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ESTIMATING VOLUME OF DOUGLAS-FIR BUTT LOGS

by

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ABSTRACT

Describes development of equation to get close estimates of cubic-foot volume of flaring butt logs (S.E. = 3 percent) based on measurement of length and top and butt inside bark diameters.

Several methods of estimating cubic-foot volume of butt logs are commonly used. Most are based on three measurements or on two measurements and an estimate. The needed information includes length, top diameter inside bark (d.i.b.), and a butt diameter inside bark. This last item is usually estimated, by either guessing the taper, eyeballing what the diameter would be with the butt flare removed, or estimating the diameter a given distance above the butt cut. These estimates are often inaccurate and volume estimates might be improved if a measurement of butt diameter were used. Use of a butt measurement requires an allowance for butt flare when calculating volume.

Since butt logs are often fluted as well as flared, the only truly accurate measurement of volume is by displacement of water. This is generally impracticable so some substitute is needed. Measurement of diameter outside bark (d.o.b.) and bark thickness at short intervals gives close estimates of volume but is too expensive for most purposes. Also, it requires allowance for fluting. This allowance can be made on the butt cut by visualizing an irregular closed curve that excludes as much wood in projections as it includes air and bark in fissures. Once this allowance is made, the diameter is estimated as the average of the measured long axis of the closed curve and the one at right angles to it. A model was sought that would give good volume estimates based only on length, top d.i.b., and this adjusted butt d.i.b.

The classical Neiloid[1/] is often suggested as a figure that exhibits butt flare. If this were true, butt logs would resemble frustums of Neiloids. It was found that the middle diameter of a Neiloid with top diameter 60 percent of base diameter is 79.15 percent of the base--imperceptibly smaller than the 80 percent middle diameter of the frustum of a cone with the same end diameters. Since butt log shapes are more like bells of trumpets than ice cream cones, neither the Neiloid nor cone alone will serve as a model.

Several models that describe a flaring curve were tested but found unsuitable for empirical least square regression fitting. The model finally used came from the notion of joining a cylinder to the frustum of a cone. The cylinder approximates the upper part of the log and the cone the basal flare. The equation for the volume of such a figure is $V = b\, D_u^2 L_u + b\, (D_u^2 + D_u D_b + D_b^2)\, L_b/3$

where V = volume

D_u = upper diameter

D_b = basal diameter

L_u = upper length

L_b = basal length

$b = 5.454 \times 10^{-3}$ (with length in feet and diameter in inches)

1/ A Neiloid is a solid of revolution generated by revolving Neil's parabola written in the form: $d = ah^{3/2}$ around its h-axis. The locus of this equation on a plane is the profile of the figure. If h is height and d is diameter, h is measured from the top of the tree downward.

Since the relative lengths of the upper and basal ends of such a figure that would best simulate a butt log were unknown, this equation was fit in the form $V = b_1 D_u{}^2L + b_2 D_uD_bL + b_3 D_b{}^2L$ (where L is total log length). Usually, when this was fit[2] to logs of a single length, the third term did not significantly reduce residual variation. The two terms most often significant were $D_u{}^2L$ and D_uD_bL. When these were fit to data for logs grouped by lengths, the b_i coefficients varied linearly with length. This suggested adding the cross products of the independent variables with length to the model fitted to logs of all lengths. Stepwise regressions led finally to the model:

$$V = b_1 L D_u{}^2 + b_2 L D_uD_b - b_3 L^2 D_uD_b$$

This equation corresponds to a composite solid comprised of a cylinder with the diameter of the small end and a concave solid of revolution resembling a truncated conoid (fig. 1). Obviously, this will be smaller than a butt log at the juncture of the cylinder and the conoid. This appears to compensate for the butt of the log being more concave than the conoid. Fitting this regression model by least squares determines coefficients that best make these compensations.

In these regression fits, the sum $(b_1 + b_2 - b_3L)$ was close to 5.454×10^{-3}. The third term in the equation adjusts for the lower percent of log volume in butt flare in longer logs.

This model and several others were tested on stem profiles of 112 Douglas-fir butt logs. Forty-eight of these profiles were based on tree measurement data on file at the Pacific Northwest Forest and Range Experiment Station. The data included stump d.i.b., d.b.h.i.b., and upper stem d.i.b. measurements at 10-foot intervals. The other 64 profiles were based on measurements of the bottom five veneer bolts of trees in mill recovery studies by the Timber Quality and Wood Utilization Project of the Pacific Northwest Forest and Range Experiment Station. The range of d.b.h. was 15 to 75 inches.

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[2] In regression analyses of such data, V is transformed to a dimensionless ratio by dividing by $D_u{}^2L$, D_uD_bL, or $D_b{}^2L$ to avoid high correlations among variables with the same order of dimensions. Such high correlations sometimes lead to acceptance of oversimplified functions. In these transformations, all variables for each sample unit are divided by the same factor. These transformations have the added effect of reducing or eliminating correlation of error variance with independent variables.

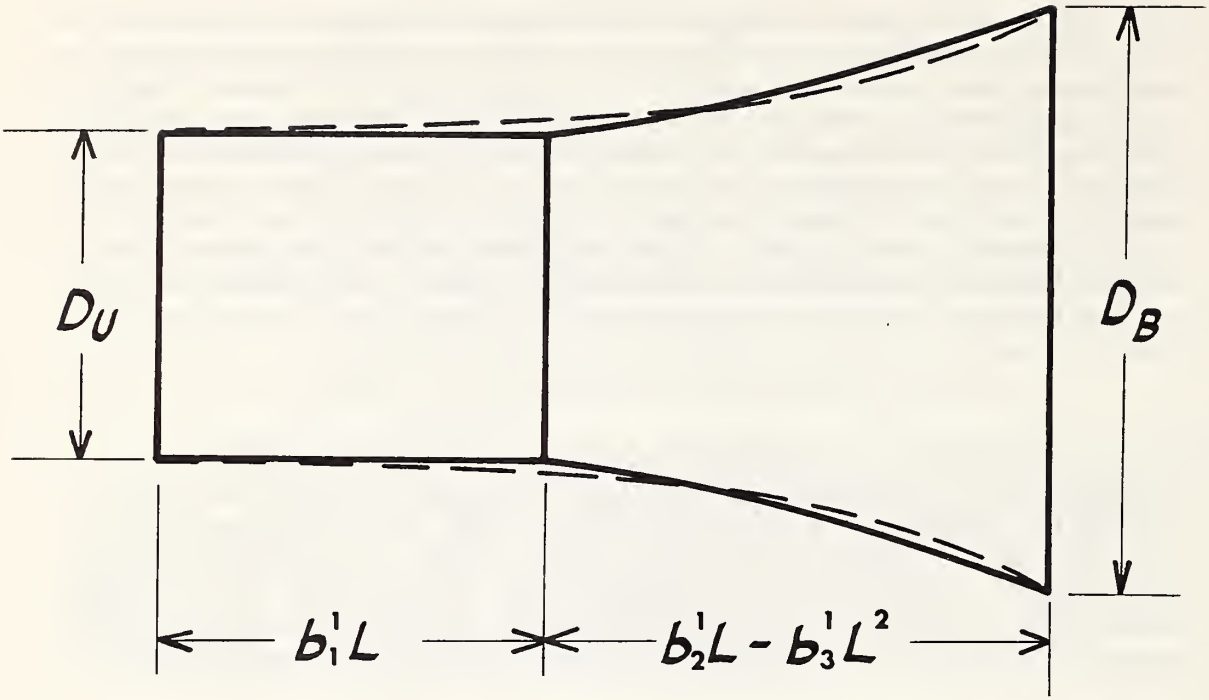

Figure 1.--Profile described by butt log equation (dashed line indicates one possible butt log profile).

Diameters on these profiles were measured at 2-foot intervals, and volumes were calculated as described by Johnson[3/] using the Smalian formula.[4/] Volumes were calculated for the butt 8-, 16-, 24-, and 32-foot logs of each tree.

[3/] Johnson, Floyd A. A technique for calculating cubic volumes in butt logs of trees. J. Forest. 55: 666, illus. 1957.

[4/] Grosenbaugh, L. R. Tree form: definition, interpolation, extrapolation. Forest. Chron. 42: 444-457, illus. 1966.

This article states that concavity or convexity becomes relatively unimportant in sections short enough that the smaller diameter is at least 80 percent of the larger diameter. With this diameter ratio, the volume of a Neiloid frustum is 98.9 percent of the volume of a paraboloid frustum; with an end ratio of 90 percent, the volume ratio is 99.7 percent. Of course, butt logs are much more concave than Neiloids, but few 2-foot sections have end ratios under 90 percent, so the timesaving use of Smalian's equation appears justified.

The Timber Quality and Wood Utilization Project data were from widely separated locations--Medford and Mapleton, Oreg., and Darrington, Wash. Analysis of covariance indicated no significant differences among regressions fitted to data from these three areas. These regressions were then compared with the regression for the older data. It was found that regression surface shapes varied more than could be attributed to chance.

Volume estimates from various regressions were compared. Estimates based on the regression for old data were higher for logs with extreme taper and lower for logs with little taper than estimates based on the pooled regression for recent data. With average taper (81 percent end ratio), estimates were the same. The pooled regression based on all data gave estimates within 1.5 percent of the other equations for all logs and within 1 percent for 98 percent of the logs. It has a standard error of 3 percent. This pooled equation appears to be sufficiently accurate for both groups of data and, therefore, is likely to do a good job of estimating volumes for other Douglas-fir butt logs. The equation is:

$$V = 2.233 \times 10^{-3} \times D_u^2 L + 3.687 \times 10^{-3} \times D_u D_b L - 5.654 \times 10^{-6} \times D_u D_b L^2$$

To get accurate cubic-foot volume estimates with this equation, diameters should be recorded to the nearest one-tenth inch and actual lengths (no trim allowance) to the nearest one-tenth foot if possible. With large, rough logs, the smallest feasible units of measurement should be used.

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