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MAGNETIC INDUCTION IN IRON AND OTHER METALS

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BY

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PREFACE.

DURING recent years, and especially during the last ten, our knowledge of the physical facts of Magnetisation has made a marked advance. Perhaps no subject has profited more by the beneficent reaction of Practice on Science. The labours of a number of observers have made it possible to present a connected account of the phenomena of magnetic induction and of the distinctive qualities of the magnetic family of metals. There are still, of course, many questions for experiment to answer ; but a text-book of the subject may now be written with some degree of continuity and completeness.

In attempting this task, the author has not approached the matter from the standpoint of the scientific historian. He has been more concerned to tell of things discovered than of discoverers. In many instances, therefore, the work of early observers is passed over with no mention, or with the briefest, because later experiments are found to deal with the same points in a more conclusive or more exhaustive way.

The author's aim has been to present the subject in sufficient detail to satisfy scientific students, as well as to meet the wants of those who may turn to the book in quest of data for application to matters of practice. Particulars, which will facilitate reference to the original memoirs in which researches are described, have in all cases been given for the assistance of those who may wish to pursue the subject further than a short text-book can well take them.

After an introductory chapter, which attempts to explain the fundamental ideas and the terminology, an account is given of the methods which are usually employed to measure the magnetic qualities of metals. Examples are then quoted, showing the results of such measurements for various specimens of iron, steel, nickel, and cobalt. A chapter on Magnetic Hysteresis follows, and then the distinctive features of induction by very weak and by very strong magnetic forces are separately described, with further description of experimental methods, and with additional numerical results. The influence of Temperature and the influence of Stress are next discussed. The conception of the Magnetic Circuit is then explained, and some account is given of experiments which are best elucidated by making use of this essentially modern method of treatment. The book concludes with a chapter on the Molecular Theory of Magnetic Induction; and the opportunity is taken to refer to a number of miscellaneous experimental facts, on which the molecular theory has an evident bearing.

Throughout the book the author has endeavoured to familiarise the student with the notion of intensity of magnetisation (**I**) as well as with the notion of magnetic induction (**B**). It has been urged by some writers that the alternative which is in this way offered is unnecessary and confusing, and that if we keep "**B**" we may dispense with "**I**." The scientific value and the practical utility of "**B**" are so obvious that no one proposes to avoid using that. It is "**I**" that we are told must go. In this cry the author is by no means disposed to join. It is not too much to say that in stating the magnetic qualities of a metal the quantity "I" is of primary importance. The facts of saturation, the molecular theory, and the phenomena of magneto-optics, all demonstrate its physical reality and its fundamental interest.

The author would take this opportunity to repeat an acknowledgment, already made elsewhere, of the assistance most willingly and ably rendered by a number of his pupils in carrying out experiments on some of the subjects with which this book deals. Messrs. Tanakadate, Fujisawa, Tanaka, and Sakai, in Japan, and Messrs. W. Low, Cowan, D. Low, and Frew, in Dundee, have been skilful and sympathetic collaborators, whose interest was as lively as their patience was inexhaustible.

A reminder of how far the subject still is from finality comes, as the last pages are passing through the press, in the announcement by Prof. J. J. Thomson of his demonstration that iron continues to be strongly susceptible to magnetisation by such rapid alternations of magnetic force as occur in a Leyden-jar discharge; and that the damping-out of the electric oscillations when the discharge traverses a coil with an iron core proves magnetic hysteresis to play an important part, notwithstanding the excessive frequency of the reversals. Independent experiments made by Prof. Trowbridge point to the same conclusion. Prof. Thomson's use of vacuum-globes without electrodes as induction secondaries opens up new possibilities of magnetic research, which he has himself been the first to turn to account.

Cambridge, *Nov. 28, 1891.*

CHAPTER VII.—MAGNETISM IN STRONG FIELDS.

CHAPTER VIII.—EFFECTS OF TEMPERATURE.

2.5

LIST OF ILLUSTRATIONS.

CHAPTER I.

INTRODUCTORY.

§ 1. **Introductory.**—Though all substances show some magnetic quality, there are three that form a group distinct from all others in this respect. In other metals and non-metals a feeble magnetisation may be induced with difficulty; iron, nickel and cobalt take magnetism readily, and take it in amounts that are relatively enormous. In other substances we have no evidence that there is such a thing as permanent magnetism, but these three can retain magnetism strongly. Their capability of being magnetised, which is more or less shared by alloys in which one or other of them is contained (and also by the magnetic oxide of iron), is so conspicuously great, in comparison with that of any other substance, that they may properly be said to stand apart as the magnetic family of metals. Our purpose is to give some account of the properties that entitle them to this name.

Before proceeding to speak of experiments and their results, it may be well to recall the various conventions according to which magnetic quantities are expressed. Most of this introductory matter is, of course, familiar to the student, but parts of it, perhaps, are less familiar, and some confusion is apt to be felt on account of the variety of ways which one may follow in stating facts about magnetic quality. The magnetisation of an iron bar, for example, may be specified by its magnetic moment, by its intensity of magnetisation, or by its magnetic induction; and its readiness to be magnetised may be measured either by what is called its magnetic permeability, or by another not quite identical quantity called its magnetic susceptibility. The student is liable to feel that there is an *embarras* *de richesse* in magnetic ideas and phraseology. The various lights in which the magnetism of a piece and its magnetic quality may be regarded are, of course, consistent with one another, and are related in a simple enough manner. Some forms of expression have the advantage that they fit in best with modern conceptions of the magnetic state; others survive because they are more convenient in special cases. The magnetic circuit of a dynamo is most simply treated by using one set of terms; another set of terms come readier to hand when we have to speak about the properties of a magnetised steel bar. The student will, therefore, do well to master the meaning of all the magnetic terms in common use, and should accustom himself to look at magnetic phenomena from various points of view.

§ 2. **Magnetic Poles, Axis, and Moment.**—An old and still useful way of looking at the matter is to think of the action of a magnet as due to the existence of two quantities of hypothetical magnetic substance, or "free magnetism," equal in amount and opposite in kind, which are distributed in the neighbourhood of the two ends. These hypothetical positive and negative substances have the property that two portions of like kind repel each other, and two portions of unlike kind attract each other, with a force which is proportional to the product of the amounts of the substances, and inversely proportional to the square of the distance between them. In an ordinary bar magnet the free magnetism is distributed partly over the surface at and near the ends of the bar, and partly throughout the interior of the bar, especially near the ends. The action of the magnet upon anything at a considerable distance from it is much the same as it would be if the free magnetism were concentrated at two points, near the ends, which are called the *poles*. Strictly speaking, there are no precise poles in a magnet—that is to say, there are no two points at which we might imagine the positive and negative free magnetism to be gathered, and find the magnetic action on external things to be quite unchanged. It is only when the bar is very thin and uniformly magnetised (§ 7, below) that we come near to realising the idea of two definite centres of force at the ends of the bar where the positive and negative free magnetism is concentrated. The idea of poles in a magnet has therefore to be employed with much caution, but it is too useful to be altogether abandoned.

The *strength* of a pole is the whole amount of magnetism which is to be taken as gathered there. *Unit pole*, or unit quantity of free magnetism, is that quantity which repels or attracts another quantity equal to itself with unit force when the two are placed at unit distance from each other. It is now a universal practice to express magnetic quantities in terms of the centimetre-gramme-second system of units. We may, therefore, define the unit pole as that which acts on another pole of equal strength with a force of one dyne when the two are placed one centimetre apart. The *axis* of a magnet is the line joining its poles. The *moment* of a magnet is its pole-strength multiplied by the distance between the poles. According to this conception of a magnet, the magnetic action of the bar is due to the free magnetism at the poles; the middle portion of it is neutral, and merely serves to hold apart the ends, in which the free magnetism resides.

§ 3. **Magnetic Field and Magnetic Force.**—For many purposes this conception of poles is a very serviceable one. It is especially serviceable when we have to treat of the influence which the magnet exerts throughout the space in its neighbourhood, or throughout what is called the *magnetic field*. To examine the magnetic field, we may think of the force which the magnet N S (Fig. 1) will exert on an imaginary particle of magnetic substance, P, placed anywhere in its neighbourhood. The two poles of the magnet will exert two forces, F_1 and F_2, which are proportional to $\frac{1}{(\mathrm{N\,P})^2}$ and $\frac{1}{(\mathrm{S\,P})^2}$: one pole will tend to pull the particle towards it, the other pole will tend to push the particle from it. These forces will have for their resultant a single force, R, which is the whole force exerted by the magnet upon the particle. The direction of this resultant is definite at any point in the field, but its amount will depend on the amount of magnetic substance in the particle. Suppose, now, that we take a particle in which the amount of magnetic substance is one unit (as defined in § 2), and observe the force exerted upon it when it is placed anywhere in the field, we now find a force which is definite (at any one point of the field) both in magnitude and direction. This force measures the intensity of the field at the point in question, and is called the *magnetic force* at that point. Instead of one magnet only there may be any number of magnets, the poles of which contribute to the production of magnetic force at any point of the field in which the magnets lie. We may still think of each pole as exerting its own component of force on the imaginary particle of unit strength, and then combine all these components to find a single resultant which measures the magnetic force.

Electric currents also give rise to magnetic force in their neighbourhood, so that, in considering the value of the magnetic force at any place, we have to take their action into account as well as the action of the poles of neighbouring magnets. But whatever currents or magnets contribute to the production of the magnetic field, the magnetic force at any point of space has a definite direction and value, which may be expressed by stating the mechanical force which would be felt by a unit magnetic pole when placed there. The direction of the magnetic force is the direction in which the unit particle will tend to move, and the value of the magnetic force is the value in dynes of the mechanical force which tends to move the particle.

FIG. 1.

§ **4. Lines of Magnetic Force.**—If we allow the particle to move so that the direction it takes is, at every instant, the direction in which the magnetic force acts upon it, and if we mark the course it takes through space, we shall trace out what is called a *line of magnetic force.* In general these lines are curved, for the direction of the magnetic force varies as

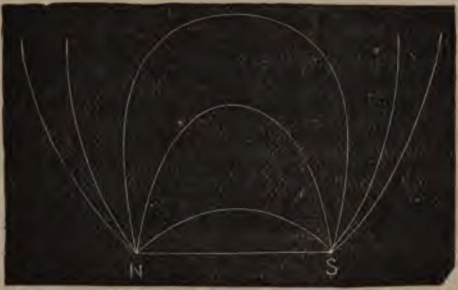

FIG. 2.

we pass from point to point through the field. If the magnetic field is produced by a single pair of opposite poles, the lines of force start from the positive pole and spread in curves, which bend round through space, and all

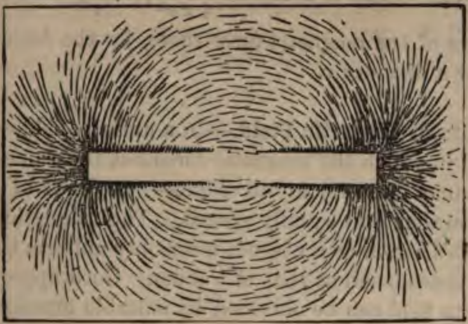

FIG. 3.

converge on the negative pole (Fig. 2). The well-known curves in which iron filings group themselves when scattered near a magnet represent approximately the forms taken by the lines of force. In the field produced by an actual bar magnet (Fig. 3) the lines do not converge as in Fig. 2 to a single pair of points, for the positive and negative quantities of free magnetism are each distributed over a considerable part of the length of the bar. Where the lines come close to one another the magnetic force is intense: where they lie far apart the field is weak. If we pass along lines of force from one place to another in the magnetic field, we shall find that the intensity of magnetic force at each place is proportional to the number of lines of force which cross an imaginary surface of unit area, placed there and set so that it stands at right angles to the direction of the lines. We can make the number of lines which cut such a surface not only proportional, but numerically equal to the magnetic force, throughout the whole field, by adhering to a proper convention as to the whole number of lines to be drawn. The convention is that the number of lines of force which start from any pole of strength m is $4\pi m$. Consider a sphere of radius r in centimetres enclosing a magnetic pole of strength m. According to the convention, the number of lines of force which radiate from the pole and cut the surface of the sphere is $4\pi m$. But the area of surface of the sphere is in square centimetres $4\pi r^2$. The number of lines of force per square centimetre at the surface of the sphere is therefore $\frac{4\pi m}{4\pi r^2}$, or $\frac{m}{r^2}$ and this is also the measure of the magnetic force there, for the force is due to a pole m at a distance r.

§ 5. **Uniform Magnetic Field.**—In a uniform field—that is to say, a field in which the magnetic force has the same direction and the same intensity at all points—the lines of force are straight, parallel, and equally spaced. The magnetic field due to the earth's action as a magnet is sensibly uniform throughout any small space, such as that of a room. Good approximations to a uniform field can be obtained by suitable arrangements of magnets, or of conductors carrying electric currents. Thus, if we take a long uniform solenoid or helix of wire, wound so that the diameter is constant and the number of turns is the same in each unit of the length, and pass a current through it, a magnetic field will be produced which is very nearly uniform throughout the whole space within the solenoid, except close to the ends. The value of the magnetic force in this field due to the current is

$4\pi C n$, where C is the current in absolute electro-magnetic units,* and n is the number of turns in the winding per centimetre of the length of the solenoid. Reducing this to practical units, the magnetic force within the solenoid is 1·257 times the number of ampere-turns per centimetre of the length.

Again, a nearly uniform field may be produced by taking two similar magnets with flat ends and placing them in line with their flat ends parallel, so that the north pole of one nearly touches the south pole of the other. In the narrow gap between the ends facing one another there is a strong magnetic field, through which the lines of force pass almost straight across from one face to the other. The field is very nearly uniform except at the edges.

§ 6. **Continuity of the Magnetic State.**—We have seen that the magnetism of a magnetised bar may be described by reference to its ends, where the imaginary positive and negative magnetic substance is accumulated, the middle parts being inactive. From another point of view the magnetisation extends throughout the whole substance. We may think of every portion of the magnet as *polarised;* that is to say, every particle or elementary piece of the bar may be regarded as a separate magnet. Throughout the middle portion of the length these elementary pieces are grouped so that each pole of one touches the opposite pole of its next neighbour, and the result is that the middle portion of the bar shows no positive or negative magnetism; but at the ends the poles of the elementary pieces are no longer neutralised, and the poles of the pieces there become the poles of the bar. This is the modern view of the matter, and is in many ways an advance on the simple polar view. It is borne out by the fact, experimentally observed, that when a magnet is cut up into pieces, however small, each piece is a separate magnet.

§ 7. **Intensity of Magnetisation.**—From this point of view we are to regard the magnetic state as existing continuously throughout the bar. If this state is uniform from end to end

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* The absolute electro-magnetic unit of current in the C.-G.-S. system is equal to 10 amperes.

of a bar, the metal is said to be *uniformly magnetised.* If we could cut up such a bar by cross-sections into short lengths without disturbing the uniformity of the magnetisation, we should find every part to be a magnet with the same pole-strength as the original bar. If we could split it by longitudinal sections we should find the pole-strength of each part to be proportional to the area of cross-section in that part. In other words, if we could cut up the bar in any manner (always without altering the magnetic state of the metal) we should find that the pieces were separate magnets of which the moments were proportional to the volumes of the pieces. The magnetic moment of each piece will be the same fraction of the magnetic moment of the uncut bar as the volume of the piece is of the volume of the uncut bar. The magnetic state which existed throughout the bar before it was cut, and exists throughout each piece after cutting, may be specified if we state the *moment per cubic centimetre* of the metal. This quantity is called the *intensity of magnetisation*, and is usually denoted by **I**.

§ 8. **Relation of I to Pole-Strength.**—Let M be the moment of a uniformly magnetised straight bar; let l be the length of the bar in centimetres, s its area of cross-section in square centimetres, and m its pole-strength.

Then $$M = m\,l.$$

The volume of the bar is $s\,l$; hence **I**, which is M divided by the volume, is $\frac{m\,l}{s\,l}$, or $\frac{m}{s}$.

We might, therefore, have defined **I** as the *pole-strength per square centimetre of sectional area.* It is useful to remember that **I** has also this meaning, but the essential idea implied in the phrase "intensity of magnetisation" is better conveyed by the definition of **I** given above. We are to think of **I** as the measure of a polarised state which has a true existence everywhere in the substance of the metal, though it manifests itself only at the ends, so far as external action is concerned.

§ 9. **Ring Magnet.**—The usefulness of this idea will be at once apparent if we consider what happens when a uniformly magnetised rod is bent round into a ring until the ends meet. There are now no poles: those that existed in the rod have met and have neutralised each other; there is now no magnetic moment, and the ideas of poles and moment will no longer serve as means of stating the magnetic state of the ring. But the ring is still magnetised: if we were to cut it in pieces we should find the pieces to be magnets. The magnetic state expressed by the quantity I still exists within the metal: there is a definite "intensity of magnetisation" throughout. If we were to cut a narrow gap through the ring we should find on one side of the gap a positive pole, and on the other side a negative pole, and the strength of each would be $\mathsf{I}\,s$.

§ 10. **Lines of Magnetisation.**—Suppose such a gap or crevasse to be cut, the number of lines of force which cross the gap is $4\pi\mathsf{I}\,s$ (by § 4), and hence the magnetic force within the gap (which is the number of lines of force per square centimetre) is $4\pi\mathsf{I}$; so far, that is to say, as the magnetic force there is due to the magnetism of the ring. Of course there may be additional magnetic force within the gap due to other magnets or to electric currents in the neighbourhood; but for the present we may confine our attention to the force that exists there on account of the magnetisation of the ring itself alone. The same lines $4\pi\mathsf{I}\,s$ which cross the gap may be conceived as extending continuously round the ring through the substance of the metal. Each line forms a closed curve: a short part of it is in the gap, the greater part is in the metal. We may call the parts of the lines which lie within the metal *lines of magnetisation.* The name "lines of force," which is applicable to the lines in the gap, is inappropriate to those parts of the lines which lie within the metal, because the lines within the metal do not form a measure of the magnetic force there.

§ 11. **Lines of Magnetisation** (*continued*).—Fig. 4 illustrates the supposed case of a narrow gap or crevasse, A B, cut across the substance of a magnetised ring at any place in its circumference. Within the metal we have the lines of magnetisation which are shown by dotted lines in the figure. The number of these, per square centimetre of cross-section, is $4\pi\mathsf{I}$. These lines are continuous closed curves, and pass across the gap, forming lines of force there. If we measure the magnetic force within the crevasse, we shall find it equal to this quantity, $4\pi\mathbf{I}$, together with whatever other magnetic force may act there in consequence of electric currents or magnets in the neighbourhood. That part of the magnetic force within the crevasse which is represented by $4\pi\mathbf{I}$, is directly due to the breach which we have made in the continuity of the magnetised ring. It may be regarded as existing there in consequence of the fact that lines of magnetisation within the metal are necessarily continuous with lines of force outside the metal.

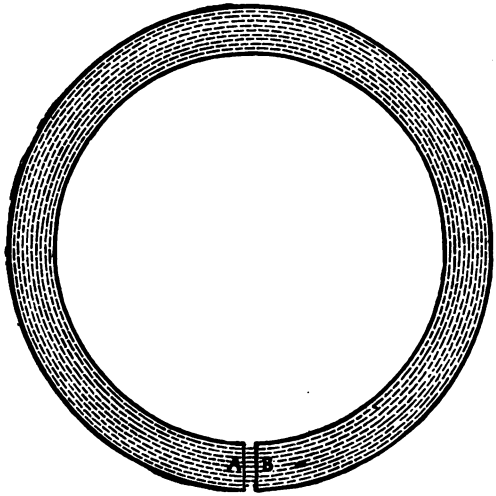

FIG. 4.

Take another way of looking at the matter. We may think of this force within the crevasse A B as due to free magnetism on the surfaces A and B. When we cut the crevasse in the magnetised ring, we bring into existence a positive pole which is distributed over the surface at A, and a negative pole which is distributed over the surface at B. The strength of each of these poles is $\mathbf{I}\,s$, and the surface-density of free magnetism—that is to say, the amount of free magnetism per square centimetre of surface—is $\mathbf{I}$. By a well-known proposition in the theory of attraction, a plate on which the surface-density is I attracts a unit particle placed close to it with a force equal to $2\pi\mathsf{I}$ (except near the edge, where the force is less). Let a unit positive magnetic pole, then, be placed in the crevasse. The plate of free magnetism on A repels it with the force $2\pi\mathsf{I}$; the plate of free magnetism on B attracts it with an equal force. The whole force at a point in the crevasse, due to the magnetisation of the ring, is the joint force exerted by the two plates; in other words, it is $4\pi\mathsf{I}$.

Suppose now that the uniformly magnetised magnetic ring is stretched out to form a straight bar. If we imagine a crevasse to be cut across it at any part of its length, we shall still find in the crevasse the lines $4\pi\mathsf{I}$ per square centimetre due to the continuous magnetisation of the bar, in addition to whatever lines of force may exist there in consequence of electric circuits or free magnetism in the neighbourhood (excluding, of course, the free magnetism on the faces of the crevasse, to which the lines $4\pi\mathsf{I}$ are directly due). The whole field within the imaginary crevasse may therefore be thought of as made up of two components, namely, (1) the lines of magnetisation, the number of which is $4\pi\mathsf{I}$ per square centimetre; and (2) the magnetic force due to external causes, namely, that which is due to electric currents and free magnetism in the neighbourhood. Amongst the causes of this magnetic force is to be included the free magnetism at the ends of the bar itself, as well as the poles of any other magnets which may be near enough to produce any sensible effect.

§ 12. **Magnetic Force within the Metal.**—The magnetic force due to external causes—that is, to magnets or electric currents in the neighbourhood—which has just been described as constituting one part of the magnetic force which we should measure in a crevasse, is to be thought of as acting also within the uncut substance of the metal itself. It constitutes the whole magnetic force there. We shall denote the magnetic force by H. It must be borne in mind that in reckoning the value of H at any point within the substance of a piece of magnetised metal, account is to be taken not merely of the forces due to electric currents and to terrestrial magnetism and to the poles of other magnets, but also of the forces which are contributed by the poles of the piece itself.

§ 13. **Magnetic Induction.**—The whole group of lines which cross the crevasse is to be conceived as existing within the metal before the crevasse was cut, partly as lines of magnetisation and partly as lines of force. The whole group of lines which cross the imaginary crevasse consists (per square centimetre) of the resultant of $4\pi \mathsf{I}$ and H. This resultant is called the *magnetic induction* within the metal, and is denoted by B. The quantities $4\pi \mathsf{I}$ and B are vectors, having direction as well as magnitude, and are to be compounded as forces or velocities are compounded. If H and I happen to have the same direction, B is numerically equal to the sum of $4\pi \mathsf{I}$ and H. In any case the equation

$$\mathsf{B} = 4\pi \mathsf{I} + \mathsf{H}$$

is true when understood in the vector sense, that B is the resultant of $4\pi \mathsf{I}$ and H. In most of the cases that are of practical interest H has either the same direction as B or it has the opposite direction, so that the above equation holds good in the numerical sense when the proper sign ($+$ or $-$) is given to H, according as it assists or opposes the magnetisation.

§ 14. **Distinction between Magnetic Induction and Magnetic Force within the Metal.**—The lines of magnetic induction (B) within the bar are continuous with the lines of magnetic force in the space outside—that is to say, every Line of Force outside is completed, so that it forms a closed curve, by a Line of Induction inside. For many purposes B is the most important quantity by which the magnetisation of a magnet may be specified. In a dynamo, for instance, it is the value of B in the armature core that determines the strength of the magnetic circuit. The analysis of B into two components, H and $4\pi \mathsf{I}$, is no doubt highly artificial, but it is of service when we have to deal with the relations which exist between the magnetism of a magnet and the influences which are affecting its magnetism from outside. The student will find it useful to picture to himself the state of a magnet at any point of its substance by thinking of two groups of lines as passing through the metal, namely, $4\pi \mathsf{I}$ and H, which combine to form a resultant group **B**. To obtain **B** directly we have only to imagine a narrow crevasse cut across the magnet: **B** is measured by the force a unit pole would experience if placed in such a crevasse; in other words, it is the number of lines which cross the crevasse per square centimetre of cross-section. If, on the other hand, we wish to isolate the magnetic force **H** that acts at any point within the metal, we may imagine a hole drilled through the magnet from end to end in the direction of magnetisation, and passing through the point at which **H** is to be measured. The force which a unit pole would experience if placed in the hole at that point is **H**. That this is so will be evident when it is remembered that there is no free magnetism on the sides of the hole, because it is supposed to be drilled in the direction of magnetisation, and the force within it is, therefore, due solely to the outside influences which give rise to the magnetic force **H**, as defined in § 12, namely, the free magnetism at the ends of the magnet and any other magnets or currents that are near.

It is only at points inside the metal that we need distinguish the magnetic force **H** from the magnetic induction **B**. Outside, at points in non-magnetisable space, the magnetic induction is identical with the magnetic force. There is no discontinuity in the lines of induction where they pass into the metal or out of it.

§ 15. **Particular Cases.**—The following illustrations may help to make these definitions intelligible. Take a ring electro-magnet consisting of an iron core, wound with a solenoid of n turns per centimetre, and let a current, C, flow in the solenoid. The magnetic force within the solenoid, due to this current, is approximately equal to $4\pi C n$ at all points. If there are no neighbouring magnets or other sources of magnetic force, this is the value of **H** which acts on the metal of the ring. Next, let the ring be cut and straightened into a bar, with the solenoid still on it, through which the current C flows. The magnetic force due to the current is still sensibly equal to $4\pi C n$ (except near the ends). But we now have another term to consider in reckoning **H**. The free magnetism residing at the ends of the bar produces magnetic force at all points in the interior, as well as at points in the space outside, and **H** is the resultant of this force, together with the force $4\pi Cn$ due to the current. The force due to the free magnetism at the ends is opposite in direction to that due to the current; hence H at any point within the metal is less than $4\pi Cn$ by an amount which depends on the distance of the point considered from the ends of the bar. The longer the bar is the more nearly will H be identical with $4\pi Cn$, and if the bar is very long, so that the ends are too far removed to have any material influence, we may take the magnetic force on central portions as sensibly equal to $4\pi Cn$.

Again, in a permanent bar-magnet there is at any point a certain magnetic force, H, due to the free magnetism at the ends, and opposite in direction to the lines of magnetisation within the metal. We may call this the *self-demagnetising force exerted by the bar*, since its tendency is to reduce the bar's magnetisation.

Again, a long piece of straight iron wire stretched in the direction of the lines of force of the earth's magnetic field is acted on by a magnetic force, H, equal to the force of the earth's field. If the wire is hung vertically it is convenient to treat the earth's field as consisting of a horizontal and a vertical component. The former is a magnetic force which acts across the wire; the latter is in this case much the more important of the two, for it constitutes the whole longitudinal part of the magnetic force H, and, as we shall presently see, it is upon this almost wholly that the magnetisation of the wire depends.

§ **16. Magnetic Permeability.**—In general, when a substance is placed in a magnetic field it becomes magnetised. The connection between the magnetism it acquires and the magnetic force which acts upon it may be expressed in two ways.

One of these ways is to compare the magnetic induction B which is produced in the metal with the magnetic force H to which that induction is due. For many purposes this is the most convenient way.

To fix our ideas let us think of a very long uniform rod placed in a uniform field of magnetic force, with the direction of its length parallel to that of the lines of force. When the rod becomes magnetised its ends disturb the field of force, but we can get rid of any trouble about the ends by thinking of the rod as indefinitely long, or so long that the influence which the ends have on the value of the magnetic force is negligible. Let the uniform magnetic field exert a certain force, **H**, on the rod. This produces within the rod a certain induction, **B**, the value of which might be measured by cutting a narrow crevasse across the rod at any place, and measuring the number of lines per square centimetre which cross the crevasse.

If the rod is of iron, nickel, or cobalt, it will be found that the number of lines of induction **B** per square centimetre within the rod is much greater than the number of lines per square centimetre in the field. This fact may be expressed by saying that the material of the rod is more *permeable* with respect to lines of magnetic induction than is the space or medium surrounding it. In Faraday's expressive language, the material of rod has greater conductivity for the lines of induction than the surrounding space or medium has. We may think of the lines as crowding by preference into the rod, finding an easier path through it than through the surrounding medium.

The quality in virtue of which the material of the rod conducts the lines better than empty space conducts them, is called its *magnetic permeability*. This phrase was introduced by Sir William Thomson in his mathematical development of the subject, as a synonym for Faraday's "Conducting power of a magnetic medium for lines of force."

In the case we have supposed, of an indefinitely long rod, the magnetic force at any point within the metal has the same value as the magnetic force at any neighbouring point in the space outside, since the force is not disturbed by the magnetisation of the rod. In such a case we might define the permeability as the number (per square centimetre) of lines of induction **B** in the rod to the number (per square centimetre) of lines of force in the space outside. But if we wish a definition which will be of more general application—applying to short rods as well as to long ones, and to other forms of magnet—we have to bear in mind that the surrounding field is generally disturbed by the magnetisation of the piece. What has to be compared is the induction at any place in the metal with the magnetic force which is in operation there; in other words, we may define the permeability as the ratio of the induction

B at any point of the metal to the magnetic force H which acts within the metal at that point. The permeability is usually denoted by μ; so that we have

$$\mu = \frac{B}{H}.$$

In this definition it is to be understood that B, the magnetic induction, has been produced by subjecting the material to a magnetic force, H.

§ 17. **Permeability of Paramagnetic and Diamagnetic Substances.**—A paramagnetic substance is one in which the permeability is greater than that of empty space. In other words, when such a substance is placed in a magnetic field it will become magnetised in such a way that B is greater than H. The lines of force of the surrounding field will converge more or less towards it, preferring it to the neighbouring space as a magnetic "conductor." Iron, nickel, and cobalt are paramagnets with exceedingly great permeability.

In a diamagnetic substance, on the other hand, the permeability is less than that of empty space. When such a substance is placed in a magnetic field, the lines of force more or less avoid it as a bad "conductor," preferring the space outside. No substance is more than slightly diamagnetic. Even in bismuth, which is the most highly diamagnetic substance known, the magnetic permeability is very little less than unity: its value is about 0·99982.

The permeability of air is sensibly the same as that of empty space. Hence, when a magnetic field is formed in air, the lines of induction are indistinguishable from the lines of force. It is only when the lines pass into a substance which is either paramagnetic or diamagnetic that the distinction between magnetic force and magnetic induction must be maintained.

§ 18. **Illustrations of Permeability.**—By way of illustrating the behaviour of paramagnetic and diamagnetic substances when placed in a magnetic field, Fig. 5 and Fig. 6 have been copied from one of Sir William Thomson's Papers.* In Fig. 5 a magnetic field which was originally uniform has been disturbed

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* Reprint of Paper on "Electrostatics and Magnetism," pp. 489 and 491.

by having a sphere of exceedingly permeable material placed in it. Before the sphere was placed in the field the lines of force were straight, parallel, and equally spaced. The effect of introducing the sphere is to make them converge upon it in the manner which has been exactly represented in the figure from which Fig. 5 is copied. Outside the sphere the lines may be called indifferently lines of induction or lines of force (§ 14). The lines inside, which have been added in this copy, are continuous with them, and are lines of induction. The mag-

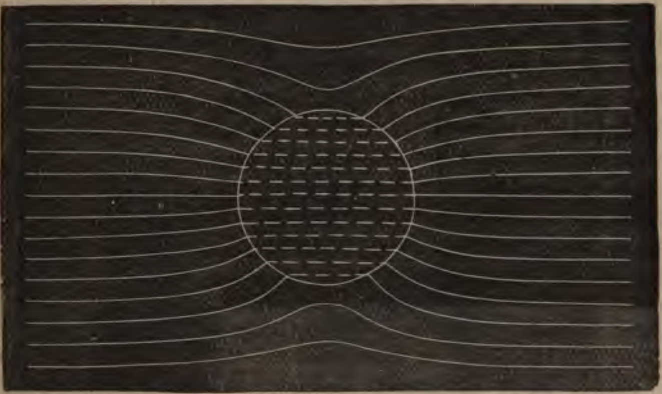

FIG. 5.—Disturbance of an originally uniform magnetic field by the introduction of a soft iron sphere.

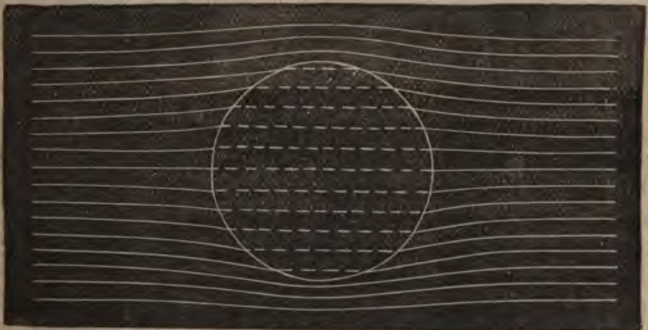

FIG. 6.—Disturbance of an originally uniform magnetic field by the introduction of a sphere of strongly diamagnetic material.

netic induction within the sphere is uniform. Fig. 5 may be taken to represent what happens when a homogeneous spherical ball of soft iron is placed in an originally uniform magnetic field.

Fig. 6 shows in the same way how an originally uniform field of force is disturbed by the introduction of a sphere of diamagnetic material. The material here is a purely imaginary one, with permeability barely one-half that of the surrounding medium, and is far more highly diamagnetic than any actual substance.

The student will not fail to notice that the convergence or divergence of the lines of induction, illustrated by these typical cases, depends on whether the permeability of the body is greater or is less than the permeability of the medium in which it is placed. If the surrounding medium were itself a paramagnetic substance, the case shown in Fig. 6 might be realised by choosing for the material of the spherical ball a substance whose permeability was about half (more exactly 0·48 times) that of the substance surrounding it.

We shall return to these figures later, in speaking of the influence which the form of the body that is placed in a magnetic field exercises on the amount of magnetic induction within the body.

§ 19. **Magnetic Susceptibility.**—When a substance is magnetised by subjecting it to the action of magnetic force, the relation of the induction B to the force H measures, as we have seen, the permeability of the substance. But instead of expressing the magnetisability of the substance by stating the relation of the induction B to the force H, we may state it in a different way by giving the relation of the intensity of magnetisation I to the force H. The ratio of the intensity of magnetisation to the magnetic force producing it is called the *magnetic susceptibility* of the substance, and is usually denoted by κ; thus

$$\kappa = \frac{\mathsf{I}}{\mathsf{H}}.$$

§ 20. **Connection of the Ideas of Permeability and Susceptibility.**—We have seen (§ 13) that

$$\mathsf{B} = 4\pi\mathsf{I} + \mathsf{H};$$

and by definition of the susceptibity κ, $\mathsf{I}=\kappa\,\mathsf{H}$;

Hence
$$\mathsf{B}=4\pi\kappa\mathsf{H}+\mathsf{H}$$
$$=(4\pi\kappa+1)\mathsf{H}.$$

But by definition of the permeability μ, $\mathsf{B}=\mu\,\mathsf{H}$;

Hence
$$\mu=4\pi\kappa+1,$$

and
$$\kappa=\frac{\mu-1}{4\pi}.$$

In a substance such as air, in which the permeability is unity, the magnetic susceptibility is zero. In a paramagnetic substance, in which μ is greater than 1, the susceptibility is positive. In a diamagnetic substance, in which the permeability is less than 1, the susceptibility is negative.

In other words, a paramagnetic substance when subjected to magnetic force acquires a magnetisation I, which is in the same direction as the force, and so makes B greater than H. A diamagnetic, on the other hand, acquires a magnetisation I, which is opposite to the force, and so makes B less than H.

§ **21.** A word of caution is, perhaps, desirable here as to the application of the equations which have just been given. It has been assumed that the material to which the magnetic force H has been applied, has no magnetism except what the force itself induces. If other forces had acted before, leaving residual magnetisation, the ratio $\frac{\mathsf{B}}{\mathsf{H}}$ would not be a true measure of the permeability, nor would the ratio $\frac{\mathsf{I}}{\mathsf{H}}$ be a true measure of the susceptibility.

Again, it has been assumed that the material is magnetically isotropic—that is to say, that a lump of it is equally capable of taking magnetisation in all directions. If this were not so, if the magnetic properties of the substance were different in different directions (as would, for instance, be the case to some extent in a piece of iron cut from a rolled plate), it would be necessary, if we wished to specify fully the relation of the magnetisation to the magnetic force, to resolve the force into components along axes chosen in the directions which give greatest susceptibility and least susceptibility, find the component magnetisation in each of those directions by multiplying each component force by the value which the susceptibility has in that direction, and then compound these components of magnetisation to find the resultant value of I. In such a case the direction of the resultant magnetisation will not in general coincide with that of the resultant magnetic force, and the equation $\mathsf{B} = 4\,\pi\,\mathsf{I} + \mathsf{H}$ will be true only when interpreted in its vector sense.

But in the cases of magnetisation in iron which have ordinarily to be dealt with, it is not necessary to take account of this consideration, for the material is either sufficiently nearly isotropic, or the direction of the applied magnetic force coincides with an axis of greatest or least magnetic susceptibility, and the effect is that I and B have the same direction as H.

§ 22. Influence of the Form of Bodies on the Magnetisation induced in them.—When a body is placed in a magnetic field the degree to which it becomes magnetised depends not only on the original strength of the field and on the permeability of the substance, it depends also (often in very great measure) on the form of the body. This is because the body, in becoming magnetised, generally disturbs the field, causing the magnetic force at any point within or near the body to be different from the force that existed there before the body was introduced. The free magnetism which is developed by the body's magnetisation contributes to produce magnetic force, and so affects the resultant value of the force at any point, inside the body or outside, that is not too far off to be sensibly affected. With iron and other very susceptible materials this disturbance of the field is often so great that the original value of the magnetic force is not even a rough approximation to the value the force assumes when modified by the magnetisation of the body. The intensity of magnetisation at any point within the body depends on the actual value which the magnetic force assumes at that point, and this in its turn depends partly upon the magnetisation of the body as a whole. When we wish to examine the magnetic susceptibility or permeability of a substance, we require to know the actual value of the magnetic force within it, for the purpose of comparing that with the intensity of magnetisation, or with the magnetic induction there. The permeability is measured by the proportion which the induction **B** bears to the strength which the magnetic force **H** actually has at the same place, not to the strength which it may have had there before the body was introduced, nor to the strength which it may still have in external parts of the field.

We have, therefore, to take account of what may be called the reaction of the magnetised body upon the magnetising field.

In very many cases the reaction of the body upon the field is too complex to allow a mathematical examination of it to be practicable. With bodies of irregular form it is out of the question to calculate beforehand what will be the magnetic force and the magnetic induction at internal points, having given the original strength of the external field and the permeability of the substance. The problem is determinate, but too difficult to attack. Even so apparently simple a case as that of a short cylindrical iron rod with flat ends, placed lengthwise in an originally uniform field, presents difficulties so formidable that no exact solution has been given. The difficulty in the case of such a rod is aggravated by the fact that even though the rod be perfectly homogeneous to begin with, the susceptibility or the permeability is not uniform throughout when the rod becomes magnetised. This is because the magnetisation is not uniform, and, as we shall see later, the permeability of iron depends to a considerable extent on the intensity of magnetisation. The reaction of the rod upon the original field tends to reduce the magnetic force at internal points, but this effect is unequal at different parts of the length. It is least at the middle of the length; hence the magnetic force, and consequently the induction also, is greatest there and is less near the ends.

§ 23. Long Rod placed Lengthwise in a Uniform Field.— When the rod is long in comparison with its breadth and thickness the effect of its free magnetism in reducing the magnetic force is less than when the rod is short, especially in the middle region of the length, because the ends, in which the free magnetism chiefly resides, are too far off to have much influence. The amount of magnetic induction is consequently greater in a long rod than in a short one of the same breadth and thickness, the original strength of the field and the permeability of the substance being the same in both cases. When a very long rod is placed lengthwise in a uniform field the influence of the ends becomes almost insensible, and the actual magnetic force at points within the rod is then almost the same as at points outside, except near the ends. The magnetisation will be practically uniform throughout the middle region, but will fall off towards the ends.

When the substance of the rod is very permeable, the rod must be very long relatively to its transverse dimensions before we may neglect its reaction upon the magnetic field, and before we may treat the magnetic force at internal points near the middle as sensibly equal to the force at external points, and the magnetisation as nearly uniform. When the substance is less permeable a shorter length will give an equally good approach to uniform force and uniform magnetisation.

§ 24. **Analogy of Induced Magnetisation to Electric Conduction.**—The concentration of magnetic induction which takes place when a permeable body is placed in a magnetic field is analogous to the concentration of electric flow which may be brought about by immersing a piece of copper in a tube full of mercury, through which an electric current is passing. Let the tube be wide and long, and let the current in it be uniformly distributed over the whole cross-section: we have in this the analogue of a uniform magnetic field. Suppose a short piece of copper wire to be inserted and held lengthwise anywhere near the axis of the tube. The lines of electric flow, which before were straight and parallel, converge more or less towards the piece of copper, preferring to crowd into it because its conductivity is much greater than that of the surrounding medium. The whole current is divided between the copper and the mercury around it, the copper taking a share that is greater than the proportion which its cross-section bears to that of the whole conducting tube. The current enters and leaves the copper not at the ends merely, but also along the sides, especially near the ends. If the piece of copper is short, there can be no more than a slight convergence of the flow into it. For instance, to take an extreme case, a little disc of thin copper plate placed in the mercury, so that it faces in the direction of the flow, has little more conduction through it than through an equal area of the surrounding liquid. In other words, the disc produces but a slight disturbance of the distribution of flow in the tube. On the other hand, a long thin copper wire set lengthwise will gather much of the flow into itself, and if the wire be very long its share of the whole will be greater than the amount taken by an equal section of the mercury in the proportion in which the conductivity of copper is greater than that of mercury. Substitute magnetic permeability for electric conductivity, and magnetic induction for electric flow, and we have a nearly perfect analogue of what happens when an iron rod or wire is placed in a magnetic field.

There is, however, this important difference, which makes the magnetic case less simple than the other. The electric conductivity of the copper is a constant quantity, independent of the strength of current in the metal; whereas the permeability of iron depends on the actual intensity of magnetisation, and consequently varies (in general) to some extent throughout the piece.

§ 25. **Cases in which the Magnetisation is Uniform: Ellipsoid.**—In certain special cases it happens that when a magnetisable body is placed in a uniform magnetic field, the magnetic force at all points inside the body is uniform, though its value there is not the same as at external points. A very important instance in which this is true occurs when the form of the body is that of an ellipsoid, the material being homogeneous, so that the permeability has the same value throughout. In such a case it may be shown that the effect of an originally uniform external field is to produce a strictly uniform magnetisation.*

Let the ellipsoidal body be made of a paramagnetic material, such as iron, and let it be placed in a uniform field: then the originally straight and parallel lines of the field become bent, so that they converge on it, as the lines converge on the sphere

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* *See* Maxwell's "Electricity," Vol. II., §§ 437-438.

in Fig 5. The reaction of the body on the field is such that the magnetic force at outside points near the body is no longer uniform. But at internal points the effect of the reaction is different. The force becomes uniform there, with a value, however, which is less than the value it had in the undisturbed field. This uniform internal force implies uniform induction and uniform intensity of magnetisation—that is to say, each of the quantities **H** and **I** and **B** is uniform throughout the whole of the body; but it must be borne in mind that **H** differs, and often differs greatly, from the value which the force had originally, and still has in distant parts of the field. The amount of this difference will depend on the shortness of the ellipsoid and the intensity of its magnetisation. For brevity we shall use **H′** to designate what may be called the external force—that is, the original value which the force had before the field was disturbed, or, what is the same thing, the value which the force still has at distant external points; and we shall keep **H** to mean, as usual, the actual magnetic force at points within the metal.

§ 26. **Magnetisation of an Ellipsoid** (*continued*).—The case of an ellipsoid subjected to the action of an originally uniform field is of so much practical interest that it is worth while to state here some of the results of calculation which are applicable to it.

Suppose the ellipsoid to be set with one of its axes pointing in the direction of the magnetic force. Let c be half the length of this axis, and a and b half the lengths of the other axes, which point in directions that are perpendicular to the direction of the force. It will suffice to take the case of an ellipsoid of revolution, in which $a = b$.

The original external force being **H′** and the force actually operative being **H**, we have

$$\mathbf{H} = \mathbf{H}' - \mathbf{N}\,\mathbf{I},$$

where **N** is a number depending on the relation of the length of the ellipsoid to its transverse dimensions. We may express **N** in terms of the eccentricity e. When the ellipsoid is of the prolate or elongated form, Fig. 7 (the polar diameter $2c$ or C C′ greater than the equatorial diameter $2a$ or A A′),

$$e = \sqrt{1 - \frac{a^2}{c^2}},$$

and
$$\mathsf{N}=4\pi\left(\frac{1}{e^2}-1\right)\left(\frac{1}{2e}\log_e\frac{1+e}{1-e}-1\right) \quad . \quad . \quad (1)$$

When the ellipsoid is much elongated this expression approximates to the following simpler form :—

$$\mathsf{N}=4\pi\frac{a^2}{c^2}\left(\log_e\frac{2c}{a}-1\right) \quad . \quad . \quad . \quad . \quad (2)$$

When the ellipsoid is of the oblate or flattened form, Fig. 8 (the polar diameter $2c$ less than the equatorial diameter $2a$), the eccentricity e is $\sqrt{1-\frac{c^2}{a^2}}$, and

$$\mathsf{N}=4\pi\left(\frac{1}{e^2}-\frac{\sqrt{1-e^2}}{e^3}\sin^{-1}e\right) \quad . \quad . \quad . \quad (3)$$

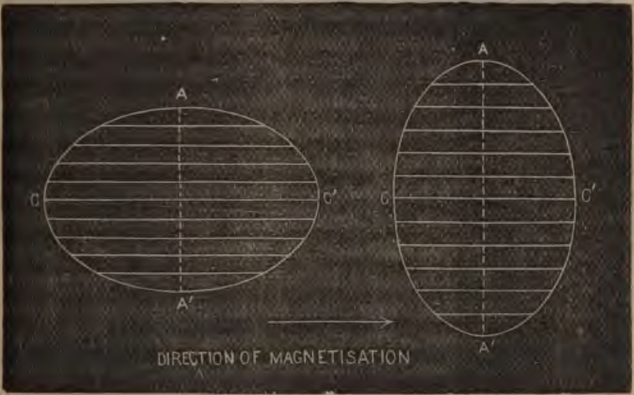

FIG. 7. FIG. 8.

§ 27. **Distribution of Free Magnetism in the Uniformly Magnetised Ellipsoid.**—Within the ellipsoid the lines of force **H**, of magnetisation **I**, and of magnetic induction **B**, lie, as in Figs. 7 and 8, straight and parallel to the lines in the undisturbed field. Since the magnetisation is uniform the free magnetism resides wholly on the surface. To see how it is distributed over the surface we have to remember that **I** is the surface density of free magnetism per square centimetre on every part of an imaginary *surface formed by taking an end elevation of the ellipsoid*—that is to say, by projecting it upon a plane which is perpendicular to the direction of magnetisation. We may therefore obtain a diagram showing the true surface density of free magnetism on the actual surface of the ellipsoid, by supposing the ellipsoid shifted through a very small distance in the direction of magnetisation, so that a meniscus is formed on either side of the middle, between the old and the new positions. Then the thickness of the meniscus is proportional to the surface density of the free magnetism. Thus in Fig. 9 the original ellipsoid is C A C′ A′. By shifting it through the small distance C D or C′ D′, we get the positive meniscus C′ D′, which is a diagram of the surface density of positive free magnetism on one half, and the negative meniscus C D, which in the same way represents the negative free magnetism on the other half. The free magnetism, though densest at the ends, extends towards the middle, and it is only at the equatorial line that there is none. It is easy to show, by referring to the geometrical properties of an ellipsoid, that the distribution which has been described results in making the total quantity of free magnetism on any narrow zone taken perpendicular to the direction of magnetisation proportional to the width of the zone and to its distance from

FIG. 9.

FIG. 10.

the equator A A′. In other words, what we may call the linear distribution of free magnetism in the direction of the axis C C′ is correctly represented by the height of the lines above and below C C′ in Fig. 10.

Again, in regard to linear distribution, it follows that what we may call the centre of the negative magnetism is at Q, two-thirds of the distance A C from A, and the centre of the positive magnetism is at Q′, two-thirds of the distance A C′ from A. The distance Q Q′ is two-thirds of C C′, or $\frac{4}{3} c$.

§ 28. **Moment of Ellipsoid.**—The whole quantity of positive or of negative magnetism is $\pi a^2 \mathsf{I}$—namely, I multiplied by the area of the equatorial section, which is a circle in the special case we have taken, the case of an ellipsoid of revolution with $a = b$. On *distant* external points the action of the magnetised ellipsoid will be the same as if this quantity of positive magnetism were gathered at Q′, and an equal quantity of negative magnetism at Q. The magnetic moment of the ellipsoid is therefore

$$\pi a^2 \mathsf{I}\ \overline{\mathrm{Q\,Q'}} = \frac{4\pi}{3} a^2 c\, \mathsf{I}.$$

But we might have obtained this result more directly. Since I is, by definition, the moment per unit of volume (§ 7), and the volume of the ellipsoid is $\frac{4}{3}\pi a^2 c$, the moment is $\frac{4}{3}\pi a^2 c\,\mathsf{I}$, as above.

§ 29. **Application to the Case of a Sphere.**—When c is equal to a, the eccentricity e is 0, and the ellipsoid becomes a sphere. We then have the case of which Figs. 5 and 6 furnish illustrations. The sphere is uniformly magnetised, but even when the material of which it is made is exceedingly permeable the magnetisation is by no means strong, because the free magnetism which becomes developed on the surface causes the true magnetic force H in the interior to be much less than the original magnetic force H' due to the external field.

By applying the general formula of § 26 to find the value of N in the expression

$$\mathsf{H} = \mathsf{H}' - \mathsf{N}\,\mathsf{I},$$

it may be shown that for a sphere $\mathsf{N}=\frac{4}{3}\pi$, so that

$$\mathsf{H}=\mathsf{H}'-\frac{4\pi}{3}\mathsf{I}.$$

Dividing by H we have

$$1=\frac{\mathsf{H}'}{\mathsf{H}}-\frac{4}{3}\pi\frac{\mathsf{I}}{\mathsf{H}}=\frac{\mathsf{H}'}{\mathsf{H}}-\frac{4}{3}\pi\kappa.$$

Hence, the proportion which the true magnetic force H bears to the force in the undisturbed field is

$$\frac{\mathsf{H}}{\mathsf{H}'}=\frac{1}{\frac{4}{3}\pi\kappa+1}.$$

This shows that when the material is very susceptible, so that κ is large, the true force H is only a small fraction of H'. To take a practical instance, the susceptibility of soft iron to weak magnetic forces, such as those produced by the earth's field, is about 20. Assigning this value to κ, we have $\frac{\mathsf{H}}{\mathsf{H}'}=\frac{1}{85}$ approximately. Thus the true magnetic force within a spherical ball of soft iron placed in the earth's field is only about the $\frac{1}{85}$th part of the force in undisturbed parts of the field, and the magnetisation I which the ball will take up is only about $\frac{1}{85}$th of that which would be taken by a very long rod of the same material set lengthwise in the direction of the lines of terrestrial magnetic force.

Again, as to the magnetic induction in the sphere and its relation to the permeability, we have (remembering that the permeability $\mu=4\pi\kappa+1$)

$$\mathsf{B}=\mu\mathsf{H}=\frac{\mu\mathsf{H}'}{\frac{4}{3}\pi\kappa+1}=\frac{3\mu}{\mu+2}\mathsf{H}'.$$

When μ is exceedingly large, the factor $\frac{3\mu}{\mu+2}$ approximates to 3. Hence, in a sphere of very permeable material, the number of lines of induction through the sphere (per square centimetre of section) is nearly three times the number of lines in the undisturbed field. This is the case in the sphere of Fig. 6 (the proportion of the closeness of the lines inside to that of the lines outside at a distance from the sphere being $\sqrt{3}$ to 1, as seen on the plane of the diagram). The student should note that when the permeability of the sphere is great, its exact value has very little influence on the number of lines of induction that pass through the sphere, and hence a spherical ball would be a very bad form of body to select if we wished by measuring the induction to determine the permeability of the material. A small error in the form of the sphere would, in fact, have more influence in altering the amount of the induction than a large difference in the value of μ or of κ; so that, as Prof. Chrystal has well put it, the experimenter would be testing the accuracy of his instrument-maker rather than the magnetic susceptibility of his material.*

§ 30. The same objection would apply, though in a slightly less degree, to a *short* ellipsoid. By way of illustrating this

FIG. 11.—Short ellipsoid of infinitely permeable material in a uniform field.

further, Fig. 11 has been drawn to show in a general manner the induction through an ellipsoid, and the distortion which it produces in an originally uniform field, when the axes have the proportion of 4 to 1, the material being assumed to have indefinitely great permeability. With this proportion between the axes, **N**, by the formula (1) of § 26, is 0·946, and for every line of force (per square centimetre) in the undisturbed field there are 13·3 lines of induction (per square centimetre) within the ellipsoid. The space between the lines within the body is therefore narrower than the space between the lines in any distant part of the field in the proportion of 1 to $\sqrt{13{\cdot}3}$. The permeability might vary widely without materially affecting

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* Article "Magnetism," *Encyc. Britannica*, Ninth Edition.

the amount of induction, and the figure may be accepted as representing very nearly what would happen if the ellipsoid were of soft iron.*

§ 31. Application to the Case of a Long Cylindrical Rod of Circular Section Magnetised Transversely in a Uniform Field.— This case, of which an example is furnished by a long wire stretched in the earth's field in a direction perpendicular to the lines of force, is deducible from the general case of the ellipsoid by making one of the axes infinite.† This gives $N = 2\pi$, so that $H = H' - 2\pi I$.

Hence, $$\frac{H}{H'} = \frac{1}{2\pi\kappa + 1}, \text{ and } \frac{B}{H'} = \frac{2\mu}{\mu + 1}.$$

Thus, when μ is very large, as it is in soft iron, the transverse induction B across the wire approximates to a value which is twice that of the external field. This is a very small induction compared with that which the same wire would take longitudinally if it were set lengthwise instead of crosswise in the field (compare § 15 above). If we assume κ to be 20, the proportion of the induction in the two cases is about 1 to 127.

It follows from this that when we hang a wire vertically in the earth's field, the transverse magnetisation due to the horizontal component of the earth's field is so small that account

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* Generally, to find the proportion of the induction B within an ellipsoid to the force H' in the undisturbed field, we have:—

$$\begin{aligned} H' &= H + N\,I, \\ &= H + \frac{N}{4\pi}(B - H), \\ &= B\left(\frac{1}{\mu} + \frac{N}{4\pi}\left(1 - \frac{1}{\mu}\right)\right). \end{aligned}$$

When the permeability of the substance is very great, the expression within brackets approximates to $\frac{N}{4\pi}$, giving $B = \frac{4\pi}{N} H'$. In the case considered in the text, $\frac{4\pi}{N}$ is 13·3.

† And using a formula (not quoted in § 26) which refers to magnetisation in the direction of an equatorial axis. *See* Maxwell, *loc. cit.*

need not in general be taken of it, and the same thing is true of the transverse magnetisation of a wire laid horizontally in the earth's field.

§ 32. **Case of a Thin Disc Magnetised in the Direction of the Thickness by a Uniform Field.**—We may find the true magnetic force within a disc or large thin plate magnetised normally in a uniform field from the fact that the lines of induction B within the disc are continuous with the lines of force H' in external space, and if the disc is very wide in comparison with its thickness, the lines go straight through it without sensible distortion. Thus $\mathsf{H}' = \mathsf{B} = 4\pi\mathsf{I} + \mathsf{H}$, so that $\mathsf{H} = \mathsf{H}' - 4\pi\mathsf{I}$, $\frac{\mathsf{H}}{\mathsf{H}'} = \frac{1}{\mu}$, and the induction within the disc is the same whatever be the permeability of the material. The same result may be derived from equation (3) of § 26, by making a indefinitely great in comparison with c. This gives $e = 1$ and $\mathsf{N} = 4\pi$.

§ 33. **Long Ellipsoid: Influence of the Length on the Magnetising Force.**—Returning now to the general case of a long ellipsoid of revolution placed longitudinally in a uniform magnetising field, it is interesting to notice to what extent the uniform magnetisation of the ellipsoid itself affects the magnetic force, when we assume various values as the ratio of length ($2c$) to transverse diameter ($2a$).

In the formula

$$\mathsf{H} = \mathsf{H}' - \mathsf{N}\,\mathsf{I},$$

we may write $\frac{\mathsf{B} - \mathsf{H}}{4\pi}$ for I (by § 13), and if the material is very permeable, so that B is large compared with H, this will be very nearly equal to $\frac{\mathsf{B}}{4\pi}$ simply. Hence in an ellipsoid made of very permeable material, such as iron,

$$\mathsf{H} = \mathsf{H}' - \frac{\mathsf{N}}{4\pi}\,\mathsf{B}, \text{ very nearly.}$$

The following values of N and also of $\frac{\mathsf{N}}{4\pi}$ have been calculated by means of the expressions in § 26 for ellipsoids in which the ratio of length to breadth is 50, 100, 200, 300, 400, and 500 respectively.

Ratio of Length to Breadth $\left(\frac{c}{a}\right)$	N	$\frac{\mathsf{N}}{4\pi}$
50	0·01817*	0·001446
100	0·00540	0·000430
200	0·00157	0·000125
300	0·00075	0·000060
400	0·00045	0·000037
500	0·00030	0·000024

Since $\mathsf{H} = \mathsf{H}' - \mathsf{N}\,\mathsf{I}$, $\frac{\mathsf{H}'}{\mathsf{H}} = \frac{\mathsf{N}\,\mathsf{I}}{\mathsf{H}} + 1 = \mathsf{N}\,\kappa + 1$. The proportion which the resultant force H bears to the original force H' in the undisturbed field,

$$\frac{\mathsf{H}}{\mathsf{H}'} = \frac{1}{\mathsf{N}\,\kappa + 1}.$$

By the help of the above table it is easy to find this proportion, for an assigned ratio of length to breadth, when the susceptibility of the material is known.

As an example, we may take $\kappa = 200$ as a representative value of the susceptibility in soft iron when subjected to a moderately strong magnetic force. Suppose that the ellipsoid is 100 diameters long, then

$$\frac{\mathsf{H}}{\mathsf{H}'} = \frac{1}{0{\cdot}0054 \times 200 + 1} = \frac{1}{2{\cdot}08}.$$

In other words, the magnetic force actually operative within the metal—as reduced by the magnetism of the piece itself—is in that case rather less than one-half the force due to external causes.

§ 34. **Residual Magnetism and Retentiveness.**—When a piece of any one of the strongly magnetisable metals—iron, steel, nickel, or cobalt—is magnetised by applying magnetic force, and the externally-applied force is then withdrawn, it is found that the magnetisation does not wholly disappear. What

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* The approximate formula (2) of § 26 gives 0·01812. For the longer ellipsoids the values of N calculated from it may be taken as correct.

remains is usually called the residual magnetism, and metals which retain residual magnetism when the external magnetic force is withdrawn are said to possess retentiveness.

We shall see later that this retention of residual magnetism, when the externally applied magnetising force is withdrawn, is only one instance of a general tendency which these metals exhibit to resist any change in their magnetic state.

§ 35. **Self-Demagnetising Force.**—In connection with the subject of retentiveness it is of the first importance to notice that though the externally applied magnetic force be withdrawn from a magnetised piece, there is in general some magnetic force in action. This force is due to the residual magnetism itself, and its tendency is to *reduce* the residual magnetisation. In a bar magnet, for instance, the residual magnetism at and near the ends of the bar produces a magnetic force acting in the direction of the length and tending to demagnetise the bar. In a ring magnet uniformly magnetised we get rid of this self-demagnetising force by having the ends, so to speak, brought together. In an exceedingly long bar the self-demagnetising force becomes insignificant because the ends are far removed from most parts of the bar. The residual magnetism in a ring or a very long rod will therefore be greater, other things being equal, than in a short rod. Indeed, so much is this the case that, in dealing with soft annealed iron, we shall find almost no residual magnetism if we experiment with rods the length of which is only 10 or 20 times their diameter, because in these rods the self-demagnetising force is sufficient to remove the residual magnetism almost completely, whereas a rod 400 or 500 diameters long will be found to retain a very large proportion of its induced magnetism when the inducing force is withdrawn. Hence the term residual magnetism has one meaning when it is used to describe the magnetism that remains when magnetic force is completely withdrawn without any reverse force being applied, an experiment which can be made if we use an exceedingly long rod or a ring magnet; and it has another and quite different meaning when it is used to describe the magnetism which a bar or other short piece will retain in opposition to the demagnetising force which it exercises upon itself.

§ 36. **Self-Demagnetising Force in Ellipsoids.**—In the case of an ellipsoid, uniformly magnetised, the self-demagnetising force is uniform throughout the body, and its value is

$$\mathsf{N}\,\mathsf{I},$$

where N has the same meaning as in § 26, and I is the *residual* intensity of magnetisation.

To get an idea of what this may amount to in actual cases, we may take 1,000 C.-G.-S. units as a residual value of I which is commonly enough found in the magnetisation of iron. When an ellipsoid is 200 times as long as it is broad the value of N is 0·00157 (by § 32), and a residual intensity (I) of 1,000 would therefore produce a self-demagnetising force of 1·57. The experimental results which will be given later will show that a force of this magnitude is by no means insignificant, and that it would, in fact, be sufficient to remove a large part of the residual magnetism. It is only when the length is as much as 400 or 500 times the transverse diameter that the self-demagnetising force in a material so susceptible as iron becomes nearly negligible.

CHAPTER II.

MEASUREMENTS OF MAGNETIC QUALITY: THE MAGNETOMETRIC METHOD.

§ 37. **Methods of Measuring Magnetic Quality.**—From what has been said about the influence which the ends of magnetised bodies exercise by reacting on the magnetising field, and by exerting a self-demagnetising force when the external force is withdrawn, it will be clear that when we attempt to measure the permeability, or the susceptibility, or the retentiveness of a magnetic metal, we must either arrange the conditions of the experiment in such a way that the influence of the ends may be calculated and allowed for, or else choose pieces which are actually or practically *endless*.

We may use long ellipsoids (short ones will not do, for the reason already explained—that in them the magnetisation depends too much on the form and too little on the quality of the piece), and, having observed I, calculate the true magnetic force within the metal by subtracting $\mathsf{N\ I}$ from the externally applied force H'.

Again, if the piece tested is a very long cylindrical rod or wire, the influence of the ends may be approximately allowed for by treating the piece as an ellipsoid. And by making the length great enough (400 times the diameter or so if iron is being tested and if the rod is straight) we may reduce the influence of the ends so much that it may generally be neglected. In a rod long enough to be practically endless, the magnetic force within the metal is sensibly the same as the force in the field when the rod is withdrawn from it.

The condition of endlessness can be completely secured by giving the piece to be magnetised the form of a ring. If we take a ring of uniform section, and wind the magnetising coil uniformly all round it, we secure a magnetic field which is uniform throughout the length of the ring, and nearly uniform throughout each cross-section. The magnetic force operative on the metal is quite independent of the magnetism of the piece itself. The ring has no poles; it does not react on the magnetising field; and when the external force is withdrawn it exerts no demagnetising force upon itself.

Ellipsoids, long rods, and rings have all been used in testing the permeability and other magnetic qualities of iron. Recent experiments have, as a rule, been made either with rings (or pieces equivalent to rings), or with very long cylindrical rods. From some points of view, long ellipsoids would be the most satisfactory of all forms of specimen, but the difficulty of shaping them correctly is a serious obstacle to their use.

§ 38. Classification of Methods: Magnetometric and Ballistic.—The magnetisation produced in a specimen by applying magnetic force, or, more generally, the change of magnetism produced by any change in the force, is usually measured in one or other of two ways.

In one—the magnetometric method—the magnetism of the piece is measured by observing the deflection of a magnetic needle suspended near it, called a magnetometer. This method is applicable when we deal with ellipsoids and rods, but obviously cannot be used with rings, since a uniformly magnetised ring exerts no magnetic force in the neighbouring space.

In the other method, any change in the magnetic induction within the specimen is determined by measuring the transient current which is induced in a surrounding coil when the change of induction takes place. The coil acts like the secondary wire of an induction coil or transformer. When any change takes place in the number of lines of induction surrounded by the coil, a transient current is produced, the whole quantity of which (that is to say, its time-integral) is proportional to the change. This transient current is measured by passing it through a "ballistic" galvanometer, which is a galvanometer with a needle that swings slowly enough to allow practically the whole of the transient current to pass before the needle has stirred to any sensible extent from its position of rest. This "ballistic" method is applicable to rings as well as to rods of any form, and is in fact the only method by which we may examine the magnetic quality of a ring. When applied to rings it serves to measure sudden changes of magnetism only, such as may be caused by making, breaking, reversing, or suddenly increasing or reducing the current in the magnetising coil. When applied to rods the ballistic method may be modified, so that it will measure the actual magnetic state of the piece, and not sudden changes merely. This is done by winding the secondary coil in such a way that it may be suddenly slipped off the magnetised piece, and removed far enough from it to be out of the reach of magnetic influence, remaining, however, all the while in circuit with the ballistic galvanometer. Slipping off the coil gives a transient current which corresponds to the sudden removal of all the lines of induction.

We shall now describe the two methods in some detail, and give examples of their application.

§ 39. **Magnetometric Method.**—In this method the rod or other piece whose magnetisation is to be measured is fixed near the magnetometer, the needle of which is already directed by some known magnetic force. Generally, the needle hangs horizontally under the control of the earth's magnetism alone, so that the directing force is the horizontal component of the earth's magnetic field. The magnetised piece is fixed in such a position that the magnetic force which it produces at the magnetometer, or what we may call the deflecting force, acts at right angles to the directing force. The tangent of the angle of deflection then measures the proportion of the deflecting to the directing force. Thus, in Fig. 12, if the magnetised body is placed so that it produces a magnetic force F_2 at the magnetometer needle $a\,b$, and the directing force is F_1, the needle, which originally pointed in the direction of F_1, is deflected through an angle θ such that

$$F_2 = F_1 \tan \theta.$$

Knowing F_1 and observing θ, we determine F_2, and from that, knowing the position and dimensions of the magnetised piece, we calculate its intensity of magnetisation.

Suppose, for instance, that the magnetic bar is an ellipsoid of revolution (with polar axis $2\,c$ and equatorial axes $2\,a$) placed in the position shown by the line C C′ in Fig. 13. The bar lies horizontally, level with the magnetometer needle O, which is opposite the middle of the bar and points to it when in the undeflected position. In its action on a distant point* the ellipsoidal bar is,

Fig. 12.

Fig. 13.

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[*] It is only when the point O is at a considerable distance from C C′ that the effect of the bar's distributed magnetism is approximately the same as that of poles at Q and Q′. If O is near the bar, the formula in

as we have seen in § 27, equivalent to a positive pole of strength $\pi a^2 \mathsf{I}$ at Q and an equal negative pole at Q′, the distance Q Q′ being $\frac{4}{3}c$. The deflecting force F_2 which the bar produces at the magnetometer is the resultant of the equal forces

$$\frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q}}^2} \text{ and } -\frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q'}}^2}$$

Its direction is parallel to the bar, and, as the diagram shows,

$$F_2 : \frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q}}^2} :: \mathrm{Q\,Q'} : \mathrm{O\,Q},$$

or

$$F_2 = \frac{\pi a^2 \mathsf{I}\, \overline{\mathrm{Q\,Q'}}}{\overline{\mathrm{O\,Q}}^3} = \frac{4\pi a^2 c \mathsf{I}}{3\overline{\mathrm{O\,Q}}^3}.$$

And, since $F_2 = F_1 \tan\theta$, we may put this result in the following form, suitable for finding the intensity of the bar's magnetisation :—

$$\mathsf{I} = \frac{3\overline{\mathrm{O\,Q}}^3 F_1 \tan\theta}{4\pi a^2 c}.$$

§ 40. **Magnetometric Method** (*continued*).—Another position for the specimen to be magnetised is shown in Fig. 14. The plane of the sketch is vertical. O is the magnetometer, the needle of which, when undeflected, points at right angles to the plane of the paper. The bar is fixed behind it, standing vertically with its upper pole at the level of the magnetometer. The deflecting force F_2 is mainly due to the upper pole: its value is—

$$\frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q}}^2} - \left(\frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q'}}^2}\right)\frac{\mathrm{O\,Q}}{\mathrm{O\,Q'}} = \frac{\pi a^2 \mathsf{I}}{\overline{\mathrm{O\,Q}}^2}\left\{1 - \left(\frac{\mathrm{O\,Q}}{\mathrm{O\,Q'}}\right)^3\right\}.$$

And since this is equal to $F_1 \tan\theta$, we have

$$\mathsf{I} = \frac{\overline{\mathrm{O\,Q}}^2 F_1 \tan\theta}{\pi a^2\left\{1 - \left(\frac{\mathrm{O\,Q}}{\mathrm{O\,Q'}}\right)^3\right\}}.$$

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the text is not applicable; but it is in any case practicable to calculate the deflecting force at O, since the distribution of free magnetism along C C′ is known.

This arrangement is particularly applicable where we have to deal with a very long cylindrical rod (diameter $2a$). In such a rod the position of the effective poles Q and Q′ is uncertain, and, indeed, varies when the intensity of magnetisation is varied. But the method has this advantage, that a change in the position of Q along the rod produces very little change in its distance from the magnetometer, and has little effect in altering the deflecting force, while, as to the other pole Q′, its total effect is so small that a movement of it is also without much influence. The best height at which to set the rod will be found by giving it a moderate degree of magnetisation, and finding by trial how high the upper end should stand above the level of the magnetometer to make the deflection a maximum. In this way the position of Q, and therefore of Q′ also, may be found with sufficient accuracy to allow the formula to be applied to a very long cylinder. For brevity, we may distinguish this as the "one-pole" method, seeing that the deflection of the magnetometer is mainly caused by one of the bar's poles.

FIG. 14.

§ 41. **Details of Magnetometric Method.**—A form of magnetometer, which is exceedingly convenient for observations of this kind, and can be made by anyone at a very trifling cost, is shown in Fig. 15. The suspended "needle" consists of one of the mirrors used in Sir W. Thomson's reflecting galvanometers, with small magnets cemented on the back. The mirror M is hung from a pin, S, at the top of a wooden upright, by a silk fibre three or four inches long. A groove is cut in the upright to allow the fibre to hang free, and at the bottom a round hole, closed in front by a lens, forms a chamber in which the mirror hangs. A glass plate is cemented on the back of the upright to cover the hole and the groove. The upright is fixed to a horizontal plate furnished with three levelling screws. The deflection of the mirror is read in the usual way by means of a lamp and scale.

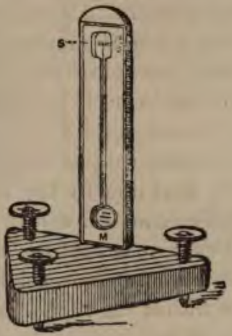

FIG. 15.

In using a mirror magnetometer, the angle through which the needle is deflected may generally be kept so small that no account need be taken of the difference between tan θ and θ, and consequently the scale readings may be taken as proportional to the deflecting forces. Thus, if we have a scale 50cm. long, set one metre from the magnetometer, a deflection of the needle amounting to only about 7deg. will make the spot of light travel from the middle to the end of the scale. (It must be remembered that the angular deflection of the mirror is only half that of the beam of light.) Even for this largest deflection the error caused by taking scale divisions instead of tangents is barely a half of one per cent., and for smaller deflections the error is very much less. In such a case, therefore, instead of tan θ we may write θ, which is equal to $\frac{\delta}{2\,D}$, δ being the deflection as measured on the scale, and D the distance of the scale from the mirror expressed in scale divisions.

Fig. 16 illustrates an arrangement for examining the magnetic quality of long thin rods or wires by the "one-pole" variety of the magnetometric method. The specimen is slipped into a tube, A, which is clamped in a vertical position behind the magnetometer B, the distance being adjusted by trial to make the deflection conveniently large. Over the tube a magnetising solenoid is wound, extending a little way above and below the wire core, so that the magnetising force inside may be sensibly uniform, except in so far as it is affected by the ends of the specimen itself. (When only one wire is to be tested, the magnetising solenoid may conveniently enough be wound on the wire itself, instead of on a tube.) Owing to the vertical position of the specimen, it is exposed to the vertical component of the earth's magnetic force. For many purposes it is desirable to eliminate this, so that the only force acting along the wire may be that due to the magnetising solenoid. To secure this a second solenoid is wound upon the tube, and through it a constant current is kept up, the strength of which is adjusted (by a method to be described later) until the magnetic force it produces within the tube just balances the earth's vertical force. In the sketch, the single gravity Daniell cell C and the resistance box D give the means of maintaining and regulating this constant current.

In circuit with the main solenoid and behind the specimen is a coil, E, consisting of a few turns of wire wound on a wooden frame which can slide towards or from the magnetometer, its axis passing through the magnetometer at right angles to the undeflected direction of the needle. This "compensating coil," as we shall call it, serves to neutralise the direct action of the magnetising solenoid upon the magnetometer. Its position is adjusted thus: Before putting the specimen to be magnetised into the magnetising solenoid, pass a fairly strong current through the solenoid and the compensating coil, and push the coil backwards or forwards until the magnetometer shows no deflection. The adjustment remains correct for all currents, and its effect is that when the specimen

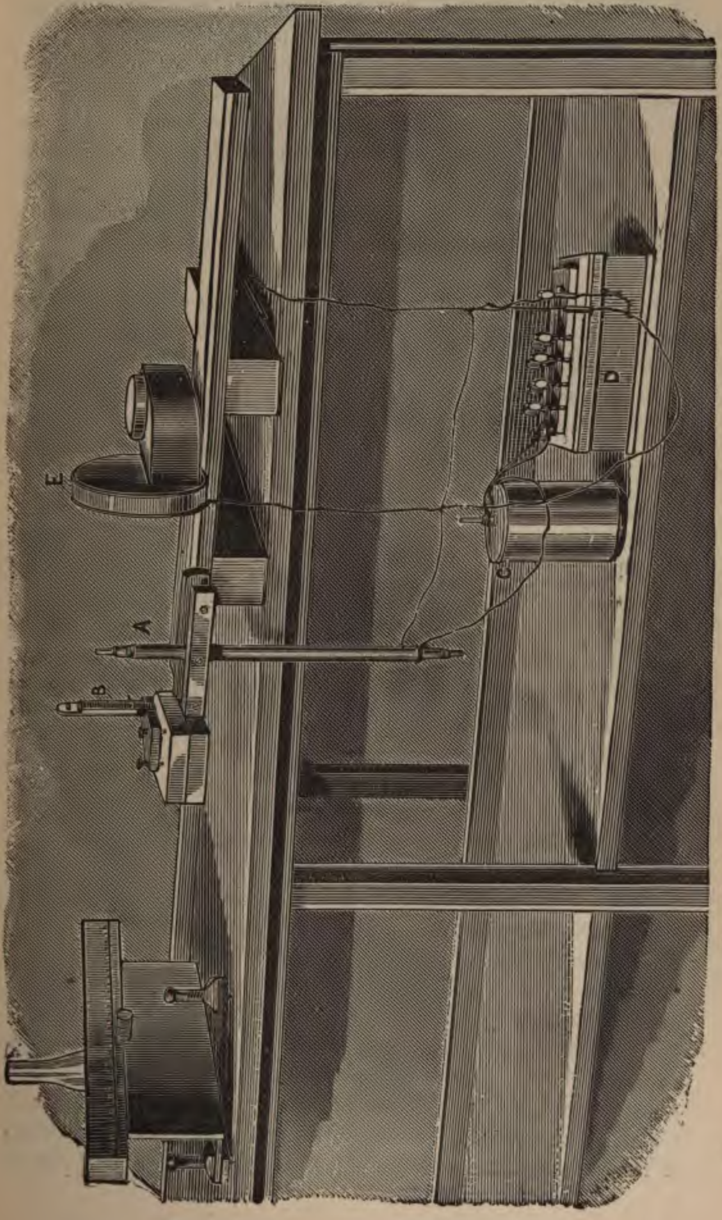

FIG. 16.—Arrangement for Examining the Magnetic Quality of Long Thin Rods.

is put in no deduction has to be made from the observed deflection on account of the magnetising solenoid.

We may of course allow for the effect of the solenoid without using a compensating coil, by observing what deflection the solenoid itself produces with a given strength of current when the specimen is removed, and then making a proportional deduction for other currents. The compensating coil, however, has a great advantage over this in point of practical convenience, and has other uses besides, of which examples will be given later.

In each part of the connections the leading wires are twisted together—a very necessary precaution to prevent their acting on the magnetometer.

In examining the permeability of a specimen, a weak magnetising current is first applied, and this is increased step by step or continuously, observations of the current strength being taken along with observations of the magnetometric deflection. A storage battery forms the most convenient source of current; if that is not available, a battery of gravity Daniell cells will do well. To observe the current strength, any good form of galvanometer or ampere-meter may be kept in circuit with the magnetising solenoid. A plan which is as good as any is to use a low-resistance mirror galvanometer, strongly controlled by a fixed permanent magnet, and test its sensibility by passing a current through it from a gravity Daniell cell; the strength of the current in amperes may be taken as $\frac{1{\cdot}1}{R}$, where R is the total resistance of the circuit in B.A. units. Care must be taken to set up the galvanometer far enough away from the magnetometer to prevent one from acting on the other.

In many magnetic experiments it is desirable to have the means of altering the magnetising current continuously instead of by steps, between zero and its highest value. This is conveniently effected by using the liquid rheostat, or potential slide, shown in Fig. 17. A tall glass jar of fairly uniform bore, two inches or so in diameter, is filled with dilute solution of sulphate of zinc. Three blocks of amalgamated zinc, *a*, *b*, and *c*, are fitted in the jar, one lying at the bottom, another fixed at the top, and the third hung between them so that it may be raised or lowered by the cord *d* which passes over a pulley above to the little winch at *e*. The blocks are connected to three terminals at *f*, insulated wires being led up through the liquid from the middle and lower blocks. The battery is connected to *a* and *c*, so that the liquid

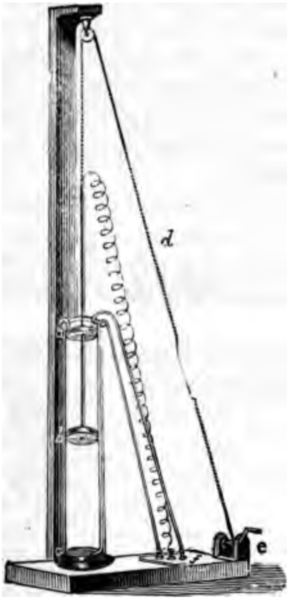

Fig. 17.—Liquid rheostat used for the purpose of continuously altering the strength of the magnetising current.

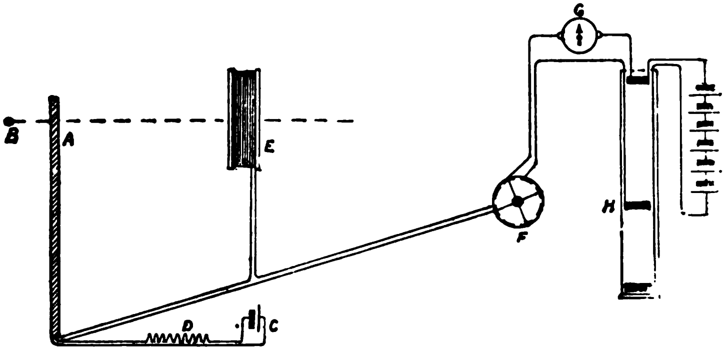

Fig. 18.

column forms a shunt to it, and a part of its E.M.F. is taken off to produce current in the magnetising solenoid by connecting the ends of the solenoid to one of the fixed and one of the moving blocks, say *a* and *b*. Thus, when *b* is raised into contact with *a* no current passes through the solenoid, and when *b* is gradually lowered the current increases, reaching its highest value when *b* touches *c*. With this slide it is easy to adjust the current to any intermediate value, and to keep it constant for as long as may be wished.

Fig. 18 is a general diagram of the connections. The letters A, B, C, D and E refer to the same parts as in Fig. 16. F is a revolving commutator, G a galvanometer for measuring the magnetising current, and H is the slide described above.

§ 42. **Demagnetising by Reversals.**—The liquid slide gives a handy means of performing a process which is resorted to when we wish to rid the specimen of any initial magnetism it may possess, or to wipe out the residual effects of previous operations. The process of "demagnetising by reversals" consists in applying a numerous series of magnetic forces alternating in direction, and gradually diminishing to zero. A commutator or rapid reversing key is inserted either between the battery and the slide or between the slide and the magnetising solenoid. Working it rapidly with one hand, and turning the winch-handle of Fig. 17 very slowly with the other, the operator applies a long series of alternating magnetising currents, each a very little weaker than the one before it, and the result is, when the process is carefully conducted, to remove all trace of residual magnetism, provided the strongest current of the series is at least as strong as the current by which the piece had been previously magnetised.

§ 43. **Adjustment of the Current Required to Balance the Vertical Component of the Earth's Field.**—The operation of demagnetising by reversals will not be completely successful unless the earth's vertical force is very exactly balanced, otherwise there will be a one-sidedness in the alternate opposite magnetic forces, which will show itself by leaving a persistent residue of magnetism in one direction or the other, the direction depending on whether the constant current which is applied to balance the earth's force is too strong or too weak. This affords an excellent criterion by which we may adjust the current. It has to be strengthened or weakened until, when the process of demagnetising by reversals is performed, the demagnetisation is complete. The more susceptible the material within the solenoid is, the more sensitive is the test, and it is well to keep at hand, for the purpose of adjusting the current in this way, a core of soft annealed iron, which may be slipped into the solenoid when the test is to be made. In order to increase the sensibility further, when a fine adjustment is required, the solenoid should be set a good deal closer to the magnetometer than it is set when we are afterwards measuring the magnetism of a wire or rod within it.

§ 44. To Find the Directing Force at the Magnetometer.—In measuring magnetism by the magnetometric method, we must know the force F_1 which directs the needle when it hangs in the undeflected position. Even when no special directing magnets are used, it is not safe to assume that F_1 is identical in value with the horizontal component of terrestrial magnetism, for the earth's field is often seriously altered within a room by the magnetic influence of iron pipes, beams, and so forth. So long as these disturbing bodies are not liable to be moved about, or to have their temperature much altered, their effect in modifying the magnetic field—though it may be considerable—will be nearly constant, and in that case an occasional measurement of F_1 will suffice. If there are iron heating pipes or stoves in the neighbourhood, the utmost care is necessary to see that F_1 does not vary. Fixed masses of iron at the atmospheric temperature are not a very objectionable feature in a magnetic laboratory; but it is difficult to exaggerate the nuisance that may be caused by an iron stove or steam-pipe liable to quick changes of temperature.

We may make an entirely independent measurement of F_1, following the well-known method which is used in measuring the horizontal component of the earth's field,* and taking care to

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* Full directions for the determination of the horizontal component of the earth's field will be found in Prof. A. Gray's "Absolute Measurements in Electricity and Magnetism."

swing the deflecting magnet in the place where the magnetometer is to stand.

But in general all that is required is to go through as much of this process as will serve to find the *relative* values of F_1 and the horizontal field F at a place where there is no local magnetic disturbance. In most places F is sufficiently well known from the results of recent magnetic surveys, so that the absolute value of F_1 may be deduced when we know the ratio it bears to F.

To compare the two, take a short straight piece of permanently magnetised steel wire, and suspend it to hang horizontally within a glass vessel by a little cradle and a silk fibre 3in. or 4in. long attached to the cover, so that it is free to swing. Put it where the magnetometer is to stand, and set it swinging torsionally (*not* in pendulum fashion). This is most easily done by bringing a bar magnet near it, and then drawing that away, keeping the two poles of the bar equally distant from the hanging wire. When the swings have subsided so that the motion is no more than 5deg. or so to either side, begin to count them. Note with a watch the instant at which the magnet swings past its middle position towards one side, count 30 or 40 complete swings, and again note the time the magnet swings past its middle position towards the same side. Find in this way the time t_1 (in seconds) required for one complete swing. Then take the swinging magnet to some place (outside) where there is nothing to interfere with the terrestrial magnetic field, and repeat the counting there to find the time that is required to make one complete swing when the only directing force is the horizontal component F of the earth's field. The directing force is inversely proportional to the square of the period of swinging, hence the directing force at the place where the swings were first counted

$$F_1 = \frac{F t_1^2}{t^2}.$$

When the magnetometer is furnished with a "compensating coil" (§ 41) the following is a good way to find F_1. Remove the magnetising solenoid and set the compensating coil at a known distance, O A (Fig. 19), behind the magnetometer. Pass a known current, C,* through it and observe the deflection θ of the magnetometer. A B is the mean radius of the coil, and O A is measured to the middle of its width. Let q be the number of turns in the coil; then the deflecting force which is produced at O by the current in the coil is

$$\frac{2\,\pi\,C\,q\,\overline{A\,B}^2}{(O\,A^2+A\,B^2)^{\frac{3}{2}}}, \text{ or } \frac{2\,\pi\,C\,q\,\overline{A\,B}^2}{\overline{O\,B}^3};$$

and, since this is equal to $F_1 \tan\theta$, we have

$$F_1 = \frac{2\,\pi\,C\,q\,\overline{A\,B}^2}{\overline{O\,B}^3 \tan\theta}.$$

FIG. 19.

§ 45. **Example of a Test of Iron by the Magnetometric Method.**—Before proceeding to describe the ballistic method of measuring magnetisation, it may be useful to illustrate the magnetometric method by giving the particulars of an actual experiment on a piece of wrought-iron wire.

The diameter of the wire (d) was 0·077 cm. The length of the specimen was 30·5 cms., or 400 diameters. It was annealed or softened before the test by drawing it through a lamp flame so slowly that each part of the length, in succession, was heated to bright redness and then cooled slowly as it passed away from the flame. The "one-pole" arrangement (§40) was adopted. A preliminary trial showed that the effective "poles" lay very near the ends of the wire. The upper one was set at a distance (O Q) of 10 cms. behind the magnetometer; the distance O Q′ to the lower pole was 31 cms.

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* Here, as elsewhere, the current is expressed in absolute electromagnetic (C.-G.-S.) units. If its value is known in amperes we must divide the number of amperes by 10 to find C.

The directing force at the magnetometer F_1 was 0·299 in C.-G.-S. units. The deflections were read in millimetres, and the scale was set at a distance of 1 metre from the magnetometer. Hence one scale division of deflection corresponds to a value of $\frac{1}{2000}$ for θ or $\tan\theta$.

Substituting these values in the expression of § 40,

$$\mathsf{I}=\frac{4.\overline{\mathrm{O\,Q}}^2.\,\mathrm{F}_1\tan\theta}{\pi d^2\left\{1-\left(\frac{\mathrm{O\,Q}}{\mathrm{O\,Q'}}\right)^3\right\}},$$

we have I for one scale division of magnetometer deflection

$$=\frac{4\times\overline{10}^2\times 0{\cdot}299}{3{\cdot}1416\times\overline{0{\cdot}077}^2\times 0{\cdot}9665\times 2000}=3{\cdot}32.$$

Again, the magnetising solenoid contained 69 turns per centimetre of its length. Its magnetising force for one ampere of current was, therefore,

$$\frac{4\pi\times 69}{10}=86{\cdot}7.$$

The current was measured by a mirror galvanometer, which was found to give a deflection of 575 scale divisions, with a current of 0·235 amperes. This corresponds to 0·000408 amperes per scale division. Hence the magnetising force for one scale division of the galvanometer was

$$86{\cdot}7\times{\cdot}000408=0{\cdot}0354.$$

After the independent current (in a separate solenoid) which was required to balance the vertical component of the earth's force had been adjusted, the process of demagnetising by reversals was gone through to wipe out any traces of magnetism the wire might have acquired in handling. Readings of the magnetometer and galvanometer were then taken, while the current was slowly increased step by step from zero till the magnetic force reached a value of 22·27 units. Then the current was slowly and step by step reduced to zero, the magnetism retained by the specimen being observed at each stage, and then a negative current was applied, giving a reversed magnetic force, which was slowly increased until the residual magnetism began to become reversed. The results of the experiment are stated in Table I. Column 1 gives the observed galvanometer deflections, and column 2 the magnetising force calculated from them. This is the force produced by the solenoid; in the notation of § 25 it is **H**′, and is a little greater than the true magnetic force **H**, which is diminished by the action of the ends of the specimen when it becomes magnetised (see § 47 below). Column 3 gives the observed magnetometer deflections (due to the wire alone), and column 4 gives values of **I** calculated from them.

Table I.—*Magnetisation of Annealed Iron Wire.*

(1) Magnetising current (Gal. readings).	(2) Magnetising Force.	(3) Magnetometer Readings.	(4) **I**
0	0	0	0
9	0·32	1	3
24	0·85	4	13
39	1·38	10	33
59	2·18	28	93
79	2·80	89	295
99	3·50	175	581
119	4·21	239	793
139	4·92	279	926
159	5·63	304	1,009
189	6·69	327	1,086
239	8·46	348	1,155
289	10·23	359	1,192
342	12·11	365	1,212
441	15·61	373	1,238
574	20·32	378	1,255
629	22·27	380	1,262
464	16·42	379	1,258
239	8·46	375	1,245
139	4·92	372	1,235
89	3·15	369	1,225
39	1·38	363	1,205
0	0	350	1,162
− 11·5	−0·41	342	1,135
− 23	−0·81	329	1,092
− 31	−1·10	318	1,056
− 41	−1·45	295	979
− 51	−1·80	253	840
− 62	−2·20	166	551
− 71	−2·51	70	232
− 81	−2·87	−12	−40

§ 46. **Magnetisation Curve.**—A convenient way of representing such results graphically is to draw a curve showing the relation of the magnetising force to **I** or to **B**. In Fig. 20 a curve showing the relation of the magnetising force of the solenoid to **I** is drawn from the above table. O A B is the ascending limb, got by applying and increasing a magnetising current, the iron being originally in a non-magnetised and perfectly neutral state. From B to C the magnetising current is being reduced to zero; from C to D an increasing negative current is being applied.

This example is thoroughly characteristic of the behaviour of annealed wrought iron. The ascending limb of the curve may be divided, broadly, into three portions. In the first, under feeble magnetic forces, the gradient of the curve is very small, which means that at this stage there is (comparatively) very little magnetic susceptibility.* Later, as the force increases, the curve becomes exceedingly steep, and nearly straight; this is the region of great susceptibility. Then, lastly, the curve rounds off until the rate of ascent again becomes small, so that the susceptibility diminishes, and any considerable addition to **I** can then be brought about only by applying a very strong magnetising force.

This third stage is a necessary consequence of the well-known phenomenon of magnetic "saturation." We shall see later that the value of **I** has a definite limit which cannot be exceeded no matter how high the magnetic force be raised.

§ 47. **Residual Magnetism and Coercive Force.**—In the descending limb of the curve it is interesting to notice how little of the magnetism disappears as the magnetic force is withdrawn. Even when the solenoid current is reduced to zero, the residual magnetism O C is in this case 1,162 C.-G.-S. units, which is no less than 92 per cent. of the value reached when the current was in action (1,262 units). This residual magnetism is, however, very feebly held. Applying a reverse

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* The comparatively small susceptibility of iron to feeble forces seems to have been first clearly pointed out by Stoletow (*Phil. Mag.*, Vol. XLV., 1873, p. 40), whose observations on the relation of magnetisation to magnetic force were confirmed and greatly extended by Rowland (*Phil. Mag.*, Vol. XLVI., 1873, p. 140; and Vol. XLVIII., 1874, p. 321).

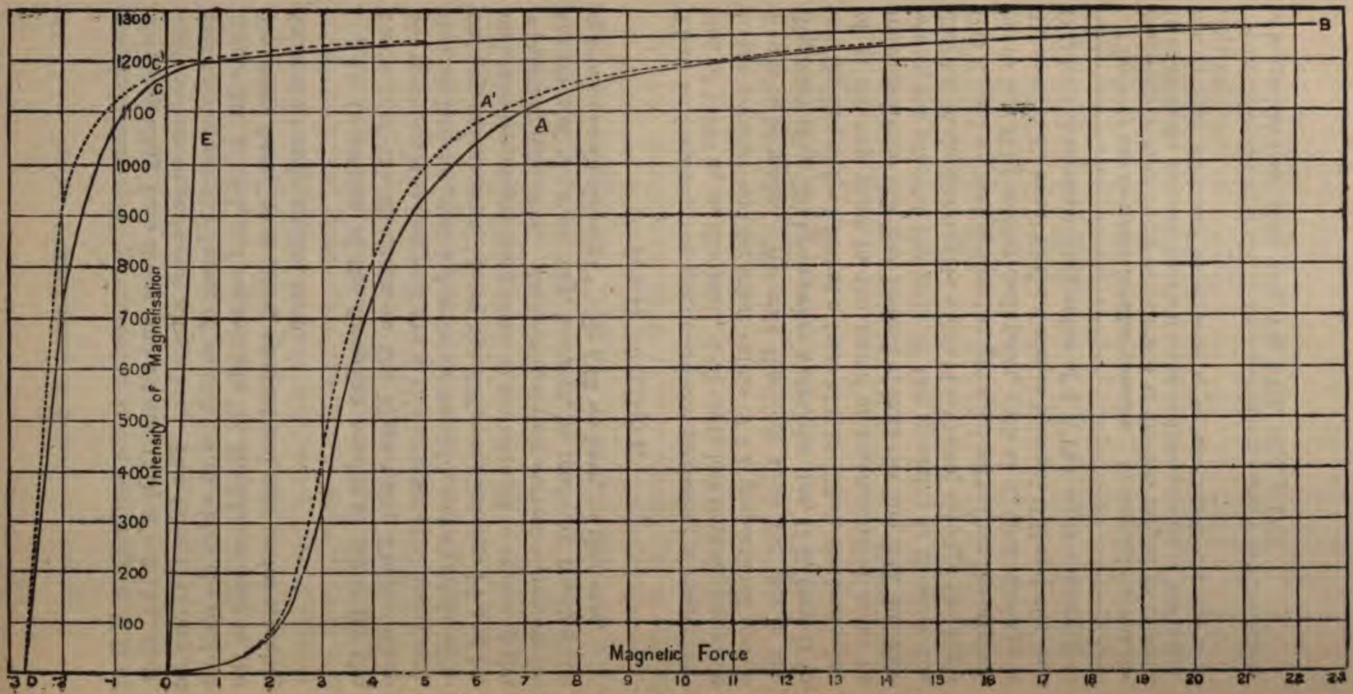

Fig. 20.—Curve of Magnetisation in Annealed Wrought Iron.

magnetic force quickly removes it, as the continuation C D of the descending limb shows, and a force, O D, of $-2·75$ C.-G.-S. units suffices to destroy it altogether. This force O D may be said to measure the degree of stability with which the residual magnetism is held, and accordingly Dr. Hopkinson calls it the "coercive force," thus giving an exact and very useful meaning to an old loosely applied term.

§ 48. **Correction of the foregoing results to allow for the reaction of the Specimen on the Magnetising Field.**—When the specimen is so long as it was in this instance (400 diameters), its ends have no great influence on the field, and we might without any serious error ignore the difference between H' and H, and take the magnetising force of the solenoid to represent the whole magnetic force. An approximately correct allowance for the effect of the ends may, however, be made by treating the specimen as an ellipsoid.* By § 33 we have, in that case,

$$\mathsf{H} = \mathsf{H}' - 0·00045\ \mathsf{I}.$$

Thus, for instance, the magnetic force producing a magnetisation of 1,000 is, on this basis, 0·45 units less than the force due to the solenoid, and for other values of I a proportional correction is to be made. We may therefore allow for this in the diagram† (Fig. 20) by drawing a straight line, O E, from O to cut the line of $\mathsf{I} = 1{,}000$ at a force of 0·45. Then the true value of H for any point in the curve of magnetisation is to be measured from this line, instead of from the line O C. In other words, the effect of the ends of the specimen is to *shear* the diagram in the direction in which H is drawn, through the angle C O E; and we may deduce the corrected curve from the original curve O A B by setting each point, such as A, back through a distance, A A′, equal to the distance between O E and the axis O C for the corresponding value of I. The same construction

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[*] Probably this correction is rather excessive. In a cylindrical rod the magnetism is more nearly concentrated at the ends than in an ellipsoid. Its effect on the magnetic force is unequal at different parts of the length, but its mean effect may be expected to be less than in the case of an ellipsoid.

[†] This construction is used by Lord Rayleigh in a Paper on "The Energy of Magnetised Iron" (*Phil. Mag.*, Vol. XXII., 1886, p. 175).

obviously applies to the descending limb. The dotted curves O A′ B and B C′ D have been drawn in this way, and they may be accepted as giving a more nearly correct representation of the relation of **I** to **H** than is given by the original curves.

One effect of this change is to make the measured susceptibility greater: its maximum value (which is found by drawing a tangent from O to the curve) rises from 189 to 209. Another effect is to increase the residual magnetism, making it 93·8 per cent. of the induced magnetism instead of 92·1 per cent. Another effect is to increase the steepness of the gradient in the steep part both of the ascending and of the descending limb.

Table II. gives the results of the same experiment (for the ascending process) on the above supposition that the correction of **H** to be made on account of the ends of the specimen does not sensibly differ from the correction which would have to be made in an ellipsoid 400 (equatorial) diameters long. Values of **B**, μ and κ are given as well as **I** and **H**. **B**, which is $4\pi\mathbf{I}+\mathbf{H}$, is so nearly equal to 4π **I** that we might adapt the curve of Fig. 20 to exhibit the relation of **B**, instead of **I**, to **H**, by simply altering the scale of the ordinates, so that 100 of **I** should represent 1257 of **B**.

TABLE II.

H	**I**	$\kappa=\frac{\mathbf{I}}{\mathbf{H}}$	**B**	$\mu=\frac{\mathbf{B}}{\mathbf{H}}$
0	0	—	0	—
0·32	3	9	40	120
0·84	13	15	170	200
1·37	33	24	420	310
2·14	93	43	1,170	550
2·67	295	110	3,710	1,390
3·24	581	179	7,300	2,250
3·89	793	204	9,970	2,560
4·50	926	206	11,640	2,590
5·17	1,009	195	12,680	2,450
6·20	1,086	175	13,640	2,200
7·94	1,155	145	14,510	1,830
9·79	1,192	122	14,980	1,530
11·57	1,212	105	15,230	1,320
15·06	1,238	82	15,570	1,030
19·76	1,255	64	15,780	800
21·70	1,262	58	15,870	730

§ 49. Differential Susceptibility and Differential Permeability. In many cases we are less concerned to know the actual ratio of I or of B to H than to know the rate at which I or B is increasing or diminishing with respect to H—in other words, the gradient of the magnetisation curve. We have seen that the gradient $\frac{d\mathsf{I}}{d\mathsf{H}}$ begins by being small, then becomes very large, and then again becomes small as the region of saturation is approached. Prof. Knott has proposed to call this quantity the "differential susceptibility"; similarly $\frac{d\mathsf{B}}{d\mathsf{H}}$ may be called the differential permeability. In the example which has been quoted, the differential susceptibility (after applying the ellipsoidal correction) has a maximum value in the ascending limb of 530, which is sensibly constant, while I changes from, say, 150 to 650. The corresponding differential permeability is 6,660. In the descending limb the greatest differential susceptibility is 1,660, and this is sensibly constant, while I changes from 700 to 0 (and to -700, as we shall see later in other examples). The corresponding differential permeability is 20,850.

§ 50. Supplementary Remarks on the Magnetometric Method. In testing the magnetisation of soft iron, especially if the specimen be at all thick—it will be found necessary to pause after each increase in the magnetising current, and to keep the current constant for some seconds, or even minutes, before the iron takes its full magnetisation. The "creeping up" of the magnetometer deflection which takes place after each increase of magnetising force will be spoken of more fully in a later chapter.

If the specimen is placed rather near the magnetometer and the deflection threatens to become greater than the scale will measure, the magnetometer needle may be brought back towards its undeflected position by using a permanent magnet (a hard steel wire will do well) to counterbalance a part of the deflecting force exerted by the specimen under examination. This compensating magnet must be placed so that it exerts no force at the magnetometer except in the direction exactly opposite to that of the force the specimen exerts. In other words, it must exert deflecting force, not directing force (§ 39), and to secure this it should be placed before or behind the magnetometer (pointing towards it) in the line B A C of Fig. 18. When the compensating magnet is introduced, the number of scale divisions through which it causes the needle to return is to be noted, and this is to be added to subsequent scale readings in reckoning the virtual deflection. The compensating magnet is specially convenient if we wish to examine the effect of applying or removing a small amount of magnetic force when the specimen is already somewhat strongly magnetised.

Fig. 21.

A useful method of getting increased sensibility is to use a compensating coil (§ 41) to balance a part (or even the whole) of the deflection produced by the magnetisation of the specimen itself. Suppose, for instance, that it is desired to examine particularly the form of the magnetisation curve under moderately weak magnetic forces—say the part O A B of the curve (Fig. 21)—we may set the specimen near the magnetometer, and at the same time advance the compensating coil so that it counterbalances a large part of the deflection. Thus, let P M be the part of the deflection balanced by the coil, when the magnetising current is O M: the whole virtual deflection is got by adding this to P A, which is the observed deflection. For any stronger or weaker current the part balanced is represented by the corresponding ordinate of the straight line

O P Q. The slope of this line is found by observing the deflection given by the coil and solenoid when the specimen is taken out and a known current is applied. When the specimen is put in and the process of magnetisation is gone through, the actual deflections of the magnetometer are, of course, limited to the (positive or negative) portions of the ordinates intercepted between the straight line O Q and the curve. By adjusting the position of the coil so that O Q is nowhere far from the curve a high degree of sensibility becomes possible, for the whole range of the magnetometer scale may then be used in exhibiting the differences P A, Q B, &c. This method should be specially serviceable in dealing with ellipsoidal specimens of moderate length.

CHAPTER III.

MEASUREMENTS OF MAGNETIC QUALITY: THE BALLISTIC METHOD.

§ 51. **The Ballistic Method.**—The ballistic method, briefly alluded to in § 38, was invented by Weber, was used by Thalen, Stoletow, Rowland and others, and received its name from Sir William Thomson.* It determines any sudden change of magnetic induction by measuring the quantity of electricity in the transient current which is induced in a coil wound over the magnetised piece. Let a coil, which for brevity we may call the secondary coil, be wound on the bar or ring or other specimen which is to be magnetised. The coil need not extend over the whole length of the specimen, and in the case of a bar a short coil is best, wound over the central part only, where the magnetisation is most nearly uniform. The coil is to be put in circuit with a galvanometer, the needle of which has a considerable moment of inertia (in relation to the directive force acting on it), so that it swings slowly. An ordinary mirror galvanometer is easily adapted to serve as a ballistic galvanometer by fixing a small weight to the mirror. If the specimen is wound with a magnetising solenoid sudden changes of its magnetism may be produced by applying a magnetising current, by increasing it by steps, by reversing it, and so on; and each of these will cause a "throw," or impulsive deflection of the ballistic needle, which will be proportional to the whole change of magnetic induction within the secondary coil. The "throw" is proportional to the whole quantity of electricity which passes in the transient current, and this in its turn is proportional to the change of magnetic induction within the

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* *Phil. Trans.*, Vol. CLXVI., p. 693.

coil. Let Q be the total number of lines of magnetic induction within the secondary coil, and ΔQ any sudden change which this number undergoes. Let N_2 be the number of turns in the secondary coil, and R_2 the whole resistance of the secondary circuit (in ohms), including, of course, the resistance of the ballistic galvanometer. Then the whole quantity of electricity in the corresponding transient current is

$$\frac{N_2 \Delta Q}{R_2}.$$

The observed throw of the galvanometer measures this, and the simplest way to calculate ΔQ from it is to compare this throw with that which occurs when the number of lines of induction within a coil in the secondary circuit is changed by a known amount. In other words, we may most conveniently standardise the ballistic galvanometer by finding what throw a known change of induction causes.

§ 52. **Earth Coil.**—Suppose, for instance, that there is included in the secondary circuit another coil, consisting of a number of turns of wire wound on a pretty large frame, and that this is laid flat on a horizontal table, so that it may be suddenly turned over. By turning it over we cause the direction of the vertical component of the earth's magnetic force in it to be reversed, and thus induce a throw of the ballistic galvanometer due to a known change in the number of lines of induction within the circuit, from which it is easy to interpret the throws that are produced by changes in the magnetism of the specimen.

This "earth coil," as it may for brevity be called, was first used in magnetic researches by Rowland.* Instead of lying horizontally it may stand vertically, facing towards the magnetic north and south, so that when turned over it will cut the horizontal component of the terrestrial field, or it may be set at right angles to the dip, so that the whole terrestrial field acts upon it. The horizontal or the vertical position is, however, more convenient. For the former a light wooden frame lying on the table answers well. Fig. 22 is engraved from a photo-

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* *Phil. Mag.*, Vol. XLVI., 1873.

graph of an earth coil, which the writer has found serviceable; the coil is wound on a large brass ring mounted on trunnions and furnished with projecting stops, which strike against the post P and allow the coil to turn just 180 degrees. When the post P is standing up, as in the figure, the coil lies horizontally, and the vertical component of the earth's force is then the active field; but the post can fold down by means of a hinge so that the coil stands vertically, and it may then be set to cut the horizontal component.

Let N_1 be the number of turns in the earth coil, A_1 its area in square centimetres, and F the (known) value of that component of terrestrial magnetic field which acts upon it. A sudden turning over of the coil changes the number of lines within it by the amount 2 A_1 F, and the whole quantity of the transient current is

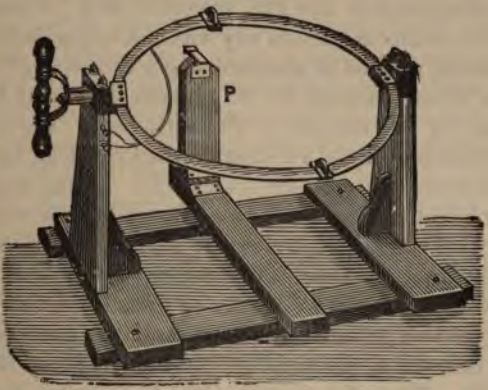

FIG. 22.—Earth Coil for use in Ballistic Measurements.

$$\frac{2 N_1 A_1 F}{R_1},$$

R_1 being the resistance of the secondary circuit at the time when the observation with the earth coil is made. It is convenient and generally quite practicable to keep the earth coil continuously in the secondary circuit, in which case $R_1 = R_2$. Let d_1 be the ballistic throw produced by the earth coil, and let

d_2 be the throw produced by the magnetic change ΔQ, which we wish to evaluate:

then, $$\frac{N_2 \Delta Q}{R_2} : d_2 :: \frac{2 N_1 A_1 F}{R_1} : d_1,$$

from which $$\Delta Q = \frac{2 N_1 A_1 F R_2 d_2}{N_2 R_1 d_1}.$$

§ 53. Use of a Solenoid and Current for Standardising the Ballistic Galvanometer.—To use the earth coil successfully we must know with sufficient accuracy the horizontal or the vertical component of the local magnetic field. These are apt to vary in a rather capricious way within a magnetic laboratory. The following method of standardising a ballistic galvanometer (due, the writer believes, to Sir William Thomson) is a good substitute for the method of the earth coil. The results it gives are independent of variation in the local field, but depend on the absolute measurement of a current. A long magnetising coil is to be uniformly wound on a brass tube or a wooden rod, or some other non-magnetic core, the diameter of which must be accurately known. Over this primary, at the middle of its length, a short secondary coil is to be wound, and put in circuit with the ballistic galvanometer. Let A_3 be the mean area of the primary coil, and n_3 the number of turns in it per centimetre of the length. Then, if a current, C (C.-G.-S. units), be made to pass through it, the magnetic force (or induction) within it (at any place near the middle) is $4 \pi C n_3$ per square centimetre, and the whole number of lines of force (or induction) which the current introduces within the coil is $4 \pi C n_3 A_3$. If N_4 is the whole number of turns in the secondary coil, and R_4 the resistance of its circuit, the quantity of electricity in the transient current that passes when the primary current C is made or broken is

$$\frac{4 \pi C n_3 A_3 N_4}{R_4}.$$

Let d_4 be the throw which this produces, then

$$\Delta Q = \frac{4 \pi C n_3 A_3 N_4 R_2 d_2}{N_2 R_4 d_4}.$$

Still another way of standardising the ballistic galvanometer is to discharge through it a known quantity of electricity, namely from a condenser of known capacity, charged to a known potential. This has no particular advantage over the methods already described, and it is less likely to be accurate in practice.

§ 54. Damping and Calibration of the Ballistic Galvanometer.—In some uses of the ballistic galvanometer it is important that there should be little "damping"—in other words, that the swinging of the needle should subside very slowly. But in magnetic observations of the kind now under description—when what we deal with is merely the comparison of different ballistic effects—this is not necessary: it is, in fact, desirable, as a matter of convenience, to have a good deal of damping, provided always there is not so much as to prevent the throws from being proportional to the changes of magnetic induction. To test whether this condition is satisfied, a series of successive currents of increasing strength should be made and broken in the primary coil of § 53, while the corresponding throws are observed and compared with the strength of the primary current, to see that the two vary together. Another plan is to have a small induction coil (in circuit with the ballistic galvanometer) slipped upon a long bar magnet. Pull the coil quickly off the magnet, and observe the throw; then reduce the number of turns in the coil by unwinding one or more of them, and observe the reduced throw when the coil is again pulled off, and so on, until the number of turns and the throw is greatly reduced. The observed throws should be proportional to the successive numbers of turns in the coil.

To save time between ballistic readings, it is convenient, especially when the damping is not very considerable, to follow Rowland's plan of including in the secondary circuit a small coil slipped upon a magnet.* By pulling it off or slipping it on at the right moment, while the needle is swinging, the observer may, with a little practice, succeed in giving the needle a check, which brings it quickly to rest. Care must, of course, be taken that this coil is not moved while observations are being made.

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* *Phil. Mag.*, Vol. XLVI., 1873, p. 147.

§ 55. Ballistic Tests of Rings and Rods.—Fig. 23 illustrates the ballistic method as applied to a magnetic ring. The ring A is wound all over with a primary or magnetising solenoid, the current in which is measured by G_1, and can be subjected to sudden variations by putting in or drawing out the plugs of the resistance-box B_1, or can be made, broken, or reversed by the key K. G_2 is the ballistic galvanometer, in circuit with a secondary or induction coil (wound over a part or the whole of the ring), with the resistance box B_2, by which the amount of the throws may be varied, the earth coil E and the small coil D

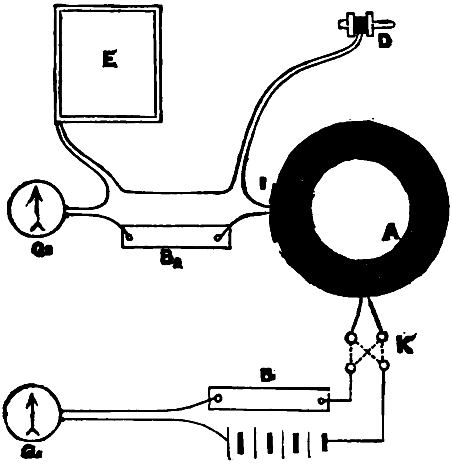

Fig. 23.—Diagram of Connections for Ballistic Method.

which is used to check the swinging of the needle. In addition to the parts shown, it is convenient to include in the primary circuit the arrangement of liquid slide and rapid reversing key for demagnetising by reversals, as explained in § 42. By this means we can ensure that the ring is in a magnetically neutral state to begin with.

To test the permeability and to determine the form of the magnetisation curve, one or other of two plans may be followed: (1) *By Steps.* A weak current may be applied and the throw noted, then the resistance at B_1 may be suddenly reduced, and the additional throw noted, and so on, each throw measuring the magnetic effect of a sudden increase in the magnetising current. The whole magnetism acquired at any stage is then to be estimated by summing up the throws. The same process evidently allows us to trace the individual and cumulative effects of successive diminutions in the strength of the magnetising current, and thus to trace the magnetisation curve throughout any step-by-step process of applying, removing, or reversing magnetising force. This is its advantage: on the other hand, it has the practical drawback that if an error happen to be made in measuring the throw at any step it is carried forward and affects all the subsequent values of the magnetisation. (2) *By Reversals.* Another plan is to suddenly reverse the current in the primary coil. Half the throw is then taken as measuring the actual magnetisation. Breaking the current also allows the permanent magnetism to be calculated by showing the amount that disappears in the withdrawal of the magnetic force. As to the effect of each reversal, care must be taken that the currents are progressively increased, and even then the assumption that half of that effect measures the total magnetism is not quite accurate, especially in the case of hard iron or steel, which is less ready to be magnetised by a force of one sign after a force of the opposite sign has been applied than if the opposite force had not acted. In soft iron the curve of magnetisation as determined by this process of reversals is not materially different from the curve determined by the process of steps.

In applying the ballistic method to long rods or ellipsoids, or other specimens with ends, either of these processes may, of course, be used, and in addition a third plan is practicable—namely, to have the secondary coil arranged so that it may be suddenly slipped off the magnetised piece. The effect of slipping it off is to reduce the lines of induction within it to zero, provided the coil be at once drawn far enough away to get practically out of the magnetic field, and the throw of the galvanometer therefore measures the whole magnetisation which existed just before the coil was removed. This method is often useful, but it must be borne in mind that the mechanical disturbances caused by pulling off the coil may, especially with soft iron, alter very seriously the amount of magnetism associated with any assigned value of the magnetic force. This will be evident later when reference is made to the effects of vibration on the magnetic susceptibility of iron. We cannot, therefore, use this plan to trace the effects of successive currents of ascending and descending strength. After the step-by-step process, however, has been applied to a long rod, slipping off the coil affords a useful test of the accuracy with which the summation of the steps has been carried out.

§ 56. **Calculation of B from Ballistic Measurements.**—We have seen how the ballistic measurements serve to determine Q, the whole number of lines of induction within the secondary coil. If the secondary coil is wound close upon the iron, very nearly the whole of these lines lie within the iron, and we then have $\mathbf{B} = \frac{Q}{S}$, where S is the area of cross-section of the iron in square centimetres. If, however, the area of the secondary coil includes any sensible air space (or other non-magnetic space) in addition to the iron, a suitable deduction must be made from Q before dividing by the area of cross-section of the iron to find **B**. Thus if the secondary coil is outside the primary, and the mean area of the primary coil is S′, we shall have $(S' - S)\,\mathbf{H}$ lines enclosed within the secondary coil, but outside of the iron, and this number will fall to be subtracted from Q. Even when the secondary coil is wound directly upon the iron, its mean area is necessarily somewhat greater than the section of the core, and there is consequently a small correction to be applied (namely, the difference of these areas multiplied by **H**), but the amount of this correction is generally insignificant.

§ 57. **Magnetic Force in Rings.**—Though the magnetising solenoid be uniformly wound over the whole ring, so that its effect at any one cross-section is the same as at any other, the magnetic force is not quite uniform throughout. It is strongest at the inner side—the shortest length—of the ring, and decreases towards the outer side in proportion as the radius of the ring increases. This is because the number of turns per centimetre is greatest at the inner side and least at the outer. Let N be the whole number of turns of the magnetising solenoid, the number per centimetre at any radius r is $\frac{N}{2\pi r}$ and the magnetic force is $\frac{4\pi C N}{2\pi r}$, or $\frac{2 C N}{r}$. Thus the magnetic force varies from $\frac{2 C N}{r_1}$ at the inside to $\frac{2 C N}{r_2}$ at the outside (Fig. 24). So far as it goes, this is a drawback to the use of ring-shaped specimens.

To prevent this objection from having much weight the thickness of the ring should be small compared with its radius. The form shown in Fig. 25 allows a small ring to be used without excessive variation of magnetic force over the section, and without unduly reducing the sectional area.

In dealing with weak magnetic forces it is desirable to place the ring in such a position that the earth's magnetic field does not affect the uniformity of its magnetisation, namely, in the plane perpendicular to the direction of the lines of terrestrial force. For the sake of homogeneity in the metal itself, a ring turned out of a solid block is to be preferred to one that is forged from a bar.


FIG. 24.

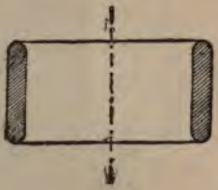
FIG. 25.

§ 58. **Bar and Yoke.**—The condition of endlessness, which is realised perfectly in a ring of uniform section, uniformly wound (or, rather, is realised as perfectly as the imperfect homogeneity of the metal will allow), can be approximated to, even when the sample has the form of a short bar, by a very interesting and useful method, invented by Hopkinson.* Let the ends of the bar be sunk in holes in a massive yoke (Fig. 26) which has an area of cross-section many times greater than that of the bar, and is made of the most permeable

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* "Magnetisation of Iron," *Phil. Trans.*, 1885, p. 455.

material available, namely, soft annealed wrought iron. The yoke is so much better a conductor of lines of magnetic induction that the lines which proceed from either end of the bar nearly all pass back through it to the other end, instead of escaping into surrounding space. This closing of the magnetic circuit through the yoke prevents the bar from exercising almost any self-demagnetising force upon itself; and if the bar is wound with a magnetising solenoid throughout its whole clear length within the yoke,* the magnetic force actually operative on it is only a very little less than the whole force due to the solenoid.

The amount of the difference will be more easily discussed when we come, later, to speak of the magnetic circuit as a whole, and the relation of the induction in it to the whole number of ampere-turns in the magnetising coil. Meanwhile, it may suffice to say that the magnetism of the bar and yoke reacts to a small extent on the magnetic force, reducing it by amounts which are proportional, or nearly so, to the magnetisation. The effect is like that which has already been described as occurring in a long rod or long ellipsoid, not so extremely long as to be virtually endless, and the curve of magnetisation is consequently sheared over (§ 48): the apparent susceptibility and the residual magnetism are reduced. This makes the method of the yoke unsuitable for accurate determination of the susceptibility and retentiveness of a very susceptible metal,

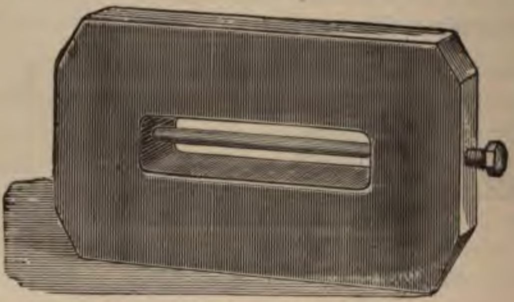

FIG. 26.—Yoke for the Ballistic Tests of Bars.

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* In the figure the bar is shown in its place, but the magnetising solenoid and the induction coil are omitted.

like soft wrought iron; but there is no serious error in the case of hard iron or steel.

The ends of the bar should be sunk for a considerable distance into the yoke, and should fit in the holes without shake, for if there is any appreciable clearance its effect in producing self-demagnetising force may be considerable.

§ 59. Hopkinson's Application of the Bar and Yoke.—In applying the ballistic method to a bar in a yoke we may, of course, proceed by steps or by reversals, as with a ring. In Hopkinson's original use of the yoke, however, a different procedure was followed. The bar was made in two parts, C and C′ (Fig. 27), which abutted against one another near the middle, where the secondary coil D was slipped on, in a space between two halves of the magnetising solenoid B B. A clutch fixed on the projecting end of the rod C enabled it to be suddenly drawn away from C′ sufficiently to allow D (which was pulled sideways by a spring) to leap out of the field. This gave a ballistic throw which measured the actual magnetic state of the bar at the moment when C was drawn out. The plane of section between C and C′ is a rather objectionable feature in this arrangement; for, as will be shown later, its influence on the general permeability of the bar is by no means immaterial, even when the abutting surfaces are accurately faced.

FIG. 27.

§ 60. Double Bars and Yokes.—Fig. 28 illustrates an arrangement which will serve well when two equal bars of the material to be tested are available. The bars should be of considerable length—twenty or more times the diameter, and the yokes should be short and thick. Equal magnetising solenoids are wound over the two bars, and are connected to give opposite directions of magnetisation. The secondary coil is preferably distributed over the middle region of both bars. If it is wished to measure the actual magnetic state at any time, one of the yokes may be arranged so that on pulling it away it brings the secondary coils with it.

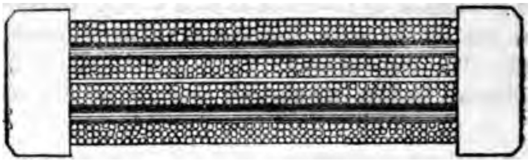

FIG. 28.

§ 61. **Example of the Ballistic Method.**—In the following example* the specimen was a ring welded out of a piece of moderately soft annealed iron wire :—

Diameter of wire forming the ring......	0·248 cms.
Area of section of iron......................	=0·0483 sq. cms.
Mean radius of ring 5·0cms.	Mean circumference 31·4cms.
Number of turns in magnetising coil...	474
Number of turns in secondary coil......	167
Area of earth-coil	1216 sq. cms.
Number of turns in earth-coil............	10
Earth's force, cut by earth-coil	0·34
Ballistic throw on turning over earth-coil	42·9 scale divisions.

The mean value of the magnetic force per ampere was, therefore :—

$$\frac{4\pi \times 474}{10 \times 31{\cdot}4} \text{ or } \frac{2 \times 474}{10 \times 5{\cdot}0} = 18{\cdot}96.$$

The resistance of the secondary circuit was not altered throughout the experiment, and the correction for air-space within the secondary coil was negligible. Hence, from the above data, the change of the induction **B** per square cm. in

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* *Phil. Trans.*, 1885, pp. 530-532.

the iron corresponding to one scale division of ballistic throw was—

$$\frac{1216 \times 10 \times 0{\cdot}34 \times 2}{0{\cdot}0483 \times 167 \times 42{\cdot}9} = 23{\cdot}89.$$

The experiment consisted in applying first a weak magnetic force, and increasing it by a series of sudden steps to 9·14 C.-G.-S. units, and then removing and finally re-applying the same force, all by steps, while the ballistic throws were observed. The following Table (III.) gives the results, μ, **I**, and κ having been calculated from **B** and **H**.

Table III.—Annealed Wrought-Iron Ring.

H.	Ballistic Throw.	Sum of Throws.	**B.**	μ.	**I.**	κ.
0·13	1·1	1·1	26	...	2	...
0·26	1·1	2·2	53	...	4	...
0·30	0·5	2·7	65	...	5	...
0·40	0·8	3·5	84	...	7	...
0·53	1·0	4·5	107	...	9	...
0·71	2·1	6·6	158	...	12	...
0·93	2·9	9·5	227	...	18	...
1·31	3·9	13·4	320	245	25	19
1·69	9·2	22·6	540	320	43	25
1·89	6·9	29·5	705	370	56	30
2·78	77·5	107·0	2,560	920	203	73
3·36	78·7	185·7	4,440	1,320	353	105
4·01	82	267·7	6,400	1,600	509	127
4·95	91·5	359·2	8,580	1,740	683	138
5·86	57	416·2	9,940	1,700	791	135
7·20	57	473·2	11,300	1,570	899	125
8·10	23·5	496·7	11,870	1,460	944	116
9·14	24	520·7	12,440	1,360	989	108
7·83	− 4·4	516·3	12,330	...	981	...
6·21	− 6·7	509·6	12,170	...	968	...
4·75	− 7·1	502·5	12,000	...	955	...
2·70	−14·0	488·5	11,670	...	929	...
0	−33·2	455·3	10,880	...	866	...
2·78	15	470·3	11,240	...	894	...
4·95	14·2	484·5	11,570	...	921	...
6·21	11·9	496·4	11,860	...	943	...
8·00	14·5	510·9	12,170	...	971	...
9·14	10	520·9	12,440	...	990	...

The relation of B to H in this experiment is shown in Fig. 29. It will be seen that the curve of magnetisation presents the same characteristics as in the former example. The residual value of B is 88 per cent. of the induced value.

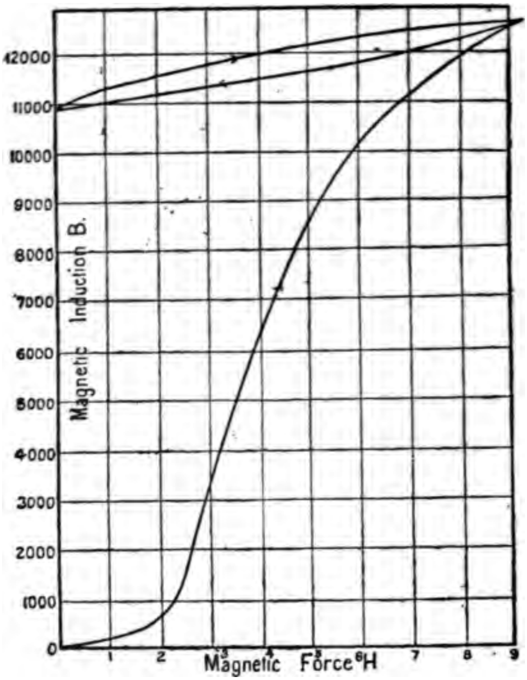

FIG. 29.—Wrought-Iron Ring.

CHAPTER IV.

EXAMPLES OF MAGNETISATION.

§ 62. Ballistic Method using Reversals: Magnetisation of an Iron Ring (Rowland).—A few more examples may be quoted in further illustration of the relation of magnetisation to magnetising force in iron.

In the following experiment by Rowland* the specimen was a welded and annealed ring of "Burden's Best" wrought iron, 6·77cm. in mean diameter and 0·916 sq. cm. in section. **B** was measured by reversing the magnetising current and taking half the ballistic effect. The ballistic effect of breaking the magnetising current was also noted. This, subtracted from half the effect of reversal, gave the residual magnetism at each stage in the magnetising process. The results, reduced to C.-G.-S. units, are given in Table IV.,† where the residual values of the magnetic induction appear in the third column under the heading B_r.

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* *Phil. Mag.*, Vol. XLVI., 1873, p. 151.

† The dimensions of **H** and of **B** and of **I** are $\dfrac{(\text{Mass})^{\frac{1}{2}}}{(\text{Length})^{\frac{1}{2}}\,(\text{Time})}$. Rowland, in the paper cited, uses *metre*-gramme-second units in expressing the induction. His numbers (called Q in the Paper) have therefore to be divided by 10 to bring them to C.-G.-S. units. With regard to **H**, he gives (under the heading M) numbers which are equal to the magnetic force divided by 4π. These have accordingly to be multiplied by $\frac{4\pi}{10}$ to reduce them to C.-G.-S. units of **H**.

TABLE IV.—ANNEALED WROUGHT-IRON RING.

H	B	B_r	μ
·18	71	18	390
·69	600	211	869
·86	967	439	1,129
1·27	2,460	1,570	1,936
1·41	2,920	1,940	2,078
1·45	3,080	2,060	2.124
2·04	4,960	3,630	2,433
2·22	5,480	3.810	2,470
2·34	5.780	4,010	2,472
2·72	6,650	4,750	2,448
3·16	7,470	5,430	2,367
4·05	8,940	6.270	2,208
5·31	10,080	6,840	1,899
8·48	12,270	7,500	1,448
10·23	12,970	7,670	1,269
11·99	13,630	7,520	1,137
17·69	14.540	7,990	824
34·17	15,770	8,130	462
46·02	16,270	7,850	354
64·33	16,600	7,890	258

Fig. 30 shows these values of B and B_r in relation to H, for forces up to 10 C.-G.-S. Beyond that force, the residual magnetism becomes very nearly constant. The proportion of residual to induced magnetism is considerably smaller here than in experiments with long wires or with wires welded into rings. Probably this is due less to any specific difference in the material than to a difference in the conditions of the experiment. It was shown long ago by Von Waltenhofen that when magnetic force is suddenly removed from an iron rod it leaves less residual magnetism than when gradually removed.* This is notably the case when the specimen is comparatively thick. In a thick rod or ring the sudden withdrawal of magnetic force sets up oscillating circumferential currents in the substance of the metal, which have an effect not unlike that which is produced in the process of "demagnetising by reversals" (§ 42). With very long wires or rings of small section one commonly finds 80 or 90 per cent. of the induced mag-

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* Pogg. *Ann.*, CXX., 1863. *See also* Wiedemann's *Elektricität*, Vol. IV.

netism survive the removal of magnetising force, especially when the force is reduced by small steps or quite continuously. In the present case the force was removed suddenly, and the ring was thick.

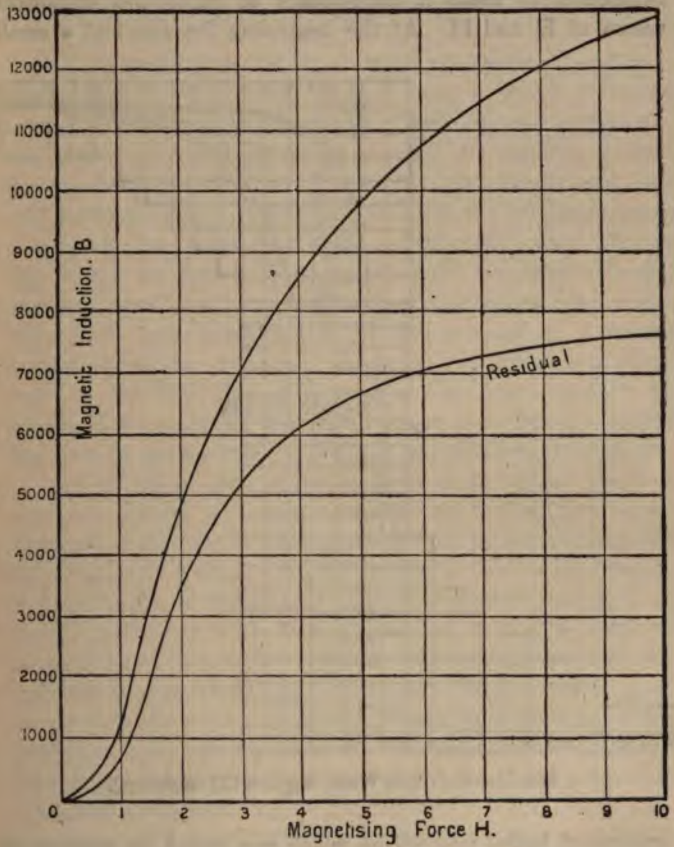

Fig. 30.—Wrought-Iron Ring (Rowland).

§ 63. Cyclic Process of Magnetisation : Long Iron Wire.— In this instance the specimen—a straight wire of very soft annealed iron, 0·158 cms. in diameter and 64 cms., or 400 diameters, long—was tested by the step-by-step ballistic method,* the magnetic force being first raised from zero to 17·26 units, then reversed to −17·26, then reversed again to +17·26, then reduced to zero, and finally restored to +17·26. The effects of these cyclic processes are exhibited in Fig. 31 sufficiently to make it unnecessary to quote the numerical values of **B** and **H**. At the beginning the wire had a small amount of initial magnetism, which was found by slipping off the secondary coil.

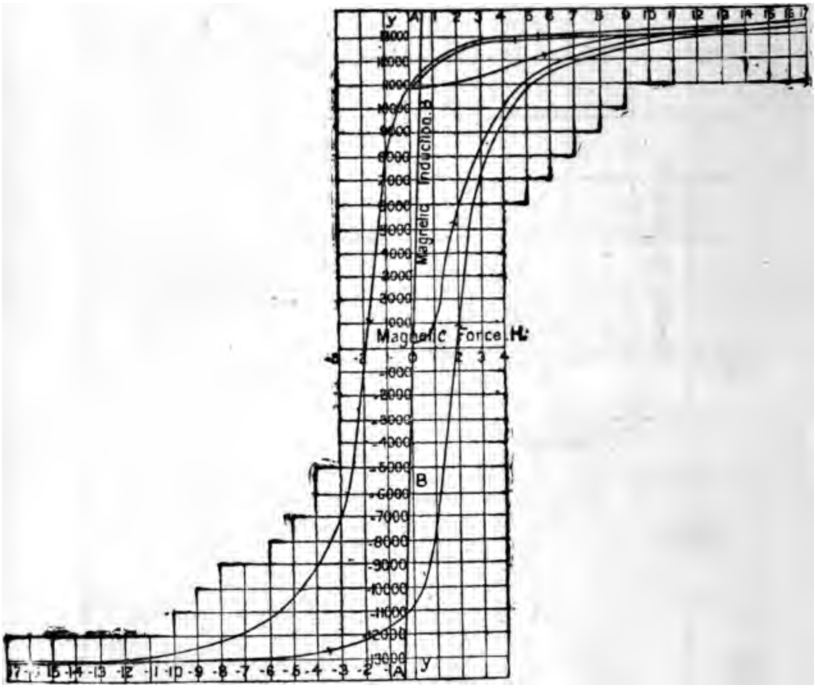

FIG. 31.—Soft Iron Wire (length = 400 diameters).

In this figure the magnetising force of the solenoid is accepted as the whole magnetic force **H**, no allowance having been made for the influence of the ends. If we treat the rod

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* *Phil. Trans.*, 1885, p. 539.

as equivalent to an ellipsoid 400 diameters long, **H** is to be measured from the line O A instead of from O Y.*

On both sides of the figure the residual magnetism is 82 per cent. of the induced, and the reversed magnetic force required to remove it—the "Coercive Force," § 47—is 1·9. This figure is thoroughly typical of the behaviour of soft wrought iron when subjected to cyclic reversals of magnetic force.

§ 64. **Magnetisation of Iron Rods of Various Lengths.**— The curves of Fig. 32 are selected from a group of experiments† in which an annealed wire of soft wrought iron, originally 300 diameters long, was tested by the ballistic method, first in its full length and then after the length had been reduced successively to 200, 150, 100, 75, and 50 diameters by cutting off equal portions (in each case) from the ends. The central part of the length, through which the magnetic induction was measured, remained unchanged throughout the series. After each magnetisation the rod was reduced to a neutral state not by the process of reversals, but by taking advantage of the fact that a soft iron wire loses sensibly the whole of its residual magnetism when it is briskly tapped. Soft iron is extraordinarily sensitive to the effect of vibration; to tap it when the magnetic force is in action increases the permeability very greatly, and to tap it when the force is removed does away almost completely with its retentiveness. So sensitive is it that when the magnetic force is removed the lightest touch of the fingers suffices to destroy much of the residue, and after brisk tapping only one or two per cent. will in some cases be found to remain. The residual magnetism of soft iron is in fact very insecurely held. So long as the metal is left perfectly at rest it does not appear to suffer loss through the mere lapse of time; but any variation of temperature, or mechanical disturbance of whatever kind, reduces it with remarkable rapidity.

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* By the table in § 33, the value of $\frac{N}{4\pi}$ for an ellipsoid 400 diameters long is 0·000037. Hence the line O A is drawn at such an inclination as to make the reaction of the magnetism upon the field equal to a force of 0·37 when **B** is 10,000.

† *Phil. Trans.*, 1885, p. 535.

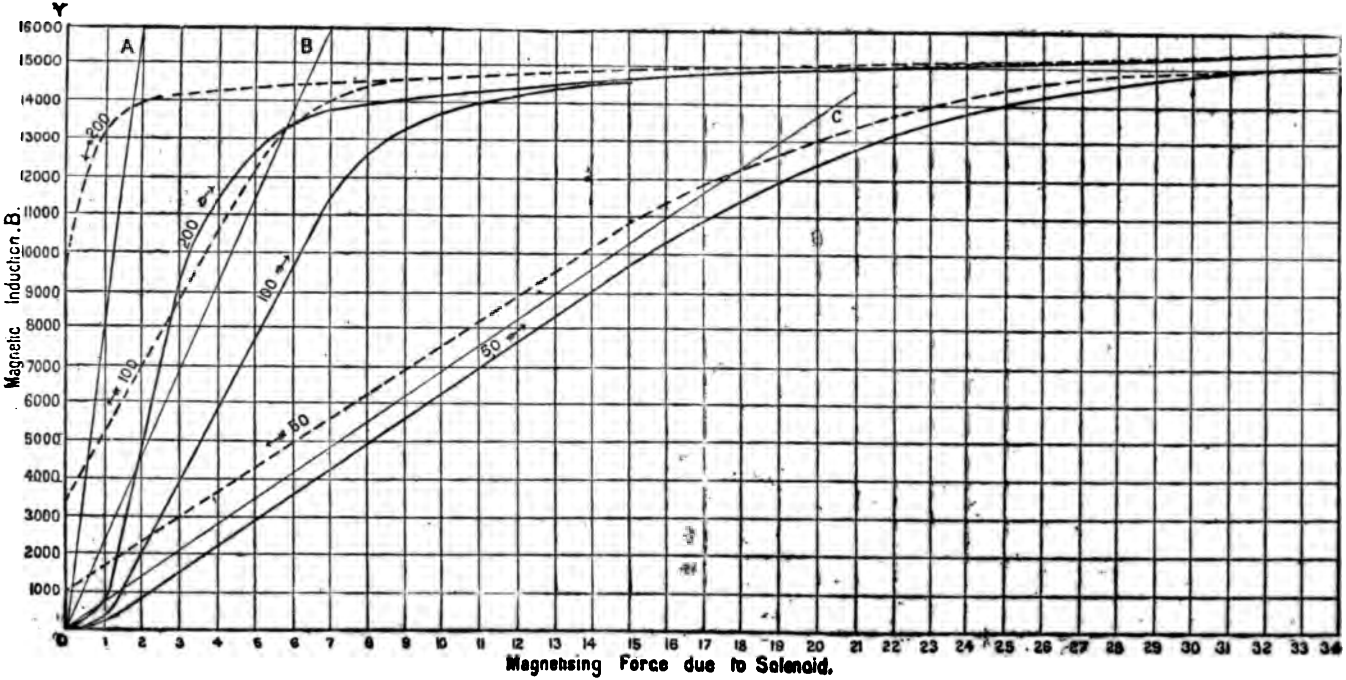

Fig. 32.—Magnetisation of Soft Iron Rods of various lengths.

The three curves selected for reproduction in Fig. 32 refer to the cases where the length was 200, 100, and 50 times the diameter respectively. The abscissæ give the magnetising force exerted by the solenoid, not the true **H**, as affected by the ends of the specimen. To give some idea of the true **H**, the lines O A, O B, and O C have been drawn; these show the reactions which ellipsoids of 200, 100, and 50 diameters respectively would exert. By measuring the magnetic force from them instead of from O Y, we get an approximation to the true value of **H**. The approximation is a very fairly correct one for the rods of 200 and 100 diameters; the curves for them, when rectified by taking abscissæ from O A and O B respectively, agree well with one another, and with curves for longer rods or rings of the same material. The diagram shows well what has already been sufficiently explained—how it is that soft iron shows little retentiveness when tested in the form of a short rod, though it shows much when tested as a long rod or as a ring. The broken lines show the gradual reduction which the magnetism suffered as the magnetising force of the solenoid was reduced to zero. By producing them past the axis O Y to cut O X produced we find that the "coercive force" of the material was 1·9, as in the experiment of Fig. 31, § 63, which dealt with another specimen of the same annealed iron wire.*

§ 65. **Wrought-Iron Bar.**—Fig. 33 is copied from a Paper by Hopkinson,† and refers to a ballistic test of annealed wrought iron by the method of the yoke (§ 59). The magnetic force was raised to 240, then reversed and re-reversed; but in the figure the negative magnetisation and the parts relating to high forces are omitted. A comparison of this figure with those that have been already given suggests that the condition of endlessness was imperfectly realised (in great part, no doubt, through the action of the plane of section referred to in § 59), and that the curves might be approximately rectified by measuring **H**

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* In further illustration of the effects of length in the magnetisation of rods, see a Paper by A. Tanakadate, *Phil. Mag.*, November, 1888, where experiments are described dealing with a series of rods shorter than those referred to in the text.

† *Phil. Trans.*, 1885, plate XLVII.

from a line such as O A (which has been added in copying the figure). This would make the bar within the yoke equivalent as regards endlessness to a bar (with free ends) about 150 diameters long. The coercive force in this sample has a value almost identical with the value found in soft iron wire, which strengthens the view that it is to imperfect endlessness, rather than to any specific difference in the quality of the iron, that one is to ascribe the comparatively small retentiveness of this bar.

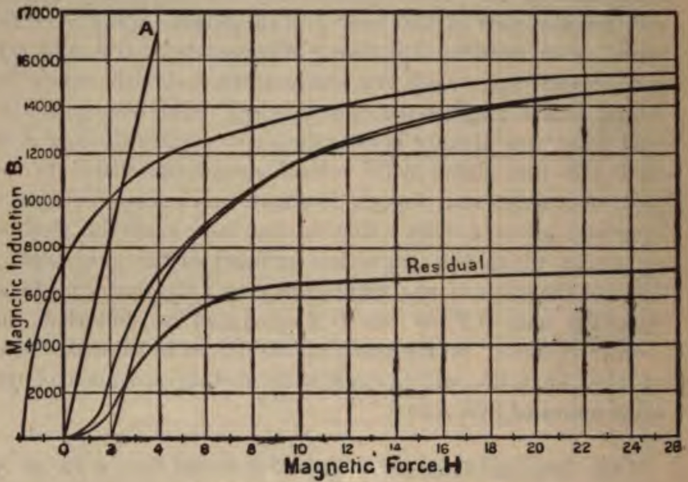

FIG. 33.—Wrought-iron Bar in yoke.

§ 66. **Magnetisation of Mechanically Hardened Iron.**—In all the examples which have been given above the iron was annealed or softened by heating to redness. Iron which has been mechanically hardened—by hammering, rolling, wire-drawing, or straining in any way beyond the limit of elasticity—shows much less permeability and susceptibility, much less residual magnetism (when tested in the form of an endless specimen), and considerably more coercive force. Thus the retentiveness of hardened iron, when in the form of a very long rod or a ring, is less than that of soft iron; but a short rod holds more residual magnetism when hard than when soft, on account of its greater coercive force.

These differences combine to give the curves of magnetisation of hardened iron a roundness of outline by which they are readily distinguished from those of soft iron.

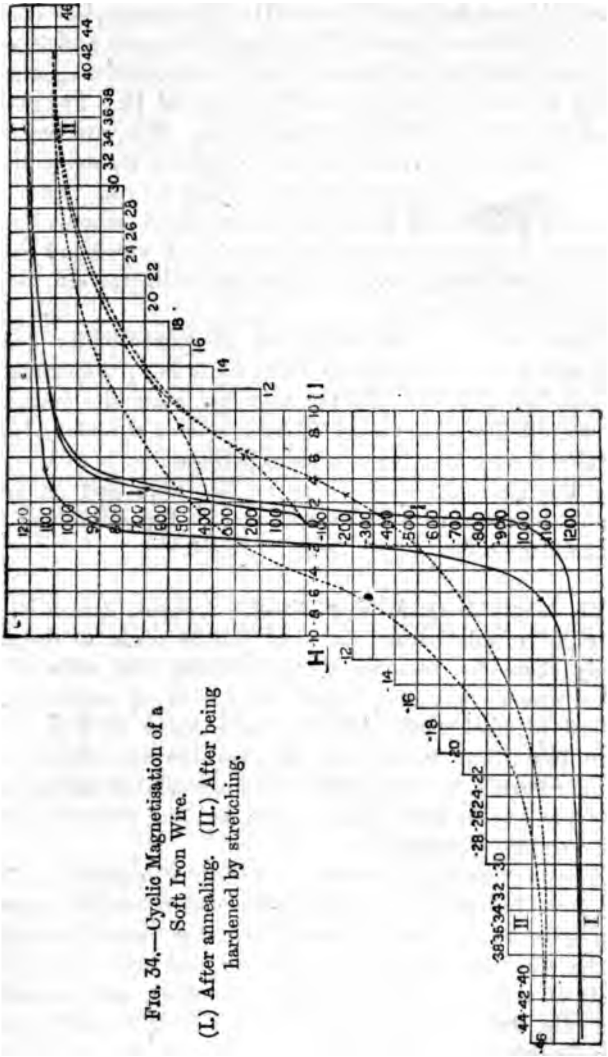

FIG. 34.—Cyclic Magnetisation of a Soft Iron Wire.

(I.) After annealing. (II.) After being hardened by stretching.

The altered characteristics of the curves are well seen in Fig. 34, which shows the results of two experiments made (by the one-pole magnetometric method) on the same piece of iron wire.* In the first the wire (0·158 cm. in diameter and 60 cms. long) was annealed, and was subjected to a cyclic magnetising process between the limits +46 and −46 of **H**. The results are shown by the full lines of the figure. The wire was then demagnetised by reversals, and was hardened by stretching it through about 10 per cent. of its original length. After the stretching weight had been removed, a cyclic process of magnetisation was gone through, the results of which are shown by the dotted lines. In this figure the ordinates are the intensity of magnetism **I**.

In the soft state, the maximum of susceptibility occurs early, at a force of 2·6, and its value (κ) is 245; the maximum permeability is 3,080. In the stretched state the maximum of susceptibility occurs much later, at a force of about 11, and its value is only 53: the maximum permeability is 670.

In the stretched state there is less than half as much residual magnetism as in its soft state. But stretching has increased the coercive force from 1·7 to 4·5.

§ 67. **Magnetic Qualities of Steel.**—Speaking generally, the curves of magnetisation for steel can be made to resemble closely those for iron by simply altering the scale of **H**. Under strong magnetic forces the region of saturation is reached in steel with much the same value of **I** or of **B** as in iron; but to reach it requires the application of a stronger force. At every stage the susceptibility and permeability are less in steel than in iron, and the coercive force is correspondingly greater.

The name "steel" covers as large a variety of magnetic qualities as it does of mechanical. Beyond those differences which result from difference in chemical composition, the range is extended by the effects of mechanical treatment, and above all by the effects of annealing, hardening by quenching, and tempering. As a rule, steel which is mechanically soft or "mild" is magnetically soft—in other words, its permeability is comparatively

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* *Phil. Trans.*, 1885, p. 547.

high, and its coercive force is low; and steel which is mechanically hard is magnetically hard. Thus, if we compare samples differing in their percentage of carbon, we find, in general, corresponding differences of magnetic hardness; the harder samples—that is to say, those with more carbon—are less susceptible and have more coercive force. Again, as regards the effects of temper, specimens which have been hardened by quenching from a red heat are magnetically much harder than specimens of the same composition which have been annealed.*

Other constituents than carbon affect the magnetic quality, often very greatly. Chromium and tungsten increase the coercive force immensely; and tungsten, in particular, is a usual constituent in magnet steel. In soft iron, as we have seen, the coercive force is about 2, or sometimes even less. In chrome steel, hardened by quenching in oil, it is 40, and in tungsten steel it may exceed 50.† These numbers are taken from a Paper by Hopkinson, which contains the most important data at present available regarding the magnetic qualities of different steels. The value of his results is much enhanced by the fact that a chemical analysis of each sample is given. We shall have occasion to recur to them later: meanwhile, it may suffice to illustrate the magnetisation of steel by a pair of examples taken from another source.‡

§ 68. **Magnetisation of Pianoforte Steel Wire.**—Figs. 35 and 36 show the results of cyclic processes of magnetisation (with positive and negative magnetic forces ranging up to nearly 100) applied to two pieces of the same pianoforte steel wire—one (Fig. 35) softened by annealing; the other (Fig. 36) glass-hardened by quenching in water from a red heat. The coercive force in the latter is scarcely inferior to that of tungsten steel. The maximum permeability is only 118; in Fig. 35 it is 295.

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* The influence which differences of temper exert on magnetic retentiveness has been exhaustively examined by Barus and Strouhal. Their results are published as a Bulletin of the U.S. Geological Survey, No. 14, 1885.

† Hopkinson, *Phil. Trans.*, 1885, p. 463.

‡ *Phil. Trans.*, 1885, pp. 546–7.

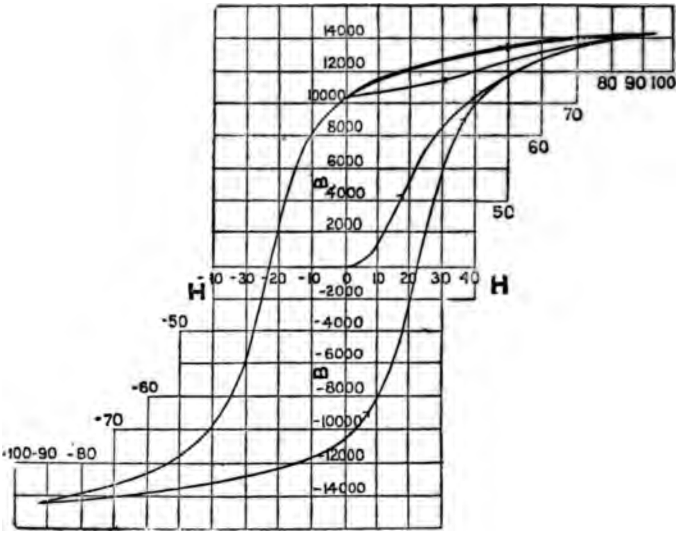

Fig. 35.—Pianoforte Steel Wire, annealed.

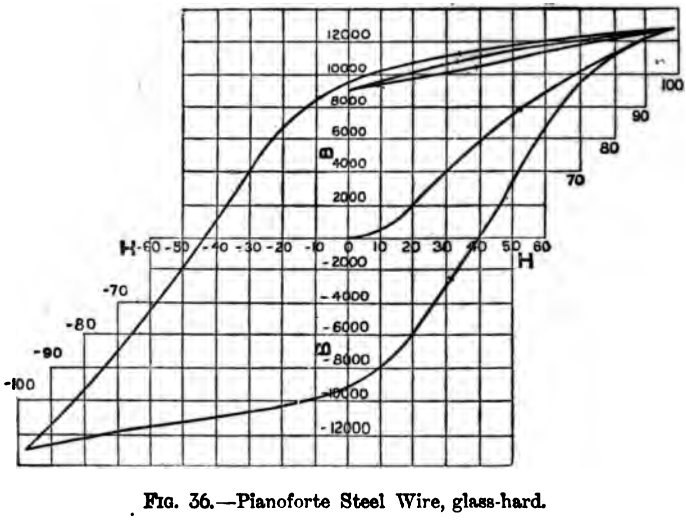

Fig. 36.—Pianoforte Steel Wire, glass-hard.

§ 69. **Cast Iron.**—Cast iron reaches a somewhat lower magnetisation than wrought iron or steel, even under strong forces. The intensity, when saturated, is about three-quarters that of wrought iron. In permeability under moderate magnetising forces, and in coercive force, it generally resembles mild steel. Fig. 37 (from Hopkinson) exhibits half of a cyclic process of magnetisation, for what is probably an exceptionally soft specimen of grey cast iron, in which the coercive force is barely double that of annealed wrought iron. The specimen was a short bar tested by the method of the yoke (§ 59).

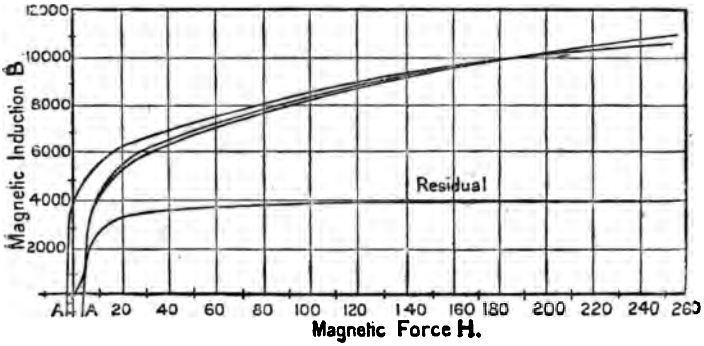

FIG. 37.—Cast Iron.

§ 70. **Non-Magnetic Steels.**—In certain alloys of iron there is a remarkable absence of magnetic quality. The presence of manganese in large quantities deprives the metal of nearly all its susceptibility. A notable instance occurs in the "manganese steel" of Mr. Hadfield, which contains about 12 per cent. of manganese and 1 per cent. of carbon. The permeability of this alloy is only about 1·3 or 1·5, and is sensibly constant in strong and weak magnetic fields. There is sensibly no residual magnetism, even after a very powerful magnetising force has been applied. A still more curious case is that of "nickel steel." Hopkinson* has found a specimen containing 25 per cent. of nickel to be practically non-magnetic under ordinary conditions of temperature, its

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* *Proc.* Roy. Soc., Dec. 12, 1889; May 1, 1890.

permeability being constant and equal to about 1·4. Here we have two materials, nickel and iron, each strongly magnetic, becoming non-magnetic when combined. What makes this alloy peculiarly interesting is the further fact that when cooled to a very low temperature, it becomes strongly magnetic, and remains so after the temperature is again allowed to rise to ordinary atmospheric values. The effects of temperature on magnetic susceptibility will form the subject of a later chapter.

§ 71. **Nickel.**—Fig. 38 gives curves showing the cyclic magnetisation of a long piece of nickel wire (0·068 cm. in diameter, and 25·4 cms. long) first in the annealed state (full lines) and next after being hardened by stretching beyond the limit of elasticity (dotted lines).* The curves give **I**, not **B**. They show that under strong forces the magnetisation reached by nickel is greatly inferior to that reached by wrought or cast iron or ordinary steels. (The saturation value of **I** in nickel is $\frac{1}{3}$ or $\frac{1}{4}$ the saturation value in wrought iron.) The following numbers refer to the experiment of Fig. 38 when the wire was in the soft state :—

Annealed Nickel Wire.

H	I	κ
0	22	
4·0	36	
6·5	83	12·8
8·0	177	22·1
9·5	223	23·5
10·9	251	23·0
12·3	273	22·2
24·6	325	13·2
52·6	371	7·1
79·7	392	4·9
100·4	401	4·0
0	284	
−7·5	0	

The last numbers in columns 1 and 2 of the table show the residual magnetism and coercive force. The greatest susceptibility ($\kappa = 23·5$) corresponds to $\mu = 283$. In the test with

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* *Phil. Trans.*, Vol. 179A, 1888, p. 327.

hardened wire the maximum susceptibility κ was only 8·3 ($\mu = 105$) and the coercive force was 18. The curves for annealed and mechanically hardened nickel differ in much the same way as the corresponding curves for annealed and hardened iron.

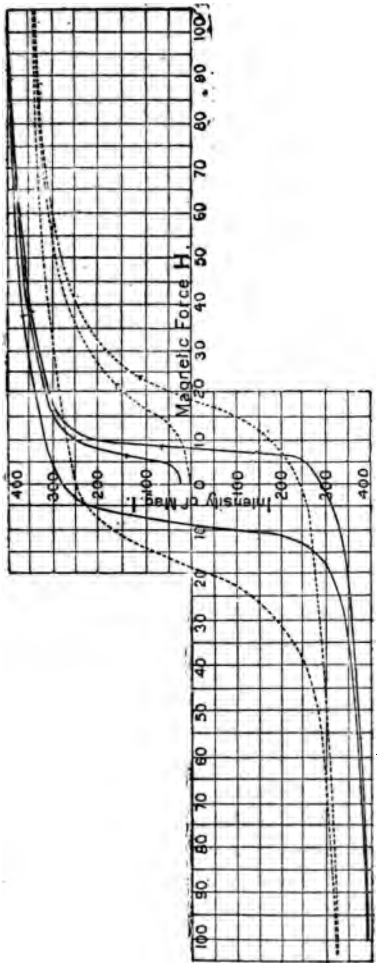

FIG. 38.—Nickel Wire.
Annealed——— Hardened by stretching............

Rowland,* using a ring of cast nickel, found a maximum susceptibility of 17·6 (permeability 222), and reached a value of **I** equal to 434 with a force **H** of 104.

§ 72. **Cobalt.**—Cobalt has decidedly more capacity for magnetisation than nickel. Under the action of a strong field it takes up about as much magnetism as cast iron; it has, however, comparatively little susceptibility when the magnetising force is weak.

Fig. 39 exhibits an experiment on the cyclic magnetisation of a cobalt rod (containing about 2 per cent. of iron), cast and turned, and tested, within a yoke, in the manner described in § 58, the magnetism at each stage being determined by summing the ballistic effects of successive steps. There was a small amount of initial magnetism, not removed when the experiment began. The greatest permeability was found when the force was about 25; its value is 174, which corresponds to a susceptibility of 13·8. Rowland, using a cast cobalt ring, found a maximum susceptibility of 11·2.

The curves for cobalt have a rounded outline recalling those for hardened iron. One effect of this is that the residual magnetism is comparatively small. The coercive force in Fig. 39 is 12.

§ 73. **Curves of Permeability and Susceptibility.**—The behaviour of magnetic metals during the imposition of magnetic force is sometimes exhibited graphically in other ways. Instead of drawing a curve to show the relation of **B** or of **I** to **H**, as has been done in the examples already given, we may draw a curve showing the relation of κ or of μ to **H**. This method of representing the results of experiment was used by Stoletow.† Another, and better plan, due to Rowland,‡ is to represent μ in relation to **B**, or κ in relation to **I**. These may be called permeability curves and susceptibility curves respectively. The following are a few examples:—

§ 74. **Susceptibility Curves for Wrought-Iron Wire.**—Fig. 40 shows two curves of κ and **I** for the experiment described in

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* *Phil. Mag.*, November, 1874. † *Phil. Mag.*, January, 1873.

‡ *Phil. Mag.*, August, 1873.

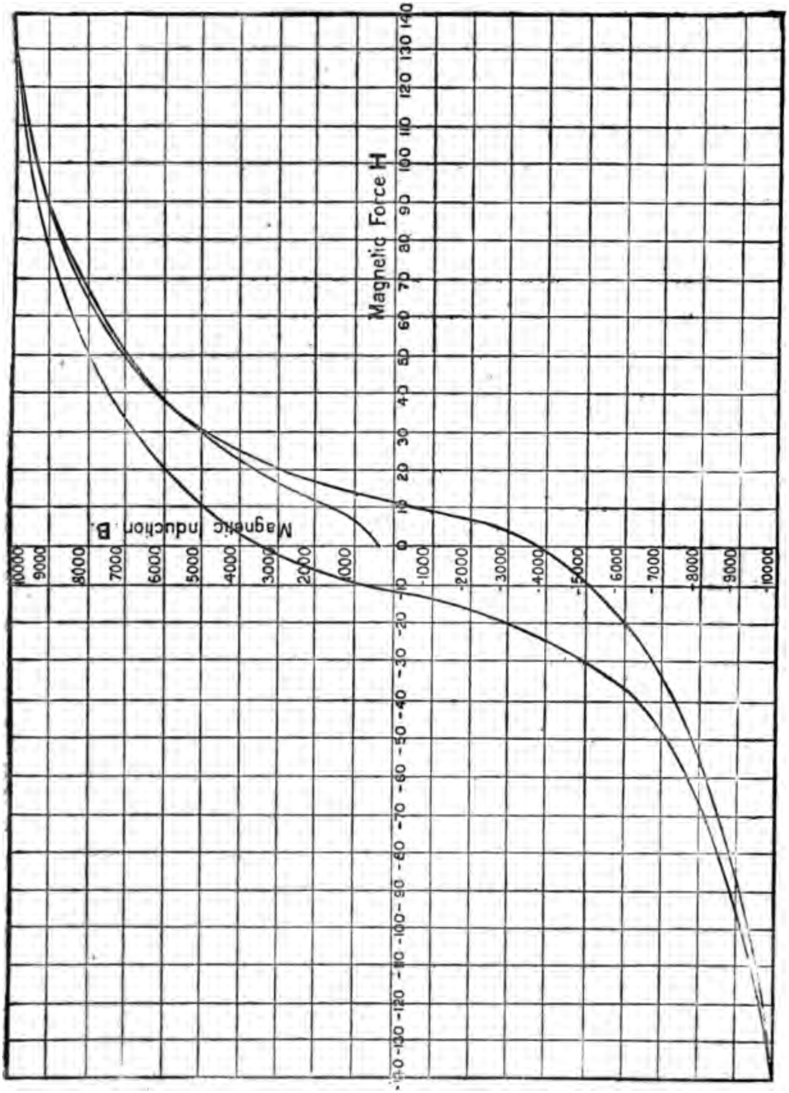

FIG. 39.—Cyclic Magnetization of Cobalt. (Rod : 2 per cent. iron.)

§ 66, where the same piece of wrought iron was tested, first in the soft annealed state, and again after being hardened by stretching. Curves of μ and **B** would have the same form, since in wrought iron μ is almost exactly $4 \pi \kappa$, and **B** is almost exactly 4π **I**.

The approximate symmetry which a curve of this type exhibits about an inclined straight line through the apex was noticed by Rowland, and led him to devise an empirical formula, from which, by extrapolation beyond the limits of experiment, a limiting or saturating value of **B** or of **I** might be deduced. It has, however, been shown by other observers that when the magnetic force is sufficiently raised the curves cease to be even approximately symmetrical; the empirical formula then fails, and it is not possible by producing the curve beyond experi-

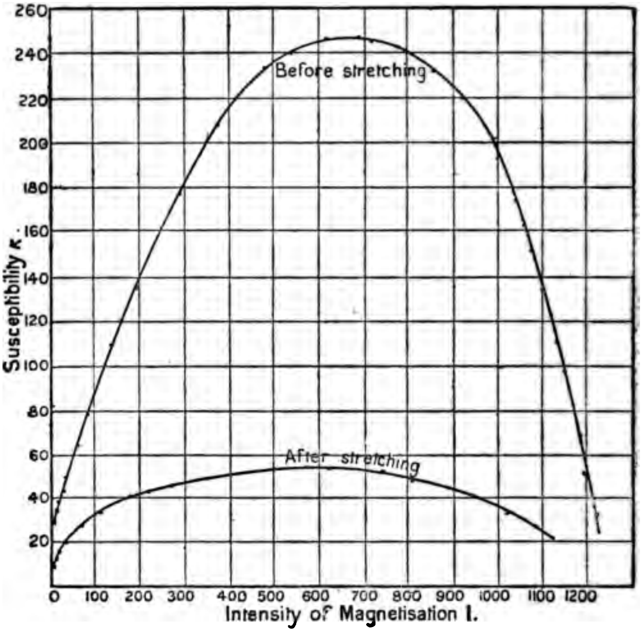

FIG. 40.—Relation of κ to **I** in Iron Before and After Stretching.

mental values to find a limiting intensity of magnetisation. There is a true saturation value of **I** (not of **B**), as will be shown later; but it cannot be found in the manner suggested by Rowland, because the curve of κ and **I** or of μ or **B** bends out under high forces, becoming concave on its upper side. This feature will be seen below in the corresponding curves for nickel and cobalt.

§ 75. **Permeability Curves for Nickel.**—Fig. 41 gives three permeability curves for a nickel rod in the annealed state, tested within a yoke:—The lowest, shown by a full line, is the curve got when the rod was tested under ordinary conditions; the other two, shown by dotted lines, relate to tests made when the rod was subjected to compressive stress. Some account will be given later of the effects of stress on the magnetic qualities of iron, nickel, and cobalt; meanwhile it may suffice to explain that nickel is extremely sensitive to stress, its susceptibility being greatly reduced by tensile stress, and greatly increased by compressive stress. The upper and lower dotted curves relate to compressive stresses of 6·8 and 3·5 kilogrammes per square millimetre respectively.

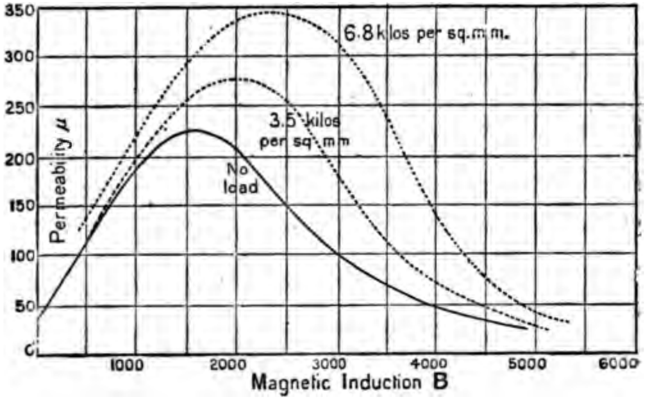

FIG. 41.—Permeability of Nickel in the Annealed State.

§ 76. Permeability Curves for Cobalt.—Fig. 42 shows in the same way two permeability curves for a rod of cast cobalt, tested in a yoke. In this experiment the rod was tested first in ordinary condition of no stress, and then under a series of loads producing various amounts of compressive stress. The full line is the curve for no load; the dotted line is for a load of 16·2 kilogrammes per square millimetre. The curves cross, showing that under weak magnetic forces cobalt has its permeability increased by the presence of compressive stress; but under sufficiently strong forces the reverse is the case.* In a later chapter it will be shown that a reversal of the effects of stress also occurs in iron.

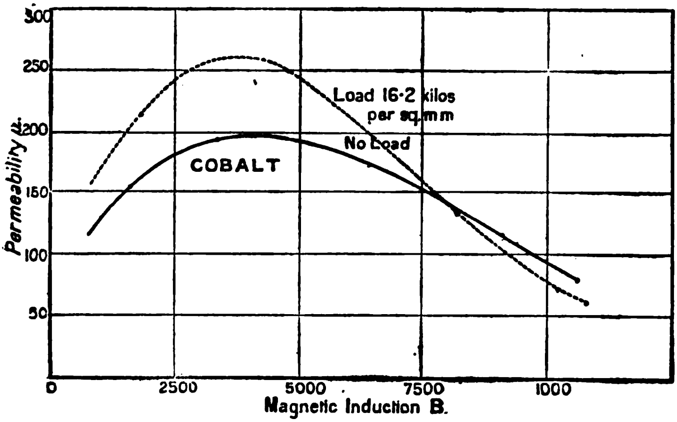

FIG. 42.—Permeability of Cast Cobalt.

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* In a Paper by Mr. C. Chree, published in abstract in *Proc.* Roy. Soc., December 19th, 1889, the same conclusion is stated, along with other results of experiment on the influence of pressure on the magnetic qualities of cobalt. The experiments with cobalt described in the text, and illustrated in Figs. 39 and 42, were made in 1888 by the writer and Mr. W. Low, They have not been previously published.

CHAPTER V.

MAGNETIC HYSTERESIS.

§ 77. Magnetic Hysteresis.—The curves which have been drawn to show the effects of cyclic magnetising processes in iron, steel, nickel, and cobalt, have this important feature in common, that there is a tendency on the part of the metal to persist in any magnetic state which it may have acquired. This tendency is specially obvious whenever an alteration begins to be made in the character of the magnetising process. Thus, when the magnetising force has been raised to its highest value, we find, on beginning to reduce the force, that the magnetism tends to remain. It does not all remain, but the rate at which it disappears during withdrawal of the magnetising force is notably less than the rate at which magnetism was being acquired during imposition of the force, especially at the beginning of the withdrawal. The existence of residual magnetism when the force is wholly withdrawn is one result of this reluctance on the part of the metal to change its magnetic condition. But the results of this tendency go further. If, for example, after withdrawing the magnetising force, we begin to re-apply it, we find in the early stages of the process the same reluctance to change; the metal begins to regain magnetism, but not so fast as it was losing magnetism during the last stages of the removal of the force. The rate, however, improves, and when the force has been completely restored we find that the piece has recovered all, or nearly all (sometimes even a little more than all), the magnetism it lost while the force was being withdrawn. The curve of magnetisation comes again to the same, or nearly the same, point as that from which it started; but its path during the process of return differs entirely from its path during removal of the force. The two curves form a loop, and any intermediate value of the magnetic force is associated with different values of the magnetisation during the two processes.

Moreover, this description applies equally to the effects of *any* cyclic variation of magnetic force, provided the range through which the force is varied be not exceedingly small. Starting from any condition of magnetism and of magnetising force, if we remove and re-apply a part of the force, or if we apply and remove a supplementary force, and repeat the process until its effects become cyclic, we find that the two stages of the process may be represented by two curves, which do not coincide, but differ in a way that may be concisely described by saying that there is a tendency, at each change of process, for the preceding magnetic condition to persist. The changes of magnetism lag behind the changes of force. This tendency has received the name of *magnetic hysterēsis*, from ὑστερέω, to lag behind.*

§ 78. **Effects of Hysteresis.**—Figs. 43 and 44 give further illustrations of the effects of magnetic hysteresis in causing a loop to be formed on the curves of magnetisation when the magnetising force experiences any cyclic change. Fig. 43 refers to a ballistic test of a ring of very soft annealed iron. It shows, in addition to the large loop produced by reversal of the magnetising force, a smaller loop produced by its removal and re-application, and also two small loops formed by pausing at points on the steep part of the demagnetisation curve and removing and re-applying the force there. In Fig. 44 the effects are shown of removing and re-applying the force at a number of successive points during the magnetisation of a long wire of soft annealed iron. Many other experiments have shown that similar loops are formed when there is partial instead of complete withdrawal of magnetising force, followed by its re-application, and that steel, nickel, and cobalt yield results of the same kind. The form of the curves is found to be not materially different whether the changes of magnetising force are made to occur at a moderate rate or excessively slowly. In other words, the hysteresis shown by these loops is persistent

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* *Proc.* Roy. Soc., No. 216, 1881, p. 22; *Phil. Trans.*, 1885, p. 524.

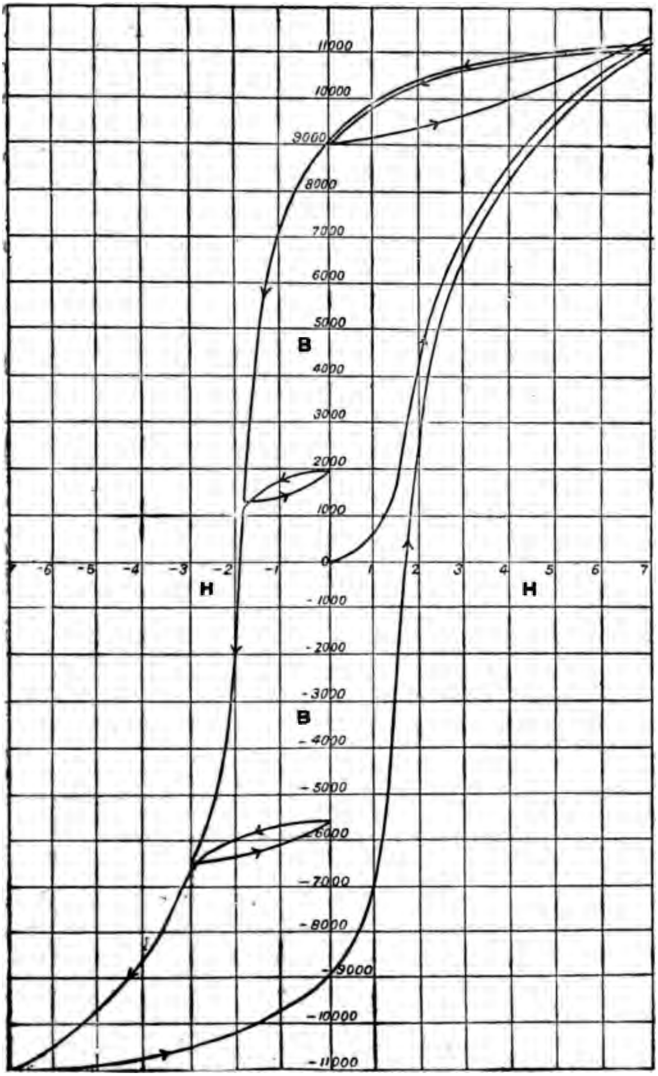

FIG. 43.—Very Soft Iron Ring.

with regard to time. Even prolonged pauses, during which the magnetising force is kept constant at values midway between the two extremes of the cycle, do not cause the differences of magnetism due to hysteresis to disappear, or even to become sensibly lessened.

It is at steep places of the magnetisation curve that the effects of hysteresis are most apparent. Starting from a point such as *a* (Fig. 44), removal and re-application of the force augments the magnetism; this is because the acquisition of magnetism in the original ascending process was retarded by hysteresis, with the result that any species of disturbance (such as the removal and re-application of the force) causes an increase. If we were to repeat the cyclic disturbance by again removing and re-applying the same force, we should find a small further increase; and it is only after several repetitions of the cyclic change of force that its magnetic effects become strictly cyclic.

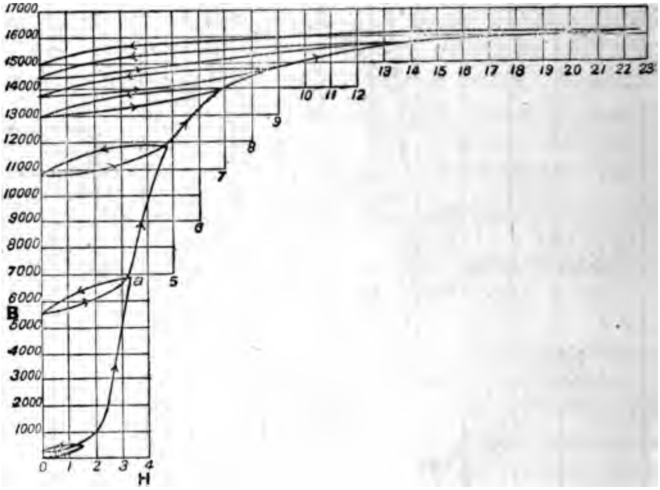

FIG. 44.—Annealed Iron Wire. Length = 400 diameters. Results of removing and reapplying the magnetising force.

Every loop in these diagrams shows that whenever the process of altering the magnetic force is reversed from a process of increment to a process of decrement, or *vice versâ*, the magnetism begins to change very slowly relatively to the change of H, no matter how fast it may have been changing (in the opposite direction) immediately before. So much is this the case that the curves, when drawn to a scale so small as the scale of these diagrams, appear to start off tangent to the horizontal line whenever the change of H is reversed in sign. It will be shown, however, in the next chapter that the initial gradient of these curves is not really zero, but a small positive quantity. At the steepest part of the great cycle in Fig. 43 the value of $\frac{d\,\mathsf{B}}{d\,\mathsf{H}}$ is no less than 14,500; in the initial slope of one of the small loops $\frac{d\,\mathsf{B}}{d\,\mathsf{H}}$ is probably less than 200. In other words, if during the reversal of magnetism we pause at the steepest part of the curve and begin to remove the magnetic force, the gradient of the new curve may be some 70 or 80 times less steep than that of the curve from which it springs.

An obvious effect of hysteresis is to prevent any simple relation from existing between H and B, or H and I. To specify the magnetisation, we must know not only what value the magnetic force actually has, but what changes it has undergone in reaching that value. Associated with any one value of the force there is a wide range of possible values of B or of I. By a suitable choice of processes in the application and removal of H, we may carry the magnetisation curve through any point whatever within the wide area enclosed between the curves which correspond to reversal and re-reversal of a strong magnetising force. Hence the definition of permeability, as the ratio of B to H, or the definition of susceptibility, as the ratio of I to H, requires to be limited (as was indicated in § 21) by the conditions (1) that the piece is neutral to begin with; and (2) that the magnetising force, with reference to which the permeability or susceptibility is expressed, is to be applied by simple increment from zero, without passing at any stage through higher to lower values.

Not only may the magnetisation curve be made to pass through any specified point in the area enclosed by the large reversal curves, but it may have more than one gradient in passing through the point. The following is an interesting example of this effect of hysteresis. Suppose that on the descending limb, P Q (Fig. 45), of the main reversal cycle we have stopped increasing the negative magnetic force at a point Q, so chosen that when the force is removed the curve Q O passes through the origin. When the process Q O is completed, the piece has no magnetism, and it lies in a field of no force. Tested in any ordinary way it might seem to be in a perfectly neutral state; but its condition is far from being the same as that of a virgin piece, or from that of a piece which has been made neutral by the process of "demagnetising by reversals." Such pieces show no directional difference; their susceptibility is the same whether the first magnetising force be positive or negative.

Fig. 45.

But in this case a positive force would give the curve O R, which is a continuation of Q O; whereas a negative force would give the wholly different curve O S. The initial susceptibility in the former case is much greater than in the latter. In consequence of hysteresis the piece, although destitute of actual magnetism and not acted on by any magnetising force, retains latent traces of the magnetic changes it has passed through, which cause it to show a striking want of directional symmetry when it is subsequently magnetised in one or the other direction. Though there is no external evidence that the piece is anything but neutral, it is much more ready to take magnetism of an opposite sign from that which it last held than to take magnetism of the same sign.

§ 79. Dissipation of Energy through Magnetic Hysteresis.— One very important consequence of magnetic hysteresis is that changes of magnetisation (on the part of iron and the other magnetic metals, all of which exhibit hysteresis) involve a dissipation of energy. When the magnetism is carried through a cyclic series of values, by cyclic changes of the magnetising force, the curves showing the relation of $\mathbf{I}$ to $\mathbf{H}$ form a loop, and the area of that loop, in other words, the integral $\int \mathbf{H}\, d\, \mathbf{I}$, measures the amount of energy dissipated during the cycle through hysteresis.*

Perhaps the simplest way to prove this is to think of the magnetisation as being produced by a current in a magnetising solenoid, and to consider the work done by the current when the magnetism changes. To fix the ideas, take as the core of the solenoid a ring or very long rod, of length l and cross-section s, wound with a solenoid of n turns per centimetre, so that the whole number of turns is $l\,n$. Say that its magnetic induction is increased by an indefinitely small amount, $d\,\mathbf{B}$, in an indefinitely small time, $d\,t$, by increasing the magnetising current to an indefinitely small extent. Then the whole number of lines of induction within the solenoid is increased by the number $s\,d\,\mathbf{B}$, and the time-rate of this increase is $s\,\frac{d\,\mathbf{B}}{d\,t}$. This induces in the surrounding solenoid an electromotive force equal to $l\,n\,s\,\frac{d\,\mathbf{B}}{d\,t}$ in the direction opposite to that of the current. The current has accordingly to do work in overcoming this opposing electromotive force, over and above whatever further quantity of energy it expends in heating the conducting wire. With this last source of loss we need not concern ourselves: we wish to find the energy which is spent in producing magnetisation. Let C be the mean value which

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* This was first shown by Warburg in an important paper dealing with several effects of magnetic hysteresis, *Wied. Ann.*, XIII. (1881), p. 141. He proved it by supposing the magnetic force to depend upon the position of permanent magnets, and by calculating the work spent in carrying these magnets through the necessary cyclic changes of position. It was afterwards discovered independently by the writer (*Proc.* Roy. Soc., May, 1882, No. 220, p. 39; *Phil. Trans.* 1885, p. 549). The method of proof followed in the text is substantially Hopkinson's, *Phil. Trans.*, 1885, p. 466; *see also* Lord Rayleigh, *Phil. Mag.*, Vol. XXII., p. 176.

the current has during the time dt. The opposing electromotive force, due to the change of magnetic induction in the core, when multiplied by the current and by the time dt, gives the quantity of work done by the current. Hence the work done by the current in producing the change of magnetism $d\mathsf{B}$ is

$$lns\frac{d\mathsf{B}}{dt}C\,dt, \text{ or } lnsC\,d\mathsf{B}.$$

The fact that dt disappears shows that this work does not depend on the time-rate at which the change of induction takes place; we have the same quantity of energy used in the process whether the change $d\mathsf{B}$ takes place fast or slowly. Since the volume of the core is ls, we may write dW, the work done by the magnetising current per unit of volume (that is, per cubic centimetre), in bringing about the change $d\mathsf{B}$, as

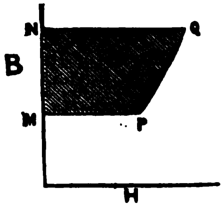

Fig. 46.

$$nC\,d\mathsf{B}.$$

But the magnetising force H is $4\pi Cn$, hence

$$nC=\frac{\mathsf{H}}{4\pi}, \text{ and } dW=\frac{1}{4\pi}\mathsf{H}\,d\mathsf{B}.$$

To obtain the work done per cubic centimetre of the metal when B is changed by any finite amount by changing the magnetising force from (say) a value H_1 to another value H_2 we have to integrate this expression, finding

$$W=\frac{1}{4\pi}\int\mathsf{H}\,d\mathsf{B}$$

between the limits H_2 and H_1.

Thus, in Fig. 46, if P and Q are any two points in the curve of B and H, the work done per cubic centimetre as the magnetic state alters from P to Q is the area M P Q N divided by

4π. For example, in magnetising a piece which has no magnetism to begin with, the curve followed being O P (Fig. 47), the work done in reaching P is equal to the area O P M divided by 4π. If we then remove the magnetising force (curve P R) we *recover* a quantity of work equal to the area R P M, divided by 4π; the net expenditure of energy in the whole process is, therefore, equal to the shaded area O P R, divided by 4π.

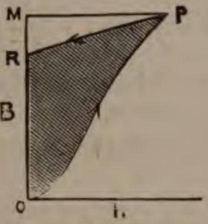

FIG. 47.

In this case the final state of the metal at R is different from its initial state at O, and it is therefore not immediately obvious how much of this energy has been spent irrecoverably. But let a *cyclic* process be followed, so that at the end the magnetisation, as well as the magnetising force, is brought back to the value it had at the beginning; in that case there can be no accumulation of recoverable energy at the end of the cycle.

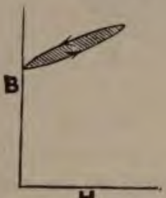

FIG. 48.

The whole difference between what is spent during one part of the process and what is recovered during the other part is therefore *dissipated*, and simply goes to heat the metal. Thus, when we carry the metal through any cyclic series of magnetic changes—such, for instance, as are represented in Fig. 48, where the magnetising force is removed and reapplied, or in

Fig. 49, where it is reversed and re-reversed—there is a quantity of energy dissipated in each cycle which is equal, per cubic centimetre of the metal, to $\frac{1}{4\pi}\int \mathsf{H}\, d\, \mathsf{B}$; in other words, it is equal to the shaded area enclosed by the curves connecting H and B, divided by 4π.

Moreover, since $d\mathsf{B} = 4\pi d\mathsf{I} + d\mathsf{H}$,

$$\frac{1}{4\pi}\int \mathsf{H}\, d\mathsf{B} = \int \mathsf{H}\, d\mathsf{I} + \frac{1}{4\pi}\int \mathsf{H}\, d\mathsf{H}.$$

But in a cyclic process, $\int \mathsf{H}\, d\mathsf{H}$ vanishes; and the energy dissipated in a cycle is, therefore,

$$\int \mathsf{H}\, d\mathsf{I}.$$

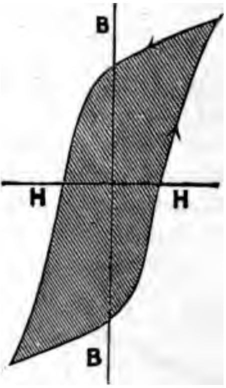

FIG. 49.

Thus in Figs. 48 and 49, if we represent the magnetism by I instead of by B, the shaded area measures the amount of energy dissipated in each cubic centimetre of the magnetised metal during the cyclic process which the curves represent.

As C.-G.-S. units are used in expressing H and I, the area within the curves gives the energy dissipated (per cubic centimetre) in C.-G.-S. units of work, or ergs.

§ 80. **Heating Effect of a Cyclic Process.**—The dissipated energy takes the form of heat: hence iron and other metals in which there is magnetic hysteresis become warmed when their magnetism is successively reversed or varied in any way—the effect of reversal being much more marked than the effect of simple removal and reapplication of a magnetising force, because of the much greater area of the reversal loops.

The iron of transformers and the cores of dynamo-armatures are familiar instances in point. The heating which occurs as a consequence of hysteresis has, of course, nothing to do with the additional heating which Foucault or eddy currents may cause when quick changes of magnetism are made to take place in iron which is not sufficiently laminated. Hysteresis causes heating however slowly the magnetism changes, and however minute are the subdivisions of the core.

To find the rise of temperature which a magnetic metal suffers when its magnetism is cyclically varied, we have to reduce the value of $\int \mathsf{H}\, d\mathsf{I}$ to thermal units, and divide by the number of grammes in a cubic centimetre, and by the specific heat. Using centigrade degrees, there are 41,600,000 ergs in a thermal unit. In iron, the specific heat is 0·11, and there are 7·7 grammes in a cubic centimetre. Hence, the rise of temperature caused by a magnetic cycle is—

$$\frac{\int \mathsf{H}\, d\mathsf{I}}{41{,}600{,}000 \times 7\cdot7 \times 0\cdot11} = 2\cdot84 \times 10^{-8} \times \int \mathsf{H}\, d\mathsf{I}.$$

§ 81. Values of $\int \mathsf{H}\, d\mathsf{I}$.—In soft annealed iron the value of $\int \mathsf{H}\, d\mathsf{I}$, for each double reversal of a condition of strong magnetization, is about 10,000 ergs. Nearly 4,000 double reversals would therefore be necessary to raise the temperature of a soft iron core by 1° C., if the influence of eddy currents could be excluded.

Since there are 7·7 grammes of iron in 1 cubic centimetre, and 453·6 × 2240 grammes in a ton, the energy dissipated in taking one ton of iron through a magnetic cycle is

$$\frac{453\cdot6 \times 2240 \times \int \mathsf{H}\, d\mathsf{I}}{7\cdot7}.$$

Suppose that there are n cycles per second: the work done in ergs per second is then

$$\frac{n \times 453\cdot6 \times 2240 \times \int \mathsf{H}\, d\mathsf{I}}{7\cdot7}.$$

We may reduce this to horse-power by dividing by $7{\cdot}46 \times 10^9$, which is the number of ergs per second in 1 horse-power. Hence the horse-power consumed through magnetic hysteresis when one ton of iron is taken repeatedly through a set of cyclic changes of magnetism at the rate of n cycles per second is

$$0{\cdot}00001769\, n \int \mathsf{H}\, d\mathsf{I}.$$

Applying this to the case of soft annealed iron, where $\int \mathsf{H}\, d\mathsf{I}$ for double reversals of strong magnetisation is about 10,000 C.-G.-S. units, the horse-power per ton, for 100 cycles of double reversal per second, is 17·7.

In harder specimens of annealed wrought-iron the value of $\int \mathsf{H}\, d\mathsf{I}$ for a double reversal of strong magnetism may be as much as 16,000. Hardening the metal by mechanical strain increases the area within the curves, as a reference to Figs. 34 and 38 will show. In mild steel Hopkinson's experiments[*] show that the value ranges from that found in wrought iron up to 40,000 or even 60,000; it increases in a general way with increase in the percentage of carbon, and is greater in specimens which are hardened by quenching than in those which have a lower temper. In high carbon steels the value may exceed 60,000. In pianoforte steel wire 94,000 has been obtained when the metal was annealed, 116,000 when in its commercial state, and 117,000 when hardened by quenching in water from a red heat. Chrome steel (containing about 1 per cent. of chromium) ranged from about 65,000 (annealed) to 167,000 (oil-hardened). In tungsten steel Hopkinson found even higher values; an oil-hardened French specimen containing 3·4 per cent. of tungsten, 0·5 per cent. of carbon, and 0·6 per cent. of manganese, consumed 216,800 ergs, or more than twenty times the amount consumed in soft wrought-iron. The dissipation of energy in a cycle of double reversal is roughly equal to four times the coercive force multiplied by I.

In cast-iron, values of 30,000 to 40,000 appear to be usual; but in one sample of soft grey cast-iron Hopkinson has found so low a value as 13,000. In nickel, a hard-drawn wire gave 25,000, which was reduced to 11,000 when the wire was annealed. Thus the dissipation of energy in nickel when a strong magnetising force is reversed is much the same as that

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[*] *Phil. Trans.*, 1885, p. 463.

in wrought-iron, the greater coercive force of nickel being counterbalanced by the lower intensity of magnetism it is capable of reaching, even when "saturated." As regards cobalt, the experiment with a cobalt rod containing two per cent. of iron, described in §72 and shown in Fig. 39, gives 30,400 as the value of $\int \mathsf{H}\, d\,\mathsf{I}$.

§ 82. **Dissipation of Energy by Reversals of Moderately Strong Magnetisation.**—When the intensity of magnetism at which reversal takes place is reduced, the energy dissipated is, of course, less than has been stated in § 81, where the numbers given for $\int \mathsf{H}\, d\,\mathsf{I}$ refer to reversals of a magnetic state approaching saturation. Fig. 50 shows the effect of subjecting a piece of soft annealed iron wire to a graded series of reversals beginning with weak forces, and gradually increasing the force till the limits of H were ±75 C.-G.-S.* Parts of the curves relating to strong forces are omitted in the figure. The wire was 0·078cm. in diameter and 29cm. long, and was tested by the direct magnetometric method; between 300 and 400 observations of the relation of H to I were required to define the curves in the ten successive processes of double reversal which are represented in the figure. In Table V. the numbers in the first, second and third columns are the values of H, of B, and of I, between which the successive double reversals took place; the next column gives the energy dissipated per cycle in ergs per cubic centimetre, found by measuring the areas enclosed within the curves, and the last shows the rise of temperature which a complete cycle should produce.

These results are shown in Fig. 51 by plotting the measured values of $\int \mathsf{H}\, d\,\mathsf{I}$ in terms of the induction, B, at which each double reversal took place. It will be seen from this curve that the waste of energy increases rapidly as B is raised, which is a reason for avoiding high induction in the cores of transformers, and in the armatures of alternate-current dynamos. With low intensities of magnetism the waste is less than proportionally small. Table VI. gives numerical values taken from the curve of Fig. 51, along with the horse-power wasted per ton of iron,

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* *Phil. Trans.*, 1885, p. 555.

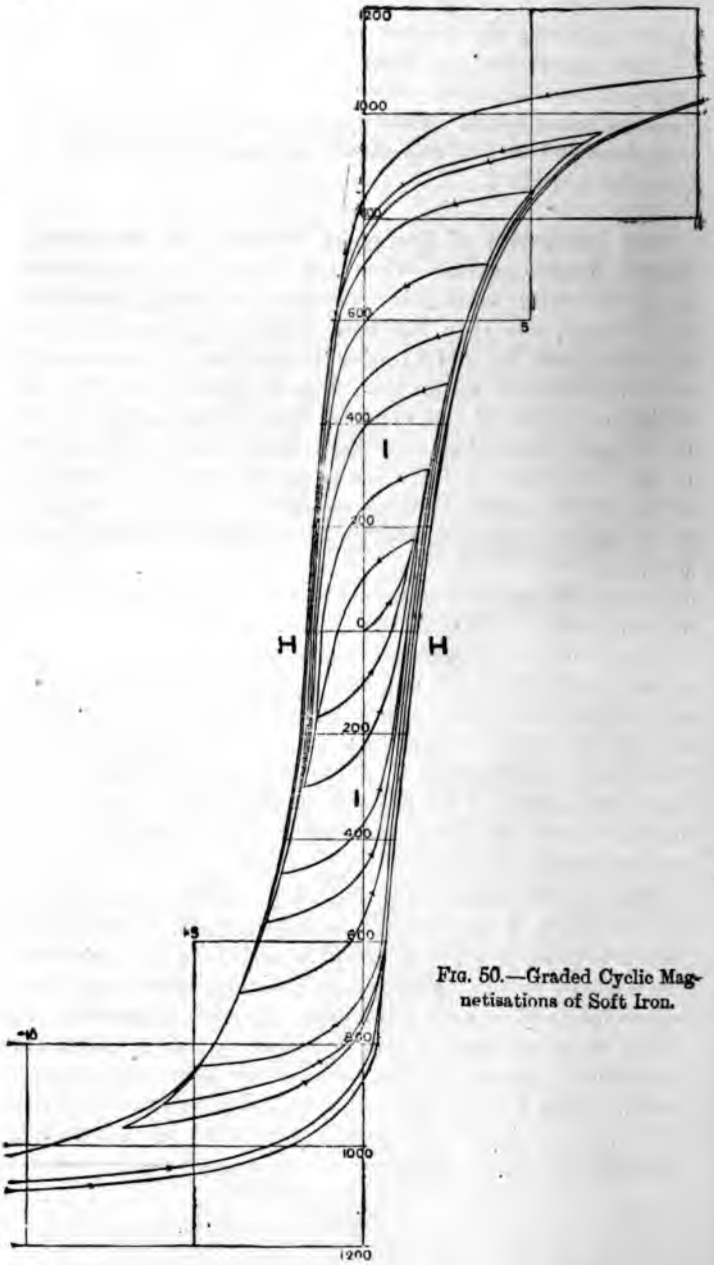

Fig. 50.—Graded Cyclic Magnetisations of Soft Iron.

if 100 cycles (that is, 200 separate reversals) are completed per second.*

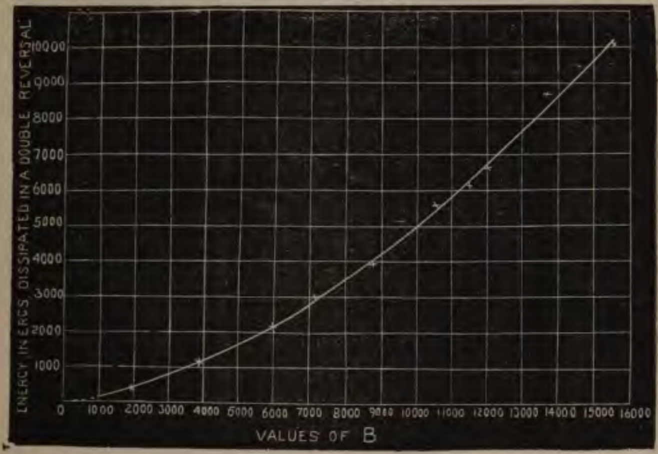

Fig. 51.—Dissipation of Energy in Soft Iron through Magnetic Hysteresis in Double Reversals of Magnetisation.

Table V.—*Graded Cyclic Magnetisations of Soft Iron.*
(Fig. 50.)

H	B	I	$\int H\,dI$	Calculated rise of temperature.
			ergs.	deg. C.
1·50	1,974	167	410	0·000012
1·95	3,830	304	1,160	0·000033
2·56	5,950	473	2,190	0·000062
3·01	7,180	571	2,940	0·000083
3·76	8,790	699	3,990	0·000113
4·96	10,590	842	5,560	0·000158
6·62	11,480	913	6,160	0·000175
7·04	11,960	951	6,590	0·000187
26·5	13,720	1090	8,690	0·000247
75·2	15,560	1230	10,040	0·000285

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* *See also* Mr. Kapp's Paper on "Alternate-Current Machinery" (*Min. Proc.* Inst. C.E., Feb., 1889), where a similar table is given, calculated from the same experiment.

TABLE VI.—*Dissipation of Energy by Double Reversals of Magnetism in Soft Iron.*

B	$\int$ H d I (ergs).	Horse-power wasted per ton assuming 100 cycles per second.
2,000	420	0·74
3,000	800	1·41
4,000	1,230	2·18
5,000	1,700	3·01
6,000	2,200	3·89
7,000	2,760	4·88
8,000	3.450	6·10
9.000	4.200	7·43
10,000	5,000	8·84
11,000	5.820	10·30
12,000	6,720	11·89
13,000	7,650	13·53
14,000	8,650	15 30
15,000	9,670	17·10

Fig. 52* shows the results of a corresponding experiment made with a specimen of annealed pianoforte steel wire. Here much the same features present themselves. When the magnetisation is feeble there is but little dissipation of energy, but as the range of **I** is extended the area of the loops increases fast.

Fig. 53† exhibits, in a different way, the results of these two experiments on iron and steel. The heating effect of a cycle (calculated from $\int$ **H** d **I**) is shown in relation to the value of **H** which was reversed. At first the heating effect of reversal is much less in steel than in iron, with a given value of **H**, for the smaller susceptibility of steel makes the whole magnetic change comparatively small. But with stronger fields its greater coercive force begins to tell, and the heating effect becomes at last very much greater in steel than in iron.

§ 83. **Influence of Speed on Magnetic Hysteresis.**—Experiments are wanting to show whether the speed at which a cycle

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* *Phil. Trans.* 1885, p. 556.

† Copied from a Paper by A. Tanakadaté "On the Thermal Effect due to Reversals of Magnetisation in Soft Iron," *Phil. Mag.*, Sept., 1889.

of magnetisation is performed has, in general, any material effect on the value of $\int \mathsf{H}\, d\, \mathsf{I}$. In certain cases speed is known to have an effect. In the next chapter results will be described which show that when bars of soft iron are subjected to very small cyclic changes of **H** the corresponding magnetic changes depend very largely upon the speed at which **H** is varied. There does not appear to be anything like so serious a dependance on speed when the magnetic changes are considerable; but it is still

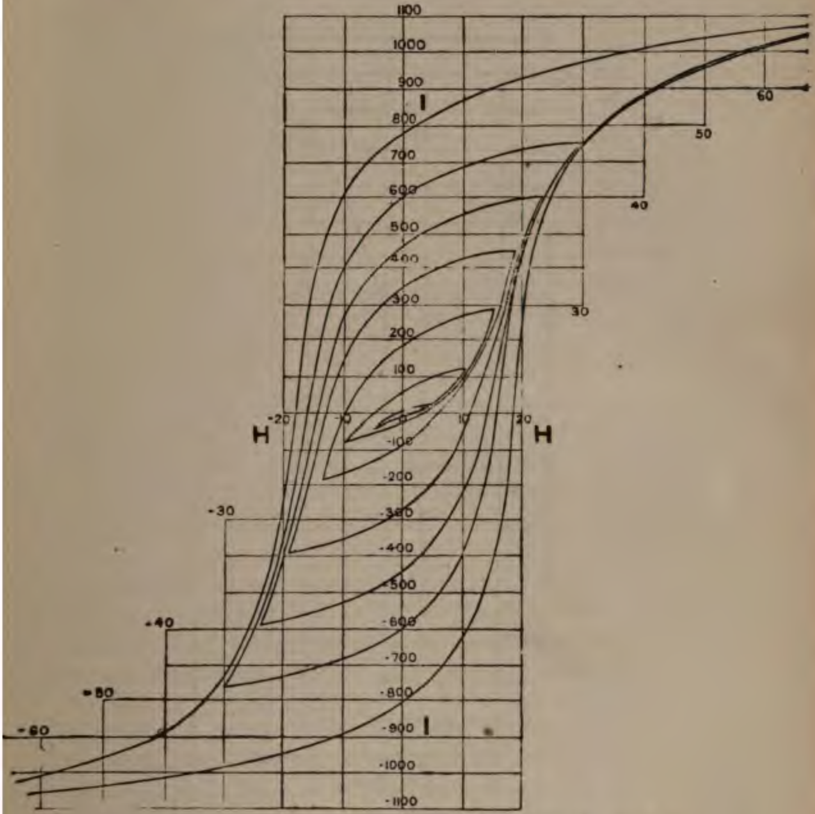

FIG. 52.—Graded Cyclic Magnetisations of Annealed Pianoforte Steel Wire.

doubtful to what extent the results given above, calculated as they are from the observed relation of I to H when H is changed very slowly, are applicable when H is changed fast. With regard to small changes of magnetising forces, at least, soft iron exhibits what may be called magnetic viscosity—that is to say, the changes of magnetism follow somewhat sluggishly the changes of magnetising force, just as in the stretching and unstretching of a rod of india-rubber by applying and removing weights, the changes of length follow sluggishly the changes of load. If this property exists to any considerable degree in

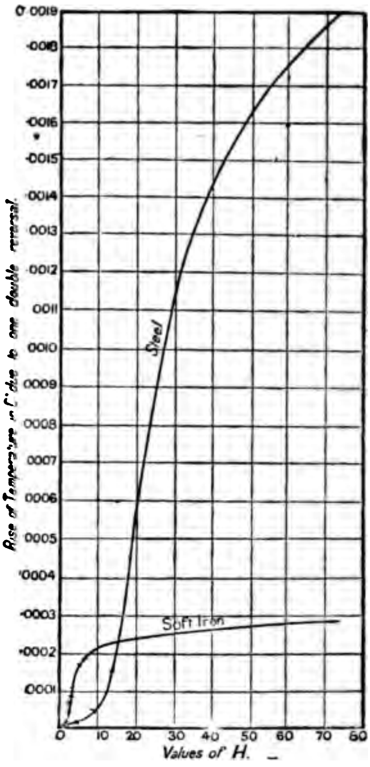

FIG. 53.—Heating Effect of Reversals of Magnetising Force in Iron and Steel.

cases where the range of magnetic change is wide, it may have the effect of causing $\int \mathsf{H}\, d\, \mathsf{I}$, in a quickly performed cycle, to have a value very different from the value observed in such experiments as have been described.

The whole question of magnetic viscosity is one of great practical interest. Probably it is to direct calorimetric measurements of the heat generated by magnetic reversals that we may most hopefully look for its solution, but direct calorimetric measurements present a good deal of difficulty. They have been made by more than one observer, but the experiments hitherto carried out cannot be said to settle the question raised above. Warburg and Hönig,[*] experimenting with bundles of fine wires (to get rid of Foucault currents), found that the heating effect of reversals, as measured in a calorimeter, was about two-thirds of the value of $\int \mathsf{H}\, d\, \mathsf{I}$, as calculated from magnetic observations in slow cycles. Tanakadaté,[†] using a multiple ring of cotton-covered soft-iron wire, and measuring the heat developed in it by reversals, by noting the rise in temperature of a thermo-electric junction placed under the magnetising coil, found the heating effect of quick cycles to be equivalent to about 80 per cent. of the slow-cycle value of $\int \mathsf{H}\, d\, \mathsf{I}$. He observed, further, that the heating effect was practically independent of the frequency of the reversals when that was varied between the limits of 28 and 400 cycles per second. A chief difficulty in observations of this class is to determine what is the actual value which H reaches during rapid alternations of the magnetising current. Though these results are subject to some uncertainty, they concur in making it probable that, for a given value of alternating H, the range of magnetisation is less in quick alternations than in static experiments (where the reversal goes on slowly or by steps with pauses between), and hence that the dissipation of energy is less in a quick cycle on account of this diminished range of magnetic change.

§ 84. Effects of Vibration.—The influence of vibration and mechanical disturbance generally upon magnetic quality has

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[*] Wied. *Ann.*, 1883, Vol. XX., p. 814.

[†] *Phil. Mag.*, Sept., 1889.

been shortly referred to in § 64; it may be succinctly described by saying that vibration lessens those differences of magnetic condition to which hysteresis gives rise. Thus, if we tap a piece of iron during the application and removal of magnetising force, we find at each stage of the application that tapping increases the susceptibility, and at each stage of the removal it reduces the retentiveness. Whatever be the exact nature of the molecular rearrangement which constitutes magnetisation it is facilitated by vibration, which may be imagined to act by setting the molecules momentarily free, more or less, from the constraint in which they ordinarily lie. An analogy may be drawn to the way in which iron turnings scattered on a table near a magnet are freed to range themselves along the lines of magnetic force when the table is tapped; but it must not be inferred that the constraint of the magnetic molecules has anything of the quality of mechanical friction. What that constraint probably is will be discussed in a later chapter.

In strong fields the influence of vibration is scarcely felt; in weak fields it is often enormous. The effect in a weak field is well shown by the familiar experiment—described by Gilbert nearly three hundred years ago—of magnetising a bar of iron by hammering it while it is exposed to the earth's magnetic force. Let the bar, for instance, be held upright: the vertical component of the terrestrial field is too weak to produce more than the feeblest trace of magnetism so long as there is no mechanical disturbance. When sharply tapped, however, it becomes a fairly strong magnet, and the magnetism taken up in this way will persist after the bar has been withdrawn from the field, until it is expelled by further tapping or by the application of a moderately strong magnetic force of the opposite sign. The magnetism acquired by an iron ship in building is another instance in point, and still another is the magnetism which the shock of rupture produces in a specimen of iron or steel broken in a testing machine. Vibration affects all the magnetic metals more or less, but it is in soft annealed iron wire that its influence is most remarkable. Gentle rubbing will give much magnetism to a soft iron wire suspended in the terrestrial field, or will take away much of the large residue which persists after a strong magnetising force has ceased to act. Much care is, in fact, necessary in experiments on the susceptibility or retentiveness of this material to avoid serious errors through accidental disturbance of the specimen. The effects of hysteresis almost entirely vanish in the magnetisation of soft iron wire, if the piece be briskly tapped during application and removal of the magnetising force. The curves of I and H or B and H in the two processes become nearly coincident, and the relation of magnetism to magnetising force becomes comparatively determinate. Two experiments may be quoted to show these effects.*

§ 85. **Experiments on the Effects of Vibration in the Magnetisation of Soft Iron Wire.**—The wire was a piece of very soft annealed iron, 0·158cm. in diameter, and 64cms., or 400 diameters long, of the same quality as that tested in the experiments of § 64. The test was made by the ballistic method; the magnetising force was raised by steps, and after each step the wire was vigorously beaten against the table, and the magnetism was then measured by slipping off a movable induction coil. Observations were made in the same way at a series of stages during the removal of the force. Table VII. gives the values of B found after tapping, first, during application, and then during removal of the force, when the magnetising force due to the solenoid had the values stated in the first column.

TABLE VII.—*Magnetisation of Soft Iron Wire with Vibration.*

Magnetising Force due to Solenoid.	B During Application.	B During Removal.
0	240 (initial)	400
0·04	840	1,440
0·15	3,370	—
0·31	5,370 ↓	5,850 ↑
0·62	8,260	8,500
0·96	9,540	9,860
1·60	10,740	11,200
2·92	12,040	12,400
5·04	13,140	13,000
7·00	13,460	13,550
16·8	14,750 →	

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* *Phil. Trans.* Roy. Soc. 1885, p. 564.

A glance at these figures will show the enormous increase of susceptibility brought about by tapping. A force of 0·96 in the solenoid, with tapping, brings **B** up to 9,540; but another experiment on the same piece of wire showed that without tapping the value of **B** under the same force was only 550. In Fig. 54 curves are drawn, with a very open scale of **H**, to illustrate the portions of this experiment which deal with feeble magnetic forces. The full line O P refers to the application of magnetic force, and the dotted line above it to the removal of the force, both with tapping; while the line O Q refers to the application of magnetic force without tapping. The magnetic force plotted here is that due to the solenoid alone, but it is important to notice that this is by no means the true total force in the experiment made with vibration. Though the wire is 400 diameters long, it cannot be treated as sensibly endless. The reaction of the ends becomes very important on account of the excessively great susceptibility. The real field is much less than the field due to the solenoid—how much less may be judged from the line O A, which is drawn (in the manner

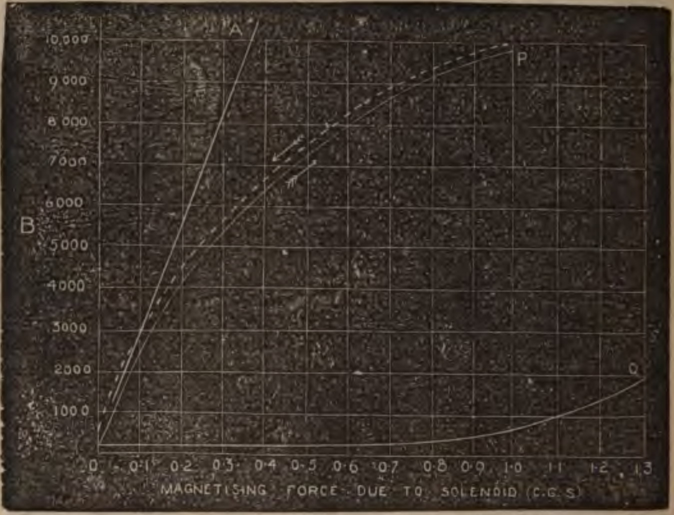

FIG. 54.—Magnetisation of Soft Iron Wire; O P, with vibration, O Q, without vibration.

described in § 48) on the supposition that the wire may fairly be treated as an ellipsoid 400 times as long as it is broad. On this supposition the true magnetic force is to be found by measuring the horizontal distance of any point in the curves from the line O A. Even neglecting this correction of the magnetic force the ratio of B to the (solenoid's) force is not less than 20,000 in the initial part of the curve; and after allowing for the influence of the ends of the specimen by measuring the magnetic force from the line O A the permeability is found to have the enormous value of about 80,000. The permeability is greatest at or near the beginning of the magnetising process; the concavity, which is a feature in the early part of curves determined without tapping, has nearly, if not quite, disappeared.

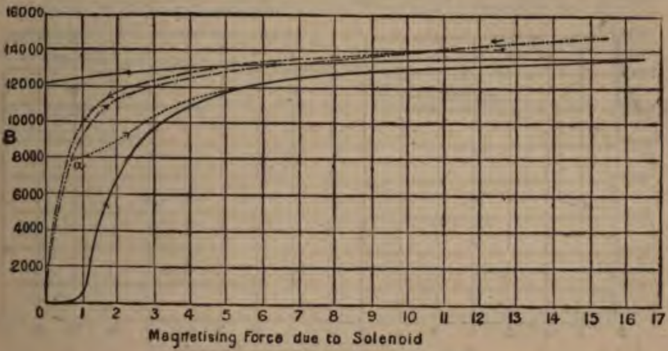

FIG. 55.—Magnetisation of Very Soft Annealed Iron Wire. Without tapping, ———; with tapping, -—-—-—-—; continuation, without tapping, after reaching (with tapping) the point *a*, ...

The complete experiment is shown in Fig. 55. The curves shown by full lines were obtained by applying and removing a magnetising force of nearly 17 units without vibration. The curves shown thus —-—-— - refer to the same process performed with vibration. Finally, after magnetising again to the point *a with* vibration, the application of magnetic force was continued *without* vibration, and the results of this are shown by the dotted curve .... It is interesting to notice how the effects of hysteresis immediately re-assert themselves when, after tapping, we continue the magnetising process with the specimen at rest.

In another experiment, with the same piece of wire, the magnetic force was raised to a certain value, without vibration, while B was determined ballistically; then the wire was smartly tapped, and the change which B underwent through the tapping was measured by slipping off the induction coil; then the coil was replaced, and the force was raised by steps to a higher value; then the wire was again tapped, and so on. The wire had an initial magnetism (B) of 170, which rose to 190 when a force of 0·32 was applied without tapping; then, while this force continued to act, tapping brought up the value of B at a bound to 6,620. Again, under a force of 1·61 tapping changed B from 7,120 to 11,600, and under a force of 3·4 it changed B from 11,940 to 12,960. On coming down the effects were equally well marked. When the force had been reduced from a fairly high value to 0·33, tapping brought B down from 11,260 to 6,880, and finally when the force was 0 the residual value of B, amounting to 6,880, was reduced by tapping to 320. The forces whose values are stated here are those due to the solenoid without allowing for the reaction of the specimen itself upon the magnetising field. The complete results of this experiment are shown in Fig. 56, where the full lines show those parts of the process which were performed without tapping; and the changes of magnetic state brought about by tapping, while the external field was kept constant, are shown thus: —-—-—-.

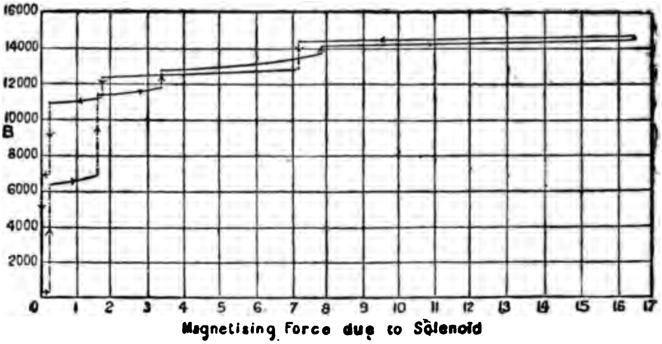

FIG. 56.—Magnetisation of Very Soft Annealed Iron Wire. Effects of tapping shown thus, -—-—-—-—.

In experiments of the same class with hard iron or with steel vibration produces effects of the same general kind; but its influence in destroying hysteresis is far less complete than in soft iron. In a piece of iron wire of the same quality as the last, but not annealed, where a residual magnetism (B) amounting to 7,000 was left after applying a force of 17, the residue fell to 2,500 when the specimen was smartly tapped.

Magnetic hysteresis exhibits itself in other changes of magnetism as well as in the changes that are brought about by varying the magnetic force. It is a prominent feature in the effects of stress upon magnetic quality, but the consideration of it in this aspect will be more conveniently reserved for a later chapter.

CHAPTER VI.

MAGNETISM IN WEAK FIELDS.

§ 86. Permeability with respect to Small Magnetic Forces.—The instances which have been set forth in earlier chapters may suffice to give a general notion of the behaviour of iron and the other magnetic metals when exposed to magnetic fields of moderate strength. It remains to give some account of experiments dealing with the two extremes of very weak and very strong magnetisation. The effects of weak fields will be taken up first.

A glance at the curves of **B** and **H** or of **I** and **H** for any of the examples which have been already given will serve to show that the initial permeability—that is to say, the permeability at the beginning of the process of magnetisation—is so comparatively small that special means are required to examine its value. The arrangements for measuring this early magnetism, whether they are ballistic or magnetometric, must be much more sensitive than those that serve when we have to deal with later portions of the curve. So small, indeed, is the permeability under very feeble forces, compared with the permeability found later, that without special appliances one might readily fall into the error of supposing it to be initially zero. Experiments made by Baur, Lord Rayleigh, and others are conclusive, however, in showing that this is not the case. They show that the initial permeability has a finite value which is applicable, without sensible change, so long as the magnetising force remains very small. In other words, the magnetisation curve starts with a definite gradient, and its very early portion is nearly straight. Lord Rayleigh has carried his investigation of the action of weak forces further, showing that the permeability has a finite value with respect to any small cyclic change of magnetic force when that is frequently repeated, whether the piece be otherwise magnetised or not—a value which is sensibly constant when the range of change is varied, provided the range be kept very small, and which is approximately independent of the mean condition as to force and magnetisation, provided the magnetic state does not approach saturation.

Baur's experiments were made ballistically with a ring of soft iron, the cross-section of which had a diameter of a little over two centimetres. Reduced to C.-G.-S. measure, his results for one trial are as follows :—*

H	I	κ
0·0158	0·263	16·5
0·0308	0·547	17·6
0·0708	1·633	23·0
0·1319	3·815	28·9
0·230	9·156	39·8
0·384	22·487	58·6

When these values of the susceptibility κ are plotted in relation to H, they are seen to lie on what is practically a straight line. By producing the straight line backwards to cut the axis, the value of κ corresponding to $\mathsf{H} = 0$ is found to be 14·5. This is, therefore, the susceptibility with respect to indefinitely feeble forces; the corresponding initial permeability, μ, is 182. Moreover, with respect to forces which are still feeble though not indefinitely small, the susceptibility and permeability may be expressed by the equations

$$\kappa = 14{\cdot}5 + 110\ \mathsf{H},\dagger$$
$$\mu = 183 + 1382\ \mathsf{H},$$

which apply with much accuracy within the limits of H used in the experiment. With any considerably higher force, however, these formulas would not apply. It follows that the relation

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* C. Baur, Inaugural Dissertation, Zurich, 1879. Wied. *Annalen*, XI., 1880, p 399.

† Baur gives $\kappa = 15 + 100\ \mathsf{H}$, but the constants given in the text seem to the writer to agree better with the numerical results of the tests.

of magnetisation to magnetic force for feeble forces may be expressed thus:

$$\mathsf{I} = 14{\cdot}5\,\mathsf{H} + 110\,\mathsf{H}^2,$$
$$\mathsf{B} = 183\,\mathsf{H} + 1382\,\mathsf{H}^2.$$

These particular numerical constants are, of course, to be taken as applying to the specimen of soft iron tested by Baur; but similar parabolic formulas may be constructed with different constants for any specimen of any of the magnetic metals. In other words, the curve of I and H or of B and H is sensibly a parabola in its earliest stages, starting, however, with a finite inclination to the axis of H. For excessively feeble forces it is virtually an inclined straight line, the term involving H^2 being then negligible.

§ 87. **Lord Rayleigh's Experiments.**—The inference drawn by Baur as to the value of κ when H is zero depends on the legitimacy of extending the straight line connecting κ and H backward beyond the region of actual experiment to cut the axis of κ. It has been entirely confirmed by the experiments of Lord Rayleigh,* who has examined the action of much feebler magnetic forces, and has found that the proportionality of magnetic induction to magnetic force continues to hold good when the force is excessively reduced.

In his experiments a bar or wire of iron was tested magnetometrically with one end very near the magnetometer, and with a compensating coil adjusted to balance the magnetism which a feeble magnetising current induced in the bar. The specimen under examination being a piece of Swedish iron wire (not annealed), the compensating coil was adjusted so that there was no movement of the magnetometer needle when a magnetising current was made or broken, the strength being such as to give a field of 0·04 C.-G.-S. Then the strength of the current was gradually reduced till the magnetic force fell to about 0·00004, and it was found that the compensation remained perfect. In other words, within these limits the induced magnetism was proportional to the inducing force: κ and μ were constant. "In view of this," says Lord Rayleigh, "neither theory nor observation give us any reason

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* *Phil. Mag.*, March, 1887.

for thinking that the proportionality would fail for still smaller forces." Quite similar results were obtained with other specimens of unannealed iron and of steel. The range through which κ and μ are sensibly constant is much less in annealed than in hard iron. Within this range of force there is no retentiveness; the magnetising process begins like the straining of a solid body with an elastic stage within which there is no "permanent set." When the magnetising force was increased above 0·04 the compensation failed to remain exact, and the deviations followed the parabolic law stated above. The formulas

$$\kappa = 6{\cdot}4 + 5{\cdot}1\,\mathsf{H}$$
$$\mu = 81 + 64\,\mathsf{H},$$

agreed well with the results of experiment for values of H ranging up to 1·2 C.-G.-S. unit. (In comparing these with the formulas given in the last paragraph, it must be remembered that these refer to hard iron, the others to annealed iron: the initial susceptibility is less here, and the deviation from the initial value is very much less rapid.) With another specimen of hard-drawn iron wire the initial value of μ was 87.

Lord Rayleigh has also examined the effect of alternately applying and removing a small amount of magnetic force, when the piece is kept more or less strongly magnetised by means of a constant force. So long as the constant force is moderately small, and the mean magnetisation consequently not very strong, the susceptibility with regard to alternate applications and removals of a small part of the force is not materially different from the initial susceptibility of the same piece when unmagnetised. But as the mean magnetisation is raised, the susceptibility with respect to small changes of force becomes reduced. In a piece of hard iron a steady force of 29 C.-G.-S. had the effect of reducing the susceptibility with respect to small alternations by about 40 per cent. of its original value; and in a piece of annealed iron the reduction due to the same steady force was more than 80 per cent.

§ 88. **Magnetic Viscosity under Small Forces.**—Allusion has already been made (§ 50) to the fact that after any change has taken place in the magnetic force acting on a piece of soft annealed wrought iron, some time elapses before the corresponding change of magnetic state is complete.* This magnetic viscosity is most noticeable when we have to deal with feeble forces or with small changes of force, and when the specimens tested are of considerable size. In such cases the time-lag in magnetisation may be so great that the ballistic method, which, of course, omits to take note of slow continuous changes, is not properly applicable.

In describing the experiments which were referred to in the last paragraph, Lord Rayleigh remarked that when small magnetic forces were applied to *hard* iron or steel it was possible to adjust the compensating coil, so that neither at the moment of closing the magnetising circuit nor afterward was there any deflection—which means that, so far as the magnetometer can decide, these metals take their full magnetism at once. With annealed wrought iron, however, the effects were more complicated. "When the coil was so placed as to reduce as much as possible the instantaneous effect, there ensued a drift of the magnetometer needle in such a direction as to indicate a continued increase of magnetisation. Precisely opposite effects followed the withdrawal of the magnetising force. The settling down of the iron into a new magnetic state is thus shown to be far from instantaneous."

Following Lord Rayleigh's plan of balancing the instantaneous effect by means of a compensating coil, and then observing the drift, the writer examined this time-lag in the magnetisation of a thick wire of annealed wrought-iron 0·404 cm. in diameter and 39·6 cms. long.† The wire was demagnetised by reversals to begin with, and feeble magnetising forces were used, not at first exceeding 0·1 C.-G.-S. So long as the force was less than this it was found that one adjustment of the compensating coil served to balance the instantaneous effect of making or breaking or reversing the current. When the compensation was correct the magnetometer needle began to drift slowly over as soon as the magnetising force was either applied or removed; and

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* *Phil. Trans.*, 1885, p. 569.—"When the magnetising current was applied to long wires of soft iron, either gradually or with more or less suddenness, there was a distinct creeping up of the magnetometer deflection after the current had attained a steady value. This action was sometimes so considerable as to oblige me to wait for some minutes before taking the magnetometer reading."

† *Proc.* Roy. Soc., June 20, 1889.

by observing the drift and adding that to the amount neutralised by the compensating coil, the total magnetism after any time was readily deduced. A force of 0·044 C.-G.-S. was applied, the instantaneous effect of which was to produce a value of **I** equal to 0·44; in five seconds this crept up to 0·58, and in 60 seconds to 0·67. Then the magnetising current was broken; the instantaneous effect on **I** was to remove 0·44, leaving 0·27; in five seconds this residue fell to 0·09, and before 60 seconds it had completely disappeared. Next a magnetising force of 0·084 was applied. The value of **I** reached at once was 0·85; in five seconds it crept up to 1·20, and in 60 seconds to 1·40. On breaking the current, **I** fell at once to 0·55, after five seconds to 0·23, and after 60 seconds to 0·07. Possibly this small residue, or part of it, was permanent. These results are shown in Fig. 57. Precisely similar results were obtained by reversing feeble magnetic forces, the initial gradient of the lines being the same when the force was reversed as when it was applied and removed. If we measure the initial susceptibility by the immediate effect of applying or reversing **H** it is 10; if we measure it by the effect after one minute it is about 15.

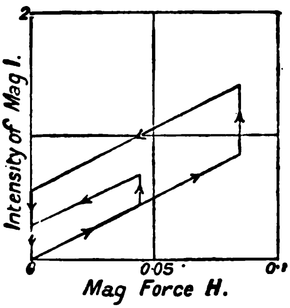

FIG. 57.

Fig. 58 shows the results of another experiment, in which successive forces were applied, ranging up to about 0·34 C.-G.-S., the compensating coil being adjusted for each force to give an instantaneous balance, so that the effect of the subsequent creeping up might be observed. Before applying each force the specimen was completely demagnetised. The three curves,

Fig. 58, show the amounts of magnetism taken (*a*) at once, (*b*) after five seconds, and (*c*) after one minute. In noting the

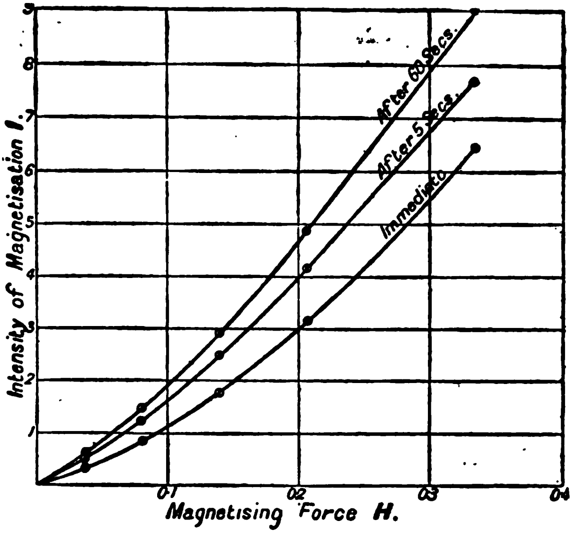

FIG. 58.—Effects of applying Feeble Magnetising Forces to a Soft Iron Rod.

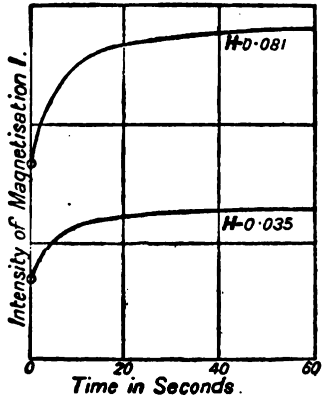

FIG. 59.—Growth of Magnetism after applying Feeble Magnetising Forces.

gradual growth of magnetism after each force was applied, readings of the magnetometer were taken every five seconds, and the two curves of Fig. 59 have been drawn from these, to show the time rate at which the process of creeping up went on under the action of magnetising forces equal to 0·035 and 0·081 respectively.

§ 89. **Further Experiments on Time-Lag in Magnetisation.** Similar differences between the immediate and ultimate action of magnetic force on soft iron present themselves when we examine the effects of small increments of the magnetic force at any stage in the process of magnetisation. In another experiment, which was made with the same specimen of annealed wrought iron, the magnetising force was applied in a series of small steps—each step being produced by a rapid but not quite sudden augmentation of the magnetising current. The immediate effect of each step was balanced by means of the compensating coil, and after each step a pause of one minute was made during which the gradual growth of magnetism was observed. The results are shown in the full lines of Fig. 60; the dotted line has been added to show that the points reached after the pauses of one minute lie in a continuous curve. As the experiment was continued into higher parts of the magnetisation curve, the compensating coil had to be pushed a little

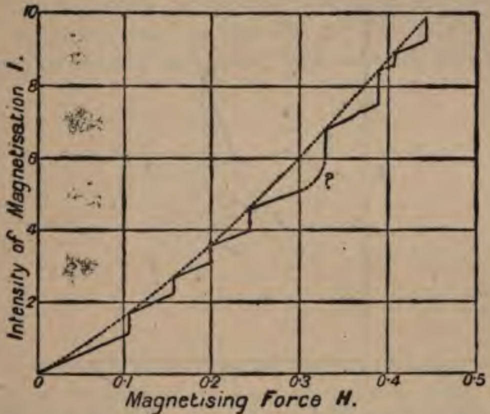

FIG. 60.—Effects of Steps in the Magnetisation of a Soft Iron Rod.

nearer the magnetometer to procure a perfect balance: in other words, the immediate effect of the step became somewhat greater. At the beginning, the instantaneous value of $\frac{dI}{dH}$ was about 10; but when the experiment of Fig. 60 was extended until the force produced by the magnetising solenoid was 3 C.-G.-S. or so, and I was about 320, the instantaneous value of $\frac{dI}{dH}$ rose to 13. In that region of the curve, the creeping-up of magnetism after a very small step-up of the current was enormous; in the course of one minute it amounted to six or seven times the immediate effect of the step. Fig. 61 illustrates the kind of action which is observed when a small increment of magnetising force is made to take place quickly after a pause anywhere in the steep part of the magnetising process, the metal dealt with being soft wrought iron. The dotted line is the normal slope of the magnetisation curve when the process of magnetising is performed slowly. A very small increment of H rapidly performed after a pause at P produces an immediate effect, P Q, which is followed by the slow creeping up Q R. It is only when the step is a very small one that P Q correctly represents the immediate effect.

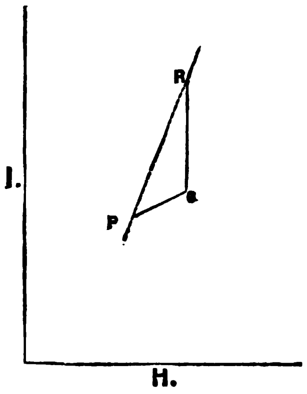

FIG. 61.—Effects of a sudden small increase of Force in the steep part of the Magnetisation Curve.

Very interesting results are obtained in examining how the time-rate of creeping up after a step is affected by the length of the pause (under constant force) which preceded the step. When the preceding pause is long the creeping up which follows a step goes on much more slowly than when the preceding pause is short.* In an experiment with the same specimen of soft iron the effects of two equal small steps were compared, both made at the same part of the magnetisation curve, one after the magnetising force had been kept constant for three minutes, and the other after it had been kept constant for an hour. The immediate effects were the same; but the subsequent creeping up, which was observed during no less than ten minutes, went on so much faster in the former case that it amounted in ten minutes to 531 scale divisions of the magnetometer, as against 320 scale divisions in the latter.

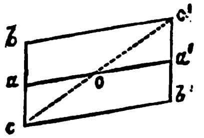

FIG. 62.

The effects of an alternate small step up and step down, performed at any stage in the process of magnetisation, are quite like those that have been shown in Fig. 57. After the steps have been repeated often enough to bring about a cyclic set of changes, the instantaneous value of $\frac{dI}{dH}$ becomes approximately the same as at the initial part of the curve—namely, about 10 in the particular specimen examined—unless the whole magnetisation approaches saturation, in which case the value of $\frac{dI}{dH}$ is distinctly less. The diagram (Fig. 62) represents in a general way the change of magnetism which takes place when any very small periodic variation of magnetic condition is made to occur in a soft iron bar, about a mean condi-

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* *Loc. cit.*, p. 280.

tion O. If the changes of force occur fast and without pauses the cycle is shown by the lines *a a'* and *a' a*. They enclose no area, and there is no dissipation of energy. If, on the other hand, the changes of force occur gradually and very slowly the cycle is shown by the lines *c c'* and *c' c*. They also enclose no area; and again there is no dissipation of energy. But if the changes of force take place quickly, with pauses at the extreme values, the cycle is *b' c' b c*, and an amount of energy is dissipated which is to be measured by the area of that parallelogram. In most actual cases in which the force varies periodically it does so not suddenly with pauses at the extreme values, but in such a manner that a loop will be formed instead of the parallelogram. When the frequency of the alternations is very great, the loop will flatten itself into the straight line *a a'*; when the frequency is very small it will again flatten itself into the straight line *b b'*. With any frequency lying between these extremes there will be dissipation of energy, and when the limits and mode of variation of the force are specified, there must be some particular frequency which will make the amount of energy dissipated in the cycle a maximum.

In hard iron and in steel the phenomenon of time-lag in magnetisation occurs, but so slightly as to be scarcely observable. A piece of the same wire as had been used in the above experiments was hardened, after being annealed, by stretching it a little beyond the limit of elasticity. Scarcely a trace of creeping could be detected when a feeble magnetic force was applied to the wire in this hardened state, but it was possible to produce a measurable amount of creeping by first applying a moderately strong magnetising force, and then making a small step up after a pause. The initial instantaneous value of $\frac{d\,\mathrm{I}}{d\,\mathrm{H}}$ for a small step was 52.

The whole phenomenon depends much on the size of the specimen that is tested. In the experiments which have been described the iron was a rod four millimetres in diameter. Smaller rods showed much less magnetic "creeping," and when a bundle of fine annealed iron wire was substituted for the rod, nearly all trace of creeping disappeared. The cause of this difference is at present obscure.

§ 90. **Molecular Accommodation.**—Closely related to the experiments which have been detailed in this chapter are results recently published by H. Tomlinson.* Examining the action of feeble magnetic forces, in the region within which the relation of B to H may be expressed (§ 86) in the form

$$\mathsf{B}=a\,\mathsf{H}+b\,\mathsf{H}^2,$$

he has discussed the influence of temperature and other conditions on the constants a and b. The constant a is of course the initial permeability, and it is on the value of b that the dissipation of energy depends. Some of the more interesting of Tomlinson's results may be briefly stated in his own words:—

"The internal friction of iron, nickel, and cobalt in any complete cycle may be decreased by repetition of the cycle; the molecules are said to be 'accommodated' by this process.

"The molecular 'accommodation' of freshly annealed iron can be largely aided by repeatedly raising the metal to 100°C, and then allowing it to cool.

"The 'accommodation' of the molecules of iron, nickel, and cobalt is disturbed by very slight mechanical shocks, by small change of temperature, or by magnetisation beyond certain limits; under such influences the internal friction may for a time, or even permanently, be considerably increased.

"The values of a and b for iron are temporarily increased when the temperature is raised from 0°C. to 100°C."

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* *Proc.* Roy. Soc. Dec. 5, 1889.

CHAPTER VII.

MAGNETISM IN STRONG FIELDS.

§ 91. **Magnetisation in Strong Fields.**—We pass now to speak of the opposite extreme of the magnetising process. In studying the relation of magnetism to magnetising force by any of the methods which have been described in earlier chapters, it is scarcely practicable to raise the force H beyond a few hundreds of C.-G.-S. units at the most. Formidable difficulties present themselves, one of which is the heating effect of the magnetising coil. Special methods have therefore to be resorted to when we wish to examine the behaviour of iron or other magnetic metal in very strong fields.

It is true that the most important parts of the magnetising process lie within the range of those forces which may easily be produced by means of a magnetising coil. Within that range the permeability or the susceptibility passes through its great changes, increasing quickly from a small finite initial value to a maximum ten or fifteen times as great, and decreasing almost as quickly to a value smaller than the first. Within that range, too, the residual magnetism apparently reaches the full value it is capable of reaching. It is within that range that the most prominent features in the influence of vibration, of temperature, and of stress, manifest themselves. And it is probably true that whatever knowledge of magnetic quality is wanted for application to the practical ends of electrical engineering can be obtained by experiments within that range.

But still the action of stronger fields is of very great interest, especially in relation to the molecular theory of magnetism propounded by Weber. According to Weber's theory the molecules of iron or any other magnetisable metal are always magnets. These point anyhow in the unmagnetised piece, so that the sum of their moments, resolved in any direction, amounts to zero, and the piece, therefore, has no magnetism as a whole. But when a magnetising force acts the molecular magnets tend to turn so that their axes may point more nearly in the direction in which the force acts; and thus the piece, as a whole, becomes a magnet. The intensity of magnetisation **I** is the sum (per unit of volume) of the moments of the molecular magnets resolved in the direction of the magnetising force. We shall discuss this theory more fully in a later chapter. Meanwhile, one obvious deduction from it may be pointed out. When all the molecular magnets are turned round to face exactly in the direction in which the force acts, no further magnetisation in that direction will be possible, however much the force may be increased. In other words, the theory points to this—that the intensity of magnetisation **I** has a saturation value which cannot be exceeded, though it points to no limit to the value which **B**, the magnetic induction, may reach.

In experiments made with moderately strong magnetising forces both **B** and **I** are increasing slowly at the last; and it is impossible to infer, from the results of such experiments, whether **B** or **I** or either of them is approaching a finite limit. The curves of permeability or of susceptibility in relation to **B** or to **I** (such as have been given in Figs. 40, 41, and 42) do not help us to a conclusion; we cannot produce a curve of this kind beyond the region of experiment until it cuts the axis of **B** or of **I**, because (as Figs. 41 and 42 show) the curve bends out when the magnetising force is sufficiently increased. This characteristic of the curve of κ and **I** or of μ and **B** was first pointed out by Fromme,* and has been commented on by a number of other experimenters. In some of the writer's experiments it appeared when **B** exceeded about 15,000.† Figures given by Bosanquet‡ for experiments with iron and steel rings, in one of which the induction was pushed as high as 19,300, show when plotted a similar inflexion in the curve of μ and **B**, occurring when **B** is about 15,000. The same feature is well shown in Fig. 63, which is copied from a Paper by Bidwell,§ describing experiments with soft wrought iron, in which the

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* Fromme, *Gött. Nahr.*, 1875, p 500. Wied. *Ann.* XIII., p. 695, 1881; *see* also J. Haubner, *Wien. Anz.*, October 21, 1880; Wied. *Beiblätter*, V., 1881, p. 205.

† *Phil. Trans.*, 1885, Part II., p. 567.

‡ Bosanquet, *Phil. Mag.*, February and May, 1885.

§ Bidwell, *Proc.* Roy. Soc., Vol. XL., 1886, p. 486.

induction was raised to 19,820, with the result of reducing μ to 33·9. To produce this the force **H** was 585, and the resulting magnetisation **I** was 1,530. These numbers give some idea of the extent to which experience has shown it is practicable to go in experiments of the ordinary class, using the magnetising force of a current in a coil.* To answer the question whether there is any finite limit to **I** or to **B**, we have to go far beyond this range.

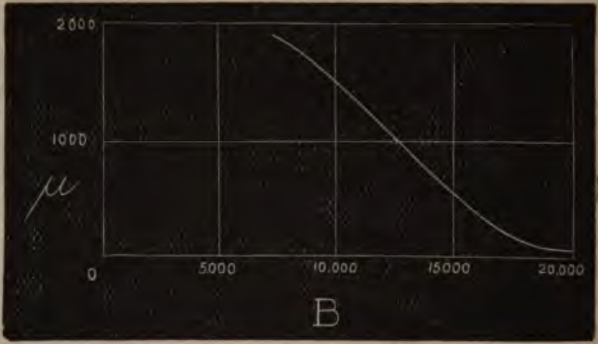

FIG. 63.—Permeability of Wrought Iron when Strongly Magnetised.

§ 92. **The Isthmus Method.**—This name has been given to a method introduced in 1887 by the writer and Mr. W. Low,† which has allowed the magnetisation of iron to be raised to greatly higher values, with the result of showing that, while **B** has apparently no limit, there is a finite limit to **I**, as Weber's molecular theory predicts.

In the air-space between the pole-pieces of a strong electromagnet, we have a magnetic field of much greater intensity than any that can easily be produced by the direct action of the electric current. If a small test-piece of the metal which is to be magnetised be placed across this space, so that it forms an isthmus between the two pole-pieces, it will become strongly magnetised. In becoming magnetised, however, it disturbs this field, and the force acting on it may be very

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* In recent experiments by du Bois, described in § 107 *infra*, a coil was used to produce magnetic forces which ranged up to 1,300 C.-G.-S.

† *Proc.* Roy. Soc., March 24, 1887; *Phil. Trans.*, 1889, A, p. 221.

different from the force which existed in the empty space. If it is a short cylinder extending lengthways from pole-piece to pole-piece, its magnetism will be very unequal. At the ends the induction will have the same value as it has in the pole-pieces themselves; at the middle it will be stronger, owing to the convergence of lines of induction from neighbouring parts of the pole-pieces, which find their way into the test-piece through its sides. Evidently we may increase the induction in the middle by furnishing the specimen with spreading ends, which will present an easier path along which the lines of induction may converge. Moreover, when the test-piece takes the form of a bobbin, with a short, narrow, central neck, from each end of which a cone extends, spreading over the face of the pole-piece, it becomes possible (by giving a proper form to the cone) to secure that the central neck will be uniformly magnetised, and that the magnetic force which acts on it will have the same value as the magnetic force in the immediately surrounding air-space. The magnetic force and the magnetic induction within the neck then admit of being measured, and the permeability, susceptibility, and intensity of magnetisation under exceedingly strong forces are readily deduced.

§ 93. **Early Experiments, using the Isthmus Method.**— Figs. 64 and 65 show two forms of bobbin which were used in the first application of the isthmus method. The dimensions are marked in millimetres. The central neck was wound with an induction coil consisting of a single layer of fine wire, and its magnetism was measured by the ballistic method. With bobbins of the shape shown in Fig. 64, the induction was measured by suddenly slipping the bobbin out from its place between the pole pieces while the electro-magnet was excited. An objection to this is that it takes no direct account of the residual induction; it shows only the magnetism that is lost when the bobbin is withdrawn from the field. The residue is small, and it may be separately measured and allowed for; but a better arrangement is shown in Fig. 65, where the bobbin may be turned suddenly round so that its magnetism is reversed; half the ballistic effect of this reversal of course measures the magnetic induction. To measure the field in the air-space immediately surrounding the neck, a second induction coil was wound over the first, but at a little distance from it, so that a narrow ring of non-magnetic space—about 1·3mm. wide—was

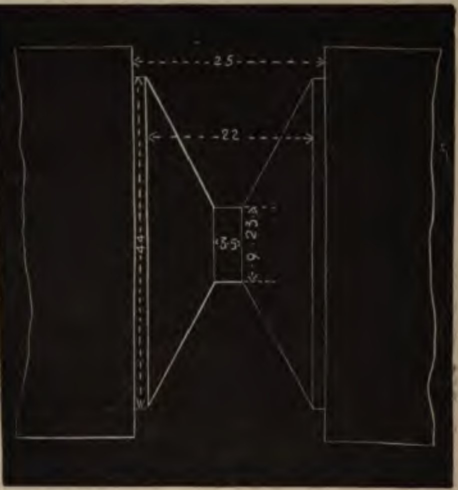

FIG. 64.

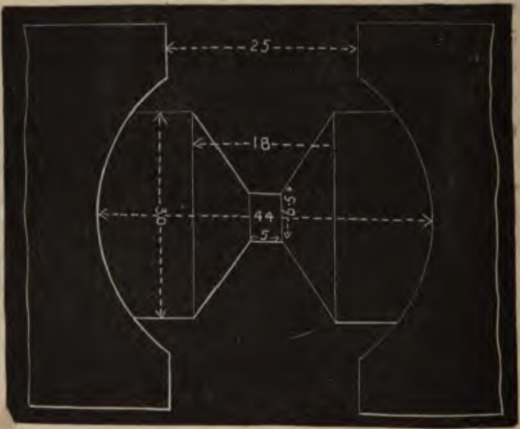

FIG. 65.

included between the two. The magnetic force in this space was calculated from the observed difference in the ballistic effects of the inner and outer coil. The knowledge of it allowed a proper correction to be made, by which the whole induction within the inner coil was reduced to allow for those lines of induction which lay within it but not within the iron.

With the bobbins shown in Figs. 64 and 65 the outside field —that is to say, the magnetic force in this narrow ring of space surrounding the neck—was probably a very little stronger than the mean force within the metal of the neck itself. Still, the outside field was so nearly equal to H that the quantity $\frac{\mathsf{B} - \text{outside field}}{4\pi}$ approximated closely to the value of I, and the quantity $\frac{\mathsf{B}}{\text{outside field}}$ approximated closely to the value of the permeability, μ.

The results in Table VIII. were obtained with a bobbin of soft Swedish wrought iron in the annealed state.

TABLE VIII.—*Swedish Wrought Iron in Strong Fields.*

Outside field ($=\mathsf{H}$ nearly).	B	$\frac{\mathsf{B} - \text{outside field}}{4\pi}$ ($=\mathsf{I}$ nearly).	$\frac{\mathsf{B}}{\text{outside field}}$ ($=\mu$ nearly).
6,690	27,960	1700	4·18
8,900	29,730	1660	3·34
9,510	30,820	1700	3·24
10,000	31,210	1690	3·12
10,360	31,630	1700	3·05
10,810	31,720	1670	2·94
10,880	32,060	1690	2·95
11,200	32,360	1690	2·90

These figures show that in the very strong fields with which this experiment deals, the quantity in the third column, which is approximately equal to the intensity of magnetisation I, becomes practically constant. Such variations as occur in the numbers are irregular and come from errors of observation. The iron is here in a condition of true saturation; I has reached a value which refuses to undergo any sensible increase, though the strength of the field be doubled; but the field itself may be increased without limit, and consequently there is no sign of any limit to the value of B.

Table IX. gives the results of a similar experiment made with a bobbin of annealed Lowmoor wrought iron, and with a wider range of magnetic forces. The apparent decrease of I in the strongest field, which is shown by the last numbers in the third column, is due to the fact that the outside field was rather greater than the true magnetic force within the metal. When the bobbin is so shaped that this source of error is avoided, the apparent decrease disappears, and I is then found to be as nearly constant as casual errors of observation allow it to be.

TABLE IX.—*Lowmoor Wrought Iron in Strong Fields.*

Outside field (= H nearly).	B	$\frac{\text{B - outside field}}{4\pi}$ (= I nearly).	$\frac{\text{B}}{\text{outside field}}$ (= μ nearly).
3,630	24,700	1680	6·80
6,680	27,610	1670	4·13
7,800	28,870	1680	3·70
8,810	29,350	1630	3·33
9,500	30,200	1650	3·18
9,780	30,680	1660	3·14
10,360	30,830	1630	2·98
10,840	31,370	1630	2·89
11,180	31,560	1620	2·82

TABLE X.—*Cast Iron in Strong Fields.*

Outside field (= H nearly).	B	$\frac{\text{B = outside field}}{4\pi}$ (= I nearly).	$\frac{\text{B}}{\text{outside field}}$ (= μ nearly).
3,900	19,660	1250	5·04
6,400	21,930	1240	3·42
7,710	22,830	1200	2·96
8,080	23,520	1230	2·91
9,210	24,580	1220	2·67
9,700	24,900	1210	2·57
10,610	25,600	1190	2·46

A noticeable feature in these results is the reduction of the permeability that is brought about by continuing to increase the magnetising force after a state of saturation has been reached. With wrought iron such as was used here the initial value of μ for exceedingly small forces is nearly 200; and the maximum of μ, reached generally with a magnetising force of two or three units, may be as much as 3,000. Here, with a magnetising force of 10,000 units or so, μ has fallen to less than 3.

Table X. gives the results of a similar experiment with cast iron. In it, as in the two last cases, saturation has been reached even with the lowest value of **H** within the range of the observations. The saturation value of **I** in this cast iron is about 1,240—a value distinctly less than that found in wrought iron. Fig. 66 exhibits in the form of curves of permeability the results given in Tables IX. and X. These are in effect an extension into regions of strong force of curves of the type shown before in Figs. 41 and 42 and in Fig. 63.

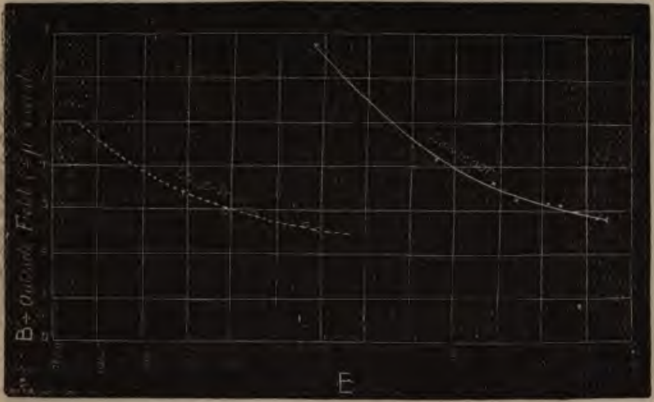

FIG. 66.—Curves of Permeability for Wrought Iron and Cast Iron very strongly magnetised.

§ 94. **Later Experiments, using the Isthmus Method.**—In subsequent experiments* the induction in iron was forced to much higher values by using a larger electro-magnet and by turning down the neck of the bobbin. The extent to which concentration of induction in the neck may be carried depends on the proportion which the sectional area of the neck bears to that of the pole from which the lines converge. In the follow-

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* Ewing and Low, *Phil. Trans.*, CLXXX., 1889, A, p. 221; *Rep.* Brit. Assoc., 1887, p. 585.

ing experiment the section of the neck was reduced until it was finally only $\frac{1}{1500}$ that of either pole. The magnet—an exceptionally powerful one, belonging to the Physical Laboratory of Edinburgh University—was excited with 64,000 ampere-turns, and its force was concentrated from poles about 10 cms. square upon a neck or isthmus 2·66 mm. in diameter and 3·5 mm. long. Fig. 67 is a full-size sketch of the poles with the bobbin in its place after its neck had been reduced to the smallest diameter. The dimensions are entered in millimetres. The bobbin *c* was the same bobbin of annealed Lowmoor wrought iron as had been used in the earlier experi-

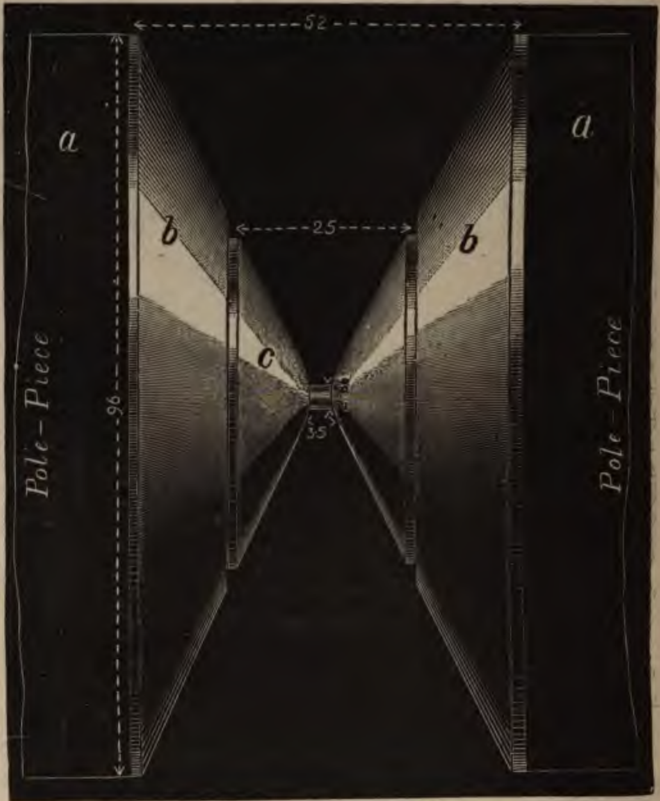

FIG. 67.—Pole-pieces and Bobbin used in the Isthmus Method.

ments, a pair of separate conical pieces *b b* being interposed to connect its ends with the pole-faces *a a*. With each reduction in the size of the neck a higher value of **B** was reached; finally, when its diameter was 0·266 mm., the induction **B** was 45,350, and the force in the space immediately surrounding the neck was 24,500. From other experiments we may infer that this was, as nearly as possible, equal to the actual magnetic force within the metal: hence the result may be written thus:—

H	B	I	μ
24,500	45,350	1,660	1·85

No attempt was made to reduce the neck further, and this is the highest induction that has hitherto been recorded in any experiment. There is no reason to doubt, however, that higher values of **H** and of **B** might be obtained by using an electro-magnet of greater size and power.

§ 95. Theory of the Isthmus Method: Form of Cone to give Maximum Concentration.*—Consider an imaginary section through the middle of the neck, at right angles to the axis of the bobbin. It is clear that there is no discontinuity between the magnetic force, at points in this plane, inside and outside the metal, for there is no free magnetism on the surface of the neck at the middle of its length. We have to consider the conditions which will make the magnetic force as nearly uniform as possible over this medial section in order that the force just outside the neck, which we are able to measure, may be fairly representative of the force within the substance of the neck itself.

The magnetic force in the space between the pole-pieces is made up of two parts: (1) the electro-magnetic force directly produced there by the current in the magnet coils; and (2) the force due to free magnetism, distributed for the most part over the pole-faces. The first of these forms a comparatively small part of the whole; and its value is sensibly uniform at such small distances from the axis as those with which we are now concerned. In considering the conditions which will secure the greatest strength or the greatest uniformity in the field at the

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* Parts of this and the succeeding paragraphs are taken from the Paper cited (*Phil. Trans.*, 1889, A, p. 221).

neck, we need only deal with that part of the force which is produced by free magnetism.

The free magnetism of the pole-faces may be treated as made up of a series of co-axial circular rings in planes normal to the axis of the bobbin. Calling M the whole free magnetism of one of these rings (Fig. 68) and r its radius, the magnetic force F due to it at a point in the axis at a distance x from the plane of the ring is $\frac{\mathrm{M}x}{l^3}$ where $l=\sqrt{r^2+x^2}$. This force will be a maximum when $\frac{d\mathrm{F}}{dx}=0$, that is, when

$$\frac{1}{l^3}-\frac{3x^2}{l^5}=0,$$

which occurs when $x=\frac{r}{\sqrt{2}}$; $\tan\theta=\sqrt{2}$; $\theta=54^\circ\ 44'$ Hence a series of co-axial rings will be most advantageously disposed

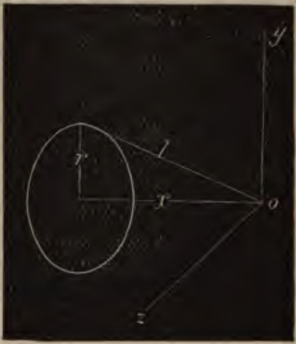

FIG. 68.

for producing force at a point on the axis if they lie on a cone having its vertex at the point in question, with a semi-vertical angle of 54° 44′.

The greatest force will be produced when the pole-pieces are themselves saturated, so that **I** reaches its limiting value in all parts of the metal. In that case the distribution of density from ring to ring is uniform. The surface density of free magnetism at any point of a sloping pole-face is $\mathbf{I}\sin\theta$, where θ is the slope of the face to the axis of magnetisation. The whole quantity in each ring is I multiplied by the area of the ring projected upon a plane normal to the axis—a quantity which is independent of the slope of the cone. We have, therefore, the same series of attracting rings to deal with whatever be the slope of the convergent faces, and whether that slope be uniform or not. Given, then, a certain diameter for the neck of the bobbin to be magnetised, the greatest magnetic force will be produced at the middle of the axis of the neck when the pole-pieces are saturated and when we make the expanding ends and pole-faces in the form of cones, with a semi-angle of 54° 44′, and with their vertices at the middle of the neck.

This determines what may be called the cones of maximum concentrative power. In practice cones intended to produce as great a concentration as possible should have a somewhat greater semi-angle—say 60° or so—because of the defective saturation of the pole-pieces.

§ 96. **Greatest Magnetising Force producible by Means of Cones.**—With a cone of any semi-angle θ, magnetised to a uniform intensity I_0, the surface density of free magnetism is $I_0 \sin\theta$, and the force at the vertex due to a ring at an axial distance x, of radius r, and of length dl, measured along the slope, is

$$2\pi r\,dl \,.\, I_0 \sin\theta \,.\, \frac{x}{l^3}, \text{ or } 2\pi I_0 \sin^2\theta\cos\theta\frac{dr}{r}.$$

The whole force at the vertex is

$$2\pi\sin^2\theta\cos\theta\int_a^b \frac{I_0\,dr}{r},$$

a being the radius of the neck on which the cone converges, and b the radius of the base to which it spreads.

Hence (treating I_0 as uniform), with a pair of truncated cones, joined by a neck at the middle of which they have their common vertex, the whole force there is

$$F = 4\pi I_0 \sin^2\theta\cos\theta\log_\epsilon\frac{b}{a},$$

which, for convenience of calculation, may be written

$$F = 28{\cdot}935\, I_0 \sin^2\theta\cos\theta\log_{10}\frac{b}{a}.$$

Applying this to the cones of maximum concentrative power (§ 95), in which $\sin\theta = \sqrt{\frac{2}{3}}$ and $\cos\theta = \frac{1}{\sqrt{3}}$.

$$F_{max.} = 11 \cdot 137 \; \mathrm{I}_0 \log_{10} \frac{b}{a},$$

and the greatest value of the force will be obtained when I^0 has the saturation value (of say 1,700 C.-G.-S. units for soft wrought iron), in which case

$$F_{max.} = 18930 \log_{10} \frac{b}{a},$$

an expression which measures the greatest possible force which the isthmus method of magnetisation can apply at a point in the axis of the bobbin (over and above the small force which is directly produced by the magnet coils). It is not practicable to produce quite so large a force, because the magnet poles cannot be fully saturated.

§ 97. **Form of Cone to give Most Uniform Field.**—The cone of maximum concentrative power is not the form best suited for producing a uniform magnetic force throughout the neck. It makes the field rather stronger at places near the axis than on the axis itself. To make the field as nearly uniform as possible in and close to the neck we must slope the cone at such an angle that $\frac{d^2 F}{d x^2} = 0$, a condition which secures that $\frac{d^2 F}{d y^2}$ and $\frac{d^2 F}{d z^2}$ shall also be zero. This condition is satisfied when

$$\frac{9x}{l^5} - \frac{15\,x^3}{l^7} = 0,$$

which makes $x = r\sqrt{\frac{3}{2}}$; $\tan\theta = \sqrt{\frac{2}{3}}$; $\theta = 39^\circ\,14'$.

In other words, the best approximation to a uniform field (the pole-pieces being saturated) is reached when the pole-faces are cones converging upon the middle of the neck, with a semi-vertical angle of 39° 14′. When the cones have this form, and the neck is very narrow in comparison with the base, the field is so nearly uniform that the magnetic force in a narrow ring of space round the neck and close to it may be taken to represent, without sensible error, the force within the neck itself, and there is no practical variation of the force in the neck from end to end, or from side to centre.

With cones of this form the concentration of force upon the neck is less than in the former case. Using the same notation as before the force is

$$15{,}240 \log_{10} \frac{b}{a},$$

in the event of the poles being of wrought iron and fully saturated.

FIG. 69.

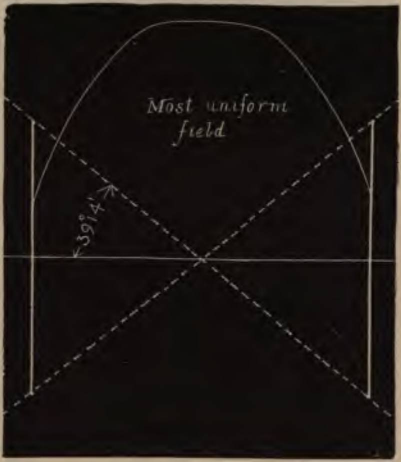

FIG. 70.

The difference between the two cases is illustrated by Figs. 69 and 70, where curves are drawn to show the force exerted at various points on the axis by a single pair of rings, forming parts of conical pole-faces which have a common vertex. In Fig. 69 the rings are parts of cones of maximum concentrative power; in Fig. 70 they are parts of cones shaped to produce the best possible approximation to a uniform field. The rings are taken equal in both cases, so that the height to which the curves rise in the middle will serve for comparison of the forces: the flat-ness of the curve in Fig. 70 shows the superiority of that form of cone in respect to uniformity of field. With actual conical pole-pieces, the force produced in the neck is, of course, made up of the sum of the forces due to pairs of rings like these distributed over the whole conical surface.

§ 98. **Further Experiments with Wrought Iron.**—In the following experiments bobbins were used of a shape suited to give a fair approximation to a uniform field, and hence the outside field close to the neck is taken as the measure of **H** :—

TABLE XI.—*Lowmoor Wrought Iron.*

H	B	I	μ
3,080	24,130	1,680	7·83
6,450	28,300	1,740	4·39
10,450	32,250	1,730	3·09
13,600	35,200	1,720	2·59
16,390	36,810	1,630	2·25
18,760	39,900	1,680	2·13
18,980	40,730	1,730	2·15

TABLE XII.—*Swedish Iron,* "L^{s}Lancash." *Brand.*

H	B	I	μ
1,490	22,650	1,680	15·20
3,600	24,650	1,680	6·85
6,070	27,130	1,680	4·47
8,600	30,270	1,720	3·52
18,310	38,960	1,640	2·13
19,450	40,820	1,700	2·10
19,880	41,140	1,700	2·07

TABLE XIII.—*Fine Swedish Iron,* Ⓛ *Brand.*

H	B	I	μ
5,310	25,670	1,620	4·83
17,680	38,080	1,620	2·15
19,240	39,540	1,620	2·06

In this last iron, which is described as the finest and most expensive iron used in commerce, made by the Walloon process, the saturation value of I seems to be specifically rather less than in other brands. The saturation value usually found in wrought iron may be stated to be, in round numbers, 1,700. The state of saturation is practically reached, in soft metal, with a force of, say, 2,000 C.-G.-S. units; from this force upwards no material change can be observed in I, though the force be increased ten-fold.

§ 99. Cast Iron and Steel in very Strong Fields.—In cast iron the highest value to which B was pushed in these experiments was 31,760, with the result of reducing the permeability to 1·9. The saturation value of I in the sample tested was 1,240, and saturation was practically complete under a force of 4,000.

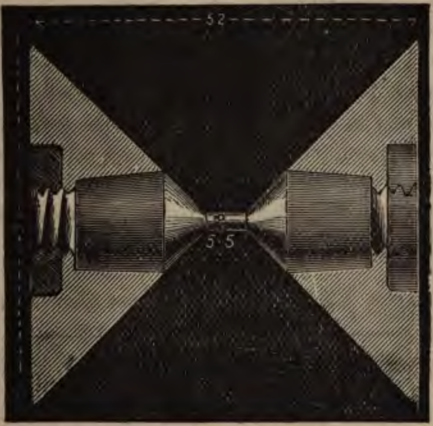

FIG. 71.—Experiments on Vickers' Tool Steel.

In hard steel the state of complete saturation is not so easily reached. The following test, which was made with a sample of Vickers' tool steel possessing much coercive force, exemplifies this. The test piece formed the central part of a bobbin with wrought-iron cones, built up in the manner shown in Fig. 71. By removing one of the cones, a loose coil on the neck could be slipped off to determine the residual magnetism, which in this case formed a considerable part of the whole. (The residual induction in the neck was about 8,000.) It may be doubted whether saturation was complete even in the strongest field.

TABLE XIV.—*Vickers' Tool Steel.*

H	B	I	μ
6,210	25,480	1,530	4·10
9,970	29,650	1,570	2·97
12,120	31,620	1,550	2·60
14,660	34,550	1,580	2·36
15,530	35,820	1,610	2·31

There appear, however, to be specific differences in the saturation values of I in different steels. In the following Table a summary of the results of experiments with other steels is given, showing in each case the highest force applied and the highest induction reached, along with (approximate) corresponding values of I and μ.

TABLE XV.—*Steel of Various Qualities.*

Description of Steel.	Outside field (= H nearly).	B	$\frac{\text{B} - \text{outside field}}{4\pi}$ (= I nearly).	$\frac{\text{B}}{\text{Outside field}}$ (= μ nearly).
1. Bessemer steel, containing about 0·4 per cent. of carbon......................	17,610	39,880	1,770	2·27
2. Siemens-Martin steel, containing about 0·5 per cent. of carbon	18,000	38,860	1,660	2·16
3. Crucible steel for making chisels, containing about 0·6 per cent. of carbon...	19,470	38,010	1,480	1·95
4. Finer quality of crucible steel for chisels, containing about 0·8 per cent. of carbon............	18,330	38,190	1,580	2·08
5. Crucible steel, containing 1 cer cent. of carbon ...	19,620	37,690	1,440	1·92
6. Whitworth fluid-compressed steel	18,700	38,710	1,590	2·07

§ 100. **Hadfield's Manganese Steel in Strong Fields.**—Reference has been made in § 70 to the remarkable absence of magnetic susceptibility shown by this steel, which contains about 12 per cent. of manganese and 1 per cent. or less of carbon. In fields of ordinary strength this alloy has a sensibly constant permeability of about 1·3, as Hopkinson's experiments have shown.* Application of very strong fields, by means of the isthmus method, shows that the permeability, even under very great forces, remains constant as nearly as may be judged. One might expect that a material which resists magnetisation so strongly would show much coercive force; the reverse, however, is the case. Even the strongest force is unable to produce more than a trace of residual magnetism. The following is one of several experiments which agree in showing that the permeability of manganese steel, under any force up to 10,000 C.-G.-S. units, is practically constant with a value of about 1·4. This permeability is so low that when the field is weak, the metal takes up scarcely any magnetism; on the other hand, since the permeability retains the same value in very strong fields, a respectably high intensity of magnetisation may be produced by applying a sufficiently strong force. The variations of μ in Table XVI. are irregular, and are no greater than may be ascribed to errors of observation.

TABLE XVI.—*Hadfield's Manganese Steel.*

H	B	I	μ
1,930	2,620	55	1·36
2,380	3,430	84	1·44
3,350	4,400	84	1·31
5,920	7,310	111	1·24
6,620	8,970	187	1·35
7,890	10,290	191	1·30
8,390	11,690	263	1·39
9,810	14,790	396	1·51

§ 101. **Nickel and Cobalt in Strong Fields.**—With nickel and cobalt a state of complete saturation is reached without

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* *Phil. Trans.*, 1885, p. 462.

difficulty, as the following observations show. In the two specimens of nickel tested (Tables XVII. and XVIII.) the saturation values of I were about 400 and 515 respectively; the difference is perhaps due to differences in the amount of iron present: neither specimen was pure. The saturation value of I in cobalt (Table XIX.) appears to be 1,300, which is a little greater than the value in cast iron.

TABLE XVII.—*Hard-drawn Nickel* (*with* 0·56 *per cent. of Iron*).

H	B	I	μ
2,220	7,100	390	3·20
4,440	9,210	380	2·09
7,940	12,970	400	1·63
14,660	19,640	400	1·34
16,000	21,070	400	1·32

TABLE XVIII.—*Annealed Nickel* (*with* 0·75 *per cent. of Iron*).

H	B	I	μ
3,450	9,850	510	2·86
6,420	12,860	510	2·00
8,630	15,260	530	1·77
11,220	17,200	480	1·53
12,780	19,310	520	1·51
13,020	19,800	540	1·52

TABLE XIX.—*Cobalt* (*with* 1·66 *per cent. of Iron*).

H	B	I	μ
1,350	16,000	1,260	12·73
4,040	18,870	1,280	4·98
8,930	23,890	1,290	2·82
14,990	30,210	1,310	2·10

§ 102. Summary of Conclusions from Isthmus Experiments.—To sum up the results which have been arrived at by means of the isthmus method, the concluding paragraph of the Paper from which these figures are taken may be quoted.*

Under sufficiently strong magnetising forces the intensity of magnetisation, **I**, reaches a constant or very nearly constant value in wrought iron, cast iron, most steels, nickel, and cobalt. The magnetic force at which **I** may be said to become practically constant is less than 2,000 C.-G.-S. units for wrought iron and nickel, and less than 4,000 for cast iron and cobalt. In stronger fields the relation of magnetic induction to magnetic force may be expressed by the formula

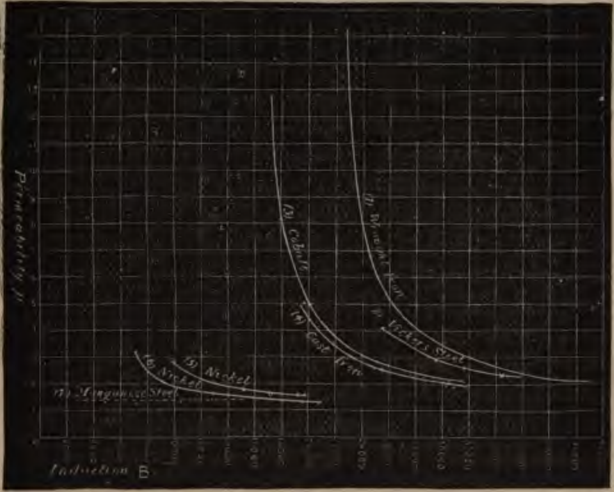

FIG. 72.—Permeability of Magnetic Metals when very strongly magnetised.

$$\mathsf{B} = \mathsf{H} + \text{constant}.$$

For the particular specimens tested, the value of this constant ($4 \pi \mathsf{I}$) is about 21,360 in wrought iron, 15,580 in cast iron, 5,030 and 6,470 in nickel, and 16,300 in cobalt.

The experiments give a definite meaning to the term "saturation," as applied to magnetic state. When magnetism is

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* *Phil. Trans.*, 1889 A, p. 242.

measured by the induction **B**, the term saturation is inapplicable; there is apparently no limit to the value to which the induction may be raised. But, when we measure magnetisation by the intensity of magnetism **I**, we are confronted with a definite limit—a true saturation value, which is reached or closely approached by the application of a comparatively moderate magnetic force. There is nothing to show that the approach to this limit is not asymptotic; but in wrought iron it is practically reached before the magnetic force rises to 2,000 C.-G.-S., and after that a ten-fold increase in the force produces no material change in the intensity of magnetism.

The results are further summarised in Fig. 72, which gives curves showing the relation of the permeability μ to the induction **B** drawn from the data supplied by experiments on—

(1.) Swedish wrought-iron (Table XII.).
(2.) Vickers' tool steel (Table XIV.).
(3.) Cobalt (Table XIX.).
(4.) Cast iron (Table X., and other data).
(5 and 6.) Nickel (Tables XVIII. and XVII.)
(7.) Hadfield's manganese steel (Table XVI.).

§ 103. **Apparatus for Applying the Isthmus Method.**—In applying the isthmus method it is desirable to be able to turn the bobbin round suddenly between the magnet poles, in order to determine the ballistic effect produced by reversal of its magnetism. An arrangement used by the writer for this purpose is shown in Figs. 73 and 74. Fig. 73 shows the electro-magnet as a whole, and Fig. 74 is a sectional sketch of the pole-pieces and bobbin and bobbin-holder. The poles, which are four inches in diameter, admit of having the distance between them adjusted, and a brass piece *a a* is fitted between them, having hollow cones turned out of its ends, into which the conical pole-pieces fit exactly. This holds the pole-pieces at the proper distance apart. Through the brass piece, *a a*, a cylindrical hole is bored, extending through from side to side, and removing the points of the conical pole-pieces. Into this hole the bobbin-holder, *c c*, with the bobbin, *d*, is slipped from one side. The part which projects outside of *a* has a shoulder turned on it, which, when it is pressed home, brings the axis of the bobbin just into line with the axis of the

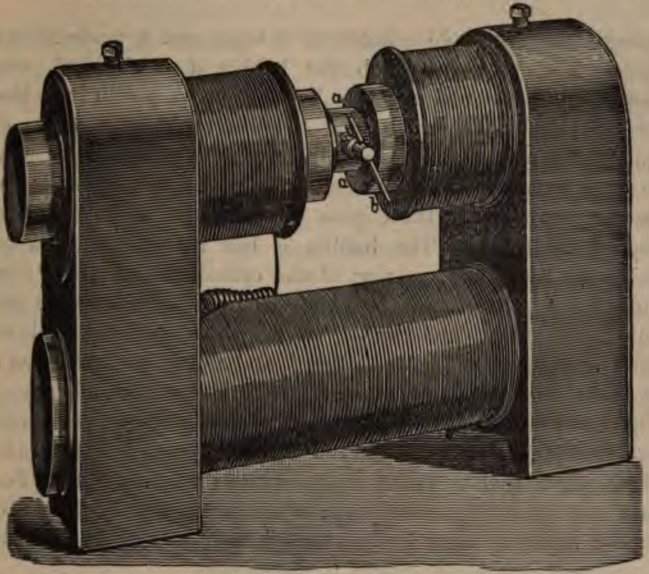

FIG. 73.—Electro-Magnet for the Isthmus Method.

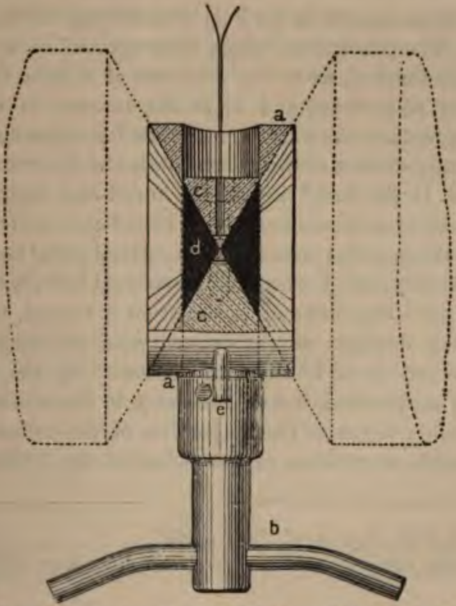

FIG. 74.—Section through Bobbin and Bobbin-holder.

pole-pieces. The bobbin-holder is of brass, and is made of two pieces, *c c*, between which the bobbin *d* is clutched, the pieces being fastened together by long screws put in from the end of the holder, which pass clear of the bobbin on each side of the neck. There is a little clearance round the neck to give room for the induction coils to be wound, and the leading wires from these pass out through a hole in the end of the holder. The bobbin *d* has its ends turned so that they virtually form part of the cylindrical surface of the holder, and it fits exactly into the cylindrical hole between the pole-piece ends. The handle *b* attached to the holder allows the bobbin to be suddenly reversed, and a stop is provided at *e* to make the movement exactly 180°. The electro-magnet of Fig. 73 is furnished with two pairs of conical pole-pieces, one pair sloped to give maximum concentrative power (§ 95) and the other pair to give maximum uniformity in the field (§ 97). For each pair a bobbin of the iron, or other metal to be tested, is turned with the same slope, and for each pair there is a distance-piece and bobbin-holder to correspond.

§ 104. Experiments by du Bois with Strong Fields. Optical Method.—The conclusions which were arrived at by means of the isthmus method, as to the existence of a finite limit to the intensity of magnetism, and as to the manner in which that limit is approached when strong magnetic forces are applied, have received independent confirmation from the later experiments by H. E. J. G. du Bois,* in which a novel and highly interesting method of measurement was introduced and used with excellent effect. The method is an optical one, based on Dr. Kerr's discovery,† that when plane polarised light is reflected by a magnet pole the plane of polarisation is turned, through an angle which depends on the intensity of the magnetisation. Before this fact could be turned to account for the purpose of measuring magnetism, it was necessary to know exactly what relation holds between the magnetism of the reflecting metal and the angle of rotation of the polarised ray. This question

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* Du Bois, *Phil. Mag.*, April, 1890, p. 293.

† Kerr, Brit. Ass. *Report*, 1876, p. 40 ; *Phil. Mag.*, May, 1877, p. 321.

was made the subject of a preliminary investigation by du Bois,* who answered it by examining the rotation of the ray when reflected from small surfaces, ground flat, and polished, on ellipsoids of iron, steel, nickel, and cobalt, the ellipsoids being magnetised by means of a surrounding coil. He found that the relation is of a very simple kind: the rotation of the polarised ray is proportional to the intensity of magnetism **I**, and may, therefore, be written equal to K **I** when K is a constant† coefficient, to which du Bois gave the name of Kerr's constant. He determined numerical values of K for iron, steel, nickel, and cobalt. Knowing these, it is, of course, possible to invert the process, and use the measurements of optical rotation to determine values of **I** in cases where they are not otherwise known.

This du Bois has done in a way that will be readily understood by reference to Fig. 75 (taken from his second Paper). P_1 P_2 are the poles of a powerful electro-magnet, made conical, as in the isthmus method, for the purpose of concentrating magnetic force in the neighbourhood of the apex. Through one of them (P_2) a hole is bored to allow the polarised light to come to, and, after reflexion, to return from, the polished plate M, which is a small piece of the metal whose magnetism is to be examined, and is in contact with the magnet pole P_1. When the electro-magnet is excited, M is very strongly magnetised, and the value of **I** in it is measured by observing the angle of rotation of the polarised ray and dividing that by the known value of Kerr's constant, previously determined by experiments with a magnetised ellipsoid of the same material. J J is a steam jacket which was used to maintain the plate at a temperature approaching 100° C in some of the experiments.

We have said nothing yet about the manner of finding the magnetic force **H** acting on the plate M. It was not found directly; what was directly found was the induction **B**. When **I** and **B** are known, **H** may of course be deduced by means of the equation $\mathbf{H} = \mathbf{B} - 4\pi \mathbf{I}$. Now, in determining **B** in the plate,

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* *Phil. Mag.*, March, 1890, p. 263.

† Constant, that is to say, for any one metal, and for any one wave-length of light. K differs much for light of different wave-lengths.

it is to be borne in mind that there is no discontinuity in lines of induction. The lines of induction pass out of the plate normal to its surface, and **B** within the plate has the same value as the induction (or what is there the same thing, the magnetic force) in the air immediately in front of the plate. This fact was taken advantage of in determining **B**. It might have been measured ballistically by slipping out an induction coil laid on the face of M, or wound round the circumference of M; but the plan actually used by du Bois was an optical one. A thin glass plate G, with a silvered back S, could be interposed in the path of the ray, immediately in front of the plate M—that is to say, at the place where a determination of the magnetic field was wanted. A plane polarised ray passing through a plate of glass in a magnetic field suffers rotation, as Faraday originally showed, and the amount of this rotation is proportional to the magnetic force and to the thickness of the plate. In the present case the ray passed twice through the

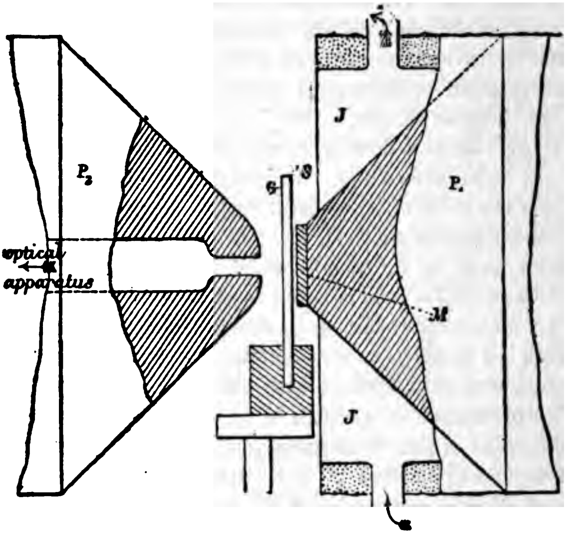

FIG. 75.—Optical Method of Measuring Magnetism in Strong Fields (du Bois).

glass plate. The plate was standardised by comparing the magneto-optic rotation in it with that in bisulphide of carbon, the value of which is well known.

Thus, by means of two independent optical measurements, there were determined, first, the value of **I** in the strongly magnetised plate M of iron or other magnetic metal (by observing the rotation of a polarised ray reflected by M), and, second, the value of **B** in the same plate, this last being equal to the magnetic force acting on the glass plate when the glass plate was put in front of the other, and being measured by observing the rotation of a polarised ray reflected from the silvered back of the glass.

§ 105. **Results of Optical Measurements.**—The general results which the optical method has yielded in the hands of du Bois, as to the action of strong fields, are in complete agreement with those obtained by means of the isthmus method and narrated above. The magnetic force was not pushed to such high values, but the values were high enough to show that a close approach to a limiting maximum of **I** had been reached. With nickel, the force **H** was raised to nearly 13,000, with cobalt to 8,500, with steel to 4,500, and with soft iron to 2,500. The limiting values towards which **I** tended appeared to be 530 in nickel, 1,200 in cobalt, and 1,630 in steel (a hard English cast steel). In the case of iron, the experiments were rather less satisfactory, but pointed to a limit between 1,700 and 1,750. These values are given for observations made when the specimens were at a temperature of 100°C.; at ordinary temperatures the values would be rather greater, as was shown by comparative tests at 100° C. and at 0° C. It is clear that these numbers are in good general agreement with those that have been stated already, §§ 98–102.

The following data are taken from du Bois' Paper. Table XX. relates to a specimen of cast cobalt, tested at 100° C., containing 5·8 per cent. of nickel, and 0·8 per cent. of iron. An additional observation, made in the strongest field, showed that **I** at 0° C. was 1,232. Table XXI. relates to a specimen of hard-drawn best nickel wire, stated to contain 99 per cent. of nickel. Here, again, a low-temperature observation in the strongest field made the value of **I** at 0° C. to be 579.

TABLE XX.—*Cobalt in Strong Fields at 100° C.*

H	B	I	μ
860	14,180	1,060	16·49
2,500	16,750	1,134	6·70
4,800	19,550	1,174	4·07
6,870	21,710	1,181	3·16
8,350	23,330	1,192	2·79

TABLE XXI.—*Nickel in Strong Fields at 100° C.*

H	B	I	μ
550	6,420	453	11·67
3,410	9,920	518	3·12
6,290	12,850	522	2·57
9,600	16,250	527	1·69
12,620	19,220	525	1·52

On making optical observations with a specimen of Hadfield's manganese steel—the non-magnetic steel spoken of in § 70—it was found that the amount of magneto-optic rotation of the polarised ray varied considerably when the ray was reflected from different parts of the same polished surface, from which result du Bois infers that this material is essentially heterogeneous, having relatively strongly magnetic layers interposed between non-magnetic or feebly magnetic portions of the mass. He supposes the structure to be laminar, but so fine-grained that to ordinary tests it appears homogeneous.

§ 106. **Magnetisation of Magnetite.**—Du Bois has applied his optical method to obtain absolute measurements of the magnetisation in strong fields of a crystal of magnetite (the magnetic oxide of iron, $Fe_3 O_4$), which is the only substance that shares with iron, nickel, and cobalt the distinction of being strongly magnetisable. The results show that there is a limiting maximum of **I** in magnetite with the value of about 350, and that saturation is practically complete with a force **H** of 1,000 or 1,500 units.*

§ 107. **Experiments with Ellipsoids.**—Reference has been made to the preliminary experiments with ellipsoids by means of which du Bois determined the values of Kerr's constant in specimens of the same metals as were afterwards tested in stronger fields by the optical process. The experiments with ellipsoids were important, not only as a means towards that end; they are interesting in themselves because they deal with a portion of the range in regard to which we have no other experimental data, namely, the portion which extends from **H** = 200 or so to **H** = 1,200 or 1,300. With respect to higher forces, we have the results of the isthmus and of the magneto-optic methods; with respect to lower forces, we have, of course, a mass of data; but between the limits named there is a gap which these experiments with ellipsoids are the first to bridge.

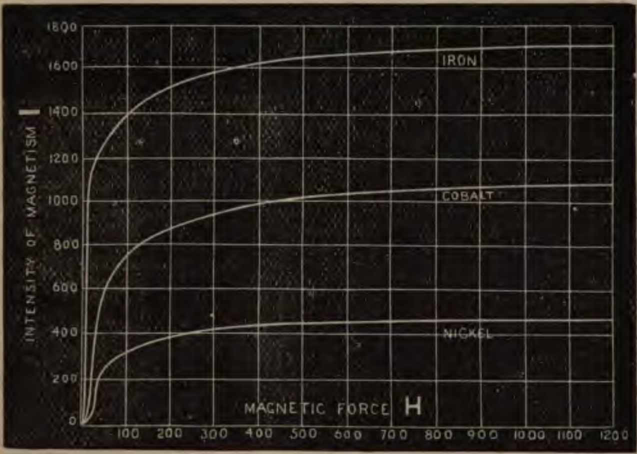

FIG. 76

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* *Phil. Mag.*, April, 1890, p. 301.

The magnetic force was applied by means of a coil, and the specimen was an ovoid or prolate ellipsoid of revolution 18cms. long, and 0·6cm. in diameter, which was kept at a uniform temperature of 0°C. or 100°C. by applying ice or steam. Its magnetisation was measured by the ordinary magnetometric method, a compensating coil being used to balance the greater part of the action of the magnetising coil upon the magnetometer. The ratio of diameters being 1 : 30, a correction of 0·052 **I** had to be subtracted from the magnetic force due to the coil to find the true magnetic force (*see* § 26).

Du Bois gives the results in the form of curves connecting **H** with the magnetic moment per unit of mass—that is, per gramme. It will be more convenient for us to adhere to the usual practice of stating magnetisation by the quantity **I**, which is the moment per unit of volume—that is, per cubic centimetre. The results are accordingly reduced to this form in the curves of Fig. 76 and in Table XXII., the numbers in which are calculated from measured values of the ordinates in du Bois' curves.* The iron tested was soft Swedish wrought iron, carefully annealed.

TABLE XXII.—*Iron, Cobalt, and Nickel, in Moderately Strong Fields.*

Magnetic Force, **H**.	Intensity of Magnetism, **I**.		
	Iron at 0°C.	Cobalt at 100°C.	Nickel at 100°C.
100	1,410	—	313
200	1,520	856	375
300	1,580	933	406
400	1,627	988	428
500	1,658	1,018	441
600	1,677	1,032	450
700	1,689	1,048	456
800	1,697	1,056	459
1,000	1,705	1,080	467
1,200	1,710	1,090	471

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* *Phil. Mag.*, April, 1890, Plate VIII, Fig. 1.

The observations with cobalt and nickel were made at 100°C., but within the range of magnetic forces that is dealt with here the difference between 100°C. and atmospheric temperature has but little influence on the magnetisation.

In Fig. 76 the same results are given, and the curves are completed to the origin (in an approximate fashion) by sketching in, from other data, the parts that relate to lower forces. The gradient of the cobalt curve at the upper end shows how cobalt needs a stronger field than the others to make its magnetisation approach closely to a state of saturation.

CHAPTER VIII.

EFFECTS OF TEMPERATURE.

§ 108. Effects of Temperature on Magnetic Quality: Loss of Magnetic Quality at a High Temperature.—It has been known from the time of Gilbert that when iron or steel is heated to bright redness it loses the power of either retaining magnetism or having magnetism induced in it, but recovers its susceptibility on cooling. The same thing happens at a higher temperature with cobalt and at a lower temperature with nickel. In general, the change from the magnetic to the non-magnetic state occurs somewhat suddenly as the temperature is raised. Thus, in one of the experiments of Hopkinson—to be referred to presently in more detail—a piece of wrought iron, subjected to the action of a weak magnetic force, was found to be highly susceptible so long as the temperature did not exceed 775°C. In fact, up to this point the effect of heating was to increase the magnetic susceptibility, and at the temperature 775°C. it was many times greater than when the iron was cold. But with further heating an extremely rapid loss of magnetic quality ensued; when the temperature had risen only 11deg. higher, to 786°C., the iron had become practically non-magnetic. Its permeability was then only 1·1, whereas at 775°C. it had been no less than 11,000. If the test be made with a strong magnetic force instead of a weak one the change from the magnetic to the non-magnetic state is less abrupt, but it is equally complete, and the same temperature as before makes the iron non-magnetic.* Hopkinson calls this the *critical temperature*. The

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* Reference should be made in this connection to the experiments of Baur, in the Paper "Experimentelle Untersuchungen über die Natur der Magnetisiringungsfunction," already cited. (Wiedemann's *Annalen*, 1880 Vol. XI.)

value of the critical temperature varies in different specimens: in samples of ordinary iron and steel it has been found to range from 690°C. to 870°C.* In an impure specimen of nickel examined by Hopkinson the critical temperature was 310°C.†

§ 109. **Change of Physical State at the Critical Temperature.**—The change from the magnetic to the non-magnetic state which iron or steel undergoes at the critical temperature is only one of several evidences that the metal then suffers a profound change of constitution. One evidence of this change is furnished by the fact, observed in 1869 by Gore, that an iron wire, cooling from a bright red heat, suffers a momentary elongation (at a dull red) and then goes on contracting as before. The change shows itself in the alteration of other physical qualities as well as those that have to do with magnetism. Thus Tait ‡ has found that the thermo-electric quality of iron alters in a remarkable way at a red heat. The alteration takes place suddenly, and there is no reason to doubt that it is associated with other changes that are brought about by raising the temperature to the critical value. Again, as the experiments of W. Kohlrausch § and Hopkinson ‖ have shown, the critical temperature is marked by a sudden change in the coefficient which expresses the effect of temperature upon the electrical resistance of iron. The same thing is true of nickel. Perhaps the most striking evidence that when iron reaches the critical temperature it passes—more or less suddenly—from one condition to another widely different condition, is furnished by Barrett's discovery of "recalescence." ¶ Let a piece of iron or steel be heated to bright redness and allowed to cool slowly; at a certain stage (coincident with that at which Gore's phenomenon occurs), the process of cooling experiences a sudden check. Heat is generated within the substance of the metal as a consequence of the change which the molecular constitution suffers at this

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* Hopkinson, "Magnetic and other Physical Properties of Iron at a High Temperature." *Phil. Trans.*, 1889, A., p. 443.

† Hopkinson, "Magnetic Properties of an Impure Nickel." *Proc.* Roy. Soc., Vol. XLIV., 1888, p. 317.

‡ Tait, *Trans.* Roy. Soc. Edin., 1873.

§ Kohlrausch, Wied. *Ann.*, Vol. XXXIII, 1888.

‖ Hopkinson, *loc. cit.*

¶ Barrett, *Phil. Mag.*, January, 1874.

critical point; the cooling is arrested, and the temperature may even rise, though the loss by radiation is going on as before. It is in hard steel that the phenomenon is most marked. So much heat is generated while hard steel passes from one molecular state to another at the critical point, that there is a very visible reglow; the surface of the cooling metal turns for a few moments from a very dull to a much brighter red, after which the colour continues to fade. The point at which recalescence takes place is the point at which the cooling metal returns from the non-magnetisable to the magnetisable state. This fact, which was surmised by Barrett, has been proved by the experiments of Hopkinson, who has measured the amount of heat liberated during the change, and has, moreover, given further proof that recalescence has an intimate connection with the recovery of magnetic quality, by showing that it does not occur at all in non-magnetisable manganese steel.*

§ 110. **Effects of Temperature below the Critical Point.**—In studying the effects of temperature we may adopt one or other of two distinct methods. We may note the changes of magnetism which are brought about by varying the temperature, while the magnetic force is kept constant; and as a special case of this we may note the changes of residual magnetism which are brought about by varying the temperature when there is no magnetic force in action. Or we may compare the amounts of magnetism which are acquired at one and another temperature when the specimen is brought to the temperature in question before the magnetic force is applied. In other words, we may determine the form which the curve of I or B and H assumes, when the one or another temperature is

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* Hopkinson, *loc. cit.* A corresponding perturbation, involving absorption instead of evolution of heat, is observed during the heating of steel when the magnetic state changes to non-magnetic. In regard to the general subject of recalescence, reference should be made to the important investigations of Osmond ("Transformations du fer et du carbone," *Mém. de l'artillerie de la marine*, 1888), which deal especially with the temperature at which the phenomena of recalescence occur. A general account of the associated phenomena will be found in the Report of a Committee of the British Association (B. A. Report, 1890). See also papers by H. Tomlinson and H. F. Newall, *Phil. Mag.*, 1887, vol. xxiv., pp. 256 and 435.

maintained constant throughout the process of magnetisation, and may compare the curves got in this way at various temperatures. The two methods do not yield identical results, because of the tendency which the magnetic metals exhibit to oppose magnetic change—the property, namely, which gives rise to those effects which are included under the general name of magnetic hysteresis. On account of this property, which all the magnetic metals share more or less, the magnetic condition that is arrived at if we heat the specimen to any assigned temperature first, and then apply any assigned magnetic force, is in general different from the condition that is reached when the order of these two operations is reversed. The same remark applies with respect to the changes of magnetic quality that are brought about by altering the state of stress, or any other physical condition on which the magnetic state of the specimen depends. In the most complete investigations which have yet been made of the effects of temperature on magnetic quality, the plan has been followed of varying the temperature first, and then studying the effects of applying magnetic force. In other words, what have generally been observed and compared are the susceptibilities of the same specimen at different temperatures.

The experiments of Rowland, Baur, and Hopkinson are of this kind. Rowland,* examining the susceptibility of nickel at two temperatures (5°C. and 230°C.), found that at the higher temperature there was much more susceptibility with respect to weak magnetic forces than at the lower temperature, but less susceptibility with respect to strong forces. In other words, when the magnetisation was sufficiently high the effects of temperature upon susceptibility became reversed. The maximum susceptibility, occurring as it does when the magnetic force is tolerably low, was greater at the higher temperature (some 70 per cent. greater at 230°C. than at 5 deg.). In cobalt, again, he showed that the susceptibility with respect to low forces is increased by heating—a specimen in which the maximum susceptibility (κ) was 11·2 at 5°C. had its maximum susceptibility raised to 18·7 at 230°C. The magnetic forces used by Rowland were not strong enough to reverse this effect of temperature in cobalt, but it is now known that under sufficiently strong force the effect is reversed in that metal, as it is in nickel. Baur † has shown

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* Rowland, *Phil. Mag.*, Nov., 1874. † Baur, Wied. *Ann.*, 1880, Vol. XI.

that iron behaves in the same way. If we compare the susceptibility of iron at two temperatures we find that the susceptibility is greater at the higher temperature provided the magnetic force does not exceed a certain value, but less at the higher temperature when the force does exceed that value. It is with respect to weak forces that the influence of temperature is most conspicuous. The most complete experiments on the subject are those of Hopkinson, who has given in the two Papers cited above (one dealing with iron and steel and the other with nickel) a series of curves of magnetisation for each metal at various temperatures, ranging up to the critical temperature at which magnetic quality disappears. A few of his results may be quoted as the best means of giving some account of the connection between magnetic quality and temperature.

§ 111. **Hopkinson's Experiments on the Magnetisation of Iron at Various Temperatures.**—In these experiments the specimens were rings, and the magnetisation was measured ballistically by reversing the magnetising force. The primary and secondary coils were insulated with asbestos paper; the ring was placed in a cast-iron box, and was heated by a gas furnace, and its temperature was inferred from the resistance of the secondary coil, which was measured before and after each magnetic experiment.

A ring of soft wrought-iron, for which the critical temperature had been found to be about 785°C., was examined in successive experiments, at various temperatures, the curve of B and H being determined in each case, while the temperature was kept as nearly constant as was practicable. The results show that heating the iron to a high temperature (short of the critical temperature) augments its susceptibility with respect to small magnetic forces very greatly. On the other hand, it reduces greatly the effect of strong magnetic forces. For example, a force H of 0·075 C.-G.-S. was found to give the following values of B at the temperatures noted :—

Temp.	10°C.	378°C.	494°C.	603°C.	670°C.	722°C.	744°C.	763°C.	775°C.	778°C.
B	17	41	45	59	120	144	203	294	494	512

At 788°C., the critical point having been passed, the induction had practically sunk to zero. These figures show well the enormous increase of permeability which heating causes at early stages of the magnetising process. On the other hand, a force of 50 units or so gave less than half as much induction at the upper as at the lower end of this range of temperature. Thus the curves of **B** and **H** taken at any two temperatures cross each other, the one for the lower temperature lying at first below and afterwards above the other. Hopkinson has expressed his results in curves of this kind, some of which are copied in Figs. 77 and 78. Curve I. in these figures

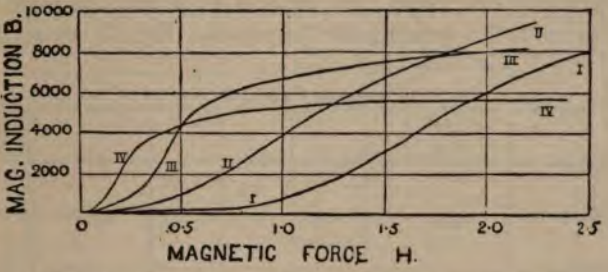

Fig. 77.—Magnetisation of Iron at Various Temperatures.

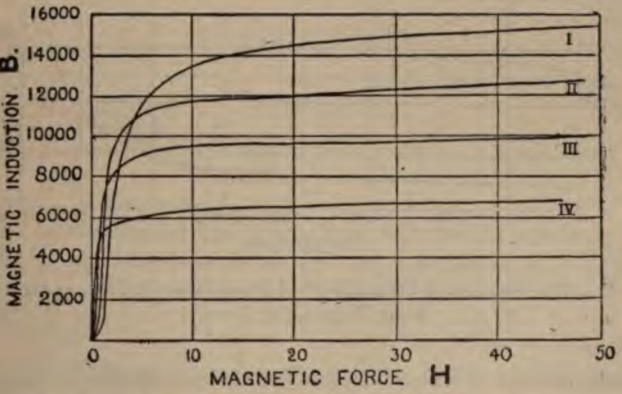

Fig. 78.—Magnetisation of Iron at Various Temperatures.

is for a temperature of 10°C.; curve II. for a temperature of 670°C.; curve III. for a mean temperature of about 742°C. (it varied a few degrees during the observations); and curve IV. for a mean temperature of about 771°C. In Fig. 77 the early portions only are shown; the scale of $\mathbf{H}$ is wide, in order to display well the crossing of the curves. Fig. 78 shows the

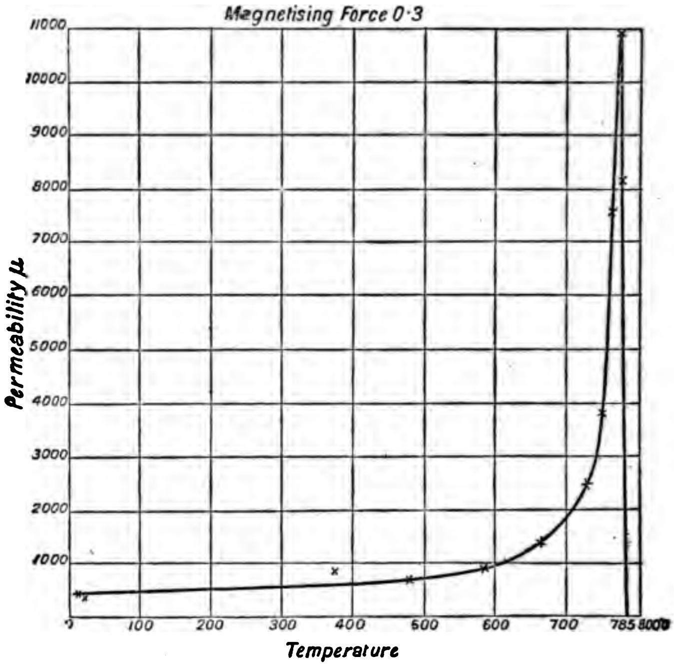

Fig. 79.—Relation of Permeability of Temperature in Iron, under a Weak Magnetising Force.

whole process of magnetisation (in the same group of experiments) with a twenty-fold smaller scale of $\mathbf{H}$. The rapid rise and low apparent saturation value in curve IV., where the temperature approaches most closely to the critical value, are to be noted. The same results are shown in a different manner in Figs. 79, 80, and 81, also copied from Hopkinson's paper. These give the permeability μ in relation to the temperature for three specified values of the magnetising force (0·3, 4·0, and 45). Fig. 79 shows in a striking way the suddenness with which susceptibility to small magnetic force is lost at the critical point, and how this is preceded by an enormous augmentation of susceptibility; the other two curves show how much more gradual is the passage from the magnetic to the non-magnetic state when we have to deal with stronger forces. In the curve of Fig. 79 the permeability at atmospheric temperatures is 367 ; as the temperature rises it increases at first slowly and afterwards with great rapidity, reaching the maximum already mentioned of 11,000 at 775°.

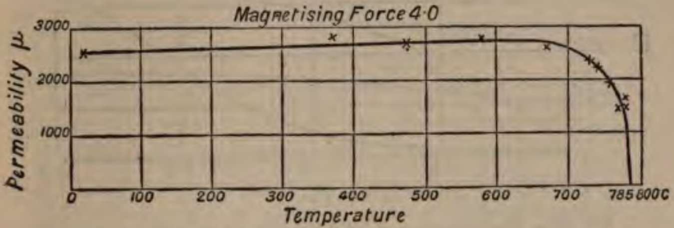

FIG. 80.—Relation of Permeability to Temperature in Iron, under a Moderate Magnetising Force.

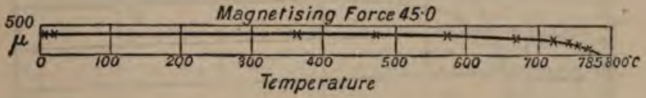

FIG. 81.—Relation of Permeability to Temperature in Iron, under a Strong Magnetising Force.

§ 112. **Whitworth's Mild Steel.**—Figs. 82 and 83 give a corresponding selection of Hopkinson's results, for a specimen of mild steel contained 0·126 per cent. of carbon. Its critical temperature was 721°C., above which the permeability was only 1·12. The curves reproduced here correspond to the following temperatures :—Curve I., 12°C. ; curve II., about 620°C. ; curve III., about 715°C. It will be seen that these present the same characteristics as the corresponding curves for iron. With a magnetising force of 0·3 the highest permeability is over 9,000, and this is found at a temperature only a very few degrees below the critical point.

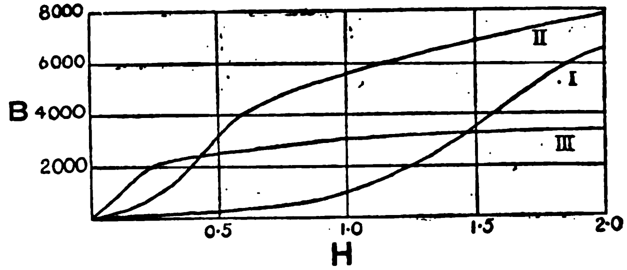

Fig. 82.—Magnetisation of Mild Steel at Various Temperatures.

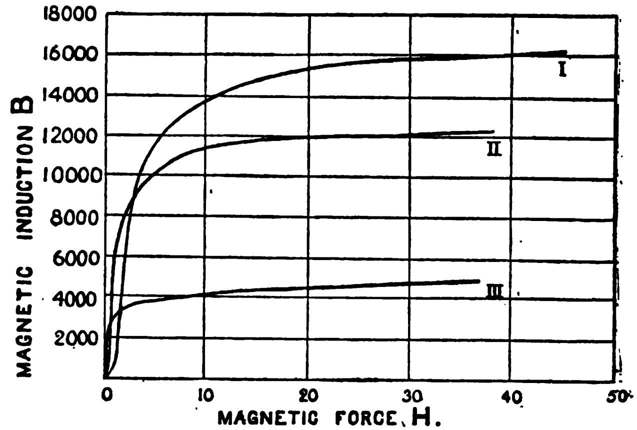

Fig. 83.—Magnetisation of Mild Steel at Various Temperatures.

§ 113. **Whitworth's Hard Steel.**—Fig. 84, taken from the same source, relates to a sample of hard steel containing 0·96 per cent. of carbon. The sample was annealed before the observations were made. The three curves, I., II., and III., are for three temperatures, 9°C., about 522°C., and about 678°C. respectively. Fig. 85 is the curve of μ and temperature for the same sample, the magnetising force being 1·5.

The loss of magnetic quality at the critical temperature is scarcely so sudden as in wrought iron and mild steel, and the influence of heating is more uniformly distributed at lower temperatures; but the same general characteristics are again obvious.

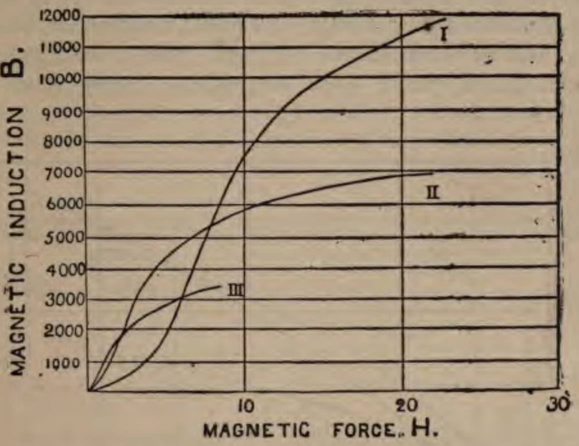

Fig. 84.—Magnetisation of Hard Steel at Various Temperatures.

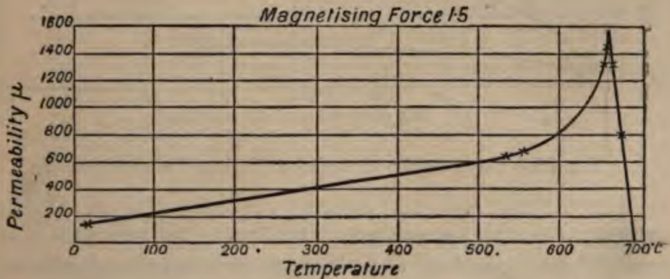

Fig. 85.—Relation of Permeability to Temperature in Hard Steel.

§ 114. **Hopkinson's Experiments with Nickel.**—In dealing with nickel, Hopkinson* has pursued the same method of experiment, using a ring, and finding, by the ballistic method, curves of magnetisation, while the ring was maintained at one

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* *Proc.* Roy. Soc., Vol. XLIV., 1888, p. 317.

or another of a series of temperatures which ranged up to the critical point at which magnetic susceptibility disappears. The specimen tested was impure, containing 95 per cent. of nickel, with about 1 per cent. (each) of iron, cobalt, and carbon, and 1½ per cent. of copper. Its critical point was about 310°C. A little below that temperature the susceptibility diminished very rapidly with rise of temperature, though there was no such excessively rapid loss of susceptibility as iron shows (under weak magnetising force) when the critical point is approached. At lower temperatures the susceptibility was observed to increase with rise of temperature when the magnetic force was low, but to decrease with rise of temperature when the magnetic force was high, in accordance with what has been already described as characteristic of the effects of temperature upon all the magnetic metals. Thus, taking curves of **B** or of **I** and **H** at any two temperatures (both well below the critical point) it is found with nickel, as with iron, that the curve for the lower temperature lies at

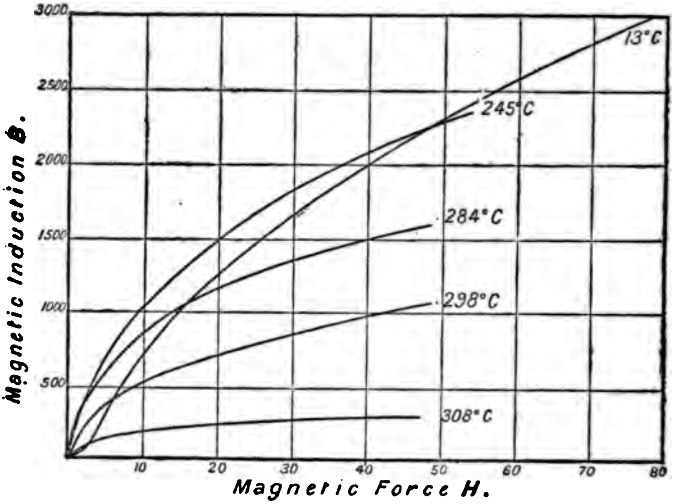

FIG. 86.—Magnetisation of Nickel at Various Temperatures.

first below and afterwards above the other. Figs. 86 and 87 give a selection of Hopkinson's curves. In Fig. 86 the curves of **B** and **H** are drawn for five temperatures: one is an ordinary atmospheric temperature, and the other four are high temperatures tending towards the critical point. At the first of these (245°C.) there is a marked gain of susceptibility for forces lying below 45 or 50. The whole group illustrates well how the loss of magnetic quality supervenes when the temperature is sufficiently raised.

The same results are shown in a different manner in Fig. 87. There the induction **B** is represented as a function of the temperature—the induction, namely, that is reached when the metal is magnetised at constant temperature by the application of the force which is specified separately for each curve. These curves should be compared with the curves of μ and temperature which have been given for iron and steel (Figs. 71—81, and 85).

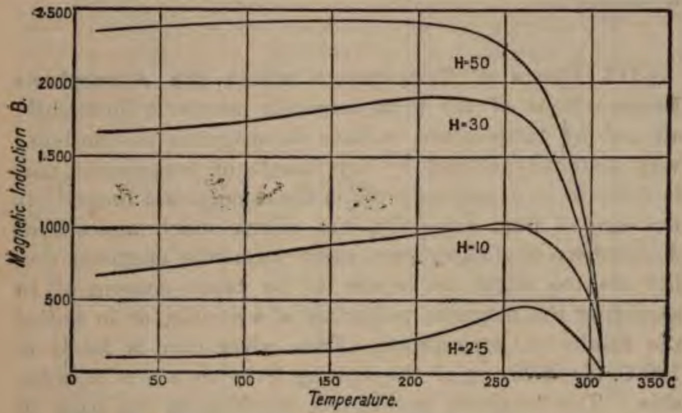

FIG. 87.—Magnetisation of Nickel at Various Temperatures.

The main points of difference in the magnetic behaviour of nickel and iron with respect to temperature are, that in nickel the effects of temperature, when the temperature is low, are more considerable than they are in iron; that in nickel the critical point is much lower; and that in nickel the change from the magnetic to the non-magnetic state is much less abrupt than in iron. Perhaps for this reason the change is not associated with any such striking physical changes as accompany it in iron. Nickel does not recalesce, and an experiment of Hopkinson's* shows that the change from the non-magnetic to the magnetic state, as the metal cools, is attended by no sudden liberation or absorption of heat. Notwithstanding the fact that the specimen tested by Hopkinson was not pure the critical point found with it appears to be fairly representative of the critical point in nickel. In another sample tested by du Bois,† the critical point again occurs about 300°C.

§115. **Effects of Temperature within the Atmospheric Range.**—None of the three magnetic metals is sufficiently affected by temperature to have its magnetic susceptibility very materially altered by any change of temperature that is liable to be experienced within the atmospheric range. In the case of iron, especially, the effects which atmospheric fluctuations of temperature exert upon the magnetic quality are too slight to require to be taken account of in specifying the magnetic properties of a sample, or in stating the results of experiments. Even when iron is raised to 100°C. the influence of the heating is by no means considerable. This is shown in Fig. 88, which gives two pairs of curves of I and H, one referring to iron wire in the soft annealed state, and the other to the same wire after it had been hardened by stretching beyond the limit of elasticity.‡ The full line in each pair is a curve of magnetisation taken at atmospheric temperature (7° or 8°C. in this case), and the dotted line is a curve of magnetisation taken while the wire was maintained at a temperature of 100°C. by enclosing it in a tube through which a current of steam was kept up. The curves cross at much the same value of I for both conditions of the metal, though at very different values of H.

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* Hopkinson, *loc. cit.*, p. 319.

† Du Bois, *Phil. Mag.*, April, 1890.

‡ *Phil. Trans.*, 1885, p. 637.

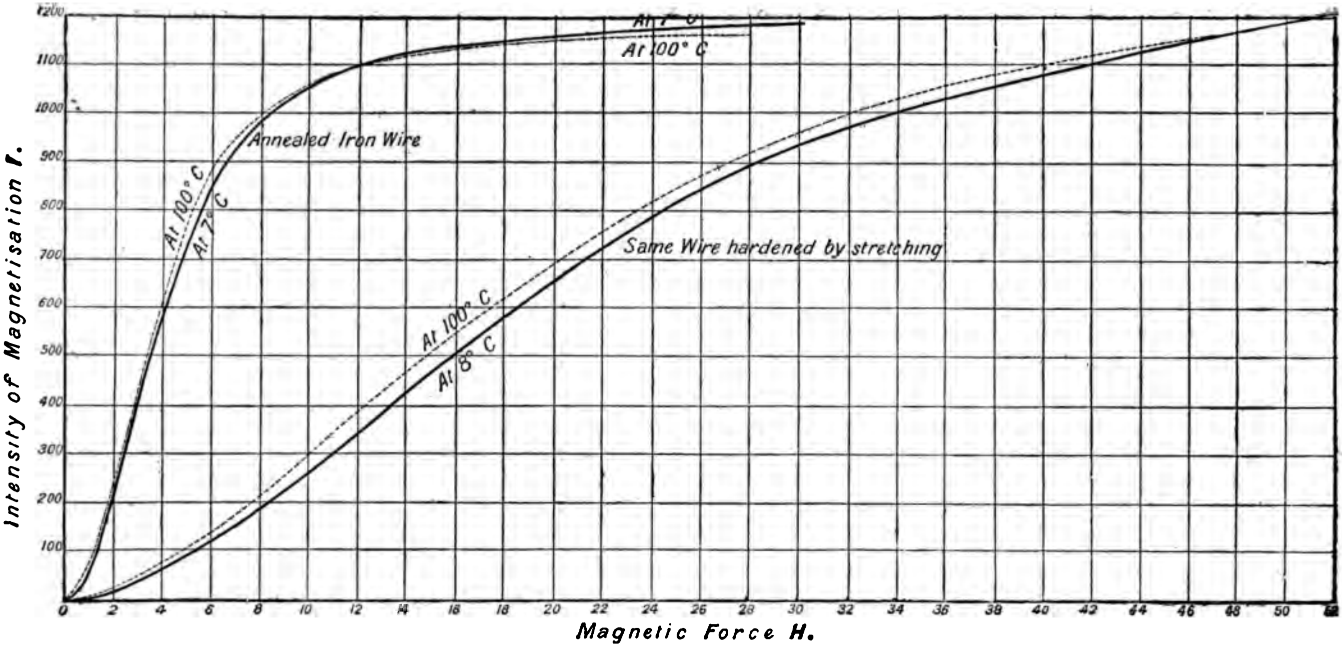

FIG. 88.—Effects of Heating to 100°C. on the Magnetic Susceptibility of Soft and Hard Iron.

§ 116. **Effects of Varying Temperature, the Magnetic Force being Constant.**—In the experiments which have been noticed above, the temperature was kept constant and the magnetic force was varied. If we keep the force constant and vary the temperature, a rather complex series of effects will be observed. In the first place, there is in general an effect which is not reversible—that is to say, which would not be undone if the temperature were brought back to its initial value. The first effect of any heating is like the effect of vibration: it produces a permanent change in the magnetisation; but whether that change will be an increase or a decrease will depend on the previous history of the magnetised piece. The reason of this should be apparent when we come to discuss molecular theories of induced magnetism: in effect it is this, that at any (not extreme) stage in the process of magnetisation there are groups of molecules verging on instability, which are precipitated into instability when the temperature begins to change. This effect is distinct from—and may be much greater than—the reversible changes of magnetism which are caused by alternate heating and cooling. But when any alternation of heating and cooling is sufficiently often repeated, a cyclic *régime* is established; the magnetism will then fluctuate between two values, but whether the higher or the lower value will correspond to the hotter state will depend on whether the magnetism is below or above a certain value. In other words, the effects of temperature, when tested in this way (by repeated alternate heating and cooling), become reversed when the magnetisation is sufficiently strong. When there is but little magnetisation heating augments, and cooling reduces the amount of magnetism, whether that be either residual or induced by the action of a constant magnetising force; when there is much magnetisation the reverse happens. The reversal of effect which is observed in experiments of this class is evidently to be anticipated in connection with the crossing of the magnetisation curves in experiments of the class described above. But, on acoount of complications proceeding from magnetic hysteresis, it is not possible to infer, from results of experiments of the one kind, where the reversal should occur in the other.

That the first effect of any change of temperature on the magnetism of iron or steel is not reversible has been shown by many experiments, probably first by those of Wiedemann.[*] If the magnetism that is dealt with is residual, and there is no acting magnetic force, a cyclic process of heating and cooling, or of cooling and heating, results in a reduction of the magnetism. If, on the other hand, the magnetism that is dealt with is that which has been reached by applying a magnetising force which is kept in action while the temperature is changed, then a cyclic process of heating and cooling or of cooling and heating results in an increase of magnetism. In either case it is, in general, only after many repetitions of the temperature cycle that the change of magnetism becomes cyclic. In the earliest cycles we find, superposed upon what may be called the legitimate or differential variations of magnetism, a progressive shaking in of magnetism if that is induced, or a progressive shaking out of magnetism if that is residual. It is in the first cycle that this is most conspicuous, but it can often be traced in the second and even in later cycles. By repeating the temperature cycle often enough we may get rid of these progressive changes, and may then study the differential effects of heating and cooling. One or two experiments of this class will be briefly referred to.

§ 117. **Experiments in Alternate Heating and Cooling of Magnetised Iron.**—In the first of these[†] a hardened iron wire was tested, surrounded by a tube, on which a magnetising coil was wound, and through which currents of steam or of cold water could be alternately passed. The resulting magnetic changes were observed by means of a magnetometer. The wire was at first demagnetised, and then from time to time its magnetism was increased a step by passing a weak current through the magnetising coil; but after each such step the current was stopped, and the only force in action was the vertical component of the earth's field. The first series of heatings and coolings (between 100°C. and 6°C.) made $\mathbf{I}$ fluctuate between about 2·14 (cold) and 2·23 (hot). At this stage heating increased $\mathbf{I}$. Later the fluctuations of $\mathbf{I}$ were from 3·51 (cold) to 3·56 (hot); the effect still had the same sign. Later still it became reversed; the fluctuations of $\mathbf{I}$ were between 8·69 (cold) and 8·67 (hot); and later still the

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[*] "Galvanismus," II., § 522 *et seq.*

[†] *Phil. Trans.*, 1885, p. 633.

reversed effect was more marked, the range being from 9·09 (cold) to 9·04 (hot).

A similar experiment with a piece of annealed iron wire* showed that the reversal of effect took place in that case when **I** was about 20. At an early stage **I** ranged from 4·77 (at 6°C.) to 4·95 (at 100°C.); at a later stage (after reversal) **I** ranged from 37·53 (at 6°C.) to about 36·77 (at 100°C.). In both cases the reversal occurred at a very early point in the process of magnetisation.

In another case † the magnetisation of the specimen was examined at intermediate points, during heating and during cooling, to see whether there was hysteresis in the relation of magnetism to temperature. The specimen—a long iron wire—was fixed inside a glass tube which could be connected at one end to any one of three small boilers capable of supplying a steady current of steam, of alcohol vapour, and of sulphuric ether vapour, or to a cistern supplying cold water. Steam and cold water (at 14°C.) were alternately passed through the tube many times until the magnetic state of the wire was observed to change from one to the other of two nearly steady values. Then readings of the magnetometer were taken during the passage through the tube of (1) cold water, (2) ether vapour, (3) alcohol vapour, (4) steam, (5) alcohol vapour, (6) ether vapour, (7) cold water. This completed a cycle of temperature changes in which two intermediate points (35°C. and 78½°C.) were fixed during the process of heating and cooling. The method was adopted in order to secure that the iron should be exposed sufficiently long to an atmosphere of definite temperature to give it time to take that temperature throughout, and so avoid any possibility of error proceeding from the sluggishness with which changes of temperature take place. The stream of vapour was kept up in every case until the magnetometer reading became steady.

The iron was magnetised to begin with sufficiently strongly to make the heating cause a diminution and the cooling cause an augmentation of magnetism. The only magnetic force in action during the heating and cooling was the earth's vertical field. In the following statement of observed results the

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[*] *Loc. cit.* p. 635.

[†] *Loc. cit.*, p. 631.

numbers are proportional (on an arbitrary scale) to the intensity of magnetisation. The arrows show the sequence of the changes:

	Water.		Ether vapour.		Alcohol vapour.		Steam.
Temp.	14°C.		35°C.		78½°C.		100°C.
	17,416	→	17,382	→	17,304	→	17,262
	17,418	←	17,382	←	17,304	←	

The whole change was about 0·9 per cent. of the whole magnetism. It is clear from the readings at 35°C. and 78½°C. that the changes occurred without any perceptible hysteresis; the magnetisation at intermediate points was, as nearly as can be judged, the same during heating as during cooling.

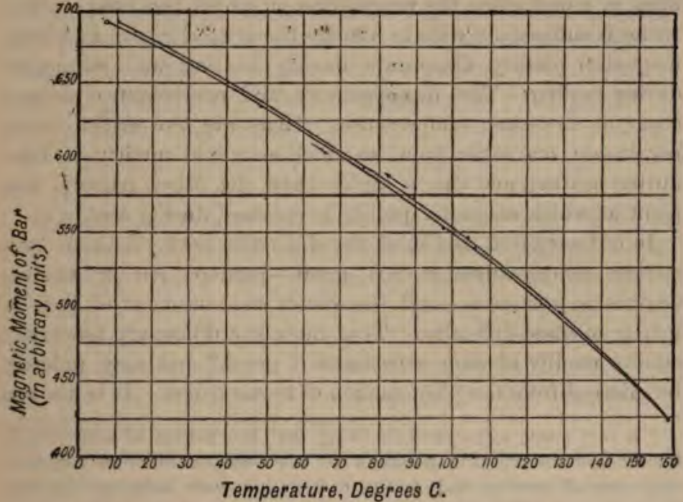

Fig. 89.—Effects of Heating and Cooling a Steel Bar Magnet.

Fig. 89 shows the result of another experiment of the same class,* in which a steel bar magnet was heated and cooled in a bath of oil, through a considerably wider range of temperature (between 10°C. and 158°C.). The temperature of the oil was altered very slowly—so slowly that it took more than 17 hours to complete the cycle of changes shown in the figure—in order to let the bar's temperature be sensibly uniform with that of

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* *Loc. cit.*, p. 638.

the oil, which was observed by means of two thermometers. Several heatings and coolings preceded the cycle in which the observations were made. Here, as in the former case, there was no sensible hysteresis in the relation of magnetism to temperature.

§ 118. **Hysteresis in the Relation of Magnetic Susceptibility to Temperature.**—Although no hysteresis appears in the experiments which have just been described—the magnetic condition depending, apparently, only upon the actual temperature and not upon past temperatures—it is still possible to experiment under conditions which show hysteresis in the relation of magnetic quality to temperature. Hysteresis of this kind is found when the range through which the temperature varies is sufficiently wide to include the critical region in which magnetic quality disappears during heating and reappears during cooling. The disappearance and reappearance do not occur at the same temperature. There are two critical temperatures: one is the point at which magnetic quality is lost during heating, and this is higher than the other, namely, the point at which magnetic quality is regained during cooling.

In ordinary iron and steel the difference between these two critical temperatures is not great—perhaps ten or twelve degrees in soft iron—and the direct measurement of it is a matter of some difficulty. That there is a difference, however, admits readily of easy experimental proof,* and may, indeed, be inferred from the phenomenon of recalescence. It is known

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* A very pretty experiment, showing that the change of state which iron or steel undergoes in passing a red heat occurs at different temperatures during heating and during cooling, has been described by Mr. H. F. Newall (*Phil. Mag.*, June, 1838), and also (*Rep.* Brit. Assoc., 1889, p. 517; *Proc.* Roy. Dublin Soc., 1886) by Mr. F. T. Trouton. A lamp flame, held under an iron or steel wire (which is in circuit with a galvanometer), so that a short portion of the wire becomes red hot, is made to travel slowly under the wire, and it is found that a current appears in the galvanometer, the direction of the current depending on the direction in which the flame travels. The current is due to difference in thermo-electric quality between that part of the iron which has changed its state by passing the critical point and that part which has not changed its state, and depends on the fact that on the side which is being heated the change of state is occurring at a higher temperature than on the side which is being cooled.

that in recalescence there is, superposed upon the process of cooling, a real rise in temperature, and it is known that the change of magnetic state occurs simultaneously with the recalescence. If the change of constitution which then takes place depended upon the actual temperature alone, and not upon preceding temperatures, this rise in temperature would be impossible, for it would undo the very change which causes it. To make it possible, the altered state of the material must be able to stand some elevation of temperature without changing back again; in other words, the change of constitution must show hysteresis with respect to temperature.

Osmond,* who has made a very full investigation of the temperatures at which perturbations take place during the processes of heating and cooling, finds that with electrolytically deposited iron there is a marked evolution of heat during cooling, about 855°C., and a marked absorption of heat during heating, about 867°C. There is no reason to doubt that it is at and about these temperatures that the changes from the non-magnetic to the magnetic state and from the magnetic to the non-magnetic state respectively occur.

In hard steel Osmond's experiments show a wider difference between the two critical temperatures. The principal evolution of heat during cooling occurs at 674°C., and the corresponding absorption of heat during heating occurs at 705°C. At any temperature between these limits we should expect to find this steel magnetic if the immediately preceding temperature had been lower, but non-magnetic if the immediately preceding temperature had been higher.

A much more remarkable difference of the same kind is found in what is called nickel-steel. Hopkinson has examined several alloys of iron and nickel, and has discovered, by direct magnetic tests, that in some of them the metal may retain either a magnetic or a non-magnetic condition throughout an extraordinarily wide range of temperature.† His results as to other physical properties of these alloys, as well as their magnetic properties, are of the highest interest.

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* Osmond, "Transformations du fer et du carbone," *Memorial de l'Artillerie de la Marine*, 1888.

† Hopkinson, *Proc.* Roy. Soc., December 12, 1889; January 23, 1890; May 1, 1890.

§ 119. **Hopkinson's Experiments with Nickel-Iron Alloys.**—The samples were tested ballistically, in the form of rings, the temperature being inferred from the resistance of the secondary coil. A sample containing 4·7 per cent. of nickel and 0·22 per cent. of carbon gave a magnetisation curve, at the temperature of the atmosphere, resembling the curves given by ordinary mild steel. When this specimen was heated, the magnetic quality being tested by reversals of a magnetising force of 0·12 C.-G.-S., it was found to lose susceptibility as the temperature approached 800°C., and not to regain it on cooling until the temperature had fallen to 650°C. or 600°C. Fig. 90 shows the changes which took place during heating and cooling; it gives the induction B produced by reversing the force H of 0·12, in terms of the temperature. It will be seen that there is a clear range of about 150 degrees within which the metal may exist in either of two states: in one state it is as susceptible as ordinary mild steel; in the other it is practically non-magnetisable, the permeability being, in fact, only about 1·4. Under stronger magnetising forces the magnetic quality appears and disappears at about the same two points. Further, an experiment in which the time rates of heating and of cooling were observed showed that the same two temperatures were marked by perturbations such as occur at the critical temperature in iron, the higher temperature being associated with an absorption of heat in the process of heating, and the lower temperature with an evolution of heat in the process of cooling. The heat which was liberated in cooling, at the temperature where

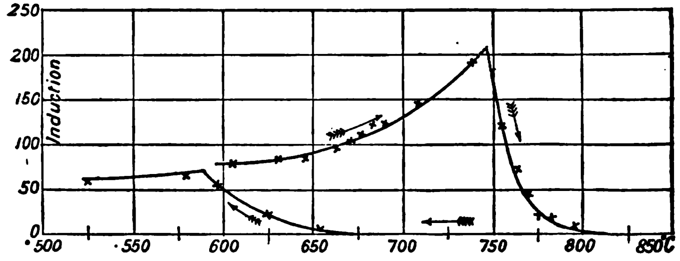

FIG. 90.—Steel with 4·7 per cent. Nickel. Magnetising Force 0·12.

magnetic quality returned, was found to be about 150 times the quantity which would raise the temperature of the piece by one degree.

Still more striking results were obtained by Hopkinson with a specimen containing 25 per cent. of nickel. This was non-magnetisable at ordinary temperatures in its primitive state, but on being cooled in a freezing mixture it became magnetisable at a temperature a little below the freezing point. Rendered magnetisable in this way, it retained its magnetic quality on being warmed until the temperature rose to 580°C. At that temperature it became again non-magnetisable, and remained so on cooling down to the ordinary temperature of the air. Within a range of about 600 degrees this steel is capable of existing, quite stably, in either state. Figs. 91 and 92 show the

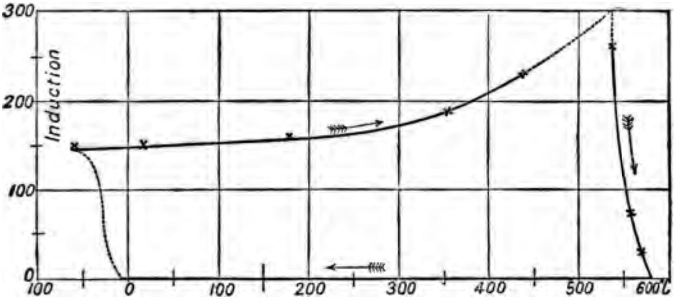

Fig. 91.—Steel with 25 per cent. Nickel. Magnetising Force 6·7.

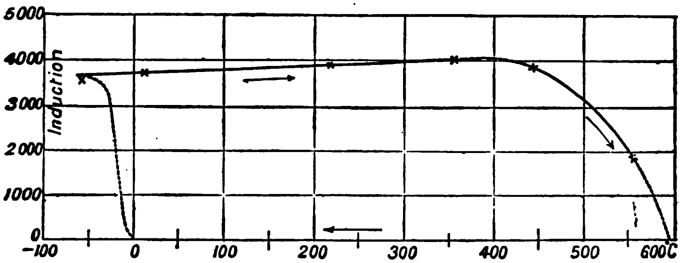

Fig. 92.—Steel with 25 per cent. Nickel. Magnetising Force 64.

induction **B** (produced by reversals of magnetic forces equal to 6·7 and 64 respectively) in terms of the temperature. In the non-magnetisable state the permeability is only 1·4; in the magnetisable state the permeability resembles (but falls rather short of) that of hard nickel. The curve of magnetisation (at 13°C.) is copied in Fig. 93. Hopkinson has also shown that other physical properties of this alloy change along with its magnetic properties. The electrical conductivity is markedly different in the two states: at 0°C., for instance, the specific resistance is only 0·00052 if the substance has been brought into its magnetisable state by applying a freezing mixture, but is 0·00072 if it has been brought into the non-magnetisable state by previous heating above 600°C.

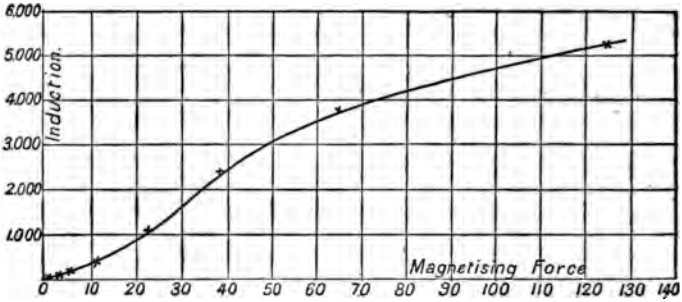

FIG. 93.—Steel with 25 per cent. Nickel. Curve of **B** and **H**.

Equally pronounced differences are found with regard to extensibility and strength. In the non-magnetisable state this metal is comparatively soft; wires show an elongation of 30 per cent. or more before rupture, and break with a load of about 50 tons per square inch. In the magnetisable state it is much harder; there is only 7 or 8 per cent. of extension, and the strength is as much as 85 tons per square inch, or even more. "If," says Hopkinson, "this material could be produced at a lower cost these facts would have a very important bearing. As a mild steel the non-magnetisable material is very fine, having so high a breaking stress for so great an elongation at rupture. Suppose it were used for any purpose for which a mild steel is suitable on account of this considerable elongation at rupture:

if exposed to a sharp frost its properties would be completely changed—it would become essentially a hard steel until it had actually been heated to a temperature of 600°C." It is interesting to notice that specimens of the non-magnetisable metal when broken in the testing machine pass into the magnetisable state; the change occurs along with the mechanical hardening which the metal suffers in being drawn out.

This remarkable power of assuming one or other of two widely different physical states is less noticeable when the percentage of nickel in the alloy is further increased. Two other nickel-iron alloys, containing respectively 30 per cent. and 33 per cent. of nickel, Hopkinson found to be much more permeable, and to show very much less hysteresis with respect to temperature in changing between the magnetisable and non magnetisable states, and to change at a comparatively low temperature. Fig. 94 shows the results of magnetising the 33 per cent. sample with a force **H** of 1·0. The curves, which correspond to rising and falling temperatures, are not far apart, and the change takes place at temperatures lying near 200°C. In the 30 per cent. sample the critical temperatures are lower (about 140°C. in heating and 125°C. in cooling). Finally, a sample containing 73 per cent. of nickel showed no material

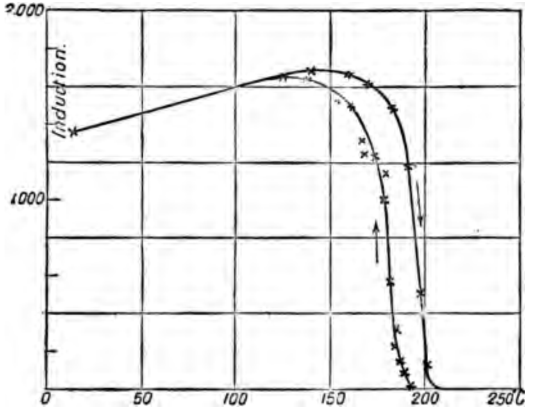

Fig. 94.—Steel with 33 per cent. Nickel. Magnetising Force 1·0.

difference between the critical points for heating and cooling; its critical temperature was 600°C.

These observations make it not unlikely that a substance such as manganese steel, which is nearly non-magnetic in all conditions of temperature in which it has hitherto been tested, would become magnetic if the temperature were sufficiently lowered. And it is even possible that other metals than iron, nickel, and cobalt are non-magnetic only because all our dealings with them are at temperatures above a "critical point."

CHAPTER IX.

EFFECTS OF STRESS.

§ 120. **Effects of Stress: Introductory.**—No part of our subject is more interesting than that which deals with the effects of mechanical stress in altering the susceptibility, the retentiveness, and other qualities of the three magnetic metals. The matter is not, at least as yet, one of practical moment, for it has at present no direct bearing on any of the applications of magnetism; but its importance on the theoretical side is not easily overrated. The effects of stress form a fascinating subject of inquiry to the physical student, and are likely to play a considerable part in revealing the molecular structure which makes magnetisation possible. The subject is a large one, and the results that have been already obtained are too intricate to permit more than a very general account of them to be given here. It will be most convenient to state the salient facts, without much regard to the historical order of their discovery. The first inquirer in this field appears to have been Matteucci,[*] who noticed an increase of magnetism in a magnetised iron bar when the bar was pulled lengthwise. Villari[†] made the important discovery that the character of this effect became reversed when the bar was sufficiently strongly magnetised: let the iron bar be weakly magnetised, and the effect of pull is to increase the magnetism; but let the bar be strongly magnetised, and the effect of pull is to reduce the magnetism. This "Villari reversal" (as it is now called) of the magnetic effects of stress in iron was rediscovered by Sir W. Thomson in the course of an inquiry which may be said to have

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[*] *Comptes Rendus*, 1847; *Ann. de Chimie et de Physique*, 1858.

[†] *Pogg. Ann.*, 1868.

laid the foundation of exact knowledge in this subject.* Thomson studied the effects of longitudinal stress by loading and unloading iron wire and steel wire in magnetic fields of various strengths; he extended the same method of investigation to nickel and cobalt. He found by experiment with a steel gun-barrel under hydraulic pressure that the effects of transverse stress were opposite in kind to those of longitudinal stress. Comparing the results of longitudinal and transverse pull, he pointed out that the effect of a simple pulling or pushing stress was to develop a difference of magnetic susceptibility in directions lying along and across the line of pull or push; and he applied this consideration to the case of torsional strain, deducing results which were verified by experiment, and discussing earlier experiments by Wiedemann, who, it may be added, has made the relations of torsion and magnetisation the subject of much detailed study.† The work of Thomson has been followed up and extended by others, particularly in the direction of investigating the forms which the magnetisation curve (the curve of I and H) assumes when the piece under test is subjected to various kinds and degrees of stress; and also of investigating, by continuous magnetometric observations, the manner in which a loaded piece gradually acquires or loses magnetism when the loads are varied, a constant magnetising force being kept in action. The effects of hysteresis, which present themselves at every turn in experiments on this subject, do much to complicate the results: and it is only by following both methods of inquiry—that is to say, by examining the consequences of changing the magnetic force while the state of stress is kept constant, and also those of changing the stress while the magnetic force is kept constant—that we can obtain a tolerably clear connected view of the phenomena.

§ 121. Effects of Longitudinal Pull on the Susceptibility and Retentiveness of Nickel.—It is most convenient to begin with nickel, because the effects of stress are—for the

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* Sir W. Thomson, "Effects of Stress on Magnetisation," forming Parts VI. and VII. of his great series of Papers on the "Electro-Dynamic Qualities of Metals" (*Phil. Trans.*, 1875, 1878; Reprint of Papers, Vol. II., pp. 332—407).

† *See* Wiedemann's *Elektricität*, Vol. III., § 762, *et seq.*

most part—much greater in it than in the other metals, and are also simpler in one very material respect. There is nothing in nickel that corresponds to the Villari reversal in iron. If we apply pull to a magnetised rod or wire of nickel, we find—as Thomson first showed*—that pull diminishes the magnetism, and relaxation of pull increases the magnetism; and this effect is still observed, however strongly or weakly the piece be magnetised.

If we magnetise nickel while it is kept in a state of longitudinal tension by means of a constant load, we find an enormous reduction in its susceptibility. This is well shown by the curves of Fig. 95, which show the magnetisation of a long piece of annealed nickel wire under various amounts of longitudinal pull.† The wire was 0·068cm. in diameter, and 374 diameters long; its section was 0·363 sq. mm., so that each kilogramme of load produced a stress of 2·75 kilogrammes per sq. mm. The curves drawn in full lines show the relation of **I** to **H** when there was no load, and also when the load was 2 and 12 kilogrammes, corresponding to 5·5 and 33 kilos. per sq. mm. respectively. The effect of tensile stress in depressing the magnetisation curve is very marked. With no load the maximum susceptibility is fully 15, with 2 kilos. it is only about 8, and with 12 kilos. the resistance to magnetisation has become so great that the maximum of susceptibility has not been reached even by raising **H** to 100 C.-G.-S.

Great as the effects of stress are upon the magnetic susceptibility, they are even greater on the retentiveness. In the same figure (95), three other curves have been drawn in broken lines, thus: —-—-—, to show the residual magnetism that was found on withdrawing **H** at each of a series of stages during the process of magnetising under each load. The presence of load reduces the residual magnetism even more than it reduces the total induced magnetism. The residual value of **I**, after applying a force, **H**, of 100, is nearly 300 when there is no stress; under 2 kilos. it is reduced to 150; and under 12

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* Reprint of Papers, Vol. II., p. 382.

† This and a number of the succeeding figures are taken from two papers, on the "Magnetic Qualities of Nickel" (*Phil. Trans.*, 1888, pp. 325 and 333), in one of which the author had the collaboration of Mr. G. C. Cowan.

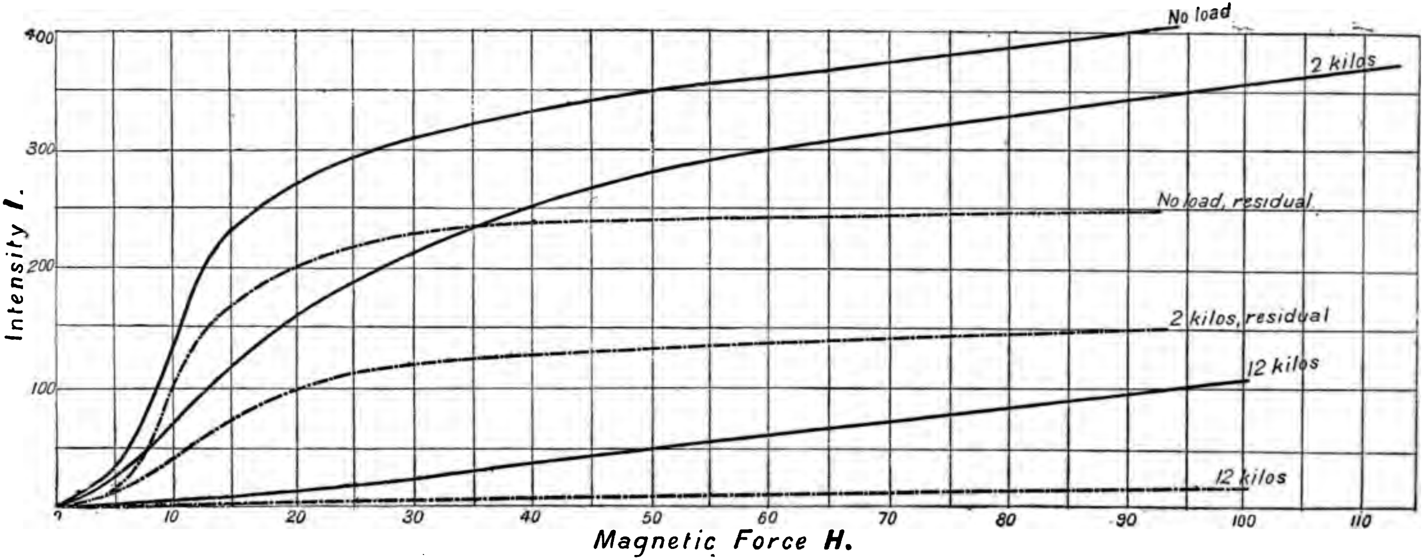

FIG. 95.—Magnetisation of annealed Nickel Wire under various amounts of longitudinal pull.

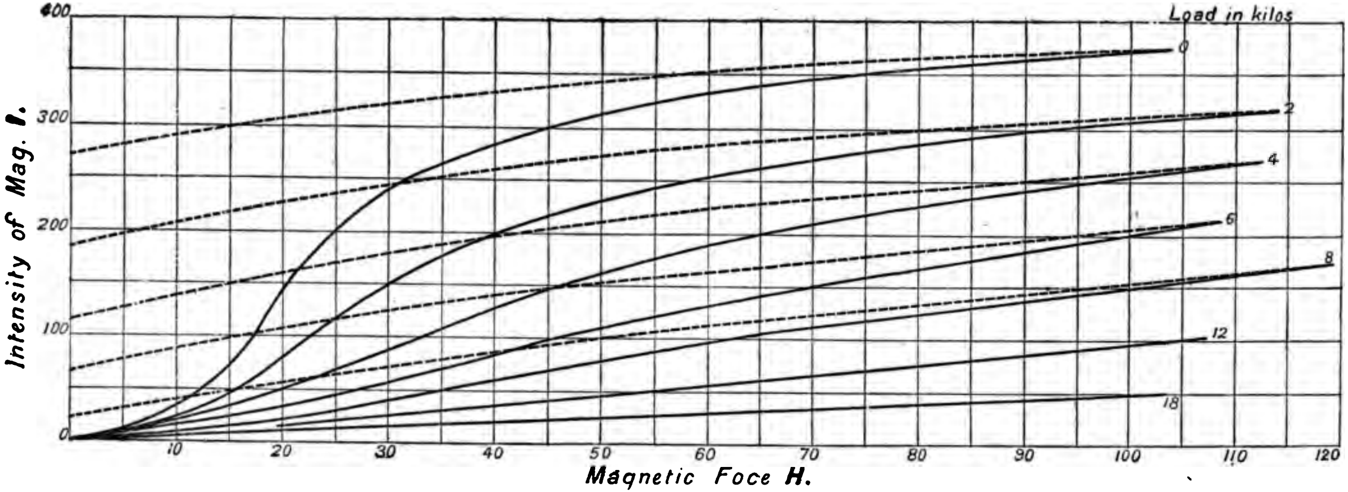

FIG. 96.—Nickel Wire hardened by stretching, magnetised under various amounts of longitudinal pull.

kilos. it is only 16. The proportion of residual to total induced magnetism has a maximum of 0·76 under no load; but under 2 kilos. it is reduced to 0·61, and under 12 kilos. to 0·19. The amounts of magnetism which disappear when **H** is removed, under various loads, form a greater proportion of the whole the more the load is increased, although (owing to the reduction in the total magnetism) the absolute amount that disappears when a strong force is removed is greater for a small load than it is for no load, and then less again for a large load.*

The presence of a *small* amount of load may, therefore, be said to *increase* the susceptibility of nickel with respect to that part of the magnetism which comes and goes when **H** is alternately applied and removed, provided **H** is strong; when **H** is weak the effect of any load is only to reduce this susceptibilty.

Fig. 96 gives the results of a similar experiment in which the same piece of nickel wire, after being hardened, however, by a slight amount of stretching beyond its limit of elasticity, was magnetised under a succession of pulling loads, ranging up to 18 kilos., or about 50 kilos. per sq. mm. With no load the maximum susceptibility of this hardened wire was about 8. Under the highest load the susceptibility was practically constant within the range of **H** used (up to 100 C.-G.-S.), and its value was only about 0·5 (permeability about 6·3). In this condition of stress the residual magnetism is almost *nil*. The dotted lines in this figure show the effect of gradually removing the strongest value of **H** which had been reached in the process of magnetising; they illustrate well how the residual magnetism becomes smaller, not only absolutely, but as a fraction of the whole magnetism, when heavier loads are used.

§ 122. **Effects of Longitudinal Push on the Susceptibility and Retentiveness of Nickel.**—The reduction of susceptibility and retentiveness in nickel by longitudinal tensile stress is associated with an equally striking augmentation of susceptibility

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* This fact has been noticed independently and commented on in a recent Paper by H. Tomlinson (*Phil. Mag.*, May, 1890).

and retentiveness by longitudinal compressive stress. Fig. 97 shows an arrangement by which nickel rods have been tested,* under compression, within a yoke of wrought iron, by means of the method described in § 58, the total magnetisation being determined ballistically by reversing **H**, and the residual magnetisation by deducting the ballistic effect got by removing **H** from half the ballistic effect got by reversing **H**. The influence of a number of loads was examined, ranging up to 19·8 kilos. per sq. mm. Every addition of load produced a decided increase of susceptibility, and caused an increasing fraction of the whole magnetism to be retained on the withdrawal of the magnetising force, until finally, under the heaviest load, the magnetisation curve rose with remarkable steepness, and the maximum proportion of residual to total induced magnetism reached the astonishingly great value of 0·96. In this group of experiments the nickel rod was in a hard (unannealed) state.

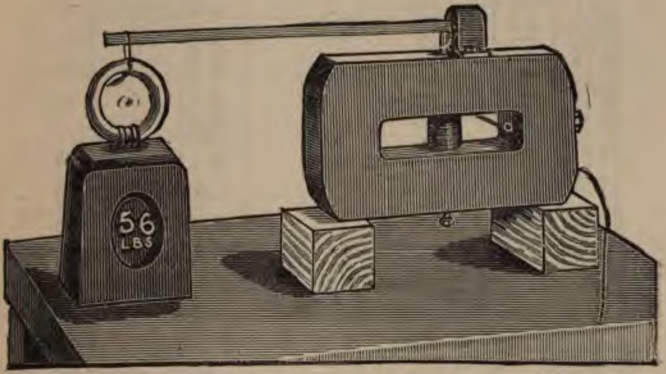

Fig. 97.—Arrangement for Testing the Magnetisation of Metals under Compression.

The results of the observations are shown in Figs. 98 and 99.

Fig. 98 gives the induced magnetism **I** in terms of **H**, under each amount of longitudinal compressive stress; and Fig. 99

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* *Phil. Trans.*, 1888, A, p. 333.

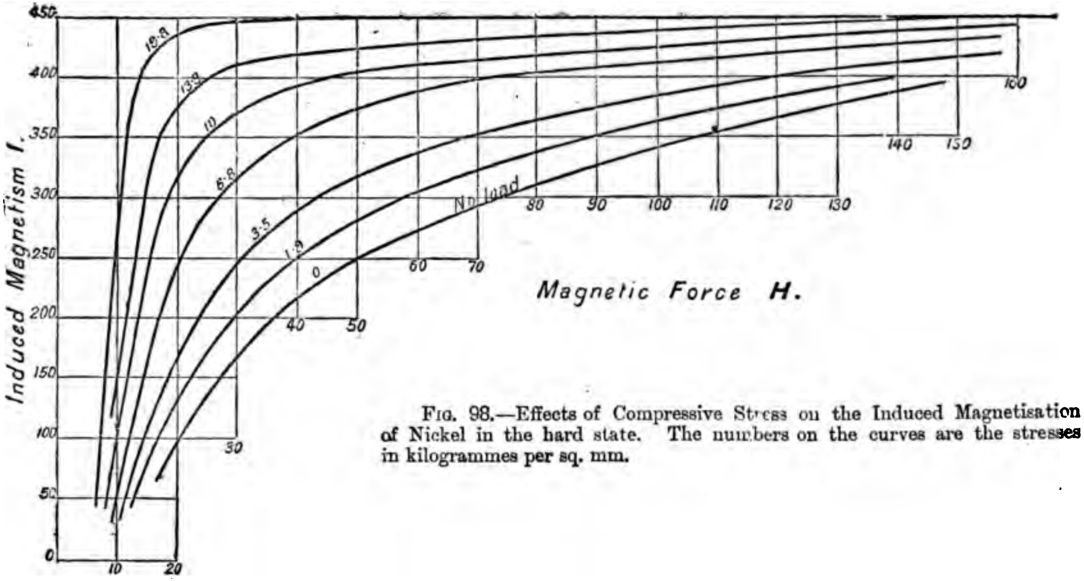

Fig. 98.—Effects of Compressive Stress on the Induced Magnetisation of Nickel in the hard state. The numbers on the curves are the stresses in kilogrammes per sq. mm.

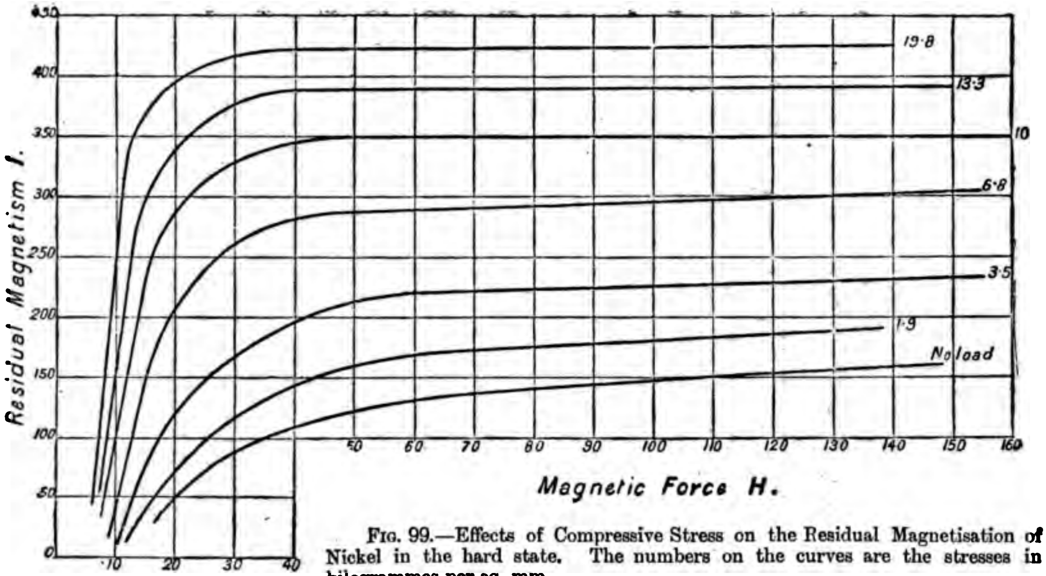

Fig. 99.—Effects of Compressive Stress on the Residual Magnetisation of Nickel in the hard state. The numbers on the curves are the stresses in kilogrammes per sq. mm.

gives the residual magnetism, which was observed in the usual way by withdrawing H at a number of stages during the taking of each magnetisation curve. Especially to be noted is the sharpness with which the curve of induced magnetism, under the heaviest stresses, bends over when H is about 20. The approach towards saturation is extremely rapid, and the change from a highly susceptible state to an insusceptible—because nearly saturated—state is remarkably abrupt.

Fig. 100 shows the result of the same experiment in a different way: the permeability μ is plotted there in relation to B for three conditions of stress which are specified on the curves.

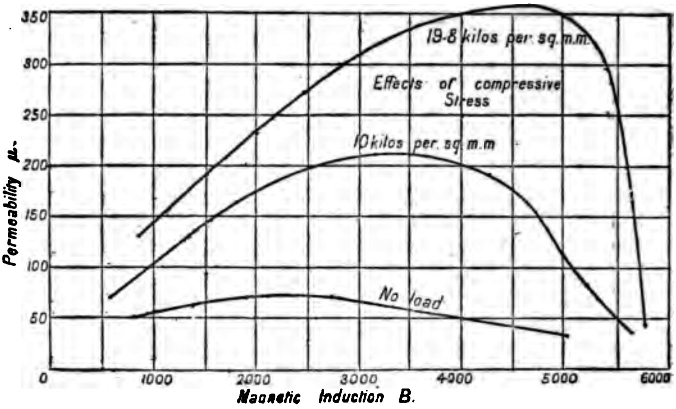

FIG. 100.—Permeability of Nickel in the hard state.

Fig. 101 records a corresponding set of observations made on a nickel rod in the annealed state, under compressive stresses ranging up to 6·8 kilos. per square mm. The curves of μ and B which relate to this experiment have already been shown in Fig. 41, § 75.

§ 123. **Effects of Cyclic Variation of Longitudinal Stress on the Magnetism of Nickel.**—As might be anticipated from the curves that have been given above, a magnetised nickel wire subjected to cyclic variations of pull by loading and unloading

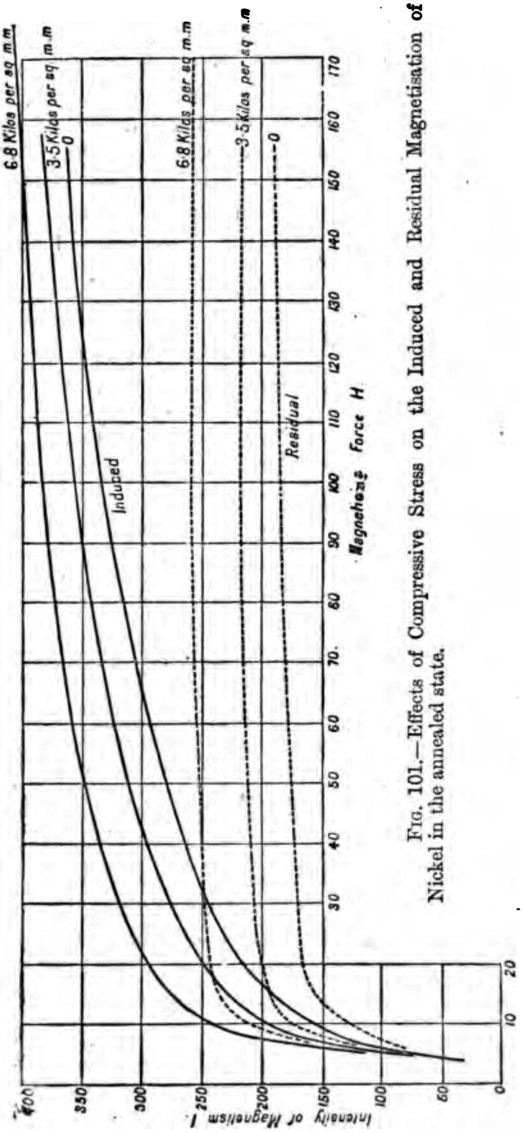

FIG. 101.—Effects of Compressive Stress on the Induced and Residual Magnetisation of Nickel in the annealed state.

it with suspended weights suffers much reduction of its magnetism when the weights are put on, and much increase of its magnetism when the weights are taken off. This happens whether the magnetism be induced or residual.

In Fig. 102 a number of curves are drawn to show the observed effect (upon I) of applying and removing loads while the magnetising force specified in the right-hand margin of the figure remained continuously in action. The dotted curves in the same figure show how the residual magnetism which was left after the action of the strongest force (116 C.-G.-S.) was affected by loading and unloading. In this experiment each kilogramme of load corresponds to a stress of 2·75 kilos per square mm. When these curves are compared with corresponding curves for iron, which will be given later, it will be seen that there is comparatively little hysteresis of magnetism with respect to stress in these.

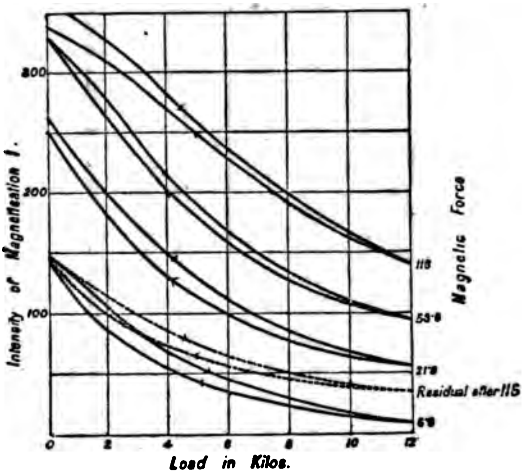

FIG. 102.—Effects of Loading and Unloading Nickel Wire in Various Constant Fields.

There is, however, some hysteresis; the curve for the process of loading invariably lies above the curve for the process of unloading, even when the cyclic variations of stress are repeated often enough to make the magnetic changes become strictly cyclic. With hardened nickel wire, tested under a wider range of stresses, there is even less hysteresis than here.*

§ 124. **Effects of Longitudinal Pull in Iron.**—Turning now to iron, we find that much more complex variations of magnetic quality are produced by longitudinal stress. We have to distinguish between two cases, that of soft annealed iron, and that of iron which has been hardened by a mechanical operation such as stretching, which has given it a permanent set. With hardened metal the effects of stress are in general much greater than with annealed metal. Both cases have this in common, that the presence of any moderate amount of longitudinal pull increases the susceptibility when the magnetisation is weak, but reduces the susceptibility when the magnetisation is strong. We have here the phenomenon of the Villari reversal to which allusion has already been made. But in the case of hard metal, where it is possible to apply a stronger pull without permanently altering the characteristics or structure of the piece, it appears that the presence of a sufficiently great amount of stress may be unfavourable to magnetisation, even in the earliest stages of the magnetising process. These, as well as other effects of stress, will be best appreciated by means of a careful study of curves which exhibit the process of magnetisation in iron wires pulled by various amounts of hanging load. The wires, in the experiments to be described, were of such a size that each kilogramme of load corresponded to a stress of about 2·2 kilogrammes per square mm.

§ 125. **Annealed Iron under Pulling Stress.**—Fig. 103 shows, by curves of **I** and **H**, the magnetisation of a wire of soft annealed iron under various amounts of longitudinal pull (no load, 2 kilos, and 6 kilos).† The curve for no load lies at first lowest, and finally highest. Each curve, in fact, lies at first lower, and afterwards higher, than a curve for any greater amount of load. Thus, the presence of load is favourable to magnetisation when **I** is small, but unfavourable when **I** is great. And the curves obtained by removing the magnetising force (which are shown to the left in the figure) preserve throughout their whole course the relative places with which they start,

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* *Phil. Trans.*, 1888, A, p. 331.

† *Phil. Trans.*, 1885, plate 64.

the differences between them becoming only accentuated as the magnetising force is reduced to zero. Thus, the presence of pulling load is unfavourable to the residual magnetism left

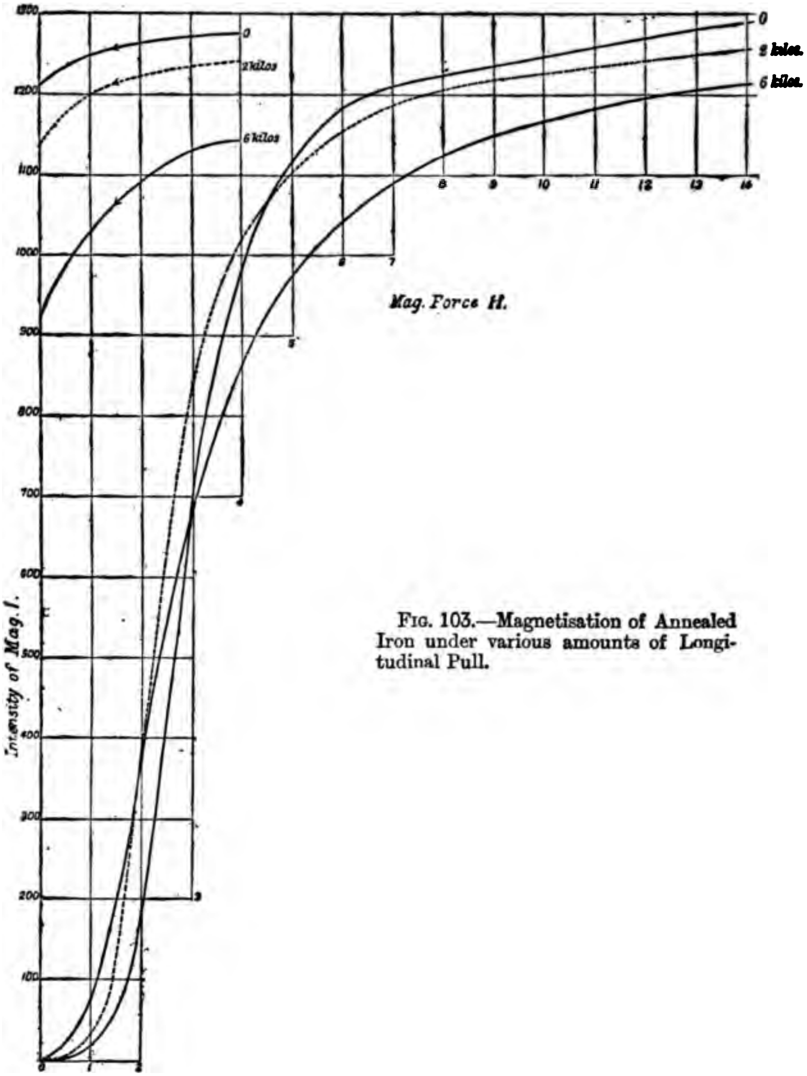

FIG. 103.—Magnetisation of Annealed Iron under various amounts of Longitudinal Pull.

after a strong field has been applied; though, as another experiment has shown, it is favourable to the residual magnetism that is left after magnetisation by a weak field. Its

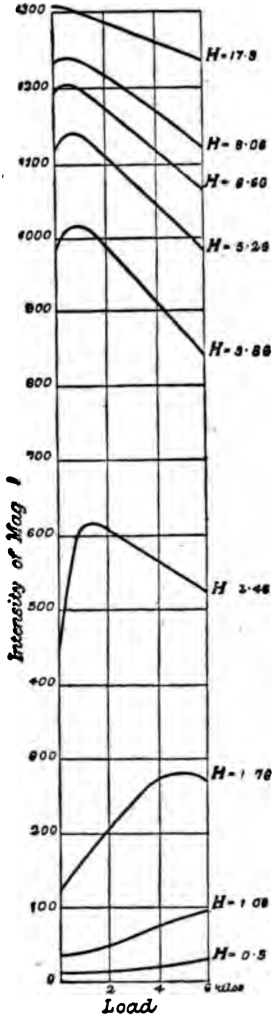

Fig. 104.—Magnetisation of Annealed Iron under Various Amounts of Longitudinal Pull.

influence on the residual magnetism is, in fact, of the same kind as its influence on the induced magnetism; both suffer reversal when the magnetisation is sufficiently increased. The curves of residual magnetism (which are not drawn in the figure) cross each other in the same manner as the curves of induced magnetism. The results of this experiment are shown in a different manner in Fig. 104. A series of curves are drawn there, each relating to a particular value of the force **H**, to show the relation of the value of **I** reached by applying that force, to the amount of load which was present when the force was applied.

This figure shows very clearly that, except under the strongest magnetising force that was applied in the experiment, the presence of a very small amount of pulling load increases the susceptibility; and further, that except in the weakest fields, the presence of a fairly large amount of pulling load reduces the susceptibility. Except at very low and again at high magnetisations, there is maximum of a susceptibility occurring with a particular load; and the value of this load becomes smaller as the magnetisation is increased. This maximum disappears in the lowest fields, no doubt only because the load is insufficiently great to show it.

§ 126. **Hardened Iron under Pulling Stress.**—Figs. 105 and 106 show the effects of various amounts of longitudinal pull on iron wire which had been previously hardened by stretching beyond the elastic limit. Fig. 105 gives the induced magnetism, and Fig. 106 gives the residual magnetism, both in relation to **H**, the process of magnetising being performed, as in previous examples, while a constant load hung from the wire.

The first thing to observe here is the immense effect which a moderate amount of pull has in augmenting the susceptibility with respect to feeble magnetising forces. On the other hand, when a condition approaching saturation is reached, the presence of load is unfavourable to magnetisation; in other words, we have, as before, the Villari reversal. But it is now to be noticed that even in the weakest fields the susceptibility is increased only when the amount of the load is moderate: to apply stress beyond a certain amount is prejudicial, whether the magnetisation be strong or weak. This is shown by the fact that the curve for 14·8 kilos lies below the curves for 5 and 10 kilos thoughout its whole course.

The same remarks apply to the residual magnetism (shown in Fig. 106). The influence of stress on it is even greater.

Fig. 107 shows, in the same way as Fig. 104, the results of

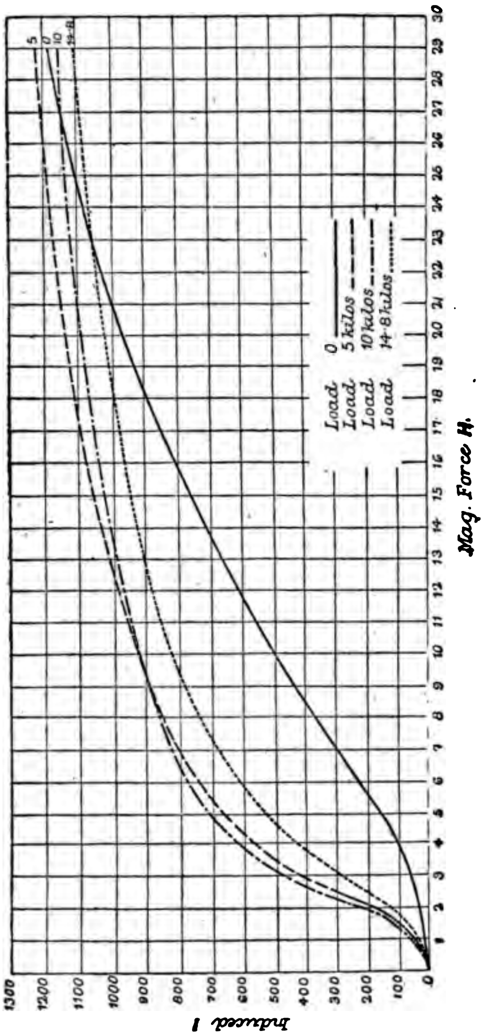

FIG. 105.—Magnetisation of Hardened Iron under various amounts of Longitudinal Pull.

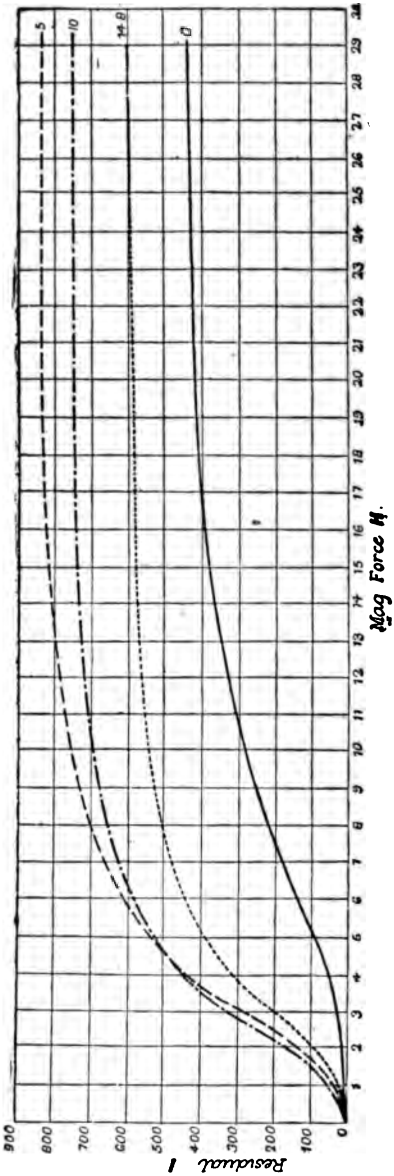

FIG. 106.—Residual Magnetisation of Hardened Iron under various amounts of Longitudinal Pull.

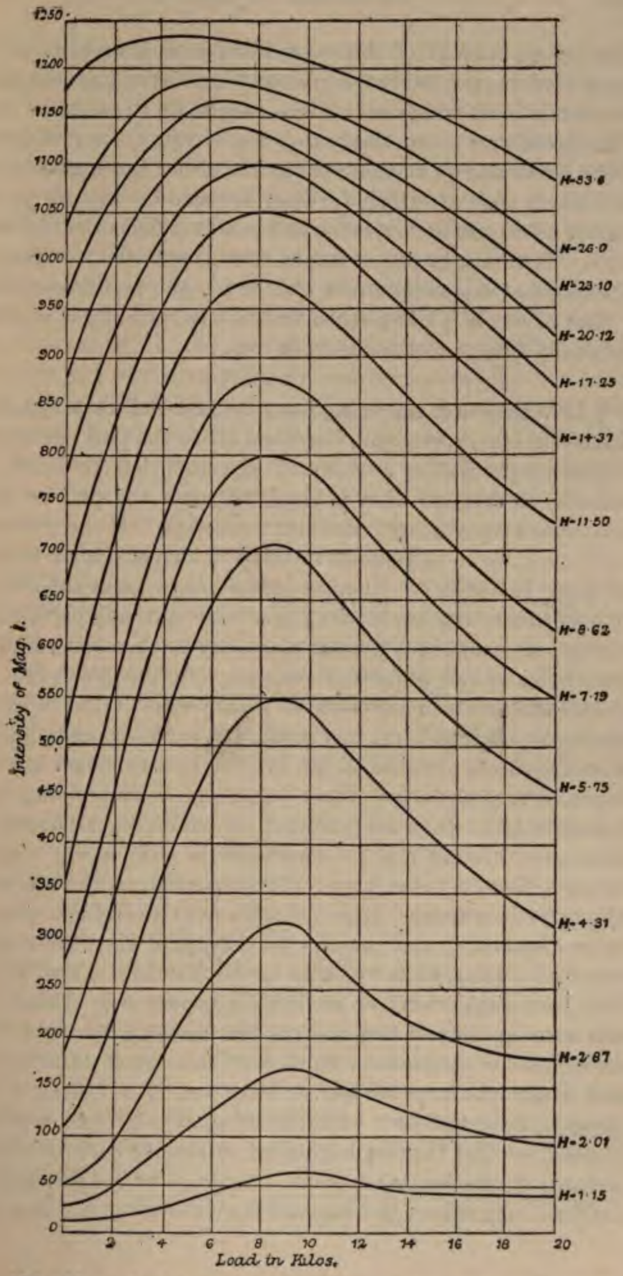

Fig. 107.—Magnetisation of Hardened Iron under various amounts of Longitudinal Pull.

another experiment of the same kind, in which a piece of the same iron wire, also hardened by stretching, was magnetised under a series of loads which in this case ranged up to about 19 kilos. This figure shows very clearly that a moderate amount of load is more favourable to magnetisation than either less load or more; the exact amount which is most favourable depends on the degree of magnetisation, being less in strong fields than in weak ones. It varies, in this example, from about 10 to 5 kilos, for the range of magnetic forces with which the experiment deals.

The effects of pulling stress on the susceptibility of steel are generally similar to the effects in iron.

§ 127. **Effects of Applying Longitudinal Pull to Magnetised Iron.**— In the experiments described above the pull was applied before magnetisation began, and was then left constant. It remains to describe what is observed when the pull is varied while the magnetising force is kept constant. If there were no hysteresis, we should obtain in this way curves similar to those of Figs. 104 or 107. In consequence of hysteresis the changes of magnetism that are actually produced by changing the load, though maintaining a general similarity to these curves, differ from them in two important respects. In the first place, the *initial* effects which are observed when we first begin to change the stress are in general very great, and are to be distinguished from the effects obtained after a cycle of stress changes has been repeated once or twice. These initial effects of applying stress resemble those that are produced by vibration, although the process of loading may be conducted in such a way that no actual vibration takes place. They proceed, as the molecular theory to be discussed later indicates, from a condition of molecular instability; and they do not disappear when the stress is removed. Thus, when we begin for the first time to load an iron wire, to which a weak or moderately strong magnetising force has been applied, we find that the first loads are associated with an increase of magnetism, which may be so great as to increase the whole quantity ten-fold. Moreover, if a load has been hanging from the wire while the magnetising force was being applied, we find that on beginning to remove it an increase of induced magnetism takes place. Again, if we are dealing with residual magnetism, the first effect of changing the load after the magnetising force has been removed (whether by way of increasing or decreasing the load) is in general to reduce largely the amount of the residue. It is only after applying and removing any load several times that the magnetic effects of the stress-changes become cyclic—that is to say, after several repetitions of the operation, the magnetism will be found to alter from one to another of two definite values when the load is put on and when it is taken off. But even then the effects of hysteresis are manifest; for any intermediate value of the load is found to be associated with very different values of the magnetism during loading and during unloading. These features are well seen when we examine curves drawn to show the changes of magnetism in relation to the changes of load, of which Figs. 108 and 109 are examples.* They refer to an iron wire, hardened by previous stretching beyond its elastic limit, of such a size that each kilo of load corresponds to a stress of about 2·3 kilos per sq. mm. The cycle of stress consisted in applying and removing 15 kilos.

Beginning at the bottom of Fig. 108, at the point marked *a*, we have the wire, free from any load and previously demagnetised by reversals, exposed to a magnetising force of 0·34 C.-G.-S. In this state there was very little magnetisation. Then loads were applied, and the effects of the first application and removal are shown by the dotted lines *a b c*. The full lines immediately above them show the effects of the *second* application and removal of load, by which time the magnetic changes had become nearly cyclic. It is clear that in the first loading we have to deal with a progressive augmentation of magnetism superposed on cyclic changes of the character shown by subsequent cycles of loading—that is to say, we have an initial effect superposed on the cyclic effect.

Next, the wire was demagnetised, and then a stronger field (2·49 C.-G.-S.) was applied, while there was no load. The effects of the first loading in this field were enormous; they are shown by the dotted line which starts from the point *d* in the figure. Here, again, a repetition of the process of loading and unloading brought the magnetic changes into a nearly cyclic state, which is shown by the full lines at the top of the figure.

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* *Phil. Trans.* 1885, plate 63, p. 603.

Next, a stronger field still (18·65 C.-G.-S.) was applied (Fig. 109). The curve for first loading still shows a considerable permanent augmentation of magnetism; but a cyclic state is reached sooner than in weaker magnetic fields. In still stronger magnetic fields the curves become more and more

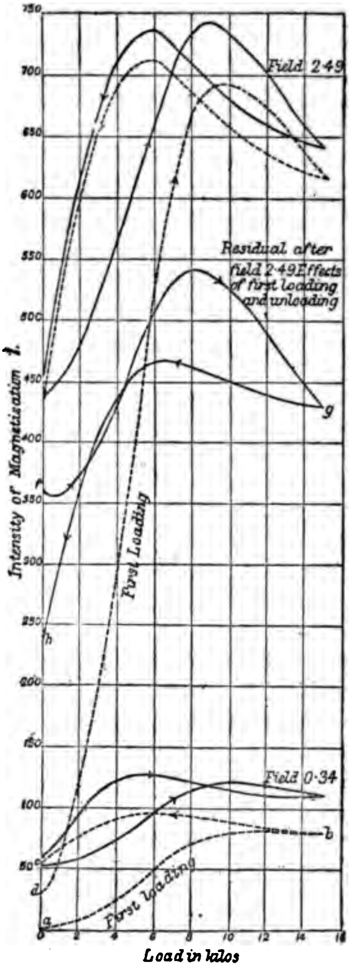

FIG. 108.—Effects of applying Pull to Magnetised Iron.

flattened down into a form in which the application of load causes a diminution of magnetism throughout.

Finally, to show how the residual magnetism is affected by change of stress, the residue left after applying a field of 2·49 units and subjecting the wire to loads in that field, was made the subject of the experiment shown by the lines *f g h* in Fig. 108. These curves show how (starting from the point *f*) the residual magnetism suffered changes due to loading and unloading, which may best be described as a progressive decrease of magnetism superposed upon cyclic changes of the same character as those which are shown in previous figures. If we repeat the cycles of load on a piece in which there is only residual magnetism, we find, in fact, cyclic changes of the same general kind as those that are found when a magnetising force is in action.

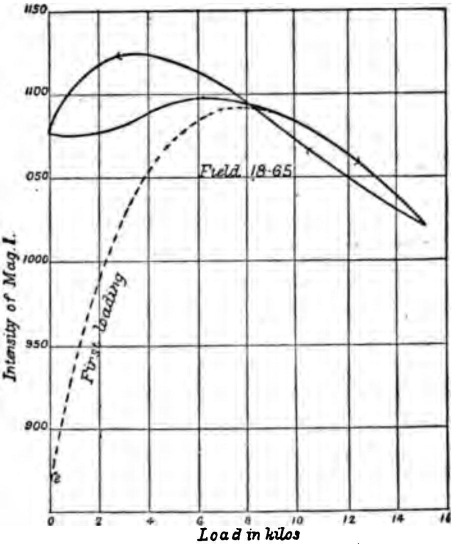

FIG. 109.—Effects of applying Pull to Strongly Magnetised Iron.

§ 128. **Hysteresis in the Effects of Stress.**—The hysteresis of magnetism with respect to changes of load, which is clearly exhibited by these curves, is static in character—that is to say, it does not depend on the time-rate at which loads are applied nor on the intervals which are allowed to elapse before readings of the magnetisation are taken. After any condition of load is reached, the magnetism does not change with the lapse of time, except possibly to a very insignificant extent.

During each loading, after a cyclic condition has been established, the magnetism is at first increased; but a maximum is passed as more load is added, and later additions of load reduce the magnetism. A similar maximum is seen during unloading; but owing to hysteresis the maximum comes at different loads in the two cases; each maximum is shifted, through hysteresis, to a later place in the operation than it would otherwise have.

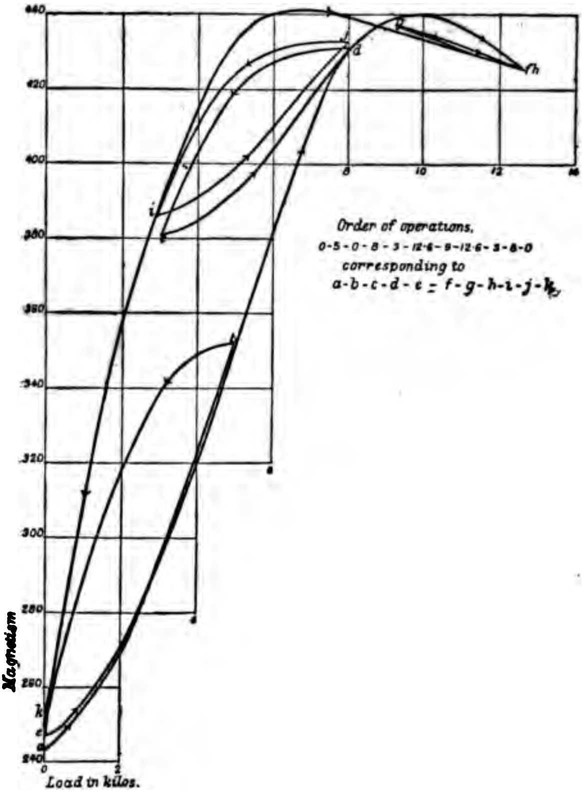

FIG. 110.—Effects of Pull on a Stretched Iron Wire.

Another manifestation of hysteresis is seen in the easy gradient with which each curve begins, as the process of loading is changed to that of unloading, or *vice versâ*. In a weak field the initial gradient of each curve is so small that the curve appears to set out tangent to the line of loads.

Fig. 110 may be referred to in further illustration of the presence of hysteresis in changes of magnetism caused by changes of load.* It shows the effect of superposing on a principal cycle of pulling stress changes several minor cycles, in each of which hysteresis is very apparent. The order in which the loads were applied was this:—0, 5, 0, 8, 3, 12·6, 9, 12·6, 3, 8, 0. The wire dealt with here was of iron, and had been hardened by stretching: it hung in a constant field the force of which was 0·34 C.-G.-S.

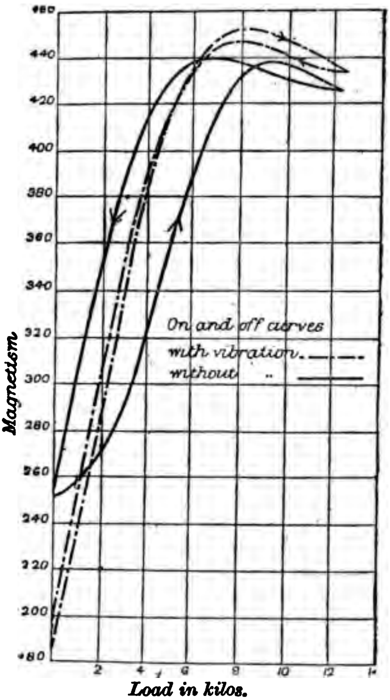

FIG. 111.—Influence of Vibration on Effects of Loading and Unloading.

§ 129. **Influence of Vibration on the Effects of Stress.**—These indications of hysteresis disappear almost entirely if we submit the piece under test to mechanical vibration either during or after the changes of load. As modified by vibration the curves for loading and unloading become nearly coincident. The whole amount of magnetic change is increased. A maximum point is still found, which lies, as regards load, between the two maximums that are observed when the processes are gone through without vibration. Tapping the wire at any stage in the process produces, in general, a large change in its magnetism; but if loading or unloading is then resumed, without further tapping, the presence of hysteresis is at once conspicuous. Fig. 111 (page 209) illustrates the influence of vibration, by showing the curves got by repeated loading and unloading of an iron wire, suspended in a weak magnetic field, first without vibration, and also with smart vibration before each reading of the magnetometer was taken.

§ 130. **Effects of Loading Annealed Iron.**—On applying loads to an annealed iron wire hanging in a magnetic field, we find at first the same extreme sensitiveness, the result of molecular instability. Repetition of the loading, if repeated often enough, brings about a cyclic state in which there is much less total change of magnetism than is found in the corresponding experiment with hardened metal. As to the character of the change, it depends on the magnitude of the load. With a sufficiently light load, loading produces increase and unloading produces decrease of magnetism; with a moderately heavy load these effects are reversed.*

§ 131. **Effects of Longitudinal Stress in Cobalt.**—Sir W. Thomson, testing a cobalt bar hung vertically in the earth's magnetic field, found that pulling decreased and relaxing the pull increased the induced magnetism. The effects of

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* For examples of the curves got by loading and unloading annealed iron see *Phil. Trans.*, 1885, plates 62 and 64. Many of the effects of stress, both in annealed and in hardened metal, will be found exhibited there, by means of curves, more completely than it is possible to exhibit them here. A few examples of the effects of *compressive* stress on the curves of **I** and **H** for iron will be found in a paper in the *Phil. Mag.* for September, 1888. The presence of compressive stress lowers the curve, as might be anticipated from the raising of it by tensile stress, shown in Figs. 103 and 105.

longitudinal pressure on the magnetisation of cobalt have been examined by Mr. C. Chree,* who found a reversal of effect, as the magnetisation was increased, resembling the Villari reversal in iron, but *opposite* to it in character. In iron, as we have already seen, after the first effects of stress are past, pressure will reduce magnetism in weak fields, but will increase it in strong fields. In cobalt the reverse happens; pressure increases magnetism in weak fields, but reduces magnetism in strong fields. This may be shown either by magnetising when the pressure is on, and again when it is off, or by applying and removing pressure while a constant magnetising force is in action. If the latter plan is followed, we have, of course, to exclude the initial effects, which, as Mr. Chree has pointed out, occur in cobalt as they occur in iron. The first application of pressure in weak fields causes a large increase of induced magnetism, just as, we may anticipate, the first application or removal of stress of any kind would do; but repetition of the process soon establishes a cyclic state.

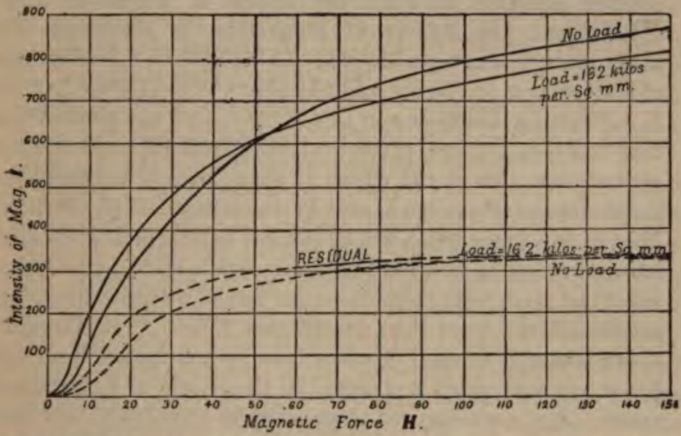

FIG. 112.—Induced and Residual Magnetisation of Cobalt with and without Compressive Stress.

The effects of longitudinal pressure in modifying the magnetisation curve of cobalt are illustrated in Fig. 112 (from an experiment by the writer and Mr. W. Low). The full lines are two

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* *Phil. Trans.*, 1890, A, p. 329; *Proc.* Roy. Soc., December, 1889.

curves of induced magnetism for a rod of cast cobalt, tested (within a yoke) without stress, and also with a compressive stress amounting to 16·2 kilogrammes per square millimetre. The broken lines are the corresponding curves of residual magnetism. The induced curves cross, illustrating the reversal described by Mr. Chree. The residual curves do not cross within the limits of field used here; but other experiments, made with the same rod but with heavier loads, show a crossing in them also. Curves of the permeability in terms of **B**, drawn from the data of the same experiment, have already been given in Fig. 42, § 76.

§ 132. **Relation between the Effects of Stress on Magnetism, and the Effects of Magnetism in Changing the Dimensions of Magnetic Metals.**—In his book on "Applications of Mathematics to Physics and Chemistry" (p. 47 *et seq.*), Prof. J. J. Thomson has discussed this subject, and has pointed out that it is possible, from theoretical considerations, to predict the general character of the effects of stress from a knowledge of the changes of dimension caused by magnetisation. Mr. Shelford Bidwell, in a Paper which will be referred to later in more detail,* has shown that an iron rod lengthens when it is magnetised, provided the magnetising force does not exceed a certain limit, but shortens if the force does exceed that limit. Prof. Thomson shows that this reversal of effect is to be anticipated from the Villari reversal which is observed in the effects of longitudinal stress. Again, a nickel rod shortens when magnetised, and continues to shorten under high magnetic forces; this agrees with the fact that in nickel there is no Villari reversal, and that longitudinal pull diminishes the magnetism, whether that is weak or strong. Again, with cobalt Bidwell has found effects opposite to those found in iron, namely, that weak magnetisation shortens a cobalt rod and strong magnetisation lengthens it. Applying his equations to this result, Prof. Thomson has anticipated what the character of the effects of stress in cobalt should be. Mr. Chree's experiments have verified his conclusions, by showing that the effects of stress in cobalt are the reverse of the effects of stress in iron, tension diminishing weak magnetism but augmenting strong magnetism.†

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* *Phil. Trans.*, 1888, A, p. 205.

† See the introduction to Mr. Chree's Paper, *Phil. Trans.*, 1890, A, p. 329

§ 133. Residual Effects of Stress applied before Magnetising.—Perhaps the most interesting of all the effects of stress are those that occur in unmagnetised iron. To apply and remove load before beginning to magnetise a piece of iron has been found to affect the magnetic susceptibility, even when the load is well within the elastic limit, and when the piece is perfectly free from magnetisation during application and removal of the load. We have, in fact, evidence that even in unmagnetised iron the process of loading and unloading causes changes of molecular configuration which are not reversible. These changes exhibit hysteresis with regard to the loads which cause them. They affect more than one physical quality of the metal; in particular, they produce upon the magnetic susceptibility an effect which becomes obvious when the piece is magnetised. These residual effects of past loads may be wiped out by subjecting the piece to the operation of demagnetising by reversals. They may also be wholly, or almost wholly, removed by tapping the piece smartly and so causing vibration.

Hence, in experiments designed to show the differences of susceptibility of iron or steel when subjected to different amounts of load, the piece should be passed through the operation of demagnetising by reversals after the load has been put on. This procedure was, in fact, followed in the experiments that have been described above.

The residual effects of stress, occurring in the absence of any actual magnetisation, are of very great interest in their bearing on any theory of the molecular constitution of magnetic metals. One or two experiments may be cited to show their general character.*

Let an iron wire be subjected to pulling stress, and let the load be removed before beginning to magnetise. Then, provided the load which has been applied lies within the elastic limit, or is less than some load by which the wire has been previously stretched, we observe no mechanical change of any ordinary kind as the result of applying and removing the load. And if, before beginning to take a curve of magnetisation, we put the wire through the process of demagnetising by reversals, we shall find nothing in the curve to show whether there has or has not been any application of load before that. But suppose,

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* *Phil. Trans.*, 1885, Part II., pp. 612-619.

after the process of demagnetising has been gone through, we apply and remove some load before beginning to magnetise. Though there has been no immediately obvious mechanical change, the wire has undergone a change of structure which shows itself in the form assumed by the curve of magnetisation. We find the magnetic susceptibility, especially under low forces, much greater in this than in the former case. The whole difference in procedure may be no more than this, that in one case the load is removed before the process of demagnetising is performed; in the other case, the process of demagnetising is performed before the load is removed. So slight a difference in procedure might, perhaps, be expected to have no influence on the form of the curve; in fact, however, it has a large influence. The curve of magnetisation depends not merely on the load actually present: it is affected, especially in its early portion, by any changes of load which have taken place since the preceding demagnetisation. For instance, it has been observed that if a curve be taken with (say) a pull of 3 kilos on an iron wire, and if, after complete demagnetisation, the load be raised to 4 kilos and 1 kilo be removed, and a second curve be then taken, the second curve will differ very sensibly from the first, in spite of the fact that the wire may have previously been subjected to many times that amount of load, and was, therefore, in a mechanically stable state.

§ 134. Experiments showing Residual Effects of Stress.—In the following case an iron wire* (previously hardened by permanent strain) was loaded with a weight of 18·5 kilos, or 42·5 kilos per sq. mm. This weight was repeatedly applied and removed, then finally removed; the wire was demagnetised by reversals, and the magnetising process was then gone through, giving the magnetometer readings stated in column I. of Table XXIII. Then the wire was demagnetised: the weight of 18·5 kilos was applied and removed, and then the process of magnetising was again gone through, giving the magnetometer readings in column II. Finally, the same thing was repeated, but with this difference, that the wire was briskly tapped after the load had been removed before beginning to magnetise; the results of this are given in column III.

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* *Loc. cit.*, p. 614.

TABLE XXIII.—*Magnetisation of Iron under the influence of previous loads.*

H	Magnetometer readings.		
	I. After demagnetisation with no load.	II. After the cycle 0 – 18½ – 0.	III. After the cycle 0 – 18½ – 0 and then vibration.
0	0	0	0
1·15	5	8	5
2·01	11	19	10
2·87	19	40	17
4·31	44	73	35
5·75	78·5	110	70
8·62	149	176	150
11·50	212·5	230	214
14·37	267	278	268
17·25	314·5	321	314
20·12	355	358·5	354
23·00	390	394	388
25·87	420	420	422
33·12	472	472	471

Comparing the three columns, it will be clear that in the first and third case the metal is in substantially the same condition as to susceptibility. In the third case its susceptibility with respect to low magnetic forces, and even to moderately great forces, has been notably raised, as a consequence of the molecular change brought about through application and removal of the load. The same change had occurred in the other two cases, but it had been undone by the demagnetising process in one, by vibration in the other.

Experiments of this kind lead to the conclusion that when we apply and remove stress in iron, even when the magnetic state is perfectly neutral, we cause some kind of molecular displacement in the relation of which to the applied stress there is hysteresis. When any load is applied and removed the changes of molecular configuration lag behind the changes of stress. We accordingly find, if we stop at any intermediate value of the load and examine the susceptibility, that the result is not the same when the stoppage is made during the process of loading, as when it is made, at the same amount of load, during the process of unloading. Magnetic susceptibility may, of course, be thought of as a physical property of the metal, apart from the existence of any actual magnetisation. During the loading and unloading of an unmagnetised piece the susceptibility changes in a manner that involves hysteresis, just as the magnetism changes when we load and unload a magnetised piece.

TABLE XXIV.—*Magnetisation of Iron under the influence of previous loads.*

Galvanometer readings. (To reduce to H multiply by 0·0575.)	Magnetometer readings.			
	I. Demagnetised with no load. Then 0 – 18½ – 3. Load = 3 kilos.	II. Demagnetised with no load. Then 0 – 18½ – 0 – 3. Load = 3 kilos.	III. Demagnetised with no load. Loaded to 18½, unloaded to 3 kilos, and *tapped* before magnetising. Load = 3 kilos.	IV. Demagnetised with no load. Loaded to 3 kilos and *tapped* before magnetising. Load = 3 kilos.
0	0	0	0	0
25	22	13	11	10
50	70	14	36	34
75	139	109	103	100
100	198	176	174	168
125	242	226	227	219
150	276	265	268	259
200	328	323	328	320
250	...	365	369	365
300	398	398	403	400
350	424	425	429	427
450	461	462	467	466
588	491	494	499	498
0	274	275	277	276

In Table XXIV. four magnetisations of the same iron wire are exhibited, each under a pulling load of 3 kilos.* In I., the load had been previously raised to 18½ kilos, then reduced to 3 kilos. In II., the condition of load had been reached by applying 3 kilos, after there had been no load. In III. and IV. these differences of procedure were repeated, but the wire was subjected to vibration before the magnetising process began. It will be seen that between I. and II. there is a marked differ-

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* One kilo of load here corresponds to a stress of 2·3 kilos per sq. mm.

ence, especially in the early portion of the curve; but in III. and IV. this difference has practically disappeared, the effects of hysteresis being destroyed by vibration.

Again, Fig. 113 shows two pairs of curves, two (I. and II.) taken under no load, and two (III. and IV.) taken under a load of 3 kilos. In I., the wire was demagnetised immediately before the curve was taken. In II. it was demagnetised, then loaded with 15 kilos, and then completely unloaded. In III. it was loaded with 10 kilos, and unloaded down to 3 kilos. In IV. it was completely unloaded from 10 kilos, then reloaded up to 3 kilos. Very similar differences in effect have been observed when annealed iron (not previously hardened by stretching) has been tested under corresponding varieties of condition in regard to previous stress.*

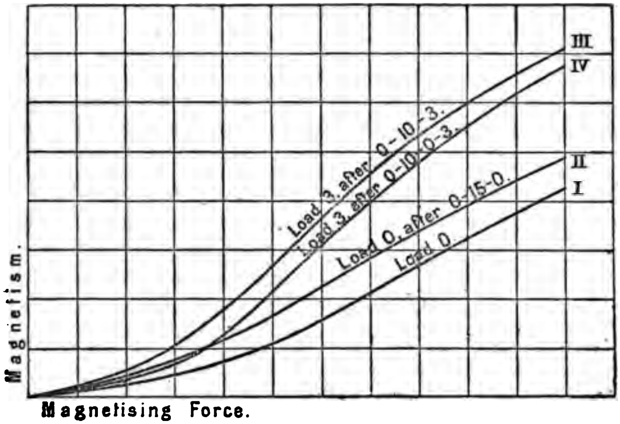

FIG. 113.—Residual Effects of Previous Loads.

The changes in molecular structure which, as these results show, are going on in iron or steel during the process of applying and removing stress sometimes result in producing a small amount of magnetism in a piece which, after being magnetised, has been brought into an apparently non-magnetic state by the application of a reversed force. There are, in such a case, superposed magnetisations which originally neutralise each

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* *Loc. cit.*, p. 618.

other so far as external effect is concerned, but the balance is disturbed through the unequal action of the stress upon them.

§ 135. **Other Evidences of Hysteresis in the Effects of Stress.**—These experiments show that the structure of iron changes, under variation of stress, in a manner that exhibits hysteresis, that is to say, the changes of structure lag behind the changes of stress. We may therefore anticipate that we shall find traces of hysteresis in other physical qualities besides magnetic susceptibility when we examine the variation of those qualities under variations of stress.

A remarkable instance is furnished by the thermo-electric quality of iron. Under variations of pull the thermo-electric quality of iron varies in a manner which strikingly resembles those variations of magnetic quality which have been described in this chapter. This is not a secondary effect, resulting from changes of magnetism, for it occurs even when care is taken to keep the iron wholly free from magnetisation during the experiment. Curves drawn to represent the relation of thermo-electric quality to load show a very remarkable general resemblance to the curves of Figs. 108-110, which show the relation of magnetism to load. There are also interesting points of difference, but a discussion of these would be out of place here. The main point, which was discovered by E. Cohn*, and afterwards, independently, by the writer†, is that there is much hysteresis of thermo-electric quality with respect to stress—a result, no doubt, of the irreversible changes of molecular structure to which allusion has just been made. We shall see later, in connection with molecular theories of magnetism, how these irreversible changes probably occur.

Further, but slighter, evidence of the occurrence of irreversible molecular changes during the loading and unloading of an iron wire is found when we examine the amount of the extension in relation to the load. Though the amount of load be restricted so that it lies well within the so-called limit of elasticity, it is found that there is no exact proportionality of strain to stress; and when a cyclic process of loading is repeated often enough to make the elongation and retraction become also

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* Cohn, *Wied. Ann.*, 1879, VI., p. 385.

† *Proc.* Roy. Soc., 1881, XXXII., p. 399; *Phil. Trans.*, 1886, p. 361.

cyclic, it is found that, at any intermediate value of the load, the wire is longer during unloading than during loading. In other words, there is hysteresis in the relation of strain to stress. The amount of this hysteresis is small; but when means are taken to magnify the extension sufficiently it may be observed without difficulty. The amount of difference in length between the length at the mean load in loading and the length at the mean load in unloading, may be $\frac{1}{300}$ of the change of the whole extension. The effect in question has to be distinguished from quasi-plastic changes of length, which depend on the time-rate at which the loads are applied. It has been observed in wires of copper and brass, as well as iron and steel.* One obvious consequence of it is that any process of loading and unloading involves some dissipation of energy.

§ 136. **Effects of Torsion on Magnetic Quality.**—The influence of twisting strain on the magnetic quality of metals has engaged the attention of many experimentalists, beginning with Matteucci,† who, in 1847, examined ballistically the change of magnetism undergone by an iron rod when it was twisted back and forth, while a magnetising current was kept up in a surrounding solenoid. Wertheim, E. Becquerel, and Wiedemann followed on the same lines,‡ and the subject was taken up by Sir William Thomson in one of the sections of his investigation of the electro-dynamic qualities of metals.|| More recently a number of other workers have pursued the matter in great detail. The results of their investigations are much too complicated to admit of anything like full statement here; we must be content with an account of some of the more conspicuous facts.

The general result of early experiments was to show that when a rod of soft iron, exposed to longitudinal magnetising force, was twisted, its magnetism was reduced, by torsion in either direction. In this effect, as in all effects of stress, we have to distinguish between the irreversible initial effect of the

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* Brit. Assoc. *Rep.*, 1889, p. 502.

† *Comptes Rendus*, Vol. XXIV., p. 301.

‡ For an abstract of these researches, *see* Wiedemann's *Elektricität*, Vol. III., p. 671, *et seq.*; *see* also Wiedemann, *Phil. Mag.*, 1886.

|| *Phil. Trans.*, 1878; Reprint of Papers, Vol. II., p. 374.

first application (due to molecular instability) and the effect which becomes manifest when a cycle of strain is repeated. The initial effect of torsion will depend on the past history of the piece, but the cyclic effect is, in soft iron, of this character, that twisting, to either side, reduces the induced magnetism, and untwisting increases it. But this effect is very small for small angles of twist. Moreover, as with other effects of stress, the changes of magnetism exhibit hysteresis. This was pointed out by Sir William Thomson, who has given curves showing the manner in which the magnetism induced in an iron wire by a constant magnetic field changes as one end of the iron wire is twisted to and fro while the other end is held fixed. The typical form into which the curves settle after repeated twistings is shown in Fig. 114, which is copied from his Paper. From the form of these curves it is clear that if the effects of hysteresis were eliminated—as they no doubt might be, at least in part, by vibrating the wire—we should have a single curve resembling a parabola with its vertex at the top of the diagram. Thus in the absence of hysteresis we should find the influence of torsion in reducing the induced magnetism to be indefinitely small for small angles of twist, and to increase initially in proportion to the square of the twist.

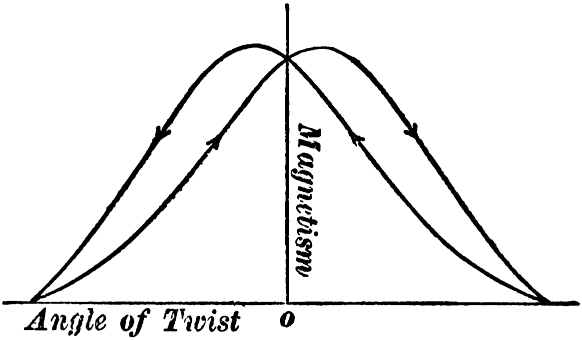

FIG. 114.—Effect of Twist on the Magnetism of Iron.

§ 137. **Effects of Torsion due to Magnetic Aeolotropy.**—Sir William Thomson has, in fact, pointed out that these results are to be anticipated from what is known regarding the effects of simple torsion and simple compression on the magnetic susceptibility of iron.* Experiments in which the metal is subjected to longitudinal pull or push and to transverse pull, have shown that a simple pulling stress or a simple pushing stress develops an æolotropic quality in respect of magnetic susceptibility, producing (in iron) greater susceptibility along than across the lines of pull, or less susceptibility along than across the lines of push, provided the magnetisation be not so strong as to pass the Villari critical value. Now in torsional strain, each portion of the twisted rod experiences a simple shearing stress, which may be regarded as made up of a pulling stress in a direction inclined at 45deg. to the direction of the length, and an equal pushing stress also inclined at 45deg. and at right angles to the pulling stress. Thus, if *a b c d* (Fig. 115) is a particle anywhere in the front half of the rod, which is twisted in the manner shown by the arrows, the twisting produces a shearing stress in *a b c d* that is equivalent to a pull on the faces *a b* and *c d*, combined with an equal push on the faces *d a* and *b c*. The effect is to increase the magnetic susceptibility along the direction *p p* and to reduce it along *p′ p′*. For small stresses these effects are no doubt equal. Hence in the direction of the length of the

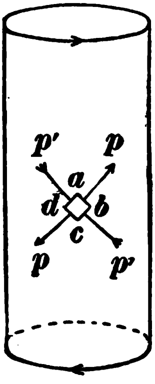

FIG. 115.

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* Reprint of Papers, Vol. II., p. 374.

rod, which is equally inclined to $p\ p$ and $p\ p'$, there is, virtually, no change of susceptibility.

The effect of torsion is to give a helical quality to the magnetisation, producing a circular component which is superposed upon the original longitudinal magnetisation. The lines of magnetisation are no longer coincident in direction with the lines of magnetic force; they become in the case considered above right-handed screws. The effect of this on the magnitude of the longitudinal component is at first indefinitely small, but as the angle of torsion increases the growth of the circular component begins to detract from the longitudinal magnetism, for magnetisation in one direction is prejudicial to magnetisation in other directions, as the molecular theory and the phenomenon of saturation suggest.

This consideration of the magnetic æolotropy produced by the pull and push into which torsional stress may be resolved supplies a key to many of the observed facts about magnetism and torsion. At the same time it fails to explain many of the facts. The influence of æolotropy is, no doubt, always present in the phenomena of torsion, but other considerations of a less obvious kind also enter, and these become in some instances so influential that the effects of æolotropy are entirely masked. This is notably the case with nickel. With soft iron, on the other hand, most of the observed effects of torsion admit of fairly complete explanation in the lines suggested by Sir William Thomson, especially when allowance is made for the complications to be anticipated from hysteresis.

§ 138. Production of Longitudinal Magnetism by Twisting a Circularly Magnetised Wire.—From the foregoing account of how a circular component of magnetisation is developed by torsion in a longitudinally magnetised wire or rod, it will be evident that the converse action should occur, namely, that twisting a circularly magnetised rod should make it develop longitudinal magnetism. This fact was observed in 1858 by Wiedemann, who found that an iron wire conducting an electric current, and therefore circularly magnetised, becomes a magnet when twisted.* Following Thomson, we may ex-

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* *Elektricität*, Vol. III., p. 680.

plain this observation as a consequence of æolotropy by resolving the magnetising force, whose direction is O A (Fig. 116), into components along the lines of pull, O *p*, and push, O *p'*. Taking the case of iron, below the Villari critical point, and twisted in the manner shown in the diagram, the susceptibility is greater along the lines of pull, O *p*, than along the lines of push, O *p'*. Hence the resultant magnetisation will be less inclined to O *p* than to O *p'*; in other words, it will take some direction, O R, which gives a longitudinal component of magnetisation directed towards the bottom of the rod. This is, in fact, the kind of longitudinal magnetism which is found.

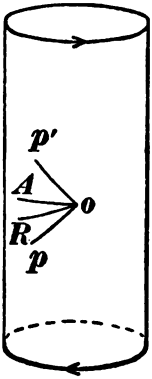

FIG. 116.

It might, however, be supposed, in view of the Villari reversal, that under sufficiently strong circular magnetisation the longitudinal component developed by twisting would become reversed. Experiment shows that this does not happen even when a very strong current traverses the wire. The explanation appears to lie in the fact that the stresses of pull or push due to torsion act not on the whole intensity of circular magnetisation but on components inclined at 45deg. Hence, though the circular magnetising force be strong enough to bring about saturation, the components of magnetisation on which the pull and push act remain below the Villari critical value, so that the effect of pull is still to augment and of push to diminish the components on which the pull and push act.*

These effects of torsion are found in dealing with residual magnetism as well as with induced. In Wiedemann's experiment the same result (namely, the production of longitudinal magnetism by torsion) is noticed though the wire be not twisted until the current has ceased to pass. There is then a strong residual circular magnetism which is affected by torsion, just as might be anticipated from the fact that the residual magnetism of a bar magnetised in the usual way is affected like induced magnetism by pull and push.

§ 139. **Torsional Strain produced by Combining Circular with Longitudinal Magnetisation.**—A similar explanation applies to another discovery of Wiedemann's, namely, that if an iron wire or rod be both circularly and longitudinally magnetised, it becomes twisted, though no external mechanical force be used. The superposition of the two magnetisms turns the lines of magnetisation into screws, and the consequent expansion along the lines of the screws and contraction across these lines causes the rod to twist. In iron the effect of magnetising (unless the magnetising force be very strong) is to lengthen the metal in the direction of magnetisation. The direction which the twist is observed to take agrees with this.

In nickel, on the other hand, the effect of magnetising is to shorten the metal in the direction of the lines of force. The twist taken by a nickel wire, subjected to superposed longitudinal and circular magnetising forces, is accordingly opposite to that of iron, as Prof. Knott has shown† by making a current traverse a nickel wire, which was at the same time exposed to the action of a magnetising solenoid.

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* This absence of reversal is referred to by Sir William Thomson as a difficulty ; but the difficulty disappears when it is recognised that the Villari reversal depends rather on the value **I** in the direction of pull and push than on the value of **H**. Though the components of **H** along directions inclined at 45deg. to the axis may be indefinitely increased by increasing the whole magnetising force, the components of **I** along these lines remain too small to allow pull to produce reduction of magnetism.

† *Trans.* Roy. Soc. Edin., Vol. XXXII. (1883), p. 193.

§ 140. **Transient Currents produced by Magnetising Twisted Rods, or by Twisting Magnetised Rods.**—The sudden development of circular magnetism when a longitudinally magnetised rod is suddenly twisted, or when a longitudinal magnetising force is suddenly applied to a rod that is held in a state of torsion, is well shown by connecting the ends of the rods to a galvanometer, when it will be found that a transient current is induced along the rod. A still more effective experiment may be arranged by substituting a tube for the solid rod, and by placing within it an insulated wire in circuit with the galvanometer.* In experiments of this class the existence of hysteresis is shown in an interesting way by making back and forth twisting take place in a series of steps, when, by summing the transient currents, it is at once seen that the circular magnetisation exhibits hysteresis with respect to the angle of twist—a result which is of course to be anticipated from the known effects of pull and push. Thus in Fig. 117 an iron wire rather strongly magnetised in the direction of its length was twisted alternately to opposite sides, but the twist-

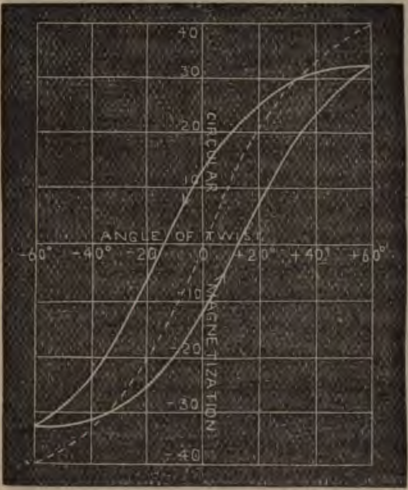

FIG. 117.—Circular Magnetisation produced by Twisting Magnetised Iron.

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* *Proc.* Roy. Soc. 1883, p. 117; 1881, p. 21.

ing was done in a series of steps, and the transient current for each step was noted.

Summing up the transient currents we obtain the circular magnetisation in arbitrary units. The full lines of the figure show how the circular magnetisation was cyclically reversed by reversing the twist, but the change of circular magnetism lagged behind the change of twist.* The dotted line in the same figure exhibits the amount of circular magnetism found by first applying a given torsion and then reversing the longitudinal magnetising force. This procedure, of course, eliminates the hysteresis which appears in the other.

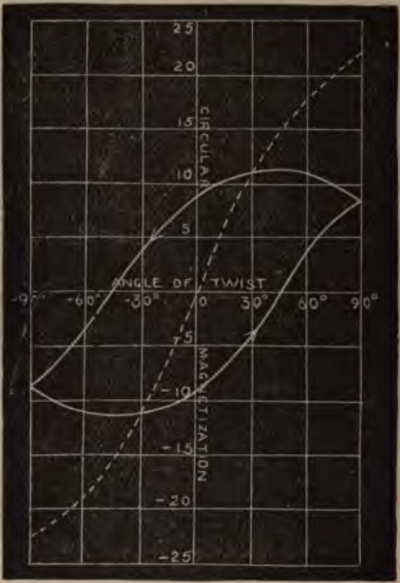

Fig. 118.—Circular Magnetisation produced by Twisting Magnetised Steel.

Fig. 118 gives the results of a similar experiment made with a piece of pianoforte steel wire in its usual condition of temper. The dotted line has the same meaning as in Fig. 117.

In these cases the process of back and forth twisting had

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* It was in connection with this instance of lagging, one of the first which the author met with in his experiments, that the word "hysteresis" was originally introduced. (*Proc.* Roy. Soc., 1881, p. 22.)

been repeated often enough to bring about a cyclic *régime* before the observations were taken. It is interesting to notice the manner in which the cyclic state is reached. This is shown in Fig. 119, which relates to the same wire as Fig. 118. Starting from the condition that had been reached by reversing the longitudinal magnetisation, when the angle of twist was +90deg., the changes shown in the diagram were brought about by twisting back to −90deg., again to +90deg., back to −90deg., and again to +90deg.

In all these cases the direction of the circular magnetisation was such as would correspond to *increase* of magnetism by pull. The current flows from the North to the South pole when the wire or tube is twisted like a common or right-handed screw. And a careful inquiry has shown that the effect of torsion is always of this character in iron: that is to say, the effect is not reversed, though the longitudinal magnetising force be made very great. What happens in that event is that the transient currents due to torsion become exceedingly small; but their sign does not change. Here, again, the explanation is that the intensity of magnetism on which the pull and push of

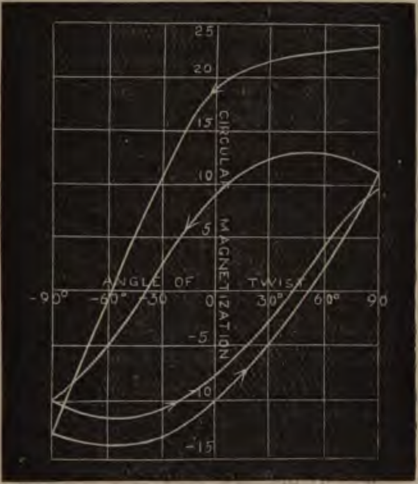

Fig. 119.—Effects of Twist.

the torsion act, is the component at 45 degrees to the axis, and this remains below the Villari critical value, even when the whole magnetism approaches saturation* (*see* § 138, *ante*).

When this longitudinal magnetism is residual instead of induced, torsion still produces transient currents of the same general character, but the effects are complicated by a progressive shaking out of the magnetism.†

Using a telephone in place of a ballistic galvanometer, Hughes has observed the production of transient currents in a twisted wire, when the current in a surrounding solenoid is rapidly interrupted or reversed. He has also illustrated the production of longitudinal out of circular magnetism in a twisted wire, by passing an interrupted current through the wire, and putting a telephone in circuit with a coil wound round the wire.‡

In nickel, the effects of simple pull and push are, as we have seen, opposite in sign to the effects in iron, provided the magnetisation of the iron is not so great as to cause the Villari reversal. Hence we may expect the transient currents produced by twisting a longitudinally magnetised nickel rod or tube to take the opposite direction to that which they take in iron. This fact has been verified by the independent experiments of Zehnder § and Nagaoka,‖ who found that when a nickel wire is twisted as a right-handed screw the transient current flows from the south to the north pole.

§ 141. Effects of Combined Pull and Torsion on the Magnetisation of Iron and Nickel.—The same observers have recently examined, in much detail, the changes of magnetism in iron and nickel, which occur when a rod is subjected at the same time to pull and twist, and Nagaoka has also determined the curves of magnetisation which are given by nickel wires when kept in this complicated condition of stress. Many of the

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* *Proc.* Roy. Soc., 1883, p. 129.

† *Loc. cit.*, p. 126.

‡ *Proc.* Roy. Soc., Vol. XXXI.

§ *Wied. Ann.*, 1889, Vol. XXXVIII., p. 68.

‖ *Jour.* Coll. of Science, Imperial Univ. of Japan, Vol. III., 1890, p. 335.

results are of great interest, and space must be found for a brief notice of some of them here.*

In the magnetisation of any of the magnetic metals we may distinguish broadly between three successive stages in the process. There is, first, the early stage, during which the susceptibility is comparatively small: the curve of magnetisation shows at the beginning a comparatively easy gradient. Then there is the middle stage, a stage of high susceptibility, when the curve has bent upwards and rises rapidly towards the "wendepunct." Lastly, there is the third stage, in which the condition of the specimen may be spoken of, rather loosely, as nearly saturated. In the third stage the curve has passed the "wendepunct," and has again taken an easy gradient: the susceptibility rapidly diminishes.

In specimens which are free from stress during the process of magnetisation these three stages are to some extent blended, but are still fairly distinguishable, as a reference to any of the figures which have been given for iron, steel, nickel, or cobalt will show. By applying torsion, and still more by applying both torsion and longitudinal pull, it is possible to differentiate the stages to a very remarkable degree. This is shown by Nagaoka's experiments on nickel wires, which are illustrated in the following figures.†

Fig. 120 shows the influence of simple torsion. The curve *a a* is the ordinary magnetisation curve of a long nickel wire, annealed and tested (without torsion) by applying and removing a magnetising force of about 30 C.-G.-S. units. The curve *b b* was taken while the wire was held twisted, the amount of the twist being 3° per centimetre of length.

As the diameter of the wire was 1 millimetre, this amount of twist corresponds to an angle of shear of $\frac{3 \times \pi \times 0{\cdot}05}{180}$, or 0·0026 radians at the circumference, where the shearing strain is greatest.

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* *See* Papers by Nagaoka, *Jour.* Coll. Science, Imp. Univ. Japan, Vol. II., 1888, p. 283, p. 304; Vol. III., 1889, p. 189; Zehnder, *Wied. Ann.*, 1890, Vol. XLI., p. 210; also Papers by Prof. Knott, *Jour.* Coll. Science, Imp. Univ. Japan, Vol. III., 1889, p. 173; *Proc.* R. S. E., Vol. XVII., 1890, p. 401, and Vol. XVIII., 1891, p. 124.

† *Jour.* Coll. Science, Imp. Univ. Japan, Vol. II., p. 304.

The curve taken when the wire was under torsion exhibits some striking differences from the other. In the first place, the initial susceptibility (with respect to feeble magnetic forces) is greatly lowered by torsion. The first part of the magnetising process is sharply distinguished from the second stage. When the second stage is reached, the twisted wire has very great differential susceptibility. Again, the "wendepunct" in it is comparatively sharp. Finally, by comparing the curves got during the removal of magnetising force, we see that the twisted wire possesses much more retentiveness than the other; the ratio of residual to induced magnetism in it has the remarkably high value of 0·97, whereas in the untwisted wire the ratio of these

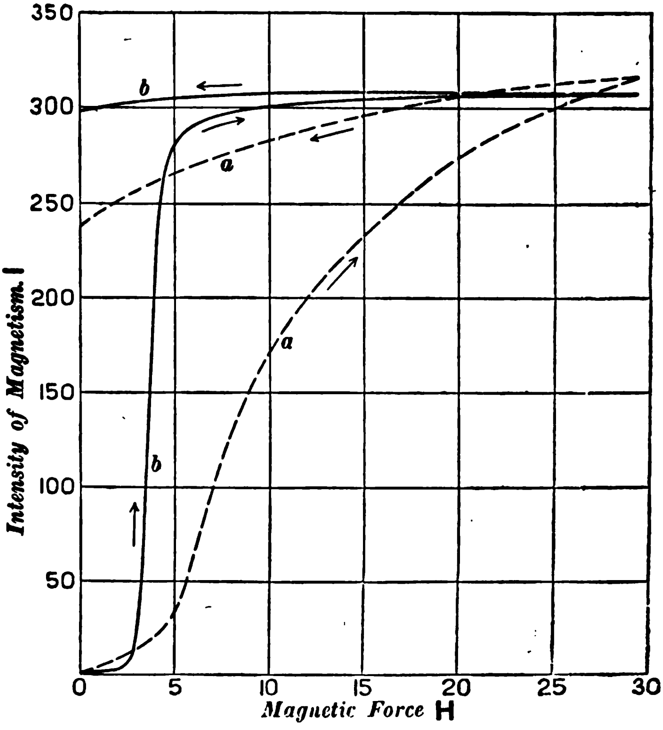

FIG. 120.—Magnetisation of Nickel: *a a*, without torsion; *b b*, with torsion.

quantities is only 0·75. If the comparison of residual magnetisms were made at earlier points in the magnetising process, this difference would be still more marked.

Mr. Nagaoka's experiments further prove that when the angle of twist is considerably increased the curve shows a slight tendency to revert towards the normal type (for untwisted wire). It must, however, be borne in mind that any large amount of torsion complicates the conditions of the experiment by making the strain pass the limit of elasticity.

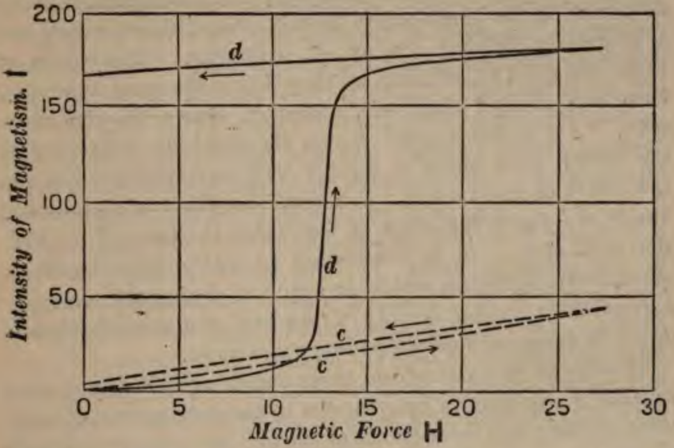

FIG. 121.—Magnetisation of Nickel: *c c*, with longitudinal pull only; *d d*, with longitudinal pull and torsion.

More curious still are the results of combining torsion with longitudinal pull. The application of pull, by itself, has (as was shown in § 121) the effect of lowering the magnetisation curve of nickel. When twist is superposed upon pull the initial part of the curve is still further lowered, but at a moderately great value of the magnetising force a sudden change takes place, the differential susceptibility becomes enormous throughout a narrow range of values of the magnetising force; then comes a somewhat sharp "wendepunct," and the second stage is followed by a third in which there is a slow approach to saturation. Fig. 121 is selected from Nagaoka's curves to illustrate these effects. The wire, which was the same specimen of nickel as before, was loaded with 10 kilos., and the curve *c c* was taken while there was no twist. Here, as the results of § 121 lead us to expect, there is low susceptibility throughout and exceedingly little retentiveness. Next, a steady twist of 3° per centimetre was given to the loaded wire. The curve of magnetisation was then found to take the extraordinary form shown in *d d*, with reduced initial susceptibility, which lasts through a wide range of force,—followed by an abrupt rise of magnetism in a field of about 12 to 13 C.-G.-S., and then high retentiveness. We have here a quite exceptionally sharp definition of the three stages in the magnetising process, and a singularly striking display of hysteresis. The curves of Figs. 120 and 121, relating, as they do, to the same specimen, form one group; they are, moreover, drawn to the same scale. *a a* is the normal curve, showing the behaviour of the metal when there is neither pull nor twist; *b b* shows the effect of twist alone; *c c* shows the effect of pull alone; finally, *d d* shows the effect of combining the twist of *b b* with the pull of *c c*. It is interesting to notice that the whole amount of magnetism which is acquired during the second or abrupt stage in *d d* is only about half the amount that is acquired during the corresponding stage in *b b*.

The effects of twist which these curves exhibit do not seem capable of explanation by reference to the development of magnetic æolotropy in consequence of the pull and push components of torsional stress. The inadequacy of this explanation will be even more apparent in the experiments with which the next paragraph deals.

§ 142. **Effects of Cyclic Twisting in Nickel, when associated with Longitudinal Pull.**—The combination of torsion and pull has been found by Nagaoka to have an even more extraordinary effect on the magnetisation of nickel if the torsion be subjected to cyclic reversals, while the pull is maintained constant. Let a nickel wire be exposed to any moderately weak magnetising force in the direction of its length, and let one end be twisted to and fro while the other end is held fixed. So long as there is no longitudinal pull the effects of this alternating torsion are comparatively simple. The curve connecting magnetism with angle of twist has a symmetrical, or nearly symmetrical, form, recalling that found in iron (Fig. 114), but with the important difference that, in nickel, the magnetism increases with twist instead of diminishing as it does in iron. This difference is intelligible enough, in view of the opposite effects of pull in nickel and in iron.

But let the experiment of twisting to and fro be repeated when a weight is hung from the wire to produce a steady longitudinal pull. It is now found that the symmetry of effect is gone. The magnetism is much increased by twisting the wire to the side towards which the earliest twist is directed. Twisting to the other side does not increase the magnetism nearly so much. And if the amount of steady pull be sufficiently increased, this want of symmetry becomes more pronounced, until a very peculiar result is brought about—that, whereas twisting towards one side increases the magnetism, twisting towards the other side decreases the magnetism, and *may even decrease it so much as to reverse its sign.*

This description will become more intelligible if reference is made to Figs. 122, 123, and 124, which illustrate one of Nagaoka's experiments. The specimen was a nickel wire 1 mm. in diameter and 40 cms. long, tested in the annealed state. A surrounding solenoid allowed a magnetising force to be applied, which was kept constant with the value 2·47 throughout the experiment. In the first instance (Fig. 122) there was sensibly no longitudinal pull. Repeated twistings from +180° to −180° brought about a cyclic state of things which is represented in the figure. Here the general effect of torsion is that twisting to either side augments the magnetism. Next, a steady longitudinal pull was applied, amounting to 1·45 kilogrammes per square millimetre, and the process of twisting to and fro was repeated. The result was to establish the cycle of Fig. 123, in which the loop of the curve on the side of positive twist is much more prominent than the loop on the side of negative twist. The term positive, as used here, simply distinguishes the direction which happened to be given to the twist in the first instance. The question of which direction of torsion augments the magnetism most depends simply on which is the direction the wire is first twisted in. Next, the longitudinal pull was increased by adding more steady load, and it was found that the positive loop of the curve lengthened, while the negative loop became more insignificant. Finally, the negative loop disappeared, and with a load of 7·82 kilogrammes per sq. mm., the cyclic process took the form shown in Fig. 124, where we see the extraordinary phenomenon of a reversal of magnetic polarity occurring with every reversal of torsional strain, notwithstanding the fact that the force **H** of 2·47 units was continuously operative in one fixed direction.

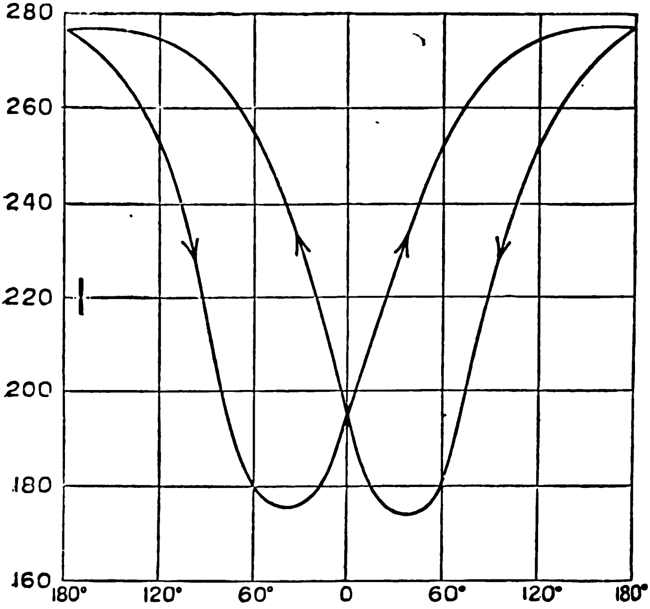

FIG. 122.

It is clear that these effects of twist are not consequences of æolotropy in the twisted material in respect of magnetic susceptibility. In fact, the inducing magnetic force plays a very subordinate part in the changes of magnetism which take place when the wire is twisted to and fro, after a cyclic *régime* is established. Its function is to set up a magnetic condition to begin with; then, as the wire is twisted back and forth, there is with each twist a profound change in the molecular configuration. This is the direct result of the twist, and may, as in the case last described, go so far as to produce reversal of magnetic polarity.

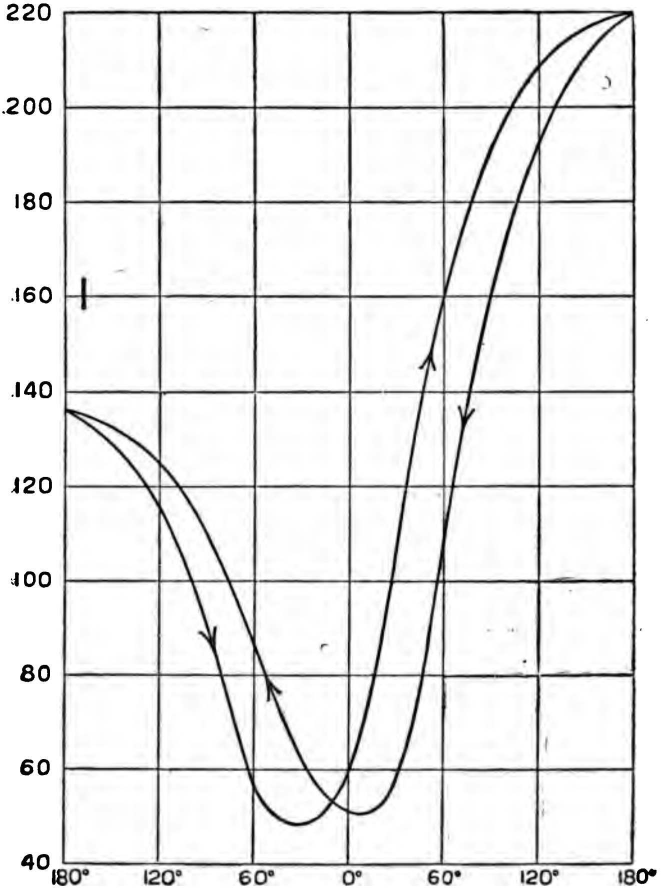

FIG. 123.

When the magnetising field is sufficiently strengthened, this reversal of polarity does not occur. The inducing force then asserts itself, and the effects of twist come to be more

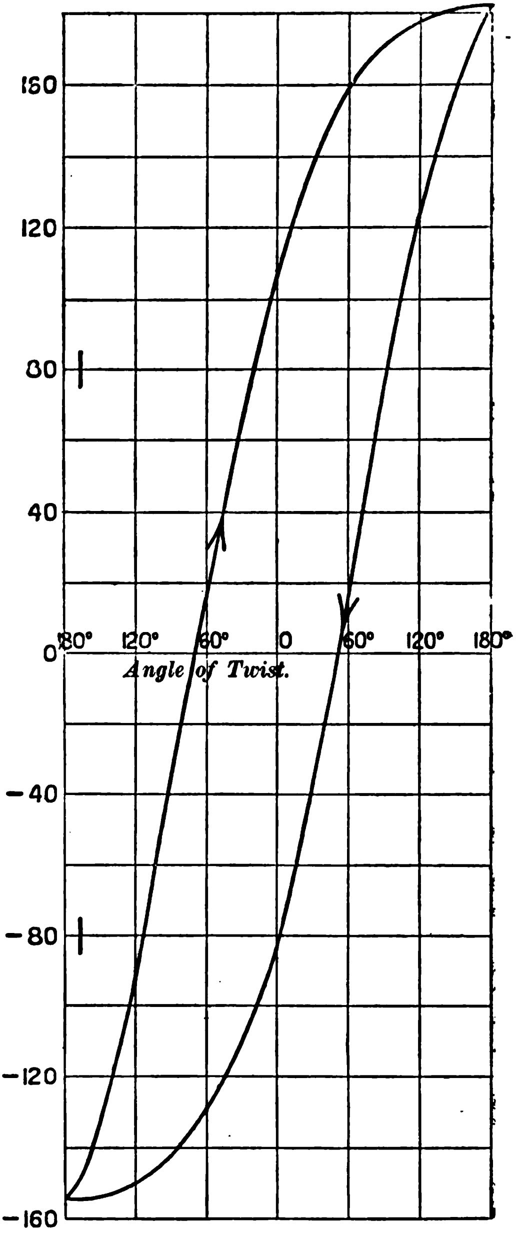
160
120
80
40
0
−40
−80
−120
−160
180°
120°
60°
0
60°
120°
180°
Angle of Twist.

Fig. 124.

nearly such as might be anticipated from the consideration of æolotropy. Iron does not reverse its polarity under the combined influence of pull and twist.

§ 143. **Strain caused by Magnetisation.**—Closely associated with the changes of magnetisation that are caused by strain are the changes of form which a piece of iron, or other magnetic metal, is observed to undergo when it is magnetised, or when its magnetism is changed. The fact that strain alters magnetism involves this converse, that change of magnetism is accompanied by strain.* The earliest experiments on the subject were those of Joule,† who found that the length of a soft iron rod was increased by the application of magnetising force, within the limits of force to which his experiments extended. The extension was accompanied by lateral contraction, with the result that the volume of the rod did not sensibly change. To show this, the experiment was made of magnetising the rod within a tube full of liquid, which was closed, except for an extended portion with a narrow bore, the rise or fall of the liquid in which would have indicated any change of volume on the part of the iron. Later experiments on the extension of rods were made by Mayer,‡ who dealt specially with steel; and by Barrett,§ who extended the inquiry to nickel and cobalt, finding that nickel retracted when magnetised. It is unnecessary for our purpose to refer to these early experiments in detail, for in recent years the matter has been exhaustively examined by Shelford Bidwell, whose results have harmonised much that was apparently contradictory in the statements of previous investigators. Dealing with rings as well as rods (to secure uniform magnetisation and determinate magnetising forces), Bidwell has tested iron, steel, nickel, and cobalt throughout a very wide range of magnetising force, and has found that when the force is pushed to high values the character of the action becomes greatly changed. He has also

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* *See* Prof. J. J. Thomson's "Applications of Dynamics to Physics and Chemistry," Chap. IV.

† Joule, *Phil. Mag.*, 1847, Vol. XXX., pp. 76, 225. Reprint of Papers, p. 235.

‡ Mayer, *Phil. Mag.*, Vol. XLVI., p. 177.

§ Barrett, *Nature*, 1882, Vol. XXVI., p. 585.

examined the modifying influence of externally imposed tensile stress on the change of length caused by magnetisation. The following is a brief summary of the more important of his results.*

The method of experiment is shown diagrammatically in Fig. 125, where S is the ring to be tested. The change of length along the lines of magnetisation was deduced by observing the change in the diameter of the ring which occurred when the magnetising current was applied. The ring S was placed between a fixed support E, and a long lever B, pivoted on a fixed fulcrum at A. The long end of the lever at C tilted a small mirror M hinged on a fixed support by a knife edge at D. The deflection of the mirror was read by means of a distant scale, the sensibility of the arrangement being such that readings could be taken corresponding to about one ten-millionth of the length of the specimen. The ring was jacketed with wood to exclude, as far as possible, the heating effect of the magnetising coil, and with the same object the circuit was never allowed to remain closed for more than a fraction of a second. A similar arrangement was

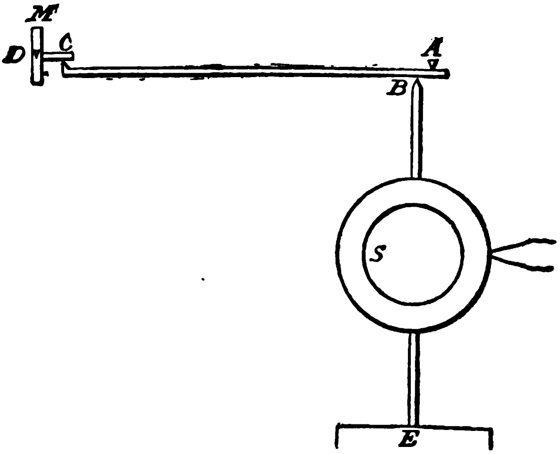

FIG. 125.

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* Shelford Bidwell, *Proc.* Roy. Soc., 1886, Vol. XL. pp. 109 and 257; *Phil. Trans.*, 1888, p. 205; *Proc.* Roy. Soc., 1890, Vol. XLVII., p. 469.

used for tests of rods. The ring or rod was first demagnetised by reversals, then a current of known strength was passed for a moment through the magnetising coil and the deflection was noted, the dead-beat character of the apparatus allowing this to be done almost instantaneously. Then the specimen was again demagnetised and another current was applied, and so on. In some of the experiments magnetising forces approaching 1,500 C.-G.-S. units were employed.*

Beginning with small magnetising forces, Bidwell found his iron rods and rings elongate when magnetised, by amounts which appear to increase at first in something like simple proportion to the degree of magnetisation. But as the magnetising force increases, the elongation of iron passes a maximum, becomes reduced, and vanishes when the magnetising force is about 300 C.-G.-S. With higher forces still, the iron retracts instead of elongating when the magnetising force is applied, and this retraction appears to tend to a finite limit as the force is further increased. The maximum amount of elongation, which is observed in comparatively moderate fields (say about 100 C.-G.-S.), varies in different specimens; it ranged in Bidwell's experiments from about $\frac{1}{200,000}$ to $\frac{1}{400,000}$ of the length. The amount of retraction under very strong force may be as much as $\frac{1}{150,000}$ of the length. These figures refer to iron. Steel behaves in much the same way, but suffers less elongation than iron under moderate forces.

In nickel, on the other hand, there is retraction from the first, and the amount apparently tends to a fixed limit as the magnetising force is raised to high values.

In cobalt the action is less simple. Weak magnetising forces cause sensibly no change of length. Stronger forces cause retraction, but the amount of that passes a maximum, and vanishes with further increase of the force, after which, with stronger force still, there is *extension*, the amount of which was still increasing fast in the strongest field to which the experiments were carried. These results are well shown by Figs. 126 and 127, taken from Bidwell's principal Paper. In Fig. 126 the magnetic force ranges up to 800 units. In Fig. 127 it is carried to nearly twice that value. The specimens of metal tested were different in the two cases; the general

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* *Phil. Trans., loc. cit.* p. 227.

results, however, agree well. The nickel used in the experiment of Fig. 126 shows more retraction than the other. The amount of retraction under the strongest magnetic force is about $\frac{1}{40,000}$ of the original length.

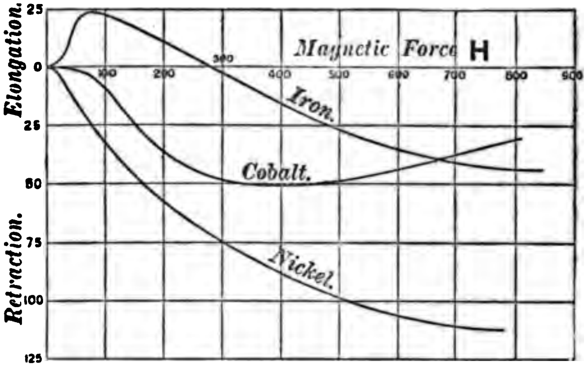

FIG. 126.—Elongation and Retraction of Iron, Nickel, and Cobalt through Magnetisation (Bidwell). The elongations and retractions are stated in ten-millionths of the length.

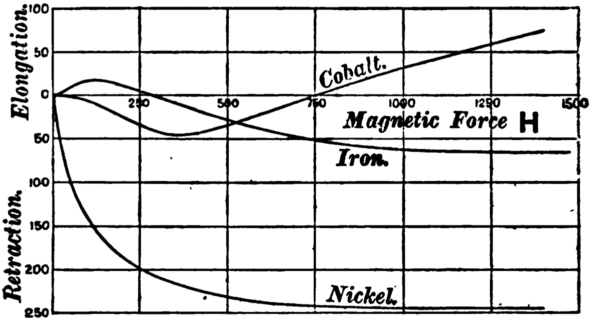

FIG. 127.—Elongation and Retraction of Iron, Nickel, and Cobalt through strong Magnetisation (Bidwell). The elongations and retractions are stated in ten-millionths of the length.

§ 144. **Modification of the Results by applying Tensile Stress.**—In a later Paper,* Bidwell has described experiments made with rods of iron, nickel, and cobalt, in which the change

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* *Proc.* Roy. Soc., 1890, p. 469.

of length caused by magnetisation was made to take place while a load was hanging from the specimen. The results for iron are shown in Fig. 128, the numbers attached to the curves being the values of the externally applied stress in kilogrammes per square cm. The effect of tension in iron is to lower the curve, reducing the maximum extension which magnetisation causes, and finally making it vanish. Under the greatest loads used in these experiments, the iron retracted from the first as the magnetising force was increased.

In nickel the effect of tensile stress is to raise the curve in its earlier stages, making the amount of retraction caused by magnetisation less if the test is made while there is pull than if it is made while there is no pull. This applies to magnetising forces of moderate strength. But in stronger fields the magnetic contraction is increased by the presence of small amounts of pull, and decreased by the presence of large amounts of pull.

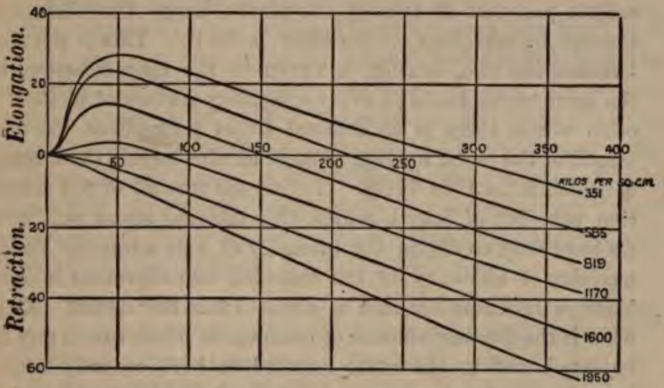

Fig. 128.—Changes of Length caused by magnetising Iron under various amounts of longitudinal pull (Bidwell). The elongations and retractions are stated in ten-millionths of the length.

In cobalt, the changes of length which the metal undergoes in being magnetised were found to be almost, if not quite, independent of the presence of pulling stress. Cobalt stands in striking contrast in this respect to iron and nickel, in both of which the modifying influence of pull is conspicuous.

§ 145. **Internal Stress in a Magnetised Rod or Ring, due to the Magnetisation.**—Some part of the retraction which is observed in experiments such as these is to be ascribed to the state of internal stress into which the metal is thrown by its magnetisation. Imagine a cross-section to be taken through the substance of a ring or very long rod, the intensity of magnetism in which is $\mathbf{I}$. The surfaces which are opposed to each other at the imaginary plane of section act as a pair of plates of attracting matter, the surface density of which is $\mathbf{I}$. The force which each plate exerts on unit quantity of the other is (by a well-known proposition in the theory of attraction) $2\pi\mathbf{I}$. But there are $\mathbf{I}$ units of attracting matter per unit of area of the plate: hence the whole force exerted by one plate on the other is $2\pi\mathbf{I}^2$. This is the stress between the two, existing in virtue of the magnetisation, and the same stress exists at every imaginary surface of section. In other words, there is an internal stress throughout the whole length of the piece, tending to contract the length, the intensity of which is $2\pi\mathbf{I}^2$ in dynes. To find the amount of the contraction per unit of length which this internal stress will cause, we have only to divide the intensity of this stress by Young's modulus of elasticity for the material, also expressed in dynes. Young's modulus for iron is about $1{\cdot}8 \times 10^{12}$ dynes. To see what is the greatest amount of contraction which can in any case be attributed to the cause referred to here, we may take for $\mathbf{I}$ its limiting value in iron, namely, 1,700 (*see* § 98), and the result is $\frac{2\pi \times 1{,}700^2}{1{\cdot}8 \times 10^{12}}$ or $\frac{1}{100{,}000}$ very nearly. In the experiments which have been described, $\mathbf{I}$ did not, of course, reach so high a value as 1,700. It is clear, then, that the effect in question is at least of the same order of magnitude as the contraction which actually occurs in iron. Bidwell, who has discussed the matter from the same point of view, has, however, come to the conclusion that the internal stress due to magnetisation is by no means the sole cause of the retraction which iron undergoes when it is strongly magnetised. Comparing the values of the retraction at two points in his curve, just when retraction has begun to take the place of extension, with the values of the internal stress as measured by the force required to separate two portions of a divided bar magnet, he contends that the difference in the amount of internal stress accounts for only about one-fifth of the actual change of length.*

§ 146. **Tractive Force in Divided Magnets.**—The remarks which have been made in the preceding paragraph may fitly be followed up by a reference to the attraction which subsists between two portions of a magnet when, instead of an imaginary plane of section, there is an actual surface of separation. Let a ring or long bar magnet be cut into portions which have their ends carefully faced to be true planes, and let these abut against one another. The force between the faces may be determined by measuring the amount of pull which is required to draw them asunder.

Measurements of the tractive force between the parts of a divided magnet were made by Joule,† who showed that the amount of the force required to separate two parts of a divided magnet varied as the area of cross-section, and found that the tractive force might be as great as 175lb. per square inch. Shelford Bidwell,‡ using a divided ring electro-magnet of iron, found that the weight which could be sustained per square centimetre of the cross-section was related to the magnetising force in the manner shown by Table XXV. :—

Table XXV.—*Tractive Force of a Divided Ring Electro-Magnet.*

Magnetic Force H.	Tractive force in grammes weight per square cm.	Magnetic Force H.	Tractive force in grammes weight per square cm.
3·9	2,210	145	12,800
5·7	3,460	208	13,810
10·3	5,400	293	14,350
17·7	7,530	362	14,740
22·2	8,440	427	15,130
30·2	9,215	465	15,275
40	9,680	503	15,365
78	11,550	557	15,600
115	12,170	585	15,905

As one gramme per square centimetre corresponds to 0·00142lb. per square inch, the highest tractive force reached

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* Bidwell, *Phil. Trans.*, 1888, pp. 217-218.

† *Phil. Mag.*, 1852, Vol. III., p. 32, or Reprint of Papers.

‡ *Proc.* Roy. Soc., 1886, Vol. XL., p. 486.

in this experiment was 226lb. per square inch. In these experiments the values of the magnetisation were not directly observed. They may, however, be inferred from the values of the tractive force in a manner which will be explained presently.

Further experiments on the same subject have been made by Bosanquet,* who used bar magnets, and employed a small induction coil (encircling the bar close to the surface of division) to determine the value there of **B** at the instant the two portions of the bar parted company (Fig. 129). In this case

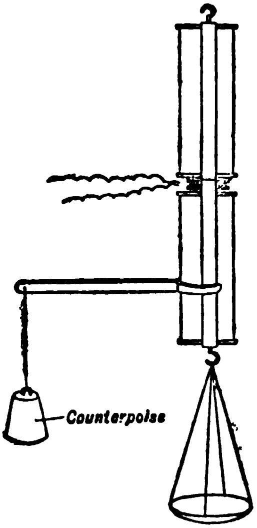

FIG. 129.—Bosanquet's Arrangement for Measuring Magnetic Traction.

it was possible to compare the actual values of the tractive force with the values which were to be anticipated from the known values of **B**. Bosanquet has made this comparison, and has found a fair agreement except in the early stages of the experiment. When **B** had any value less than about 5,000 the observed tractive force was greater than the calculated force. This is possibly to be ascribed in part to friction in the appliance by which the lower magnet was guided, and in part to a supplementary attraction between the mag-

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* *Phil. Mag.*, 1886, Vol. XXII., p. 535.

netising coils, which was then more considerable than when B was greater. With an induction B of 18,500 Bosanquet observed a tractive force equivalent to 207lb. weight per square inch of section of his iron core.

§ 147. **Relation of Tractive Force to Magnetisation.**—In connection with these experiments it is interesting to inquire what is the relation that theory would lead us to anticipate between the magnetisation of the core of a rod or ring electromagnet, and the tractive force necessary to overcome magnetic attraction at the abutting faces if the core is cut in two. This matter has been the subject of some discussion,* and it appears that a sufficiently careful distinction has not always been drawn between different conditions which, to some extent, affect the result. Consider the case of an indefinitely long rod, or a ring, wound throughout with a solenoid which produces a uniform magnetic force H. Let the intensity of magnetisation be I and the induction B, as usual. Let the rod or ring be cut across, the cut faces be scraped to form true planes, and placed in contact, so that the whole behaves, magnetically, as nearly as possible like an undivided core. At the junction there is an indefinitely narrow crevasse, on each side of which the surface density of magnetism is I. The result of that is (as was shown in § 145) that the opposing surfaces pull one another together with a stress the amount of which per square centimetre of surface is $2\pi \mathsf{I}^2$ in dynes. Since $\mathsf{B} = 4\pi \mathsf{I} + \mathsf{H}$ this quantity may be written $\dfrac{(\mathsf{B}-\mathsf{H})^2}{8\pi}$. If the magnetism that is dealt with is residual (that is to say, if H is zero) this is the whole force that must be overcome in separating the surfaces. So that if s is the area of surface in contact the whole tractive force is in that case equal to $2\pi \mathsf{I}^2 s$. But suppose H is not equal to zero—in other words, suppose that some current is circulating in the solenoid—then the separation of the opposing faces involves the movement through a field H, in the direction of the field, of a quantity of free magnetism the amount of which (per centimetre of surface) is I. There is, therefore, a supplementary force required, the value of which is $\mathsf{H}\,\mathsf{I}$.

But this is not necessarily all. The solenoid may itself be

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* *See* a Paper by Prof. S. P. Thompson, *Phil. Mag.*, 1888, p. 71.

wound in two portions—one on each part of the divided core—so that the separation of the core involves the separation of the two parts of the solenoid. In that case we have to exert enough additional tractive force to overcome the attraction of one coil on the other. The amount of this attraction will depend on the area of the coils. Take the simplest possible case, that of a solenoid so closely wound upon the core that the area of the coil may be considered identical with the area of the core. The two coils, considered alone, then behave like magnets having poles whose surface density is $n\,C$, where n is the number of turns per centimetre, and C is the current. The attraction between them, per sq. cm., is, therefore, $2\pi n^2 C^2$. Since H is equal to $4\pi n\,C$, this attraction may be written $\frac{H^2}{8\pi}$. We have here a third term which has to be added to the other two in the case considered, namely, when the solenoid (closely wound on the core) is parted along with the core. It must be borne in mind that the second term $H\,I$ occurs in this case, as well as in the case where the solenoid remains undivided, and the two parts of the core alone part company. For in the case of a divided solenoid each half of the solenoid pulls the opposing half core with a force which, per unit of area, is $2\pi n\,C\,I$, or $\frac{1}{2}H\,I$. There are two such forces to be overcome, namely, between the lower core and the upper solenoid, and between the lower solenoid and the upper core, and the two make up $H\,I$ as before.

In the case, then, of a divided electro-magnet, in which the magnetising coil parts along with the core, and in which the coil has no superfluous area (which would add still further to the tractive force), the whole force is made up of three parts—

$$2\pi I^2 + H\,I + \frac{H^2}{8\pi}.$$

This may be written

$$\frac{1}{8\pi}(16\pi I^2 + 8\pi H\,I + H^2)$$

$$= \frac{1}{8\pi}(4\pi I + H)^2 = \frac{B^2}{8\pi}.$$

This is the expression commonly used in calculating the relation of the tractive force at the cut to the magnetism.

In the exact consideration, however, of any given case, the particular disposition of the magnetising coil should be taken account of. If it is in one length, or carried independently of the core, so that the pull of its two parts on one another does not have to be overcome, the tractive force is less. If, on the other hand, the coil is separated along with the core, and has an area larger than the core itself, the force to be overcome expressed per centimetre of the core will be even greater than $\frac{B^2}{8\pi}$. In practice, especially when dealing with iron, the term expressing the mutual attraction of the magnetising coils is usually insignificantly small, since B is enormous compared with H, for such values of magnetic force as are usually employed.

By considering on general principles the state of stress in a magnetic medium which would give rise to the mechanical forces that are observed in the magnetic field, Maxwell* showed that where there is no magnetisation† there is a tension along the lines of force equal to $\frac{H^2}{8\pi}$. Within the indefinitely thin crevasse of air which separates the two opposed faces of a cut magnet there is no magnetisation, and the value of H there is that of B in the substance of the metal. This expression for the tractive force between the faces is therefore equivalent to the expression which has been deduced above in a more elementary fashion, namely, $\frac{B^2}{8\pi}$.

§ 148. **Determination of Magnetisation by Measuring the Tractive Force.**—Once a relationship is established between the tractive force and B or I, measurements of the tractive force may be resorted to as a means of determining magnetisation. This gives a fourth type of magnetic measurements, distinct from the ballistic, the direct magnetometric, and the optical methods which have been described in earlier chapters. The traction method of determining magnetisation has been used with good effect by Bidwell, who determined curves of magnetisation by observing the relation of the tractive force required to separate the halves of a divided iron ring to the

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* "Electricity and Magnetism," Vol. II., §§ 641-646.

† *Loc. cit.*, § 643.

intensity of magnetising force produced by a divided solenoid with which the halves of the ring were wound.[*] In reducing the results he used $2\pi \mathsf{I}^2 + \mathsf{H}\,\mathsf{I}$ as the equivalent of the tractive force. In the actual circumstances of his experiment the addition of the third term $\frac{\mathsf{H}^2}{8\pi}$ (making $\frac{\mathsf{B}^2}{8\pi}$ in all) would have been proper;[†] but the effect of this change on the numerical values of **B** or of **I** deduced from his experiments is quite trifling. His magnetic forces ranged up to 585, and produced at their highest value an amount of attraction from which **I** was calculated to be 1,530, **B** 19,820, and μ 33·9.[‡]

More recently Prof. S. P. Thompson has proposed the use of a simple traction-measuring instrument as a workshop appliance for determining permeabilities.[§] This "permeameter," as he terms it, is shown in Fig. 130. The specimen to be tested is a rod which slips through a hole in the top of a substantial iron yoke, and through a bobbin on which the magnetising coil is wound. The lower end of the sample is faced true, and rests on a part of the yoke which is also scraped to have a truly plane surface. The force required to detach the sample from the surface of the yoke is measured by means of a spring balance. In consideration of the fact that the magnetising coil is left *in situ*, Prof. Thompson takes $\frac{(\mathsf{B}-\mathsf{H})^2}{8\pi}$ as the quantity that represents the tractive force, and from this the practical rules are derived:—

$$\text{Pull in lbs.} = \frac{(\mathsf{B}-\mathsf{H})^2 \times s \text{ (square centimetres)}}{11{,}183{,}000},$$

or,
$$\mathsf{B} = 3344\sqrt{\frac{\text{Pull in lbs.}}{s \text{ in sq. cms.}}} + \mathsf{H},$$

or,
$$\mathsf{B} = 1317\sqrt{\frac{\text{Pull in lbs.}}{s \text{ in sq. inches}}} + \mathsf{H}.$$

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[*] *Proc.* Roy. Soc., 1886, Vol. XL.

[†] As Mr. Bidwell has himself remarked, *Phil. Mag.*, 1890, XXIX., p. 440.

[‡] To calculate **B** from the tractive force F we have $\mathsf{B} = \sqrt{8\pi F}$, F being expressed in dynes, or

$$\mathsf{B} = \sqrt{8\pi \times 981 \times \text{Force in grammes}} = 157\sqrt{\text{Force in grammes}}.$$

This formula will serve to determine **B** from the values of the tractive force given in Table XXV.

[§] *Jour.* Soc. Arts, Sept. 12, 1890.

It may be questioned whether the place chosen for the plane of contact in the "permeameter" is the best possible. The distribution of induction is rather unequal where the bar meets the yoke, and better results might be obtained by making the sample in two pieces with a plane of contact at the middle. Apart from this, however, no traction method can be regarded as a very satisfactory means of examining the magnetic quality of a metal. The presence of tensile stress itself affects the quality which is undergoing measurement, and, as will be shown later, a divided rod or ring does not behave magnetically quite like a whole rod, even when the ends are surfaced as carefully as is practicable. The existence of a cut lessens the permeability of the piece.* The traction method is at the best inexact, but it affords a ready means of making rough measurements, especially for purposes of comparison.

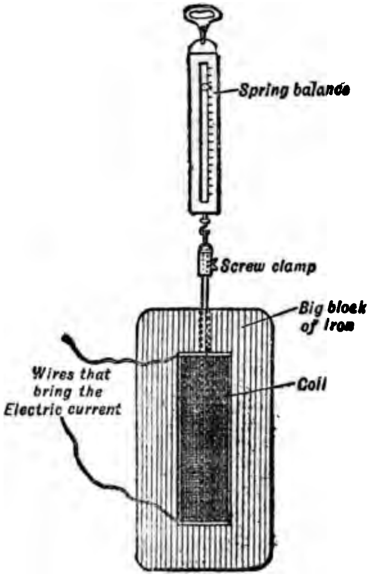

FIG. 130.—The Permeameter.

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* *Phil. Mag.*, Sept., 1888.

CHAPTER X.

THE MAGNETIC CIRCUIT.

§ 149. **The Magnetic Circuit.**—For many purposes, the most convenient way of treating the magnetisation of iron is to consider what is happening at a point within the metal. This is, in fact, the basis on which our exposition of the subject in earlier chapters has been developed. We have learnt to conceive of a magnetic force H acting in a definite direction at the point considered, and also of a magnetic induction B at the point. If the material is isotropic, and has no residual magnetism superposed upon the magnetism which H induces, the direction of B is the same as that of H. The ratio of B to H is the permeability μ. Passing from point to point of the metal, we may in certain cases find that H and B do not change; more generally they do change. Thus, in a uniformly wound circular ring magnet, of uniform section and material, H has the same value at all points on any circle co-axial with the ring. In a long straight bar magnet the value of H is nearly uniform, except in the neighbourhood of the ends. Whether H be uniform or not, it has a single definite value and definite direction at each point, and the same is true of B. At points where there is no magnetisable substance, the value and direction of B are the same as the value and direction of H; this applies, for instance, to all points in air. The value of H at any point is determined by finding the resultant of the force produced at that point by (1) all the conducting circuits, and (2) all the free magnetism in the neighbourhood; that is, by finding the resultant mechanical force which would be felt by a unit pole of free magnetism if placed at the point in question.

From this point of view, when our object is to discuss the magnetisation of a piece of metal, we have first to consider what is the value of **H** at each point within the piece. Thus, in dealing with a uniformly-wound ring, we treat the case by finding that the magnetic force **H** is $4\pi C n$, when C is the current and n is the number of turns in the magnetising coil per centimetre of the ring's length. And in dealing with a uniformly-wound rod, we find **H** to be $4\pi C n$ *minus* a certain quantity due to the free magnetism at and about the ends, which becomes unimportant when the rod is exceedingly long. Many problems in magnetism are best treated in this way, namely, by considering the condition of things at individual points in the magnetised piece.

But there is another way of regarding the matter, not in the least antagonistic to this, but sometimes more convenient. Instead of thinking about what happens at individual points, we may view the magnetism of the piece as a whole, by considering what is called the *magnetic circuit.* This is the method which has been applied by J. and E. Hopkinson* and by Kapp,† to pre-determine the magnetism of dynamos. Its applicability to dynamos and transformers gives it peculiar importance on the practical side; moreover, apart from that, the conception of the magnetic circuit has much interest as an alternative standpoint from which the facts of electro-magnetism may be viewed, and as suggesting methods of experimental enquiry.

§ 150. Tubes of Magnetic Induction. Definition of Magnetic Flux and of a Perfect Magnetic Circuit.—The lines of magnetic induction, as has been already pointed out (§ 14), are continuous through space, whether the space be filled with magnetisable or non-magnetisable substance, or partly with one and partly with the other. There is no discontinuity of **B**—no sudden change in its value or direction—when the lines pass from metal to air or from air to metal. Each line of induction is a continuous curve; moreover, it is a *closed* curve—that is to say, if traced along its whole course it returns to the point at which the tracing began. We may conceive of

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* *Phil. Trans.*, 1886, p. 331.

† *Jour.* Soc. Tel. Eng., 1886, p. 518.

all space as filled with sheafs of lines of induction, or (which is the same thing in other words) as partitioned into tubes, the boundaries of which are formed by lines of induction. Every such tube contains a number of lines of induction, and if we follow the tube along its whole length until it returns into itself we find everywhere the same number of lines of induction in it. We may take a large sheaf or a small one to constitute the tube, but, whatever be the number of lines in it to start with, the same number is present at every part of its length. Its cross-section may vary; the tube may widen or contract from place to place along its length, but if this happens it is by the lines spreading out or coming closer; the number of the lines does not change. At places where the induction **B** is strong, the tube is contracted; at places where the induction is weak, the tube is expanded. But if we take any cross-section (s) of the tube perpendicular to the direction of **B**, the product $\mathbf{B}\,s$ (or, to be more exact, the surface-integral $\int \mathbf{B}\,d\,s$ taken over the section, since **B** is not necessarily the same over all parts of s[*]) is a constant quantity for any one tube. At any sections s and s', the values of the induction **B** and $\mathbf{B}'$ are such that $\int \mathbf{B}\,d\,s = \int \mathbf{B}'\,d\,s'$. It is convenient to have a name for this constant quantity, which is the whole number of lines of magnetic induction in the tube. Following the usage of several recent writers we shall call it the *magnetic flux* in the tube.

Any tube of magnetic induction, considered as a whole—that is to say, considered as a circuit which returns into itself—may be called a *perfect magnetic circuit.* The perfect magnetic circuit is analogous to a perfectly insulated electric circuit conducting a current. The lines of induction correspond in this analogy to lines of flow of current. The cross-section of the conductor may vary from place to place, but the current density varies in inverse proportion to the cross-section, so that the product of current density into area of cross-section —which is simply the whole current—is constant at all sections, just as the flux $\mathbf{B}s$ is constant in the perfect magnetic circuit.

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[*] The cross-section, over which this integral is calculated, is taken so that every element of the surface is perpendicular to the lines of **B** which cut it. Thus, if the lines of **B** in the tube are not parallel, the surface forming the cross-section will be curved.

§ 151. **Imperfect Magnetic Circuit.**—An imperfectly insulated electric circuit allows some of the lines of flow to enter it or to leave it through the sides. We have the magnetic analogue of this when we have to deal with the magnetisation of a material ring of any form, in which the sides of the ring do not coincide with the sides of a tube of induction. This means that there are places where some of the lines of induction leak out, so to speak, from the substance of the ring through its sides into the surrounding medium. It is often convenient, especially when the greater part of the whole flux remains in the ring, still to speak of such a ring as a magnetic circuit. We shall distinguish it from a true tube of induction by calling this leaky ring an *imperfect magnetic circuit.* It is imperfect, inasmuch as it leaves out those portions of the surrounding medium through which some of the lines of induction stray, the inclusion of which would be necessary in order to make the tubes of induction complete. Examples of imperfect circuits will be given presently.

§ 152. **Line-Integral of Magnetic Force, or Magnetomotive Force.**—We have now to express the relation of the magnetic flux in a perfect magnetic circuit to the whole magnetising agency acting on the circuit, just as in a perfectly insulated electric circuit we express the relation of the current to the whole electromotive force that is operative throughout the circuit.

We have in the magnetic circuit an agent to which (when we are dealing with induced magnetism) the magnetic flux is due, which corresponds to the electromotive force of the electric circuit. To this agent Bosanquet* has given the name of *magnetomotive force.*

One way of defining the electromotive force of an electric circuit is to say that the electromotive force is the amount of work which would be done in carrying unit quantity of electricity completely round the circuit.

In the same way we may define the magnetomotive force of a magnetic circuit as the amount of work which would be done in carrying a unit magnetic pole completely round the circuit. At any point of its path the unit pole is acted on by a mechanical force which is equal to and in the direction of the magnetic

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* Bosanquet, *Phil. Mag.*, 1883, Vol. XV., p. 205.

force H, and it is against this mechanical force that the work is done.

Another name for the same quantity is the *Line-Integral of Magnetic Force*, taken round the circuit.* Conceive the path along which the unit pole is moved to be made up of a great many short pieces, any one of which is so short as to be sensibly straight and to have a sensibly uniform value of H from end to end of it. Let the length of any short piece be δl, and let its inclination to the direction of H be ϵ. Then the work done in moving the unit pole along this short piece of the path is measured by the product of the length of the path (δl) into the component of the force H along the path; that is to say, it is $\mathsf{H} \cos \epsilon\, \delta l$. The whole work done in moving a unit pole along the path is got by summing up the work done at each short piece; that is to say, it is $\Sigma\, \mathsf{H} \cos \epsilon\, \delta l$, or $\int \mathsf{H} \cos \epsilon\, d l$ when the elements into which the path is divided are indefinitely numerous and indefinitely short. The expression $\int \mathsf{H} \cos \epsilon\, d l$ is the line-integral of the magnetic force along the path.

We may integrate the magnetic force in this manner along any curve whatsoever. The term line-integral of magnetic force is not in the least restricted to cases in which the integration takes place round a magnetic circuit. Let the path through which the unit pole is supposed to be carried extend through space in any manner, the line-integral, namely, $\int \mathsf{H} \cos \epsilon\, d l$, measures the work done in carrying the pole along it.

In cases where the direction of the line coincides at all points of its course with the direction of H, $\cos \epsilon$ is everywhere unity, and the expression for the line-integral of the magnetic force becomes $\int \mathsf{H}\, d l$. This is generally† the case when the line in question is a line of magnetic induction, which it may always be when the line-integral of magnetic force is calculated for a perfect magnetic circuit.

§ 153. **Value of the Line-Integral of Magnetic Force.**— When the integration is extended all along any closed curve— in other words, when the imaginary unit magnetic pole makes

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* Maxwell, "El. and Mag.," Vol. II., § 401.

† Namely, when the medium is isotropic and has no residue of previous magnetisation in a direction inclined to the direction of H.

a complete journey along any path which returns into itself—it may be shown that the value of the line-integral of magnetic force admits of very easy calculation. If the curve along which it is reckoned does not thread its way through any circuit in which a current is flowing, then the value of the line-integral of magnetic force along the closed curve is zero. If the curve does thread its way once through a circuit in which a current C is flowing, then the value of the line-integral of magnetic force along the closed curve is $4\pi C$; and if the curve threads its way N times through such a circuit, the value of the line-integral is $4\pi C N$. For, example, if the line along which the line-integral of magnetic force is reckoned is any closed curve which is threaded through the interior of a coil of N turns, the line-integral is $4\pi C N$, for the line is interlinked with the current circuit as many times as there are turns in the coil.

The principle that the line-integral of magnetic force is equal to $4\pi C N$, when taken along any closed curve is an absolutely general one. It is true whatever be the position and direction of the curve, whether it lie along a line of force or no, and whether it lie wholly or partly in a non-magnetisable substance, such as air, or wholly or partly in a magnetisable substance, such as iron. If the closed curve threads through more circuits than one, the sum of the terms $4\pi C N$ is to be taken.

Two simple cases will serve as instances. Suppose we have a uniform solenoid of n turns per centimetre, and l centimetres long. Let the ends be bent together so that it forms a closed ring. The length of a closed curve in the centre of the solenoid is l. The magnetic force H is uniform all along that line, and is equal to $4\pi C n$. The value of $\int \mathsf{H}\, d l$ is, therefore, $\mathsf{H}\, l$ or $4\pi C n l$, or $4\pi C N$, since N the whole number of turns $= n l$.

Again, consider the magnetic force around a long straight conductor (the remainder of the circuit being supposed to lie so far off as to be uninfluential) and integrate $\int \mathsf{H}\, d l$ along the circumference of a circle of which the conductor is axis. The force at any distance r from the axis of the conductor is $\frac{2C}{r}$. This is uniform throughout the circular path, and is in the direction of the path. The length of the path is $2\pi r$. The line-integral of magnetic force round the path is, therefore,

$\frac{2\,C}{r} \times 2\,\pi\,r$ or $4\,\pi\,C$. In this case the path along which the line-integral is taken is interlinked with the circuit only once.

The principle set forth in this paragraph may be stated thus:—The line-integral of magnetic force along any closed curve is equal to $0{\cdot}4\pi$, or $1{\cdot}2566$, into the number of *ampere-turns* in the coil or coils which are threaded by the curve.

§ 154. **Equation of the Magnetic Circuit.**—Returning now to the case of a perfect magnetic circuit, we have to consider the connection between the magnetomotive force or line-integral of magnetic force along the circuit and the magnetic flux. Suppose the circuit to be divided up into a number of tubes of induction, in each of which the cross-section is small, so that B and H may be taken as uniform over any one cross section of the (small) tube. The relation which we establish for each small tube may easily be extended to apply to the whole magnetic circuit, which is built up of such small tubes placed side by side. Let s be the area of cross-section at any part of the small tube, and B the magnetic induction there. The flux in the tube is $\mathsf{B}\,s$. If μ be the permeability of the substance, the magnetic force H at the same place is $\frac{\mathsf{B}}{\mu}$; hence,

$$\frac{\text{flux}}{\mu\,s} = \frac{\mathsf{B}}{\mu} = \mathsf{H}.$$

Multiply each side by an indefinitely short length of the tube $d\,l$—

$$\text{flux} \times \frac{d\,l}{\mu\,s} = \mathsf{H}\,d\,l.$$

Integrate both sides, remembering that the flux is constant at all sections;

$$\text{flux} \times \int \frac{d\,l}{\mu\,s} = \int \mathsf{H}\,d\,l = \text{magnetomotive force},$$

when the integration is extended round the whole circuit;

hence, $$\text{flux} = \frac{\text{magnetomotive force}}{\int \frac{d\,l}{\mu\,s}}.$$

The meaning of the denominator may be most readily seen if we write ρ for $\frac{1}{\mu}$ and call ρ the *specific magnetic resistance* of the substance. Then $\frac{\rho\, d\, l}{s}$ is evidently the magnetic resistance of that portion of the magnetic circuit which has the length $d\, l$ and the cross section s. The idea of magnetic resistance is introduced here in a sense strictly analogous to the idea of electric resistance in the electric circuit. The specific magnetic resistance ρ is the analogue of the specific resistance to conduction—namely, the resistance of a piece of the conductor of unit length and unit area of cross-section. The quantity $\int \frac{\rho dl}{s}$ is simply the sum of the resistances of successive short portions of the length of the circuit. We may, therefore, write the equation of the perfect magnetic circuit thus—

$$\text{flux} = \frac{\text{magnetomotive force}}{\text{magnetic resistance of the circuit}},$$

which is the magnetic analogue of the familiar equation of conduction—

$$\text{current} = \frac{\text{electromotive force}}{\text{conduction resistance of circuit}}.$$

There is, however, this reservation to be borne in mind in pursuing the analogy. In the conduction circuit the specific resistance of the material is not a function of the current—that is to say, its value is independent of the amount of the current. In the magnetic circuit ρ and μ are functions of the flux, for they depend on the value of B. More than that, they may have many possible values even when the value of B is assigned, for they depend not only on the existing magnetic induction, but on the previous magnetic history of the piece. But the equation of the magnetic circuit will be correct and intelligible if we define μ as nothing more or less than the value which the quotient $\frac{\mathsf{B}}{\mathsf{H}}$ happens to have at that place in the circuit to which reference is made; and if we define ρ as the reciprocal of that quantity, or $\frac{\mathsf{H}}{\mathsf{B}}$, we may have a magnetic circuit in which there is no magnetomotive force, but in which there is (residual) magnetic flux. In that case the "magnetic resistance" of the circuit must vanish, and the mean value of ρ must be zero. We may even have a magnetic circuit in which the direction of the flux is (on account of past magnetisation) opposite to the direction of the magnetomotive force, which implies a negative value of μ and of ρ.

In most of the cases to which the conception of the magnetic circuit may be usefully applied, the effects of previous magnetisations are absent or negligible, so that the values of μ which are to be used are the permeabilities which are derived from the ordinary curve of magnetisation (for the particular material of the circuit)—that is to say, from the curve which expresses the relation **B** to **H** when **H** is progressively increased from zero and the metal is free of magnetism to begin with.

In many instances the circuit may be treated as (very approximately) made up of a series of portions, in any one of which μ is constant and s is constant. Thus, calling l_1 the length of one of these portions, μ_1 its permeability, and s_1 its cross-sectional area, l_2 the length of the next, μ_2 its permeability, and s_2 its sectional area, and so on, we have

$$\text{flux} = \frac{\text{magnetomotive force}}{\frac{l_1}{\mu_1 s_1} + \frac{l_2}{\mu_2 s_2} + \frac{l_3}{\mu_3 s_3} + \text{\&c.}},$$

as many terms being taken in the denominator as are needed to complete the circuit.

And if the object is to express the value of the induction at any place in the circuit in terms of the magnetomotive force, we have only to divide the flux by the area of cross-section there. Thus, if it is wished to express $\mathbf{B}_1$, the induction in the first portion of the circuit, where the area of section is s_1, we have

$$\mathbf{B}_1 = \frac{\text{flux}}{s_1} = \frac{\text{magnetomotive force}}{\frac{l_1}{\mu_1} + \frac{l_2}{\mu_2} \cdot \frac{s_1}{s_2} + \frac{l_3}{\mu_3} \cdot \frac{s_1}{s_3} + \text{\&c.}}.$$

Or, again, if what is wanted be to calculate the number of ampere turns which are required to produce a stated magnetic flux in a magnetic circuit made up of a series of portions of which the lengths, sections, and permeabilities are known, we may find the magnetomotive force from the formula

$$\text{magnetomotive force} = \text{flux} \times \left(\frac{l_1}{\mu_1 s_1} + \frac{l_2}{\mu_2 s_2} + \frac{l_3}{\mu_3 s_3} + \text{\&c.} \right),$$

and then find the number of ampere turns by dividing the magnetomotive force by $0{\cdot}4\pi$. This is, in effect, the problem which is attacked in calculating the winding of the field-magnets of a dynamo. The problem is analogous to that of finding the electromotive force necessary to drive a stated current through a circuit composed of a series of conductors of which the specific resistances, the lengths, and the cross-sections, are assigned.

§ 155. **Particular Cases: Continuous Ring wound uniformly and otherwise.**—The utility of the idea of the magnetic circuit will be apparent when we consider one or two examples. Take first the case, already familiar, of a uniform ring uniformly wound with a magnetising coil of N turns. Let l be the length of the ring, measured round any circle within the ring parallel to the sides. The magnetic force at all points of such a circle is $\frac{4\pi C N}{l}$, and the line-integral of this, or the magnetomotive force, is $4\pi C N$. If s be the area of cross-section, the magnetic resistance of the ring is $\frac{l}{\mu s}$, and the flux, which is equal to the magnetomotive force divided by the resistance, is

$$\frac{4\pi C N}{\frac{l}{\mu s}}.$$

We might, of course, have derived this expression for the flux otherwise, namely:—

$$\text{flux} = \mathsf{B}\, s = \mu\, \mathsf{H}\, s = \frac{4\pi C N \mu s}{l}.$$

The line-integral of magnetic force has the same value for all lines that thread through the magnetising coil. Moreover, the magnetic force H is itself constant at all points of the circle l, parallel to the sides of the ring; so that the line-integral is $\mathsf{H}\, l$. To compare the values of H at different points in the substance of the ring, at distances r_1, r_2, &c., from the axis of the ring, we have $\mathsf{H}_1\, l_1 = \mathsf{H}_2\, l_2$, $l_1 = 2\pi r_1$, and $l_2 = 2\pi r_2$; hence, $\mathsf{H}_1\, r_1 = \mathsf{H}_2\, r_2$. In other words, the magnetic force due to uniform winding on a uniform circular ring varies across any section in inverse proportion to the distance of the point from the axis of the ring. (*See* § 57, *ante.*)

In the case of a uniformly-wound uniform ring there is no special advantage in applying the conception of the magnetic circuit. The results to which it leads are obtained, with no less ease, by considering the magnetisation and magnetic force at any individual point of the metal. But it should be noted that the conception of the magnetic circuit makes it possible to avoid any use of, or reference to, the quantity **H**. We have derived the notion of magnetomotive force from that of magnetic force, by taking the line-integral of **H** round the magnetic circuit. But that is by no means a necessary order of ideas; nor is the notion of **H** indispensable in the treatment of the subject. The magnetomotive force may be defined without reference to it, and the flux may be stated in terms of the magnetomotive force and magnetic resistance, so that all use of **H** may be excluded. It is even theoretically possible to treat all cases of magnetisation in the same way. With a magnetised bar, for instance, the magnetic circuit is completed through the surrounding non-magnetic medium, and a sufficiently powerful analysis might determine the resistance of the circuit, and so allow the relation of magnetic flux to magnetomotive force to be treated without any allusion to the magnetic force at individual points of the bar. But to apply this method universally, though theoretically possible, is quite impracticable; and there are very many problems in regard to which the older modes of viewing the subject, described in earlier chapters, are infinitely more convenient. The student must not think to abandon the consideration of magnetic force and magnetisation at individual points because he finds that the notion of the magnetic circuit is remarkably useful in certain cases, and has, in theory, no limits to its application. Its real value lies in the fact that by its help problems which would otherwise be intractable may be solved with sufficient exactness for practical purposes. To trace, for example, from point to point in the core of a transformer or the field magnets of a dynamo the value of **H**, and so determine the magnetisation, would be a task the difficulty of which would be prohibitive. But by applying in such cases the method of the magnetic circuit, a solution is readily arrived at—not, indeed, a rigorous solution, but one that satisfies the requirements of the electrical engineer.

In the case dealt with above—that of a uniform ring uniformly wound—the metal of the ring forms a perfect magnetic circuit. None of the lines of induction stray into surrounding space; the ring itself is a tube of induction, and the flux is constant at all cross sections.

Suppose, however, that, instead of being uniformly wound, part of the ring, Q, is bare and the magnetising coil is heaped up on the other part P (Fig. 131). In that case the flux through P is greater than the flux through Q, for some of the lines of induction which thread through the coil close themselves by passing not through the bare part of the ring but through surrounding space in the manner indicated by the dotted lines. The ring is now an *imperfect* magnetic circuit. If, however, the material is very permeable, like soft wrought iron (the magnetic permeability of which, when not too strongly magnetised, is some two or three thousand times as great as permeability of air), and if the ring is short—that is to say, if its diameter is not too great in comparison with the dimensions of its section—this leakage of lines of induction into surrounding space will take place to only a

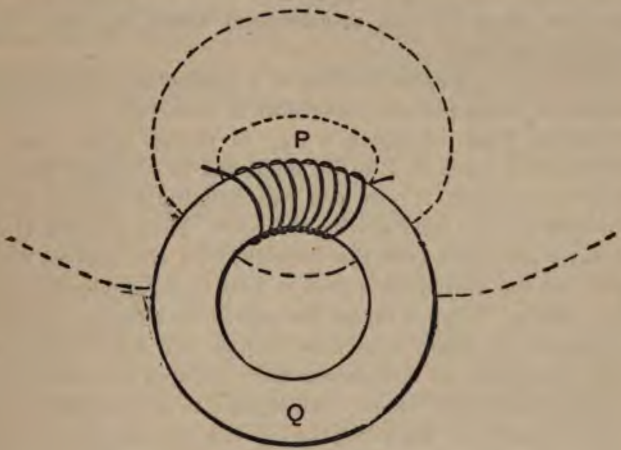

FIG. 131.

very limited extent; by far the greater number of the lines through P will complete their circuit within the substance of the ring, and the flux at Q will be only a very little less than the flux at P. We may, therefore, in such a case, as a first approximation, treat the flux in the ring as constant, and apply the equation of the perfect magnetic circuit, flux $= \frac{4\pi CN\mu s}{l}$, to find it. This quantity is, in fact, slightly less than the flux in the part P, because the resistance of the actual magnetic circuit is a trifle less than that of the ring, through the "shunting" of a part of the ring by the surrounding air. On the other hand, the flux, as calculated above, is greater than the true flux at Q.

The case is analogous to that of a conducting circuit, which instead of being perfectly insulated is immersed in a poorly conducting fluid. Imagine a ring of copper with a seat of electromotive force at P to be immersed in a liquid, the conductivity of which is only one two-thousandth or one three-thousandth of the conductivity of copper. The current at Q will be only a little less than the current at P; the current which leaks into the surrounding fluid will be an inconsiderable part of the whole. We must repeat the proviso that the ring is short; in other words, that the surface through which leakage occurs is not very great in comparison with the area of cross section through which what we may call legitimate conduction occurs.

The advantage of regarding the iron ring as a magnetic circuit, nearly, though not quite, perfect, is at once apparent when one considers how difficult it would be to determine directly the magnetic force **H** at individual points. In the case of a uniformly wound ring there is no difficulty in determining **H**, because the magnetic force is then wholly due to the magnetising coil. In the present case **H** is by no means due to the coil only. The coil acting alone would produce a strong magnetic force at points within and close to it, and would produce very little magnetic force in more distant portions of the ring. But we know that **H** must actually be pretty nearly uniform throughout the ring, because the magnetisation is pretty nearly uniform. What tends to equalise **H** is the free magnetism in the ring itself—the free magnetism which exists in consequence of the very fact that the flux is not quite uniform.

The magnetic force at any point is due partly to the action of the free magnetism and partly to the action of the coil; at points within and near the coil the free magnetism diminishes **H** by opposing the action of the coil, but at points on the bare side of the ring augments **H.** It is just because the ring is not a perfect magnetic circuit—because there is some leakage of the flux into surrounding space—that the magnetic force (and, therefore, the induction) is fairly uniform all round. In a short ring of very permeable substance, a slight variation in the flux from point to point of the ring implies the existence of enough free magnetism to correct very nearly that excessive inequality in the magnetic force which is produced by the magnetising coil; in other words, the circuit then establishes itself with but little leakage.

§ **156. Ring Magnet with an Air Gap.**—We shall next consider a magnetic circuit consisting of a uniform iron ring, in which a narrow radial crevasse has been cut. When the ring is magnetised there is some leakage of lines of induction through its sides into surrounding space, especially near the crevasse, but most of the lines go directly across the crevasse. We may conceive the magnetic circuit of the ring to be completed —though not quite perfectly—by a plate of air filling the crevasse, of the same area of cross-section as the ring itself. The lines of induction spread somewhat in crossing the crevasse, and a closer approximation to the condition of a perfect circuit would, therefore, be reached by supposing the plate of air to have an area of cross-section rather larger than the cross-section of the ring, the extent of this enlargement being dependent on the thickness of the crevasse. In the case which we postulate, however, the crevasse is very narrow, and it will suffice to take its area of section as no more than equal to that of the ring. Let s be the area of section, l the mean length of the complete ring (before the crevasse is cut), and δl the (small) mean thickness of the crevasse. Let the ring be magnetised, as before, by a coil of N turns, carrying a current C. The permeability of the ring is μ, and that of the gap is unity. Then,

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{magnetic resistance}} = \frac{4\pi \mathrm{C}\mathrm{N}}{\dfrac{l-\delta l}{\mu s} + \dfrac{\delta l}{s}} = \frac{4\pi \mathrm{C}\mathrm{N}\mu s}{l+\delta l(\mu-1)}.$$

If there had been no gap, the flux would have been $\frac{4\pi C N \mu s}{l}$. The effect of removing a short length, δl, of the iron, and substituting air as the material through which the magnetic circuit is completed, is to increase the resistance of the circuit as much as it would be increased by the addition of a length of iron equal to $\delta l (\mu - 1)$.

§ 157. **Comparison of a Split-Ring with an Ellipsoid.**—It is interesting to compare the case of a ring in which there is a gap with that of an ellipsoid of finite length.* In the ellipsoid, as we have already seen (§ 26), the free magnetism produces a self-demagnetising force, which is proportional to the amount of magnetisation, and opposes the action of the magnetising coil. If we call H the true magnetising force acting on the metal, and H' that part of the magnetising force which is due to the action of the coil alone, then

$$\mathsf{H} = \mathsf{H}' - \mathsf{N}\,\mathsf{I},$$

where I is the intensity of magnetisation, and N is a numerical factor, the value of which depends on the relation of the length of the ellipsoid to its transverse dimensions. We shall see that a precisely similar formula may be obtained for the ring with a gap by treating it as a nearly perfect magnetic circuit.

Since the magnetisation of the cut ring is very nearly uniform, the actual magnetic force in the iron, which is the resultant of that due to the coil and that due to the free magnetism, must also be very nearly uniform. Call this force H, and call H' the magnetising force due to the coil alone, which (on the supposition that the coil is uniformly wound) is $\frac{4\pi C N}{l}$.

Then $$\mathsf{H} = \frac{\mathsf{B}}{\mu} = \frac{\text{Flux}}{\mu s} = \frac{4\pi C N \mu s}{\{l + \delta l (\mu - 1\}\mu s} = \frac{4\pi C N}{l + \delta l (\mu - 1)};$$

and $$\mathsf{H}' = \frac{4\pi C N}{l}.$$

Therefore, $\mathsf{H}'\, l = \mathsf{H}\, \{l + \delta l\, (\mu - 1)\}$,

$$\mathsf{H}' = \mathsf{H}\left\{1 + \frac{\delta l}{l}(\mu - 1)\right\},$$

$$= \mathsf{H}\left(1 + \frac{4\pi\kappa\,\delta l}{l}\right),$$

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* See a Paper by H. E. J. G. du Bois, *Phil. Mag.*, Vol. XXX., 1890, p. 335.

since $\mu = 4\pi\kappa + 1$, κ being the magnetic "susceptibility," or $\frac{\mathsf{I}}{\mathsf{H}}$.

Hence,
$$\mathsf{H}' = \mathsf{H} + \frac{4\pi\delta l}{l}\mathsf{I},$$

and
$$\mathsf{H} = \mathsf{H}' - \frac{4\pi\delta l}{l}\mathsf{I}.$$

The factor $\frac{4\pi\delta l}{l}$ therefore takes the place of the factor N in the formula for ellipsoids. Its magnitude depends on the proportion of the width of the crevasse to the whole length of the circuit.

Taking the case of a circular ring, this proportion may be expressed by reference to the angular aperture of the crevasse —that is to say, the angle subtended by the crevasse at the centre. Calling this angle α in degrees, $\frac{\delta l}{l} = \frac{\alpha}{360}$ and $\mathsf{N} = \frac{4\pi\alpha}{360}$. The following table has been calculated by Du Bois, to show what aperture of crevasse in a circular ring produces the same self-demagnetising force as exists in ellipsoids of certain stated elongations :—

Ratio of Length to Diameter of Ellipsoid.	Factor N.	Equivalent Aperture in Circular Ring (degrees).
20	0·0848	2·41
30	0·0432	1·22
40	0·0266	0·76
50	0·0181	0·52
100	0·0054	0·15

It is scarcely necessary to add that the self-demagnetising force which is introduced by the presence of the crevasse affects the residual magnetism of the ring as well as the induced magnetism, precisely as it does in the ellipsoid. When the magnetising-circuit of the split-ring is broken, the residual magnetism causes a reversed force to act on the metal, the value of which is $\frac{4\pi\delta l}{l}\mathsf{I}_r$, where I_r is the residual intensity of magnetism. This prevents the residual magnetism from being nearly so great as it would be were the ring complete—

indeed, a very narrow crevasse is sufficient almost wholly to destroy the (otherwise very great) residual magnetism of a soft iron ring. For example, in a ring of soft annealed iron, which, when uncut, would retain, after being strongly magnetised, a residual induction, B_r, of 12,000 units, the presence of a gap only half a degree wide will reduce the residual value of the induction to about 1,000.

§ 158. **Graphic Representation of the Influence of a Narrow Gap.**—The influence of a narrow gap, both in resisting magnetisation and in promoting demagnetisation, is best seen by resorting to the graphic construction which has been already explained in relation to ellipsoids and long rods (§ 48). Let *a*, *a*, *a* (Fig. 132) be curves of magnetisation (curves of I and H) for the iron of which the ring is composed. Find the factor N, equal to $\frac{4\pi\delta l}{l}$, and draw the line O A, so that A M (drawn parallel to the axis along which H is measured, and interpreted on the scale of H) shall be to O M as $\frac{4\pi\delta l\mathsf{I}}{l}$ is to I. Then the intercepts between O M and O A represent the values of the self-demagnetising force, due to the corresponding values of I, and if we wish to represent the relation of the magnetism to the magnetising force produced by the coil alone (the force which has been called H' above), we have only to draw a diagram in which the lines *a*, *a*, *a* are sheared into the position *b*, *b*, *b* by taking the abscissas from O A instead of from O M, or, in other words, by adding $\frac{4\pi\delta l\mathsf{I}}{l}$ to H in every case. Thus, any point P′ in the new curve is found from the corresponding point P by taking $P'R = PR + QR$. The residual magnetism, which was O S in the ring without a gap, is reduced to O S′ in the ring with a gap. If the object of the construction had merely been to find the residual magnetism, O S′, that could have been done more readily by drawing O T inclined at the same angle as O A, but on the other side of the axis of I, to meet the descending curve *a*, and projecting S′ from the point of intersection of O T with the curve. The same construction will, of course, serve to find the residual magnetism of ellipsoids, or of long rods, which may be treated as approximating to ellipsoids.*

In Fig. 132 we have supposed that the magnetisation of the iron is exhibited by means of a curve of I and H. If, instead of this, the curve of B and H were given, a similar graphic construction would still serve to show the effect of the gap. Since $\mathsf{I} = \frac{\mathsf{B}-\mathsf{H}}{4\pi}$, the self-demagnetising force $\mathsf{N\,I} = \mathsf{N}\,\frac{(\mathsf{B}-\mathsf{H})}{4\pi}$, which, in a very permeable substance like iron, is practically equal to $\frac{\mathsf{N\,B}}{4\pi}$, since B is very great compared with H. Substituting for N its value $\frac{4\pi\,\delta l}{l}$, this becomes $\frac{\mathsf{B}\,\delta l}{l}$. The line O A has, therefore, to be drawn, in a diagram of B and H, at such an inclination that when O M represents B, M A is $\frac{\mathsf{B}\,\delta l}{l}$.

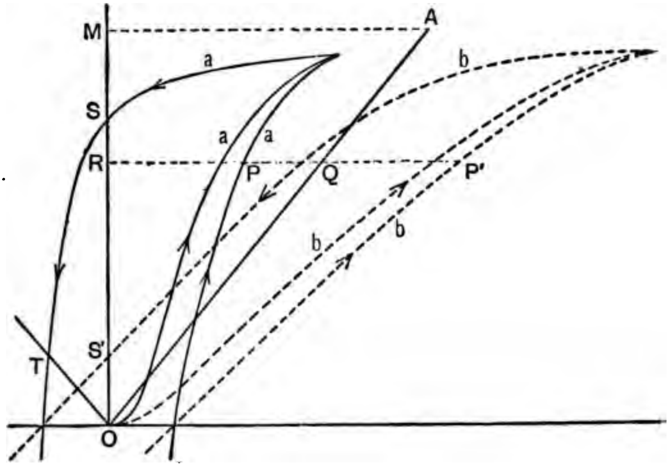

FIG. 132.

From the equation $\mathsf{H} = \mathsf{H}' - \mathsf{N\,I}$, by substituting $\frac{\mathsf{B}-\mathsf{H}}{4\pi}$ for I, we have

$$\mathsf{H} = \mathsf{H}' - \frac{\delta l}{l}(\mathsf{B}-\mathsf{H}),$$

or

$$\mathsf{H}'l = \mathsf{H}(l - \delta l) + \mathsf{B}\,\delta l.$$

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* This construction, for finding the residual magnetism of ellipsoids, is given by J. Hopkinson, *Phil. Trans.*, 1885, p. 465.

H$'l$ is $4\pi CN$; it is the line-integral of the magnetic force taken round the whole circuit, or, in other words, the magnetomotive force. **H** is the magnetic force in the iron, and **H** $(l - \delta l)$ is that part of the line-integral which is taken through metal. **B** is equal to the magnetic force in the gap, and therefore **B** δl is that part of the line-integral that is taken through air. The equation might evidently have been written down directly; it expresses the simple fact that the line integral for the complete circuit is made up of two parts, in one of which—namely, the iron, whose length is $l - \delta l$—the magnetic force has the sensibly uniform value **H**, while in the other—namely, the gap whose length is δl—the magnetic force has the sensibly uniform value **B**. We have derived it otherwise, in order to accustom the student to observe the connection between the treatment of the ring as a magnetic circuit and that other treatment which deals with the magnetic condition at individual points. In the language of the magnetic circuit, **H** $(l - \delta l)$ represents that part of the whole magnetomotive force which is used in overcoming the magnetic resistance of the iron, and **B** δl represents the remainder of the magnetomotive force, which is used in overcoming the resistance of the gap. In soft iron, if the gap is of any considerable width, its resistance is so great compared with that of the iron that nearly the whole magnetomotive force is used in forcing the induction across the gap.

§ 159. **Graphic Representation of the Relation of Flux to Magnetomotive Force.**—In dealing with the magnetic circuit as a whole, it is convenient to modify and generalise the graphic construction exemplified in Fig. 132, by drawing the abscissas to represent the whole magnetomotive force, and the ordinates to represent the whole magnetic flux in the manner first described by J. and E. Hopkinson.* Such a curve may obviously be derived for any part of a magnetic circuit from the curve of induction and magnetic force for the material by multiplying the induction by the area of section s to find the whole flux, and by multiplying the magnetic force by the length of the piece to find the magnetomotive force required for the magnetisation of that part of the circuit. Then,

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* *Phil. Trans.*, 1886, Part I., p. 331.

by graphically summing the abscissas for successive parts of any composite magnetic circuit, the whole magnetomotive force is represented in relation to the flux. An example will make the method intelligible. Take, as before, the case of an iron ring of uniform section with a gap in it, the length of the gap being δl, and that of the iron $l - \delta l$. From the curve of **B** and **H** for the metal of which the ring is formed—which we suppose to be known—draw the curve O P (Fig. 133), for the iron, in which any ordinate, P p, is the flux, **B** s, and the corresponding abscissa, O p, is **H** $(l - \delta l)$. Next draw the curve O Q for the gap, in which the ordinate Q q is again **B** s, and the corresponding abscissa O q is the magnetomotive force required *for that part of the circuit*, namely **B** δl. The line O Q is evidently straight, as it relates to a non-magnetic substance in which induction is proportional to magnetic force. Then draw the resultant curve O R, in which for each ordinate the abscissa is the sum of the abscissas of the curves already drawn, namely, O r = O p + O q. O r is the magnetomotive force required for iron and gap together, and the curve O R shows the relation of the flux to the magnetomotive force in the circuit as a whole.

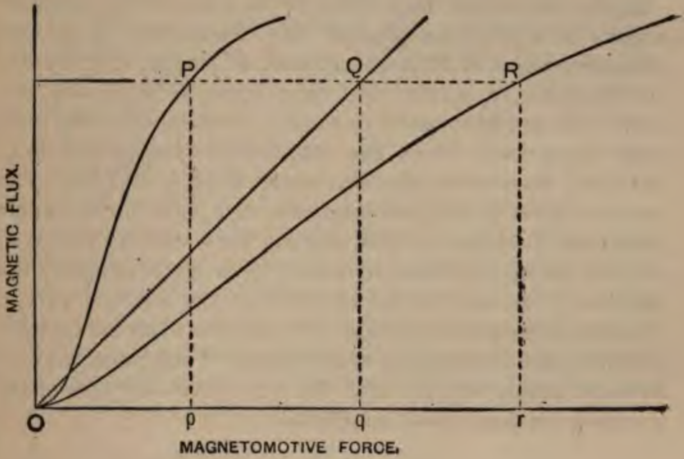

Fig. 133.

Moreover, the construction may be applied with equal facility to the descending limb of the curve, or to exhibit the behaviour of the circuit in any cycle of magnetisation. In the line O R the descending and ascending limbs coincide; in the iron part of the circuit the ascending and descending limbs have to be drawn separately, and the process of summing the abscissas has to be applied successively to each limb (as in Fig. 132) in order to determine a curve which will show the effects of hysteresis in the magnetic circuit as a whole, and all the variations of magnetic flux under cyclic variations of magnetomotive force.

Again, the method may evidently be extended to magnetic circuits of a more complicated form, containing, let us say, successive pieces of different material, of lengths l_1, l_2, l_3, &c., and sections s_1, s_2, s_3, &c. We must know, to begin with, the curve of B and H for each material. From these curves draw a set of curves in which the ordinates are respectively $\mathsf{B}\,s_1$, $\mathsf{B}\,s_2$, $\mathsf{B}\,s_3$, &c., and the abscissas are $\mathsf{H}_1\,l_1$, $\mathsf{H}_2\,l_2$, $\mathsf{H}_3\,l_3$, &c. The required curve of flux and magnetomotive force for the whole circuit will be found by compounding these curves; that is to say, by drawing a curve in which, for a given ordinate, the abscissa is the sum of the abscissas of the separate curves. The complete curve exhibiting what happens when the complex circuit is carried through a cyclic process of magnetisation may be found in this way, provided the cyclic curves for each of the materials are determined beforehand.

§ 160. **Application to Dynamos.**—A principal use of this method is to determine the magnetomotive force, and consequently the number of ampere-turns, required to produce a stated magnetisation in a circuit made up of pieces the dimensions and magnetic qualities of which are known. The method was, in fact, invented by J. and E. Hopkinson as a means of solving practical problems in the design of a dynamo, where the magnetic circuit is made up of (1) the cores of the field magnets, (2) the yoke, (3) the pole pieces, (4) the core of the armature, and (5) the non-magnetic spaces on either side of the armature core, between it and the pole pieces. This last is much the most important item in the resistance of the circuit. The magnetic circuit of a dynamo is far from perfect, and the estimation of the effective length and effective cross-section of each part is subject to some uncertainty, so that the results are no more than rather roughly approximate. To pursue this application in detail would be beside our present purpose: the student should in any case refer to the original Paper.* One point, however, must be briefly mentioned, being of general interest in relation to other magnetic circuits as well as to the circuit of the dynamo.

In the dynamo circuit the flux is by no means uniform throughout; there is much leakage. The flux is greatest in the magnet limbs, which are the seat of the magnetomotive force, and in the armature it is considerably less. Its value in the armature is, however, the matter of chief interest in the practical problem. Calling F_1 the value of the flux in the armature, the fluxes F_2, F_3, &c., in other parts of the circuit may be expressed by the use of factors q_2, q_3, &c., such that $F_2 = q_2 F_1$, $F_3 = q_3 F_1$, and so on. These factors are sometimes called coefficients of leakage; in the case considered they are greater than unity. They may be found experimentally by comparing the forces ballistically by means of induction coils wound at different places in the circuit, or by measuring directly the number of stray lines of induction in the air round about the several portions. They are not strictly constant, but tend to increase when the flux approaches saturation. When they are known, we may obtain a better approximation to the true equation of the magnetic circuit by writing

$$4\pi C N = \frac{l_1 F_1}{\mu_1 s_1} + \frac{l_2 F_2}{\mu_2 s_2} + \frac{l_3 F_3}{\mu_3 s_3} + \&c.,$$

or

$$4\pi C N = F_1 \left(\frac{l_1}{\mu_1 s_1} + \frac{q_2 l_2}{\mu_2 s_2} + \frac{q_3 l_3}{\mu_3 s_3} + \&c. \right).$$

§ 161. **Bar and Yoke.**—In speaking of applications of the idea of the magnetic circuit, we may revert briefly to the arrangement of bar and yoke, first used by Hopkinson in experiments on susceptibility, and described above in §§ 58-59, Figs. 26 and 27. The function of the yoke is to make the bar form part of

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* *Loc. cit:* see also a Paper by E. Hopkinson on the "General Theory of Dynamo Machines," Rep. Brit. Assoc., 1887, p. 614. Reference should also be made to Prof. S. P. Thompson's "Treatise on Dynamo-Electric Machines."

a nearly perfect magnetic circuit, of which the resistance of the bar itself is nearly the whole resistance. Let l_1 be the effective length of the bar, which is somewhat greater than its clear length within the yoke (since there is a gradual spreading out of the lines of induction where the bar penetrates the yoke), and let s_1 be the cross-section of the bar. Let l_2 be the length of the return path of the lines through the yoke, and s_2 the cross section of the return path, which is the sum of the cross sections of the two sides of the yoke. Let N, as usual, be the number of turns of the magnetising coil, which is wound on the bar. Then, by the principle of the magnetic circuit,

$$4\pi \mathrm{C N} = \mathrm{Flux} \times \left(\frac{l_1}{\mu_1 s_1} + \frac{l_2}{\mu_2 s_2}\right),$$

$$= \mathsf{B}_1 s_1 \left(\frac{l_1}{\mu_1 s_1} + \frac{l_2}{\mu_2 s_2}\right),$$

where B_1 is the magnetic induction within the bar. The flux, $\mathsf{B}_1 s_1$, is the quantity which the ballistic test measures. The equation may be written

$$4\pi \mathrm{C N} = \frac{\mathsf{B}_1}{\mu_1}\left(l_1 + \frac{\mu_1 s_1}{\mu_2 s_2} l_2\right).$$

Now H, the true magnetic force acting on the bar, is $\frac{\mathsf{B}_1}{\mu_1}$;

hence
$$\mathsf{H} = \frac{4\pi \mathrm{C N}}{l_1 + \frac{\mu_1 s_1}{\mu_2 s_2} l_2}.$$

The magnetic resistance of the block therefore virtually adds a small piece to the length of the rod—a piece, namely, whose length is $\frac{\mu_1 s_1}{\mu_2 s_2} l_2$, and the effect is that the actual magnetic force is equal to the force due to the magnetising solenoid $\frac{4\pi \mathrm{C N}}{l_1}$ *minus* a small correction, the amount of which may be written thus:

$$\mathsf{H} = \frac{4\pi \mathrm{C N}}{l_1} - \frac{\mathsf{B}_1 s_1 l_2}{\mu_2 s_2 l_1}.$$

This correction may be made insignificantly small by using for the yoke a material of the greatest possible permeability, and giving it an area of section very many times greater than that of the bar.*

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* *See* Hopkinson, *Phil. Trans.*, 1885, p. 458.

The amount of the correction is not constant, even when expressed as an addition to the length of the bar, for μ_1 and μ_2 are functions of the magnetic state of the bar and yoke respectively, and bear no constant proportion to each other. In all that has been written regarding the magnetic circuit, μ has simply this meaning, that it is the ratio of the induction which happens to exist at the moment to the magnetic force which happens to exist at the moment. The value of the correction depends, therefore, not merely on the magnetic force actually in operation, but on the previous magnetisation of the circuit. The correction may, of course, be very completely made by the graphic process which has been described, provided we have data from which to draw a curve of B and H for the material of the yoke.

We have spoken of this magnetic circuit as if it were wholly made up of the bar and the yoke. In fact, however, there is another constituent, the importance of which will be more apparent presently. This is the joint at each end of the bar; between the bar and the yoke. We shall see immediately that a joint, that is to say, a discontinuity in the substance of the magnetic circuit—even when there is no perceptible space separating the parts—interposes some resistance. Its effect is equivalent to that of a very narrow air-gap.

Acting like an air-gap, each joint in the circuit tends to shear over the curve of magnetisation, and one effect is that the residual magnetism of the circuit is reduced. This is a rather serious objection to the use of the yoke for determining the permeability of soft iron.

§ 162. **Magnetic Resistance of Joints.**—The fact that a joint offers magnetic resistance seems to have been first noticed by J. J. Thomson and H. F. Newall, who found that when an iron bar was cut in two, and the pieces were put in contact, the susceptibility of the bar was considerably reduced.*

In the following experiments† a tolerably full examination was made of the influence of a joint in adding magnetic resistance to an iron bar, both when the surfaces of the joint were

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* *Proc. Camb. Phil. Soc.*, 1887.

† "On the Influence of a Plane of Transverse Section on the Magnetic Permeability of an Iron Bar" (by the writer and W. Low).—*Phil. Mag.*, Sept., 1888.

placed in simple contact, and when they were pressed close by externally applied force. The bar was a turned piece of wrought iron, 0·79 cm. in diameter, and it was tested by the ballistic method within a yoke which allowed a clear length of 12·7cms. of the bar to be exposed. Over the whole of this length a magnetising coil was uniformly wound: the magnetising forces which will be stated below are the forces due to this coil. The area of cross-section which the yoke provided for the return of lines of induction outside the bar was more than one hundred times greater than the cross-section of the bar; nearly the whole magnetic resistance of the circuit was, therefore, that of the bar itself, and of the joint, or joints, in it. The magnetisation of the bar was tested by observing the transient current induced in a small secondary coil, wound at the middle of the length, when the current in the magnetising coil was reversed. Successive observations were made in this way, with magnetising currents that were progressively increased, to determine in each case a curve connecting **B** in the bar with the magnetising force of the coil.

The bar was first tested without any cut, and then when cut in the middle into two parts, the ends of which were carefully scraped to form true planes before being put into contact. The truth of the surfaces which formed the joint was tested by comparison with a Whitworth surface-plate. Notwithstanding the closeness of contact which this procedure ensured, the joint was found to offer a very appreciable amount of resistance, as the following figures will show :—

Table XXVI.—*Influence of a smooth joint in reducing the magnetic induction in an iron bar.*

Magnetising force due to solenoid.	Magnetic Induction **B**.	
	Bar uncut.	Bar cut in two pieces with surfaces of the joint faced to be true planes.
4	3,950	3,000
6	6,900	5,300
8	9,250	7,400
10	10,900	9,150
15	13,250	12,000
20	14,300	13,500
30	15,200	14,900

§ 163. **Calculation of the Equivalent Air-gap.**—The influence of a joint in adding magnetic resistance may be conveniently expressed by calculating the width of an air-gap which would have the same resistance, assuming the permeability of the metal itself to be wholly unaffected by cutting. The width of the equivalent air-gap is readily found in the following way[*] :—

Let H_1' be the magnetising force due to the solenoid when the bar is uncut, and H_2' the magnetising force due to the solenoid when the bar is cut, both for the same value of B. Let l be the length of the bar, and s its area of cross-section. Let x be the width of the air-gap equivalent in magnetic resistance to the joint. Then, by the principle of the magnetic circuit

$$\mathsf{H}_1' l = \frac{\mathsf{B} l}{\mu},$$

and

$$\mathsf{H}_2' l = \frac{\mathsf{B} l}{\mu} + \mathsf{B} x.$$

Since B is the same in both cases, μ is the same. Hence,

$$\mathsf{B} x = \mathsf{H}_2' l - \mathsf{H}_1' l,$$

and

$$x = \frac{(\mathsf{H}_2' - \mathsf{H}_1') l}{\mathsf{B}}.$$

To find x, we have therefore to draw curves of B and H_1' and of B and H_2', and measure the horizontal distance from one curve to the other, that is to say, the difference of H' for the same value of B. The quantity $\mathsf{H}_1' l$ is the magnetomotive force that suffices to produce the induction B when the bar is uncut. The quantity $(\mathsf{H}_2' - \mathsf{H}_1') l$ is the additional magnetomotive force that is required to force the same induction B through the joint.

In Fig. 134, the curves are drawn for the experiment which has just been quoted, and the values of $\mathsf{H}_2' - \mathsf{H}_1'$ are represented by the broken line at the side of the figure. This line is not far

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[*] In the Paper from which these experiments are quoted an erroneous procedure was followed in calculating the width of the equivalent air-gap. The error had the effect of making the gap appear to diminish in thickness as the magnetisation was strengthened. The figures given here are corrected.

from straight—which implies that the width of the equivalent air-gap is not far from constant throughout the range of values of B with which the experiment deals. The broken curve does, indeed, incline slightly outwards at the higher values of B, implying a greater width of equivalent air-gap in the region of strong magnetisation; but it may be questioned whether this slight deviation from straightness may not be due to errors of observation. A very slight error in the value of B in one or other of the two curves would suffice to account for it; and in another precisely similar experiment, made with another bar, the line showing values of $H_2' - H_1'$ actually bends slightly *inwards* at high values of B. It appears, then, that the joint is equivalent, in magnetic resistance, to a narrow gap of air, the width of which is at least not far from constant. The following are the widths of this gap calculated for the experiment of Table XXVI. and Fig. 134.

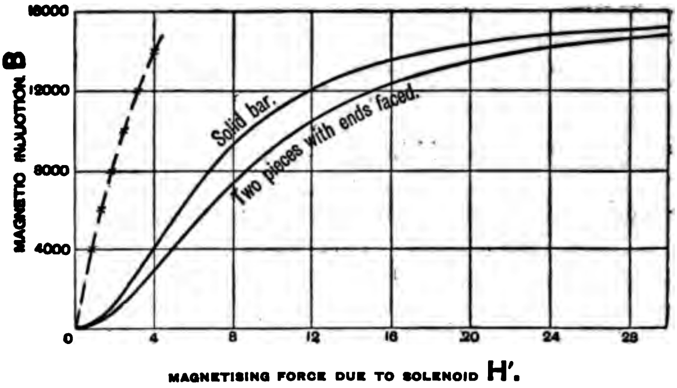

FIG. 134.—Influence of a Smooth Joint on the Magnetic Resistance of an Iron Bar.

B.	Width of equivalent air-gap in centimetres.
4,000	0·0026
6,000	0·0030
8,000	0·0031
10,000	0·0031
12,000	0·0035
14,000	0·0037

The corresponding quantities in another and quite independent experiment, made with a different iron bar, were :—

B.	Width of equivalent air-gap in centimetres.
6,000	0·0043
8,000	0·0041
10,000	0·0036
12,000	0·0030

In this case, as in the other, the ends of the bar after being cut were carefully brought to the condition of true planes.

We may take a width of 0·0033cms. to represent fairly the equivalent gap in the first case, and 0·0036cms. in the second. These figures agree with one another as well as the circumstances of the measurement would lead one to expect. The equivalent gap is not very wide, but it is difficult to believe that the surfaces of the metal were actually separated by even this narrow space. It seems more probable that the magnetic resistance of the joint is due in part to a diminished permeability in the metal itself at and close to each surface, and this conjecture receives some support, as may appear later, from the theory which ascribes the process of magnetisation to the rearrangement of molecular groups.

§164. Influence of Compression on the Magnetic Resistance of a Joint.—Other experiments in the same series* were directed to examine how the magnetic resistance of a joint is affected when the surfaces are pressed into close contact. The method of the yoke was still employed; the yoke was placed so that the bar stood vertically, and compression was applied to the bar by means of a weighted lever at the top, and a stop at the bottom, as in Fig. 97, § 122. In experiments of this kind it is, of course, necessary to remember that the permeability of the metal itself is changed by compressive stress; the influence of the joint is to be tested by comparing the resistance of the cut bar under pressure with the resistance of the uncut bar under equal pressure. It was found that the effect of pressure is to lessen the magnetic resistance of the joint, so much so, indeed, that when the surfaces composing the joint are true planes,

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* *Phil. Mag.*, September, 1888, p. 278.

a tolerably strong compressive stress almost wholly destroys the resistance of the joint, and restores the divided bar all but perfectly to the magnetic condition of an uncut bar. This effect was produced almost completely by a stress the intensity of which was 226 kilogrammes per square centimetre. Under this load a curve of **B** and **H′** taken with the cut bar was practically indistinguishable from the curve taken with the solid bar. Smaller loads only reduced the resistance of the joint without making it disappear, and a progressive reduction of the resistance could be traced as the loads were increased. The following table gives the values of **B** which were observed in an iron bar, first when solid and then when cut in two parts with faced ends, under various stresses, the magnetising force due to the solenoid being brought to the same value (5 C.-G.-S. units) in each case. This magnetising force was applied after each load had been put on.

Table XXVII.—*Influence of compressive stress in reducing the magnetic resistance of a joint.*

Stress in kilos. per sq. cm.	Magnetic Induction **B**, produced in each case by a magnetising force in the coil (**H′**) of 5 C.-G.-S. units.	
	Before Cutting.	After Cutting.
0	5,600	4,700
56·5	5,400	4,670
113	4,700	4,200
169·5	4,050	3,800
226	3,650	3,650

Here, under the highest stress, the disappearance of the joint's resistance was complete for a magnetising force of 5 C.-G.-S., but under stronger magnetising forces it was hardly so perfect.

In connection with these results it may be noted that the magnetic traction $\left(\frac{B^2}{8\pi}\right)$, amounting, as it does, to less than 1 kilogramme per sq. cm., when **B** is 5,000, is insignificant in comparison with the stress produced by external loading.

§ 165. Experiments with Rough Joints.—Others of the experiments dealt with bars which were simply cut in the lathe, without having the cut ends afterwards scraped to the form of true planes. Joints of this kind, which may by comparison be called rough, were found to offer rather more, but not very much more, resistance than a carefully faced joint, so long as the cut bar was tested without compression. But under compression the difference between a rough and a smooth joint became very manifest; the resistance of the rough joint was comparatively little reduced, and altogether refused to disappear even under the most intense stress.

Table XXVIII. shows the effect of successive cuttings in an iron bar, the joints being in every case of this comparatively rough kind. The bar was tested first in the uncut state, then when cut in two parts, then in four parts, and finally in eight parts, the ends being put in contact without compression.

Table XXVIII.—*Effect of Successive Cuttings.*

Magnetising force due to solenoid.	Magnetic Induction B.			
	Solid bar.	Bar cut in two.	Bar cut in four.	Bar cut in eight.
7·5	8,500	6,900	4,800	2,600
10	11,000	9,000	6,400	3,770
15	13,400	11,550	8,900	5,550
20	14,400	13,000	10,750	7,150
30	15,350	14,550	12,940	9,800
50	16,400	15,950	15,000	13,300
70	17,100	16,840	16,120	15,220

The results of this experiment are also exhibited in Fig. 135, where the full lines show the relation of B to the magnetising force of the solenoid when the bar was in one, two, four and eight pieces. The dotted lines in the same figures refer to a further experiment, in which a compressive stress of 226 kilos. per square cm. was applied, first to the uncut bar, and then to the bar cut in eight pieces.

Comparing the curves for the uncut bar with and without compression, we see that compressive stress lowers the permeability, except when the magnetisation is strong. In strong fields the dotted curve crosses above the plain curve. This gives incidental evidence of the existence of a reversal of the effect of compressive stress, corresponding to the "Villari" reversal of the effect of the tensile stress (*see* §§ 120, 124-126)—a result to be anticipated from what we know of the behaviour of iron under tension. For values of B below 16,000, however, compression increases the resistance of the circuit so long as the bar is uncut. But when applied to the eight pieces, compression decidedly reduces the resistance of the circuit, even when the magnetisation is weak: the dotted curve for the bar cut in eight lies considerably above the plane curve. In other

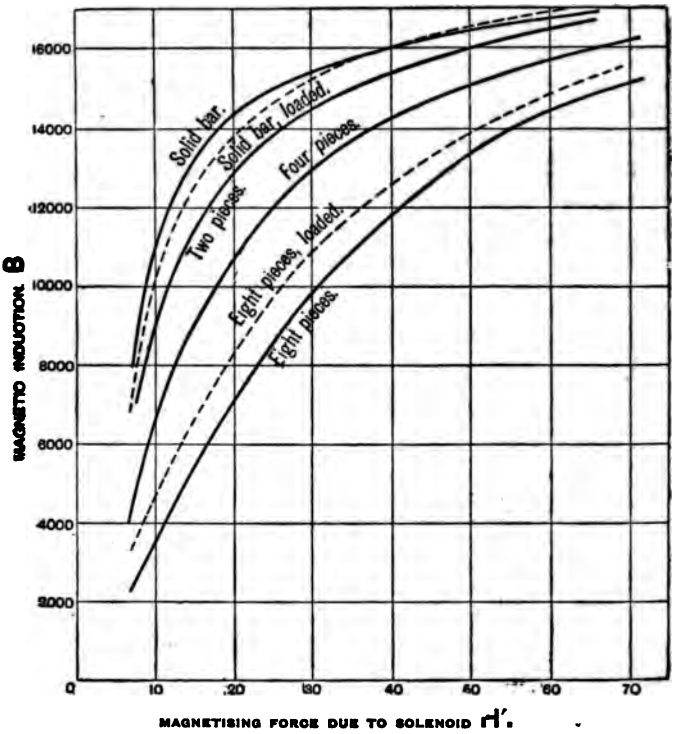

FIG. 135.—Effects of Successive Cuttings on the Magnetic Permeability of a Wrought Iron Bar.

No load . . . . ————————
Load of 226 kilogs. per sq. centim. . . . . - - - - - - - - - - - -

words, compression lowers, though it by no means destroys, the resistance of the joints, and when the joints are as numerous as they are here, its favourable effect on them more than counteracts its detrimental effect on the permeability of the metal itself. When the same test was applied to the bar in four pieces it was found that the two effects of compression came near to neutralising each other.

In the following table the width of the air-gap which would give the same resistance as the mean of the seven joints (introduced by cutting the bar into eight pieces) has been calculated by the method described above. The results are stated for both cases—with compression and without compression.

Table XXIX.—*Width of air-gap equivalent in resistance to the mean of seven joints.*

B.	Without compression.	With compression of 226 kilos. per sq. cm.
	cms.	cms.
8,000	0·0036	0·0024
10,000	0·0041	0·0031
12,000	0·0046	0·0036
14,000	0·0050	0·0041
15,000	0·0052	0·0041

It appears then that, in round numbers, the resistance of each rough joint was nearly the same as that of a film of air 0·005cm. thick when there was no compression, and that this equivalent film was only reduced to about 0·004cm. when a compressive stress was applied which would have been intense enough to practically destroy the resistance of the joint had the surfaces been carefully faced. We have seen that a joint with faced surfaces, tested without compression, is equivalent to a film of air about 0·003cm. thick. The thickness of the equivalent film in these rough joints seems to increase a little as the condition of saturation is approached. In the absence of compression a smooth joint is not very greatly better than a rough one. But compression is incompetent to produce, in a rough joint, that extreme closeness of contact which it apparently produces in a smooth joint, in consequence of which the resistance of the smooth joint almost vanishes.

CHAPTER XI.

MOLECULAR THEORY.

§ 166. **Molecular Theories: Poisson and Weber.**—We know that when a piece of iron, or other magnetic metal, is magnetised, the magnetic state permeates the whole piece. A steel bar magnet may be broken up into small pieces, and every piece is found to exhibit magnetic polarity. Assuming the structure to be molecular, it is inferred that each molecule of the magnetised bar is a magnet. Taking a row of molecules in the direction of the magnetisation, we have the north pole of one contiguous to the south pole of the next, and so on along the row—with the result that it is only at the ends of the row that free poles appear. Imagine the row to be broken into two or more parts, however, and each segment of it has free poles at its ends.

The individual molecules of a magnetised bar, then, are magnets, and the question next arises whether they become magnets only when the bar is magnetised. Does the process of magnetising consist, as Poisson suggested, in making each molecule become a magnet? Or are we to adopt Weber's view, according to which the molecules are always magnets, showing no aggregate polarity in an unmagnetised piece, only because their axes point in all directions at random, but turning into line when a magnetising force is applied? According to Poisson, there is no need to suppose the molecules capable of moving within the bar, but we must suppose that magnetic polarity can be induced in the individual molecules. In other words, the question how induction happens is only shifted from the bar to the molecule, and is brought no nearer to a solution. According to Weber, on the other hand, the molecules are to be conceived as free, more or less, to turn and take up a new alignment, very much as a pivoted compass-needle will turn when it is directed by a magnetic field; but there is no need to imagine any development of polarity within the molecule itself. The Weber molecule is a magnet before the force begins to act, and the amount of magnetism in it need suffer no change however widely the magnetism of the bar be altered. Hence Weber's theory explains the process of induction to this extent, that it makes the magnetic change of the bar be brought about by a change in the position of the molecules, and not by any change in the quality of the molecules: the magnetising process simply consists in turning the molecules to face one way. Of the two views, Weber's is the one that consorts best with our general understanding of the characteristics of molecules. Moreover, it receives strong support from certain of the known facts of magnetic induction.

§ 167. **Experimental Evidence in favour of Weber's Theory from the facts of Saturation, &c.**—It would be difficult, in Poisson's theory, to give any reason for the manner in which the magnetisation of a magnetic metal tends toward a limit as the magnetising force is increased. If the process consists in the development of magnetic polarity in the individual molecules there is no obvious reason why it should not admit of being extended without limit, nor why the relation between the magnetism of a bar and the magnetising force should have the exceedingly complex character it is known to possess. We should rather expect to find proportionality, or something like proportionality, between magnetism and magnetising force, and we should not expect to find residual magnetism or other phenomena of hysteresis. Weber's theory, on the other hand, implies that there must be a limit to the intensity of magnetisation. The limit is reached when all the molecules have become turned to face exactly in the direction of the applied magnetising force; no increase of the force beyond what is required for that can add to the magnetisation. The fact that a definite saturation value is now known to exist* adds much probability to Weber's hypothesis. Further, the

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* The evidence of this has been fully stated above (§§ 91 to 107).

process by which the molecules are supposed to turn hither and thither under varying magnetising forces, leaves ample room, as we shall presently see, for a satisfactory explanation of all the features which the curves of magnetisation are known to present, and the various manifestations of hysteresis become intelligible. Again, the effects of vibration in augmenting magnetic susceptibility are readily accounted for in consequence of the greater freedom which vibration gives the molecules to fall into line with the magnetising force. Additional evidence is furnished by experiments such as that of Beetz*, in which the effects were observed of applying a weak magnetising force to iron at a time when the molecules were peculiarly free to respond to its directive action, namely, while they were in the act of being deposited by the electrolysis of an iron salt. The iron was deposited along a line made by scribing a longitudinal scratch on a straight piece of varnished silver wire. The wire was immersed in the iron salt, and was placed in a magnetic field in such a manner that the lines of force ran in the direction of the length. The silver wire formed one pole of an electrolytic cell, and it was found that the metal deposited on the scratch was so highly magnetised that the subsequent application of a much stronger magnetic field failed to augment its magnetism more than a very little. The molecules had been ranged at the moment when they escaped from imprisonment in the salt, and before they had the opportunity of forming fresh entanglements by their action on one another; just as criminals are said to be most easily diverted into regular courses at the moment of their release from gaol, before they have time to resume the ties of their usual companionship. Not only is Weber's notion of molecular magnets strongly supported by this experiment of Beetz, but the cumulative evidence in its favour which is supplied by many facts of more recent observation may be said to give it almost conclusive proof. We may even build up a model consisting of small permanent magnets, such as Weber's theory postulates, in which all the chief characteristics of magnetic induction can be closely imitated. The study of a model of this kind leaves little room for doubt that the basis of

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* *Pogg. Ann.*, CXI., 1860, p. 107.

Weber's theory, namely, the hypothesis of permanently magnetic molecules, is essentially sound.

§ 168. Constraint of the Molecular Magnets in Weber's Theory.—It is clear that if the process of magnetic induction is to be explained as the turning of molecular magnets so that they tend to face one way, the molecules must be subject to some directive force which prevents them from responding with perfect freedom to the magnetising field. Without some such constraint they would at once take the direction of the applied field, and the weakest magnetising force would suffice to produce saturation. In fact, however, magnetisation goes on progressively as the magnetising force is increased, and at every stage the direction taken by each molecule is determined by a balance between the force of the field which tends to turn the molecule, and some other controlling force which opposes the turning.

Weber supposed that in a piece of virgin iron the axes of the molecular magnets point indifferently in all directions, and that when a magnetising force H is applied, each molecule is deflected against a directive force, which tends to restore it to its original position. He assumes this force to be that which would be exerted by a magnetic force of some constant value, K, acting in the primitive direction of the molecule's axis*. The direction in which the molecule points while the magnetising force acts is consequently the direction of the resultant of H and K, and when the external force H is removed, the molecule is brought back by K to its primitive position. This theory of the constraint of the molecules gives no explanation of residual magnetism or other manifestations of hysteresis. According to it, the magnetic susceptibility should be constant for all values of H less than K, and should diminish for higher values of H. At the stage when H becomes equal to K, and the proportionality of magnetisation to magnetising force ceases, the value of I should be $\frac{2}{3}$ of the final or saturation value. This hypothesis is inconsistent with the fact that the early part of the curve of magnetisation is not straight; that the susceptibility is small

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* *Pogg. Ann.*, LXXXVII., 1852, p. 167. See Maxwell's *El. and Mag.*, Vol. II., § 443.

at first, and increases with increasing magnetising force. This, indeed, is an example of hysteresis, and for the phenomena of hysteresis the theory, in this form, affords no room.

§ 169. **Maxwell's Modification of Weber's Hypothesis.**—To remedy this defect Maxwell suggested a further assumption based on the analogy of magnetisation to mechanical strain, with the object of admitting conditions under which the position of equilibrium of the molecular magnets may be permanently altered. He supposes that when a molecule is deflected by a magnetising force H it returns completely to its primitive position on the removal of H provided the deflection has been less than a certain value, but returns only partially if the deflection has exceeded that value. In the latter case its axis, when H is removed, remains turned through an angle which may be called the permanent set of the molecule. Maxwell has examined the consequence of this supposition at some length, assuming the molecules in a given piece to be all capable of the same or nearly the same amount of elastic deflection, and assuming a constant or nearly constant controlling force, K, to act on each in the primitive direction of its axis. This hypothesis accounts for the existence of residual magnetism, and for some of the phenomena of hysteresis; it fails, however, to explain why hysteresis should be found when, after the first application, a magnetising force is removed and reapplied, and its postulates about controlling force and the condition of permanent set are arbitrary. We shall see presently that by considering the action of the molecular magnets upon one another conclusions are reached which really embody Maxwell's idea of elastic and non-elastic deflection, though the controlling force and the amount of elastic deflection are no longer arbitrary and no longer the same or nearly the same for all the molecules.

§ 170. **Hypothesis of Frictional Resistance to the Deflection of the Molecules.**—The suggestion has been made by Wiedemann and others that the deflection of Weber's molecular magnets is opposed by a species of frictional resistance, which not only resists the magnetisation, but accounts for residual magnetism and the effects of hysteresis by tending to hold the molecules from returning after they have been disturbed. A directive force, such as that postulated by Weber, is, of course, still necessary. Several of the observed phenomena might be adduced as supporting this notion ; in particular, it harmonises well with the effects which are known to be produced by vibration and other mechanical disturbance in augmenting magnetic susceptibility, and in reducing residual magnetism; and also with the comparative suddenness with which the resistance to magnetisation breaks down when a certain stage in the magnetising process is reached. But if the molecules were held fast by friction until the applied force became sufficiently strong to start them, the susceptibility with respect to very feeble forces should be zero, whereas, in fact, it has a small positive and initially constant value (§§ 86, 87). To make the notion of frictional control agree with the facts, it would be necessary to assume some further complication, such as that a few of the molecules in any given piece are sensibly free from friction, and may begin to turn under the influence of the weakest forces.

§ 171. **The Constraint of the Molecules due to their Mutual Action as Magnets.**—The matter becomes immensely simplified if we put aside all these arbitrary postulates regarding controlling force and resistance to turning, and inquire what is the character of the constraint the molecules necessarily suffer through the forces which they exert on one another in consequence of the fact that they are magnets. It appears that this constraint is sufficient to account for the observed characteristics of the process of magnetisation, that it completely explains hysteresis, and that it at least offers a clue to those complicated variations of magnetic quality which are known to be caused by the variation of such physical conditions as temperature or stress.*

In proceeding to consider the equilibrium of the molecules under their mutual magnetic forces, it is clear that we cannot confine our attention to any one molecule. For the directive force that acts on any one molecule depends on the positions of the molecules which surround it, and becomes altered when these are disturbed. We cannot investigate the equilibrium of the

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* *See* "Contributions to the Molecular Theory of Induced Magnetism," *Proc.* Roy. Soc., Vol. XLVIII., 1890, p. 342. *Phil. Mag.*, Sept., 1890.

individual without including in the question the equilibrium of its neighbours. When an external force is applied, they, as well as it, are deflected, and the constraint they exercise on it suffers change. What must be studied is the configuration of the group as a whole, and the manner in which the group becomes distorted, broken up, and rearranged in the process of applying and removing an external magnetising force.

In seeking to find in the mutual constraint excited by the magnetic molecules, an explanation of the changes of susceptibility which are observed as a magnetising force is gradually applied to a piece of iron or other magnetic metal, it should be borne in mind that the magnetising process may be broadly divided into three stages (as was remarked in § 141), namely, the stages A, B, and C of the typical curve (Fig. 136).

Fig. 136.

These admit, in general, of being distinguished from one another without difficulty, though the passage from one stage to the next is never perfectly abrupt. In some cases, however, it is remarkably sharp, as in the curves of Figs. 120 and 121 (pp. 230, 231), which relate to nickel under torsion, and under a combination of torsion with longitudinal pull.

In the first stage the susceptibility is small, and there is almost no retentiveness. In the second stage the magnetism is acquired with great readiness, and much of it may be retained if the force be removed. In the third stage the growth of magnetism is again slow, and what is acquired in it does not contribute much to the residual magnetism. We shall see that these stages are just such as the molecular theory would lead us to anticipate.

§ 172. **Imaginary Molecular Groups.—A Single Pair.**—By way of leading up to the consideration of groups consisting of many magnetic molecules, we may begin by thinking of a group which consists of a single pair. Each member of the pair is to be conceived of as a short magnet capable of free rotation about a fixed centre. In the absence of all external magnetic force this pair of molecules will arrange themselves as in Fig. 137, with opposed poles exactly in the line joining the centres. Let an external magnetic force, H, now be applied in any direction (Fig. 138).

FIG. 137.

FIG. 138.

If H is weak the molecules will be but slightly deflected. But as H is gradually increased a stage will be reached at which the molecules part company, and fly round into a position in which the direction of the magnetic axis of each is nearly parallel to H (Fig. 139).

Except in special cases perfect parallelism with H will be reached only when H becomes indefinitely strong.

Then as H is gradually reduced there will at first be little change in the configuration, until a stage is reached at which a sudden return to the condition of Fig. 138 occurs. This will happen at a lower value of H than that which was needed to break up the group of Fig. 138; here we have, in fact, an elementary example of hysteresis. If the direction of H is chosen so that it is perpendicular to the line of centres this return will occur only when H is reduced to zero. In the more general case, illustrated by the figures, the sudden return will happen when H has a small finite value, and then the subsequent reduction of H to zero will be associated with a gradual change from the state of Fig. 138 to that of Fig. 137.

FIG. 139.

During the application of H we have three stages; there is, first, the slight deflection of the molecules (Fig. 138) which precedes what may be called the rupture of the tie that holds them in line with one another. Then there is the sudden swinging into a position of much greater deflection when that tie is broken. Finally there is the continued approach towards perfect alignment, made under stronger values of H. During each of these three stages the group is acquiring resultant magnetic polarity in the direction of H, though the magnetisation of the individual molecules is, by assumption, a constant quantity. In the first stage the process is, so to speak, perfectly elastic—

that is to say, it corresponds to the elastic stage in the straining of a solid when there is no permanent set left after the removal of the straining force. If we suppose **H** to be removed at any part of the first stage, the molecules at once return to their primitive positions. But after the critical value of **H** has been passed, which separates the first stage from the second, this is not so; there is then a tendency to retain the new configuration. We shall see presently that this tendency, which is the very essence of hysteresis, becomes much more conspicuous when we have to deal with larger groups. Finally in the third stage we have again a quasi-elastic part of the process of magnetisation.

To begin with, the equilibrium of the group is, of course, stable with respect to small displacements. Any small casual disturbance, applied and removed, will leave the magnets swinging about the position of equilibrium, shown in Fig. 137. The equilibrium continues to be stable so long as the deflecting force is weak (stage A). But as the critical point is approached, the stability becomes reduced—just at the end of stage A it is neutral, and any further increase of **H** brings about instability. The molecules then precipitate themselves into the new form (Fig. 139) in which they are once more stable so long as **H** continues to act.

To express the matter in symbols, let us suppose that each magnet may be treated as a pair of poles of strength m, separated by a distance $2r$, which is the length of the magnetic axis. Let α (Fig. 140) be the angle which the direction of the applied deflecting force **H** makes with the line of centres C C′, and let θ be the amount of deflection, which is the same for both magnets. It is assumed, in the first place, that **H** is not so strong as to produce instability.

The field **H** exerts a mechanical force $m\mathsf{H}$ on each pole, or a couple on each magnet, the distance between the parallel forces of the couple being $2r\sin(\alpha-\theta)$.

The deflecting moment which acts on each magnet is, therefore,

$$2\,\mathsf{H}\,mr\sin(\alpha-\theta),$$

and this is to be balanced by what we may call the restoring moment, due to the forces which the magnets exert on one another.

These forces are (1), the attraction of the poles P Q; (2), the attraction of the poles P′ Q′; (3), the repulsion of the poles P′ Q; and (4), the repulsion of the poles P Q′. Of these forces the moments of the third and fourth balance one another, and the moment of the second is insignificantly small compared with that of the first, provided the distance C C′ is not much greater than the length of one magnet, and the deflection is not great. Under these conditions it will suffice to consider the restoring moment as due to the first force only, namely, to mutual attraction of P and Q. Its value is

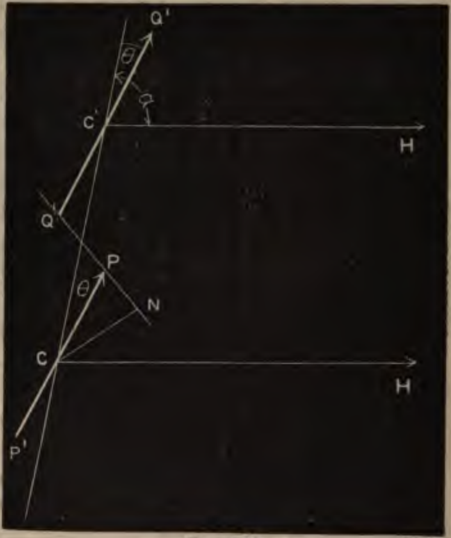

FIG. 140.

$$\frac{m^2\,\mathrm{C\,N}}{\mathrm{P\,Q}^2},$$

C N being the perpendicular distance from C to the line P Q; and the condition of equilibrium is that

$$2\,\mathrm{H}\,m\,r\sin(\alpha-\theta)=\frac{m^2\,\mathrm{C\,N}}{\mathrm{P\,Q}^2}. \quad . \quad . \quad . \; (1).$$

As θ is increased the restoring moment at first increases, but passes a maximum at a value of θ which depends on the relation of the length r, or C P, to the length C C′.

When H and θ are sufficiently increased the equilibrium becomes neutral. This occurs when

$$\frac{d}{d\theta}\left\{2\,\mathsf{H}\,m\,r\sin(\alpha-\theta)\right\}=\frac{d}{d\,\theta}\,\frac{m^2\,\mathrm{C\,N}}{\mathrm{P\,Q}^2},$$

or

$$\frac{d}{d\theta}\left\{\mathsf{H}\sin(\alpha-\theta)\right\}=\frac{m}{2\,r}\,\frac{d}{d\,\theta}\,\frac{\mathrm{C\,N}}{\mathrm{P\,Q}^2}. \quad . \quad (2).$$

From these two equations (1) and (2) it is possible to determine the values of H and of θ corresponding to the critical point in the deflection, at which the equilibrium of the deflected molecules becomes neutral. Any greater value of H will cause instability; the molecules will then swing violently round into a new position of equilibrium with their axes nearly parallel to the direction of H.

If there were a number of such pairs of magnets, of the same strength and the same pitch, all acted on by the same deflecting field, but with their lines of centres inclined at various angles to the direction of H, it is clear that some would reach instability sooner than others, as H was strengthened. The first pairs to become unstable would be those which were inclined at something more than a right angle to H, so that $\alpha-\theta$ became a right angle when the value of θ corresponding to instability was reached. Other pairs would escape passing through the unstable state altogether, namely, those pairs which lay initially in directions nearly parallel to H. How nearly parallel to H they must lie initially in order to escape instability depends on the extent by which the distance between the centres exceeds $2\,r$.

If we suppose that this excess of distance, or clearance between the poles, as one may call it, is very small, then the state of instability in pairs which lie well across the direction of H is reached approximately when

$$\frac{d}{d\theta}\,\frac{\mathrm{C\,N}}{\mathrm{P\,Q}^2}=0,$$

which happens when $\tan\phi=\frac{1}{\sqrt{2}}$, ϕ being the inclination of the line P Q to the line of centres C C′. In these circumstances the value of H which breaks up the pair is

$$\mathsf{H}=\frac{m}{12\sqrt{3}\,.\,(a-r)^2\sin\alpha},$$

where a stands for half the distance between the centres C and C′. This does not apply when the line of centres is nearly parallel to **H**. In the special case when the line of centres has the same direction as **H**, but the magnets point initially in the direction opposed to **H**, there is no stable deflection previous to the occurrence of instability. The critical point is reached in such a pair when

$$\mathsf{H} = \frac{m}{8\,(a-r)^2}.$$

The general behaviour of a crowd of groups, each consisting of two magnets, can be readily enough imagined, and still more readily examined by aid of a model. Until the first of the groups breaks up, as the field is increased, we have nothing but quasi-elastic deflection. Then the groups successively reach the critical point, so that a rapid, though not perfectly sudden, development of resultant polarity on the part of the crowd as a whole is observed. Finally, there is a slight further increase, under the action of stronger fields, as the state corresponding to saturation is approached.

Again, as the field is gradually reduced many of the groups will return to their initial state. Many others, however, will assume new forms, namely, with their poles pointing just the other way from the way they pointed at first, and the effect of these will be to contribute a resultant residual polarity which persists when **H** is reduced to zero. The application and removal of **H** will leave a majority of groups pointing, more or less, towards the direction in which the force was applied, although at first there was no preponderance in any direction.

We find, therefore, even in so simple a grouping of magnetic molecules as this—namely, a grouping in isolated pairs—many of the features which are presented in the magnetisation of iron. We find analogues of the first, the second, and to some extent the third stages, which are observed in curves of **I** and **H**, and we find evidence of hysteresis and residual magnetism. But a very much more complete reproduction of the phenomena of magnetisation becomes possible, as will be shown presently, if we suppose the molecules to be distributed either continuously or in groups consisting of a considerable number of members.

The behaviour of two-member groups would agree fairly well with what is known to happen in the first and second stages of the magnetising process in iron. It seems, however, to leave too little supplementary magnetisation to be acquired during the third stage. And a more obvious difficulty is, that though two-member groups suffice to account for the existence of some residual magnetism, they fail to explain the high retentiveness which is found in, say, soft iron, where we often find more than 90 per cent. of the induced magnetism surviving the removal of the magnetising force. To account for that, something more is needed than the constraint exercised by each member of a pair on the other member; the molecules must, in fact, form new ties after the old ones have been ruptured, and to allow of that each molecule must have more neighbours than one.

FIG. 141.

§ 173. **Group of Four Members.**—A better approximation to the facts will be obtained if we deal with a group consisting of four little magnets, with their centres at the four corners of a square (Fig. 141). When the field **H** begins to act, the members of the group are all slightly deflected, but without at first becoming unstable. If during this first stage the force **H** is removed, there is no residual displacement. But when **H** is sufficiently increased the original lines of the group break, and the members tend to pair themselves anew in lines which are more favourably inclined to the direction of **H** (Fig. 142). Finally, when **H** is further increased, the members of the group are gradually compelled to take the position sketched in Fig. 143. Next, suppose H to be removed. There will be a return from the condition of Fig. 143 to that of Fig. 142, but the pairing shown in Fig. 142 will be maintained, and this implies a large amount of residual magnetisation. If the direction of H be then reversed, and its value gradually increased, a stage will presently be reached when the resultant polarity of the group suffers an abrupt change through the reversal of the lines in Fig. 142.

FIG. 142.

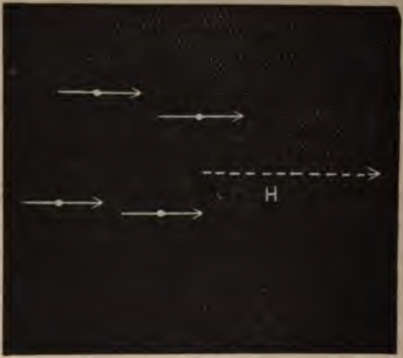

FIG. 143.

The curve of magnetisation—that is to say, the curve showing the resultant polarity in terms of H—for the single group of four members is sketched in Fig. 144.

From this it is easy to see, in a general way, what would be the form of the curve for an aggregate of many such groups, variously inclined to the direction of **H**. The transition from one stage to another will be gradual in the aggregate, for it will happen at different values of **H** in different groups. Hence the curve will assume a rounded outline in place of the sharp corners of Fig. 144.

Moreover, the curve obtained during the removal of **H** will not coincide with that obtained during the application of **H**, except the process be stopped at a very early point in the first stage. Whenever the process is extended far enough to cause any of the groups to reach the unstable state we shall find hysteresis. The two curves will not coincide, even in the third stage. Some of the members will pass through an unstable state even there. After the first re-arrangement of the group has taken place, and the lines have become directed as in Fig. 142, there may be a second breaking up and passage through instability on the way to the state of Fig. 143. This will happen when the lines of centres have a considerable inclination to **H**, and especially when the poles of the members are close together. In such an aggregate of groups we may therefore expect to

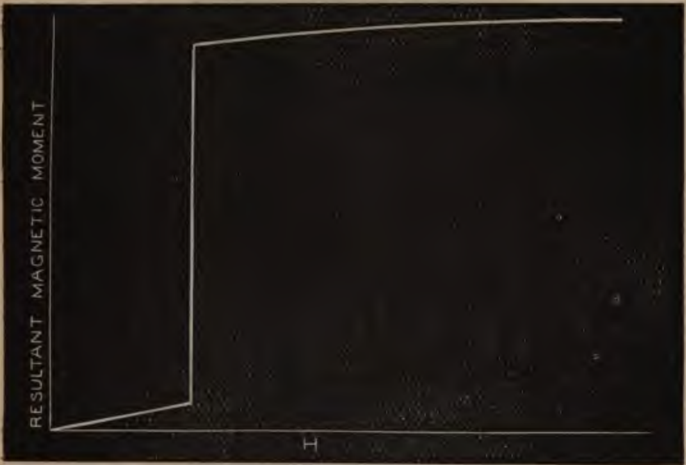

Fig. 144.

find hysteresis in all possible cyclic changes of the magnetising force. The form of the curve obtained during reversal of **H** will evidently agree with the general form given by the magnetic metals. In proportion as the corner between stages A and B in the first curve is comparatively sharp or comparatively rounded so will be the corner at which the rapid descent of the curve begins while **H** is being reversed.

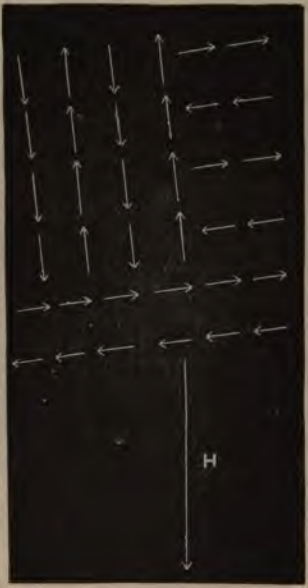

Fig. 145.

§ 174. Continuous Distribution in Cubical Order.—From these considerations regarding groups of four members it is easy to pass to the case of a manifold group or a continuous distribution of members arranged so that the lines of centres form squares. All that has been said above is still applicable. The members arrange themselves in lines, and each individual is mainly constrained by its two neighbours in the same line, instead of by one neighbour as in the case already spoken of. The equations of § 172 are readily adapted to members of a long row by substituting $2\,m^2$ for m^2 in the expression for the restoring moment. The three stages of (1) stable deflection, (2) instability, with rupture of the original lines and formation of new lines, and (3) further stable deflection are as readily traced as before, as will be evident by an inspection of Figs. 145, 146, and 147.

Fig. 145 represents an imaginary primitive arrangement. Fig. 146 is the configuration reached after the breaking up of the primitive lines, and Fig. 147 corresponds to saturation.

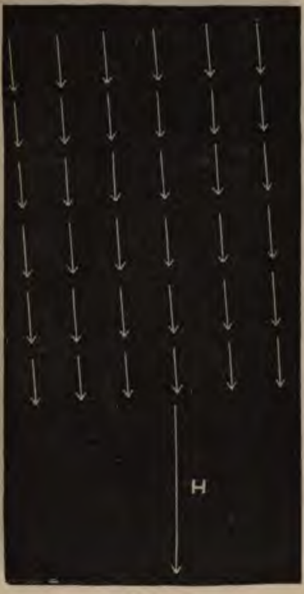

FIG. 146.

It appears, then, that the theory that the magnetic molecules owe their stability to the magnetic action of their neighbours gives results which agree with the observed facts, whether we conceive the molecular structure to consist of isolated groups, with a limited number of members in each, or to be continuous. Even with a continuous distribution the lines of molecules will, in consequence of the imperfect homogeneity of the piece, be variously inclined at various places, so that the condition necessary to give a rounded outline to the curve will in any case be present. In no piece, except perhaps in a single crystal, could we expect to find that perfect regularity of structure which would be necessary to make the transition from one stage of the magnetising process to another quite sudden, and to give the curve the form of a series of sharp steps.

Whether we picture the structure as continuous or as built up of isolated groups, special importance attaches to square patterns from the fact that the magnetic metals crystallise in the cubic system. The behaviour of pyramidal forms presents some interesting features that need not be entered into here.

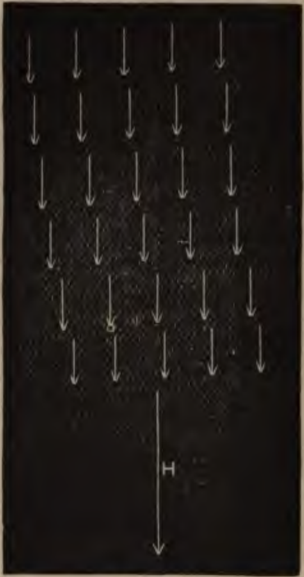

FIG. 147.

Enough has already been said to show that there is no need to assume that any arbitrary controlling forces act on Weber's molecular magnets. The theory that their constraint proceeds only from their mutual action as magnets evidently suffices to explain, generally, the characteristics of the magnetising process It may be useful, however, to point out how complete is the agreement, in point of detail, between the deductions which may be drawn from the theory and the facts which have been described in earlier chapters.

§ 175. **Agreement of the Theory with known Facts about Susceptibility.**—In the first stage there is no rupture of molecular ties until the magnetising force is sufficiently increased to bring about instability in the least stable lines or groups of molecules, and until that happens the application and removal of the force has no residual effect. Up to that point the deflections are small, and they are initially proportional to the applied force. All this is in complete agreement with Lord Rayleigh's experiments on the susceptibility of iron and steel to feeble magnetising forces (§ 87), which show that the initial value of the susceptibility is a small constant quantity, and that residual magnetism begins to show itself only when the magnetising force is so much increased that the proportionality of magnetism to force ceases. Again, it accords with the result that a small alternating change of H, superposed on a constant value of H, or acting on a piece which has residual magnetism in consequence of the action of previous forces, produces (after the first application) but a small coming and going of the molecules without breaking their ties, and that if this small alternating force is applied when the magnetisation is already strong, the changes which it causes are reduced in amount (§ 87). Again, the theory might lead us to anticipate the fact that if at any point of the ordinary magnetising process we stop increasing H and begin to decrease it, or stop decreasing H and begin to increase it, the initial rate of magnetic change or value of $\frac{d\mathsf{I}}{d\mathsf{H}}$ is very small, depending, as it does, only upon the quasi-elastic movement of the deflected molecules. Their movements through the condition of instability do not begin until the reversal of procedure has been carried some little way.

Again, in strong fields the behaviour of the little magnets accords with the gradually falling off in susceptibility which actually occurs in magnetic metals as the state of saturation is approached. To reach the state of perfect saturation would require an indefinitely strong directing force, but the alignment of the molecules is to all intents complete long before that. In view of the molecular theory it is not surprising that in iron, where many molecular groups break up under a force of no more than two or three C.-G.-S. units, a force of two or three thousand units produces (as we saw in § 102) so nearly perfect saturation that augmenting the force tenfold adds nothing perceptible to the magnetisation.

The quantity which tends to a limit when saturation is approached, is, as was shown in § 93-102, the intensity of magnetisation **I**, not the induction **B**. According to the molecular theory, **I** is the sum per unit of volume of the moments of the molecular magnets resolved in the direction of magnetisation. If n be the number of molecular magnets in unit volume, and m the moment of each, the saturation value of **I** is $m\ n$.

§ 176. **Retentiveness and Residual Magnetism.**—An equally satisfactory agreement is found when the results of experiments on retentiveness are examined in the light of the molecular theory. We shall take advantage of the opportunity which this discussion of the theory affords to describe some of these results more fully than has yet been done.

In the first stage of the magnetising process, as has been already remarked, there is no retentiveness: the magnetism that is induced under very weak forces disappears entirely when the inducing force is removed. This accords with the view that the molecular magnets are then being as it were elastically displaced from a position of stable equilibrium, without rupture of the ties by which the initial grouping maintains itself, so that each molecule simply returns to its primitive position when the displacing force is withdrawn. Theory and experiment alike show that this condition persists only so long as the susceptibility is very small.

In the second stage the susceptibility has become much increased as a consequence of the large deflection the molecular magnets suffer in breaking away from their original grouping to form new combinations. The movements they then accomplish are in great measure irreversible, that is to say, they are not undone as the magnetising force is being withdrawn. We may therefore expect to find a rapid development of residual magnetism during that part of the magnetising process in which the susceptibility is high. The theory shows that in favourable cases nearly the whole of the magnetism acquired during that stage will persist as residual magnetism. Experimental instances of this are given below.

The third stage, on the other hand, contributes little to the residual magnetism, for the molecular deflections that occur in it are for the most part undone as the magnetising force is withdrawn. A result of this is that the residual magnetism approaches saturation sooner (that is, under the action of weaker magnetising forces) than does the induced magnetism.

Another result is that the residual magnetism has a saturation value which is definitely less than the saturation value of the induced magnetism. It is indeed possible to imagine a molecular structure such that all the magnetism of saturation would be retained on the withdrawal of the force. This would be the case in a cubical formation if the lines of centres were parallel and perpendicular to the direction of the field throughout the whole piece. But the imperfect homogeneity of any actual piece of iron puts such a conception out of court, and when any of the lines of centres are inclined to the field, it is clear that the saturation value of $\mathbf{I}_r$ is less than that of $\mathbf{I}$. It will be shown presently that a continuous cubical formation with lines of centres uniformly distributed as regards inclination is a structure which gives more than sufficient possibility of residual magnetism. The value of $\mathbf{I}_r$ which the theory shows to be possible in such a structure is in fact greater than the values which are found in experiments with even the most retentive metal.

The molecular theory makes it easy to understand the difference between retentiveness and what may be called coercive capacity, by which is meant the quality that the coercive force (§ 47) measures—the quality in virtue of which a substance holds its residual magnetism so strongly that a considerable magnetic force, acting in the reversed direction, is necessary to remove it. Retentiveness, on the other hand, is the quality in virtue of which much residual magnetism is held, though it may be held very weakly. Probably no magnetic substance has so much retentiveness as soft annealed iron, and probably none has so little coercive capacity. Retentiveness, by the molecular theory, is the result of a regular molecular structure of such a kind that the molecules readily arrange themselves in lines which are but little inclined to the direction of the applied force. The molecular ties may, however, be extremely weak. Coercive capacity is a result of strong ties, such as might be formed by reducing the distances between the molecular centres or between some of them, and this condition may very well exist in a structure where the lines or groups are unfavourably arranged for retentiveness.

§ 177. Experiments on Residual Magnetism in Iron.—The following experimental results* were obtained with straight iron wires, 400 diameters long, using the direct magnetometric method. The magnetising force was gradually raised to an assigned value, then gradually withdrawn, to allow the

Table XXX.—*Induced and Residual Magnetism in a Soft Iron Wire, Annealed and Hardened by Stretching.*

Before stretching.				After stretching.			
H	I induced.	I_r residual.	Ratio of residual to induced.	H	I induced.	I_r residual.	Ratio of residual to induced.
0·42	16	3·9	0·24	0·42	3·6	0	0
0·58	24	6·6	0·27	0·99	13·1	2·9	0·22
0·70	33	9·9	0·30	1·44	21·1	6·5	0·31
0·99	62	24	0·40	1·73	26·9	11·8	0·38
1·16	91	46	0·50	2·14	41	15·3	0·38
1·30	140	85	0·61	2·88	72	32·7	0·46
1·44	195	133	0·68	3·58	116	61·7	0·53
1·58	280	209	0·74	4·20	167	98	0·59
1·76	364	283	0·78	4·90	218	132	0·61
2·02	468	380	0·81	5·76	265	167	0·63
2·14	507	418	0·82	7·20	359	225	0·625
2·28	549	455	0·83	10·78	566	327	0·58
2·51	614	513	0·84	11·90	613	348	0·57
2·74	673	568	0·85	15·20	751	381	0·51
2·88	702	598	0·85	17·50	817	399	0·49
3·16	764	650	0·85	23·61	947	414	0·44
3·58	842	711	0·85	29·81	1017	417	0·41
4·20	926	783	0·85	35·71	1078	419	0·39
5·02	984	832	0·84	41·90	1114	419	0·38
5·76	1020	848	0·83				
6·46	1050	864	0·82				
7·20	1070	877	0·82				
8·64	1110	897	0·81				
10·26	1130	910	0·80				
11·91	1150	913	0·80				
17·50	1190	929	0·79				
23·61	1195	929	0·78				
35·71	1230	933	0·76				
45·51	1230	933	0·76				

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* *Phil. Trans.*, 1885, Part II., pp. 556 *et seq.*

residual magnetism to be noted; then raised to a slightly higher value, again withdrawn, and so on; so that the values of **I** and $\mathbf{I}_r$ were ascertained, corresponding to successive steps in the magnetising process. The results will be seen to bear out what has just been stated, and to furnish strong evidence in favour of the theory that the constraint of the molecular magnets is due to their mutual magnetic forces.

Table XXX. gives the results of an experiment* in which an iron wire, 1·58mm. in diameter, was tested, first in the annealed state, and then after being hardened by stretching beyond its elastic limit. Inspection of the figures will show that the ratio of residual to induced magnetism, which is at first small in both cases, rises to a maximum. This maximum, in the annealed wire, is so high as to imply that the rate of increment of residual magnetism is then not far short of the rate of increment of induced magnetism. The ratio afterwards falls off as the magnetising process passes into its third stage.

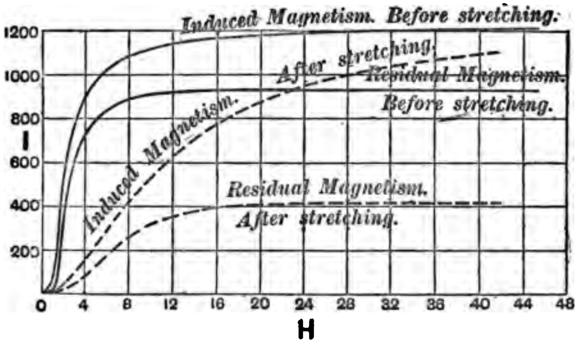

FIG. 148.—Induced and Residual Magnetism in Iron, in the soft state and hardened by stretching.

Fig. 148 is drawn to exhibit the same results. It shows well how the residual magnetism approaches its maximum faster than the induced magnetism does, notably in the hardened wire.

This mode of representing the results, where **I** and $\mathbf{I}_r$ are given in terms of **H**, is not, however, well adapted to show

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* *Loc. cit.*, § 41, pp. 559-60.

what is the saturation limit towards which I_r is tending, nor what is the relative rate of increment of the two at various stages of the magnetising process. To bring these points out we may draw a curve showing I_r in relation to I (Fig. 149). We already know the saturation value to which I tends, namely, about 1,700 C.-G.-S. units (§ 98), and it is not difficult, by extrapolation of the curve in this new figure, to deduce an approximate value for the saturation limit of I_r.

This is done in Fig. 149, where the broken lines form a conjectural extension of the curves, beyond the range of the experiment, up to the saturation value of 1,700 for the induced I. It appears from these that the saturation values of the residual magnetism in this specimen are approximately 970 when the metal is annealed, and 430 when it is hardened by stretching.

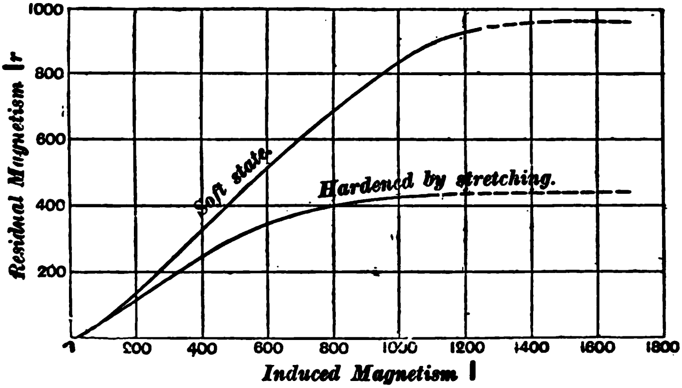

FIG. 149.—Proportion of Residual to Induced Magnetism in Iron.

An inspection of the curves in Fig. 149 will also show that after the initial stage is over, in which the residual magnetism is acquired less rapidly, the proportion which the increment of I_r bears to that of I becomes as nearly as possible constant, and remains so (in the annealed wire) throughout a large part of the whole process of magnetisation. From the point $\mathsf{I} = 150$, or so, up to 800 the curve is practically straight, and during that part of the process nearly the whole of the magnetism that is acquired goes to form residual magnetism. By Table XXX. we have

H	I	I_r
1·30	140	85
3·16	764	650
Difference....................	624	565

So that, during this time, $\frac{565}{624}$, or quite 92 per cent. of the magnetism that is being induced, contributes to the residual magnetism. After this the curve bends over rather quickly and $\frac{d\,\mathsf{I}_r}{d\,\mathsf{I}}$ becomes much reduced. In other specimens of annealed iron the value of $\frac{d\,\mathsf{I}_r}{d\,\mathsf{I}}$ during the steep stage was even more nearly unity.

This was the case in the experiment of Table XXXI., made with a piece of annealed iron wire 0·72mm. in diameter.* In this case a supplementary experiment was made to determine the

Table XXXI.—*Induced and Residual Magnetism in Annealed Iron Wire, with and without Longitudinal Pull.*

Load=0.				Load=4 kilos., or 9·76 kilos. per sq. mm.			
H	I induced.	I_r residual.	Ratio $\frac{\mathsf{I}}{\mathsf{I}_r}$	H	I induced.	I_r residual.	Ratio $\frac{\mathsf{I}}{\mathsf{I}_r}$
0	0	0	—	0	0	0	—
1·08	66	32·5	0·49	0·54	38	21	0·53
1·62	202	141	0·70	1·08	141	94	0·69
2·16	460	381	0·83	1·62	325	242	0·745
2·70	684	601	0·879	2·16	532	419	0·788
3·24	846	767	0·907	2·70	677	543	0·802
3·78	939	860	0·916	3·24	796	640	0·805
4·32	993	920	0·921	3·78	876	705	0·805
5·40	1071	994	0·928	4·37	937	754	0·804
6·48	1109	1024	0·923	4·86	978	787	0·805
7·56	1139	1046	0·919	5·51	1022	816	0·800
8·64	1157	1063	0·919	6·48	1067	856	0·800
9·72	1168	1074	0·919	8·64	1121	891	0·795
10·8	1178	1082	0·918	10·8	1162	913	0·786
13·5	1196	1095	0·916	13·5	1186	926	0·781
16·2	1210	1105	0·913	16·2	1204	933	0·775
18·9	1219	1111	0·911	18·9	1211	939	0·775
21·6	1226	1116	0·910	21·6	1219	942	0·773
25·6	1236	1119	0·905	26·2	1232	946	0·768

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* *Loc. cit.*, p. 629.

influence of longitudinal pull on the residual magnetism. After the test made in the ordinary condition of no load, a steady load of 4 kilos. or 9·76 kilos. per sq. mm., was applied (a load well within the elastic limit), and the observations in the second portion of the Table were made. The proportion of I_r to I in each case is shown in Fig. 150, where the dotted line refers to the experiment in which wire was in a state of longitudinal tension. The full curve is for no load, and is conjecturally extended to find the saturation value of I_r, which is higher in this specimen than in the last, namely, 1210. The rate of increment of I_r, relatively to I during the steep part of the curve is also greater, and the curve is practically straight throughout a wider range. The following supplementary Table will bring this out :—

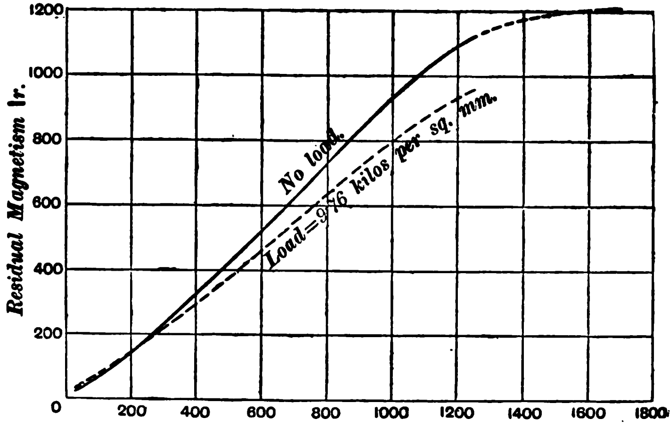

FIG. 150.—Proportion of Residual to Induced Magnetism in Soft Iron Wire, loaded and without Load.

I	I_r	Differences of I_r for 100 of I.	I	I_r	Differences of I_r for 100 of I.
300	232	—	800	722	99
400	328	96	900	822	100
500	426	98	1,000	921	99
600	524	98	1,100	1,020	99
700	623	99	1,200	1,100	80

It appears from these figures that in the stage lying between $= 300$ and $\mathbf{I} = 1{,}100$, or so, nearly 99 per cent. of the induction of magnetism was taking place by the turning round of the molecules into new lines, in which they remained when the magnetising force was withdrawn. Scarcely any of the induced magnetism was then being contributed by quasi-elastic deflections. After 1,100, the part played by quasi-elastic deflections began to be considerable.

In the test made while the wire was loaded the limit to which the residual magnetism apparently tended was about 1020. A feature to be remarked is that in the earliest stage the curve taken with load lies above the curve taken without load, crossing it when $\mathbf{I}$ is about 200. The presence of longitudinal pull, though unfavourable to the retention of magnetism by annealed iron when the magnetisation is strong, is favourable to it when the magnetisation is decidedly weak

Experiments made with other specimens gave results which agreed well with these. With another piece of annealed iron wire, 0·78mm. in diameter, the following (amongst other) readings were taken :—*

$\mathbf{H}$	$\mathbf{I}$	$\mathbf{I}_r$	$\mathbf{H}$	$\mathbf{I}$	$\mathbf{I}_r$
0·86	26	6	5·40	991	898
1·98	164	96	6·81	1,067	946
2·66	478	378	11·20	1,166	1,014
3·78	806	696	17·24	1,212	1,042

In this case between $\mathbf{H} = 2·66$ and $\mathbf{H} = 6·81$ the increment of $\mathbf{I}$ is 589, and that of $\mathbf{I}_r$ is 568 or 96 per cent. of the other.

One more experiment of the same class may be referred to in further illustration of the influence of longitudinal pull on the retentiveness of iron.† In this instance the piece tested, a wire, 0·72m. in diameter and 30·5cms. long, had been hardened by stretching beyond its limit of elasticity before the observations were made. Its retentiveness was then examined when without load and also when various amounts of steady pull were in action. The curves of $\mathbf{I}$, also of $\mathbf{I}_r$, each in rela-

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* *Loc. cit.*, p. 559, § 40. Reference to the same Paper should be made for similar experiments with steel in the soft and hard states.

† *Loc. cit.*, pp. 625-8, § 110.

tion to H, have already been given in Figs. 105 and 106 (pp. 201-202) respectively; but the points to which attention is now directed may be better seen by reference to Fig. 151, where curves of I_r in relation to I are drawn for no load and for two values of the load. These show that a moderate amount of pull is very favourable to retentiveness in hardened iron, and greatly augments the saturation limit towards which I_r tends. A stronger pull, on the other hand, is less favourable, though under the greatest pull that was used in these experiments the wire continued to be more retentive than it was in the unloaded state. In a similar experiment made with steel wire, the amount of pull was further increased, and was then found to have an unfavourable effect, that is to say, it reduced the retentiveness in the upper part of the process below the value possessed by the unloaded wire.

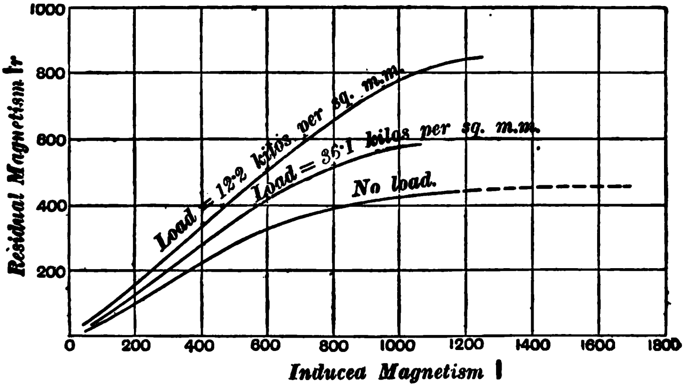

FIG. 151.—Proportion of Residual to Induced Magnetism in Hard Iron Wire, loaded and without Load.

In Fig. 151 the apparent saturation limit of I_r is about 460 when there is no load, and this is raised to 860 by the presence of a load of 12·2 kilos. per square mm. It is, of course very possible that a slightly greater or slightly less load than this would produce a still more favourable effect on the saturation value of the residual magnetism. When there is no load the rate of increment of I_r with respect to I at the steepest part of the curve is about 0·7; but the presence of a suitable amount of load raises that to at least 0·85.

§ 178. **Retentiveness of Nickel.**—A reference to the curves which were given for nickel when the effects of stress were discussed in Chapter IX. (§§ 121-122, Figs. 95, 96, 98, and 99,) will show that the presence of longitudinal *push* has a highly favourable effect on the retentiveness of that metal, and on the maximum of residual magnetisation. Pull, on the other hand, is extremely unfavourable to the retentiveness of nickel. A comparison of the results set forth in Figs. 98 and 99 shows that the value of $\frac{d\mathbf{I}_r}{d\mathbf{I}}$, which is at no stage great in unstressed nickel, rises to a maximum approaching unity when the metal is tested under the influence of strong longitudinal push. And it is, at least, highly probable that the same thing occurs at the steep stage of the magnetising process in the tests under torsion figured in Figs. 120 and 121, § 141.

§ 179. **Amount of Retentiveness possible under the Molecular Theory.**—The full bearing of these experimental results on the molecular theory is not easily traced, and it would be scarcely profitable to speculate at present on the forms in which the groups of molecular magnets may conceivably be arranged. It is, however, important to notice that the theory leaves ample room for even the high retentiveness which iron is found to possess. To show that this is so we may consider what would be the saturation value of the residual magnetism if the structure consisted of lines like those of Fig. 145, with the centres of the molecules grouped in cubical order. It would be unreasonable to postulate any particular directional relation between the lines of centres and the direction in which the piece is to be magnetised. We shall suppose that the structure is an aggregate of tribes of molecules, with a cubical formation for each tribe, but with all possible variety in the direction of the lines of centres.* In the piece as a whole the directions of the lines of centre may be taken as uniformly distributed; in other words, they might be represented by all possible radii of

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* The structure may be continuous; in other words, the transition from one direction in the line of centres (at one place in the metal) to another direction at another place may occur through very slight distortion of the cubical formation.

a sphere, drawn so that the points in which they meet the spherical surface are equally spaced.

Suppose that a very strong magnetising force **H** is applied, so that saturation is produced, and that the force is removed. We have to consider how much residual magnetism will be found when the molecules have returned into lines in which they are stable, and which are as favourably directed for giving residual magnetism as the assumed structure of the substance will permit.

Let α be the angle at which any line of molecules is inclined to the direction of **H**, before the process of magnetisation begins. Since the distribution of direction is by assumption uniform, the number of molecules whose inclinations are less than α will be to the whole number in the proportion which that part of a spherical surface cut off by a cone of semi-angle α (with its vertex at the centre), bears to the whole spherical surface. In the same way the number of molecules whose inclinations lie between α_1 and α_2 will be proportional to the area of that belt of the sphere's surface which is cut off between cones with α_1 and α_2 for semi-angle. Let the whole number of molecular magnets per unit of volume be n. Then the number whose inclinations lie between α_1 and α_2 will be

$$\frac{n}{2}\int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha.$$

Let θ be the inclination of a molecule after it has been displaced by the application and removal of **H**. If m is the moment of a single molecule, it contributes $m\cos\theta$ to the residual magnetism. The whole amount of residual magnetism contributed by those molecules whose original direction ranged from α_2 to α_1, will therefore be

$$\frac{mn}{2}\int_{\alpha_1}^{\alpha_2} \sin\alpha\cos\theta\, d\alpha.$$

And to find the whole residual magnetism we have to extend the limits of this integration to include all the initial directions, from $\alpha = 0$ to $\alpha = 180$ deg.

We have next to consider the relation of θ, the inclination after **H** has been applied and removed, to the original inclination α. Our assumption as to the structure makes the permanent deflection of the molecule necessarily either 0, or 90 deg., or 180 deg.

(1) Molecules for which α is less than 45 deg. will suffer no permanent deflection. This is because the original lines are more favourably directed than lines at right angles to them. For these molecules $\theta = \alpha$.

(2) Molecules for which α is greater than 45 deg. and less than 135 deg. will be permanently turned through one right angle. For these molecules $\theta = \alpha - 90$ deg., and $\cos\theta = \sin\alpha$.

(3) Molecules for which α is greater than 135 deg. will be permanently turned through two right angles. For these molecules $\theta = \alpha - 180$ deg.

The whole residual magnetism, therefore, consists of the sum of three terms, namely

$$\frac{m\,n}{2}\int_0^{\frac{\pi}{4}} \sin\alpha\cos\alpha\, d\alpha + \frac{m\,n}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\alpha\, d\alpha$$

$$+\frac{m\,n}{2}\int_{\frac{3\pi}{4}}^{\pi} \sin\alpha\cos(\alpha - 180^\circ)\, d\alpha.$$

The first and third terms are of equal value. The integral of the first term is

$$-\frac{m\,n}{2}\left[\frac{\cos 2\alpha}{4}\right]_0^{\frac{\pi}{4}} = \frac{m\,n}{2} \times \frac{1}{4}.$$

The integral of the second term is

$$\frac{m\,n}{2}\left[\frac{\alpha - \sin\alpha\cos\alpha}{2}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{m\,n}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right).$$

The whole residual magnetism is therefore

$$\frac{m\,n}{2}\left(2 \times \frac{1}{4} + \frac{\pi}{4} + \frac{1}{2}\right) = \frac{m\,n}{2}\left(1 + \frac{\pi}{4}\right) = 0{\cdot}8927\ m\,n.$$

This is the residual magnetism of saturation, and is to be compared with the induced magnetism of saturation, which is $m\,n$.

Assigning to $m\,n$ the value 1,700 this calculation shows that a continuous structure of the kind postulated, cubical in arrangement, is competent, on the molecular theory, to have nearly 1,500 units of residual magnetism, an amount considerably greater than experiments show even the most retentive iron to be capable of holding. It is clear, therefore, that the intermolecular magnetic forces are abundantly sufficient to account for residual magnetism, and that the actual structure of soft iron, and still more that of hard iron, steel, nickel, and cobalt, is less favourable to retentiveness than is the simple structure we have been discussing here.

§180. **Hysteresis and the Dissipation of Energy.**—The molecular theory shows that hysteresis is to be expected whenever the magnetism of iron is caused to alter through anything more than a very narrow range. It occurs when the molecular movements are sufficiently great to involve the breaking up of old ties, and the formation of new ones, on the part of some, at least, of the molecules. In other words, the necessary and sufficient condition for hysteresis is that there must be an unstable phase in the movement of some of the molecules. The change of magnetism will then lag behind the exciting cause of the change, whatever that may be.

When the change is restricted within very narrow limits there is no hysteresis, for the molecular movements are then of the quasi-elastic type, occurring without rupture of the molecular ties. A very weak magnetic force, applied and removed (whether acting alone or superposed on a steady force), or a very small change of mechanical strain, will, if it be many times repeated, cause small changes of magnetism which do not involve hysteresis, because the molecular magnets are then suffering deflections with respect to which they are stable. But when the action is extended by using larger magnetic forces or larger variations of mechanical strain, so that the molecules are deflected far enough to become unstable, hysteresis comes into play. We find hysteresis, in fact, manifesting itself in all save the narrowest cycles of magnetising force, of longitudinal pull, of torsional strain, and so on.

The dissipation of energy which has been shown to accompany hysteresis in cyclic variations of magnetism is an obvious outcome of the mechanically irreversible character of these unstable molecular movements. When the molecule's equilibrium is upset it tumbles over, and acquires kinetic energy in falling towards a new position of equilibrium. It oscillates about that, and communicates oscillations to its neighbours, until the motion is damped out by setting up eddy currents in the surrounding conducting mass, and the energy of these currents is in turn converted into heat. The damping effect of eddy currents in the surrounding medium may even be so great that there is no continued oscillation; the molecule may be as it were let down gently like a body connected to a dash-pot. In any case, the ultimate effect is that a magnetic cycle generates heat in the substance of the metal. We have seen already that when the cycle is produced by reversing the magnetising force H, this heat is the equivalent of the quantity $\int \mathsf{I}\, d\, \mathsf{H}$, and numerical values of it have been given in an earlier chapter.

But what will be the heating effect when a magnetic cycle is performed, not by varying the amount of H but by turning the iron round in a constant magnetic field? The question is of much practical interest, since that is the way that the magnetism is reversed in the core of a dynamo armature of the Gramme ring or the drum type. Experiments to answer it are as yet wanting. It appears to have been often taken for granted that the same amount of work is spent in the two cases, but there is no apparent basis for the assumption. In a series of experiments on dynamo armatures by Mr. Mordey*, when a successful attempt was made to separate the loss of power due to magnetic hysteresis for that due to Foucault currents, the loss due to hysteresis was found to be rather less than the amount of energy which would be used up in reversing the magnetism by varying the field without rotation.

The molecular theory makes it probable that the work spent in reversing magnetism will be less when the reversal is accomplished by rotation in a constant field, than when it is accomplished by reducing the magnetic force to zero, and restoring it with sign reversed. A difference of this kind is especially to be looked for when the field is strong. In fields of no more than moderate strength, the turning round of the iron will involve breaking of molecular ties, with those unstable movements to which the dissipation of energy is due. But when the field is

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* "Alternate-Current Working." *Jour.* Inst. Elec. Eng., Vol. XVIII. 1889.

very strong, so that the iron is nearly saturated, every molecule points steadily in the direction of the magnetising force all the while, and there is no opportunity for irreversible movements. This deduction from the molecular theory seems to have been first pointed out by Mr. Swinburne. The matter is well worth experimental examination.

§ 181. **Dissipation of Energy through Hysteresis in the Cores of Transformers.**—Experiments on the efficiency of transformers show that each double reversal of magnetism in the core causes a dissipation of energy, which is, at least, of the same order of magnitude as the value of $\int \mathsf{H}\, d\mathsf{I}$ between the same limits of magnetisation in a slowly performed cycle. It has, however, been asserted by several observers* that when the transformer is loaded, that is to say, when the secondary circuit is closed through a low resistance, the dissipation of energy in the core is notably reduced, even although the same limits of magnetisation are maintained. Should this result be established it would probably mean that the damping action of the secondary circuit tended to establish a *régime* in which the molecular magnets swing with some approach to unison, crowds of them keeping step, and so to a great extent giving up energy to the secondary circuit, instead of wasting it in local eddies within the iron. But, in fact, the result itself seems to be more than doubtful. In the experiments from which it was deduced the loss of energy in the transformer was determined by measuring the energy taken in and the energy given out. Both of these are very large quantities in comparison with their difference, and hence a small error in the measurement of either makes a large error in the determination of the loss. The writer has proposed a direct method of measuring the loss, which is not open to this source of uncertainty,† and experiments which have been carried out by its means ‡ show

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* See a Paper by Prof. Ryan, American Inst. of Elec. Eng., Dec., 1889 (*The Electrician*, Vol. XXIV., 1889, pp. 239 and 263). Mr. Mordey and Prof. Ayrton have also made experiments from which they have formed the same conclusion.

† See *The Electrician*, Vol. XXVII., 1891, p. 631.

‡ November, 1891. The experiments were made by the author in conjunction with Miss H. G. Klaassen.

decisively that the transformer used in [them (a plain anchor-ring of many turns of insulated iron wire) loses just as much energy through magnetic hysteresis when heavily loaded as when the secondary circuit is open, the limits of magnetisation and the frequency of reversals being kept unchanged.

§ 182. **Reduction of Hysteresis by Vibration, &c., and other Disturbances.**—We have seen (§§ 84, 85, 129) that mechanical vibration lessens the differences of magnetic conditions that are brought about by hysteresis. It makes the metal readier to respond to any influence which tends to alter the magnetism. In a soft iron wire, where its effects are most conspicuous, it practically abolishes the distinction between what we have called the first and second stages of the magnetising process, it destroys the retentiveness almost entirely, and it makes the magnetic effects of strain nearly reversible, so that the "on" and "off" curves for a cycle of loading come to be not far from coincident.

The molecular theory makes all this intelligible. Vibration, producing small periodical displacements of the molecular centres, sets the molecular magnets oscillating. The displacement of the centres, and still more, perhaps, the oscillation to which that gives rise, allows the molecules intervals of comparative freedom, and probably even goes so far as to vary the combinations in which they are grouped. Then if there is an external field the molecules yield readily to it in their freer intervals, and even when there is no external field a kind of shuffling goes on, one effect of which is to reduce residual magnetism. It may be, that in the removal of residual magnetism vibration acts in the first place locally; a cluster of molecules shaken up so that the residual magnetism of the cluster is less than that of surrounding portions will act to some extent like a cavity in the metal, producing a demagnetising field round about it. In the same way the demagnetisation of a long iron rod under vibration no doubt begins at and about the ends, where there is a self-demagnetising field, and then extends itself towards the central portion.

Any kind of disturbance that will give the molecular magnets intervals of freedom, or of diminished constraint, will tend to do away with hysteresis. Interesting examples of this will be found in a Paper by G. G. Gerosa and G. Finzi* in which experiments are described showing how cycles of reversal of magnetism become modified when a continuous, or periodically interrupted, or alternating current is made to traverse the piece under test, while slow reversal of the field goes on. The experiments dealt with iron, steel, and nickel wire in their annealed and hard state. A continuous current, traversing the wire while its longitudinal magnetism was being changed by applying and varying a longitudinal magnetic force by means of a surrounding solenoid, was found, as might be expected, to reduce the susceptibility of iron : the circular magnetisation maintained by the current in the wire left the molecules less than their usual freedom to obey the longitudinal force. When the longitudinal current, instead of being continuous, was rapidly interrupted without changing its sign, a molecular oscillation was set up which made the iron more than usually susceptible to weak longitudinal forces ; but when the field was strengthened the iron was still found to be less susceptible than when no current was passing through it. The mere make and break of the longitudinal current would, in fact, cause no more than a small variation of circular magnetisation, and would consequently do little to agitate the molecules. But when a rapidly alternating current of moderate strength traversed the wire, the susceptibility to longitudinal magnetisation was notably increased; the magnetisation curve was found in that case to lie above the normal curve everywhere except in the region of strongest magnetisation. The violent agitation which was brought about by rapid reversals of circular magnetism destroyed nearly all trace of hysteresis, and obliterated the usual distinctions between successive stages in the magnetising process. An illustration is given in Table XXXII. and Fig. 152, which relate to an experiment in which a piece of soft iron wire, 0·84mm. in diameter, was magnetised, first under the usual conditions (without any longitudinal current), and then when traversed by a rapidly alternating current of three amperes.

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* Rendiconti del R. Istituto Lombardo. Vol. XXIV., fasc. x., April, 1891. *See* also a Paper by Dr. Finzi in *The Electrician*, April 3, 1891, p. 672.

Table XXXII.—*Magnetisation of Iron with and without an alternating longitudinal current.*

Without current.		With current.	
H	I	H	I
1·43	50	0·17	75
2·24	119	0·82	290
3·62	367	4·33	803
5·76	773	12·3	1,178
12·5	1,162	42	1,537
42	1,500	0	76
0	1,121		

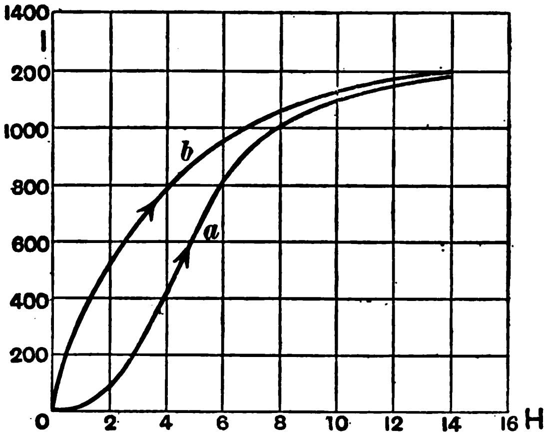

Fig. 152.

In Fig. 152 the curve *a* is the normal curve, and *b* is the curve obtained when the alternating current was in action. The table shows how little residual magnetism is left in the second case.

Fig. 153 exhibits in the same way the influence of the alternating longitudinal current on a cycle in which the longitudinal magnetism of another iron wire was reversed. The normal figure *a a* collapses, as an effect of the molecular shaking, into *b b*, which is very nearly a single curve.

Effects of the same kind were observed in steel and in hard iron, but the suppression of hysteresis was less complete.

The single curve by which the relation of **I** to **H** may be represented when hysteresis is done away with by sufficiently violent agitation of the molecules, may be expressed, with fair accuracy, by the formula

$$\mathbf{I} = \frac{\alpha \mathbf{H}}{1+\beta \mathbf{H}},$$

in which α and β are constants for a given specimen, and $\frac{\alpha}{\beta}$ is the saturation value of **I**. This formula, which was proposed by Lamont and Fröhlich as a general means of expressing the relation of magnetism to field, is, of course, of no service so long as hysteresis is operative, since **I** then depends not only on the existing value of **H** but on previous values: it will not even serve to express the curve of initial magnetisation in a virgin piece. But when hysteresis is eliminated, as in these experiments, it may be made to fit the curve reasonably well. Values of the constants α and β will be found in the Paper from which these results are quoted.

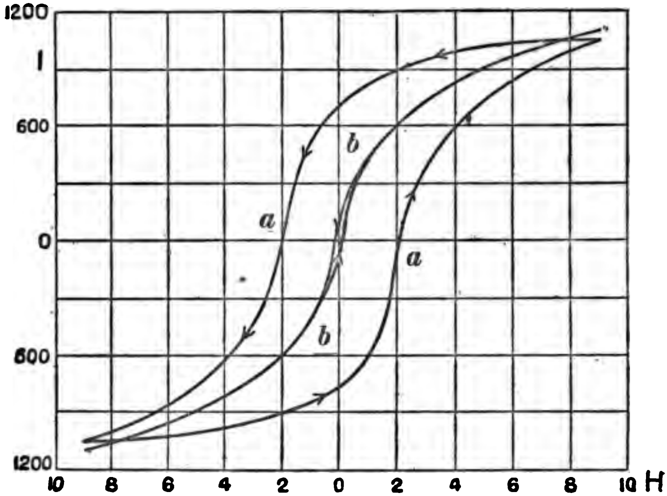

FIG. 153.

§ 183. **The Molecular Theory and the Effects of Temperature.**—To see the bearing of the molecular theory on experimental results regarding the effects of temperature on magnetic quality, we have to revert to Figs. 79, 80, and 81, § 111, which show Hopkinson's determination of the permeability of iron at various temperatures, for a small, a moderate, and a fairly strong magnetic force respectively. In the first of these figures (Fig. 79) the magnetic force is only 0·3, and consequently the susceptibility at ordinary temperatures has the comparatively small value which we expect to find in the first stage of the magnetising process. As the temperature is raised the susceptibility increases, at first but slightly, until a temperature of about 600°C. is passed. Then the rise in susceptibility becomes very rapid. It quickly increases more than ten-fold, showing that the effect of this heating is to *bring on the second stage of the magnetising process.* Finally, at a temperature of 775°C. or so there is an extraordinarily sudden fall of susceptibility, so sudden and complete that when the temperature reaches 785°C. practically all magnetic quality is lost.

Under a moderate force (of 4 C.-G.-S. units, *see* Fig. 80) there is none of the sudden rise of susceptiblity during heating which occurred when the force was weak. This is because, under the stronger magnetic force, the second stage in the magnetising process had already been entered before the piece was heated. Further, the loss of susceptibility at high temperature occurs much more gradually. Still more is this the case when the field is comparatively intense (Fig. 81).

The first effect of heating is to hasten the transition from the first to the second stage of the magnetising process, that is to say, to make this transition occur at lower values of the magnetic force. This is probably due to two causes. Heating expands the structure, and that weakens the ties between the molecules by increasing the distances between their centres. We may conjecture that it also sets up oscillations which contribute to make the ties be more easily broken. When the field is weak, so that the second stage has not been reached while the metal is cold, heating is consequently favourable to magnetisation, and with an appropriate relationship of temperature to field the metal is in a critical state, in which a small rise of temperature produces an immense augmentation of susceptibility by making groups of molecules which were stable at the lower temperature become unstable at the higher.

This effect of heating cannot occur if the field is strong enough to have upset most of the molecules before heat is applied. Hence the curve of Fig. 80 has no sharp apex like that of Fig. 79.

The case of a fairly strong field is more simple. Heating has two antagonistic influences. On one hand, the alignment of the molecular magnets is still being facilitated by the weakening of their mutual forces. On the other hand, the oscillations which they acquire have virtually the effect of reducing the moment of each molecule. Throughout a wide range of temperature the two influences nearly counterbalance one another; the curve in Fig. 80 or Fig. 81 is nearly level for a great part of its course; but as the temperature becomes rather high the prejudicial effect becomes stronger, and the curve bends down.

At this stage the molecules seem to acquire oscillation very rapidly, and a plausible conjecture to account for the complete loss of magnetic quality which ensues when the temperature rises a little higher, whether the field be weak or strong, is that the oscillation then becomes so violent as to develop into *rotation*.

The establishment of this rotation would account for the energy which we know to be absorbed during heating, while the iron passes from the magnetic to the non-magnetic state; and the rapid subsidence of this rotation into oscillations of comparatively narrow range, during cooling, would in the same way account for the energy which the iron then gives out as it recovers its power of being magnetised (§ 109).

§ 184. **Time-Lag in Magnetisation.**—The phenomena of magnetic viscosity, described in §§ 88 and 89, have some light thrown on them by the molecular theory. We saw that when a weak magnetic force is applied to soft iron, or is raised a step, the resulting change in the magnetism is not completed instantly. There is a protracted creeping up of the magnetism, which goes on long after the magnetic force has become constant. We saw that the softness of the iron and the thickness of the specimen had a great influence on the extent of this time-lagging. A piece of hard iron, or a very thin piece of soft iron, showed little or no lag; a thick piece of soft iron showed much, especially when the experiment was made at an early part of the second stage (stage B, Fig. 136) of the magnetising process.

It appears probable that an explanation of this is to be found by referring to the part that is played by the inertia of the molecules during the development of instability in molecular groups. The process of breaking up the primitive configuration takes time. The disturbance begins at a point where the primitive constraint is comparatively weak, and then slowly spreads itself even when the deflecting force is kept constant. An outlying molecule is first upset; then its neighbours, weakened by the loss of its support, follow suit, and the action propagates itself from molecule to molecule throughout the group. The surface molecules may be conjectured to be the least securely held, and, therefore, to be the first to yield. In a very thin piece of iron, such as a fine wire, there are so many surface molecules in proportion to the whole number, and consequently so many points that may become origins of disturbance, that the breaking-up of the molecular communities is too quickly completed to allow much of this lagging to be noticed. Again, when iron is hardened by mechanical strain the structure ceases to be even approximately homogeneous; the molecules become as it were parcelled out into small groups with too few members to require much time for the spreading of the disturbance through a group, and in that case also the lagging is scarcely perceptible (see § 185, below).

§ 185. **Effects of Permanent Mechanical Strain.**—It was shown in § 66 that when a piece of iron is hardened by being strained sufficiently to take permanent set, the curve of **I** and **H** assumes a rounded form which allows this condition of the metal to be readily distinguished from that of an annealed piece. The successive stages of the magnetising process, in the hardened metal, become much blended. No part of the curve has nearly so steep a gradient as we find in dealing with annealed iron. The susceptibility is less throughout, and saturation is approached with greater difficulty. There is much less retentiveness; on the other hand, there is much more coercive force. We may refer back, in illustration of these differences, to Fig. 34, § 66, where the curves for a cyclic process of reversal are drawn side by side for the same piece of iron in the annealed and hardened states.

These differences, regarded in connection with the molecular theory, seem to indicate that mechanical set resolves a structure which is relatively homogeneous and continuous into one which may be described as a patchwork of more or less distinct molecular groups. Hardening the metal by set makes only a slight change in the density, and it appears probable that it brings some of the molecules closer together, while the intervals between others are widened, with the result that groups are formed in which the intermolecular forces between members of any one group are stronger than the forces which are exerted across the wider gaps between members of different neighbouring groups. The "gaps" tend to shear over the curve of **I** and **H**, to round the outlines of the curve, and to reduce the residual magnetism. The closeness of the members within each group increases the coercive force. Thus, without any necessary change in the density of the metal, this modification of the structure would bring about the alteration in magnetic quality which is observed. Another consideration lends some support to this view. In hard metal there is exceedingly little, if any, "time-lag" in magnetisation. The explanation of "time-lag" suggested in the last paragraph seems to require that the structure of annealed iron be continuous throughout platoons of many molecules. As soon as the platoons are split up into little groups the action described there cannot be expected to take place.

In connection with this it may be remarked that any interruption of the continuity of the molecular structure tends in some measure to shear over the diagram of **I** and **H**, and, in particular, to reduce residual magnetism, by making the conditions of constraint of molecules at and near the boundary differ from those of molecules far from the boundary. It seems probable that this consideration gives a clue to the "magnetic resistance" of joints, described above in §§ 162-165. Let the separated parts of a cut bar be ever so well fitted together, the molecules at the boundary, and for some little distance from it, are not subject to the same conditions of constraint as subsist in the uncut bar.

§ 186. **Effects of Repetition of Magnetic Processes.**—Space may be found here to refer shortly to one or two of the minor phenomena of magnetisation, which the molecular theory goes far to make intelligible.

A consequence of the irreversible displacements which the molecular magnets suffer, together with the fact that the stability of each molecule depends on the configuration assumed by many molecules in its neighbourhood, is that in general a magnetising process has to be repeated more than once before its effects become strictly cyclic. In some cases a progressive change may be traced even during many repetitions of the process.

For instance, let a magnetising force be applied to a piece of soft iron, the strength of the field being regulated so that it brings the metal into what we have called the second stage of the magnetising process, when many of those molecules which are not already upset are on the verge of being upset. Let the force then be removed and reapplied. The configuration of the group during this re-application is by no means the same as it was during the first application, and accordingly we may expect that some of the molecules which were just able to stand in the first instance yield in the second owing to the changes which have meanwhile taken place in the grouping of their neighbours. The re-application of the magnetising force may therefore be expected to produce a somewhat stronger magnetisation than was produced when the force was first applied. To a less degree, a third application of the force should make the magnetisation rise a little higher still, and so on.

Similarly, the second removal of the force should leave more residual magnetism than is left after the first removal. But we may expect that the limits between which the magnetism changes when the magnetising force is applied and removed will come to be closer in each repetition of the process; the molecular "accommodation" which goes on as one after another of the doubtful molecules is upset has the effect of narrowing the range through which the magnetism alters in succeeding cycles.

That these anticipations are in accord with the results of experiment will be seen from the following paragraphs, mainly extracted from a Paper which was written without reference to the light which the molecular theory throws upon the matter.*

When a magnetising force is first applied, then removed, and then re-applied, whether suddenly or gradually, the resulting value of **I** is somewhat higher than that reached by the first application. A third application gives a somewhat higher value, and so on, the effects apparently approaching an asympotic limit. This appears to have been first shown by the experiments of Fromme.† At each removal of the magnetising force the residual magnetism is also left somewhat greater than before. And this second action (the increase of the residual magnetism) exceeds the increase of the induced magnetism, with the result that the changes of magnetism between residual and induced diminish in range with successive removals and re-applications of the magnetising force.

The following observations (Table XXXIII.) were made by the ballistic method on a long piece of soft annealed iron wire. The readings are given without reduction to absolute measure; they relate to a point which falls early in the steep part of the curve of magnetisation.

Table XXXIII.

Magnetising current.	Throw of ballistic galvanometer.	Magnetism.	
		Induced.	Residual.
First made	203	203	...
broken	-53·6	...	149·4
made	+54·2	203·6	...
broken	-47·8	...	155·8
made	+48·7	204·5	...
broken	-45·7	...	158·8
made	+46·6	205·4	...
broken	-44·9	...	160·5
made	+46·1	206·6	...
broken	-44·0	...	162·6
made	+45·6	208·2	
After many makes and breaks—			
broken	-42·6		
made	+43·1		
After many more makes and breaks—			
broken	-39·5		
made	+39·8		

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* *Phil. Trans.*, 1885, p. 570, §§ 54-58.

† Pogg. *Ann.*, Ergbd. vii., 1875, and Wied. *Ann.*, iv., 1878.

Similar results were repeatedly obtained, both with freshly annealed wires and wires from which a previous strong magnetism had been shaken out by tapping. In curves showing the relation of **B** or **I** to **H** the same thing exhibits itself in what may be called the over-closing of loops formed by removing and re-applying a given value of **H**. A good example of this is furnished by Fig. 44, § 78, which shows how much more considerable the action now spoken of is at early than at late stages of the magnetisation.

The following experiment (Table XXXIV.) dealing also with annealed iron shows that the same kind of action occurred when the current was slowly changed by the liquid rheostat of Fig. 17, § 41, and the magnetism was determined by a magnetometer :—

Table XXXIV.

Magnetising current.	**H**	Magnetometer deflection.	**I**	
			Induced.	Residual.
Gradually raised to 70	2·46	93	298	...
,, reduced to 0	0	65	...	208
,, raised to 70	2·46	97	310	...
,, reduced to 0	0	70	...	224
Then 100 sudden makes and breaks—				
Suddenly raised to 70	2·46	103	330	...
,, reduced to 0	0	80	...	256

Incidentally, this experiment illustrates another point, to which attention was long ago directed by Von Waltenhofen—that the amount of magnetisation gained or lost by applying or removing a given magnetising force is greater when the change of force is sudden than when it is gradual. Other instances of the same thing will be found in the experiment quoted below.

When a magnetising force is applied and then repeatedly *reversed*, the changes of magnetism, instead of being strictly cyclic, form what may be termed unclosed loops. An instance of this is given by Fig. 52, § 82, which shows a series of these unclosed loops in the magnetisation of steel wire. The result is, as in the case of repeated removals and reapplications of magnetising force, that successive repetitions of the process give a gradually diminishing range of magnetic change. This action, like the one just described, occurs most conspicuously at points in the early part of the curve of magnetisation. The observations in Table XXXV. were made specially to exhibit it, on a piece of annealed iron wire, 400 diameters long, by the magnetometric method.

Table XXXV.

Magnetising current.	Magneto-meter deflection.	Remarks.
0	0	
Gradually raised to ....+190	+146	Here there is gradual diminution of range. This part of the operation is shown in Fig. 154.
„ reversed to ......−190	−141	
„ „ „ ......+190	+127	
„ „ „ ......−190	−133	
„ „ „ ......+190	+120	
„ „ „ ......−190	−132	
Suddenly „ „ ......+190	+124	Here there is an increase of range due to the suddenness of these reversals.
„ „ „ ......−190	−136	
„ „ „ ......+190	+123	
Fifty double reversals, then—		But after repeating the sudden reversals often enough the range becomes smaller than ever.
Suddenly reversed to ......+190	+111	
„ „ „ ......−190	−127	
Then gradually „ ......+190	+108	And a *gradual* repetition of the cycle causes still a further reduction of range.
„ „ „ ......−190	−126	

In the first part of the above operations, during the five gradual reversals of magnetising force, intermediate readings were taken, which enabled the curves shown in Fig. 154 to be drawn. These show at a glance the manner in which the range of magnetic change diminishes. Sudden reversals, following on these, cause at first an increase of range, thus illustrating the comparative effects of gradual and sudden change of **H**, but on being repeated many times they reduce the range to a lower value than before.

The same piece of wire was next subjected to a magnetising force about five times greater than the above, and was then demagnetised by reversals. Experiments similar to the above were then made on it, when it was found that *the tendency to a diminution of range with repetition of a cyclic alteration of magnetising force had disappeared.* The diagram, Fig. 155, shows the effect of applying, reversing, and re-applying the same magnetising force as in the former case, after the wire had been demagnetised by reversals. It shows that the changes of magnetism are now cyclic. The same result was given by other specimens, which when freshly annealed gave much diminution of range, but when demagnetised by reversals after the magnetising force had been raised to a high value, were found to have lost this property. In this respect, then, a wire demagnetised by reversals differs from the same wire in its primitive annealed state. It will be seen, too, by comparing figures 154 and 155, that the unsymmetrical susceptibility with respect to forces of opposite signs which exists in the annealed wire has given place to a very perfect symmetry after demagnetisation by reversals.

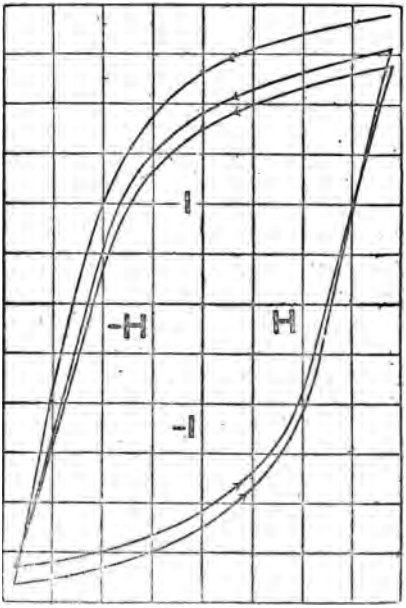

FIG. 154.—Repetition of Magnetic Cycles in Annealed Iron Wire.

Re-annealing the wire restored all the characteristics of the primitive state.

The following observations (Table XXXVI.), made with another piece of annealed iron wire at a part of the curve very sensitive to the actions now spoken of, show well the reduction of range by reversals, and then the rise of magnetism, induced and residual, which is produced by successive removals and re-applications of **H**. This last occurs in a very marked way after the range of magnetic change has been reduced by reversals of **H**. The two directions of the current will for brevity be distinguished as A and B. The changes were sudden, and the magnetism was determined by the direct magnetometric method. A want of symmetry is very noticeable here between the positive magnetisation due to the current A, which is first applied, and the subsequent negative magnetisation due to the equal and opposite current B.

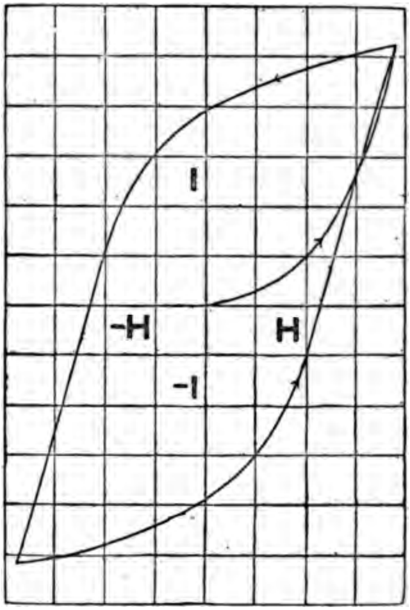

Fig. 155.—Cyclic Process in Annealed Iron Wire previously demagnetised by reversals.

Table XXXVI.

Magnetising Current.	Magnetometer deflection.	Remarks.
Made A	+232	
,, B	−110	Diminution of range by reversals.
,, A	+180	
,, B	−101	
,, A	+172	
,, B	−100	
Twenty reversals, then—		
Made B	− 95	
,, A	+158	
Broke A	+150	Rise of magnetism (induced and residual) by successive removals and re-applications of **H**.
Made A	+200	
Broke A	+193	
Made A	+206	
Broke A	+201	
Twenty makes and breaks, then—		
Broke A	+205	
Made A	+209	
Then reversals again—		The diminution of range by reversals is again conspicuous.
Made B	−105	
,, A	+178	
Forty reversals then—		
Made A	+163	
,, B	−105	
Broke and remade B	−136	
Ditto twenty times	−175	

The magnetisation of steel exhibits, even more than that of iron, reduction of range with successive reversals of **H**, and want of symmetry between the values of **I** induced by successively applied + and − values of **H**. Fig. 156 shows the changes of magnetism which were undergone by an annealed steel wire when a magnetising force of 15 C.-G.-S. units was applied, removed, re-applied, reversed, and again reversed twice. The want of symmetry between the positive and negative values of the magnetism is very marked in this example: the steel acquires a strong magnetic *set* towards the side of the first magnetisation.

§ 187. **Effects of Elastic Strain.**—In an earlier chapter (§§ 120—142) an account has been given of experiments made to investigate the effects of stress on the magnetic quality of iron and the other magnetic metals. Without attempting any full discussion of these results from the point of view which the molecular theory affords, we may refer to one or two general features where a molecular explanation seems comparatively easy.

That stress should produce an influence on magnetic quality is a probable result of the strain to which the stress gives rise. The effect of a simple longitudinal stress is, as we have seen, to make the metal, originally isotropic in its magnetic quality, become æolotropic, and it may be conjectured that this happens through differences becoming established in the pitch of the molecular magnets, in lines respectively along and across the direction of the stress, whereby old lines of molecules break up and new lines are formed. A uniform dilation or a uniform compression (with equal intensities of stress in all directions) might be expected to have a much less considerable influence on magnetic quality than a simple stress has. Experiments on the effects of such stresses are wanting; it may be antici

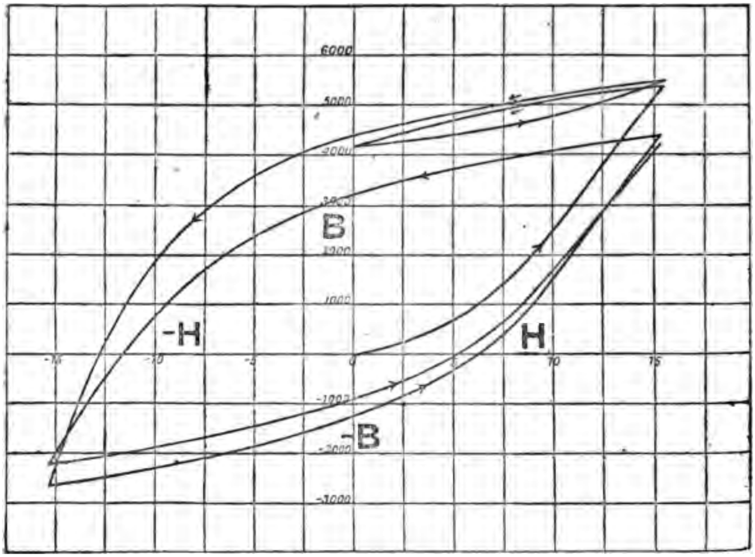

FIG. 156.—Repetition of Magnetic Cycles in Annealed Steel.

pated that effects resembling those due to change of temperature would be observed. Thus we might expect to find a uniform pressure in all directions associated with a general reduction of magnetic susceptibility. The experiment would be an interesting one to carry out, especially in nickel, where (§ 122) the susceptibility is known to be greatly increased by a single stress of compression applied in the direction of magnetisation.

A stress of simple pull will lengthen those rows of molecules which lie more or less along the axis of the stress, and will shorten those rows which lie more or less across the axis. This is enough of itself to develop differences of magnetic susceptibility in the longitudinal and transverse directions; and the difference is probably much intensified by a re-arrangement of the molecular rows, the longitudinal rows being more or less broken up and transverse rows formed. The lengthening of the longitudinal rows will tend to increase the susceptibility; the shortening of the transverse rows, and still more the secondary consequence of stress, namely, the formation of new transverse rows, will tend to reduce it. It seems that in nickel the reducing effect is the dominant one; in iron, on the other hand, we find a conflict of influences which makes pull favourable or otherwise according as the magnetisation is less or greater than a critical value.

The large magnetic changes due to torsion which are seen in experiments on nickel, such as the reversal of magnetism which Nagaoka found when a loaded nickel wire was twisted to and fro to alternate sides (§ 142)—appear to be secondary effects, due to the reconstruction of molecular rows which become unstable when the molecular centres are displaced by the strain. It is the existing magnetism of the piece that is being affected, rather than its susceptibility to induction by the field.

An obvious conclusion from the molecular theory is that there should be, as we know there is, hysteresis in the changes of magnetic quality that are associated with changes of stress, and also that the condition arrived at by first applying a load and then magnetising should in general be different from the condition arrived at by first magnetising and then applying a load. (*See* §§ 120—131.)

Another fact which the molecular theory serves to explain is the important difference which experiments reveal between the effect that is produced by the first application of a stress, and the effect that is produced when the same stress is applied after it has been previously applied and removed many times. After what has been said above in § 127, a brief reference to this matter will suffice. Provided the magnetising force is not very strong, the first application of load, when the piece hangs in a steady magnetic field, upsets molecules which were nearly upset before the load was applied. Removal of the load does not make these molecules recover the position from which the application of the load disturbed them. Thus successive loadings and unloadings, especially in a weak field, serve, as it were, to shake in the magnetism; and, if residual magnetism is dealt with, the field having been removed, successive loadings and unloadings serve to shake it out. Examples of this have already been given in Figs. 108 and 109, where the effects of a first loading and unloading are readily distinguishable from those that occur after a cyclic régime has become established by repetition of the cycle of loads. Fig. 119, exhibiting certain effects of successively applied twists to alternate sides, is also an instance in point. When we load a wire in a strong field we find, as the theory would lead us to expect, that the cyclic régime is quickly attained; a second loading is enough to show that the initial disturbing influence of the stress is exhausted. In weak fields, the loading has to be repeated many times before that is the case, and the first disturbance is sometimes immensely greater than the alteration of magnetism that accompanies each application and

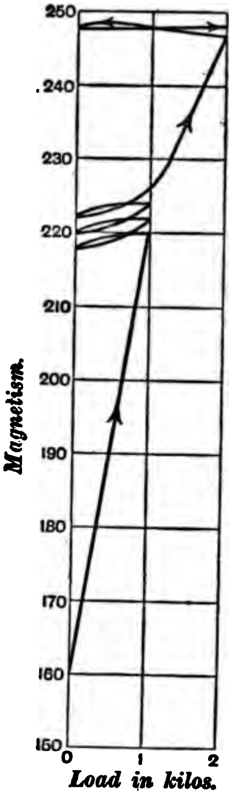

Fig. 157.— Effects of Loading a Soft Iron Wire in a Weak Magnetic Field.

removal of the load after a cyclic condition has been reached. Fig. 157 gives an example. The specimen dealt with there was a long wire of soft annealed iron, 0·76mm. in diameter, which hung in a weak field ($\mathsf{H}=0{\cdot}34$). A load of 1 kilogramme applied for the first time raised the magnetism from 159 to 220 (in arbitrary units). Removal of the load reduced it only to 218. Re-application brought it up to 222; a second removal reduced it to 220¼. A third application made it 224, a third removal 222, and a fourth application 225½. Then the load was increased to 2 kilogrammes, and the magnetism went up at a bound to 247, after which successive removals and re-applications of that load produced but slight changes which tended gradually to assume a cyclic character when the operation was repeated many times.*

§ 188. **Hysteresis in Changes of Molecular Configuration, apart from the Existence of Magnetisation.**—In §§ 133-135 experiments have been referred to which show that when iron is subjected to cyclic variation of stress, its structure undergoes changes that involve hysteresis, even when no magnetic force acts upon it, and when there is no magnetisation of the piece as a whole. The molecular theory makes the reason of this sufficiently apparent. Elastic strain brings about a rearrangement of the molecular grouping; old combinations break up and novel combinations are formed, although no magnetic forces are concerned other than the forces which the molecular magnets exert on one another. These changes of configuration involve unstable movements on the part of the molecules, and hysteresis consequently manifests itself, when the piece is carried through a cycle of strain. We find, for instance, that when an iron wire under tension is loaded and unloaded, by putting on and taking off weights, there is a distinct difference in the physical state of the metal, under one and the same intermediate amount of weight, during loading and during unloading. The difference shows itself in magnetic susceptibility, in thermo-electric quality,† and possibly in many other physical qualities of the material. It continues to be found when the cycle of loading

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* For details of this and other experiments illustrating the point now referred to, see *Phil. Trans.*, 1885, p. 594, *et seq.*

† See *Phil. Trans.*, 1886, p. 361.

is repeated, and its character is just such as the molecular theory would lead us to expect.

This hysteresis in molecular configuration, apart from all actual magnetisation, which exhibits itself when the piece is carried through a cycle of elastic strain, has one important consequence. It implies that the elasticity of the substance is not perfect. The unstable movements of the molecules, to which it is to be ascribed, result in a dissipation of energy. More work has, therefore, to be spent in stretching the piece, while loads are being put on, than is recovered when the loads are taken off—in other words, the stress that corresponds to any given intermediate value of the strain must be greater during the application of the load than during its removal. There must be hysteresis in the relation of strain to stress; and, as we have seen already (§ 135), this conclusion is borne out by experiment.

FIG. 158.

§ 189. **Experimental Study of Molecular Groups by means of Models.**—It is extremely helpful, in considering the constraint which the molecular magnets suffer in consequence of their polar forces, to experiment with a model consisting of a number of short steel magnets, pivoted like compass needles on fixed centres, and placed near enough to one another to allow their mutual control to be felt.* Such a model is readily made out of pieces of stout magnetised steel wire, bent, as in Fig. 158, to bring the centre of gravity below the pivot point. A recess for the pivot is stamped by a centre punch in the hollow of the

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**Proc.* Roy. Soc., 1890, Vol. XLVIII., p. 342; *Phil. Mag.*, September, 1890.

bend, and the pivot itself is a needle stuck, with the point upwards, in a small block of lead or of wood. Instead of a wire, a piece of steel plate may be used for the magnet, and this may have any form given to its polar extremities, from sharp points to semicircles. The magnets being of hard steel, strongly magnetised, are practically unaffected (as to the intensity of their magnetism) by the comparatively weak external magnetic forces which are applied for the purpose of turning them into line. The external force may be applied by a coil wound in an open manner over a light framework, within which the group of magnets is placed, the open winding allowing the behaviour of the magnets within to be observed. Or a larger coil placed entirely underneath the group may be used; or, better still, a pair of closely-wound short coils placed one on either side of the group. This last form is especially convenient when the behaviour of the group is to be exhibited by projecting them on a lantern screen. For that purpose short magnets are necessary, and the magnets used for small pocket compasses will be found very suitable; the pivots themselves may also be cut out of such compasses and cemented, at proper distances, on a glass plate. To exhibit the effects of strain, the pivots may be arranged on a framework of jointed wooden rods, forming two crossed sets of parallel lines; by placing the pivots at the joints, or midway between the joints, some of the effects of simple shear or simple pull and push may be studied.

A model of this kind allows the three stages of the magnetising process to be readily distinguished. The phenomena attending reversal of magnetism, the dissipation of energy in hysteresis, the conditions that promote residual magnetism, the comparative effects of slow and sudden changes in magnetic force, the primitive and final effects of strain, the influence of vibration, the existence of time-lag, are all matters of which the model gives effective illustration.

The manner in which the resultant polarity of the group of pivoted magnets changes when the field is applied, reversed, or varied in any way, is sufficiently evident on mere inspection of the group. It may, however, be determined quantitatively by using a magnetometer in the ordinary way, taking care to compensate for the action of the coil which supplies the magnetic field by placing in series with it a second coil, the position of which is adjusted so that it may annul the deflection which the first coil by itself would produce. A group of magnets examined in this way, when carried through a cycle of configuration by applying and reversing the directive force of the coil, gives what we may call curves of magnetisation, in which all the main characteristics of the ordinary curves for iron appear, though, of course, the limited number of magnets which it is practicable to use in such an experiment makes the steps of the process more jerky than they are when we have to deal with the multitudes of molecules in a piece of solid

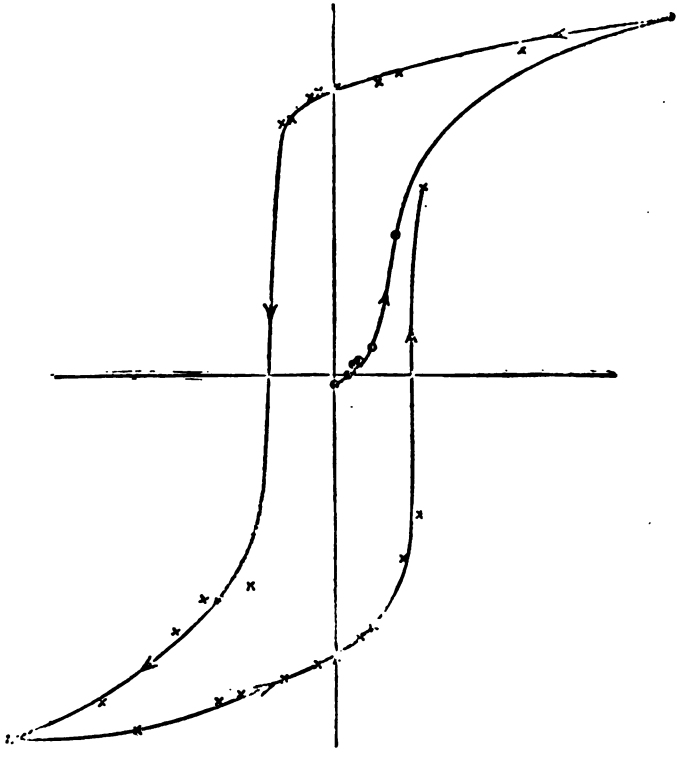

FIG. 159.—Cyclic Proccss applied to a Group of Twenty-four Pivoted Magnets.

metal. Curves obtained by this means, showing the reversal of a group of twenty-four little magnets (like the one shown in Fig. 158) under reversal of the directing field, are given in Fig. 159.*

§ 190. **Ampère's Hypothesis as to the Nature of the Magnetic Molecules.**—Granting, as we very well may (in view of the considerations summarised in this chapter), that the process of magnetising consists in turning round molecules that are already magnetic, so that their axes tend, under the directing force of the applied field, to approach a particular direction, the question still remains, to what is the primitive magnetism of the molecules due? Weber's theory does not help us to an explanation of the fact, which it postulates, that each molecule is a permanent magnet.

According to the hypothesis of Ampère the magnetism of the molecule is due to an electric current continually circulating within it—in other words, the molecule is a conducting circuit in which a current flows, and when a directing field acts, the channel in which this current flows tends to set itself at right angles to the direction of the field, just as does the coil of an electro-dynamometer. Ampère's theory, therefore, explains all the phenomena of magnetisation as consequences of the mutual action of electric currents. According to it, in magnetising a piece of iron we are dealing with the forces which exist between the current in an external conductor and the currents in molecular circuits within the metal, which are prevented from immediately putting themselves into perfect parallelism with the external circuit only because of the forces which the currents in the molecules exert on one another. In this view the model of a magnetic metal should be constructed by using not pieces of permanently magnetised steel to represent the molecules, but little coils, free to turn, in each of which an electric current flows continually.

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* Fig. 159, for which the author is indebted to Mr. Glazebrook, represents the results of an experiment by Mr. J. W. Capstick, made in answer to a question set in the practical examination of the Cambridge Natural Science Tripos, 1891. Curves of this kind were first published by Mr. Arthur Hoopes in the *Electrical World* (New York), May, 1891. See *The Electrician*, May 29, 1891.

The molecular channels must be supposed to offer no resistance as conductors, otherwise the primitive currents would require energy to be expended in maintaining them.

When a field is applied it tends to turn the molecular circuits, and it also induces supplementary currents in them. These induced currents are superposed on the primitive currents; their strength depends on the inclination of the circuit to the field; and their general effect is to reduce the primitive currents. Whether they will do so to any considerable extent depends on the area and the self-induction of the molecular circuits, and on the primitive strength of the currents in them.* Thus if the primitive currents are strong and the other conditions favourable, very little reduction of the primitive strength takes place through this induction of current by the applied field. In that case the molecular circuits are nearly equivalent to strictly permanent magnets, and merely turn in response to the field, without suffering any material loss of intensity. Probably this represents what occurs when iron or any of the other strongly magnetic metals is magnetised.

When the primitive molecular currents are weak the induction of opposing currents by the application of a magnetic field may modify the resultant strength very greatly; and in particular, when there are no primitive currents at all, but only conducting molecules ready to have currents induced in them, the application of the field will induce currents which give to the piece a polarity of the kind opposite to that which it acquires in ordinary magnetisation. By recognising the existence of these induced currents Weber thus extended Ampère's theory of molecular conducting circuits to account for *diamagnetism.*

But even when there are strong primitive currents, as we must suppose there are in the molecules of iron, the induction of opposing currents, in consequence of applying a magnetic field, will go on to some extent, and there is a stage at which its influence may be appreciable. This is when the piece is saturated—when all the molecular circuits are turned into planes perpendicular to the direction of the field. In that position they are as favourably placed as possible for the

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* *See* Maxwell's "Electricity and Magnetism," Vol. II., chap. XXII.

induction in them of currents opposed to the primitive currents. When the field is further strengthened, the resultant current in each molecular channel is reduced, and as the channels are already all perpendicular to the field, the only effect of increasing the field is to reduce the magnetisation of the piece by reducing the strength of each molecule. The Ampère-Weber theory, therefore, leads us to conceive of the magnetism of iron as tending to pass a limiting value when saturation is reached, after which a stronger magnetising force should actually weaken the magnetism. The results of experiments with very strong fields neither confirm this nor contradict it. They show that when the condition of saturation has been approached the field may be strengthened ten-fold or more without any material change in the magnetisation, either in the way of addition or loss. But the conditions under which such experiments are carried out make very accurate measurement impracticable, and a small reduction of the magnetism might pass undetected. It is probable enough that stronger fields still must be used to discover it, for the reduction which is to be expected as a consequence of induced currents in the molecular channels is slight at the most, and in the approach to saturation, which is long drawn out, the continued deflection of the molecules tends to counterbalance any effect that may be produced by a small loss of moment on the part of each.

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