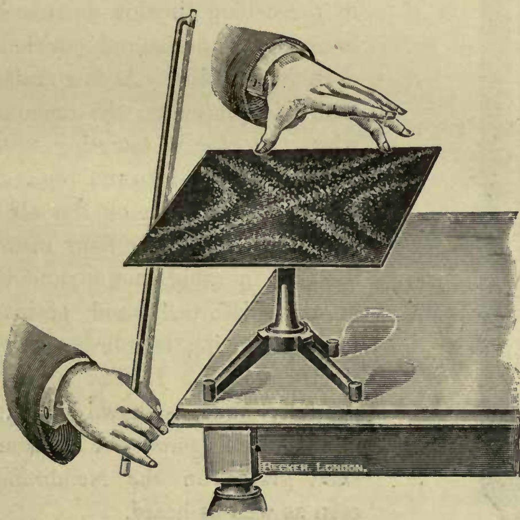

Fig. 9.—Chladni's plate.

15. **Sound produced by the Longitudinal Vibration of a Column of Air in a Pipe.**—A column of air in a pipe may be set in longitudinal vibration in various ways, but most effectively by directing a stream of air across the open end of the pipe, or across a specially constructed opening in the side of the pipe. When the column is thus set in vibration, it at once becomes a source of sound. The sounds produced by an ordinary whistle, a flute, an organ pipe, and other similar

"wind" instruments, are caused by the longitudinal vibration of the air column in the tube or pipe. In fact, in all wind instruments, the sounds produced by them are due to the vibration of the masses of air which fill the tubes of the instruments.

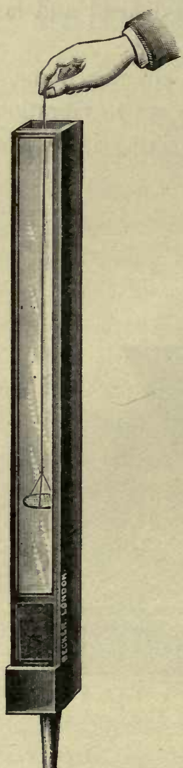

Fig. 10.—Organ-pipe sand pan.

The vibration of the air column in an organ pipe can easily be detected. A small pan, made by stretching a fine membrane over a light circular frame, is suspended by three long threads, so that it can be lowered in a horizontal position into an open organ pipe. A few grains of dry sand are placed on the membrane, and when the pipe is sounding strongly, the pan is lowered into the pipe. The up-and-down vibration of the air particles in the pipe sets the light membrane of the pan in rapid motion, and the rattle of the disturbed sand grains on the rapidly moving membrane can be distinctly heard. If the organ pipe is specially made, as shown in Fig. 10, with one side of glass, the motion of the sand grains on the membrane can be seen as well as heard.

If the motion of air particles at different points in the length of the pipe is studied by means of this simple apparatus, it will be found that all the particles in the air column conform to a definite scheme or system of vibration, which constitutes the vibration of the column as a whole.

16. **Limits of Frequency for the Production of Sound by Vibration.**—It has already been stated that a body in

vibration is not always a source of sound. It will be noticed that in all the examples, given in the foregoing articles, where the vibrating body is a source of sound, the frequency of vibration is comparatively high. On the other hand, it is noticeable that in all cases where the frequency of vibration is very low, the vibrating body is not a source of sound; thus, a long lath in transverse vibration, with one end fixed in a vice, is not a source of sound.

These facts suggest that there may be a lower limit of frequency below which a vibrating body is not a source of sound. This suggestion may be investigated by studying the vibration of a thin steel strip, with one end fixed in a vice.

Experiment 3.—Take a thin steel strip or rod, about 60 cm. long, and fix it in a vice so that a length of about 50 cm. is free to vibrate. It will be noticed that the vibrations are fairly slow, and that the motion of the rod can be clearly followed by the eye; also, no sound is heard. Now fix the rod further into the vice, so that a length of about 40 cm. is free to vibrate. It will be noticed that the frequency of vibration, although still low, is higher than in the first instance, but that no sound is produced.

Repeat this process of shortening the free length of the rod, until a decisive result is obtained. It will be found that, as the free length of the rod is reduced, the frequency of vibration increases, until, ultimately, a sound of very low pitch is produced. If the experiment is continued beyond this point, it will be found that the vibrating rod continues to be a source of sound, the only difference being that the pitch of the sound heard rises as the vibration frequency of the rod increases.

This experiment indicates that when the vibration frequency is below a certain limit, a vibrating body is not a source of sound. Careful experiments with different vibrating bodies confirm this result, and show that when the vibration frequency is below about 30 per second, the vibrating body is not a source of sound.

When the vibration frequency is above 30 per second, a sound is produced which rises in pitch as the frequency increases. If, however, the frequency becomes higher than a variable, but very high, limit no sound is heard by the human ear. This upper limit for the audibility of sound lies generally, for man, between 30,000 and 40,000 vibrations per second; it varies considerably for different persons, and is probably much higher for dogs, cats, and other animals than for men.

The upper limit of audibility cannot readily be determined experimentally by any very simple direct method. It can, however, be readily found by means of a specially constructed

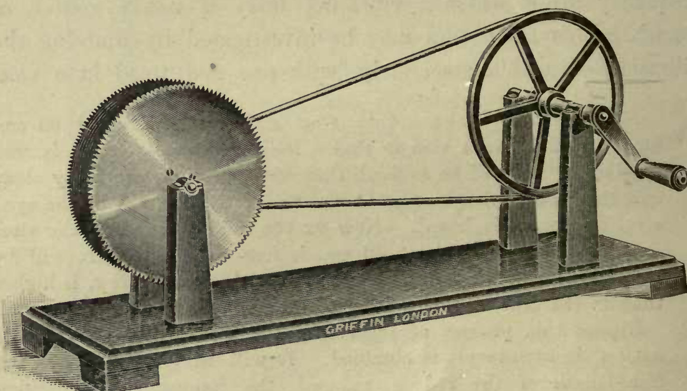

Fig. 11.—Savart's wheel.

whistle of very small dimensions. By shortening the length of the air column in the whistle, the pitch of the sound produced can be raised until it reaches the limit of audibility in any particular case. The vibration frequency of the air column in the whistle can then be determined indirectly by measuring the length of the column.

These limitations of audibility are obviously physiological limitations determined by the structure of the ear.

17. **Savart's Wheel.**—Savart's wheel is a toothed wheel mounted, as shown in Fig. 11, so that it can be set in rapid rotation. If the wheel is set in rapid rotation, and a card is

held so that the teeth of the moving wheel strike lightly on the edge of the card, a sound is produced by the regular motion of the card.

The card cannot be said to be, in the proper sense of the term, a vibrating body, but it is kept in rapid **periodic** motion by the action of the teeth of the wheel, and if the frequency of its motion is greater than 30 vibrations per second it acts as a source of sound.

Savart used this wheel as a means of determining the vibration frequency of the card as a source of sound.

It is evident that if the number of teeth in the wheel is known, and the rate of revolution of the wheel observed for any particular sound, the vibration frequency of the card can be at once determined. Thus, if a wheel with n teeth makes N revolutions per second the vibration frequency of the card must be given by Nn.

Numerical Example.—If a Savart wheel having 20 teeth makes 15 revolutions per second the vibration frequency of the card is evidently (20×15) or 300 per second.

By means of this wheel Savart determined the lower and upper limits of frequency for the production of sound by the vibration (or periodic motion) of the source.

18. **Seebeck's Siren.**—Sound can also be produced by the periodic interruption of a jet of air so that the jet is broken up into a regular succession of puffs at equal intervals of time. This periodic interruption of a steady air jet may be conveniently effected by the following simple piece of apparatus invented by Seebeck, and known as **Seebeck's siren,** or sometimes as the **cardboard siren.** A circular disc of cardboard or metal has one or more rings of equidistant holes pierced round its edge, as shown in Fig. 12, and is mounted so that it can, like Savart's wheel, be set in rapid rotation. A jet of air from a fine nozzle is directed on the disc so that the jet strikes a ring of holes as the disc rotates.

By this arrangement it is obvious that, during the rotation of the disc, the jet is broken up into a series of puffs. Every time a hole in the disc comes opposite the nozzle of the jet, a puff of air through the hole occurs, and if the disc rotates rapidly at uniform speed, a rapid succession of regularly timed puffs is thus produced.

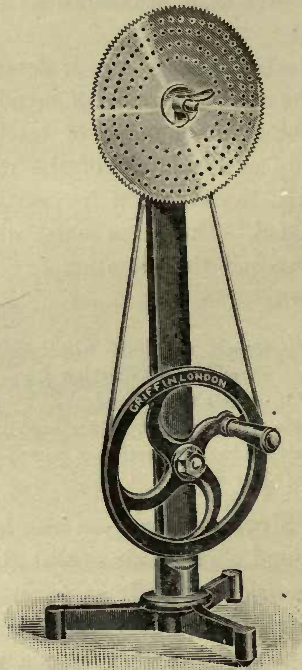

Fig. 12.—Cardboard siren.

It is found that, provided the number of puffs per second is more than thirty, a characteristic sound is produced in this way. The source of sound, in this case, is the intermittent jet of air; it cannot be said to be in vibration in the strict meaning of the word, but it has a definite **periodic** motion impressed on it by the action of the siren disc, and so is capable of becoming a source of sound.

It will be seen that the frequency of the source can be determined in this case in exactly the same way as for Savart's wheel. Thus, if the number of holes in the siren disc is n, and the disc makes N revolutions per second, the jet of air is interrupted Nn times per second, and becomes a source of sound of which Nn is the frequency.

19. **Production of Non-musical Sounds or Noises.**—It is found that in all cases, such as those already considered in this chapter, where sound is produced by the vibration or periodic motion of the source, the sound is of the character commonly called musical. Many sounds of a non-musical character are, however, of common occurrence; in fact, most of the common sounds heard in daily life are of this character. All sounds of a non-musical nature are conveniently included under the term **noise.**

A consideration of the origin of noises of various kinds suggests that they are produced by rapid and irregular or non-periodic motion at the source. When a plank is struck by a hammer a loud noise is produced; the particles of the plank can be felt to be in motion, possibly in vibratory motion, but the vibratory disturbance produced in the plank is of no regular systematic character, such as would obtain if the plank were set in vibration as a whole. The blow apparently produces merely an irregular disorganised disturbance of a vibratory character, distributed in a partial or irregular manner throughout the particles of the plank, and this irregular disturbance is the source of the noise produced.

The sound of an explosion is another case in point. When a amall quantity of gunpowder is exploded a small quantity of a solid substance occupying a small volume is suddenly converted into gases occupying a very large volume. This sudden expansion is the origin of sound produced by the explosion of the powder.

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Similarly, when an explosive mixture of gases, such as a mixture of oxygen and hydrogen or a mixture of gas and air, explodes, a mixture occupying a considerable volume is suddenly converted into a gaseous product occupying, at the instant of explosion, a very different volume to that of the initial mixture. This sudden change of volume is the origin of the sound produced by the explosion. In most cases of this kind the first effect during the explosion is probably a sudden expansion, due to the heat developed by the explosive combustion, followed by a sudden contraction, due to the condensation of the water vapour and the rapid cooling of the gaseous products of explosion.

Thunder is another case of a noise produced by sudden expansion followed by sudden contraction at the source of the sound. The air along the path of the lightning flash is intensely heated by the flash, and the very sudden expansion thus produced, followed by the contraction due to rapid cooling, is the origin of the thunder which always follows a lightning flash.

Non-musical sounds or noises may thus be said to be produced by irregular non-periodic motion or disturbance at the source of the sound. As in the case of musical sounds, however, the originating motion or disturbance must be of a sufficiently rapid or sudden character.

CHAPTER III.

WAVE MOTION.

20. **Meaning of the Term Phase as applied to Vibration.**—A particle in vibration describes each complete vibration in a definite constant time called the period of the vibration. During a complete vibration—that is, during the interval of time measured by the period—the particle passes through all the stages of the motion which constitutes a complete vibration. Any particular stage in a complete vibration is usually called the **phase** of the motion.

The phase of a particle in vibration may be specified at any instant by stating exactly its position and direction of motion in the path of vibration at that instant. It is, however, more usual and more convenient to specify the phase at any instant by stating the interval of time, expressed as a fraction of a period, between the instant of passing through the centre of its path (or any other specified starting point) and the instant considered. Thus, the phase of a particle in vibration may be specified as that reached by the particle in one-eighth, or one-tenth, or one-sixteenth, or in any other fraction of a period, after the instant of passing through the centre of its path of motion. Similarly, two particles in vibration with the same period may differ in phase by any fraction of a period. Thus, if at the instant one particle is passing through the centre of its path, the other particle, moving in a parallel path, is also passing through the centre of its path, but in the opposite direction, the difference of phase for the two particles is evidently half a period. That is, we may say that one particle is half a period in advance of the other, or that one particle is half a period behind the other in phase.

21. **Medium.**—The **medium** at any point is the substance in which the point is taken, and through which any disturbance originating in the substance at the point can be propagated. If the outer boundary of the medium is at a sufficiently great distance from the point considered, the propagation of any disturbance from the point will depend only on the physical properties of the medium, and will be free from all boundary conditions; when this is the case, the medium at the point is said to be a **free medium**.

Thus, the material medium surrounding objects at the surface of the earth is, in general, air; the material medium at a point in the sea is sea-water; the material medium at a point in a block of iron is iron; and the medium at a point in space, where no material substance exists, is the ether, which pervades all space and all matter. In the study of **sound** we have to do only with material media.

The physical properties of matter on which the propagation of any disturbance through a material medium depends are the **elasticity** and **density** of the medium.

Elasticity is that property of matter which enables any portion of it, while undergoing change of volume or change of shape, to resist that change, and by virtue of which it is able, within certain limits, to recover its original volume or shape when the force causing the change is removed.

The density of a substance is the mass per unit volume of the substance. Thus, the density of water at 4° C. is almost exactly 1 grm. per cub. cm., or about 62·5 lbs. per cub. ft., and the density of air at 0° C. is 1·293 grms. per litre, or 1·291 ozs. per cub. ft.

When the properties of a medium are the same in all directions at any point in it, the medium is said to be an **isotropic medium**.

22. **Wave Motion.**—When a body or a particle surrounded by any medium is in vibration it communicates its vibratory

motion to the layer of medium adjacent to it. This layer in turn, by virtue of the elasticity of the medium, communicates the motion to the next adjacent layer, and so on, from layer to layer. In this way the vibratory motion of the source in vibration travels outwards from the source into the medium. This transmission of vibratory motion from layer to layer through the medium constitutes what is called **wave motion** in the medium.

A body in vibration in a medium thus becomes the source or origin of wave motion travelling out in all directions from the source into the medium. If the medium is uniform and has the same properties in all directions at any point, the speed of transmission of vibratory motion along any line from the source—that is, the velocity of the wave motion along any line of transmission—must be uniform and the same in all directions.

Thus, if O (Fig. 13) be a centre of vibratory disturbance in

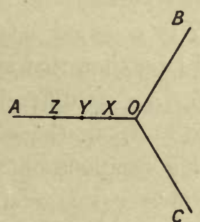

Fig. 13.

any isotropic medium, the disturbance is transmitted outwards from O into the medium with the same uniform velocity in all directions. The particles of the medium along any line of transmission, such as OA, or OB, or OC, all vibrate in exactly the same way as the source, and with the same period. It will be seen, however, that although the vibratory motion of the source is thus transmitted from particle to particle along any line of transmission, and the particles are therefore all vibrating with the same period and along similar and similarly placed paths, the particles along any line cannot all be in the same phase. The process of transmission of the vibratory motion from particle to particle occupies time, and the phase for each particle must therefore be later and later as the distance from the source at O increases. This retardation of phase is due to the time taken by the wave motion in travelling from point to point along the line, and the difference of phase between any two particles on the line is, therefore, measured by the time taken by the motion in travelling from one particle to the other.

It follows from this that the difference in phase for two particles separated by the distance which the wave motion traverses in the time of one complete vibration of the source must be a complete period. Thus, if OX, XY, YZ, . . . represent the equal distances traversed by the wave motion from the source at O, along the line OA, during the first, second, third, and successive complete vibrations of the source, then the particle at X is a complete period later in phase than the source at O, the particle at Y is a period later than that at X, the particle at Z is a period later than that at Y, and so on. Since the particles at X, Y, Z, . . . differ in phase by a complete period they may be said to be in the same phase. Similarly, particles in corresponding positions on OX, XY, YZ, . . . differ in phase by a complete period, and may also be said to be in the same phase.

It will be understood from what has been said above, that if equal lengths, such as OX, XY, YZ, . . . equal to the distance traversed by the wave motion in one complete period be taken in order from any starting point along any line of transmission in the medium, the motion of the particles in every length will be exactly the same, and the motion in any one length may, therefore, be taken as completely representative of the wave motion in the medium.

23. **Wave Length.**—In a uniform medium, in which the velocity of wave motion is the same in all directions from the source of motion, the wave disturbance along any line of trans-

mission travels over a definite constant distance during the time of one complete vibration of the source. This distance is called the **wave length** of the motion. As explained in the preceding article, this distance is also the distance between two particles which differ in phase by one complete period, and may, therefore, be said to be the shortest distance between two particles in the same phase. The wave length for wave motion in any given medium may, therefore, be defined as the distance travelled over by the wave motion, along any line of transmission in the medium, during the time of one complete vibration of the source of the motion; or it may be defined as the shortest distance between two particles in the same phase. In Fig. 13 the distances OX, XY, YZ, . . . represent wave lengths along the line of transmission OA.

24. **Relation between the Wave Length and Velocity of the Wave Motion in any Medium.**—If the wave length for wave motion in any medinm be defined as the distance traversed by the wave motion during the time of one complete vibration of the source, then the distance traversed by the wave motion in one second—that is, the velocity of the motion in the medium—is obviously given by the product of the frequency of the source into the wave length.

Thus, if λ denote the wave length and n the frequency of the source, then λ being the distance traversed by the motion in the time of one vibration, $n\lambda$ is the distance traversed in the time of n vibrations—that is, in one second. But the distance traversed in one second by the motion is the velocity of the motion in the medium. Hence, if this velocity is denoted by V we at once get the relation $V = n\lambda$, which expresses concisely the relation between the velocity V, the frequency n, and the wave length λ. It is important to remember that the quantities V and λ must refer to the same medium. The same source, or sources of the same frequency, may set up wave motion in different media, and the values of V and λ

will be different for the different media, but for any given medium the relation $V = n\lambda$ always holds.

Numerical Examples.—If a source of frequency 400 per second sets up wave motion in air of which the wave length is 3 feet, then the velocity of the wave motion in air is (400×3) feet per second, or 1,200 feet per second.

If a source of frequency 120 per second sets up wave motion in a medium for which the velocity of the motion is 1,200 feet per second, the wave length in the medium is $\left(\frac{1,200}{120}\right)$ feet, or 10 feet.

If a source of frequency 120 per second sets up wave motion in a medium for which the velocity of the motion is 8,400 feet per second, the wave length of the motion in this medium is $\left(\frac{8,400}{120}\right)$ feet, or 70 feet.

If wave motion in a given medium is found to have a wave length of 20 cm. and the velocity of the motion is known to be 33,000 cm. per second, the frequency of the source of the motion is $\left(\frac{33,000}{20}\right)$, or 1,650 vibrations per second.

25. **Longitudinal and Transverse Wave Motion.**—As already explained, wave motion in any medium involves the transmission of vibratory motion from layer to layer, or from particle to particle along any line of transmission. If the vibratory motion thus transmitted is of such a character that

Fig. 14.

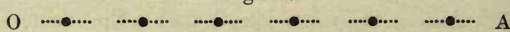

Fig. 15.

the paths of vibration of the particles are in and along the line of transmission, the wave motion is called **longitudinal wave motion.** If, however, the paths of vibration of the particles are at right angles to the line of transmission, then the wave motion is **transverse wave motion.** Thus, if the line of dots in Figs. 14 and 15 represent a few of the particles along a line of transmission OA the wave motion is **longitudinal** if the paths of vibration, shown by the dotted line through the particles, are along the line OA, as in Fig. 14, and **transverse** if the paths are at right angles to the line OA, as in Fig. 15.

26. **Longitudinal Wave Motion.**—The propagation of longitudinal wave motion in any medium evidently results from the elastic resistance which the medium offers to compression or rarefaction. When the vibratory motion of the source first reaches any layer in the medium the layer is displaced outwards or inwards in the direction of transmission. If it is displaced outwards towards the next layer the portion of the medium made up of the two layers is compressed. The medium, however, by virtue of its elasticity, resists this *compression* and tends to recover its normal state by displacing the next layer outwards. Similarly, if the layer is displaced inwards away from the next layer the portion of the medium made up of the two layers is rarefied, and tends to recover from its state of *rarefaction* by drawing the next layer inwards.

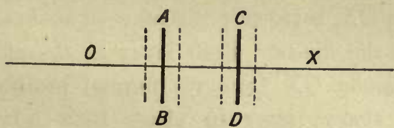

Fig. 16.

The propagation of longitudinal wave motion in a medium thus depends upon the elasticity which enables the medium to resist compression and rarefaction.

The velocity of propagation, however, depends not only upon this elasticity, but also upon the density of the medium. The greater the elasticity the greater the forces causing the displacement of the layers of the medium, but the greater the density the greater the mass of these layers. Hence it follows that the greater the elasticity and the smaller the density the greater is the velocity of propagation.

It follows from what has been said that the propagation of longitudinal wave motion in a medium must involve the development and transmission of successive states of *condensation* and *rarefaction* in the medium. Let AB and CD, in Fig. 16, represent small portions of two layers of the medium taken, a short distance apart, on a line of transmission of the wave motion from one layer to the next. If the layers AB and CD are *equally* displaced from their normal positions in the *same* direction along the line of transmission OX, the portion of the medium between the layers is neither compressed nor rarefied. If, however, the layers are *unequally* displaced so as to be brought nearer together or drawn further apart, the medium between them must be either compressed or rarefied. Now, during the propagation of longitudinal wave motion along OX each layer is in regular vibration along OX, between positions such as those shown by the dotted lines in the figure. Each layer is, therefore, subject to displacement along OX from its normal position, and as the layer CD is always later in phase than AB by the time necessary for the disturbance to travel from AB to CD, the displacements of the layers at any instant must, in general, be *unequal.*

It follows from this that the medium between the two layers must be, during the propagation of the motion, in a varying state of compression and rarefaction. Suppose the distance between the layers to be such that the difference of phase between the layers is one-eighth of a period, and then consider, as exhibited in Fig. 17, the relative position of the layers at intervals of one-sixteenth of a period during a complete vibration. It will be found that during a complete vibration the medium between the layers passes through a complete cycle of alternating states of compression and rarefaction. Each state, existing during half a period, gradually increases from the normal to a maximum and then decreases to the normal state, to be at once followed by a similar variation in the opposite state. Or it may be said that, during a complete vibration, the density of the medium between the layers changes, regularly and gradually, between a maximum and minimum limit. When the density is greater than its

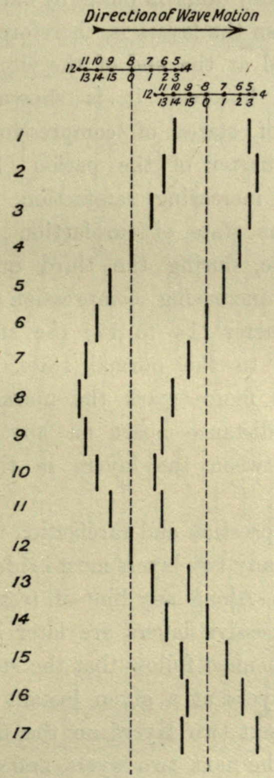

Fig. 17.

normal value the medium is subject to compression, and when the density is less than its normal value the medium is subject to rarefaction.

In Fig. 17 the paths of vibration of the layers are indicated at the top of the figure. They are shown disproportionately long and arranged to overlap, so as to make the states of compression and rarefaction more apparent to the eye. The complete vibration shown, at intervals of one-sixteenth of a period, in the figure, begins at an instant when the particles are *equally* displaced although they differ in phase by one-eighth of a period. The medium between the layers is, therefore, in its normal state at the beginning and at the end of the vibration. During the complete vibration, however, it is shown to pass through a complete cycle of states of compression and rarefaction. During the first quarter of the period (1 to 5) it is in a state of gradually increasing rarefaction, during the second quarter (5 to 9) the state of rarefaction gradually decreases to the normal state, during the third quarter (9 to 13) a state of gradually increasing compression is developed, and during the last quarter (14 to 17) the state of compression gradually decreases to the normal state. The two vertical dotted lines in the figure mark the normal positions of the layers, and their distance apart on any line indicates the normal distance between the layers in the normal state of the medium.

The states of compression and rarefaction which are developed in this way between any two layers must evidently be transmitted from layer to layer. Along any line of transmission, outwards from the source, successive layers are later and later in phase. As a result of this, it must follow that the state of the medium between any two layers at a given instant appears a moment later between the next two layers on the line, and a moment later still, between the next two layers, and so on. In this way the successive states of compression and rarefaction which are developed during a complete vibration between any two adjacent layers "travel" on, one after the other, along the line of transmission with the velocity of the wave motion.

Fig. 18 shows at intervals of one-sixteenth of a period, the variation in the state of the medium during a complete period over a wave length along a line of transmission. Along each line in the figure, layers one-sixteenth of a wave length apart, and differing, therefore, in phase by one-sixteenth of a period are shown. The first line shows the state of the medium, for a wave length, at the beginning of the period. The lines 2 to 17 show the successive states for *the same wave length* at successive instants

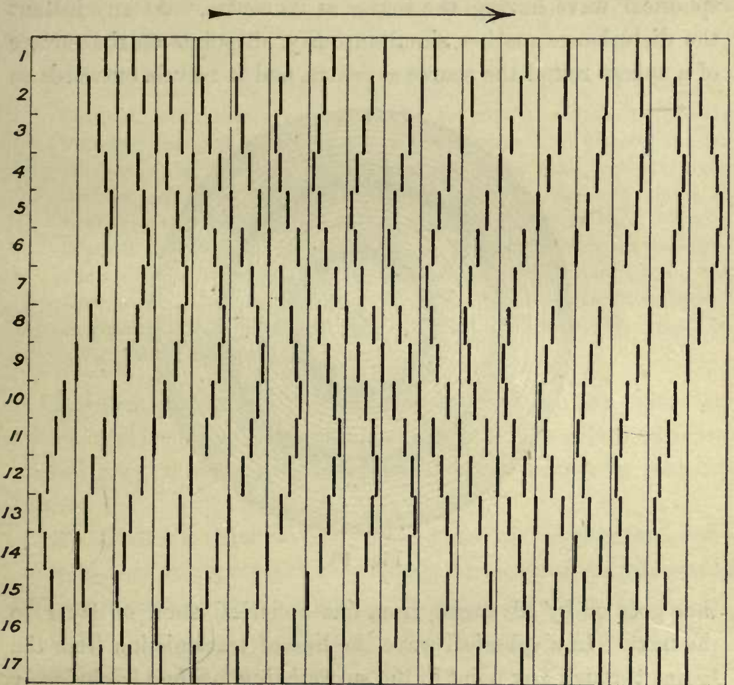

Fig. 18.

taken at intervals of one-sixteenth of a period up to the end of the period. In this figure the vibratory motion of any particular layer, the retardation of phase along the line of transmission, the transmission, from layer to layer, of the successive displacements which constitute vibratory motion, the cycle of states of compression and rarefaction developed between any two layers, and the transmission of these states from layer to layer may all be traced and studied.

If longitudinal wave motion is originated in a medium by the vibration of a particle or small source at any point in it, the disturbance spreads out from the source with the same velocity in all directions, and must, therefore, extend into the medium as a spherical wave having the source as its centre. At any instant the disturbance reaches, simultaneously, all points on the surface of a sphere round the source as centre, and it travels outwards as

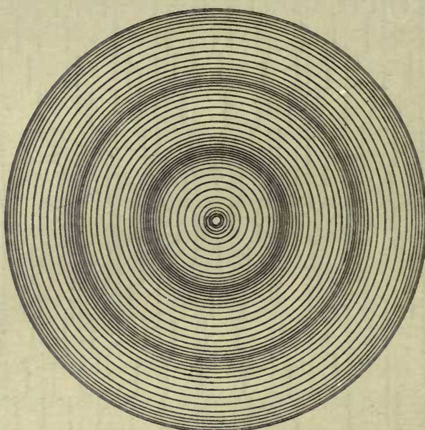

Fig. 19.

time goes on by advancing from one spherical shell or layer to the next. In a spherical wave the line of transmission from the source through any point in the surrounding medium is obviously the radius through that point. Even when the source of the motion is a vibrating body of not very small dimensions, the front of the advancing wave at a distance from the source, large compared with the dimensions of the body, is practically spherical.

Fig. 19 shows the state of the medium in a spherical wave of longitudinal wave motion, for an instant, at the end of the third complete vibration of the source.

It is important to remember that the passage of longitudinal wave motion through any medium, does not involve any actual permanent displacement of any portion of the medium. The particles of the medium at any point are set in vibration about their normal positions, and the medium at the point is subject to alternating states of compression and rarefaction, but no portion of the medium is permanently displaced from its normal position.

Experiment 4.—Set up a long, wide, glass tube in a horizontal position, and place a lighted candle so that the flame is directly opposite one end of the tube. Fill the tube with smoke, and let it stand until the ends of the tube are fairly clear, and only the middle section is filled with smoke. Now strike two pieces of wood together some distance from the free end of the tube. Notice that the longitudinal wave disturbance propagated through the tube affects the candle flame vigorously, and may even blow it out, but the smoke-laden air in the tube is not appreciably disturbed or displaced. This effect is very different to that which could be produced by blowing through the tube.

A source of sound in periodic motion gives rise to longitudinal wave motion in the surrounding medium, and, in a limited sense, sound may be said to be longitudinal wave motion in material media.

27. **Crova's Disc.**—The propagation of longitudinal wave motion may be illustrated very clearly by means of an interesting device known as **Crova's Disc.** This disc, as shown in Fig. 20, may be constructed as follows:—In the centre of a large sheet of cardboard, or drawing paper, describe a small circle about 5 mm. in radius. Take 8, 10, 12, or more equidistant points on the circumference of the circle, and with these points, taken in order round the circle for several revolutions, as centres, describe a series of circles with radii beginning at about 10 cm. and increasing, for each circle, by an amount not less than the distance between the equidistant points on the small central circle. Cut out the disc marked by the outermost of these

circles, and mount it so that it can be rotated steadily round an axis passing through the centre of the small circle.

Then take a card about 10 cm. long by 5 cm. wide, and cut a narrow rectangular slit about 8 cm. long in it. Set up this card in front of the disc, as close to the face of the disc as possible, and in such a position that the length of the slit, as shown at CD in Fig. 20, crosses the circles on the disc along a

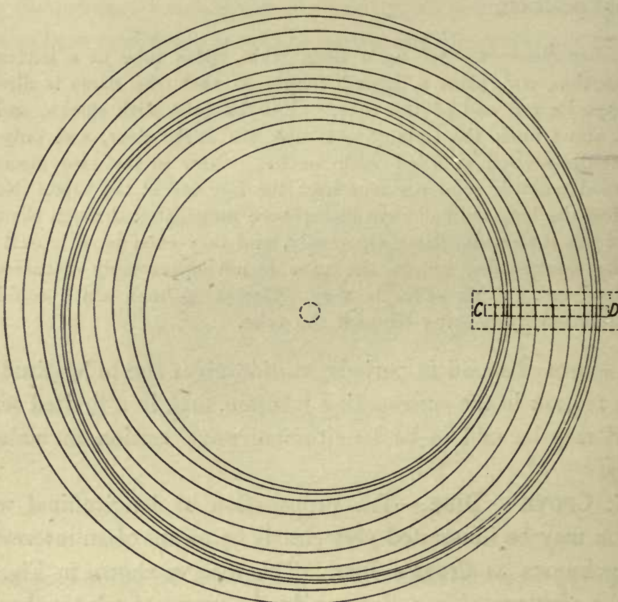

Fig. 20.

radius through the axis of rotation. A radial row of short parallel arcs can now be seen through the slit, and it will be found on rotating the disc at a uniform rate, that the motion of these arcs is such as to illustrate very effectively the propagation of longitudinal wave motion along the row.

It will be seen that, as the disc rotates, the centres of the large circles drawn on it move in a small circle round the axis of rotation. Since the radius of the central circle is comparatively small, the general result of the rotation on the motion of any arc seen through the slit is, therefore, practically the same as if the centre of the corresponding circle moved backwards and forwards, in simple harmonic motion, along the diameter of the small circle which lies in a line with the length of the slit. That is, each arc moves backwards and forwards in simple harmonic motion along a path parallel to the length of the slit; the amplitude of the motion is equal to the radius of the small central circle, and the period is equal to the time of a complete revolution of the disc.

Further, it will be seen that the arcs are not all in the same phase, but that there is a constant difference of phase from arc to arc along the row. For, since the centres of the arcs are taken in order round the small central circle, it follows that, as the disc rotates, the centres of successive arcs follow each other through any given position, at a constant interval of time, determined by the number of points on the circle; the arcs must, therefore, be subject to a corresponding retardation of phase from arc to arc. If n denote the number of points on the small circle, this retardation of phase is evidently $1/n$th of the period of the motion of the arcs, and if the disc be rotated so that the centres of the arcs follow each other through any given position in the order in which they were drawn (from the centre outwards), the retardation will also be outwards from arc to arc along the row.

The motion communicated to the arcs by the uniform rotation of the disc is thus a fairly exact imitation of the motion which would attend the propagation of longitudinal wave motion from arc to arc along the row. The harmonic motion of each arc, the uniform retardation of phase from arc to arc, and the formation and propagation of successive states of condensation and rarefaction along the row, can all be seen and followed by studying the motion of the arcs through the slit.

The velocity of propagation of the wave motion may also be determined from the fact that it travels from arc to arc in a time equal to $1/n$th the period of the motion of the arcs.

Experiment 5.—Set up a Crova's disc, with the observation slit in front of it. Turn the disc slowly, and note carefully the following points characteristic of longitudinal wave motion:—

1. Every arc moves backwards and forwards in the same way, in approximately simple harmonic motion, with a period equal to the time of revolution of the disc.

2. There is a difference of phase, from arc to arc, of $1/n$th of a period, where n is the number of equal parts into which the small central circle is divided. It will be noticed that, in one direction, the phase gets later and later from arc to arc. This direction is determined by the direction of rotation of the disc, and can be reversed by reversing the direction of rotation.

3. The direction of propagation is the direction of retardation of phase.

4. The distance between *any* two adjacent arcs decreases and increases in a systematic way, and passes through the same complete cycle of change during every complete vibration of either arc. This is illustrative of the periodic cycle of states of compression and rarefaction, which are produced during longitudinal wave motion between adjacent layers in the medium.

5. The cycle of states of "compression" and "rarefaction," which occur between any two arcs, is later and later in "phase" as we pass along from arc to arc in the direction of propagation. As a result of this, each successive stage of compression or rarefaction appears to travel on from arc to arc in the direction of propagation.

6. The velocity of propagation of the "wave motion" along the line of arcs, is approximately given by Nnd, where N denotes the number of revolutions made, per second, by the disc; n, the number of equidistant points on the small circle; and d, the common difference between the radii of the large circles on the disc. The wave length of the motion is nd.

28. **The Wave Spiral.**—The propagation of longitudinal wave motion may also be effectively illustrated by means of a long spiral of wire, suspended by threads so as to hang freely in a horizontal position. The spiral may be made by winding fairly thick brass wire (No. 18 S W.G.) on a suitable rod into a long spiral, 6 or 8 feet long and 3 or 4 inches in diameter. The

turns of the wire should not be too close together; in a spiral of the dimensions given they should be about half an inch apart. It is best mounted for use by suspending it from two fixed parallel rods, by threads looped through every fourth or fifth turn, as shown in Fig. 21.

A spiral of this kind is best adapted to illustrate the formation and propagation of the states of compression and rarefaction which attend the propagation of longitudinal wave motion through a medium. If one end of the spiral is suddenly forced inwards a compression of the turns is started at that end, and can be seen to travel along the spiral to the other end. Similarly, if the end of the spiral is suddenly pulled outwards a

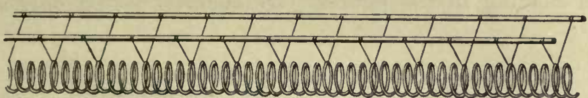

Fig. 21.

" rarefaction " is started which can also be seen to travel along the spiral.

It should be noticed that the pulse of compression or rarefaction which is thus made to travel along the spiral extends over a definite length of the spiral, and varies in degree from point to point on the pulse. The degree of compression or rarefaction is zero at the front of the pulse, it increases gradually to a maximum value at the middle of the pulse, and it then decreases gradually to zero at the rear of the pulse.

If one end of a very long spiral were set in periodic in and out motion, alternate pulses of compression and rarefaction illustrative of wave motion would be seen to form and travel along the spiral from the disturbed end. This train of pulses would quickly be complicated by reflection from the other end of the spiral, but, as first seen, each pulse would extend over half a wave length, and a complete wave length taken anywhere along the spiral between any two points in the same phase, would, in all cases, include all the elements of two complete pulses, one of compression and one of rarefaction.

29. **Transverse Wave Motion.**—In the propagation of transverse wave motion, when the vibratory motion of the source first reaches any small portion of a layer such as AB, Fig. 22, it is not displaced towards or away from the adjoining layer CD, but laterally, at right angles to the direction of propagation, as shown in Fig. 23.

The position of the medium between the layers is obviously not compressed or rarified by this displacement. It is, however, strained or deformed in the manner indicated in a general way by the shading in the figures. Of material media only solids

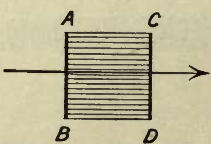

Fig. 22.

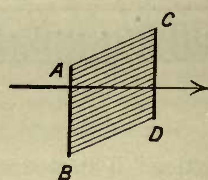

Fig. 23.

resist this form of strain which is usually called a **shear**; a fluid medium offers no elastic resistance to a shearing strain of this kind. It follows, therefore, that transverse wave motion may be propagated in a solid medium, but not in a liquid or gaseous medium.

Transverse wave motion in solid material media is of no special interest, and has not been studied experimentally. Transverse wave motion in the ether, the medium which is supposed to fill all space and all matter is, however, of the greatest interest. Every body, by virtue of the vibratory motion of its molecules, is a source of transverse wave motion in the ether. This transverse wave motion is called the **radiation** from the body.

As an illustration of the general character of wave motion, the propagation of transverse wave motion along a line of particles is of interest.

Let the dots along the line OX, Fig. 24, represent a few equidistant particles along the line of propagation, and imagine the vibratory motion of the particles to begin at O.

Let it be supposed that the particle at O vibrates up and down along the line AB taken at right angles to OX, and that this vibratory motion is transmitted from particle to particle along the line OX, with a retardation of phase determined by the time the wave motion takes to travel from particle to particle.

During the time that the particle at O describes a complete

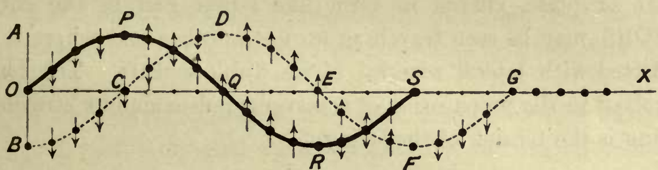

Fig. 24.

vibration over the path OBAO, the disturbance will have travelled a wave length out along OX, and will have reached a particle at some point S on this line. If this particle is the *n*th particle from O, the retardation of phase along the row of particles will be $1/n$th of a period, and at the end of the first complete vibration of the particle at O, the particles between O and S will occupy the positions shown in the figure and will be moving, each in its own path of vibration, in the directions indicated by the arrows. The positions of the particles along the wave length OS lie on the curve OPQRS, which is evidently the same as the displacement curve for the motion of the particle at O. As the motion continues the positions of the particles along the line of propagation will change, but the retardation of phase from particle to particle will remain constant, and the particles will, therefore, always lie on some portion of a curve which is made up of repetitions of the wave length section represented by OPQRS. Thus, at the instant the particle at O has completed one and a quarter complete vibrations, the disturbance will have travelled from O a distance of one and a quarter wave lengths along OX to a particle at G, and the particles will lie on the dotted curve BCDEFG shown in the figure.

A string or cord may be considered as a continuous row of connected particles, and a wave of transverse displacement may be transmitted along a stretched string. If, for example, a rope is stretched fairly tightly in a horizontal position between two posts and is plucked aside at a point near one end, a half wave or pulse, curved in form like either half of the curve OPQRS, may be seen travelling up to the other end where it is reflected with lateral reversal of its displacements. The force involved in the propagation of transverse pulses along a stretched string is the tension of the string.

Experiment 6.—Attach one end of a long piece of stout rubber tubing to the ceiling or to a beam overhead, and hold the lower end in one hand so that the tube is in a nearly vertical position Now strike the tube transversely with the other hand. A transverse half wave or pulse is produced at the point struck, and travels up the tube to the top end where it is reflected with lateral reversal of its displacements, and then returns so reversed.

30. **Ripples and Surface Waves.**—The ripples and waves on the surface of a liquid are also familiar examples of transverse wave motion. These waves are propagated, not as transverse wave motion in the liquid *as a medium*, but as waves of transverse displacement along the surface of the liquid ; and the forces involved in their propagation are not determined by the elasticity of the liquid, but by the surface tension and weight of the liquid.

If the surface of a liquid is disturbed at any point the general result is to originate up and down periodic motion of the particles of the surface at this point. The disturbance is, therefore, propagated from particle to particle along the horizontal surface of the liquid as **transverse** wave motion, and the particles along any line of propagation will, therefore, take up positions on a curve similar to that shown at OPQRS in Fig. 24. As a result of this the surface assumes the familiar trough and crest formation characteristic of waves and ripples on a liquid surface. The **crest** of a wave is evidently the locus of the particles in the wave at the highest points of their paths, and the trough is similarly, the locus of the particles at the lowest points of their paths. It follows that the horizontal distance from crest to trough in any wave is half the wave length, and the horizontal distance from crest to crest, or from trough to trough, for adjacent waves is a complete wave length.

To an observer looking at the propagation of surface waves on a liquid, each individual wave appears to be moving on bodily in the direction of transmission. This illusion is characteristic of wave motion. Each particle simply moves up and down in a fixed path, but the position occupied by any particle at a particular instant is occupied a moment later by the next particle on the line of transmission. For example, the crest of a wave which, at any instant, is formed by a line of particles at their highest points, will a moment later be formed by a line of particles a little further on in the direction of propagation. In this way the sequence of **positions** which determine the form of any wave, as seen at a particular instant, travels on in the direction of propagation, and so gives rise to the illusion that the mass of water outlined by the **travelling form** is moving on bodily.

Experiment 7.—Fix a small cone of wax or cork to the prong of a large tuning fork by attaching its base to the flat outer surface of the prong near the free end. Set the fork in vibration and hold it, so that while vibrating, the apex of the attached cone dips below the surface of mercury or water at rest in a large shallow vessel. Note the ripples produced by the vibratory motion of the cone as source.

Experiment 8.—Drop a stone into water with its surface at rest. Note the ripples produced by the vibratory disturbance caused by the passage of the stone through the surface of the liquid.

The same effect is produced in a more effective way by letting small drops of mercury fall at intervals on the surface of mercury at rest in a large shallow vessel.

Experiment 9.—Fill a long rectangular trough, such as that shown in Fig. 25, with water. Surface waves larger than ripples, and dependent upon the weight of the liquid more than upon the surface tension of the liquid for their propagation, may be produced by

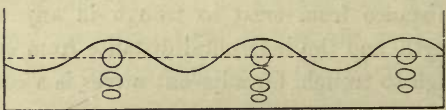

Fig. 25.

periodically raising and lowering a block of wood at one end of this trough.

Sprinkle a few pieces of cork on the surface of the water and note they are not carried backwards or forwards by the wave motion, but that each piece simply oscillates up and down in a fixed path. The path of vibration for a particle at any depth in the layer of liquid disturbed by the wave motion may be studied by means of small balls of bees wax, mixed with sand or iron dust until the balls are of the same average density as water, these balls are in equilibrium at any depth in the liquid, and may, therefore, be placed so as to indicate the path of motion at any point in the liquid. It will be found by studying the motion of the pieces of cork at the surface, and of wax balls at various depths that the paths of motion of the disturbed particles of water are, in general, as shown in the figure, closed curves in a vertical plane containing the direction of propagation.

31. **Simple Harmonic Wave Motion.**—If the source of wave motion in a medium is in simple harmonic vibration, the wave motion is called simple harmonic wave motion.

A source of wave motion must necessarily be in periodic motion, and just as simple harmonic motion is the simplest form of periodic motion, so simple harmonic wave motion is the simplest form of wave motion.

32. **Non-periodic Disturbances.**—A source of rapid irregular motion, not of a periodic character, cannot give rise to wave motion properly so called. It can, however, give rise to an irregular disturbance similar in character to wave motion. This disturbance in a fluid medium would be of the "longitudinal" type, and would consist of a sequence of "longitudinal" displacements, varying in an irregular and, it may be, very abrupt manner, accompanied by a corresponding sequence of irregularly varying states of compression and rarefaction, and it would travel into the medium from the centre of the disturbance with the velocity of regular longitudinal wave motion.

If the motion of the source is of short duration, then the "wave" disturbance is of correspondingly short length, but if the motion of the source is continuous, then the irregular wave spreads out continuously from the source as long as the motion lasts.

33. **Intensity of Wave Motion.**—A source of wave motion in a medium communicates energy to the medium. If the amplitude of vibration of the source is constant, the same quantity of energy is communicated to the medium during each complete vibration of the source, so that the quantity of energy communicated to the medium by the source in one second is constant. This energy travels out from the source into the medium, as energy of wave motion, and as it travels out the area, across which it is transmitted from layer to layer, increases as distance from the source increases. The quantity of energy which is transmitted per second across **unit area**, from any one layer to the next, must, therefore, decrease as distance from the source increases. That is, the **intensity** of the wave motion decreases as the distance from the source increases, for the quantity of energy which is transmitted per second across unit area, at any

point, is generally taken as a measure of the intensity of the wave motion at that point.

If the source be assumed to be at a point in the medium, the energy communicated by the source to the medium travels out into the medium from one spherical layer to the next, and as the total energy transmitted from layer to layer per second is constant, the quantity that crosses unit area from any one spherical layer to the next, in one second, decreases as the radius of the layers increases. If Q denote the quantity of energy communicated by the source to the medium in one second, then the quantity of energy which is transmitted per second across unit area of any spherical surface of radius r round the source as centre is $\frac{Q}{4\pi r^2}$.* That is, the intensity of the wave motion at a distance r from the source is, in this case, given by $\frac{Q}{4\pi r^2}$, and the form of the result shows that, when the source is at a **point** in the medium, the intensity of the wave motion, at any distance from the source, is *inversely proportional to the square of the distance.* When the source is of finite dimensions, the same law holds, approximately, at distances from the source, which are great compared with the dimensions of the source.

Since the intensity of the wave motion decreases as the distance from the source increases, it follows that the amplitude of vibration of the particles of the medium also decreases as the distance increases. It can be shown that the intensity of the wave motion as defined above is, at any point in the medium, directly proportional to the square of the amplitude of vibration of the particles of the medium at that point. That is, the amplitude of vibration of the particles of the medium is inversely proportional to the first power of the distance from the source.

34. **Wave Form.**—As explained above, wave motion results from the transmission of the vibratory motion of the source from

* The **area** of a spherical surface of radius r is $4\pi r^2$, where $\pi = 3{\cdot}1416$.

layer to layer out into the surrounding medium. It has also been explained that the vibratory motion of the source is transmitted from particle to particle along any line of transmission with regular retardation of phase. It follows, as a result of this, that the varying displacement of the source during a complete vibration is reproduced, wave length after wave length, in the successive displacements of the particles along the line of transmission. Thus, the displacement curve which exhibits the displacement variation of the source during a complete vibration, shows, also, subject to correction for decrease of intensity, the displacement variation from point to point along the corresponding wave length of the wave motion. This characteristic of the wave motion, being indicated by the **form of the curve**, is sometimes called the **wave form.**

The character of the variation of the displacement of the particles along a line of propagation at any instant determines, as already explained, the character of the sequence of states of compression and rarefaction, which is found along the line of propagation at that instant. Both these characteristics of wave motion are, therefore, included in the term wave form, as applied to wave motion.

35. **Wave Front.**—The wave front at a given instant for wave motion, from any source, is the locus of all points which the motion has just reached at that instant. Thus, the wave front at any instant for wave motion, from a point source in an isotropic medium, is a spherical surface having its centre at the source, and its radius equal to the distance which the wave motion has travelled from the source at the given instant. For some purposes a wave front may be defined as the locus of all points in the same phase in the same wave length.

When the wave front at any instant is known, the wave front at an instant any given interval of time later can be determined by a simple construction. In the case of an isotropic medium, in which the velocity of propagation of the motion is the same in all directions at any point in it, the construction is as follows :— Let a spherical surface of radius, equal to the distance travelled by the wave motion in the given interval of time, be described round every point on the given wave front; then the surface which encloses or envelops these spherical surfaces, and touches each one of them tangentially, is the new wave front. Thus, in Fig. 26, let AA represent the trace of a plane wave front at right angles to the plane of the paper, and let it be required to find the wave front at an instant *t* seconds later. Let a spherical surface of radius V*t*, where V is the velocity of the wave motion in the medium, be supposed to be described round every point in the wave front, and let the circles round the points *a*, *a*, *a*, . . . in the figure, be the traces of these surfaces for a few of the points in the plane of the paper. The wave front at the end of the time *t* will be the plane tangential to all these spherical sur-

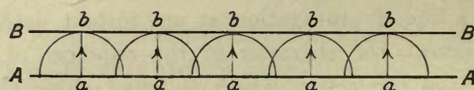

Fig. 26.

faces, and the trace of this wave front in the plane of the paper is represented by the line BB, tangential to the circles round the points *a*, *a*, *a*, . . . and touching the circles at the points *b*, *b*, *b*, . . . as shown in the figure.

The lines *ab*, *ab*, *ab*, . . . from the points *a*, *a*, *a*, . . . in the wave front AA, to the points *b*, *b*, *b*, . . . in the wave front BB, are **lines of transmission** of the wave motion, from one wave front to the other, in the sense that the disturbance from any point *a* may be considered as transmitted through the medium to the corresponding point *b* along the line *ab*.

A line of transmission, defined in this way, is, therefore, always at right angles to the wave front, and it follows that in longitudinal wave motion the paths of vibration of the particles of the medium are always at right angles to the wave front, while in transverse vibration they must be in the wave front.

CHAPTER IV.

PROPAGATION OF SOUND.

36. **Sound.**—The term sound connotes both the sensation of sound and the external physical phenomena to which this sensation is due. The subject of sound sensation belongs mainly to Physiology; the external physical phenomena to which sound sensation is due constitute what is usually understood by **Sound** or **Acoustics** as a branch of Physics.

It has already been explained that a body in vibration or in periodic motion of any kind within certain limits of frequency, or even in irregular non-periodic motion within certain limits of rapidity of motion, is a **source** of sound, either a musical sound or a noise.

It has also been explained that a body in periodic motion, or in irregular non-periodic motion, in a material medium surrounding it sets up longitudinal wave motion, or an irregular wave-like disturbance of longitudinal type, in the medium, and that this wave motion or disturbance travels out from the body into the surrounding medium.

These facts, taken together, at once suggest that the sensation of sound is due to the incidence of the wave motion or disturbance from the source of sound on the drum of the ear.

In a limited sense, sound may be defined as longitudinal wave motion in material media, and is, therefore, a form of energy.

37. **Elasticity.**—From what has been said in Chapter III., it will be understood that the essential property of a material medium for the propagation of sound as longitudinal wave motion in the medium is **elasticity.** The elasticity necessary for the propagation of sound is that property of the medium which enables it to resist compression or rarefaction, and by virtue of

which it is able, within certain limits, to recover its original state when the force causing the strain is removed. Substances like steel, glass, and ivory, which offer very great resistance to compression or extension, are said to be very highly elastic, but the limits of change of volume within which they possess the property are very narrow. Thus, if glass is compressed or extended beyond a very small extent it breaks into fragments. India-rubber, on the other hand, offers comparatively little resistance to compression or extension, and is, therefore, of low elasticity, but its limits of elasticity are very wide. Lead is an example of a substance of comparatively low elasticity, and some substances, such as clay, are practically without elasticity.

38. **The Propagation of Sound in Material Media.**— It will now be understood from what has been said above, that sound is propagated as longitudinal wave motion in material media. A sound is heard because the wave motion set up by the source of the sound in the surrounding medium travels from the source to the ear, through the intervening material media, and by its incidence on the drum of the ear produces the sensation of hearing.

It is, therefore, essential for hearing a sound, that a material medium or a succession of material media, should extend continuously from the source of the sound to the ear.

Experiment 10.—Take a small bell actuated by clockwork which will ring gently but continuously for some time, and rest it on a thick pad of felt under the receiver of an air pump. When the receiver is in position, and the pump ready for action, the sound of the bell can be heard quite distinctly. Now work the pump slowly, and notice the effect on the sound heard.

It will be found that as the receiver is exhausted of air, the sound of the bell becomes fainter and fainter, and, ultimately, if the pump gives a good vacuum, it becomes too faint to be heard.

Now turn the tap which admits air into the receiver, so that the air enters very slowly, and note the effect on the sound heard.

It will be found that as the air re-enters the receiver, the sound heard becomes louder and louder, until it attains its initial intensity.

This experiment shows that when a gap is made in the media extending from the bell to the ear, by withdrawing the air in the receiver, the sound of the bell ceases to be heard. It also shows that the loudness of the sound depends upon the density of the medium in which it is produced.

Instances of the propagation of sound by material media are readily found. The sounds that we hear in daily life reach our ears through the air. The sound produced by the wind in telegraph wires is propagated through the wood of the telegraph posts, and can be heard distinctly by applying the ear directly to a post. The sound of a distant train is propagated through the iron rails, and can be heard by applying the ear to a rail, just as the sound of the galloping of a horse can be heard through the ground by applying the ear to the ground. Sounds produced under water are propagated through the water, and can be heard very distinctly, and at great distances from the source by an observer under water, or by means of a long ear trumpet placed with its receiving end under water.

If a sounding body is small and presents a surface of small extent to the surrounding medium, the wave motion set up by the body as a source of sound may be of very low intensity, and the sound may be almost inaudible at a short distance from the source. The surface of contact between the body and the medium being very small the energy communicated to the medium at each vibration of the source is very small, and the intensity of the resulting wave motion is correspondingly low. If, however, the sounding body is put in contact with a board of light, dry wood in such a way that it sets the particles of the wood in vibration without having its own vibratory motion destroyed, the area of the surface at which vibratory motion from the source of sound is communicated to the medium is greatly extended, and the intensity of the wave motion set up in the medium is correspondingly increased. Thus, if a tuning fork is struck and held in the air the sound heard is very feeble and scarcely audible. If, however, the end of the stem of the fork is put in contact with the top of a light table the sound of the fork is at once heard loudly and distinctly at a considerable distance from it. Similarly, a string stretched between two nails in a wall gives out but a very faint sound when set in vibration. If, however, the string is mounted on a **sounding board,** or on a violin, it gives out a loud, distinct sound.

It should be noted, however, that the sound given out by a sounding body, such as the fork or string referred to above, dies away much more quickly when in contact with a sounding board than when vibrating by itself; in the former case the body gives out much more energy per second to the medium than in the latter case, and, therefore, loses much more quickly the stock of energy it initially possessed.

When sound waves pass from one medium to another reflection always takes place at the surface of separation of the two media, and the amount of energy which passes from one medium into the other, may, in certain cases, be very small. When the media differ considerably in density, sound waves in either medium, incident on the surface of separation of the media, are, even for direct incidence, almost entirely reflected, and the intensity of the wave motion transmitted across the surface from one medium to the other is, therefore, very small. This explains why sound waves do not readily pass from air into surrounding solid and liquid media, or from these media into air. Thus, under certain conditions, sound waves pass from air to water, and from water to air, but the loss of energy by reflection at the surface of separation is always great, and a sound originating in one medium and heard in the other is always much fainter than if heard in the medium in which it originated.

Similarly, sound waves do not readily pass from the earth to the air. Thus, the sound caused by water running in pipes or drains underground cannot easily be heard in the air; but if the ear be placed on the ground, or if a **sounding rod** of light, dry wood be placed between the ground and the ear, the sound can be clearly heard. These sounds could also be heard with the help of a piece of rubber tubing opening into a cup-like expansion at one end. If the rim of the cup is applied to the ground, and the other end of the tube to the ear, the sound waves are carried to the ear by the column of air in the tube, and the sounds are distinctly heard. This principle is applied in the **stethoscope** used by medical men for listening to the sounds made by the heart and lungs in action. These sounds can be heard faintly by applying the ear directly to the wall of the chest; but they are quite inaudible in the air, even when the ear is quite close to the wall.

Experiment 11.—Sound a tuning fork and apply its stem to one end of a long rod of wood. Rest the other end of the rod on the top of a light wooden stool. Notice how the sound of the fork increases in loudness when the rod rests on the stool.

Experiment 12.—Apply the ear to one end of a long beam of wood and note that the sound made by scratching the other end with a nail can be distinctly heard.

Experiment 13.—Stand a cylindrical gas jar, or a large tin canister, on a light wooden stool resting on a felt pad, and fill the jar with water. Take a fairly large tuning fork and attach a light cone of cork to one prong by fastening the base of the cone to the flat outer surface of the prong.

Set the fork in vibration and hold it so that the apex by the cone dips into the water as the fork vibrates. Note that the sound of the fork is clearly heard only when the cone is in contact with the water.

Remove the stool, stand the jar on the felt, and repeat the experiment. Note that the loudness of the sound is not now appreciably increased by putting the cone in contact with the water.

When the jar rests on the stool the wave motion originated by the motion of the cone travels through the water, the glass of the jar, and the wood of the stool to the air and then through the air to the ear. The stool acts as a sounding board; much more wave motion energy passes per second from the wood of the stool to the air than from the glass of the jar to the air.

Experiment 14.—Take two cylindrical wooden boxes, like large and rather deep pill boxes, and join them by a long piece of string or wire securely attached to the bottoms of the boxes. It will be found that words spoken into one of the boxes can be heard with some distinctness by applying the ear to the other box. The connecting line may be taken round corners by supporting it on loops of string.

This apparatus is generally called the string telephone. The sound waves originated at one end by speaking into the box travel through the bottom of the box and along the connecting line to the other box, and thence through the air to the ear.

39. **Conditions favourable to the Propagation of Sound.**—The first essential for the propagation of sound in any medium is that the medium should possess elasticity. A substance, such as clay or putty, which is practically devoid of elasticity, cannot transmit sound waves. Similarly, masses of substances, such as sand, sawdust, felt, wool, and other similar materials which have no continuity of structure, are essentially inelastic, and are, therefore, unable to act effectively as **conductors** of sound. For this reason they are generally used in the construction of sound-proof walls and floors.

Experiment 15.—Enclose a small clockwork bell in a box and cover the box with a mass of clay or freshly-made putty. It will be found that the sound of the bell cannot be heard, showing that the covering of the box is impervious to sound waves.

Experiment 16.—Enclose a clockwork bell in a box and pack it with sand, or sawdust or cork dust, in another box. It will be found that the sound of the bell does not travel through the packing.

Force a bradawl through the side of the outer box into the side of the inner one. The sound of the bell can now be heard distinctly.

The propagation of sound through any continuous elastic medium is limited in two important ways. The sound wave may lose energy in doing work against the molecular friction opposing the vibratory motion of the molecules of the medium, or the wave motion may be broken up and scattered as the result of a want of uniformity or homogeneity in the structure and properties of the medium.

If the physical properties of a substance are such that wave motion meets with appreciable molecular friction in the substance, then sound cannot be propagated through that substance to any great distance, and the substance is in that sense a bad conductor of sound. If, however, the properties of the substance are such that the molecular friction is small, then sound may be propagated through the substance to great distances, and the substance is said to be a good conductor of sound.

Even when the physical properties of a medium are favourable to the propagation of sound waves, a further condition absolutely essential for propagation without material loss of energy is the *homogeneity* of the medium. If the medium is not homogeneous or uniform throughout in structure, density, and physical properties generally, the wave motion is subject to continuous loss of energy on account of the numerous reflections and refractions which tend to break up and scatter the wave. A medium such as the air is rarely quite homogeneous over large masses; disturbance due to wind currents, the presence of convection currents due to local heating or cooling, the presence and irregular distribution of water vapour, all tend to produce a want of homogeneity which is highly detrimental to the propagation of sound through the air. The effect of the varying homogeneity of the air on the propagation of sound explains the great differences which are commonly observed in its power of transmitting sound. Under some conditions sounds "carry" only comparatively short distances; at other times, when the air is still and free from the disturbances referred to above, sounds "carry" to very great distances. This is specially the case in a fog; a fog can form only in still, homogeneous air, and is, therefore, an indication of the existence of conditions specially favourable to the propagation of sound.

Experiment 17.—Arrange two tin or cardboard tubes, each about 3 feet long and 2 inches in diameter, end to end in the same line, but with a gap of about half an inch between them. Place a ticking watch at one end of the tube and note that it can be clearly heard at the other end.

Arrange a Bunsen flame just below the gap between the tubes so that a current of hot air and gases rises through the gap. Test if the watch can now be heard as at first. Also, arrange a beaker of boiling water below the gap so that a current of steam rises through it, and test again with the watch.

It will be found that in both these cases the up current of heated gas or vapour is an effective bar to the transmission of sound along the tube.

The presence of particles in a medium does not interfere in the least with the propagation of wave motion through the medium, provided the dimensions of the particles are small compared with the wave length of the motion. Thus, although the presence of small particles of smoke and dust in the atmosphere may interfere seriously with the propagation of light, it has no effect whatever on the propagation of sound. A thick fog, for example, is practically opaque for light, but foggy air is specially favourable to the propagation of sound, for the air is then homogeneous and the fog particles are much too small to interfere at all with the propagation of the sound waves.

For the same reason, any curtain or screen which does not interrupt the continuity of the medium does not interrupt the propagation of sound waves through the medium. For example, a curtain of muslin or silk, or any loosely woven material through which air passes easily, has, in air, little or no effect in obstructing the passage of sound waves through it. If, however, the material is wet, so as to present a *continuous* film of water in the path of the waves, reflection takes place and the sound waves are practically completely reflected at the wet screen.

Experiment 18.—Arrange two tubes end to end, as in Exp. 17, and fix a screen of linen or silk transversely in the gap between the two tubes. Now test whether the ticking of a watch is audible from end to end of the tubes (*a*) when the screen is dry, (*b*) when the screen is wet.

It will be found that when the screen is dry the ticking is heard as distinctly as if the screen were not there, but when it is wet the ticking is not heard at all.

40. **Velocity of Propagation of Sound.**—It has already been explained that wave motion travels through any medium with a uniform velocity determined by the physical properties of the medium. It is, therefore, to be expected that the longitudinal wave motion which constitutes sound, travels through any given medium with a definite velocity which may be characterised as the velocity of sound in the medium.

Everyday experience proves that sound takes time to travel through air. The report of a gun fired at a distance is heard some seconds after the flash is seen, the whistle from the engine of an approaching train is heard a little after the escape of steam at the steam whistle is observed, and the thunderclap which accompanies a lightning flash is always heard after the flash is seen. Since the velocity of light is very great (186,000 miles per second), the time taken by light in travelling from the flash or jet of steam to the eye may be neglected, and the interval observed in these cases may be taken as the interval between the instant at which the sound is produced at the source, and the instant at which it is heard by the observer. This interval is therefore a measure of the time taken by the sound in travelling from the source to the observer, and it is a matter of common observation that the length of the interval depends upon the distance of the source from the observer; the greater the distance at which a gun is fired, the longer is the interval between the instants of seeing the flash and hearing the sound; the more distant the lightning flash the longer is the interval between seeing the flash and hearing the thunder. Hence, if the time taken by sound to travel over different measured distances be carefully observed, it is possible to show that sound travels in air with a uniform velocity, and also to determine the magnitude of this velocity. Thus, let a cannon be fired at a convenient centre, and let three observers be placed at three stations, A, B, and C, at distances of 1 mile, 2 miles, and 3 miles, respectively, from the cannon. Let the observer at each station measure with a stop watch, reading to fifths of a second, the interval between seeing the flash and hearing the report of the cannon, and let these intervals be $4\frac{4}{5}$ seconds, $9\frac{2}{5}$ seconds, $14\frac{3}{5}$ seconds at the stations A, B, and C, respectively. These data give the average velocity of sound over the distances between the cannon and the stations A, B, and C, as 1,100 ft. per sec., 1,123 ft. per sec., and 1,115 ft. per sec. Allowing for experimental errors, these results indicate that sound travels through air with a definite uniform velocity of about 1,120 feet per second.

Many careful determinations of the velocity of sound in air have been made by this and other methods, and it is now established that the velocity of sound in air, under ordinary atmospheric conditions, is about 1,120 feet per second.

In the same way it can be shown that sound travels through water with a definite constant velocity of about 4,700 feet per second.

It is found, too, that the velocity of sound in any medium is practically the same for all sounds; musical sounds and noises travel with the same velocity, and, within ordinary limits, this velocity is quite independent of the intensity or pitch of the sound. This result may be inferred from the fact that music played by a band is heard at any distance in correct tune and harmony; all the sounds reach the ear in the exact sequence in which they are played, and must, therefore, travel through the air with the same velocity.

There is some evidence that the velocity of sound increases with the intensity of the sound; thus, it is asserted, that in rifle practice the sound of the command to fire is sometimes heard by distant observers *after* the sound of the firing. It is also stated that the velocity of sound increases slightly with rise in the pitch of the sound. The experimental evidence in support of these statements is, however, somewhat inadequate.

As explained in Art. 21, the velocity of propagation of longitudinal wave motion in a medium, depends upon the elasticity and density of the medium. The velocity of sound in any medium must, therefore, as already explained, depend upon the elasticity and density of the medium; the greater the elasticity and the smaller the density, the greater the velocity. Any change of state, therefore, which affects the elasticity or density of a substance, may change the value of the velocity of sound in that substance.

Thus, in any gas, such as air, a change of pressure causes a change of density and a change of elasticity. The properties of a gas, however, are such that the changes in density and elasticity, due to a change of pressure, are so related that their combined effect on the velocity of sound in the gas is nothing. That is, the velocity of sound in air is not affected by change of pressure only. A change in temperature, on the other hand, causes a change in density without affecting the elasticity of the gas, and, therefore, causes a change in the velocity of sound in the gas. A rise in the temperature of the air, for example, causes a decrease in density, and, therefore, increases the velocity of sound in air. Similarly, a fall in temperature causes a decrease in the velocity of sound in air.

In the same way the presence of water vapour in the air, by decreasing the general density of the air, causes an increase in the velocity of sound in air.

In the case of gases at the same pressure, it can be shown that, in general, the velocity of sound in the gases is inversely proportional to the square root of the density. For example, if the density of one gas is *sixteen* times that of another, the velocity of sound in the denser gas is *one-fourth* the velocity in the other gas.

CHAPTER V.

CHARACTERISTICS OF SOUNDS.

41. **A Musical Sound.**—It has already been stated that a body in vibratory motion within certain limits of frequency gives rise to a **musical sound.** It will also be found that the musical sounds with which we are familiar can generally be referred to a body in vibratory motion as source. The notes of a piano are produced by the vibration of the steel wires strung on its frame, the notes of the violin have their origin in the vibratory motion of the strings, and the notes of the organ are due to the longitudinal vibration of the air columns in the organ pipes. It has further been shown, by means of Savart's wheel and Seebeck's siren, that periodic motion at the source, even if not of a vibratory character, can give rise to a musical sound.

It may, therefore, be stated generally that any source of **periodic** disturbance in a medium is, within certain limits of frequency, a source of a musical sound.

It follows from what has been said above, that the sensation which we associate with a musical sound is due to the incidence on the ear of waves of compression and rarefaction, which are all exactly equal in wave length and exactly similar in character. That is, the effect of any single wave on the drum of the ear is repeated wave after wave, and the drum is thus subjected to a **periodic** disturbance which gives rise to the sensation generally associated with a musical sound.

The distinctive characteristic of a musical sound is thus **periodicity.** The motion of the source is periodic, and the wave motion, constant in wave length and fixed in form, gives rise to a periodic stimulus in the ear.

If the motion of the source is simple harmonic motion the resulting wave motion is simple harmonic wave motion, and the musical sound produced is known as a **simple sound** or **pure tone.** If, however, the motion of the source is periodic, but not simple harmonic in character, the wave motion is not of the simple harmonic type, and the musical sound produced is known as a **complex sound** or **compound tone.** The note of a tuning fork, for example, when not sounding very loudly, is practically a pure tone; the note of a violin string, on the other hand, is complex in character and is a good example of a compound tone.

42. **Noise.**—All sounds other than musical sounds are usually grouped under the general term **noise.**

Just as periodicity is the distinctive characteristic of a musical sound, so irregularity or want of periodicity is the distinctive characteristic of noise. The motion of the source in the case of a noise is non-periodic in character. The disturbance set up by the source in the surrounding medium is, therefore, merely an irregular sequence of states of compression and rarefaction in which there is no periodic recurrence of the same sequence of states. The disturbance is, therefore, not divisible into wave lengths, and if its form were represented by a curve (Art. 34) the form of the curve would be quite irregular and entirely wanting in any element of periodicity. It will be seen, however, that irregular disturbances of this kind admit of infinite variety in form, and this accounts for the great differences which exist between the many noises which we hear around us.

The incidence of an irregular disturbance of this kind on the ear gives rise to the sensation usually associated with a noise, provided the irregular sequence of states of compression and rarefaction which constitute the disturbance come within the physiological limits of rapidity for this sensation.

The character of the noise heard depends upon the form of the incident disturbance.

43. **Characteristics of Musical Sounds.** — A musical sound, considered as a sensation, possesses three distinctive characteristics which enable us to differentiate one musical sound from another. These characteristics are **loudness, pitch,** and **quality.**

Loudness is a quality of sound sensation which belongs to all sounds, to noises as well as to musical sounds. The general meaning of the term is commonly understood ; it applies to the strength or degree of the sensation, and is applicable as a general term to sounds of all degrees of audibility. The terms soft, faint, feeble, loud, as commonly used, apply to this characteristic of a musical sound.

Pitch is the quality which enables us to distinguish between the different notes of a musical scale. On any musical scale the pitch rises from note to note as we ascend the scale, and falls as we descend the scale; notes low down on the scale are notes of **low pitch,** and notes high up on the scale are notes of **high pitch.** The terms low, deep, shrill, high, as commonly used, apply to this characteristic of a musical sound.

Quality, in the special sense in which the term is here used, is the characteristic of a musical sound by means of which we are able to distinguish between sounds of the same pitch, and, it may be, of the same loudness, but of different general character.

Thus, the differences between musical sounds which we usually refer to difference in origin or in mode of production are differences in quality. The terms clear, harsh, thin, mellow, full, round, as commonly used, apply to some special feature in the quality of a note.

A note given out by a tuning fork differs essentially in quality from a note of the same pitch given out by a stretched string; so also a note given by a closed organ pipe differs in quality from a note of the same pitch given by an open pipe.

The French term **timbre** is sometimes used instead of quality in this special sense.

Each of these three characteristics of a musical sound, as a sensation, can be referred to a corresponding characteristic of the motion of the source of the sound, and of the wave motion by which it is propagated.

44. **Loudness.**—The loudness of a musical sound may be traced to the **amplitude** of vibration of the source of sound. When a tuning fork is sounding the loudness of the sound heard decreases as the amplitude of the vibration of the prongs of the fork decreases. When a violin string is sounding, the loudness of the sound decreases as the amplitude of vibration decreases. Similarly, in the case of any other source in vibratory motion, the loudness of the sound depends on the amplitude of vibration and can be shown to be directly proportional to the square of the amplitude.

The sensation of loudness is, however, due to the effect of the wave motion incident on the drum of the ear. Loudness is, therefore, more directly traceable to the intensity of the wave motion incident on the ear; the greater the intensity the greater the loudness of the sound. The intensity of wave motion at any point in the medium is proportional to the square of the amplitude of vibration of the particles of the medium at that point. The loudness of a sound may, therefore, be referred to the amplitude of vibration of the air particles near the ear, and, if loudness be assumed to be proportional to the intensity of the wave motion incident on the ear it may be said to be proportional to the square of the amplitude of the air particles near the ear.

The amplitude of vibration of the air particles at any point in air carrying wave motion from a source of sound is directly proportional to the amplitude of vibration of the source, so that although the loudness of a sound is most directly referred to the amplitude of vibration of the air particles near the ear of the observer, it can also be referred, as stated above, to the amplitude of vibration of the source itself.

As explained in Art. 33, the intensity of the wave motion from a source of sound at any point in the medium is inversely proportional to the square of the distance of the point from the source. It follows then, from what is stated above, that the loudness of the sound heard at different distances from a *constant* source of sound varies inversely as the square of the distance of the hearer from the source. Thus, at distances of 100 yards and 200 yards from the same source of sound, the loudness of the sound heard at the more distant point is only one-fourth the loudness at the nearer point. The loudness of a sound is also found to depend upon the density of the medium surrounding the source. The greater the density the louder the sound and the smaller the density the fainter the sound.

Experiment 19.—Suspend a small clockwork bell in a large flask or receiver fitted so that it can be filled with air at different pressures, water vapour, hydrogen, or any other gaseous medium.

It will be found that the loudness of the sound heard varies with the density of the medium, and that the denser the medium the louder the sound.

The reason for this effect of the density of the medium surrounding the source of the sound is evidently found in the fact that the energy communicated by the source to the surrounding medium during each complete vibration is directly proportional to the density of the medium. As a result of this, the intensity of the wave motion at any point, and, therefore, the loudness of the sound heard at that point, are directly proportional to the density of the medium surrounding the source. Loudness as a sensation cannot be measured. It is possible only to compare, generally, the relative loudness of two sounds, and, perhaps, to decide when two sounds are equally loud.

45. **Pitch.**—The pitch of a musical sound depends upon the **vibration frequency** of the source.

Experiment 20.—Set up a Seebeck siren or a Savart wheel, and test how the pitch of the note produced by the revolution of the disc or the wheel varies with the speed of revolution.

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It will be found that if the speed is increased the pitch rises, if it is kept constant the pitch remains constant, and if it is decreased the pitch falls. That is, the pitch of the note changes when the frequency of the source changes; it rises or falls as the frequency increases or decreases.

The relation between pitch and vibration frequency indicated by this experiment may be brought out more clearly by the apparatus shown in Fig. 27.*

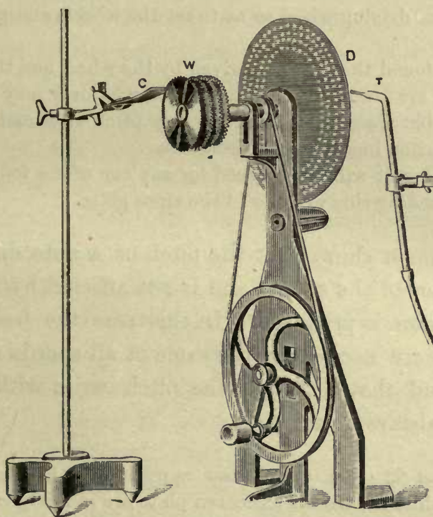

Fig. 27.

A block of four Savart wheels, W, and a siren disc, D, with four concentric circles of holes, are fixed on the same axis and mounted on a stand so that they can be rotated at any required speed by means of the driving wheel and belt shown in the figure.

The numbers of the teeth on the four wheels, taken in ascending order, are respectively the same as the numbers of the holes in the four circles of the siren plate, and these numbers are

* See *Sound* by Poynting & Thomson, p. 9.

in the ratio 4 : 5 : 6 : 8. A card, C, is held in a stand so that it can be adjusted to any one of the four wheels, and a tube, T, is held in another stand so that it can be adjusted to direct a stream of air on to any one of the four circles of holes in the plate.

Experiment 21.—Set up this apparatus and adjust the card C to one of the wheels, and the nozzle of the tube T to the corresponding ring of holes in the siren plate. Put the air jet in action and turn the driving wheel so as to set the wheels and plate in rapid rotation.

It will be found that the note given by the wheel and the note given by the siren are always of the same pitch whatever may be the speed of rotation, but that, as in Exp. 20, this pitch rises and falls as the speed of rotation increases and decreases.

The same result will be obtained for any one of the four wheels and the corresponding ring of holes in the siren plate.

This experiment shows that the pitch of a note depends only on the frequency of the source, and is not affected by the manner in which the note is produced. In this case the frequencies of the two notes are necessarily the same at all speeds of rotation, and it is found that although the pitch varies with the speed, the notes are always in unison.

Experiment 22.—Set up the same apparatus as in the last experiment, and while the wheels and siren plate are in fairly rapid rotation at a uniform rate test the sequence of notes obtained (*a*) by touching the four wheels with the card in succession, beginning with the wheel having the smallest number of teeth, (*b*) by directing the air jet in similar succession on the four circles of holes in the plate.

Provided the speed of rotation is uniform for the short interval of time necessary to get a sequence of four notes as in (*a*) or (*b*), it will be found that at all speeds the sequence obtained is the same, and constitutes the common chord on the note of lowest pitch as tonic.

This experiment shows that the difference in pitch or the **interval** between any two notes is determined, whatever may be the individual frequencies of the notes, by the **ratio** of their frequencies. In the experiment the ratio of the frequencies of the four notes is fixed, by the construction of the apparatus, at the value 4 : 5 : 6 : 8, and it is found that at all speeds—that is, for all frequencies—the intervals between the successive notes of the sequence remain constant.

The ratios corresponding to the familiar intervals which occur in the common chord, **d m s d$^{\prime}$**, should be noted. The interval **d** to **m**, measured by the ratio $\frac{5}{4}$, is a **major third**; the interval **d** to **s**, measured by the ratio $\frac{3}{2}$, is a **fifth**; and the interval **d** to **d$^{\prime}$**, measured by the ratio $\frac{2}{1}$, is an **octave**. It will be seen from the value of the ratio corresponding to an octave interval that the pitch of a note may be raised an octave by doubling the frequency of the source, or lowered an octave by halving the frequency.

Although the pitch of a sound is thus found to depend on the vibration frequency of the source, its direct cause as a sensation is the incidence on the ear of the wave motion originated by the source. Pitch, therefore, depends directly upon the number of wave lengths incident on the ear in one second. This explains why the pitch of a note from a source of constant frequency is altered by the relative motion of the source and the observer.

46. **The Diatonic Scale.**—The diatonic scale is the familiar musical scale associated with the common chord referred to above. The relative frequencies of the notes included in an octave of the scale are denoted by the numbers given below:

d	r	m	f	s	l	t	d$^{\prime}$
24	27	30	32	36	40	45	48

Thus, if we take the frequency for the middle C of the piano as 256, the **diatonic** scale (not the piano scale) on this note as tonic is given by

C	D	E	F	G	A	B	*c*
256	288	320	341	384	427	480	512

The successive intervals which occur in the scale are **tones** and **semitones**, arranged in the following sequence :—

d		r		m		f		s		l		t		d'
	Tone.		Tone.		Semitone.		Tone.		Tone.		Tone.		Semitone.	
	$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

It will be seen, however, that the tones are not exactly equal, some being measured by the ratio $\frac{10}{9}$ and others by the ratio $\frac{9}{8}$; the two semitones are equal, each being measured by the ratio $\frac{16}{15}$.

Since the interval between any two notes is measured by the ratio of the corresponding frequencies, it follows that two intervals may be *added* by taking the *product* of the ratios which measure them. Thus, the interval from **d** to **s** on the common chord is the sum of the intervals **d** to **m** and **m** to s, and it is clear that $\frac{3}{2} = \frac{5}{4} \times \frac{6}{5}$. Similarly, the difference of two intervals is obtained by dividing the ratio for the greater of the two intervals by the ratio for the smaller. Thus, the difference between the two *tones* which occur in the diatonic scale is given by $\frac{9}{8} \div \frac{10}{9}$, or $\frac{81}{80}$.

It should also be noted that two intervals are equal when the frequency ratio for each interval is the same. Thus, a note for which the frequency is 300 may be said to be midway in pitch between notes for which the source frequencies are 200 and 450 respectively, because the interval measured by the ratio $\frac{450}{300}$, or $\frac{3}{2}$, is equal to the interval measured by the ratio $\frac{300}{200}$, or $\frac{3}{2}$. In the same way it can be seen that the semitone found in the diatonic scale is not exactly *half* either of the tones of the scale; for $\frac{16}{15} \times \frac{16}{15}$, or $\frac{256}{225}$, has a value less than $\frac{9}{8}$ and greater than $\frac{10}{9}$.

The scale to which a piano is tuned is a modified or **tempered** form of the diatonic scale. The octave is divided into twelve *equal* intervals so that all the tones are equal, and the semitone is exactly half a tone.

The standard of absolute pitch is not very well defined. In England **concert pitch** requires the middle C of the piano to correspond to a source frequency of 273. The new **standard pitch** is defined by stating that standard tuning forks giving the C and A of the middle octave must have a frequency of 261 and 439 respectively, at 68° F.

47. **Quality.**—The **quality** or **timbre** of a note may be referred to the manner in which the displacement of the source varies during a complete period; that is, to the **form** of the displacement curve for the motion of the source.

The quality of a musical sound as perceived by the ear is, however, more directly referred to the **wave form** of the incident waves. The orderly sequence of states of compression and rarefaction which determine the **form** of a longitudinal wave recur, wave length after wave length, in the same *general* cycle for all waves. The manner in which state follows state through the same general cycle admits, however, of infinite variety in detail, and may, therefore, serve to differentiate one wave from another, even when the wave length and intensity of the waves are the same.

The quality of a note depends, therefore, upon the **wave form** of the wave motion incident on the ear, the wave form being determined by the special character of the sequence of states of compression and rarefaction which constitute a wave length of the motion.

It should be noticed that the three characteristics of a musical sound—loudness, pitch, and quality—may each be referred to a corresponding attribute of (*a*) the source, (*b*) the wave motion originated by the source, and (*c*) the vibratory motion of the particles of the medium carrying the wave motion.

Thus, with reference to the source, loudness depends upon the amplitude of vibration, pitch on the vibration frequency, and quality on the nature of the displacement variation during a complete vibration.

With reference to the wave motion originated by the source, loudness depends upon the intensity of the motion, pitch upon the wave length, and quality on the wave form.

With reference to the vibratory motion of the particles of the medium, loudness depends upon the amplitude of vibration, pitch upon the vibration frequency, and quality upon the nature of the displacement variation during a complete period for particles near the ear of the observer.

CHAPTER VI.

REFLECTION AND REFRACTION OF SOUND.

48. **Reflection and Refraction of Wave Motion.**—When wave motion in any medium is incident on the boundary surface separating the medium from another medium the motion is, in part, turned back or **reflected** at the surface, and, in part, transmitted across the surface or **refracted** into the second medium. In general, too, some portion of the wave motion is **absorbed** at the surface; that is, a portion of the energy of the wave motion is, in general, converted into heat at the surface.

The **reflection** of the wave motion at the surface of separation of the media depends, in character, on the nature of the surface. If the surface is rough the reflection at any point on the surface is **diffuse** in character. That is, the disturbance incident on the surface along any particular line of transmission is not reflected at the point of incidence in another definite direction back into the medium, but is reflected or **diffused** in all directions from this point.

If the surface is smooth the reflection at any point on the surface is regular in character; the disturbance incident on the surface along any particular line of transmission is reflected at the point of incidence along another definite direction related to the direction of the incident disturbance by definite laws, known as the laws of regular reflection.

The degree of smoothness necessary for regular reflection depends upon the wave length; if the inequalities of the surface are small compared with the wave length of the incident wave motion the reflection of the motion will be regular in character. The extent of surface necessary for the reflection of wave motion depends also on the wave length; the extent must be large compared with the wave length in order to obtain appreciable reflection.

In the case of a smooth surface of regular geometrical form the wave front of the reflected wave is determined geometrically by the form of the incident wave front and the form of the reflecting surface.

The **refraction** of the wave motion from one medium into another depends, also, on the nature of the surface of separation of the media. If the surface is rough the incident wave motion is, in general, practically all reflected diffusively or absorbed at the surface. If the surface is smooth the motion is partly reflected and partly refracted so that the greater the reflected portion is, the smaller is the portion refracted into the second medium.

In the case of refraction from one uniform medium into another, the disturbance travelling along any particular line in the first medium is refracted at the surface of separation along another definite line in the second medium. The direction of this refracted line of disturbance is related to the direction of the incident line of disturbance by definite laws, known as the laws of refraction. The relation involved in these laws depends mainly, as shown below, on the ratio of the velocities of the wave motion in the two media. It follows, also, from this relation that the directions of the incident and refracted lines of disturbance are not, in general, in the same straight line; that is, the line of transmission of the disturbance from one medium to the other suffers a sudden change of direction at the point where it crosses the surface of separation of the media.

The laws of reflection and refraction of wave motion may be deduced theoretically by a geometrical method due to Huyghens. A brief and incomplete outline of this method is given below for the case of the reflection and refraction of a plane wave—that is, a wave with a plane wave front—at the plane surface of separation between two uniform media.

In Fig. 28 let the plane of the wave front and the plane of the surface of separation of the media be at right angles to the plane of the paper, and let SS denote the trace of the surface and AB the trace of the wave front at the instant the point A is incident on the surface of separation. By the time the disturbance at B has travelled on to C, and the whole wave front AB has been incident on the surface from A to C, the **reflected** disturbance from A will have travelled to all points on a hemispherical surface of radius AD, equal to BC, and the

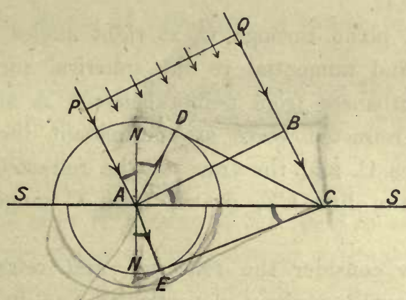

Fig. 28.

refracted disturbance from A will have travelled to all points on the surface of a hemisphere of radius AE, greater or less than AD and BC according as the velocity of the wave motion in the second medium is greater or less than in the first medium. The ratio of AE to AD or BC is evidently the ratio of the velocities of the wave motion in the media in which these distances are taken. In the figure AE is shown less than AD, thus indicating that the velocity of the wave motion in the second medium is less than in the first medium.

Similarly, the reflected and refracted disturbances from n equidistant points, intermediate between A and C, will have reached the surfaces of hemispheres of radii which decrease regularly for the reflected disturbances by $1/n$th of AD, and for the refracted disturbances by $1/n$th of AE.

It follows from this that a plane through C, at right angles to the plane of the paper and tangential to any one of the spherical surfaces for the reflected disturbances from points between A and C, is tangential to all the surfaces, and is, therefore, the front of the reflected wave at the instant the disturbance from B reaches C. Also, since the surfaces are spherical their points of contact with this plane lie in the plane of the paper, and the trace of the reflected wave front is, therefore, given by the line CD, drawn from C tangential to the circle AD.

Similarly, a plane through C, at right angles to the plane of the paper and tangential to the spherical surfaces for the refracted disturbances from points between A and C, is the front of the refracted wave at the instant the disturbance from B reaches C, and the trace of the refracted wave front is given by the line CE, drawn from C tangential to the circle AE.

Let us now consider the reflection and refraction of the disturbance travelling along a particular line in the incident wave. The disturbance at the point P travels along the line PA to the point A, so that PA may be taken as a line of disturbance incident at the point A on the surface of separation of the two media. This line of disturbance is, by the construction of Fig. 28, reflected along the line AD and refracted along the line AE. Let NAN′ be the normal to the surface at A, the point of incidence; then PAN is the **angle of incidence** for the disturbance along PA, DAN is the **angle of reflection**, and EAN′ is the **angle of refraction.** The following results can now be readily proved from the geometry of the figure :—

(*a*) The angle DAN is equal to the angle PAN.

(*b*) The angles PAN and EAN are respectively equal to the angles BAC and ECA, and the ratio of the sines of the angles is, therefore, constant for all corresponding values of the angles. For we have

$$\frac{\sin \text{PAN}}{\sin \text{EAN}} = \frac{\sin \text{BAC}}{\sin \text{ECA}} = \frac{\text{BC}}{\text{AE}} = \frac{V_1}{V_2},$$

where V_1 and V_2 are the velocities of the wave motion in the first and second media respectively.

(*c*) The lines PA, AD, AE, and NAN′ are all in the same plane.

From these results the laws of reflection and refraction can now be formulated.

For the laws of reflection we have—

(i) The angle of reflection is equal to the angle of incidence.

(ii) The incident line of disturbance, the normal at the point of incidence, and the reflected line of disturbance are in the same plane.

Similarly, for the laws of refraction we have—

(i) The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for all corresponding values of these angles.

(ii) The incident line of disturbance, the normal at the point of incidence, and the refracted line of disturbance are in the same plane.

The ratio of the sine of the angle of incidence to the sine of the angle of refraction for any two media is called the *index of refraction* for these two media, and is usually denoted by μ. Thus, when wave motion is refracted from a medium A to a medium B, if i denote the angle of incidence and r the angle of refraction, we have

$$\frac{\sin i}{\sin r} = \mu,$$

where μ denotes the index of refraction from the medium A into the medium B.

It can be seen, from the construction of Fig. 28, that when wave motion is refracted from one medium into another in which the velocity is *greater* than in the first there must be a particular angle of incidence, less than 90°, at which the radius of the circle AE is equal to AC. In this case the angle of refraction is 90°, and the direction of the refracted wave will be parallel to the surface of separation of the media. This angle is called the **critical angle** for the two media, and it is evident that, for angles of incidence greater than this critical angle, the construction of Fig. 28 is impossible and no direction can be determined for the refracted wave. This corresponds with the experimental results that, in the case here considered, wave motion incident on the surface of separation of the media is partially reflected and partially refracted for all angles of incidence up to the critical angle for the media, but that at the critical angle refraction ceases and for all angles of incidence greater than the critical angle the wave motion is **totally reflected.**

Total reflection, therefore, takes place when wave motion in any medium is incident on the surface of a second medium in which the velocity of propagation of the motion is greater than in the first, at an angle greater than the critical angle for the two media. The magnitude of the critical angle for any two media is readily determined by the fact that the critical angle is the angle of incidence which corresponds to an angle of refraction of 90°, so that if i denote the critical angle for any two media, we have

$$\frac{\sin i}{\sin 90} = \mu;$$

and since $\sin 90 = 1$, we get the relation

$$\sin i = \mu.$$

That is, the critical angle for any two media is the angle whose sine is the index of refraction for the media.

49. **Reflection of Sound.**—Sound is known to be longitudinal wave motion in material media, and should, therefore, be subject to reflection in accordance with the laws of reflection of wave motion. General experience and the results of experiments show that this is the case. Sound waves in air are reflected at the surfaces of walls and cliffs, at the surface of water, at the surfaces of clouds, at the surface of a warmer or colder mass of air; in fact, at the surface of separation of the air from any other substance or medium, in which the velocity of the waves differs from their velocity in the air in which they are travelling.

Echoes are caused by the reflection of sounds from surfaces of this kind, and the confusion of sound observed in a room ill designed for speaking or singing is due to reflection from the walls of the room. Noises and musical sounds are reflected alike, pulses of irregular longitudinal disturbance being reflected in exactly the same way as regular wave motion.

The reflection of sound cannot be studied experimentally with the same exactitude as is possible in the case of light, but it can be shown by a few simple experiments that it is subject to the general laws of the reflection of wave motion.

Experiment 23.—Take a cardboard tube about 3 ft. long and 1 inch in diameter, and fix it in the position shown at AB in Fig. 29, so that

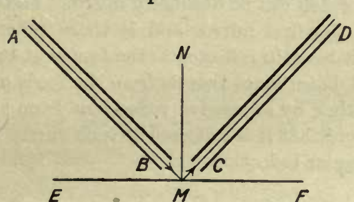

Fig. 29.

it makes an angle of about 30° with the horizontal surface of the table top shown at EF.

Now, suspend a watch just inside the tube at A, and then adjust another similar tube CD in position until the position is found in which the ticking of the watch at A can be most distinctly heard by an ear at D. When this adjustment is made it will be found that AM and MD, the axes of the tubes AB and CD are equally inclined to the normal to the surface of the table at M, and that both axes and the normal are in the same plane. That is, the wave motion transmitted along the axis AM, and incident upon the reflecting surface of the table at M, is reflected along the axis MD in a direction such that the angle of reflection NMD is equal to the angle of incidence AMN, and the two lines AM and MD and the normal MN are in the same plane.

Experiment 24.—Adapt the foregoing experiment so as to obtain reflection of sound from the surface of a large *flat* flame or from the surface of a stream of warm or cold air issuing as a thin sheet from a nozzle with a narrow slit opening.

It will be found most convenient to arrange the two tubes in the same horizontal plane at an angle of about 120°, and then to adjust the position of the plane of the flame or the sheet of air until good reflection is obtained.

It will be found when the adjustment is made that the angles of reflection and incidence are equal and in the same plane.

Experiment 25.—Set up two large concave mirrors * facing each other about 10 feet apart, and adjust their positions so that their axes are in the same straight line.

Place a ticking watch at the focus of one mirror and a small funnel, with a long piece of rubber tubing attached to its stem at the focus of the other mirror. The open end of the funnel should be directed towards the mirror.

If the free end of the rubber tubing be now applied to the ear the ticking of the watch can be distinctly heard. The sound travels from the watch to the first mirror and is there reflected to the second mirror where it is again reflected to the funnel at the focus. The fact that the sound disturbance travels from the focus of one mirror to the focus of the other by successive reflections from the surfaces of the mirrors is a proof that it is reflected at each mirror in accordance with the laws of regular reflection.

It has been explained in Art. 48 that the degree of smoothness and the extent of surface necessary for regular reflection of wave motion both depend upon the wave length.

The wave length for audible musical sounds varies from about 35 feet for sounds of the lowest audible pitch to less than

* See Chapter V. in the Section on *Light*.

half an inch for sounds of the highest pitch, so that the degree of smoothness or the extent of surface necessary for the regular reflection of sound varies within somewhat wide limits according to the pitch of the sound.

The surface of an ordinary plaster wall, for example, is smooth enough to reflect sounds of the highest pitch, but the extent necessary for efficient reflection would increase from a few square inches for sounds of high pitch to many square yards for sounds of very low pitch. A good brick wall of sufficient extent would reflect all sounds except, perhaps, those of the very highest pitch. On the other hand, the face of a quarry or a cliff, or a mountain side might be smooth enough, and would certainly be of sufficient extent to reflect sounds of very low pitch. In the case of a broken surface, such as the face of a cliff or quarry, it is possible that many of the small surfaces which make up the face may act individually as reflecting surfaces for sounds of fairly high pitch, while the surface as a whole may reflect only sounds of very low pitch, or may even be too irregular to give regular reflection of any sound.

The wave length of spoken sounds in ordinary conversation varies from about 2 feet to about 6 feet, so that a fairly rough surface will reflect these sounds, but it is obvious that for efficient reflection the extent of the reflecting surface must be fairly large.

Noises cannot be said to have pitch, but the pulse of irregular disturbance associated with a noise may be long or short, according to the character of the noise. The pulse for a sharp, clear sound, such as the stroke of a hammer on an anvil, is very short, while the pulse associated with the report of a cannon is fairly long. The conditions for the efficient reflection of a noise depend upon the length of the pulse in the same way as they depend, in the case of a musical sound, on the pitch.

50. **Echoes.**—An echo of any sound is caused by the reflection of the sound at some suitable surface, such as the face of a

wall, a cliff, or a mountain side. The reflection may even take place, as in the case of thunder, at the surface of a cloud or at the surface of a current of hot or cold air.

The conditions under which an echo may be heard, in addition to those which apply to the reflection of the sound, include the physiological fact that the hearing sensation for any sound lasts or *persists* for about one-tenth of a second after the sound wave or pulse has ceased to act on the ear. Hence, in the case of a short, sharp sound of practically no duration, such as the stroke of a hammer on an anvil, the sound itself and the echo cannot be heard as distinct separate sounds unless the echo reaches the ear at least one-tenth of a second after the wave of the initial sound has left it. That is, if the observer be placed at the point where the sound originates at some distance from a suitable reflecting surface, an echo will be heard only if the perpendicular distance of the surface from the observer is such that the sound takes at least one-tenth of a second to travel from the ear to the reflecting surface and back to the ear after reflection at the surface. That is, if we take the velocity of sound to be about 1,120 feet per second, the distance of the reflecting surface from the observer must be at least 56 feet, for the double distance from the ear to the reflecting surface and back will then be 112 feet, or the distance travelled by the sound in one-tenth of a second.

If the sound refleeted has an appreciable duration then it is evident that a *complete* separate echo of the sound will not be heard unless the distance of the reflecting surface is such that the echo of the beginning of the sound reaches the ear one-tenth of a second after the end of the sound leaves the ear. That is, if the duration of the sound is t tenths of a second, the beginning of the sound must travel for at least $(t + 1)$ tenths of a second before reaching the ear as an echo. In this time the sound will have travelled $112\ (t + 1)$ feet, and the distance of the reflecting surface necessary to give a complete and separate echo of a sound of t tenths of a second duration must, therefore, be at least

56 $(t + 1)$ feet. If the distance is less than this the echo begins before the sound ends and only the end of it is heard as a distinct sound; if the distance is greater than this there is an interval between the sound and its echo.

In the case of sounds uttered by the human voice it is found that the minimum time necessary for the distinct articulation of a short syllable is about one-tenth of a second, and that, therefore, not more than five syllables can be uttered and heard as distinct sounds in one second. The echo of a single syllable sound will, therefore, be heard immediately after the sound ends if the distance of the reflecting surface is at least (56×2) feet, or 112 feet, for the value of t in this case is 1 if we take the duration of the sound to be exactly one-tenth of a second. In the case of a shout or sound of n syllables, uttered as rapidly as possible, the distance of the reflecting surface, in order to give a distinct echo of all n syllables, must evidently be at least $(n \times 112)$ feet. If the distance is less than this the echo of the first syllable returns before the whole of the n syllables have been uttered, and the echoes of only a few of the end syllables are heard after the sound has ceased. For example, if the distance of the reflecting surface is 112 feet, the echo of only the last syllable is heard as a separate sound; if the distance is 224 feet, the echoes of only the last two syllables are heard; and if the distance is r (112) feet, the echoes of only the last r syllables of the sound are heard. Echoes which repeat more than one syllable of a polysyllabic sound after the sound ends are sometimes called **polysyllabic echoes.**

When a sound is reflected at a number of different surfaces a number of different echoes of the same sound may be heard, and if these follow each other at short intervals they may have the effect of prolonging a short loud sound into a long roll or rumble. It is possible, also, to get a very prolonged echo by successive reflection between two suitably placed surfaces. The rolling of thunder, for example, is caused in this way, by multiple and successive reflections from a large number of reflecting surfaces.

51. **Speaking Tubes.**—The action of a speaking tube is due primarily to reflection at the inner surface of the tube. The energy communicated per second to the air in the tube across the transverse layer at the mouth of the tube is passed on from layer to layer along the tube, and as the cross-section of the tube is constant in area, the area across which this energy is transmitted from layer to layer is constant. That is, the energy transmitted across unit area per second, or the intensity of the wave motion (Art. 33) transmitted along the tube, does not decrease as distance from the starting point increases, but remains constant from point to point along the tube, except

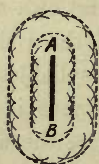

Fig. 30.

in so far as it is decreased by work done against friction in the tube.

Wave motion originated at the mouth of the tube would, *in the open air*, spread out into the medium in the manner indicated by the wave front diagram (Art. 35) shown in Fig. 30. The area of the wave front across which the energy is transmitted from a layer obviously increases as the distance from the origin increases, and the intensity of wave motion decreases as this area increases. In fact, at no great distance from the starting point the wave motion spreads out into the medium practically as a **spherical wave**, and the intensity is inversely proportional to the square of the distance.

The wave motion originated at the mouth of the tube travels through *the air in the tube*, however, practically as a **plane wave**, in the manner indicated in the wave front diagram shown in Fig. 31. The area of the wave front across which the energy is transmitted from layer to layer is constant, and the intensity of the wave motion is, therefore, constant and subject only to decrease by work done against frictional resistance as the motion travels along the tube.

It will be seen that in the transmission of the wave motion along the tube the plane wave front is prevented from spreading latterally, as in Fig. 30, by the reflection at the inner wall of the tube of disturbances originating at points in the wave front close to the wall.

It will be understood from what has been said above that sound may be transmitted through a tube for very considerable

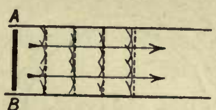

Fig. 31.

distances without undergoing any great decrease in loudness. A **speaking tube** is a tube used in this way for the purpose of speaking from one room to another some distance away. The tube need not be more than an inch in diameter and it may be of considerable length. It may also be bent at any point in any direction provided the bends are made smoothly and not too abruptly. The inner wall should be smooth to avoid loss of energy by frictional resistance.

The action of an **ear trumpet** is practically the same as that of a speaking tube. The instrument is essentially a long, conical tube, tapering towards the ear, and commonly bent into a compact form convenient for use. The wave motion entering the mouth of the trumpet travels along the tube, round the carefully constructed bends, to the ear. Since the tube tapers towards the ear, the wave fronts across which the energy is transmitted from layer to layer *decrease* in area, and the intensity of the wave motion therefore increases, and may, when it enters the ear, be considerably greater than at the point where it enters the trumpet.

The **stethoscope** used by medical men in auscultation is another instance of the application of the principle of the speaking tube.

52. **Whispering Galleries.**—In any room or enclosure of circular plan in which the inner wall surface is free from obstructions and smooth enough to reflect sounds, a sound produced at any point near the wall is transmitted round the circumference of the room by successive reflection from the wall, and may be distinctly heard at any point in the circumference.

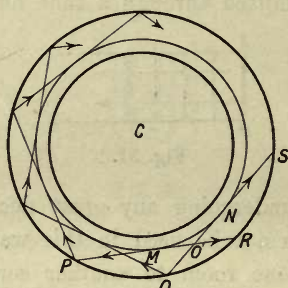

Fig. 32.

In the circular gallery round the wall in the dome of St. Paul's Cathedral, for example, a whisper at any point is distinctly audible all round the gallery.

The explanation of this effect of successive reflection at the wall of the gallery is best understood with the aid of a diagram. Let O, in Fig. 32, represent on the plan of the gallery shown in the figure, the point at which the sound is produced, and let the circle MN represent the plan of a cylindrical surface in the air, concentric with the surface of the wall and of any radius less than CO. Through O, the position of the source of sound, the lines POR and QOS are drawn tangential to the circle MN at the points M and N.

It will be clear from this construction that the wave motion diverging from O within the angle POQ will be reflected round and round the gallery, within the ring of air, lying between the circle MN and the wall of the gallery. The disturbance along the lines OP and OQ will be subjected to successive reflection from the wall, along chords of equal length tangential to the circle MN; and the disturbance along any line between OP and OQ will be reflected round along chords tangential to a circle between MN and the wall. It follows, therefore, that the wave motion diverging from O, between the lines OP and OQ is transmitted round the gallery by successive reflection from the wall, in such a way that it travels only in the air between MN and the wall. The energy transmitted from O between the lines OP and OQ is thus transmitted to a limited mass of air, and is prevented from spreading out as in an open medium. The intensity of the motion, therefore, decreases very slowly as the wave travels round the gallery, and it follows that a faint sound produced at any point may be audible at all points in the gallery.

In the same way it will be seen that the wave motion diverging from O within the angle ROS also travels round the gallery, in the opposite direction, in the air between the circle MN and the wall. That is, a sound produced at any point in the gallery is reflected round and round the gallery in opposite directions from the point at which it is produced.

53. **Musical Note Produced by Successive Reflection.** —It has already been explained that the sensation associated with a musical sound is caused by the incidence of a periodic wave disturbance on the drum of the ear. If, therefore, successive reflections or echoes of the *same sound* fall upon the ear in regular, and sufficiently rapid sequence, the sensation produced must, in some measure, correspond to that for a musical note.

The initial sound must necessarily be of short duration, not more than one-thirtieth of a second, but it may be a short sharp sound of any kind, for the periodicity of the train of pulses incident upon the ear is due to the regular sequence of reflection of the *same pulse*, and does not depend upon the form of the pulse itself. The quality of the note will, however, depend upon the form of the initial pulse, for this determines the form of the wave incident upon the ear.

This effect may be observed if a very short, sharp sound, such as may be produced by striking two pieces of metal together, is produced near a railing. If the pulse set up in the air at the production of the sound is short enough, it will be reflected at each rail, and the regular sequence of reflected pulses which are thus returned to the ear will be rapid enough to produce a musical note which is usually heard as a short musical ring immediately following the initial sound.

A musical note produced in this way can be heard very distinctly near a boarded fence in which the boards are set in the usual way with overlapping edges. A sharp sound produced near the fence is well reflected at the edges of the successive boards, and a clear musical ring following the sound can be distinctly heard.

Successive reflection from the faces of the individual steps of a flight of steps, may also give rise to a similar effect.

In cases of this kind it is evident that if d denote the distance between the reflecting surfaces and V the velocity of sound in the air, the pitch of the note heard by an observer in or near the plane of the reflecting surfaces corresponds to a frequency given by $\frac{V}{2d}$. Thus, in the case of a boarded fence, if the distance between the edges of the boards is 4 inches and the velocity of sound in the air be taken as 1,120 feet per second, the frequency for the note heard is $\frac{1{,}120 \times 3}{2}$ per second, or 1,680 per second.

That is, the note is about two octaves and a sixth above the middle C of the piano.

54. **Refraction of Sound.**—Just as sound is reflected in accordance with the laws of reflection of wave motion, so is it refracted at the surface of separation of two media in accordance with the laws of refraction. This may be verified by showing that sound waves are affected by refraction through a plate or prism or lens of any suitable medium in exactly the same general way as light.* Thus, if sound waves are refracted through a lens, the lines of transmission are made either more convergent or more divergent by the refraction, according to the form of the lens and the nature of the material of which it is made. If the waves diverge from a point on the axis of the lens it can be shown that, after refraction through the lens, they diverge from or converge to another point on the axis of the lens and that the two points are—as in the case of light—conjugate foci.

A suitable lens for the refraction of sound is made by enclosing carbon dioxide gas in a collodion bag, shaped, when full, like a large double convex lens. The velocity of sound in carbon dioxide is less than in air, so that this lens acts as a converging lens for sound waves.

Experiment 26.—Suspend a carbon dioxide lens with its axis in a horizontal position, and place a ticking watch at a point on the axis at a distance from the lens equal to about twice the radius of curvature of the faces of the lens.

Use a small funnel attached to a length of rubber tubing (as in Exp. 25), as an "ear," and find the point on the axis at which the sound pulses diverging from the watch are brought to a focus. When this point is found interchange the positions of the "ear" and the watch, and show that the positions are conjugate.

Measure the distances of the conjugate foci from the lens and calculate the focal length of the lens by the usual formula for a lens.

It is to be noticed in connection with the refraction of sound that the velocity of sound in most liquid and solid media is con-

* See Chapter IV. in the Section on *Light*.

siderably greater than in air. It follows, therefore, that sound waves in air may be totally reflected at the surface of these media, and that the critical angle in most cases will be small. Thus, in the case of air and water, the index of refraction as determined by the ratio of the velocities of sound in these two media is about ·24, and the critical angle for the media is, therefore, an angle of about 14°. That is, for all angles of incidence greater than 14°, sound waves are totally reflected at the surface of water. For most other liquid and solid media and air the critical angle is even smaller and it follows that, except in the case of nearly direct incidence, sound waves in air are totally reflected at the surface of any liquid or solid medium.

In the case of gases the velocity of sound increases as the density decreases so that total reflection takes place—as in the case of light—when sound is travelling in the rarer medium.

It should be remembered when comparing the refraction of sound and light that the terms "rarer" and "denser" as used in light do not apply in sound. Light travels in any transparent substance as transverse wave motion in the ether which permeates the substance,* and, as a general rule, its velocity increases as the density of the substance decreases. Sound, however, travels in the material media, and this general relation does not at all apply for solids and liquids, although, for the reason given in Art. 40, it happens to be true for gases.

* See Art. 5 in the Section on *Light*.

CHAPTER VII.

VELOCITY OF SOUND IN AIR AND WATER.

55. **Introductory.**—The determination of the velocity of sound in air has been the subject of experiment for over two hundred years. The early experiments were made by the simple direct method of measuring the time taken by a sound in travelling over a long measured distance. Recent determinations have also been made by this direct method, but over shorter distances and with much more elaborate and accurate methods of measuring time. Good determinations have also been made by several indirect methods.

In this chapter it is proposed to notice only some of the simpler direct methods of determination.

56. **Early Determinations of the Velocity of Sound in Air.**—One of the simplest and most direct methods of determining the velocity of sound in air is to note the interval of time which elapses between the instant of seeing the flash and the instant of hearing the report of a gun fired at a known distance from the observer. This method has been dealt with in Art. 40.

A rough determination may also be made by noting the interval of time which elapses between hearing a sound and hearing its echo from a reflecting surface at a known distance from the observer.

Thus, if an observer stationed at a point on the perpendicular from the source of the sound to the reflecting surface, and at a distance of 1,000 yards from the surface, finds, by a stop watch, that the interval between the instant he hears the direct sound and the instant he hears the echo is $5\frac{2}{5}$ seconds, the average velocity of sound between the observer and the reflecting surface is given by $\frac{6,000}{5 \cdot 4}$ feet per second, or 1,111 feet per second.

It should be noticed that a method of this kind is subject to three important sources of error. A considerable error is possible in the measurement of the distance involved, and a much larger percentage error is probable in the measurement of the comparatively short time to be observed, unless a specially accurate method of measurement is adopted. A third source of error is due to the physiological fact that every observer is liable to an appreciable error in recording the instant at which a particular observation is made; this error is sometimes called the **personal equation** of the observer, and it usually differs slightly for different observers.

In all experiments of this kind, too, the motion of the air as well as its temperature and hygrometric state (Art. 40) have to be considered. If the air is in motion—that is, if there is any wind—the velocity obtained in this way is really the resultant of the velocity of the sound in still air and the velocity of the air in the direction considered. Thus, if sound is travelling with the wind the observed velocity of sound is the sum of its true velocity in still air and the velocity of the wind. If the sound is travelling directly against the wind then the observed velocity is the difference between the true velocity of sound in still air and the wind velocity. As, however, the velocity of sound in air is over 700 miles per hour, and the wind velocity under the conditions selected for an experiment would naturally be very much lower, the error resulting from omitting the velocity of the wind from the data of the experiment would be comparatively small. The effect of the wind velocity can, however, be eliminated by the method of reciprocal observations described below.

In the earliest experiments the velocity of sound in air was determined simply by observing the time taken by the report of a gun or cannon fired at a particular station, in travelling to another station at a known distance from the first. The observer at the observation station noted by means of a clock or chronometer the time that elapsed between the instant of observing the flash and the instant of hearing the report of the gun at the firing station.

The influence of the wind was usually neglected, and the temperature and humidity of the air were not recorded as they were not supposed to have any influence on the result.

Among the early determinations, that made by Derham in 1707 holds an important place. Derham observed the time taken by the report of a cannon to travel from a firing station on Blackheath to the tower of Upminster Church in Essex, a distance of 12½ miles. A long series of observations were made under different atmospheric conditions in order to determine, and, if possible, eliminate the effect of the wind.

The effect of variation in temperature and humidity was overlooked, but the effect of the wind was conclusively established in accordance with theory. It was found that the time taken by the report of the cannon in travelling between the two stations varied between 55 seconds and 63 seconds, according to the direction of the wind. As the final result of his experiments Derham found the velocity of sound in air to be 1,142 feet per second.

57. **The Method of Reciprocal Observations.**—In later experiments for the determination of the velocity of sound in air by long distance observations of the type described above, the effect of wind velocity was eliminated by the method now known as the reciprocal observation method.

Two stations at any exactly known distance apart are selected. A gun is fixed at each of the two stations, and the time taken by the report in travelling over the intervening distance in each direction is carefully determined. The average, or mean, of the two times thus observed gives the time in which sound would travel over the distance between the stations in *still air*, provided the velocity of the wind is not great and remains practically constant during the time in which the observations are made. For if S denote the distance between the stations, V the velocity by sound in air, v the velocity of the wind along the line joining the stations, and t_1 and t_2 the observed times taken by the report in travelling with and against the wind respectively, we have—

$$t_1 = \frac{S}{V + v} = \frac{S}{V\left(1 + \frac{v}{V}\right)}$$

and
$$t_2 = \frac{S}{V - v} = \frac{S}{V\left(1 - \frac{v}{V}\right)}.$$

If the ratio $\frac{v}{V}$ is small, these results reduce approximately to—

$$t_1 = \frac{S}{V}\left(1 - \frac{v}{V}\right)$$

and
$$t_2 = \frac{S}{V}\left(1 + \frac{v}{V}\right).$$

That is,
$$\frac{t_1 + t_2}{2} = \frac{S}{V}.$$

In this relation $\frac{(t_1 + t_2)}{2}$ is the mean of the observed times, and $\frac{S}{V}$ is evidently the time taken by sound in travelling over the distance S in still air.

The effect of the wind is thus eliminated under conditions such as would prevail during a determination.

In most of the determinations by this method the temperature of the air was noted and recorded among the data of the experiments.

Some of the earliest determinations were made in France by this method. In 1738 the Academie des Sciences arranged for a careful determination of the velocity of sound in the open air between two stations near Paris. One station was at Montlhéry and the other at Montmartre, 28 kilometres distant. Cannons were fired at these stations alternately, at intervals of half an hour, and observers posted at points on the line joining the stations noted the instants at which the successive sounds were heard. Clocks with pendulums beating half seconds were used for the time observations.

The value obtained by these experiments was 337 metres per second, the temperature of the air being 6° C.

Later, in 1822, the Bureau des Longitudes promoted a determination which was made by experimenting between stations at Montlhéry and Villejuif, over a distance of nearly 18 kilometres. Cannons were fired alternately at these stations at intervals of five minutes, and the times taken by the reports in travelling over the distance between the observing stations were measured by means of special chronometers.

The results of the experiments gave the velocity of sound in air at 16° C. as 341 metres per second.

Other important determinations have been made by this method, and the general result of all the best determinations is to fix the velocity of sound in dry air at 0° C. at 332 metres per second, or 1,090 feet per second.

58. **Stone's Experiments in Cape Town.**—In 1871 a careful determination of the velocity of sound in air was made by Mr. Stone, of the Cape Town Observatory. The one o'clock gun fired at Port Elizabeth served as the source of sound. Two observers were stationed on the line joining the gun to the Observatory, one being 641 feet from the gun, and the other at the Observatory, 15,449 feet distant. Each observer signalled electrically, by pressing a key, the instant he heard the report of the gun. These signals were recorded at the Observatory on a specially constructed chronograph, and from the record

given by this instrument the time taken by the sound in travelling over the distance between the observers was accurately known.

Special precautions were taken to eliminate the personal equation of the observers. As the result of these experiments Stone gives the velocity of sound in dry air at 0° C. as 1,090·6 feet per second, or 332·4 metres per second.

59. **Regnault's Experiments.**—In 1864 Regnault carried out some important open-air experiments for the determination of the velocity of sound in air.

He adopted the method of reciprocal observations. A gun was used as the source of sound, and the instants of firing the gun and hearing the reports were recorded electrically on a suitable chronograph.

In these experiments Regnault determined the average velocity of sound in air over two distances, one of 1,280 metres and the other of 2,445 metres. He found the average velocity of sound in dry air at 0° C. over the 1,280 metres to be 331·37 metres per second, and over the 2,445 metres to be 330·7 metres per second. These results seem to indicate that as the sound wave travels on and gets less intense its velocity decreases slightly as the intensity decreases. This would explain why the average velocity over the longer distance is less than over the shorter distance.

The possibility that loud sounds travel faster than faint sounds was first suggested by the fact that, during experiments in the Polar regions on the velocity of sound in air, the report of the gun fired in the experiments was usually heard before the order to fire.

A number of other results partially confirming this conclusion have been obtained, but it is still doubtful whether sufficient care has been taken to eliminate the effect of the "wind" of the explosion, or air displacement, which usually accompanies the production of a loud sound.

60. **Effect of Change of Temperature on the Velocity of Sound in Air.**—It has been found, both by theory, as explained in Art. 40, and by experiment, that the velocity of sound in air increases with rise of temperature.

The velocity of sound in dry air at 0° C. is 1,090 feet per second, or 332 metres per second, and is found to increase by about 2 feet per second, or 0·6 metre per second, for each degree centigrade rise of temperature. Thus the velocity of sound in dry air at an ordinary temperature of, say, 15° C. is 1,120 feet per second, or 341 metres per second. In ordinary atmospheric air at this temperature the velocity would be slightly greater than this on account of the presence of water vapour in the air.

In 1889, Greely made a number of determinations of the velocity of sound in air in the Arctic regions at temperatures between −10° C. and −45° C. As a result of his experiments he found that the velocity of sound in dry air increases by about 0·6 metre per second for each degree centigrade rise of temperature.

61. **Experimental Determination of the Velocity of Sound in Water.**—The velocity of sound in water was determined experimentally by Colladon and Sturm in 1826. The experiments were made in the water of the Lake of Geneva. The time taken by a sound in travelling between two stations, a measured distance apart in the water was carefully determined by means of a stop watch, and the velocity of sound in water was deduced directly from the observed results.

The source of sound used in the experiments was a bell fixed under water at a depth of about a metre. The bell was struck by a hammer worked by a lever from the boat moored at the striking station. The same lever carried a lighted taper so arranged that it fired a charge of powder at exactly the same instant as the bell was struck by the hammer. As the experiments were made at night the flash of the powder discharge was plainly visible at any point on the lake, and thus served to signal to the observer at the receiving station the instant at which the bell was struck.

The arrival of the sound of the bell at the receiving station was detected by means of a long ear trumpet. The wide end of the trumpet, closed by a thin sheet of rubber, was immersed in the water in the proper position to receive the wave transmitted through the water, and the other end was applied to the ear in the usual way. By this means the sound of the bell could be heard through the water over very long distances; it was found during the experiments that it was distinctly audible from one side of the lake to the other, over a distance of nearly 10 miles. It was, therefore, possible to take two stations on the lake far enough apart to make the time taken by the sound in travelling from one to the other great enough to be determined with considerable accuracy.

A large number of observations were made in this way, and as the final results of their experiments, Colladon and Sturm found the velocity of sound in water to be about 1,435 metres per second, or 4,700 feet per second. The average temperature of the water was about 8° C.

CHAPTER VIII.

TRANSVERSE VIBRATION OF STRINGS.

62. **Strings.**—A **string**, as the term is used in sound, applies to a thread or filament of any material. A fibre of silk, a thread of glass, a string of catgut, and a thin steel wire are all **strings** in this general sense.

The cross-section of a string is generally assumed to be circular in form and uniform in diameter, so that for a string of any given material the mass per unit length is constant. The essential characteristic of a string is, however, **flexibility**. A string must be so flexible that the resistance which it offers to the small bending strains which it undergoes during transverse vibration is negligible.

63. **The Transverse Vibration of a String stretched between Two Fixed Points.**—If a string, AB (Fig. 33),

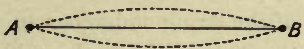

Fig. 33.

stretched between two fixed points, A and B, is pulled aside into one of the two dotted positions shown in the figure, and then let go, it is set in transverse vibration between these two dotted positions.

The motion of the string is periodic in character, each complete vibration being performed in a definite constant period. This is proved by the fact that when the vibration frequency is high enough to give a musical note the pitch of the note is found to be constant.

The amplitude of vibration decreases as the vibration continues until the string ultimately comes to rest in its initial position.

The following points characteristic of this mode of vibration of the string should be noted :—

(*a*) Every point on the string vibrates in a path at right angles to the length of the string; the string is, therefore, said to be in **transverse** vibration (Art. 10).

(*b*) The vibratory motion is the same in period and in phase for all points, but differs in amplitude from point to point along the string. The amplitude is evidently a maximum at the middle of the string and decreases gradually from the middle towards the ends to zero at each end.

(*c*) The bending or flexure to which the string is subjected during vibration varies from point to point along the string. Thus, a very small length taken at the middle of the string undergoes practically no bending during vibration; its direction remains parallel to the initial direction of the string throughout a complete vibration. A small length taken at either end of the string, however, undergoes appreciable bending; during half a period, for example, it bends through the small angle between the directions of the dotted curves at the point A or B in the figure. Similarly, a small length taken anywhere between the middle and the end of the string undergoes an amount of bending which increases, as the distance from the middle increases, from zero at the middle to a maximum at each end. The extreme bending at any point is in fact measured by the angle between the tangents at this point to the two dotted curves in Fig. 33, and this angle evidently increases from zero at the middle of the string to a maximum at each end.

The two end points and the middle point of the string in vibration thus possess two important characteristics.

Each end point is a point of minimum (zero) amplitude of vibration and maximum bending strain. A point on a vibrating body which possesses these characteristics is called a **node**. The end points of the string in this mode of vibration are, therefore, **nodes**.

The middle point is the point of maximum amplitude and minimum (zero) bending strain. A point on a vibrating body which possesses these characteristics is called an **antinode.** The middle point of the string in this mode of vibration is, therefore, an **antinode.**

A node on a vibrating body is specially noticeable because it is a point of no motion; it remains at rest throughout the vibratory motion of the body. In the case of the vibrating string here considered the end points are fixed and are necessarily nodes, but it will be seen later that nodes may form at points which are not mechanically fixed.

The distance from node to node in a vibrating body is sometimes called a **segment** or an **internode**, and the string in the mode of vibration here described is said to vibrate in a single segment.

This mode of vibration, sometimes called the **fundamental mode**, is the **simplest** mode in which the string can vibrate. It is not, however, the only mode of vibration of which the string is capable.

64. **Period of Vibration of a Stretched String.**—A string is, by definition, assumed to be so flexible that the forces which result from the slight bending which it undergoes during vibration are negligibly small.

It is assumed, too, that the stretching strains which accompany its vibration are so small that the forces due to them are also negligible. That is, the elastic properties of the material of the string are not involved in its vibration, and the only force on which the vibratory motion depends is the tension of the string. The motion of any element of the string thus depends dynamically only on the tension of the string, the length of the vibrating segment, and the mass per unit length of the string. The period of vibration, therefore, involves only these three quantities, and is quite independent of the **material** of the string except in so far as this determines the mass per unit length.

The relation between the period of vibration and the tension, length, and mass per unit length of the string can be calculated theoretically as a problem in dynamics. It can also be deduced experimentally by investigating how the period varies with each of the three quantities on which it depends taken separately.

Thus, the relation between the period and the length of the string may be determined by finding how the period of vibration of a particular string under a fixed constant tension varies with the length set in vibration. The mass per unit length and the tension are here constant, so that any observed variation of the period will be due entirely to the change in length.

The variation of the period of vibration is readily noted and measured by noting the change in the pitch of the note given out by the vibrating string. Thus, if a particular change in length causes the pitch to rise an octave, it is known at once that the vibration frequency of the string is doubled and its period, therefore, halved.

Experiment 27.—Stretch a thin steel wire on a sounding-board over two knife edges or bridges as shown at A and B in Fig. 34. The wire

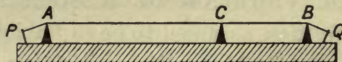

Fig. 34.

should be attached to a fixed pin, Q, at one end, and to a screw wrest pin, P, at the other end. Adjust the tension of the string by means of the wrest pin at P until the string vibrating as a single segment between A and B gives out a note of convenient pitch, about that of the middle C.

Now insert the movable bridge C (which should be very slightly higher than the bridge at A and B) between A and B, and adjust its position until the length AC, which is cut off between the bridges A and C, gives, on vibrating as a single segment, a note an octave higher than that given by the string as a whole.

It will be found that when this adjustment is made, the length AC is half of AB; that is, the frequency is doubled when the length of the vibrating segment is halved.

Similarly, if the position of the bridge C is adjusted until the string

AC, vibrating as a single segment, gives a note a fifth above that given by AB, it will be found that AC is two-thirds of AB—*i.e.*, when the length is varied in the ratio 3 : 2, the frequency varies in the ratio 2 : 3.

This result is further confirmed by verifying that if the position of the bridge C is adjusted so as to give AC successive values, equal to $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, and $\frac{1}{2}$ of AB, the successive notes are found to be the notes of the diatonic scale (Art. 46) in ascending order.

It follows from the results of this experiment that the vibration frequency of a string, vibrating as a whole, in a single segment varies inversely as the length of the string, provided the tension and mass per unit length do not change.

The relation between the vibration frequency and the tension of the string, when the length and the mass per unit length remain constant, may be established by the following experiment.

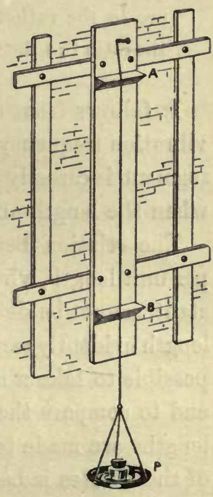

Fig. 35.

Experiment 28.—Fix a sounding-board provided with two fixed bridges A and B in a vertical position, as shown in Fig. 35, by screwing it to horizontal strips of wood nailed across vertical battens on a wall. Take a thin steel wire, attach one end to the pin at P and stretch it over the bridges at A and B, by means of weights placed in the scale pan P.

With this arrangement the tension is evidently measured by the weight carried by the wire at P, and can, therefore, be adjusted as required.

A large bucket may be used as a scale pan ; the stretching weight can then be conveniently adjusted by pouring measured quantities of water into the bucket, instead of using weights.

Adjust the tension until the length AB, vibrating as a single segment, gives a note of fairly low pitch. For subsequent reference it is convenient to adjust the tension until the note heard is in tune with that given by another string or a tuning fork. Note the amount of this tension as given by the weight carried by the wire, including the weight of the scale pan, or bucket.

Now increase the tension and note that the pitch of the note rises.

Continue to increase the tension until the pitch rises (*a*) a third, (*b*) a fifth, and (*c*) an octave, and note the amount of the tension applied to the wire in each case.

It will be found (*a*) that to raise the pitch a third the tension must be increased in the ratio 16 : 25 ; (*b*) that to raise the pitch a fifth the tension must be increased in the ratio 4 : 9 ; and (*c*) that to raise the pitch an octave the tension must be increased in the ratio 1 : 4. That is, (*a*) when the tensions of the string are in the ratio 16 : 25, the vibration frequencies of the string are in the ratio 4 : 5 or $\sqrt{16} : \sqrt{25}$; (*b*) when the tensions are in the ratio 4 : 9 the vibration frequencies are in the ratio 2 : 3 or $\sqrt{4} : \sqrt{9}$; and (*c*) when the tensions are in the ratio 1 : 4 the vibration frequencies are in the ratio 1 : 2 or $\sqrt{1} : \sqrt{4}$.

It follows from this experiment as a general result, that the vibration frequency of the string vibrating as a whole in a single segment is directly proportional to the square root of the tension when the length and mass per unit length are constant.

The relation between the vibration frequency and the mass per unit length, when the length and tension of the string are constant, cannot be determined quite so directly. The mass per unit length evidently cannot be adjusted to any desired value; it is only possible to take a number of strings differing in this particular, and to compare the pitch of the notes they give out when equal lengths are made to vibrate under the same tension. The pitch of these notes is best compared by means of a string stretched on a sounding-board, provided with a movable bridge as in Fig. 34. The length of a portion, AC, of the string is adjusted until the note it gives out is in unison with each of the notes to be compared. The vibration frequencies of the strings giving these notes can then be assumed by the result of Exp. 27, to be inversely proportional to the lengths of AC to which they correspond.

The mass per unit length can then be determined for each string by weighing a known length of the string and dividing the mass by the length.

Experiment 29.—Take several strings of catgut and metal (steel and brass wire) and find the mass per unit length for each by weighing a metre of the string and calculating the mass per cm.

Stretch the string of greatest mass per unit length as in Fig. 35, and adjust the tension until it gives out a note of rather low pitch.

Take the apparatus of Fig. 34 and adjust the tension of the string by means of the wrest pin, until it gives a note a little lower in pitch than the note given by the string under test. Then adjust the position of the movable bridge C until the note given by the part AC is in unison with this note. Measure and record the length of AC.

Now stretch each of the other strings in the same way as the first, under the *same* tension and with the same distance between the bridges. Also adjust the length AC of the reference string (*without altering its tension*) in each case until it gives a note in unison with the note given by each string in turn. Measure the length of AC in each case and record the lengths.

If now l_1 and l_2 are the recorded lengths of AC for any two strings, and m_1 and m_2 the masses per unit length for these strings, it is known from the result of Exp. 28, that if n_1 and n_2 denote the vibration frequencies for these strings $n_1 : n_2 = l_2 : l_1$.

It will be found, however, from the data of this experiment that $l_2 : l_1 = \sqrt{m_2} : \sqrt{m_1}$. It follows, therefore, that $n_1 : n_2 = \sqrt{m_2} : \sqrt{m_1}$. For example, if the masses per unit length for any two of the strings are in the ratio 4 : 9, the vibration frequencies for equal lengths of these strings under the same tension are in the ratio $\sqrt{9} : \sqrt{4}$ or 3 : 2.

The general result of this experiment may evidently be expressed by saying that the vibration frequency of a string vibrating as a whole in one segment is inversely proportional to the square root of its mass per unit length, provided its length and tension are kept constant.

This relation can be established more neatly, but perhaps less obviously, by adjusting the tension of each of the strings so that equal lengths give out notes of the same pitch and then deducing the relation from the result of Exp. 28.

Experiment 30.—Stretch each of the strings to be compared as in Fig. 35, and adjust the tension in each case so that the strings all give a note of the same pitch.

Record the tension in each case.

It will be found on comparing the data of the experiment that the tension necessary for this adjustment is, for each string, directly proportional to the mass per unit length.

It follows, therefore, that since, by Exp. 28, the vibration frequency is *directly* proportional to the square root of the tension, it must be *inversely* proportional to the square root of the mass per unit length.

The three general results or laws established by these experiments are most conveniently expressed algebraically. Thus, if n denote the vibration frequency of the string, l the length in vibration, t the *tension*, and m the mass per unit length, we have—

(i) n is proportional to $\frac{1}{l}$, when t and m are constant (Exp. 27).

(ii) n is proportional to $\sqrt{t}$, when l and m are constant (Exp. 28).

(iii) n is proportional to $\frac{1}{\sqrt{m}}$, when l and t are constant (Exps. 29, 30).

Combining these three results algebraically we get—

$$n \propto \frac{1}{l}\sqrt{\frac{t}{m}},$$

or,

$$n = k\left(\frac{1}{l}\sqrt{\frac{t}{m}}\right),$$

where k is a constant.

If n is determined for a string for which l, t, and m are known, it will be found that the value of k is $\frac{1}{2}$, and we therefore get—

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

This formula expresses concisely the three laws for the transverse vibration of a string, which have been formulated above.

It will be seen from the formula that for strings of the same length, n is proportional to $\sqrt{\frac{t}{m}}$ and is, therefore, constant if the ratio $\frac{t}{m}$ is constant. This is the relation established by Exp. 30.

This formula is sometimes expressed in somewhat different terms. Thus, if r denote the radius of cross-section of the string, and d the density of its material, we have, $m = \pi r^2 d$, and the formula reduces to—

$$n = \frac{1}{2lr}\sqrt{\frac{t}{\pi d}},$$

which may evidently be expressed verbally in four "laws."

Similarly, if s denote the stretching *stress*, or tension per unit area of cross-section, and a the area of cross-section, we have $t = as$ and $m = ad$, and the formula becomes—

$$n = \frac{1}{2l}\sqrt{\frac{s}{d}}.$$

The formula, $n = \frac{1}{2l}\sqrt{\frac{t}{m}}$, can evidently be applied to determine n directly for any string for which l, t, and m are known. This determination can readily be made with the apparatus of Exp. 28. The quantities l, t, and m are measured directly, and the value of n calculated from the formula. The units in which l, t, and m are expressed must, however, be *consistent*. Thus, l may be in centimetres, m in grammes per cm., and t in dynes; or l may be in feet, m in pounds per foot, and t in poundals.

Numerical Example.—A string stretched as in Exp. 28, is 50 cm. long, its mass per unit length is ·01 grm., and its tension is due to the weight of 8·83 kilogrms., calculate its vibration frequency.

Here, from the data of the question, we have—$l = 50$ cm., $m = \cdot 01$ grm., and $t = 8{,}830 \times 981$ dynes.

The value of n is, therefore, given by—

$$n = \frac{1}{100}\sqrt{\frac{8{,}830 \times 981}{\cdot 01}}.$$

or, $n = 294 \cdot 3.$

This method of determining the vibration frequency of a string may evidently be applied to determine the vibration frequency for any source of a musical note. For example, if the tension of the string is adjusted until it gives a note in unison with the note of a tuning fork (or any other source) the value found for n from the values of l, t, and m for the string is also the value of n for the source which gives a note in unison with the note of the string.

Numerical Example.—A string whose mass per unit length is ·0016 grm. is stretched by a weight of 3,924 grms., and it is found that a length of 19 cm. of it gives a note in unison with the note of a tuning fork. Find the vibration frequency of the tuning fork.

Here, from the data given $l = 19$ cm., $m = {\cdot}0016$ grm., and $t = 3{,}924 \times 981$ dynes.

The value of n is, therefore, given by—

$$n = \frac{1}{38}\sqrt{\frac{3{,}924 \times 981}{\cdot 0016}},$$

or, $$n = 129.$$

The pitch of the note given by the fork is, therefore, about the C below the middle C of the piano.

65. **Harmonic Modes of Vibration of a String.**—When a string AB is stretched, as in Fig. 34, over two fixed bridges at A and B, any portion of it between the bridge at A, and a movable bridge inserted at C, can be set in vibration as a single segment by plucking it or bowing it at the middle point of the segment. It is found, however, that if the length AC is an exact sub-multiple of the length AB, the vibration is not confined to the segment AC; the whole string divides into segments, each of which is equal in length to AC and vibrates in the same manner as that segment. Thus, if AC is a half, or a third, or a fourth of AB, the string vibrates as a whole in two, three, or four segments, as shown in Fig. 36. The vibration of each segment is the same in period and amplitude, but there is a difference in phase between any two adjacent segments of exactly half a period, so that at any instant the lateral displace-

ments of adjacent segments are in opposite directions. In Fig. 36, the two extreme positions between which the string vibrates are shown by a dotted line and a thin continuous line respectively, and it can be seen that the segments into which it is divided are at any instant alternately above and below the initial position of the string.

A string can be set in vibration in this manner without the use of a movable bridge. It is only necessary to **damp** the

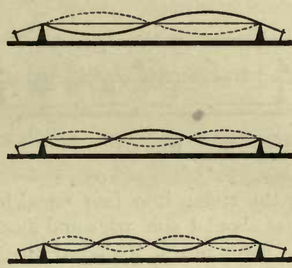

Fig. 36.

string at the point C, by touching it lightly with some suitable object, such as the finger or the edge of a paper knife, and then to pluck or bow the segment AC at its middle point. Thus if a string AB is damped lightly at a point C, such that AC is one-sixth of AB, and the segment AC plucked aside gently at its middle point, the whole string will at once break up into six segments, and will continue to vibrate in this manner for some time.

It will be noticed that when a string vibrates in this way, nodes are formed at points which are not mechanically fixed, and antinodes are formed at points which have not been plucked or bowed. Thus, if a string vibrates in four segments as shown in Fig. 36, there are, in addition to the nodes at the ends of the string, three equidistant nodes at the points where the string divides into segments ; there are also four antinodes, one at the middle of each segment.

When a string vibrates in segments the amplitude of vibration, even at an antinode, is usually very small, so that it is practically impossible to detect the existence of nodes and antinodes at particular points by mere inspection. The existence of these points can, however, be effectively demonstrated by the following experiment.

Experiment 31.—Take a fairly long string, AB, stretched over terminal bridges at A and B, as shown in Fig. 37, and mark the points

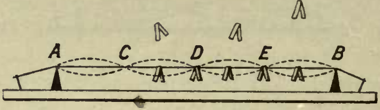

Fig. 37.

C, D, E, dividing the string into four equal lengths. Now get five small strips of paper about 1 mm. wide and 2 cm. long, and bend each strip at its middle point into a V-shaped rider. Place these riders across the string at the points D and E, and at the middle points of the segments CD, DE, and EB, as shown in the figure.

Now damp the string lightly at the point C and bow the segment AC at its middle point.

It will be found that the instant AC is bowed, the riders at the middle points of CD, DE, and EF are thrown off the string, but that the riders at D and E are not perceptibly disturbed.

This indicates that the points D and E are nodes or points of zero displacement, and the middle points of CD, DE, and EB antinodes, or points of maximum displacement.

The string is, therefore, vibrating in four segments, with nodes at the points A, B, C, D, E, and an antinode at the middle point of each segment.

In the case of a string vibrating in segments the vibration frequency of the string is the vibration frequency of its segments. A string of length l vibrating in n segments is practically n short strings of length l/n vibrating in unison.

Now it has been shown (Exp. 27) that the vibration frequency of different lengths of the same string vibrating under the same tension is inversely proportional to the length in vibration.

Hence it follows that when a string vibrates in n segments, its vibration frequency is n times the frequency with which it vibrates as a single segment. For, if l be the length of the string, the length of each segment when it divides into n segments is l/n, and the vibration frequency of this length is n times that of the length l.

If, therefore, a string vibrate in 2, 3, 4, 5 . . . n segments, the vibration frequency becomes 2, 3, 4, 5 . . . n times the vibration frequency for the fundamental mode in one segment.

The note given out by a string vibrating in the fundamental mode is called the **fundamental tone** of the string. Hence when a string vibrates in 2, 3, 4, 5, &c., segments the notes given out are, successively, an octave, an octave and a fifth, two octaves, and two octaves and a third, &c., higher in pitch than the fundamental note.

Notes for which the frequencies are, respectively, 2, 3, 4, 5 . . . n times the frequency for a given note are called the **harmonics** of that note.

When, therefore, a string vibrates in 2, 3, 4, 5 . . . n segments, the notes given out are, respectively, the **harmonics** of the fundamental note. The modes of vibration in which a string divides into segments are, for this reason, generally called the harmonic modes of vibration for the string.

66. **Compound Modes of Vibration of a String.**—When a string is set in vibration in any way, it does not usually vibrate in its fundamental mode only, or in any one harmonic mode, but generally in a complex mode compounded of the fundamental mode and a number of the harmonic modes.

The note given out by a string vibrating in this way is, therefore, a compound note made up of the simple notes corresponding to the component modes of vibration present in the string's actual mode of vibration. In some cases the existence of the component notes present in a compound tone can be detected by the ear. Thus, if the middle C of the piano be struck sharply, and the key held down to prevent damping, the presence of the first harmonic, an octave above the fundamental note, and the second harmonic, a fifth above the first, can usually be detected by a trained ear in the compound note produced.

The notes of higher pitch compounded with the fundamental note in any compound note are generally called the **overtones** of the note. The overtones in the case of a note given by a vibrating string are the harmonics of the fundamental note in unbroken sequence (up to a certain point), but it is more generally the case that the overtones of a compound note include only certain members of the harmonic series. Some notes, for example, contain only the even harmonics (the second, fourth, sixth, &c.), while others contain only the odd harmonics as overtones.

The **quality** of a musical note, as defined in Art. 47, depends very largely upon the number, order, and relative intensity of the harmonics present in its overtones.

67. Relation between the Velocity of Propagation of a Transverse Displacement along a String and the Vibration Frequency of the String.—The period of vibration of a string stretched between two fixed points depends upon

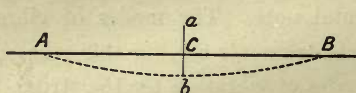

Fig. 38.

the velocity of propagation of a transverse displacement along the string.

Imagine a segment of a string, AB (Fig. 38), to be set in vibration by setting its middle point, *c*, in periodic motion along a short transverse path *ab*. As the middle point moves from *c* to *a* the transverse displacement thus impressed on the string at *c* travels along the string from *c* in both directions towards A and B. When the point *c* has reached *a*, and so completed a quarter of a complete vibration, the transverse displacement will have reached points A and B equidistant from the middle point. The length AB thus determines the length of the segment which can be set in vibration with the period of the periodic motion impressed on the string at c; and if the points A and B are fixed the segment AB will vibrate as a whole with this period.

Now, the length AB is evidently twice the distance travelled by the transverse displacement impressed on the string at c in one quarter of the period of vibration. That is, the displacement travels half the length of the string in a quarter of a period, and the period of vibration of the string AB as one segment is, therefore, equal to the time in which a transverse displacement travels twice the length of the string. Hence, if the velocity of transmission of a transverse displacement along the string be denoted by v, the period of vibration of a string of length l is $\frac{2l}{v}$, and the vibration frequency of the string is, therefore, given by—

$$n = \frac{v}{2l}.$$

It can, however, be proved that the value of v for a stretched string is given by—

$$v = \sqrt{\frac{t}{m}},$$

where t denotes the tension and m the mass per unit length of the string.

Substituting this value of v in the relation, $n = \frac{v}{2l}$, we get—

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

This is the relation already obtained in Art. 64 as the general result of an experimental investigation of the relation between the quantities involved in the formula.

CHAPTER IX.

LONGITUDINAL VIBRATION OF RODS AND COLUMNS OF AIR.

68. **Longitudinal Vibration of a Rod fixed at both Ends.** —A rod AB, Fig. 39, extending between two fixed points, A and B, can be set in longitudinal vibration, in its fundamental mode, by displacing its middle point C slightly, in the direction of its length towards one of the ends, and then letting it go. Thus, if the middle section at C be displaced into either of the dotted positions shown at *a* and *b* in the figure, one half of the rod is slightly extended, and the other half slightly compressed, so that

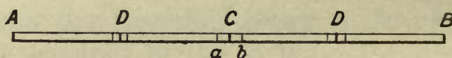

Fig. 39.

when the constraint is removed, the rod by virtue of its elasticity recovers its original unstrained state by a series of longitudinal vibrations, in which each half of the rod alternately lengthens and shortens through a gradually decreasing range until the initial state of rest is attained.

The mode of vibration of a rod under these conditions is exactly analogous to the transverse vibration of a string AB stretched between two fixed points A and B, and vibrating in one segment as described in Art. 63. Every section of the rod vibrates longitudinally, backwards and forwards on each side of its normal position, with the same period and in the same phase, but the amplitude of vibration decreases from a maximum at the middle to zero at each end. Thus, in Fig. 39, if dotted lines similar to those shown at *a* and *b* for the sections at C and D be taken to indicate the range of vibration at any point on the rod, the distance between the lines will decrease from the centre to each end, and the mode of vibration is such that all sections pass through corresponding positions such as the *a* positions, the normal positions, and the *b* positions, in the same direction at the same instant.

The longitudinal vibration of a rod is obviously accompanied by elastic strain in the material of the rod, and it will be readily understood from what has been said, that the strain is of the nature of an extension or a compression. The amount of strain at any point in the rod depends upon the *difference* in the displacement of adjacent sections during vibration. This difference is not the same for all points on the rod, so that the strain at any instant will vary from point to point along the rod. Further, the strain at any point varies from instant to instant during a complete vibration. When every section is in its normal position there is no strain at any point; and when every section is in either of its extreme positions the strain is at its maximum for that point. At the middle point of the rod the amplitudes or maximum displacements of adjacent sections are practically equal and the maximum strain at this point is, therefore, of zero value. Also, the difference between the maximum displacements of adjacent sections increases from the middle to each end of the rod, so that the maximum strain at any point increases from the middle outwards towards the ends of the rod. The range of strain during a complete vibration is thus zero at the middle point, and increases from point to point along either half of the rod to a maximum at the ends.

It has already been noted that during vibration one half of the rod lengthens while the other shortens; it follows that if at any instant all points in one half of the rod are in extension all points in the other half will be in compression. It will also be noticed that when the sections of the rod are displaced from their normal positions towards one end, the half of the rod at that end is in compression and the other half is in extension.

The end points of the rod, being fixed, are **nodes**, or points of zero displacements, but maximum range of strain; the middle point, similarly, is an **antinode**, or point of maximum displacement, but zero range of strain.

Thus, the rod AB, when its ends A and B are fixed, vibrates longitudinally in its **fundamental mode** in one **segment** with a **node** at each end and an **antinode** at its middle point.

In addition to its fundamental mode of vibration the rod is also capable of certain **harmonic modes** of vibration.

If the rod is fixed or clamped at a point C, such that the

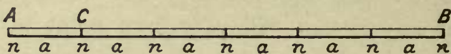

Fig. 40.

length AC is $1/n$th of AB, where n is an integer, the rod may be set in longitudinal vibration in n segments, each equal to AD, by exciting it at the middle point of that segment. The points of division of the rod into segments are **nodes**, and the middle point of each segment is an **antinode**. Thus if, as in Fig. 40, AC is one-sixth of AB, the rod, when excited at the middle point of AC, vibrates in six equal segments with nodes at the points marked n and antinodes at the points marked a.

The mode of vibration in each segment is exactly the same as that described above for the rod as a whole. It must be remembered, however, that while the phase is the same for all points in any one segment it differs by half a period for adjacent segments. Thus, while points in any one segment are always moving in the same direction, points in adjacent segments are always moving in opposite directions. That is, points on opposite sides of a node are, at any instant, moving either towards the node or away from it.

A rod fixed at both ends may thus vibrate longitudinally in any number of segments, and its actual mode of vibration is, in general, a complex mode compounded of two or more of the various modes of which it is capable.

69. **Longitudinal Vibration of a Rod Fixed at one End.**—Let AB, Fig. 41, represent a rod fixed at the end B and free at the end A. If the rod is extended slightly, so that the end A is pulled out to *b*, or compressed slightly so that the end A is pushed in to *a*, and then let go, the rod, by virtue of its elasticity, recovers its original unstrained state after a number of periodic longitudinal vibrations in which the rod alternately lengthens and shortens through a gradually decreasing range, until it finally comes to rest in its initial state.

Fig. 41.

The conditions of constraint are here such that the end B is necessarily a node, and the end A an antinode. The simplest mode of vibration possible for the rod is, therefore, that in which it vibrates as a single **half segment**, with a node at B and an antinode at A. This, then, is the **fundamental mode** of vibration of the rod.

The **harmonic modes** of vibration possible for the rod are limited by the conditions that the point B must always be a node and the point A an antinode. The rod must, therefore, in all cases divide into *an odd number of half segments*; that is, it may vibrate in *one* half segment, as in its fundamental mode, or in 3, 5, 7, 9 . . . half segments for its harmonic modes.

A rod fixed at one end may thus vibrate longitudinally in any odd number of half segments, and its actual mode of vibration is, in general, a complex mode compounded of two or more of the modes of which it is capable.

The general characteristics of the vibration of the rod in any of its modes of vibration, are exactly the same as those described in the foregoing article with reference to a rod fixed at both ends.

In the case of a rod free at both ends and fixed at the middle point, each half of the rod must evidently vibrate as a rod fixed at one end and free at the other end. The fundamental and harmonic modes of vibration of the rod, are, therefore, those of the half rod vibrating as a rod fixed at one end.

The case of a rod free at both ends and not subject to constraint at any other point evidently cannot be realised in practice. These conditions are, however, realised in the case of an air column in longitudinal vibration in a pipe open at both ends.

70. **Laws of the Longitudinal Vibration of Rods.**—The vibration frequency of a rod in longitudinal vibration cannot conveniently be determined by experiment as in the case of the transverse vibration of a string. It can, however, be deduced by the principle explained in Art. 67, from the velocity of propagation of a *longitudinal* displacement along the rod.

Thus, let it be supposed that the section at C in a uniform rod AB (Fig. 42), is set in periodic motion along a short

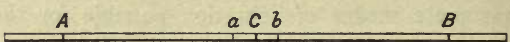

Fig. 42.

longitudinal path *ab*. As the section moves from C to *a* the longitudinal displacement thus impressed on the rod at C, travels along the rod in both directions as a pulse of compression towards A, and as a pulse of extension towards B. When the section at C has reached *a*, and has therefore completed a quarter of a complete vibration, the longitudinal displacement initiated at C will have reached two points, A and B, equidistant from the middle point. The length AB, thus determined, gives the length of the segment of the rod which can be set in longitudinal vibration with the period of the motion impressed on the rod at C; and if the points A and B are fixed, the segment AB will vibrate as a whole with this period.

The vibration frequency of a rod fixed at both ends for its fundamental mode of vibration is, therefore, such that a longitudinal displacement travels along half the length of the rod in a quarter of the period of vibration. That is, a longitudinal displacement travels along twice the length of the rod in the period of vibration of the rod. If it be remembered that a segment of a rod in any mode of vibration vibrates as a short rod fixed at both ends, this result may be stated quite generally by saying that the vibration frequency of a rod in longitudinal vibration in any mode is such that *a longitudinal displacement travels along twice the length of a segment in the period of vibration.*

Hence, if l denote the length of a segment and V the velocity of propagation of a longitudinal displacement along the rod, the period of vibration of the rod is $\frac{2l}{V}$, and the vibration frequency of the rod is given by—

$$n = \frac{V}{2l}.$$

Now V, the velocity of propagation of a longitudinal displacement along the rod, is really the *velocity of sound* along the rod, and its magnitude therefore varies with the material of the rod, but is independent of the area of cross-section.

The relation, $n = \frac{V}{2l}$, may be expressed verbally in two simple laws for the longitudinal vibration of rods. These laws are :—

(i) The vibration frequency for rods of the same material (same value of V) is inversely proportional to the lengths of the segments in which the rods vibrate.

(ii) The vibration frequency for rods in longitudinal vibration with the same length of segment (same value of

l) is directly proportional to the velocity of sound along the rod.

It will be seen that these laws are quite general, and apply to the longitudinal vibration of rods in all modes and under all conditions of vibration.

In the case of a rod fixed at both ends, vibrating in its **fundamental mode**, the length of a segment is the length of the rod, so that, if L denote the length of the rod, we have $l = \mathrm{L}$, and the vibration frequency for this mode is given by—

$$n = \frac{\mathrm{V}}{2\mathrm{L}}.$$

If the rod vibrates in a **harmonic mode** of p segments, the length of a segment is $\frac{\mathrm{L}}{p}$ and the vibration frequency is, therefore, $\frac{p\mathrm{V}}{2\mathrm{L}}$ or pn. That is, if n denote the vibration frequency for the fundamental mode of vibration, the vibration frequencies for the harmonic modes of 2, 3, 4, 5 . . . p segments are $2n$, $3n$, $4n$, $5n$ . . . pn respectively.

It follows from this that when a rod fixed at both ends vibrates in a **compound mode**, the **overtones** of the note are the harmonics of the fundamental tone.

In the case of a rod fixed at one end vibrating in its **fundamental mode**, the length of a segment is twice the length of the rod, so that $l = 2\mathrm{L}$, and the vibration frequency for this mode is given by—

$$n = \frac{\mathrm{V}}{4\mathrm{L}},$$

where L denotes, as above, the length of the rod.

When the rod vibrates in any one of its harmonic modes it always divides, as we have seen, into an odd number of half segments, so that if n denote the vibration frequency for the fundamental mode of vibration, the vibration frequencies for the harmonic modes in which the rod divides into 3, 5, 7, 9 . . . half segments are $3n$, $5n$, $7n$, $9n$ . . . respectively.

It follows from this that when a rod fixed at one end vibrates in a **compound mode**, the **overtones** of the note emitted include only the **odd harmonics** of the fundamental note.

In the case of a rod free at both ends and fixed at its middle point, each half vibrates as a rod fixed at one end and free at the other. Its fundamental and harmonic frequencies are, therefore, those of a rod of half its length, vibrating as a rod fixed at one end. That is, if L denote the length of the rod, the fundamental frequency is given by $n = \frac{V}{2L}$, and the harmonic frequencies are $3n$, $5n$, $7n$, $9n$ . . . as explained above.

Numerical Example.—The velocity of sound along a glass rod is about 15,000 feet per second; calculate the vibration frequency for a glass rod 3 feet long, fixed at one end and vibrating in its fundamental mode.

In this case the length of the segment is twice the length of the rod. Hence, in the formula—

$$n = \frac{V}{2l},$$

we have—

$$n = \frac{15,000}{6} = 2,500.$$

That is, the vibration frequency of the rod is 2,500 per second.

This result illustrates the fact that for most solid materials, the velocity of sound along a rod of the material is so great that the pitch of a note emitted by a rod in longitudinal vibration is very high, unless the rod is of considerable length.

Experiment 32.—Take four rods or tubes of glass of the *same kind*, about 20, 25, 29, and 35 inches long respectively. Fix them in turn with one end in a clamp or vice, so that the free lengths are respectively 15, 20, 24, and 30 inches. Set each rod in longitudinal vibration in its fundamental mode as a rod fixed at one end, and compare the pitch of the notes emitted. The rods are readily set in vibration by using a pad of cotton-wool moistened with alcohol as a rubber; the pad should be held so as to grip the rod lightly, about 6 inches from the free end, and then drawn smartly along and away from the rod at the free end.

It will be found, if the glass is really the same for each rod, that the notes given by the rods form the common chord of the diatonic scale on the note given by the longest rod as tonic.

That is, when the lengths of the rods are in the ratio 30 : 24 : 20 : 15 their frequencies are in the ratio $\frac{1}{30} : \frac{1}{24} : \frac{1}{20} : \frac{1}{15}$ or 4 : 5 : 6 : 8. The frequencies are therefore inversely proportional to the lengths of the rods. This result is in accordance with the first law given above.

Rods of wood cut from the *same piece of wood* may be used instead of glass rods. The free lengths of the rods might conveniently be 4, 5, 6, and 8 feet respectively. A piece of leather sprinkled with powdered resin will serve as a rubber.

Experiment 33.—Take two rods of the same length, one of good white pine and one of fir. Clamp each in turn so that it can vibrate as a rod fixed at one end, and determine the interval between the notes given by the rods.

The frequency ratio for this interval gives the ratio of the values of the velocity of sound along the rods.

The rods should be about 8 or 10 feet long, in order that the pitch of the notes emitted may be low enough to admit of the interval between them being determined with some accuracy.

Rods of glass, iron, steel, or brass may also be used.

71. Characteristics of the Vibratory Motion and Strain at any Point in a Rod in Longitudinal Vibration.—The main characteristics of the states of motion and strain at any point in a rod in longitudinal vibration, as explained above, may here be summarised.

I. Displacement.—Every cross-section of a rod in longitudinal vibration is subject to periodic displacement parallel to the length of the rod, so that it vibrates longitudinally backwards and forwards on each side of its normal position with the same period as that in which the rod vibrates.

The main characteristics of this periodic displacement or vibratory motion for points on a rod in longitudinal displacement are given below.

(i) *Period.*—The period is the same at all points in the rod ; it is the period in which the rod itself vibrates.

(ii) *Phase.*—The phase is the same for all points in the same segment, but differs by half a period for points in adjacent segments. (A segment extends from node to node.)

(iii) *Amplitude.*—The amplitude varies from point to point along the rod in the following manner :—It is zero at a node and a maximum at an antinode ; and, in each segment, it increases in the same way from its zero value at the nodes to its maximum value at the antinode. The **rate of increase** of the amplitude from node to antinode in any segment gradually decreases.

II. Strain.—Just as every cross-section of a rod in longitudinal vibration is subject to periodic displacement or vibration,

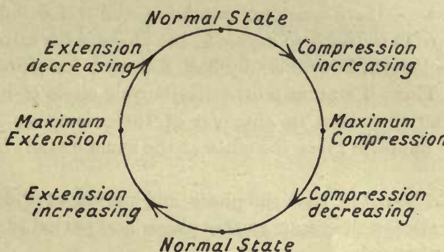

Fig. 43.

so the material of the rod at every cross-section is subject to periodic strain, and passes through a complete cycle of states of strain between extreme limits differing in opposite senses from the normal state, in the period of one complete vibration of the rod. The nature of the strain is that of linear extension (stretching) or linear compression, so that each transverse layer or slice of the rod is subject to periodic change of state within certain extreme limits of longitudinal extension and compression. The cycle of states which constitute the periodic strain in any section are indicated diagrammatically in Fig. 43.

It will be understood that when every section of the rod is in its normal position there is no strain at any point in the rod. When, however, in any half segment from node to antinode, the sections are

displaced from their normal positions towards the node, each section is subject to compression, and this compression is "increasing" or "decreasing" according as the section is **moving** towards the node or away from it. When, on the other hand, the sections in any half segment are **displaced** from their normal position away from the node, each section is subject to extension, and this extension is "increasing" or "decreasing" according as the section is **moving** away from the node or towards it.

The characteristics of this periodic strain for points on a rod in longitudinal vibration are given below.

(i) *Period.*—The period is the same at all points in the rod ; it is the period in which the rod itself vibrates.

(ii) *Phase.*—If we consider lengths on the rod equal to segments, but taken *from antinode to antinode*, the phase is the same for all points in any one length, but differs by half a period for points in adjacent lengths. Thus, if a point on the strain cycle circle of Fig. 43 gives at any instant the state in any one of these lengths, the point diametrically opposite gives the state at the same instant in the adjacent lengths.

It should be noted that the phase and period of the strain at any point are always the same as the phase and period of the vibratory motion at that point.

(iii) *Range* or "*Amplitude.*"—The range or "amplitude" of the strain—that is, the degree or extent of the extreme deviation from the normal state—varies from point to point along the rod. It is of zero value at an antinode, and of maximum value at a node ; and in each length taken from antinode to antinode it increases in the same way from its zero value at the antinodes to its maximum value at the node.

72. **Longitudinal Vibration of a Column of Air in a Pipe.**—A column of any fluid enclosed in a tube or pipe can be set in longitudinal vibration, like a solid rod, if excited by suitable means under suitable conditions. The general character of the vibration of the column is exactly similar to that of a rod in longitudinal vibration. There is, however, one important difference between the two cases. The strain in the rod is one of **linear** compression or extension in the direction of the length of the rod—that is, the material of the rod is extended or

compressed in only *one direction.* The strain in the fluid column is one of **volume** compression or expansion—that is, the fluid at any point under strain is expanded or compressed in *all directions.* The periodic strain at any point in a fluid column in longitudinal vibration is, therefore, accompanied by a corresponding periodic variation in the density of the fluid at the point, and also by a periodic variation in the pressure at that point in the fluid.

The laws of longitudinal vibration of a fluid column are the same as those already given for a solid rod. The frequency of vibration is, therefore, given by $n = \frac{V}{2l}$, where V denotes the velocity of sound in the fluid and l the length of a **segment** of the fluid column. The frequency of vibration is thus directly proportional to V and inversely proportional to l, but is quite independent of the area and form of the cross-section of the column. The dimensions of the cross-section must, however, be considerably smaller than the length of the column.

What has been said above applies to the longitudinal vibration of a column of any fluid, whether a liquid or a gas.

The longitudinal vibration of a column of air in a pipe of uniform section is, however, the only case that need be further considered.

A column of air cannot be constrained in the same way as a solid rod. It can, however, be subjected to certain constraints by means of the pipe in which it is enclosed. The length of the column is determined by the length of the pipe, and either end of the column becomes "fixed" or "free" according as that end of the pipe is closed or open.

A node in a column of air in longitudinal vibration is a point of zero displacement and maximum range of strain, accompanied by maximum changes in the density and pressure of the air at that point. The "fixed" end of a column is necessarily a point of zero displacement, and is, therefore, always a node.

An antinode is a point of maximum displacement and zero range of strain. It is, therefore, a point at which there is no change in the density or pressure of the air during the vibration of the column. The "free" end of a column is necessarily an antinode, for at a free end, the air of the column is in free communication with the outer air, and cannot, therefore, be subject to any variation in pressure or density at that point. For the same reason an antinode is determined also at any point in the air column which is in free communication with the outer air through a hole in the tube at that point.

A column of air in a pipe is usually set in vibration by producing, in some way, a variation in pressure at an "open" point in the column. Thus, the air column in a length of glass tubing can be set in vibration by blowing across the open end. The blast of air disturbs the pressure equilibrium at the mouth of the tube, and sets the column in vibration by initiating a compression or rarefaction at that point; the manner in which the vibration is maintained cannot, however, be very simply explained.

The columns of air in a whistle, a flute or fife, and in an organ pipe are all set in vibration by means of a specially directed air blast acting at an open end of the column.

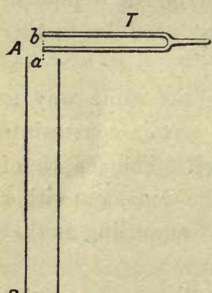

Fig. 44.

A column of air may also be set in vibration by impressing periodic motion of the proper period on the air at an "open" point in the column. Thus, if the prong of a tuning fork, T, Fig. 44, be set in vibration at a point, A, near the open end of a column of air, AB, the longitudinal displacement impressed on the air at A, during the first *quarter* vibration of the prong from A to *a*, travels down the column and reaches a point, B, just as the prong reaches the point *a* at the extreme end in its downward displacement. Now, as in the case of the string in Art. 67, this length, AB, determines the length of the half segment which can vibrate longitudinally with the period of the prong of the tuning fork, and if the column is constrained so that a node can form at B, its frequency of vibration will be the same as that of the tuning fork. For example, if the tube is closed at B the column AB vibrates in its fundamental mode with the period of the fork. During the first quarter vibration of the prong the compression accompanying the displacement caused by the motion of the prong *extends from* A *to* B, and *the column* AB is compressed to the length aB; after that the column lengthens and shortens between the limits Ba and Bb as the prong vibrates up and down between the limits a and b. The action of the prong on the air column at A is thus, at every instant during its motion, exactly timed to maintain the column AB in longitudinal vibration.

This action of the prong of the tuning fork is an example of the general principle of **resonance**, which states that periodic motion of a particular period is readily impressed on a body whose natural period of vibration is the same as that of the impressed motion.

It has already been stated that if n denote the vibration frequency of a fluid column, V the velocity of sound in the fluid, and l the length of a **segment** of the vibrating column, then

$$n = \frac{V}{2l}, \text{ or } V = 2nl.$$

Also, if V′ denotes the velocity of sound in the surrounding medium, we have—

$$V' = n\lambda$$

where λ denotes the wave length of the wave motion set up in the medium by the vibrating column as a source of sound.

It follows from this that

$$\frac{V}{V'} = \frac{2l}{\lambda}.$$

An important special case of this relation is that in which the fluid of the vibrating column and the surrounding medium are *the same.* We then have, $V = V'$, and $\lambda = 2l$. This condition is realised in the longitudinal vibration of an **air column** in **air** as the surrounding medium. In this case V and V′ are equal, each being the velocity of sound in air, and λ the wave length of the wave motion in the air surrounding the column is equal to $2l$, or to twice the length of a segment of the vibrating column.

It should be noted, however, that even in this case V is not exactly equal to V′; the velocity of sound through air along a tube depends, to a small extent, on the diameter of the tube, and is always slightly less than the velocity of sound in air as an open medium.

73. **Modes of Vibration of a Column of Air in a Tube Closed at One End.**—The modes of vibration of a column of air in a tube closed at one end and open at the other end, are exactly the same as those of a rod fixed at one end and free at the other.

The constraint imposed on the column by the tube is such that there will in all cases be a node at the closed end and an antinode at the open end. The **fundamental mode** of vibration of the column is, therefore, that in which it vibrates as a single **half segment,** with a node at one end and an antinode at the other end. Similarly, in its **harmonic modes** of vibration the column must divide into an **odd number of half segments**; that is, its harmonic modes are those in which it divides into 3, 5, 7, 9 . . . half segments. Hence, if n denote the vibration frequency of the fundamental mode, the vibration frequencies of the successive harmonic modes are $3n$, $5n$, $7n$, $9n$ . . . respectively. The notes given by these harmonic modes include, therefore, only the **odd harmonics** of the fundamental note.

The modes of vibration of the column for the frequencies n, $3n$, $5n$, and $7n$ are indicated diagrammatically in Fig. 45; the positions of the nodes and antinodes are shown in each case at n and a in the figure.

It will be seen that if L denote the length of the column, n the

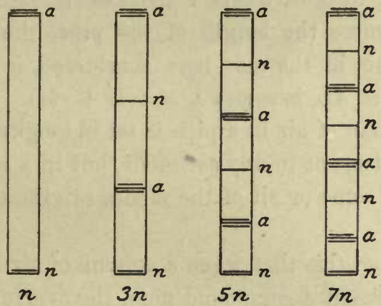

Fig 45.

frequency of the fundamental mode, V the velocity of sound in air, and l the length of a **segment** of the vibrating column, we have—

$$l = 2L$$

and
$$n = \frac{V}{2l} = \frac{V}{4L}.$$

or
$$V = n \,.\, 4L.$$

We also have—
$$V = n\lambda,$$

where λ denotes the wave length in air of the wave motion set up by the vibration of the air column in its fundamental mode.

It follows that in this case $\lambda = 4L$; that is, the wave length in air of the wave motion set up by the fundamental mode of vibration of an air column in a pipe closed at one end is four times the length of the column. It must be noted, however, that the length of the air column is not given accurately by the length of the pipe. It is found that the vibrating column always extends a little beyond the open end of the pipe, and that the vibrating column is, therefore, always a little longer than the pipe. Experiment shows that the correction for this effect of the open end depends upon the diameter of the pipe, and may generally be taken as equal to $0{\cdot}6r$, where r denotes the radius of the pipe. That is, if L denote the length of the pipe, the length of the vibrating column, in the case here considered, is $(L + {\cdot}6r)$, and the relation, $\lambda = 4L$, becomes $\lambda = 4(L + {\cdot}6r)$.

When a column of air in a pipe is set in longitudinal vibration it usually vibrates, not in any *one* mode, but in a compound mode compounded of some or all of the modes of vibration of which it is capable.

It follows from this that when a column of air in a pipe closed at one end gives out a compound note, the **overtones** of the note will include only the odd harmonics of the fundamental note.

74. **Modes of Vibration of a Column of Air in a Pipe Open at Both Ends.**—In this case the constraint imposed on the column by the pipe is such that an antinode must, in all modes of vibration, form at each end of the pipe.

The **fundamental mode** of vibration is, therefore, that in which the column vibrates in **two half segments** with an antinode at each end, and a node at the middle point of the column. Similarly, in its **harmonic modes** of vibration the column must, in all cases, divide into an even number of half segments; that is, its harmonic modes are those in which it vibrates in 4, 6, 8, 10 . . . half segments. Hence, if n denote the vibration frequency of the fundamental mode, the vibration frequencies of the harmonic modes taken in ascending order are $2n$, $3n$, $4n$, $5n$ . . . respectively. The notes corresponding to these harmonic modes of vibration are, therefore, in this case, the **harmonics** of the fundamental note.

The modes of vibration of the column for the frequencies n, $2n$, $3n$, $4n$ are indicated diagrammatically in Fig. 46; the positions of the nodes and antinodes are shown in each case at the points marked n and a in the figure.

In this case it will be seen that if L denote the length of the column, n the frequency of the fundamental mode, l the length of a segment of the vibrating column, and V the velocity of sound in air, we have—

$$l = L$$

and,
$$n = \frac{V}{2l} = \frac{V}{2L},$$

or,
$$V = n \,.\, 2L.$$

We also have as before—

$$V = n\lambda,$$

where λ denotes the wave length in the air of the fundamental note of the column. It follows, therefore, that $\lambda = 2L$; that is,

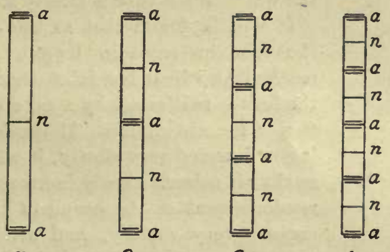

Fig. 46.

the wave length in air of the fundamental note of an air column in a pipe open at both ends is twice the length of the column.

In this case the pipe is open at both ends, so that, as explained above, the air column extends slightly beyond the pipe at each end. Hence, if L be the length of the pipe, the length of the air column which is set in longitudinal vibration in the pipe is approximately given by $[L + 2(\cdot 6r)]$ or $(L + 1\cdot 2r)$, where r denotes the radius of cross-section of the pipe.

It will be seen that the overtones of a compound note given by a pipe open at both ends are the harmonics of the fundamental note. The quality of the note must, therefore, differ from that of a note of the same pitch given by a pipe closed at one end, for in this note only the odd harmonics of the fundamental note are present as overtones.

Experiment 34.—Take a glass-tube about 150 cm. long and 2 cm. in diameter, and connect it by means of a length of rubber-tubing with a large funnel or reservoir, F, in the manner shown in Fig. 47.

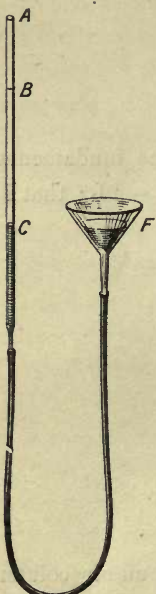

Fig. 47.

Pour sufficient water into the funnel to fill the tubes and the stem of the funnel, when the neck of the funnel is on a level with the top of the glass-tube. The level of the water in the tube can now be raised or lowered as required by raising or lowering the funnel.

Get a fairly large tuning fork of frequency about 256, and hold it, while in vibration, over the mouth of the glass-tube at A, as the level of the water in the tube is slowly lowered.

It will be found that as the column of air in the tube increases in length, a point is soon reached at which the faint sound given out by the fork is reinforced by a note due to the vibration of the air column. If the level of the water is now lowered very slowly, it will be found that as the air column slowly increases in length, this reinforcement of the sound of the fork first increases very rapidly, and attains a maximum value for a particular length of the air column, and then rapidly decreases and dies away. With a little practice the point of maximum reinforcement or resonance can be fixed with fair accuracy. Suppose this point occurs when the water in the tube is at B, then AB is the length of the air column which vibrates in its fundamental mode with the frequency of the fork. Resonance is not confined strictly to this particular length, for the fork is able to set up slight vibrations of its own period in columns a little shorter or a little longer than AB. Hence, if L denote the length of AB, r the radius of the tube, and n the vibration frequency of the fork, then λ, the wave length of the note given by the fork and the air column, is equal to $4(L + \cdot 6r)$, and V, the velocity of sound in air (along the tube), is given by—

$$V = 4n(L + \cdot 6r).$$

That is, if n is known, and l and r are carefully measured, *the velocity of sound in air can be calculated from the data of this experiment.*

Now lower the level of the water in the tube still further until a second point is found at which resonance again occurs, and let C be the point at which the level stands when maximum resonance for the note of this fork is obtained.

It will then be found that the length of the column AC is a little more than three times the length of the column AB. The length of AC is such that when vibrating in its first harmonic mode, in three half segments with a node at C and a node at B, its vibration frequency is the same as that of the column AB, and can, therefore, be set in vibration in this mode by the fork. The distance BC is an exact segment of the vibrating column, and is, therefore, greater than twice AB, which is less than a quarter segment by the correction for the open end of the pipe.

Hence, if l denote the length BC, we have—

$$\lambda = 2l,$$

and

$$V = 2nl.$$

That is, if n, the vibration frequency of the fork, is known, and l, the length of the segment BC, is carefully measured, the velocity of sound in air can be determined from the data of the experiment without the necessity for applying the correction for the open end of the tube.

If the tube were long enough it would be possible to find other points at which resonance would occur for air columns of lengths about 5, 7, 9 . . . times the length of AB; these columns by vibrating respectively in 5, 7, 9 . . . half segments would each have a node at B, and their vibration frequencies would, therefore, be the same as that of the column AB.

In this way it can be shown, experimentally, that an air column in a pipe closed at one end can vibrate in any odd number of half segments.

Numerical Example.—In an experiment of this kind the frequency of the fork was known to be 256, the radius of the tube 1·5 cm., and the lengths AB and BC were found to measure respectively 32·5 cm. and 66·6 cm.

From these data the velocity of sound in air calculated from the length of AB, with the aid of the correction for the open end, is given by—

$$V = 1{,}024\ (32{\cdot}5 + {\cdot}9)$$

or,

$$V = 1{,}024 \times 33{\cdot}4 = 342{,}016.$$

That is, the velocity is about 342 metres per second.

If the velocity is calculated from the length BC we get at once, without any correction,

$$V = 512 \times 66{\cdot}6 = 340{,}992.$$

That is, the velocity is about 341 metres per second.

It may be seen, too, from the data that the correction for the open end given by BC/2 – AB is (33·3 – 32·5) cm., or ·8 cm., which differs very little from ·6r, which is ·6 × 1·5 cm., or ·9 cm.

75. Characteristics of the States of Motion and Strain in a Column of Air in Longitudinal Vibration.—The main characteristics of the state of motion and strain in a rod or column in longitudinal vibration have already been summarised in Art. 71. It will, however, be convenient to repeat the summary here in terms specially applicable to the vibration of an air column.

I. Displacement.—Every transverse layer of an air column in longitudinal vibration is subject to periodic displacement parallel to the length of the column, so that it vibrates backwards and forwards, *longitudinally*, on each side of its normal position with the same period as that in which the column vibrates.

The main characteristics of this periodic displacement or vibratory motion at points in an air column in longitudinal vibration are given below.

(i) *Period.*—The period is the same at all points in the column ; it is the period in which the column itself vibrates.

(ii) *Phase.*—The phase is the same for all points in the same segment, but differs by half a period for points in adjacent segments.

(iii) *Amplitude.*—The amplitude varies from point to point along the column in the following manner. It is zero at a node, and a maximum at an antinode ; and in each segment it increases in the same way from its zero value at the nodes to its maximum value at the antinode. The **rate of increase** of the amplitude from node to antinode in any segment gradually decreases.

II. Strain.—Just as every layer of an air column in longitudinal vibration is subject to periodic displacement or vibration, so every layer is subject to periodic strain, and passes through a complete cycle of states of strain between extreme limits differing in opposite senses from the normal state, in the period of one complete vibration of the column. The nature of the strain is that of volume compression or rarefaction, so that each transverse layer of the column is subject to periodic strain within certain extreme limits of compression and rarefaction which vary from point to point along the column. The cycle of states which constitute the periodic strain in any section are indicated diagrammatically in Fig. 43.

It will be understood that when every layer of the column is in its normal position there is no strain at any point in the column. When, however, in any half segment, from node to antinode, the layers are **displaced** from their normal positions towards the node, each layer is subject to compression, and this compression is "increasing" or "decreasing" according as the layer is **moving** towards the node or away from it. When, on the other hand, the layers in any half segment are **displaced** from their normal positions away from the node, each layer is subject to rarefaction, and this rarefaction is "increasing" or "decreasing" according as the section is **moving** away from the node or towards it.

The main characteristics of this periodic strain for points in an air column in longitudinal vibration are given below.

(i) *Period.*—The period is the same at all points in the column; it is the period in which the column vibrates.

(ii) *Phase.*—If we consider lengths of the column equal to segments, but taken from antinode to antinode, the phase is the same for all points in any one length, but differs by half a period for points in adjacent lengths.. Thus, if a point on the strain cycle circle of Fig. 43 gives, at any instant, the state of strain in any one of these lengths, the point diametrically opposite gives the state at the same instant in the adjacent lengths.

(iii) *Range* or "*Amplitude.*"—The range or "amplitude" of the strain—that is, the extent of the extreme deviations from the normal state varies from point to point along the column. It is of zero value at an antinode, and of maximum value at a node; also, in each length taken from antinode to antinode, it increases in the same way from its zero value at the antinodes to its maximum value at the node.

76. **Organ Pipes.**—The use of pipes in an organ is the most important application of the longitudinal vibration of air columns in the construction of musical instruments.

Organ pipes are usually either wooden pipes of square section or thin metal pipes of round section. As already explained, in Art. 72, the pitch of the note emitted by a pipe varies with its length, but is quite independent of the form or area of the cross-section, provided the dimensions of the latter are not out of proportion to the length.

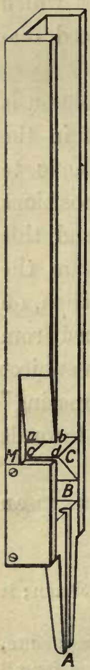

Fig. 48.

An organ pipe is set in vibration by forcing a blast of air from the bellows of the organ, through a narrow slit at the mouth of the pipe. The construction by which this is effected is shown in the cross-section of a wooden pipe shown in Fig. 48.

The mouth or **embouchure** of the pipe is shown at M. It is a narrow rectangular opening in the side wall of the pipe, and its upper edge, *ab*, is bevelled to a very thin edge sometimes called the feather edge. The cross-section of the pipe on a level with the lower edge of the opening is closed by the block in which the **flue** or passage communicating with the organ bellows is cut, and the narrow slit *cd*, shown on the top of this block, is the upper end of this passage, at the point where it opens into the pipe.

The air from the wind-chest of the bellows enters the pipe at A, and passes up the tubular opening in the stem of the flue block into the air cavity shown at B in the figure. Here the air blast is diverted by the sloping face of the block C through the slit *cd*, so that the thin ribbon-like stream of air which emerges from the slit is directed towards the feather edge of the embouchure. The direction of this stream of air is, in this way, so arranged that a very small displacement will divert it inside the feather edge into the pipe, or outside the edge into the outer air. Hence, when the air column in the tube is once set in vibration by the change in pressure caused by the action of the air blast at the mouth of the pipe, the thin stream of air in following the displacement in the air column at that point is periodically diverted in and out of the tube, and so maintains the column in longitudinal vibration with a period determined by its length.

The end of the pipe at which the embouchure is situated is evidently an open end, for the air column is in direct communication with the outer air through that opening. The other end may be open as in an **open pipe**, or closed as in a **stopped pipe.**

When an organ pipe is blown gently it gives a note compounded of its fundamental tone and its overtones. The fundamental tone is the loudest, and appears to an untrained ear to be the only tone given by the pipe; the overtones are usually fainter and fainter as their order rises, and it is difficult to detect the presence of more than the first, or the first and second by the ear. When the pipe is blown more strongly the note sounded lacks the fundamental tone, and is compounded only of the overtones, with the first overtone as the predominant note. If blown more strongly still it may be made to give a note based on the second or the third, or even on a higher overtone as the lowest component of the note.

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